Update on Energy Loss

PR#1323

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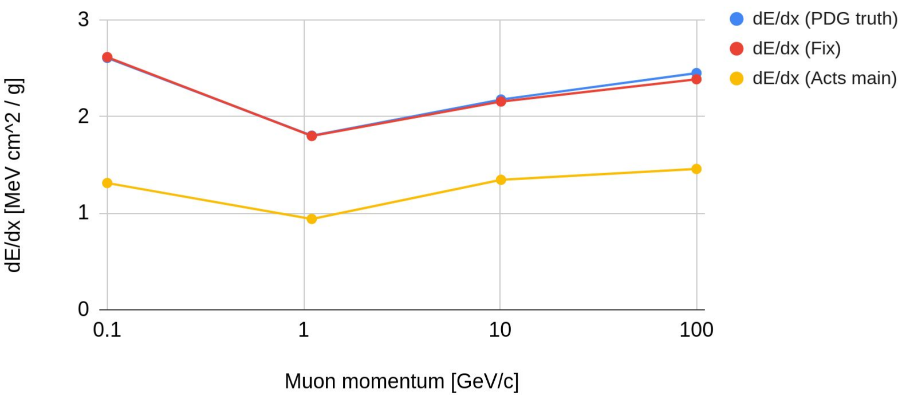
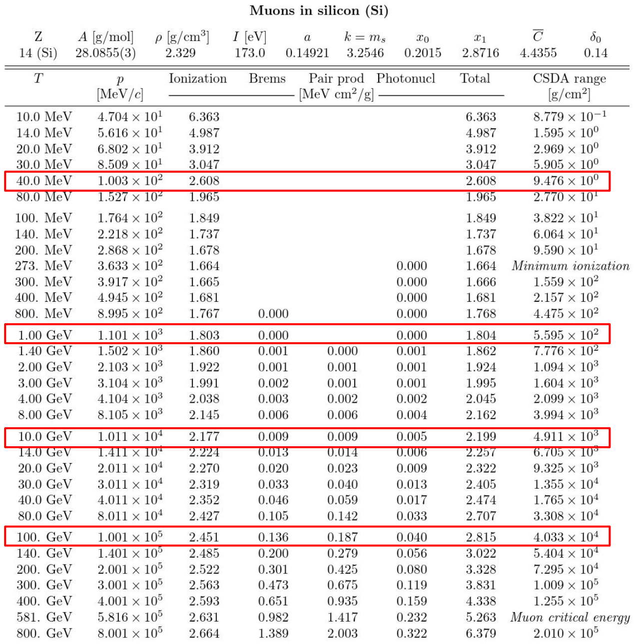
Motivation

- Current ACTS energy loss shows a huge difference when compared to the PDG truth value
- Mostly detected while I was working on detray material interactions and writing their unit tests
- Its impact is not negligible not only for physics but also for statistics (pull value distributions)

Mean Energy Loss (Bethe-Bloch)

- Comparison with PDG table for a muon in silicon: https://pdg.lbl.gov/2022/AtomicNuclearProperties/MUE/muE_silicon_Si.pdf





Muons in silicon (Si)

Z	A [g/mol]	ρ [g/cm^3]	I [eV]	a	$k = m_s$	x_0	x_1	$\overline{C}$	δ_0
14 (Si)	28.0855(3)	2.329	173.0	0.14921	3.2546	0.2015	2.8716	4.4355	0.14

T	p [MeV/c]	Ionization	Brems	Pair prod [MeV cm^2/g]	Photonucl	Total	CSDA range [g/cm^2]
10.0 MeV	4.704×10^1	6.363				6.363	8.779×10^{-1}
14.0 MeV	5.616×10^1	4.987				4.987	1.595×10^0
20.0 MeV	6.802×10^1	3.912				3.912	2.969×10^0
30.0 MeV	8.509×10^1	3.047				3.047	5.905×10^0
40.0 MeV	1.003×10^2	2.608				2.608	9.476×10^0
80.0 MeV	1.527×10^2	1.965				1.965	2.770×10^1
100. MeV	1.764×10^2	1.849				1.849	3.822×10^1
140. MeV	2.218×10^2	1.737				1.737	6.064×10^1
200. MeV	2.868×10^2	1.678				1.678	9.590×10^1
273. MeV	3.633×10^2	1.664			0.000	1.664	*Minimum ionization*
300. MeV	3.917×10^2	1.665			0.000	1.666	1.559×10^2
400. MeV	4.945×10^2	1.681			0.000	1.681	2.157×10^2
800. MeV	8.995×10^2	1.767	0.000		0.000	1.768	4.475×10^2
1.00 GeV	1.101×10^3	1.803	0.000		0.000	1.804	5.595×10^2
1.40 GeV	1.502×10^3	1.860	0.001	0.000	0.001	1.862	7.776×10^2
2.00 GeV	2.103×10^3	1.922	0.001	0.001	0.001	1.924	1.094×10^3
3.00 GeV	3.104×10^3	1.991	0.002	0.001	0.001	1.995	1.604×10^3
4.00 GeV	4.104×10^3	2.038	0.003	0.002	0.002	2.045	2.099×10^3
8.00 GeV	8.105×10^3	2.145	0.006	0.006	0.004	2.162	3.994×10^3
10.0 GeV	1.011×10^4	2.177	0.009	0.009	0.005	2.199	4.911×10^3
14.0 GeV	1.411×10^4	2.224	0.013	0.014	0.006	2.257	6.705×10^3
20.0 GeV	2.011×10^4	2.270	0.020	0.023	0.009	2.322	9.325×10^3
30.0 GeV	3.011×10^4	2.319	0.033	0.040	0.013	2.405	1.355×10^4
40.0 GeV	4.011×10^4	2.352	0.046	0.059	0.017	2.474	1.765×10^4
80.0 GeV	8.011×10^4	2.427	0.105	0.142	0.033	2.707	3.308×10^4
100. GeV	1.001×10^5	2.451	0.136	0.187	0.040	2.815	4.033×10^4
140. GeV	1.401×10^5	2.485	0.200	0.279	0.056	3.022	5.404×10^4
200. GeV	2.001×10^5	2.522	0.301	0.425	0.080	3.328	7.295×10^4
300. GeV	3.001×10^5	2.563	0.473	0.675	0.119	3.831	1.009×10^5
400. GeV	4.001×10^5	2.593	0.651	0.935	0.159	4.338	1.255×10^5
581. GeV	5.816×10^5	2.631	0.982	1.417	0.232	5.263	*Muon critical energy*
800. GeV	8.001×10^5	2.664	1.389	2.003	0.322	6.379	2.010×10^5

Most Probable Energy Loss (Landau)

- Quite tricky to validate because the most probable energy loss is not provided by the PDG table

- But at least one figure is provided in PDG review
    - 1.7 mm silicon slab
    - Muon
    - 10 GeV
    - Most probable energy loss ~ 0.525 MeV

- Landau function of Acts main gives 0.69 MeV (??)

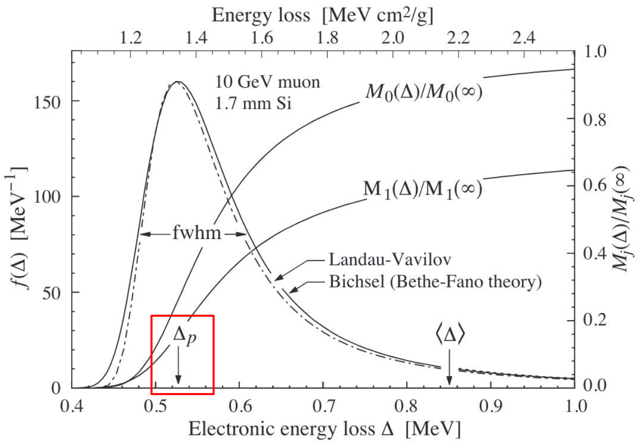


Figure 34.7: Electronic energy deposit distribution for a 10 GeV muon traversing 1.7 mm of silicon, the stopping power equivalent of about 0.3 cm of PVT-based scintillator [1, 13, 33]. The Landau-Vavilov function (dot-dashed) uses a Rutherford cross section without atomic binding corrections but with a kinetic energy transfer limit of $W_{\max}$. The solid curve was calculated using Bethe-Fano theory. $M_0(\Delta)$ and $M_1(\Delta)$ are the cumulative 0th moment (mean number of collisions) and 1st moment (mean energy loss) in crossing the silicon. (See Sec. 34.2.1). The fwhm of the Landau-Vavilov function is about 4ξ for detectors of moderate thickness. Δ_p is the most probable energy loss, and $\langle\Delta\rangle$ divided by the thickness is the Bethe dE/dx.

List of Bugs

1. A factor of two is missing in the calculation of the mean energy loss from Bethe-Bloch equation
   - Just a silly mistake

2. Incident particle mass is used in the calculation of the most probable energy loss from Landau distribution
   - It should be fixed to the electron mass

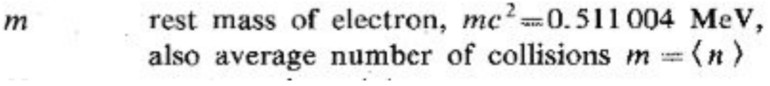
3. A factor of 1000 is missing in the log term of density effect correction
   - Should be from mm^3 -> cm^3 conversion

➢ Mean energy loss and most probable energy loss are not independent to each other because they are from the same Landau distribution - These bugs can badly affect both physics and statistics

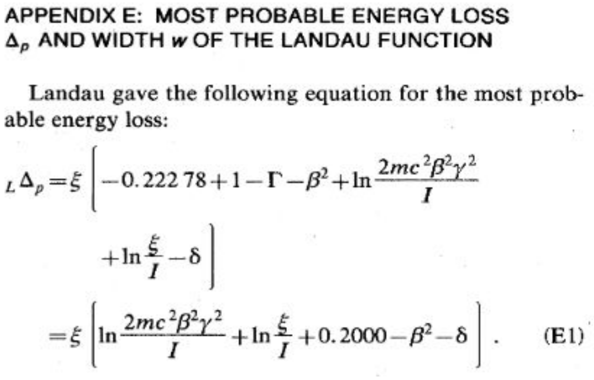
Electron Mass input for the Most Probable Energy Loss

- PDG review definitely has an error in the mass notation

Original paper: Straggling in thin silicon detectors



m rest mass of electron, $mc^2 = 0.511004$ MeV, also average number of collisions $m = \langle n \rangle$

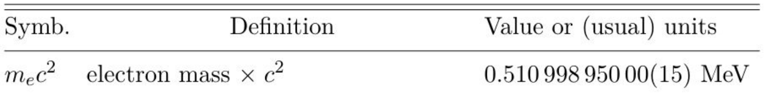


APPENDIX E: MOST PROBABLE ENERGY LOSS Δ_p AND WIDTH w OF THE LANDAU FUNCTION

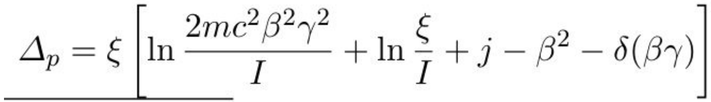
Landau gave the following equation for the most probable energy loss:

$${}_L\Delta_p = \xi\left[-0.22278 + 1 - \Gamma - \beta^2 + \ln\frac{2mc^2\beta^2\gamma^2}{I} + \ln\frac{\xi}{I} - \delta\right] = \xi\left[\ln\frac{2mc^2\beta^2\gamma^2}{I} + \ln\frac{\xi}{I} + 0.2000 - \beta^2 - \delta\right]. \qquad \text{(E1)}$$

PDG review



Symb.	Definition	Value or (usual) units
$m_e c^2$	electron mass × c^2	0.510 998 950 00(15) MeV



$$\Delta_p = \xi\left[\ln\frac{2mc^2\beta^2\gamma^2}{I} + \ln\frac{\xi}{I} + j - \beta^2 - \delta(\beta\gamma)\right]$$

Where the factor of 1000 comes from

- In density effect correction, there is an *unit-less* density term in g/cm^3
    - ACTS sets mm unit as 1 so we need to multiply 1000 here..

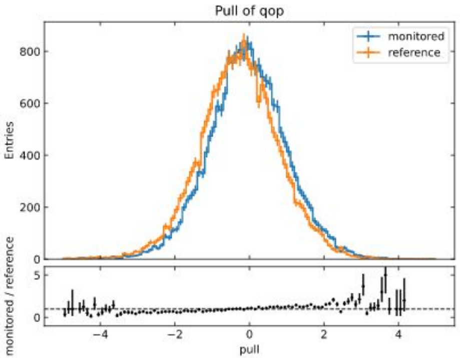
PDG review

$\hbar\omega_p$	plasma energy $\sqrt{4\pi N_e r_e^3}\; m_e c^2/\alpha$	$\sqrt{\rho\,\langle Z/A\rangle} \times 28.816$ eV $\hookrightarrow \rho$ in g cm^{-3}

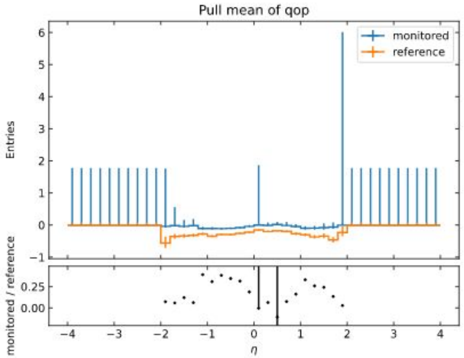
Before (reference) & After (monitored) the Fix

- Alas…

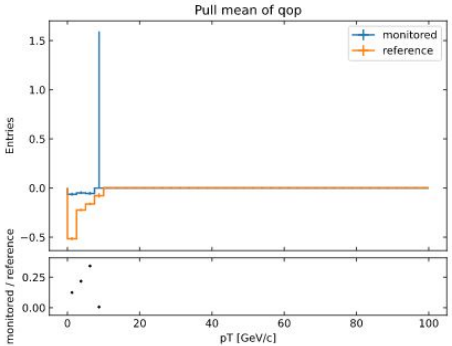
pull_qop



pullmean_qop_vs_eta



pullmean_qop_vs_pT



Outlook

- The impact is not small after the fix
  - I recommend to update the Acts version that will include the changes especially in case using Acts as an external library

- There is still a negative shift in qop pull value for low pT particles
  - Have some plans to improve it later

Backups

Pure Impact of Mass in the Most Probable Energy Loss

- Reference: electron mass
- Monitored: Incident particle mass (muon as in physmon.py)

resmean_qop_vs_pT