Accelerator Basics Part I: Principles and Main Accelerator Types

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What is an Accelerator?

Definition

OXFORD DICTIONARY: A particle accelerator is an apparatus for accelerating *charged particles* to high velocities.

- Particle accelerators are the **most complex devices**, which are widely used in research and development in the majority of subfields in Physics, Chemistry, Biology, Medicine, Archaeology, Energy research and other areas;
- Particle accelerators are widely used in industry as well. One of the most important application is the **beam lithography in microelectronics**.

Introduction

Particle accelerators are expensive "toys" producing:

- either flux of particles impinging on a fixed target (after beam extraction)
- or debris of interactions emerging from colliding particles (no beam extraction needed)

To clarify the basic characteristics and the present and future challenges in the physics and technology underlying these expensive toys are one can:

- List the technological problems
- Describe the basic physics and mathematics involved

The majority of the phenomena in a particle accelerator can be described in terms of

- Special methods of *classical mechanics and dynamical systems in general* such as nonlinear dynamics, dynamic stochasticity, etc.
- Statistical mechanics, fluid dynamics and methods of plasma physics
- Electrodynamics
- Special relativity
- Quantum mechanic is required in a couple of cases for leptons (synchrotron radiation, quantum effects in free electron lasers and others) only

Introduction Continued…

There are however some hard problems comprising serious complications:

- Many nonlinear phenomena such as nonlinear resonances of betatron oscillations, coupled synchro-betatron resonances, space-charge effects, onset of chaotic motion, which is intuitively clear that is best to be avoided
- A big number of particles interacting with each other and with complex surroundings
- The observables must be averaged over large ensembles of particles – the so-called single particle dynamics approximation is not enough for the full understanding of some important phenomena
- To handle high energy high intensity beams a complex technology is required – low temperature physics and cryogenics technology involved
- Interesting and important applications of plasma physics, non equilibrium thermodynamics and the physics of transport processes oriented towards the development of advanced acceleration methods and technology.

Some Historical Developments

- 1900 to 1925 radioactive source experiments à la Rutherford -> request for higher energy beams;
- 1928 to 1932 electrostatic acceleration ->
  - Cockcroft & Walton -> voltage multiplication using diodes and oscillating voltage (700 kV);
  - Van der Graaf -> voltage charging through mechanical belt (1.2 MV);
- 1928 resonant acceleration -> Ising establish the concept, Widerøe builds the first linac;
- 1929 cyclotron -> small prototype by Livingstone (PhD thesis), large scale by Lawrence;
- 1942 magnetic induction -> Kerst build the betatron;
- 1944 synchrotron -> MacMillan and Veksler invent the RF phase stability (longitudinal focusing);
- 1946 proton linac -> Alvarez build an RF structure with drift tubes (progressive wave in 2π mode);
- 1950 strong focusing -> Christofilos patent the alternate gradient concept (strong focusing);
- 1951 tandem -> Alvarez upgrade the electrostatic acceleration concept and build a tandem;
- 1955 AGS -> Courant, Snider and Livingstone build the alternate gradient Cosmotron in Brookhaven;
- 1956 collider -> Kerst discuss the concept of colliding beams;
- 1961 e^+e^- collider -> Touschek invent the concept of particle-antiparticle collider;
- 1967 electron cooling -> Budker proposes the e-cooling to increase the proton beam density;
- 1968 stochastic cooling -> Van der Meer proposes the stochastic cooling to compress the phase space;
- 1970 RFQ -> Kapchinsky and Teplyakov build the radiofrequency quadrupole linear accelerator;
- 1980 to now superconducting magnets -> developed in various laboratories to increase the beam energy;
- 1980 to now superconducting RF -> developed in various lab to increase the RF gradient.

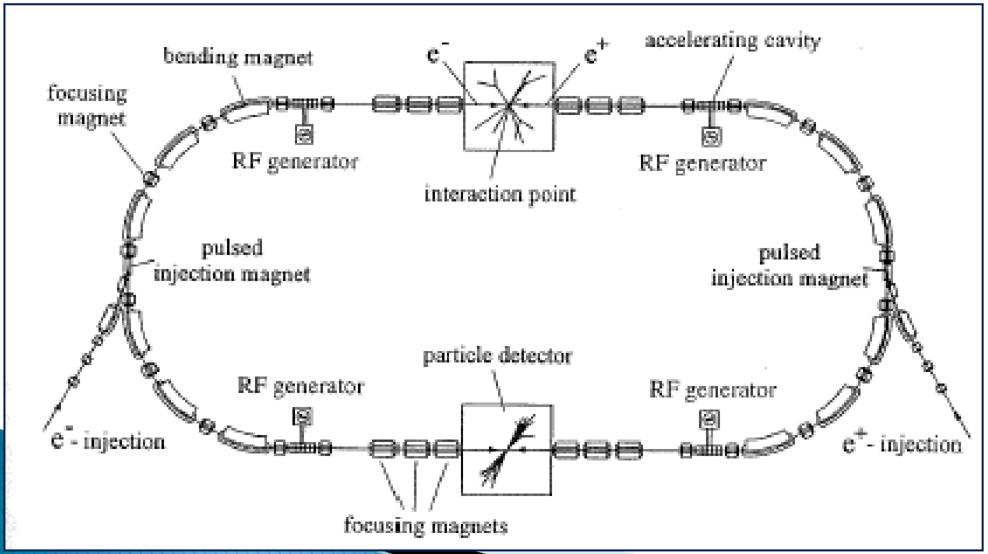
Modern Accelerators of Synchrotron Type

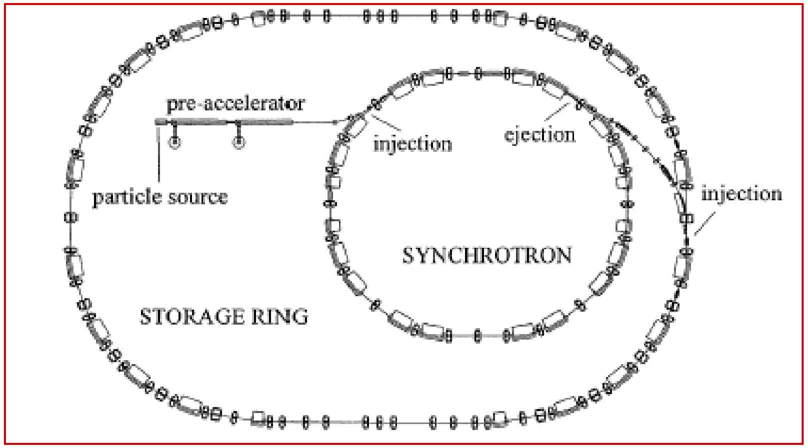
The best known accelerator for everybody is the TV monitor consisting of *electron gun*, linear accelerator of directly applied voltage, drift region and a **target** (TV screen) and *detector* (user's eye).

Interestingly enough, the basic ideology of nowadays accelerators has not moved that far…

COLLIDER Depicted on the figure is a lepton collider

ACCELERATOR CHAIN Injector + Booster + Accumulator





Electromagnetic Fields and Their Utilization

For *purely magnetic field*, we have

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \mathbf{F} = q\mathbf{v} \times \mathbf{B}, \qquad \mathbf{p} = m_0 \gamma \mathbf{v}, \qquad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2},$$

Since $\mathbf{p} = const$ and therefore $m = m_0 \gamma = const$, there is NO ENERGY GAIN. *Important conclusion*: magnetic field can be used only for **particle focusing and confinement**!

Simplest case of *magnetic dipole*: $\mathbf{B} = B_z \mathbf{k}$

$$\frac{\mathrm{d}v_x}{\mathrm{d}t} = \frac{q}{m} v_y B_z, \qquad \frac{\mathrm{d}v_y}{\mathrm{d}t} = -\frac{q}{m} v_x B_z, \qquad \frac{\mathrm{d}v_z}{\mathrm{d}t} = 0,$$

Helical motion in general $v_z(t) = v_{z0}$ but closed circles for $v_{z0} = 0$

$$v_x(t) = v_{x0} \cos(\omega t) + v_{y0} \sin(\omega t), \qquad v_y(t) = v_{y0} \cos(\omega t) - v_{x0} \sin(\omega t),$$

with *cyclotron frequency* $\omega = \frac{qB_z}{m}$

From the well-known $v = \omega R$ follows the **fundamental relation**

$$\boxed{p = qB_z R}$$

Multipole Expansion of Static Magnetic Field

You know that two-dimensional static magnetic field can be expanded in terms of *multipoles* $\mathbf{u} = x + iy = r\exp(i\varphi)$

$$\mathbf{B}(\mathbf{u}) = B_y(x,y) + iB_x(x,y) = \sum_{n=1}^{\infty} \mathcal{C}_n \left(\frac{\mathbf{u}}{\mathcal{R}_c}\right)^{n-1} \exp(-in\alpha_n),$$

where $\mathcal{C}_n$ and α_n are constants, and $\mathcal{R}_c$ is an arbitrary reference radius typically chosen to be 50-70% of the magnetic aperture.

The second term $(n = 2)$ describes the quadrupole field

$$B_x = a_1 y - b_1 x, \qquad B_y = a_1 x + b_1 y, \qquad \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = \frac{\mathcal{C}_2}{\mathcal{R}_c} \begin{pmatrix} \cos 2\alpha_2 \\ \sin 2\alpha_2 \end{pmatrix},$$

The *normal component* of a quadrupole is given by the *a*-coefficient. It has a simple physical meaning

$$a_1 = \left(\frac{\partial B_y}{\partial x}\right)_{x,y=0},$$

The *skew component* is present if $b_1 \neq 0$ that is the angle

$$\alpha_2 \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \ldots$$

Electromagnetic Fields and Their Utilization Continued…

The usual case in all types of particle accelerators is

$$\mathbf{B} \perp \mathbf{v} \parallel \mathbf{E}, \qquad F = q(E + vB),$$

Technical limitations on electric and magnetic fields:

- **Electric fields**:

$$E_{max} \approx 10^7 \; V/m \; (10 \; kV/mm)$$

- **Magnetic fields**:

$$B_{max} \approx 2 \, T \; (normal \; conducting) \; / \; 10 \, T \; (superconducting)$$

Magnetic fields are therefore used for particle *deflection (confinement)* – *bending and focusing*.

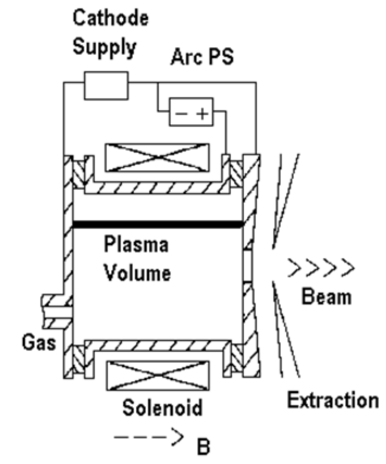
Electric fields are used for *acceleration*. In some special cases, electric fields are used for deflection as well.

Why not electric bends? $$\frac{R_{el}}{R_{mag}} \approx 300\beta \frac{B}{E} \frac{[T]}{[MV/m]}.$$ **For bending in high-energy machines electric fields are not only highly ineffective, but TECHICALLY IMPOSSIBLE to use!**

Particle Sources

Ion sources (Penning type sources commonly used):

- Positive ions sources
  - Formed by electron bombardment of a gas or solid compound
  - Ions extracted from the resulting plasma: species ranging from hydrogen to uranium (multiply charged).



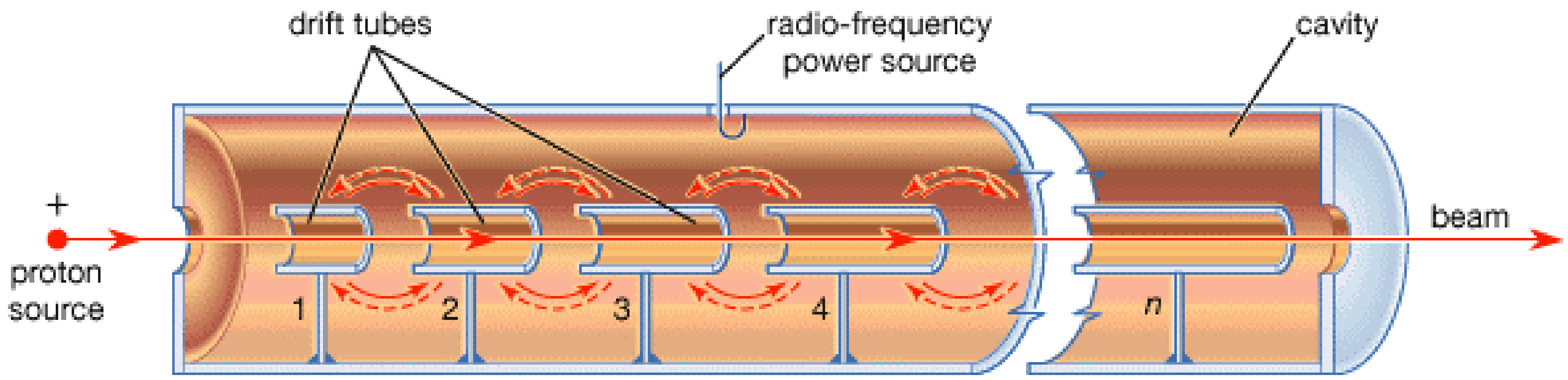
- Negative ion sources: principal interest is in H-, for charge exchange injection
  - *Surface sources*: In a plasma, hydrogen atoms pick up electrons from an activated surface
  - *Volume sources*: Electron attachment or recombination in hydrogen plasma
  - *Polarized ion sources*: e.g., optically pumped source -> some penalty in intensity, relatively high (> 65 %) polarization

- Electron sources

- Electron production mechanism:
  - *Thermo ionic emission* (pulse duration controlled by a pulsed grid) classical method
  - *Photocathode irradiation by pulsed laser* (laser pulse width determines the pulse duration).

Radio–frequency (RF) Linear Accelerators

Patented by Rolf Widerøe in 1928 and independently invented by Leó Szilárd. The discovery of the RF linac was greatly influenced by a journal article by Gustav Ising.



Energy gain for successive passages through the acceleration gaps

$$\gamma_n = \gamma_{n-1} + \frac{qU_0}{m_0 c^2} \sin\phi_s .$$

After the *n*–th passage

$$\beta_n = \frac{\sqrt{\mathcal{G}_n^2 + 2\mathcal{G}_n}}{1 + \mathcal{G}_n},$$

where $\mathcal{G}_n = \frac{qnU_0}{m_0 c^2} \sin\phi_s .$ The *length* of the *n*–th drift tube is

$$L_n = \frac{cT}{2}\beta_n \approx \frac{T}{2}\sqrt{\frac{2nqU_0 \sin\phi_s}{m_0}}.$$

Phase Stability and basic Types of Accelerating Structures

Analyse the *longitudinal dynamics* to understand the process

$$\frac{\mathrm{d}\gamma}{\mathrm{d}s} = \frac{qE_0}{m_0 c^2} \sin \Phi, \qquad \frac{\mathrm{d}\Phi}{\mathrm{d}s} = \frac{\omega}{c\beta},$$ where ω is the RF frequency.

Linearize the dynamic equations around the synchronous phase and synchronous energy: $\Phi = \phi + \phi_s, \qquad \gamma = \Delta\gamma + \gamma_s.$

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}s^2} + \left(\frac{2\pi}{\lambda}\right)^2 \phi = 0, \qquad \lambda = 2\pi \sqrt{\frac{m_0 c^3 \beta_s^3 \gamma_s^3}{\omega q E_0 \cos \phi_s}}, \longrightarrow$$ *wavelength* of

phase oscillations. Maximum energy gain occurs at $\phi_s = \pi/2$ however this process is **unstable**!

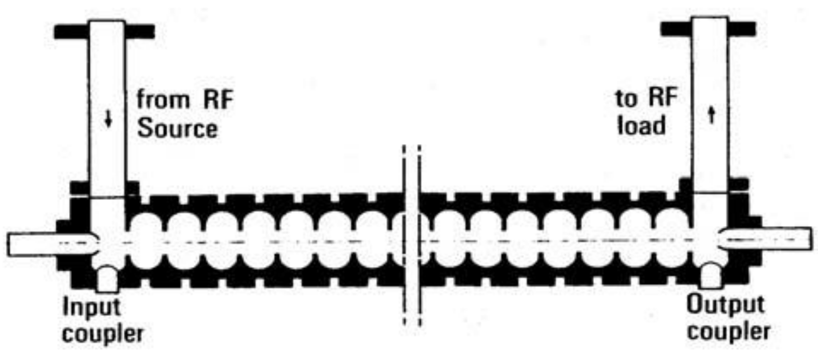
Travelling wave linac features:

▹ generally for v ≈ c (electrons)

▹ RF frequency of the order 3 GHz

▹ pulsed (few μs, 10 – 100 Hz Repetition rate)

▹ field gradient 10 – 50 MV/m



Linacs Around the World

Standing wave structures

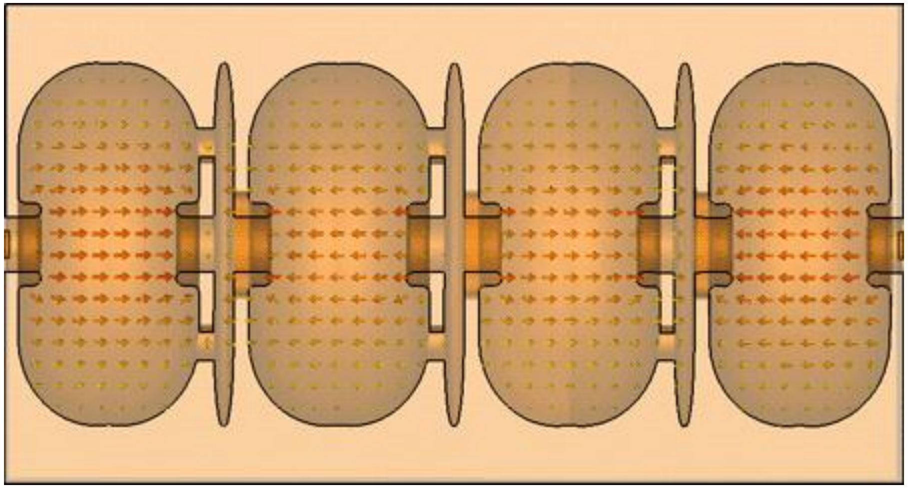
RF cavities

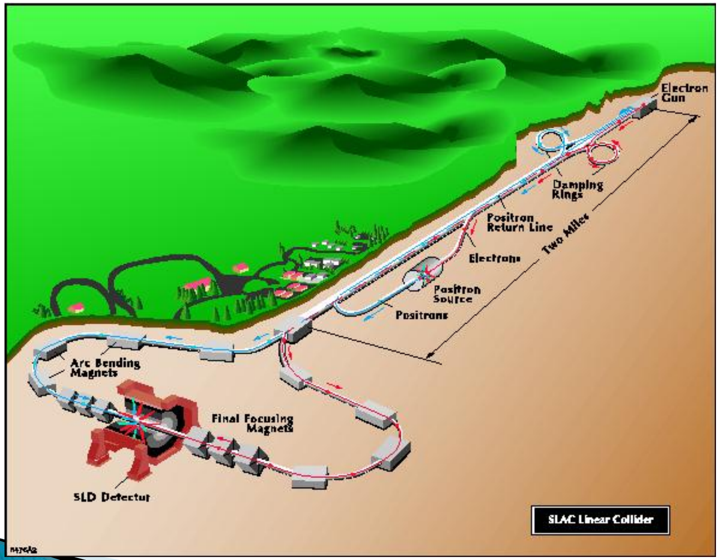
▹ continuous operation possible:

Used in circular machines

▹ frequency ~ 100 MHz – 3 GHz

▹ field gradients ~ 1 MV/m



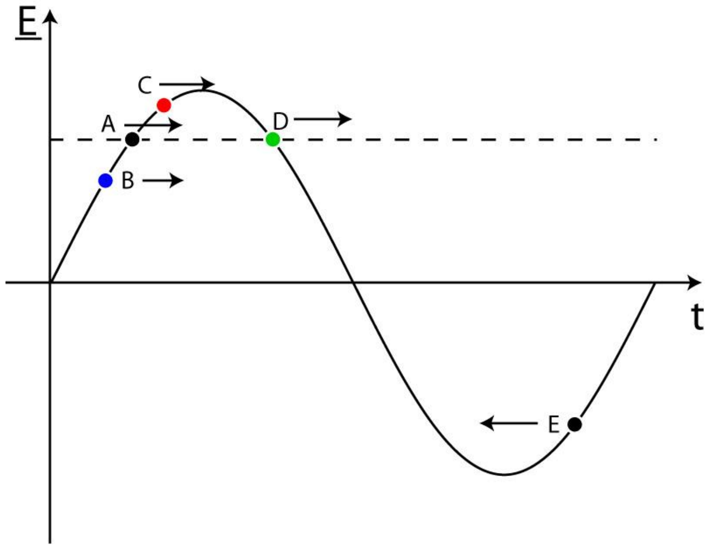


Stanford 100 GeV electron-positron linear collider – SLC

Buncher Cavities and Linacs

A ***beam buncher*** is a small linac, which operates at synchronous phase: $\phi_s \approx 0.$

$$\Delta\gamma = \frac{qU_0}{m_0 c^2} \sin\left(\omega \Delta t\right).$$



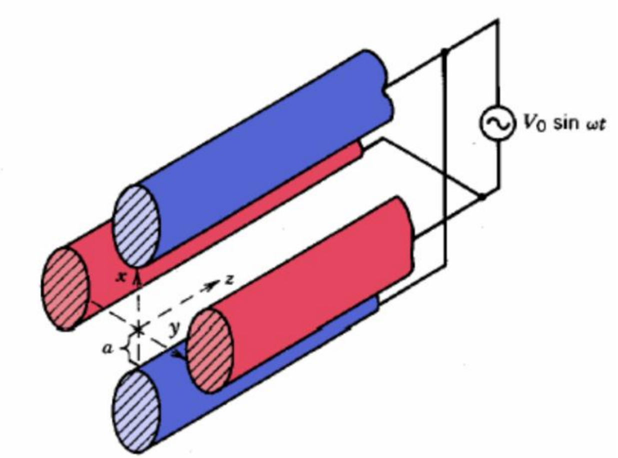
The accelerating gradient, in the form of the applied *E-field*, experienced by five different particles in an accelerating cavity as a function of the time at which they arrive at the cavity. Here **A** is the synchronous particle, **B** is a one having higher energy, **C** is a particle with lower energy, **D** is a particle on the edge of phase stability and the particle **E** experiences a *retarding potential*.

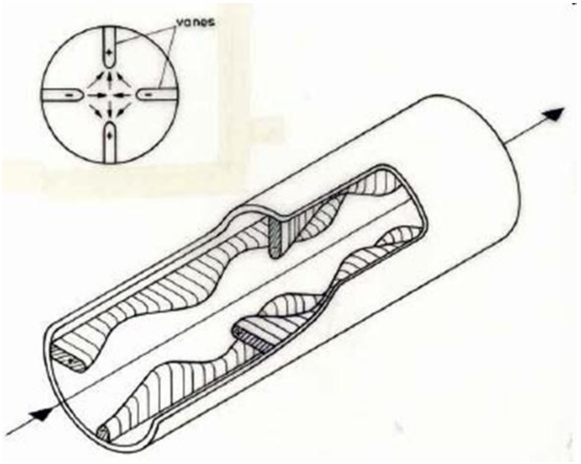
Radio Frequency Quadrupole

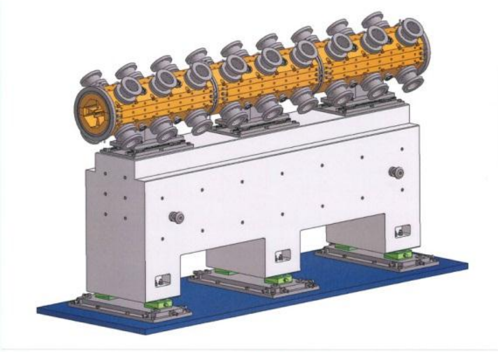
The RFQ was proposed in 1970 by Kapchinsky and Teplyakov. Interesting idea *combining the quadrupole focusing and resonant acceleration*.
The quadrupole pole shapes are modulated.
The potential function can be written as

$$U = \frac{U_0}{2}\left[A\left(x^2 - y^2\right) + BI_0(kr)\cos kz\right]\sin\left(\omega t + \phi\right).$$

Complicated beam dynamics issues in RFQ.
In practice an RFQ can bring proton (or ion) energies from a few 10 keV to a few MeV over a reasonable overall length (1 to 2 meters).

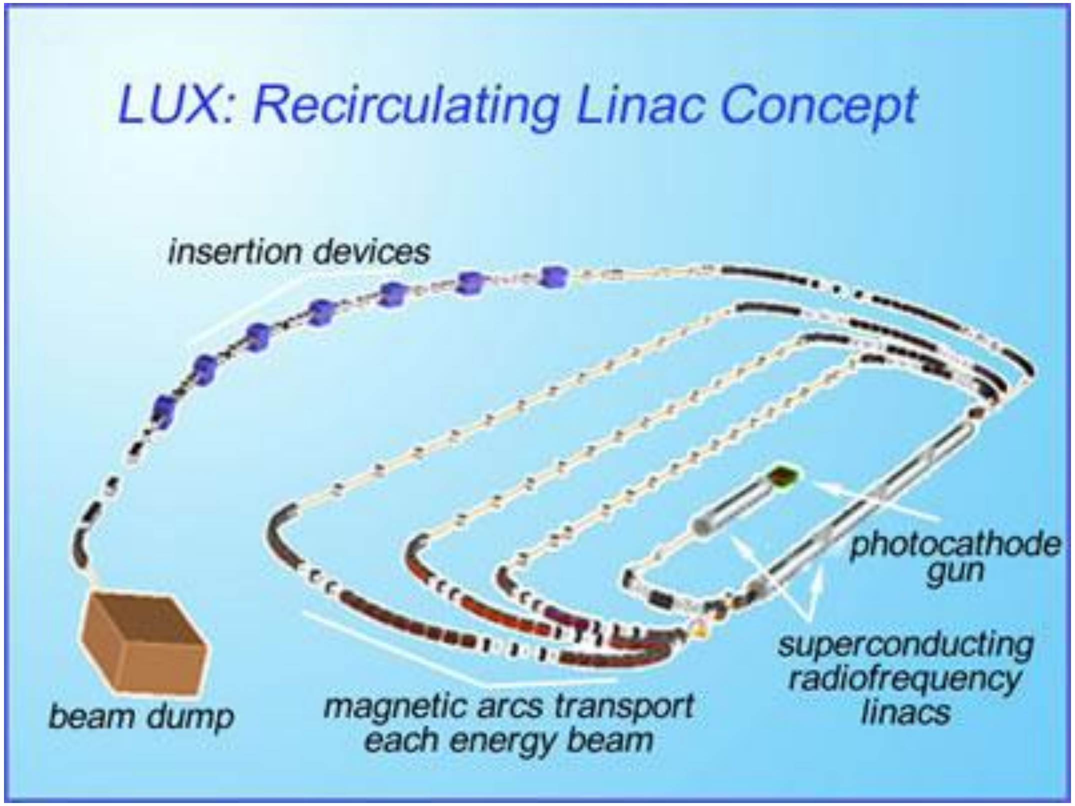






Recirculating Linear Accelerators

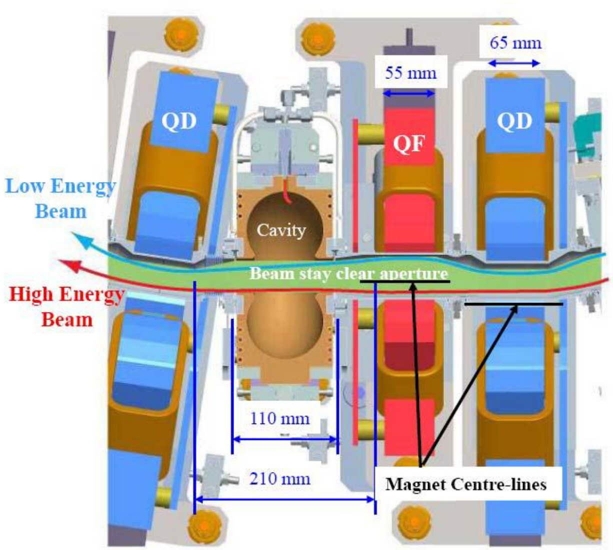
The primary component of the proposed LUX Facility at *LBNL* will be a 2.5 GeV *recirculating superconducting linear accelerator*. Electrons from a photocathode gun enter injector linac, and then a *50-meter-long main linac*, with four outward spiralling recirculating rings, plus multiple beam lines and experimental end-stations that would be coupled to lasers. The entire facility would occupy an area of about 150 by 50 meters.



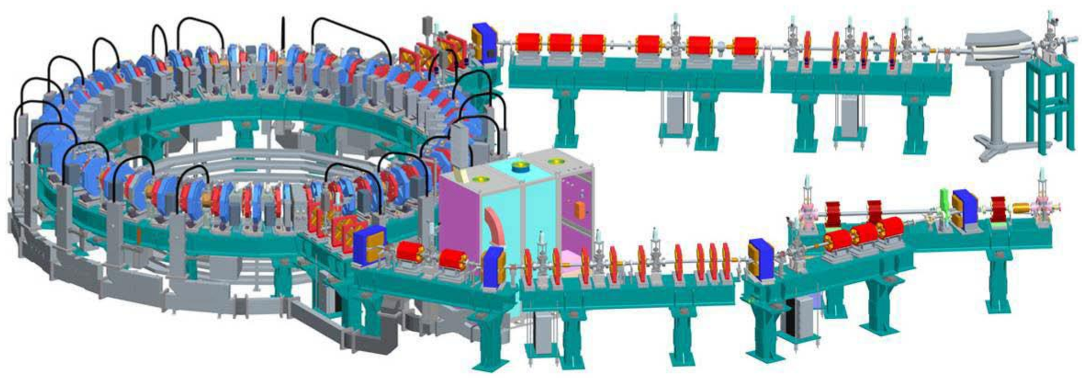
Fixed Field Alternating Gradient (FFAG) Accelerators

Prototype *Electron Machine with Many Applications* (**EMMA**). Similar to all other types of accelerators with static guiding magnetic field (cyclotron, betatron, microtron), *a unique reference trajectory* (for all energies) does **NOT** exist. Such can be found according to:

$$\frac{\mathrm{d}^2 X_e}{\mathrm{d}s^2} = \frac{q}{p_0 \beta_e \gamma_e} \left[1 + \left(\frac{\mathrm{d}X_e}{\mathrm{d}s} \right)^2 \right]^{3/2} B_z(X_e; s).$$



Important peculiarity of the non-scaling FFAG accelerator is the fast acceleration cycle!



Cyclotron

Historically, the cyclotron is the first cyclic accelerator. The first prototype was built by Livingstone in 1929. We know

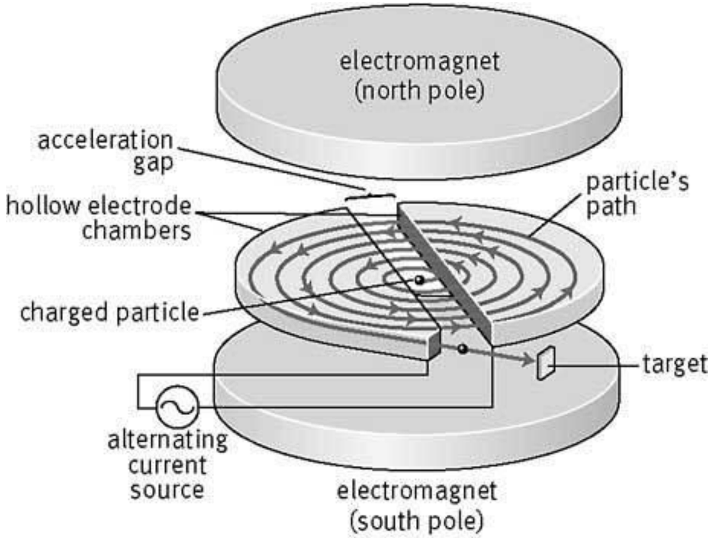
$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB_z}.$$

To maintain the *synchronism with the relativistic increase* of mass, the guiding magnetic field must <u>increase with radius</u>

$$B_z(R) = B(0)\left(1 - \frac{R^2}{R_\infty^2}\right)^{-1/2},$$

where

$$B(0) = \frac{m_0\omega_0}{q} = \frac{A}{Z}\frac{m_n\omega_0}{e}, \qquad R_\infty = \frac{c}{\omega_0}.$$



Due to the isochronism, there is practically <u>NO</u> *phase oscillations*. Frequency of RF field

$$\omega_{RF} = n\omega_0.$$

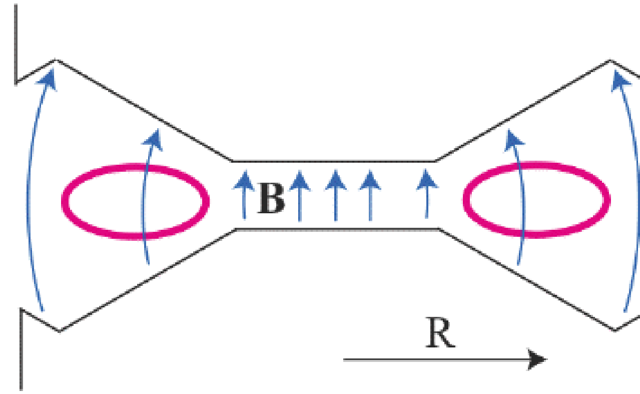
Here *n* is the <u>acceleration harmonic</u>.

$$R_{max} = R_\infty \frac{\sqrt{\gamma_{max}^2 - 1}}{\gamma_{max}}.$$

Betatron

Consider Maxwell equation $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ and apply Stokes' theorem. Therefore $E_\varphi = -\dfrac{R}{2}\dfrac{\partial \bar{B}_z}{\partial t}$. Equation of motion gives $p = \dfrac{eR\bar{B}_z}{2} + const,$ We know on the other hand how guiding magnetic field relates to momentum. Therefore,

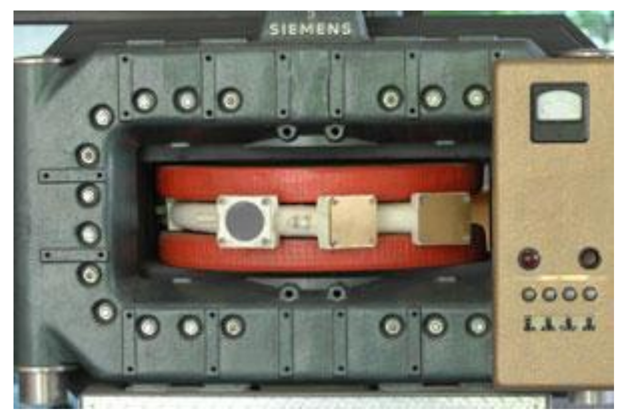
$$B_z = \frac{\bar{B}_z}{2} + const.$$



Essential relation called

THE BETATRON CONDITION !

The betatron is the *first cyclic accelerator* with a fixed (independent of energy) reference trajectory. Acceleration is *INDUCTIVE* without RF being applied.



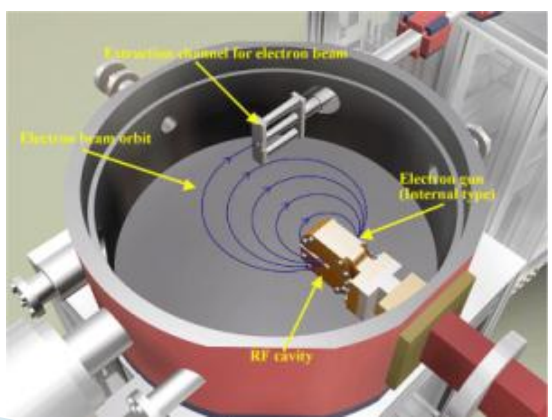
Microtron

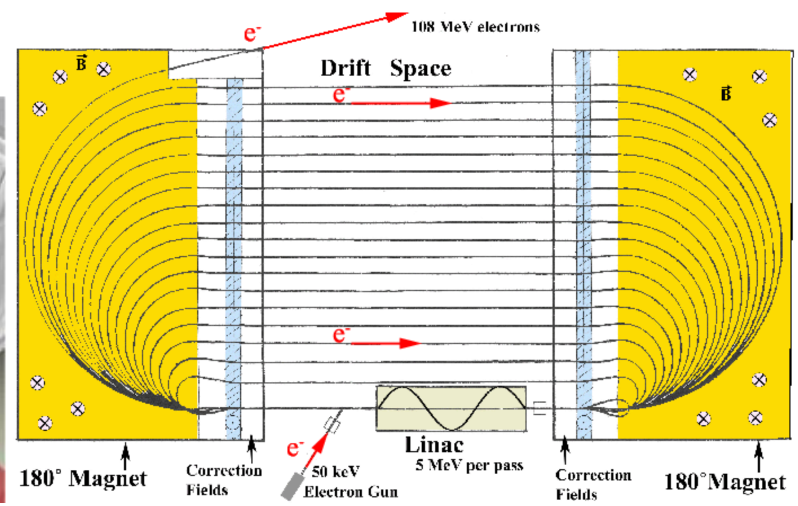
The microtron was invented in 1944 by Vladimir Veksler. It is suitable for acceleration of *electrons only* $(\beta \to 1)$. The modern version $\Rightarrow$ cutup or racetrack microtron. The time of flight for the *n*–th track is:

$$t_n = \frac{1}{\beta_n c}(2\pi R_n + 2L).$$ Since $R_n = \dfrac{E_n \beta_n}{eBc}$, we have $$t_n = \frac{2\pi E_n}{eBc^2} + \frac{2L}{\beta_n c}.$$

Time between turns: $$\Delta t = t_{n+1} - t_n \approx \frac{2\pi}{eBc^2}(E_{n+1} - E_n) = \frac{2\pi N}{\omega_{RF}}.$$

$$\boxed{\frac{\Delta E}{e}\omega_{RF} = Nc^2 B.}$$ $\Rightarrow$ This relation is the *MICROTRON CONDITION*.





Synchrotron

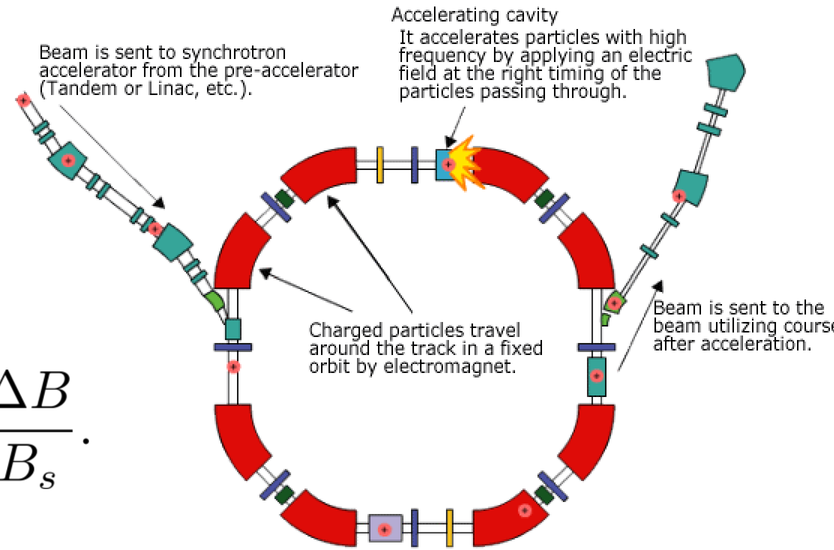
The principle was formulated in 1944 – 1945 by V. Veksler and E. McMillan.

Let a "*synchronous particle*" moves in magnetic field $p_s = qRB_s.$

Increase now the B-field by $\Delta B.$

The ***orbit radius decreases*** by $\dfrac{\Delta R}{R} = -\dfrac{\Delta B}{B_s}.$

Particles arrive **earlier** at the cavity location by $\Delta t = -\dfrac{2\pi \Delta R}{\beta_s c}.$



The particle is *accelerated*

$\Delta E = qU_0 \sin(\omega_{RF} \Delta t + \varphi), \qquad \varphi \approx \pi.$ Using $\Delta E / E_s = \beta_s^2 (\Delta p / p_s)$, we find

$$\boxed{\frac{1}{B_s}\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{qU_0\omega}{\beta_s^2 E_s}}.$$

Thus, the relative increase of the magnetic field for one turn is compensated by the energy gain in the single pass through the cavity, such that the orbit radius remains fixed.

Additional option is to allow change of the RF frequency as well. Such modification is called the SYNCHROPHASOTRON.

Free Electron Lasers

Coherent light is generated in a device called *(helical) magnetic undulator*.

Undulator radiation travels with beam and acts like accelerating RF field.

Typical process of micro bunching – bucket formation at radiation wavelength:

▷ **coherent radiation**:

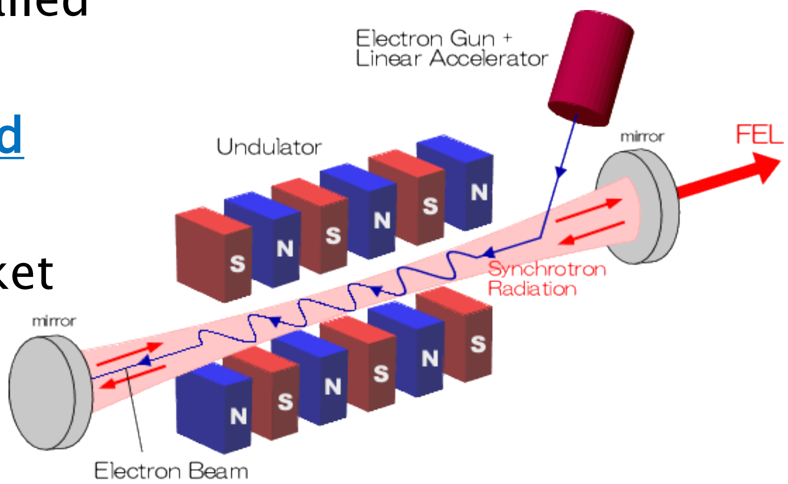
Bunch size less than the wavelength radiates like one super-particle. Radiated power with **two components**:

incoherent $P \sim Ne^2$ and ***coherent***: $P \sim N^2 e^2$

▷ **self amplification**:

Exponential increase of power with path length $P \sim \exp\left(s/L_g\right),$ where L_g is the *gain length*. Power saturation observed at lengths $\approx 22L_g.$

Peak brightness of FELs as compared to storage ring undulators.



Advanced Accelerator Concepts

Microwave-based conventional accelerator's *limitation* $E \approx 100\ MV/m.$

Electric field in laser-plasma acceleration $E = c\sqrt{\frac{m_e n_e}{\epsilon_0}} \approx 10^{-1}\sqrt{n_e}.$

For plasma densities $n_e \sim 10^{18}\ cm^{-3}$ we obtain

$$E \approx 100\ GV/m$$

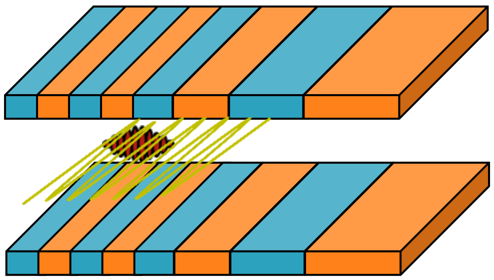
IMPRESSIVE VALUE!

Generally *two directions* in the studies on advanced accelerator concepts:

Inverse Radiation Processes;

Space-Charge Wake

The Inverse Free Electron Laser



Electrons oscillate in a transverse magnetic field.

Ponderomotive force may accelerate electrons.

The electrons move in a wiggler magnetic field

$$\mathbf{B}_w = B_0(\cos k_0 x,\ \sin k_0 x,\ 0),$$

And linearly polarized plane laser wave

$$\mathbf{E}_l = (E_0 \sin{(kz - \omega t)},\ 0,\ 0), \qquad \mathbf{B}_l = \mathbf{k} \times \mathbf{E}_l.$$

Approximate Number of Accelerators Today

CATEGORY	NUMBER
Ion implanters and surface modifications	7 000
Accelerators in industry	1 500
Accelerators in non-nuclear research	1 000
Radiotherapy	5 000
Medical isotopes production	200
Hadron therapy	20
Synchrotron radiation sources	70
Research in nuclear and particle physics	110
TOTAL	**15 000**