MAX-PLANCK-INSTITUT
FÜR PHYSIK

Cobordism Conjecture and Tadpoles

Based on 2208.01656 and 2208.01656 with R. Blumenhagen, N. Cribiori, C. Kneißl

Andriana Makridou

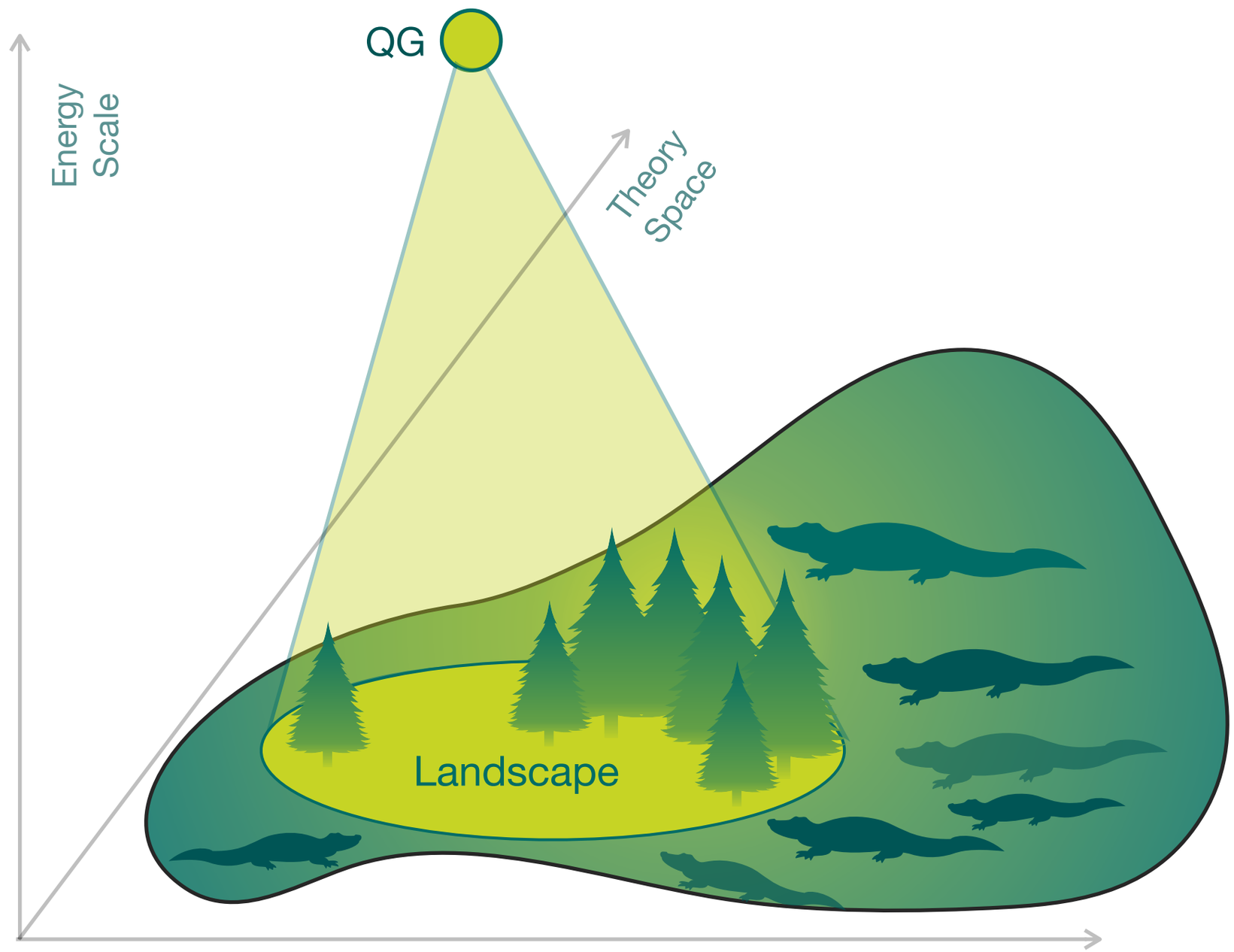
CERN, String Theory Seminar

Tuesday May 30th, 2023

[Recent reviews: 1903.06239, 2102.01111, 2107.00087, 2212.06187]

The Swampland Program

Not all low-energy EFTs can be UV-completed to Quantum Gravity



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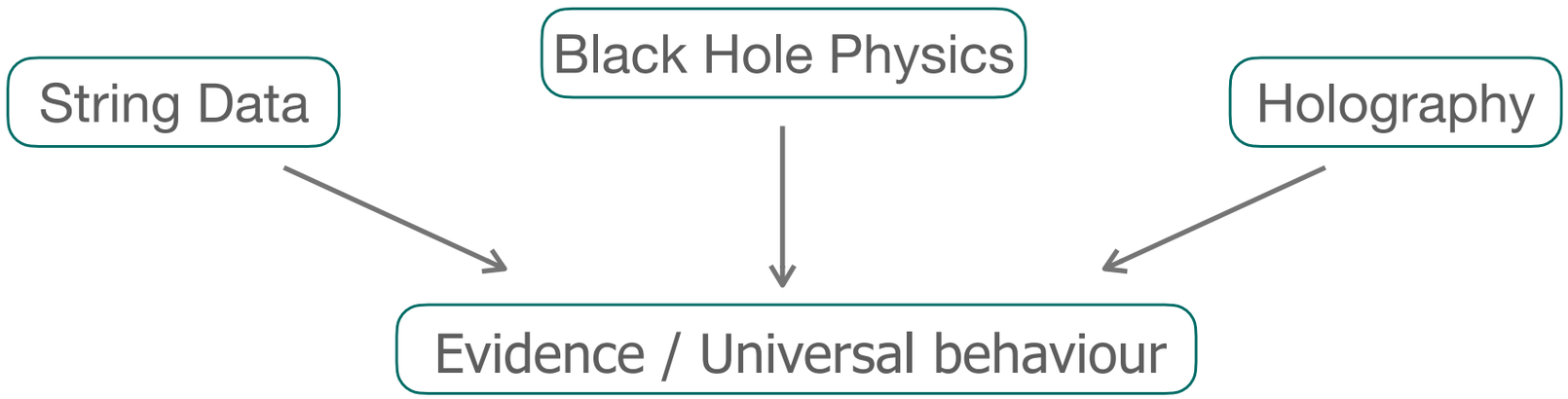

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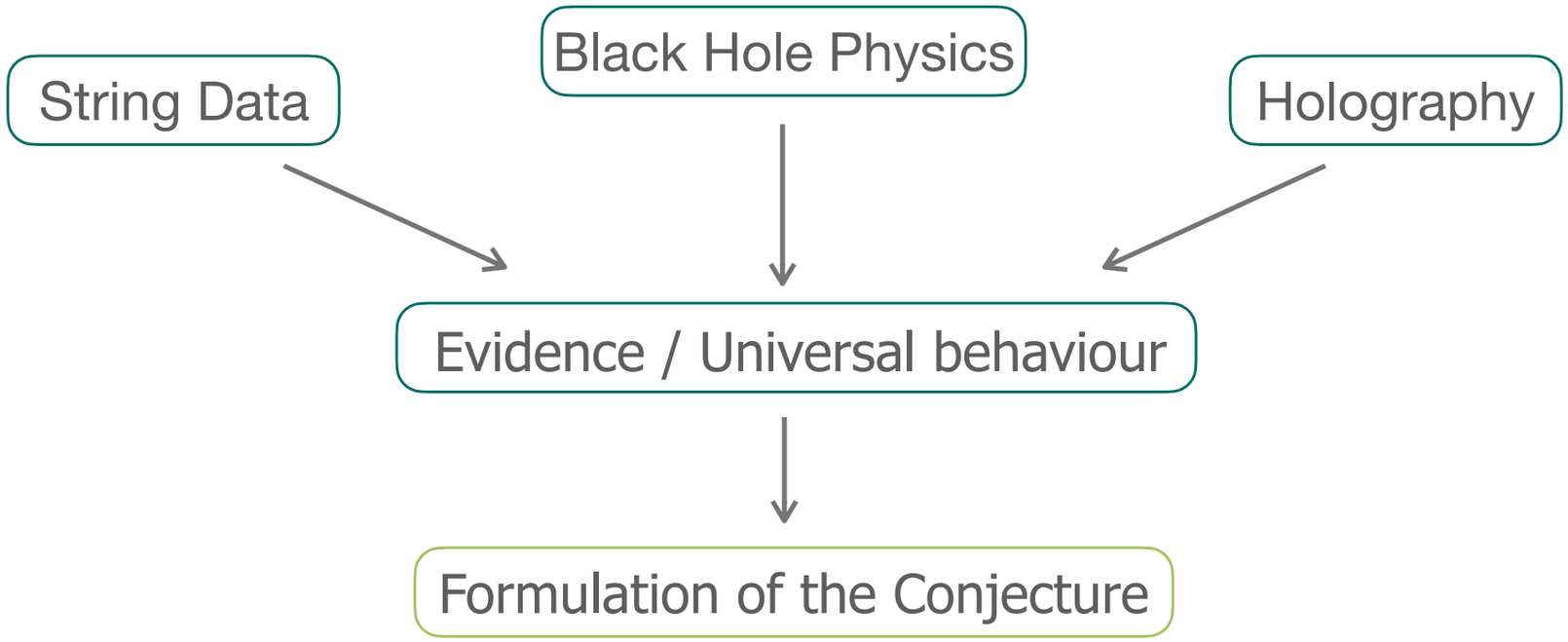

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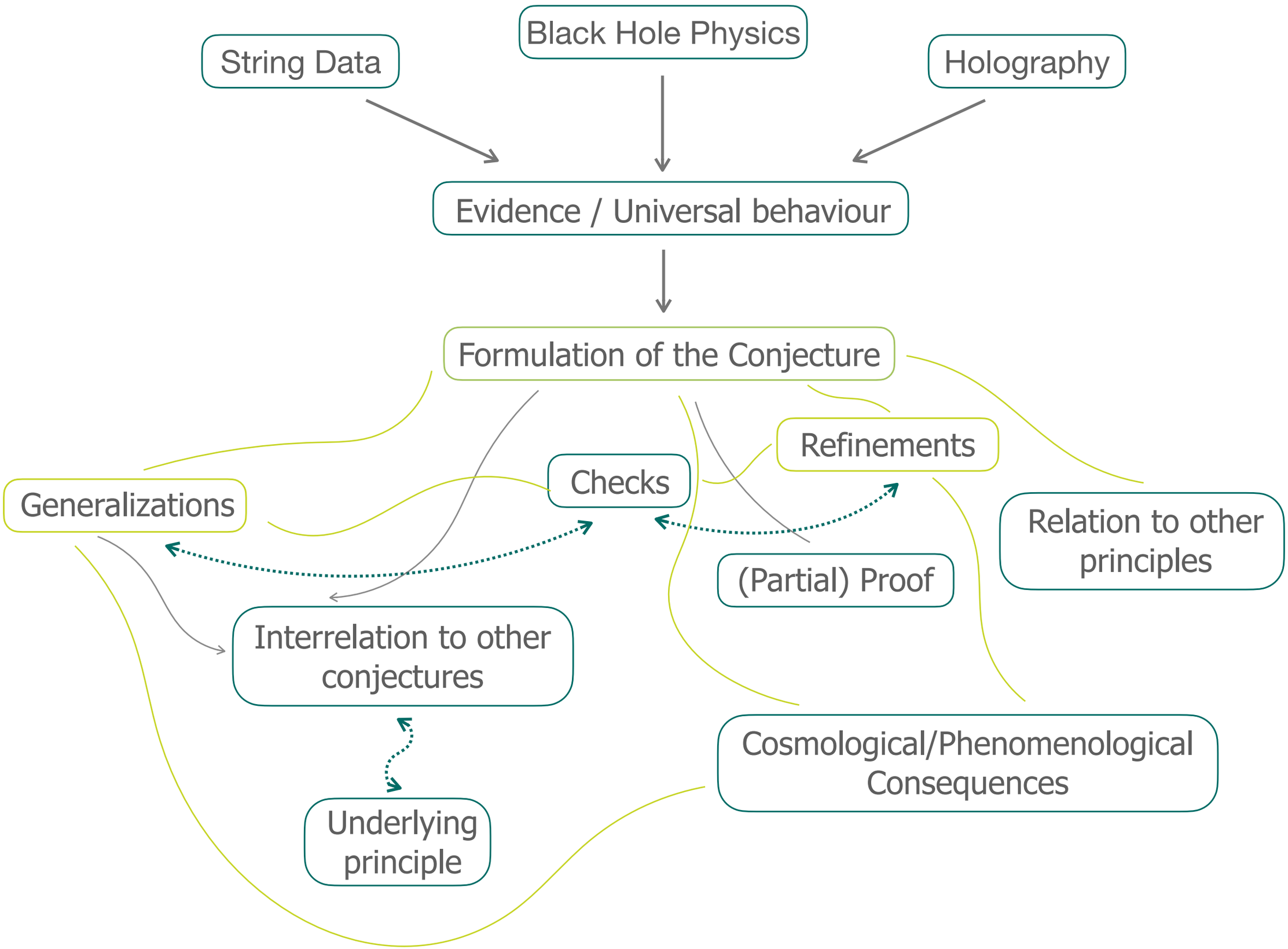
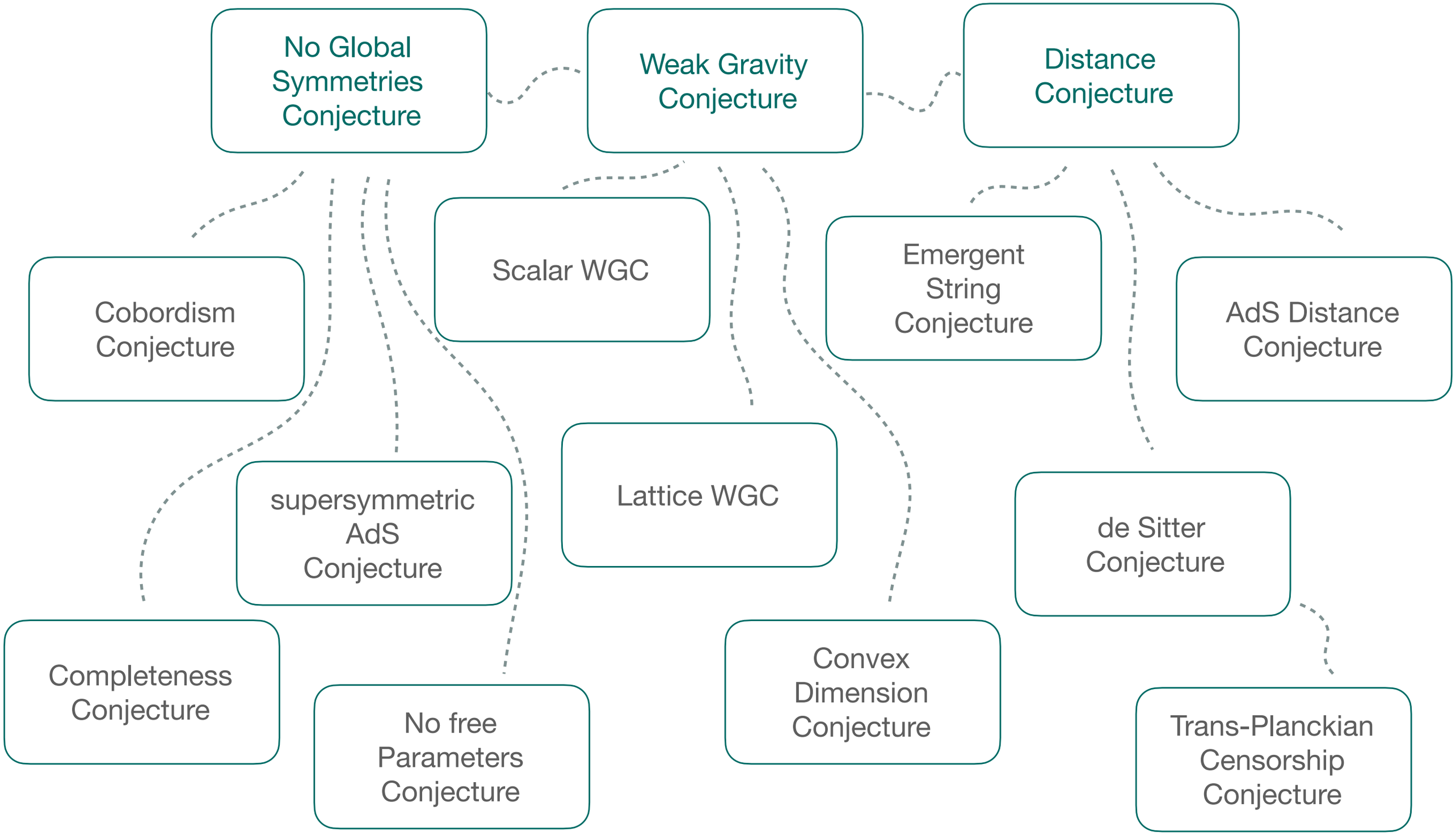


Some Swampland Conjectures



Outline

- No Global Symmetries Conjecture
- Cobordism and the Cobordism Conjecture
- K-theory and Cobordism interplay: Tadpole cancellation conditions
- Background Fixing and Dimensional Reduction
- Dynamical Tadpoles and Cobordism
- A new stringy defect

No Global Symmetries: the BH argument

No Global Symmetries Conjecture:

There should be no global symmetry in a theory of quantum gravity

[Misner, Wheeler '57; Banks, Dixon '88; Kallosh, Linde, Linde, Susskind '95; Banks, Seiberg '11; Harlow, Ooguri '18]

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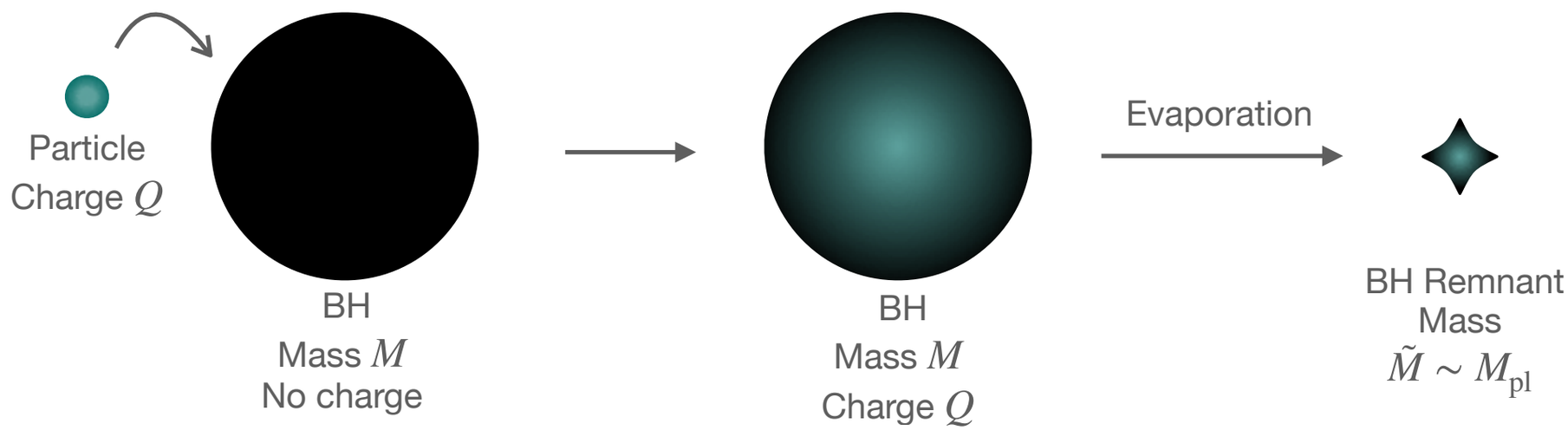

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e.g., [Susskind '95; Banks, Seiberg '11]

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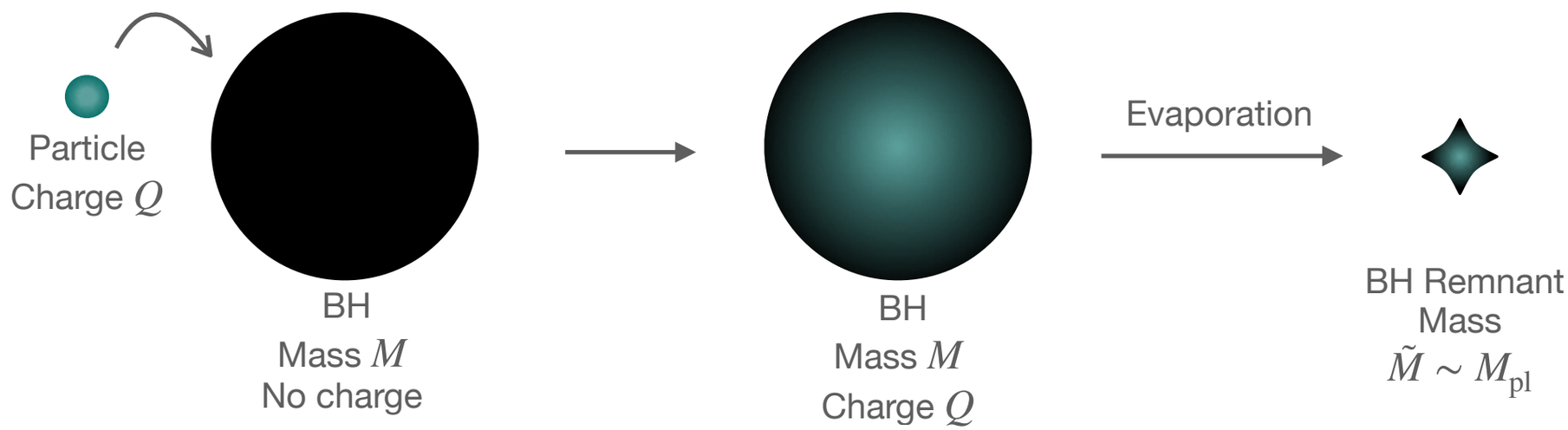

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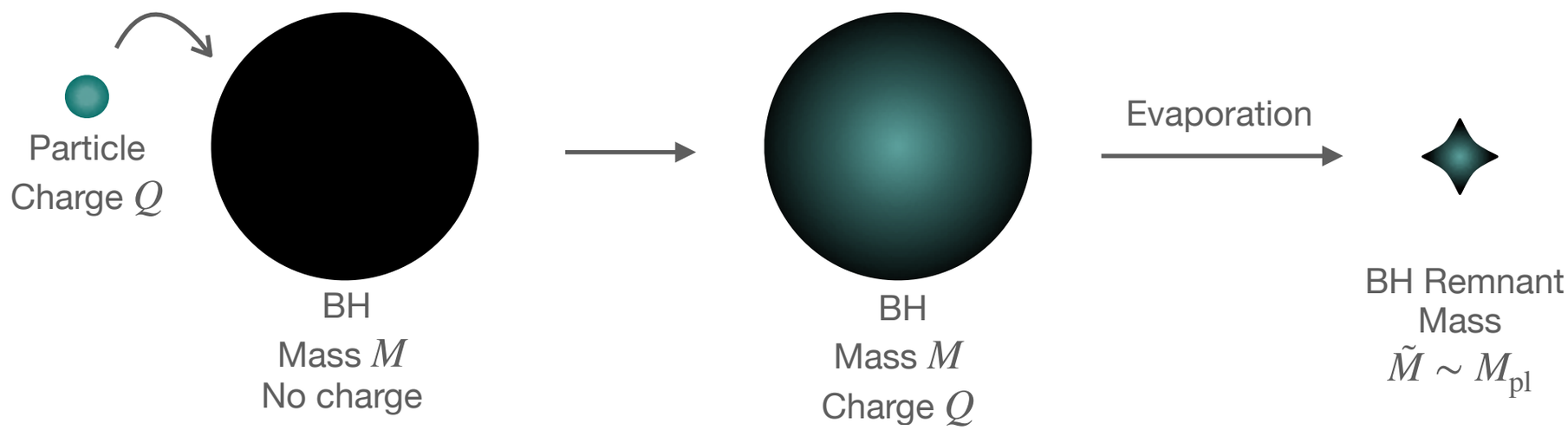

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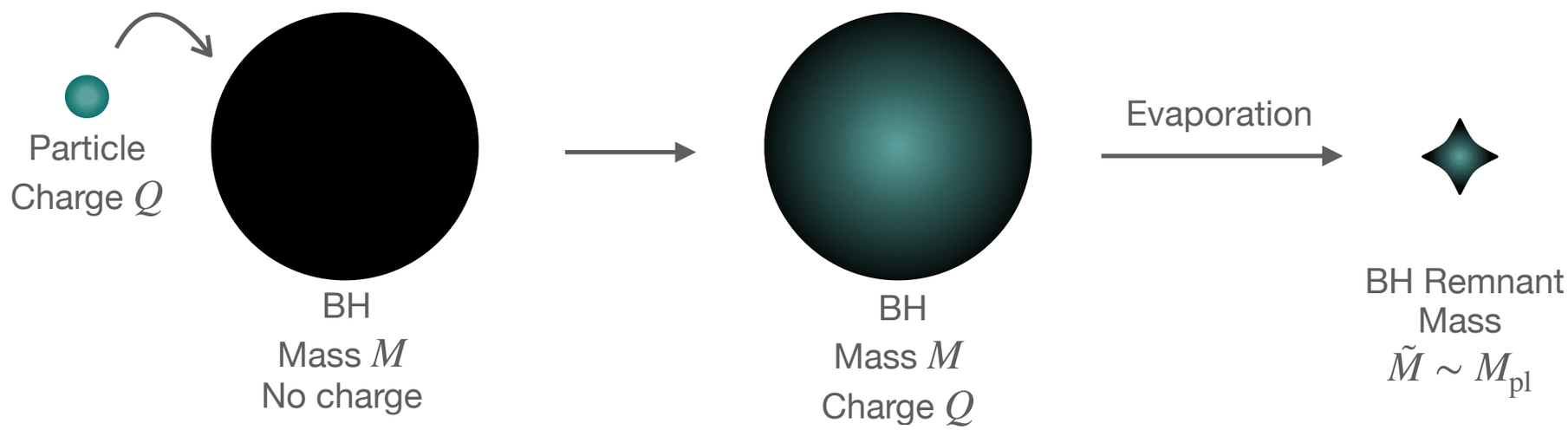

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BH argument - continuous (or infinite discrete) symmetries → Holographic argument

[Harlow, Ooguri, '18]

[Gaiotto, Kapustin, Seiberg, Willett '15]

From ordinary to higher-form global symmetries

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Ordinary 0-form Global Symmetries

Symmetry implemented by symmetry operators: $U(\Sigma^{d-1}, g)$

Observables: local operators, living on a pt

Charged objects: objects with 1-dimensional worldlines

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Conserved 1-form current $j \leftrightarrow$ Closed (d-1)-form (magnetic) current $\tilde{J}$

$$d\tilde{J}_{d-1} = d \star j = 0$$

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Symmetry implemented by symmetry operators: $U(\Sigma^{d-p-1}, g)$

Observables: operators with p spatial directions

Charged objects: objects with (p+1)-dimensional worldvolumes

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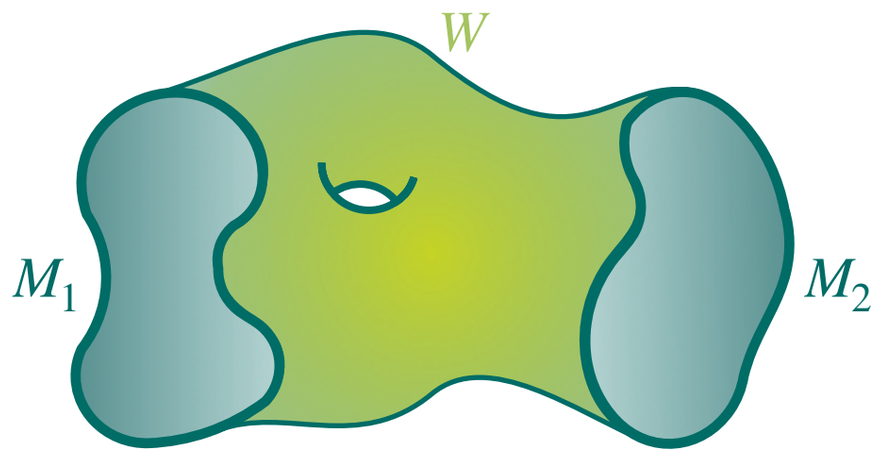
More generalised versions possible: e.g., non-invertible global symmetries,

[Heidenreich, McNamara, Montero, Reece, Rudelius, Valenzuela '21]

Chern-Weil global symmetries

[Heidenreich, McNamara, Montero, Reece, Rudelius, Valenzuela '20]

Cobordism



(Encoded in ξ)

$$M_1 \sim M_2 \Leftrightarrow \exists W \text{ s.t. } \partial \mathrm{W} = \mathrm{M}_1 \sqcup \mathrm{M}_2$$

$$\Omega_k^{\xi} = \{\text{compact, closed, k} - \text{dimensional manifolds}\} / \text{allowed topology changes}$$

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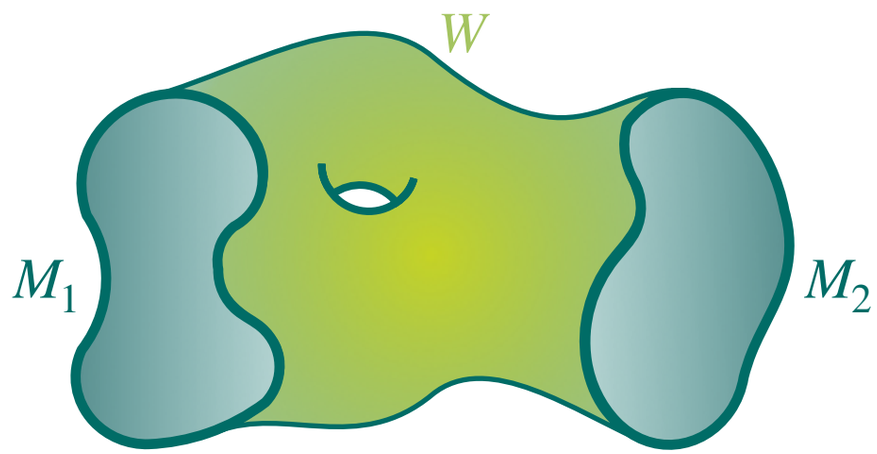

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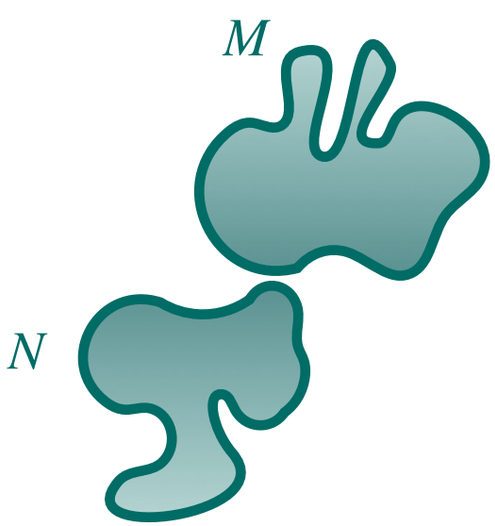
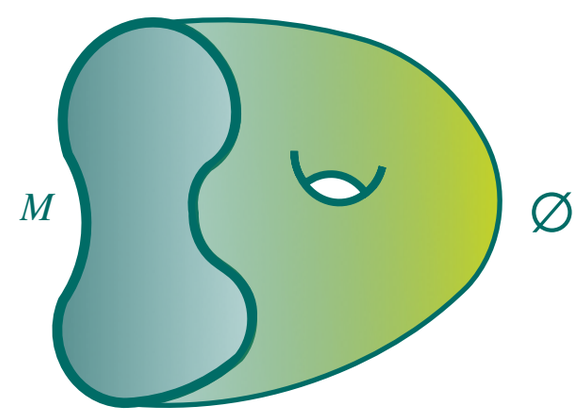


$$[M \sqcup N] = [M] + [N]$$



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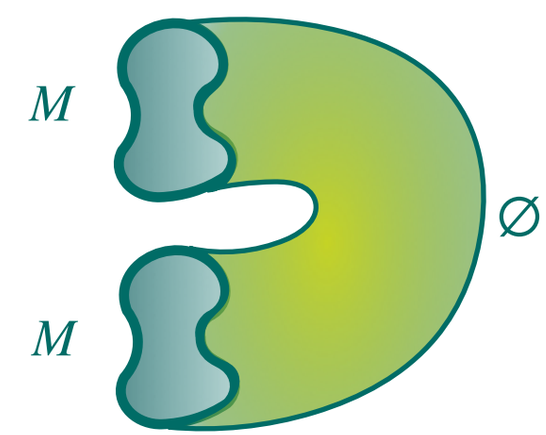

$$[M] + [M] = [M \sqcup M] = 0$$

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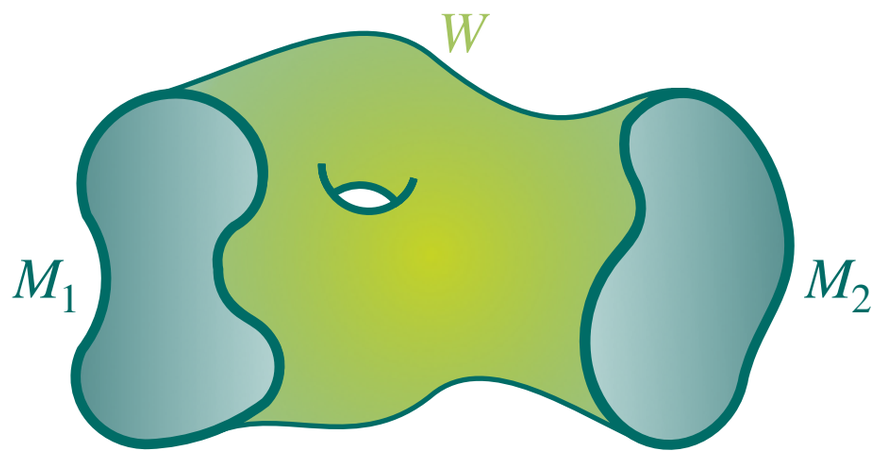

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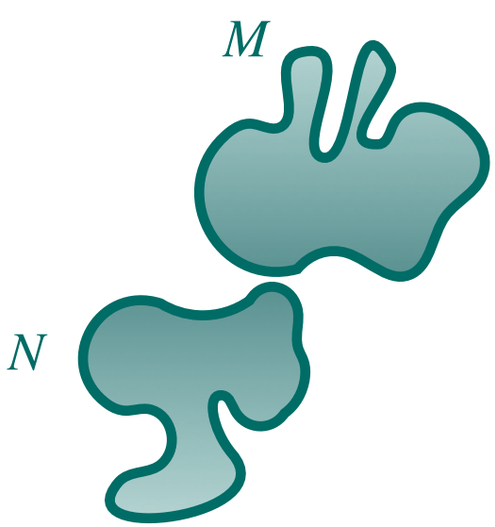

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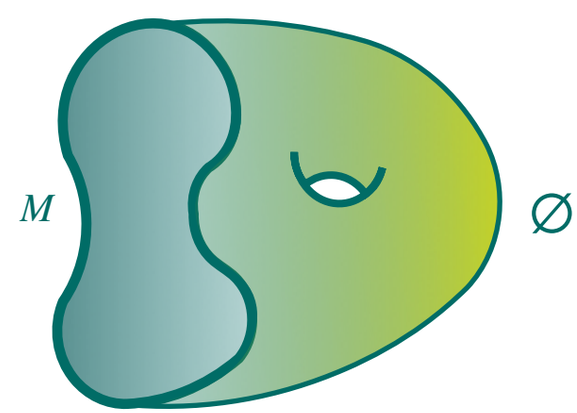

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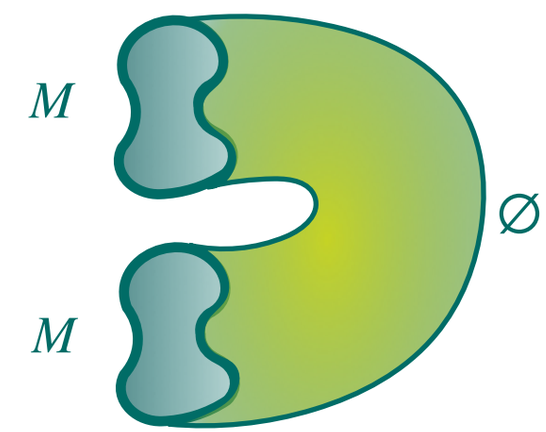

$$[M] + [M] = [M \sqcup M] = 0$$

For oriented manifolds: $[M] + [\bar{M}] = 0, \quad [M] + [N] = [M \sqcup \bar{N}]$

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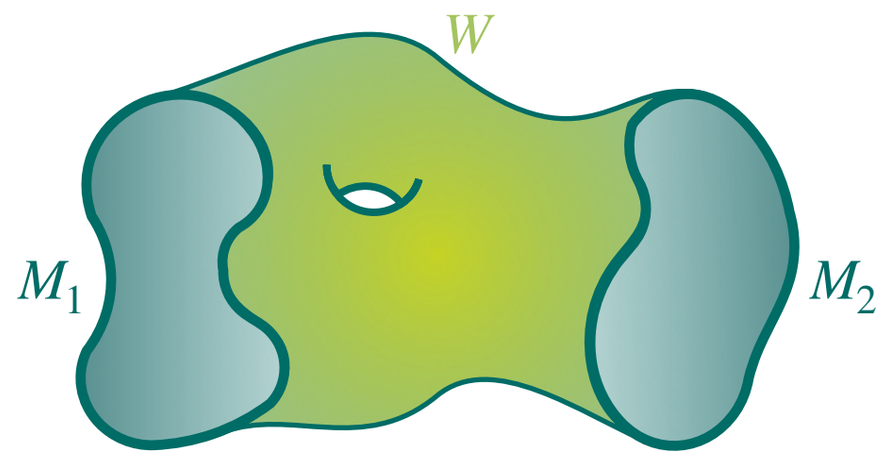

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Compactify d-dimensional theory on M^k down to D=d-k dimensions:

EFT_1 — $(d-k)$ − dimensional

Domain Wall — $(d-k-1)$ − dimensional

EFT_2 — $(d-k)$ − dimensional

→ *Finite-energy transition between EFTs*

→ *Domain walls detected by cobordism can be well-known objects, e.g. D-branes*

Cobordism as a global symmetry

Cobordism Group $\Omega_k^\xi \neq 0$ $\rightarrow$ Non-trivial cobordism classes $[M]$

Cobordism classes labelled by cobordism invariants $\mu_k : \Omega_k^\xi \rightarrow A$

For empty set: $\mu_k[\varnothing] = 0$

If cobordism class $[M] \neq 0 \leftrightarrow$ obstruction to decay into nothing

Can define a charge $Q(M) = [M] \in \Omega_k^\xi$

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[McNamara, Vafa '19]

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Charged Objects: (d-k)-dimensional gravitational solitons

[Witten '85]

Cobordism Conjecture

But: No global symmetries in quantum gravity → Cobordism Conjecture

$[M] = 0$

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All Cobordism Classes should be trivial

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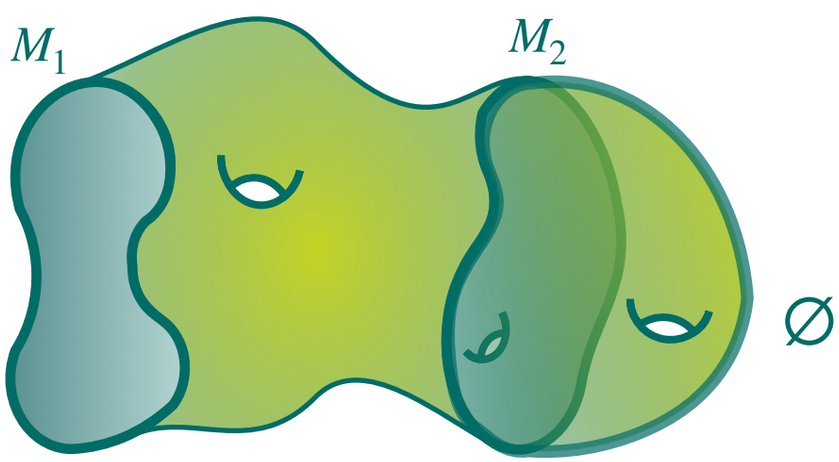

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In D=d-k dimensions:

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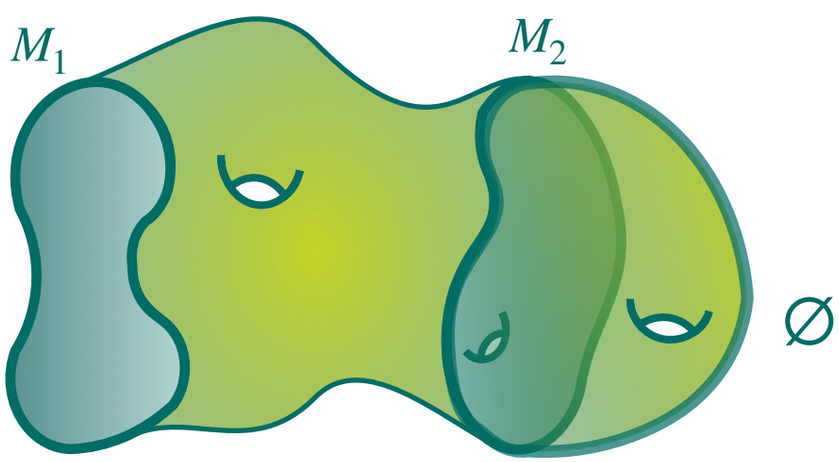

$\leftrightarrow$

In D=d-k dimensions:

Major Implication: there is a "single" theory of quantum gravity

Cobordism and String Theory

What are some* relevant structures for String Theory?

Type I $\leftrightarrow \xi = \mathrm{Spin}$: $w_2(TM) = 0$

Type IIB $\leftrightarrow \xi = \mathrm{Spin}^c$: $W_3(TM) = 0$

M-theory $\leftrightarrow \xi = \mathrm{pin}^+$: $w_2(TM) = 0 \ \& \ w_1(TM) \neq 0$

Heterotic $\leftrightarrow \xi = \mathrm{String}$: $\dfrac{p_1(M)}{2} = 0$

A possible classification scheme : Whitehead tower

[Andriot, Carqueville, Cribiori '22]

* The full duality group of a theory also becomes relevant,
see e.g. [Tachikawa, Yonekura '18], [Debray, Dierigl, Heckman, Montero '23] for discussions on type IIB.

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n	0	1	2	3	4	5	6	7	8	9	10
Ω_n^{Spin}	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	$2\mathbb{Z}_2$	$3\mathbb{Z}_2$
$\Omega_n^{\mathrm{Spin}^c}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$2\mathbb{Z}$	0	$2\mathbb{Z}$	0	$4\mathbb{Z}$	0	$4\mathbb{Z} \oplus \mathbb{Z}_2$

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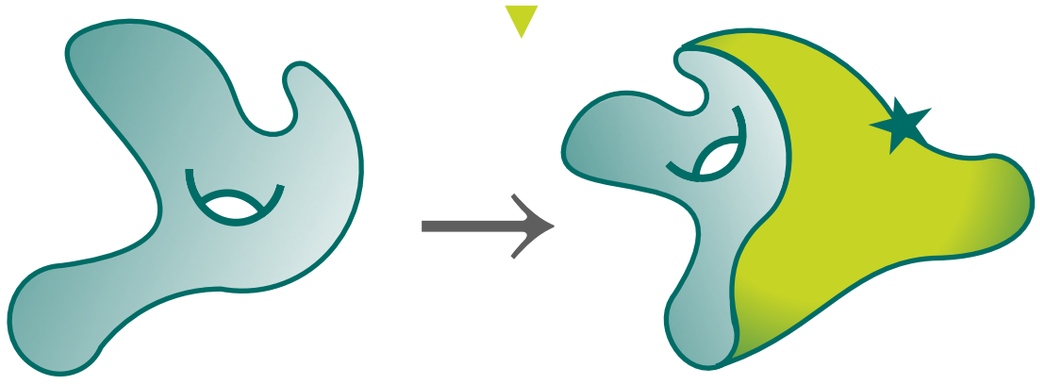
Cobordism Conjecture Consequences

$$\Omega_k^{\widetilde{QG}} \neq 0$$

Breaking

$$\Omega_k^{\widetilde{QG}} \to \Omega_k^{\widetilde{QG}+\text{defects}} = 0$$



$$0 \neq [M] \in \Omega_k^{\widetilde{QG}} \qquad [M] = 0 = \Omega_k^{\widetilde{QG}+\text{defects}}$$

Gauging

$$0 = [M] \in \Omega^{\widetilde{QG}} \neq 0$$

$$\Omega^{\widetilde{\text{QG}}+\text{g.fields}} \to \Omega^{\widetilde{\text{QG}}}$$

Possibly new defects detected!

[McNamara, Vafa '19], [Montero, Vafa '20]
[Dierigl, Heckman '21]
[Debray, Dierigl, Heckman, Montero '21]
[Blumenhagen, Cribiori, Kneissl, AM '22/1]
[De Biasio, Lüst, Velazquez '22]
[Dierigl, Heckman, Montero, Torres '22]
[Debray, Dierigl, Heckman, Montero '23]
[Blumenhagen, Kneissl, Wang '23]
[Dierigl, Heckman, Montero, Torres '23] …

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$$\Omega^{\widetilde{\text{QG}}+\text{g.fields}} \to \Omega^{\widetilde{\text{QG}}}$$

$\Omega_k \neq 0 \leftrightarrow d\tilde{J}_k = 0$: $\tilde{J}_k$ magnetic current

Breaking

$$d\tilde{J}_k = \delta^{(k+1)}(\Sigma) \neq 0$$

Gauging

$$\tilde{J}_k = dF_{k-1}$$

Cobordism Conjecture Consequences

$$\Omega_k^{\widetilde{QG}} \neq 0$$

Breaking

$$\Omega_k^{\widetilde{QG}} \to \Omega_k^{\widetilde{QG}+\text{defects}} = 0$$

Gauging

$$0 = [M] \in \Omega^{\widetilde{QG}} \neq 0$$

$$\Omega^{\widetilde{\text{QG}}+\text{g.fields}} \to \Omega^{\widetilde{\text{QG}}}$$

$\Omega_k \neq 0 \leftrightarrow d\tilde{J}_k = 0 : \tilde{J}_k$ magnetic current

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Gauging

$$\tilde{J}_k = dF_{k-1}$$

Part I: R-R tadpoles

Cobordism Conjecture Consequences

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Part II: Dynamical tadpoles

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Part I: R-R tadpoles

K-Theory and string theory

Consider complex vector bundles E, G, F over M

Identifying $(E, F) = (E \oplus G, F \oplus G)$, we get the K-theory group of M, $K^0(M)$.

Addition: $(E_1, F_1) + (E_2, F_2) = (E_1 \oplus E_2, F_1 \oplus F_2)$,

Identity: $(0,0)$, Inverse: $-(E, F) = (F, E)$

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Restricting to bundles of same rank $\rightarrow$ Reduced K-theory group $\tilde{K}^0(M)$

Applying successive reduced suspensions, we get reduced K-groups of higher degrees:

$$\tilde{K}^{-n}(M) = \tilde{K}(\Sigma^n M), \quad n \geq 0$$

Finally, the splitting lemma allows us to write:

$$K^{-n}(M) = K^{-n}(pt) \oplus \tilde{K}^{-n}(M)\,.$$

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n	0	1	2	3	4	5	6	7	8	9	10
KO^{-n}	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
K^{-n}	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$

Physical meaning: D-brane charge classification! [Witten'98]

[Blumenhagen, Cribiori '21]

Cobordism and K-Theory

n	0	1	2	3	4	5	6	7	8	9	10
Ω_n^{Spin}	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	$2\mathbb{Z}_2$	$3\mathbb{Z}_2$
$\Omega_n^{\mathrm{Spin^c}}$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$2\mathbb{Z}$	0	$2\mathbb{Z}$	0	$4\mathbb{Z}$	0	$4\mathbb{Z} \oplus \mathbb{Z}_2$

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Similarity is not an accident→ Atiyah-Bott-Shapiro orientation

[Atiyah, Bott, Shapiro '64]

$$\alpha^c : \Omega_*^{\mathrm{Spin^c}}(pt) \to K_*(pt)\,.$$

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Similarity is not an accident→ Atiyah-Bott-Shapiro orientation

[Atiyah, Bott, Shapiro '64]

$$\alpha^c : \Omega_*^{\mathrm{Spin^c}}(pt) \to K_*(pt)\,.$$

For fixed k, i.e. $[M] \in \Omega_k^{\mathrm{Spin^c}}$, we have $\alpha^c([M]) = Td([M])\,.$

α^c : cobordism invariant, surjective $\to \Omega_n^{\mathrm{Spin^c}}/\ker(\alpha^c) \cong K^{-n}.$

Can be extended beyond groups of the point!

Gauging Cobordism Charges

[Blumenhagen, Cribiori '21]

K-theory charges are gauged

[Freed, Hopkins '99; Freed, Hopkins '00]

Gauged symmetries accompanied by tadpole conditions: $0 = \int_M dF_{n-1} = \int_M J_n$

Idea: Cobordism and K-theory can be combined in single tadpole

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In practice: add K-theory + cobordism contributions

$$0 = \int_M dF_{n-1} = \sum_{j \in \mathrm{def}} \int_M Q_j \delta^{(n)}(\Delta_{10-n,j}) + \sum_{\mathrm{i} \in \mathrm{inv}} a_i \mu_{n,i}$$

D-brane contributions

Geometric contributions

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D-brane contributions

Geometric contributions

Bottom-up way to construct tadpoles without string theoretical input!
(up to constants!)

[Blumenhagen, Cribiori '21]

An example:

Focus on interplay of $K^{-6}(pt) = \mathbb{Z}$, $\Omega_6(pt) = 2\mathbb{Z}$:

Quantities entering tadpole:

D3-branes

Cobordism invariants

$$0 = \sum_i \int_M N_i \delta^{(6)}(\Delta_{4,i}) + a_1^{(6)} \frac{c_2(M)c_1(M)}{24} + a_2^{(6)} \frac{c_1^3(M)}{2} .$$

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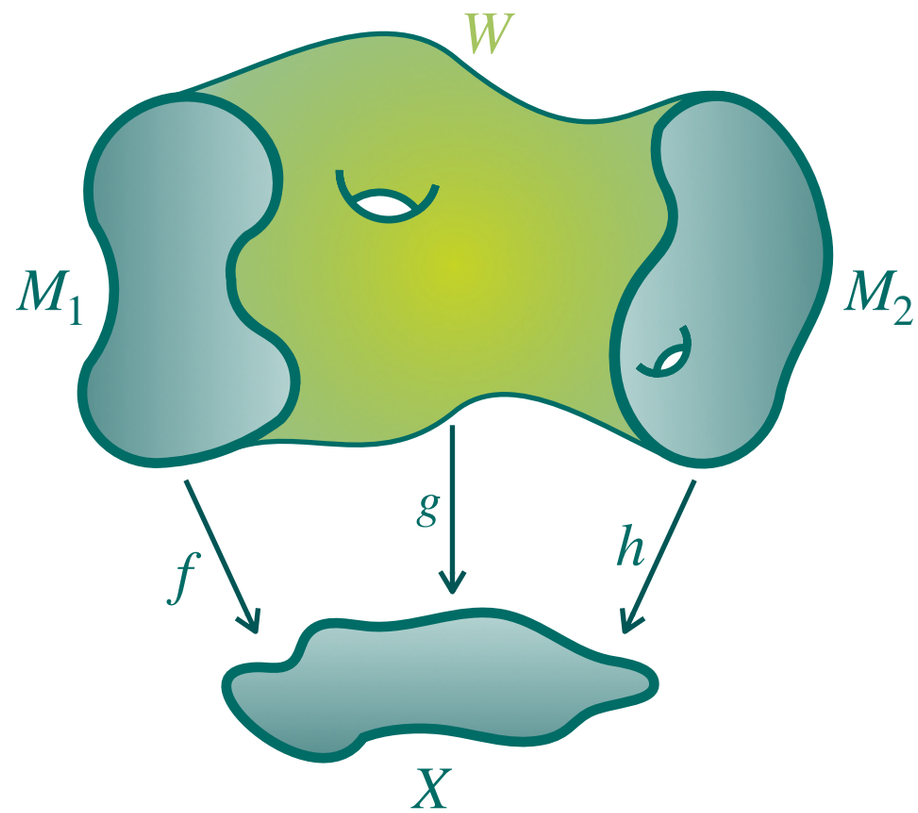
D3-tadpole for F-theory compactified on CY 4-fold Y over base B:

[Sethi, Vafa, Witten '96]

$$\int_B \sum_i \delta^{(6)}(\Delta_{4,i}) + \ldots = \frac{\chi}{24} = \int_B \left(\frac{1}{2} c_2(B)c_1(B) + 15 c_1^3(B)\right)$$

Perfectly reproduced for $a_1^{(6)} = -12, \quad a_6^{(2)} = -30$

Background fixing



$\Omega_k^\xi(X)$: cobordism groups relative to manifold X

For $X = pt$: $\Omega_k^\xi(pt) \equiv \Omega_k^\xi$.

In general: $\Omega_k^\xi(X) = \Omega_k^\xi(pt) \oplus \tilde{\Omega}_k^\xi(X)$.

→ *Cobordism group enlarged when fixing background manifold!*

Aim: How does the gauging work in this case?

Computation of $\Omega_k^{\xi}(X)$ - General technique

Tool: (Homological) Atiyah - Hirzebruch Spectral Sequence (AHSS)

Start with fibration: $F \to E \to B$, use AHSS to determine $G_n(E)$ from $G_n(B)$.

Spectral sequence data: pages E^r, differentials d^r.

$$E^{r+1} \cong H(E^r) = \frac{\ker d^r : E^r \to E^r}{\mathrm{Im} d^r : E^r \to E^r}$$

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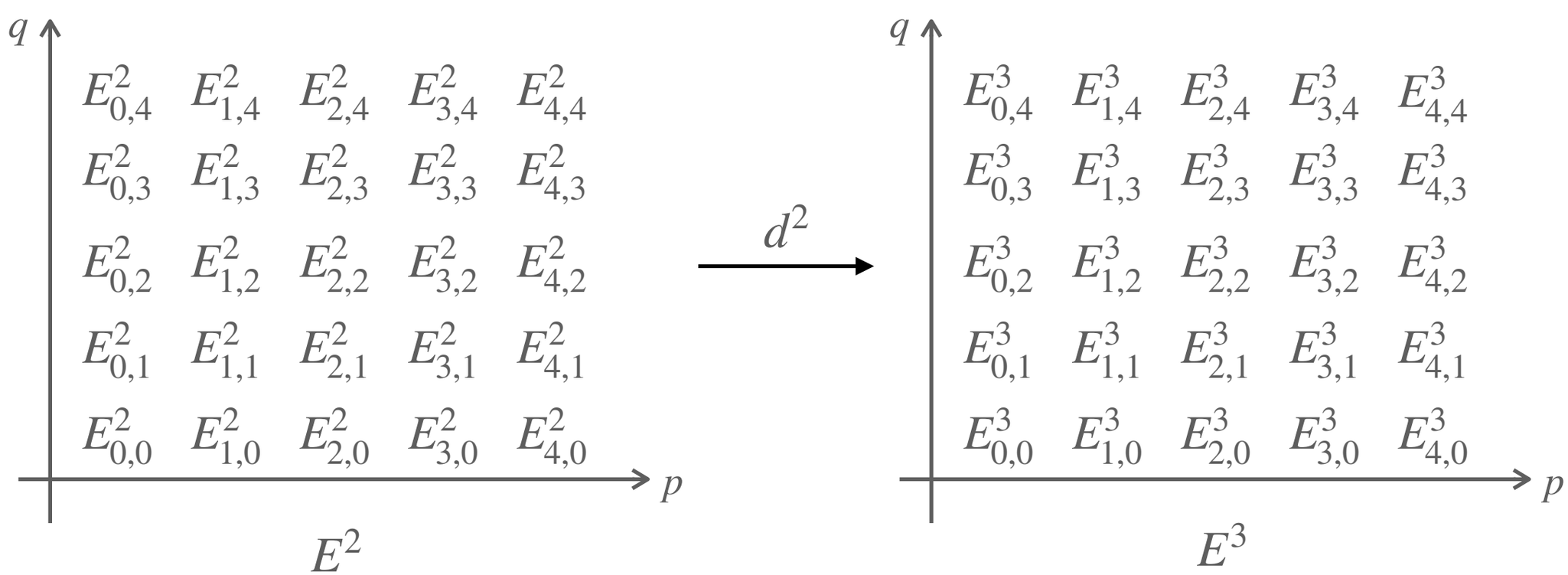

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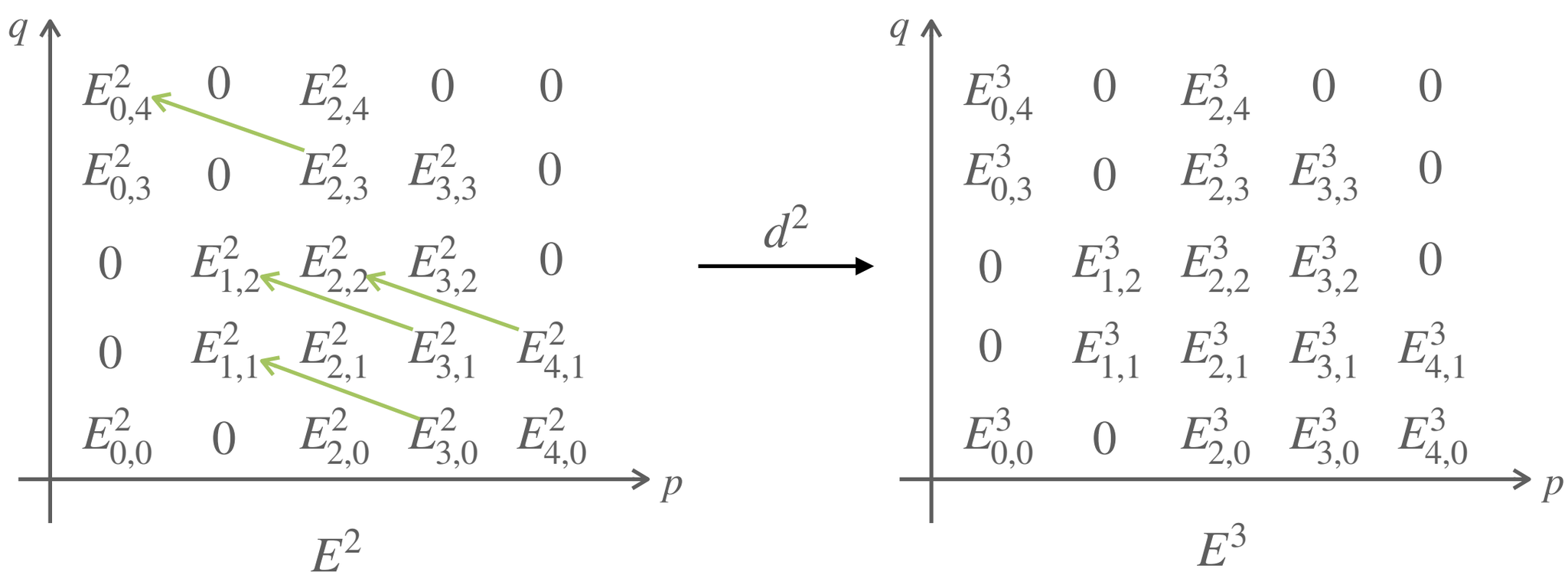

Computation of $\Omega_k^{\xi}(X)$ - General technique

Tool: (Homological) Atiyah - Hirzebruch Spectral Sequence (AHSS)

Start with vibration: $F \to E \to B$, use AHSS to determine $Gr(G_n(E))$ from $G_n(F)$.

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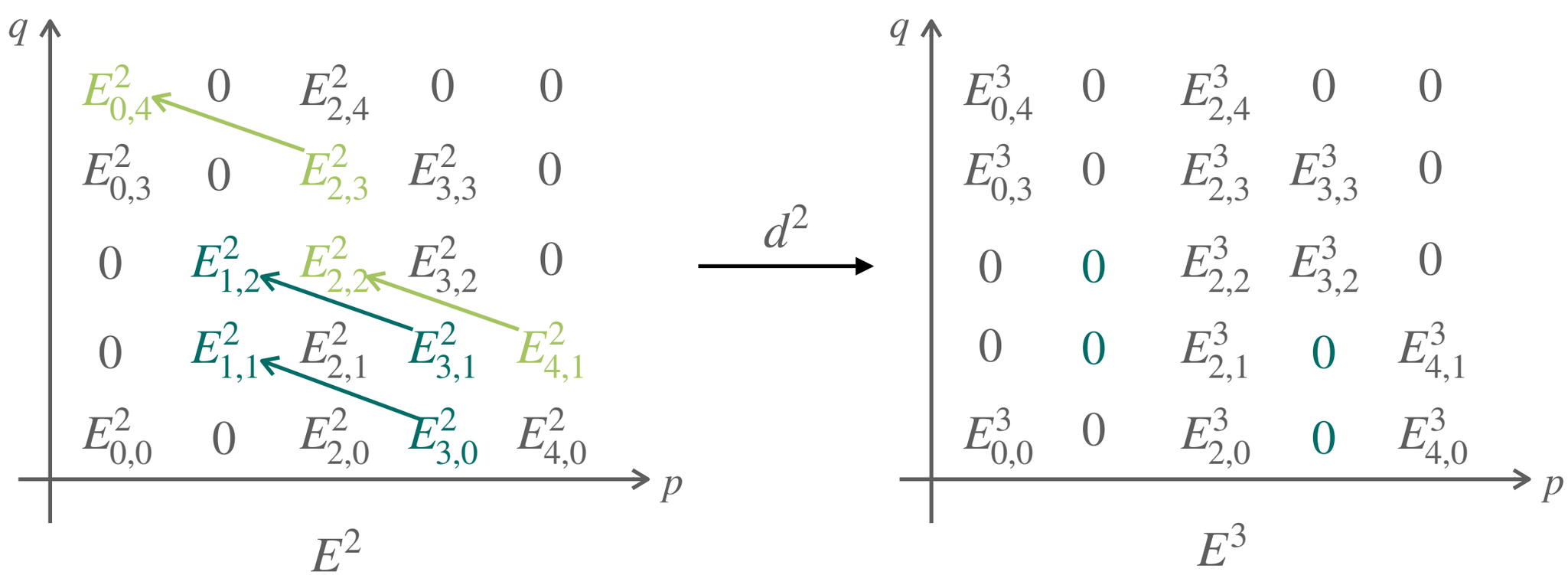

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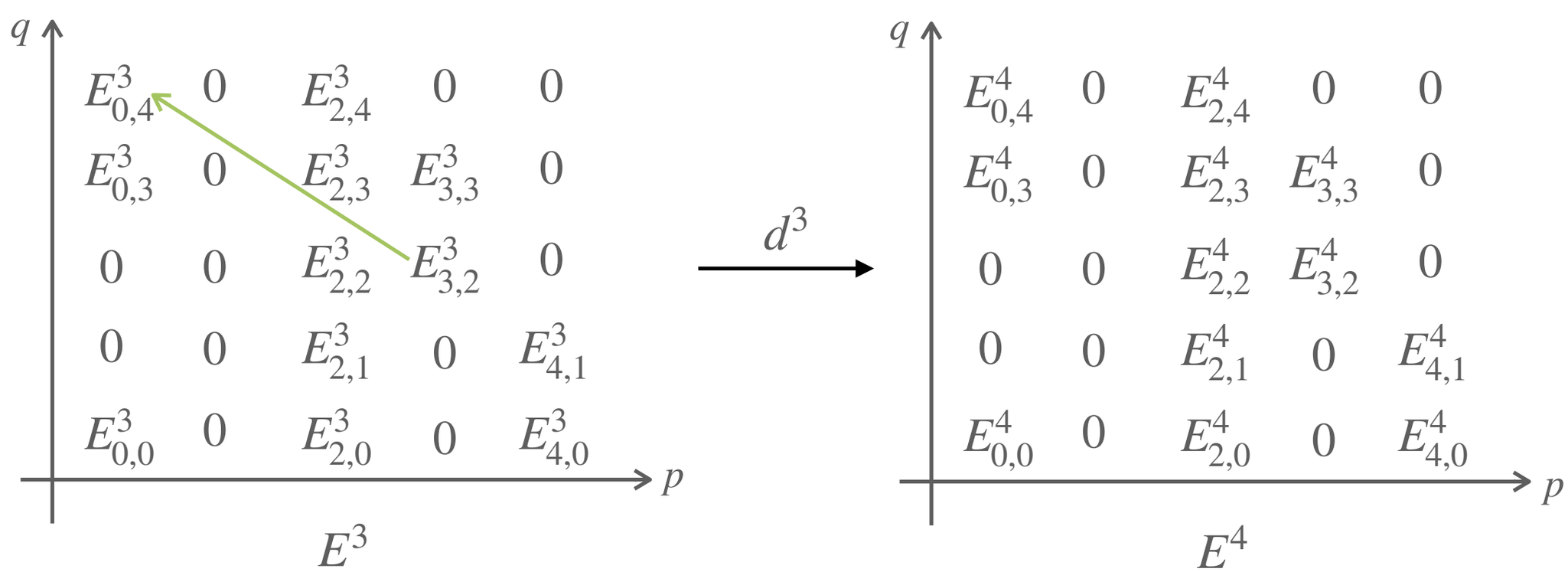

Computational Algorithm and challenges:

Compute second page → Act with d^r, until E^∞-page → Read off $Gr(G_n(E))$

$$
\begin{array}{c|ccccc}
q & & & & & \\
 & 0 & 0 & E^\infty_{2,4} & 0 & 0 \\
 & E^\infty_{0,3} & 0 & E^\infty_{2,3} & E^\infty_{3,3} & 0 \\
 & 0 & 0 & E^\infty_{2,2} & E^\infty_{3,2} & 0 \\
 & 0 & 0 & E^\infty_{2,1} & 0 & 0 \\
 & E^\infty_{0,0} & 0 & E^\infty_{2,0} & 0 & E^\infty_{4,0} \\
\hline
 & & & & & p
\end{array}
$$

E^∞

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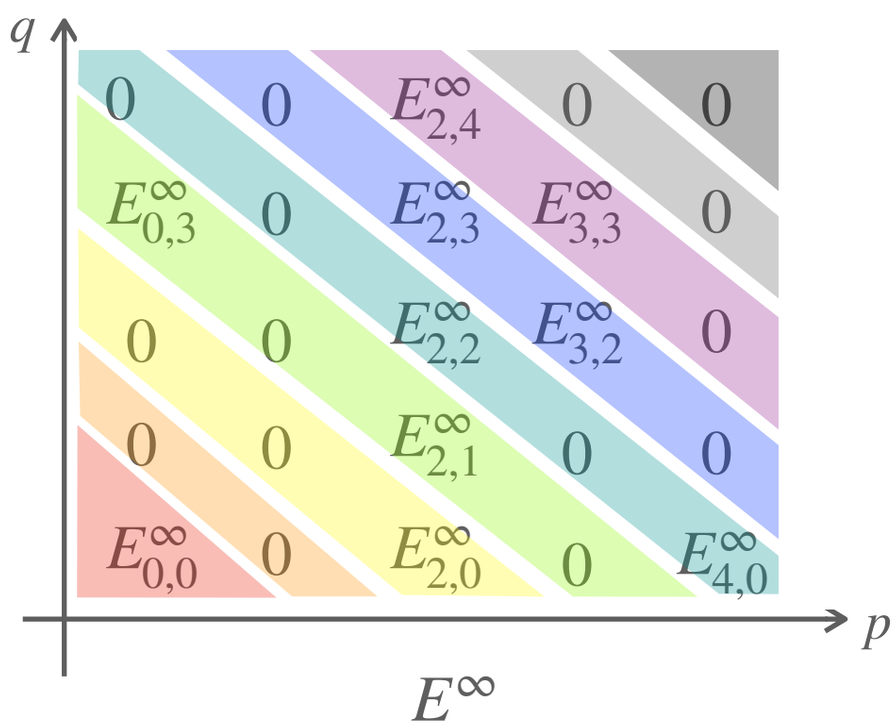

$G_0(E) = E^\infty_{0,0}$

$G_1(E) = 0$

$G_2(E) = E^\infty_{2,0}$

$G_3(E) = e(E^\infty_{0,3}, E^\infty_{2,1})$

$G_4(E) = e(E^\infty_{2,2}, E^\infty_{4,0})$

$G_5(E) = e(E^\infty_{2,3}, E^\infty_{3,2})$

$G_6(E) = e(E^\infty_{2,4}, E^\infty_{3,3})$

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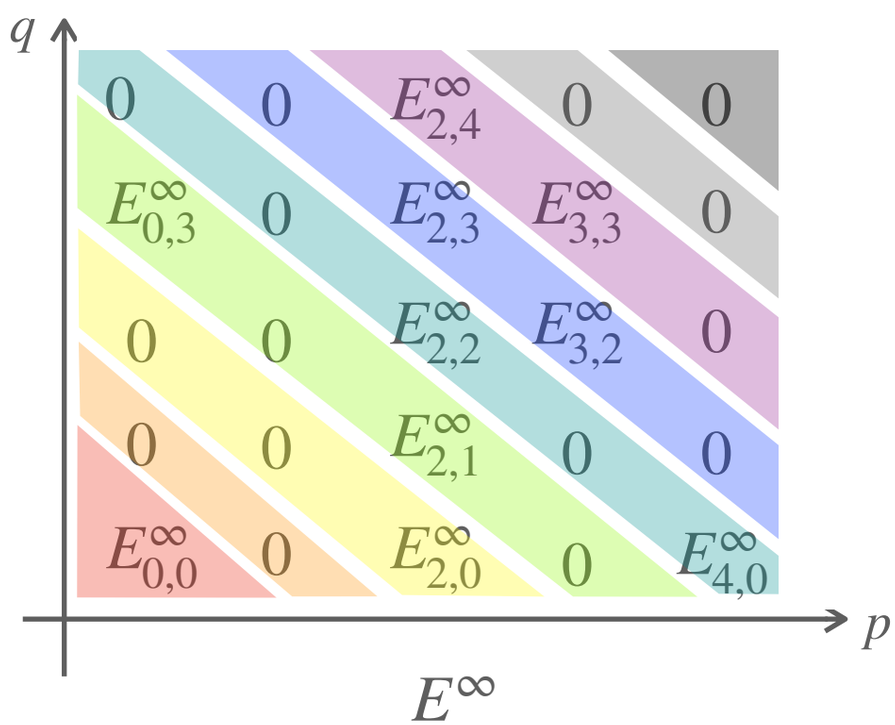

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Common Difficulties: Determination of differentials, Extension problem

There is physical info in both types of mathematical problems!

e.g.[Diaconescu, Moore, Witten ’00, Maldacena, Moore, Seiberg ’01]

K-theory general results

Now need co-homological spectral sequence - some differentials directly computable

We compute for $X = \{S^k, T^k, K3, CY_3\}$, with $k = \dim(X)$, $n \geq 0$:

$$\boxed{K^{-n}(X) = \bigoplus_{m=0}^{k} b_{k-m}(X) K^{-n-m}(pt)\,.}$$

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Physical Interpretation:

Setup: d-dim. theory, compactified on k-dim. X - total space $\mathbb{R}^{1,d-k-1} \times X$

$K^{-n}(X)$: (BPS) D-branes of codimension n in flat space $\mathbb{R}^{1,d-k-1}$

$K^{-n-m}(pt)$: (BPS) D-branes of codimension (n+m) wrapping (k-m)-cycles in X

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Extra info:

No Freed-Witten anomaly in appearing branes

[Diaconescu, Moore, Witten '00, Maldacena, Moore, Seiberg '01]

Compatibility with completeness hypothesis

Cobordism general results:

We compute for $X = \{S^k, T^k, K3, CY_3\}$, with $k = \dim(X)$, $n \geq -k$:

$$\boxed{\Omega^{\mathrm{Spin^c}}_{n+k}(X) = \bigoplus_{m=0}^{k} b_m \Omega^{\mathrm{Spin^c}}_{n+k-m}(pt)}$$

Reminder: Cobordism group of X related to continuous map $f : M \to X$, M (n+k)-dim.

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No obstruction in remaining $(n + k - m)$-directions

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Relation to K-theory:

$n \geq 0 : \Omega^{\mathrm{Spin^c}}_{n+k}(X)$ related to $K_{n+k}(X) \cong K^{-n}(X)$

$-k \leq n < 0 : \Omega^{\mathrm{Spin^c}}_{n+k}(X)$ is unphysical

Example: IIB on Calabi-Yau $CY_3 = X$

$$K^0(X) = b_6 K^0(pt) \oplus b_4 K^{-2}(pt) \oplus b_2 K^{-4}(pt) \oplus b_0 K^{-6}(pt)$$

$$= b_6 \mathbb{Z} \oplus b_4 \mathbb{Z} \oplus b_2 \mathbb{Z} \oplus b_0 \mathbb{Z}$$

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All branes of codimension 0 in $\mathbb{R}^{1,3}$ and wrapping:

entire $CY_3 \rightarrow$ D9-branes b_4 4-cycles $\rightarrow$ D7-branes b_2 2-cycles $\rightarrow$ D5-branes

nothing (point-like)$\rightarrow$ D3-branes

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$$\Omega_6^{\mathrm{Spin^c}}(X) = b_6 \Omega_0^{\mathrm{Spin^c}}(pt) \oplus b_4 \Omega_2^{\mathrm{Spin^c}}(pt) \oplus b_2 \Omega_4^{\mathrm{Spin^c}}(pt) \oplus b_0 \Omega_6^{\mathrm{Spin^c}}(pt)$$

$$= b_6 \mathbb{Z} \oplus b_4 \mathbb{Z} \oplus b_2 (\mathbb{Z} \oplus \mathbb{Z}) \oplus b_0 (\mathbb{Z} \oplus \mathbb{Z})$$

$(b_6+b_4+2b_2+2b_0)$=$(3b_2+3)$-dim. lattice of $\mathbb{Z}$-valued 3-form global charges

Plugging everything together: tadpoles

Tadpole 1: $K^0(pt) = \mathbb{Z}$ and $\Omega_0^{\mathrm{Spin}^c}(pt) = \mathbb{Z}$

9-form symmetry gauged by D9-branes

global 3-form symmetry in 4d
magnetic current: $\tilde{J}_0(M_6) = td(M_0) = 1$

$$N\delta^{(0)}(M_6) + a^{(0)} td_0(M_6) = 0$$

For $a^{(1)} = 0, -32$: Type I/IIB D9-tadpole cancellation condition

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Tadpole 2: $(\times b_4)$ $K^{-2}(pt) = \mathbb{Z}$ and $\Omega_2^{\mathrm{Spin^c}}(pt) = \mathbb{Z}$

7-form symmetry gauged by D7-branes

global 3-form symmetry in 4d:

$$\tilde{J}_2^{(a)}(M_6) = \sum_{a=1}^{b_4} j_0^{(2)a} \wedge \omega_{(2)a}$$

$$\sum_{j \in \mathrm{def}} N_j \delta^{(2)}(\mathbb{R}^{1,3} \times \Sigma_{4,j}) + a^{(2)} c_1(M_6) = 0$$

For $a^{(2)} = -24$ and M_6 base of elliptically fibered CY_4 :
F-theory D7-cancellation condition

Cobordism Conjecture Consequences

$$\Omega_k^{\widetilde{QG}} \neq 0$$

Breaking

$$\Omega_k^{\widetilde{QG}} \to \Omega_k^{\widetilde{QG}+\text{defects}} = 0$$

Gauging

$$0 = [M] \in \Omega^{\widetilde{QG}} \neq 0$$

$$\Omega^{\widetilde{\text{QG}}+\text{g.fields}} \to \Omega^{\widetilde{\text{QG}}}$$

$\Omega_k \neq 0 \leftrightarrow d\tilde{J}_k = 0 : \tilde{J}_k$ magnetic current

Breaking

$$d\tilde{J}_k = \delta^{(k+1)}(\Sigma) \neq 0$$

Part II: Dynamical tadpoles

Gauging

$$\tilde{J}_k = dF_{k-1}$$

Part I: R-R tadpoles

Tadpoles & Dynamical Cobordism

[Fischler, Susskind '86]
[Sugimoto '99]
[Antoniadis, Dudas, Sagnotti '99]
[Angelantonj '99]
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recently : [Raucci '22]
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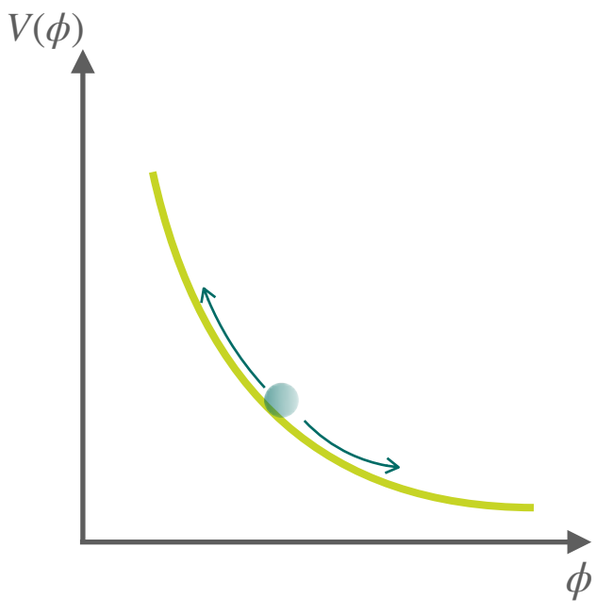

Dynamical tadpoles (vs RR tadpoles)

Naturally occurring in supersymmetry-breaking potentials

Indicate lack of maximally-symmetric vacuum

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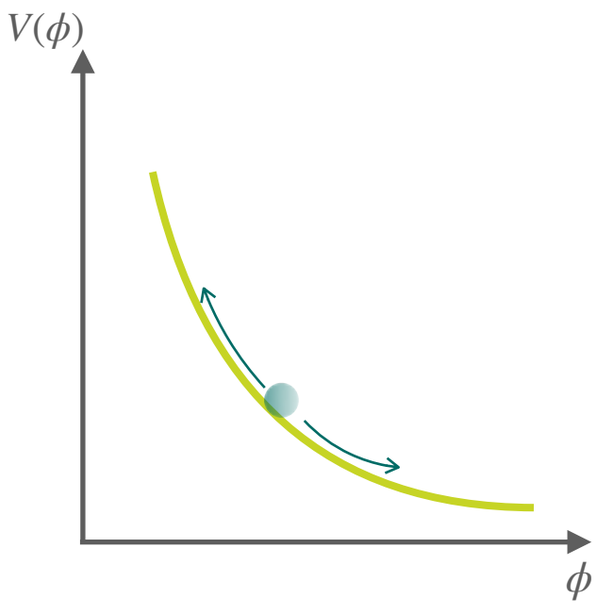

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Example: Sugimoto Model (USp(N) Type I with N $\overline{D8}$ and N $O9^+$) [Sugimoto '99]

Action:

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left(\mathscr{R} - \frac{1}{2}(\partial\Phi)^2\right) - T_9\int d^{10}x\left((N+32)\sqrt{-G}e^{\frac{3}{2}\Phi} - (N-32)A_{10}\right) + \dots .$$

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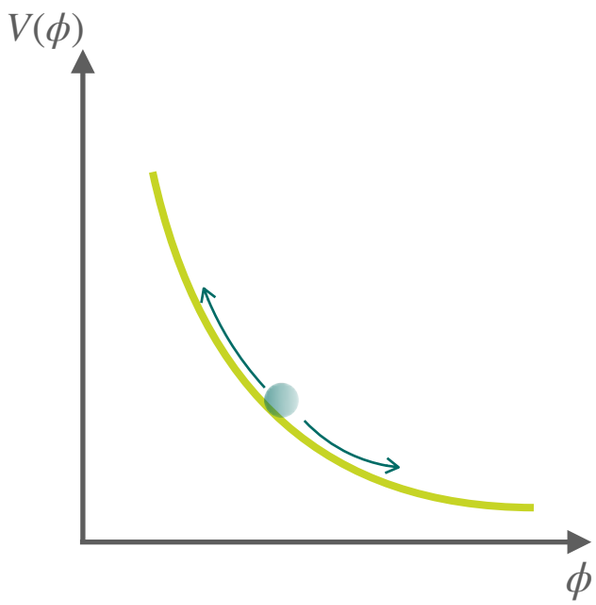

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Solution preserving 9d Poincaré invariance:

[Dudas, Mourad '00]

$$ds_E^2 = |\sqrt{\alpha_E}|^{1/9} e^{-\frac{\alpha_E y^2}{8}}\eta_{\mu\nu}dx^\mu dx^\nu + |\sqrt{\alpha_E}y|^{-1} e^{-\frac{3\phi_0}{2}} e^{-\frac{9\alpha_E}{8}y^2} dy^2, \quad \alpha_E = 64k^2T_9$$

$\rightarrow$ singularities at finite spacetime distance, spontaneous compactification to 9d

Tadpoles & Dynamical Cobordism

Tadpoles & Dynamical Cobordism

[Buratti, Delgado, Uranga '21]
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Reinterpretation in Cobordism Terms

Solution extends in finite spacetime distance Δ, with $\Delta \sim \mathscr{T}^{-n}$

Tadpole

Mechanism: "apparent singularity" = cobordism defect.

For field distance $D \to \infty$ at singularity: Wall of Nothing/End-of-the-world brane

Cobordism distance conjecture: $\Delta \sim e^{-\frac{1}{2}\delta D}, \quad |\mathscr{R}| \sim e^{\delta D}$

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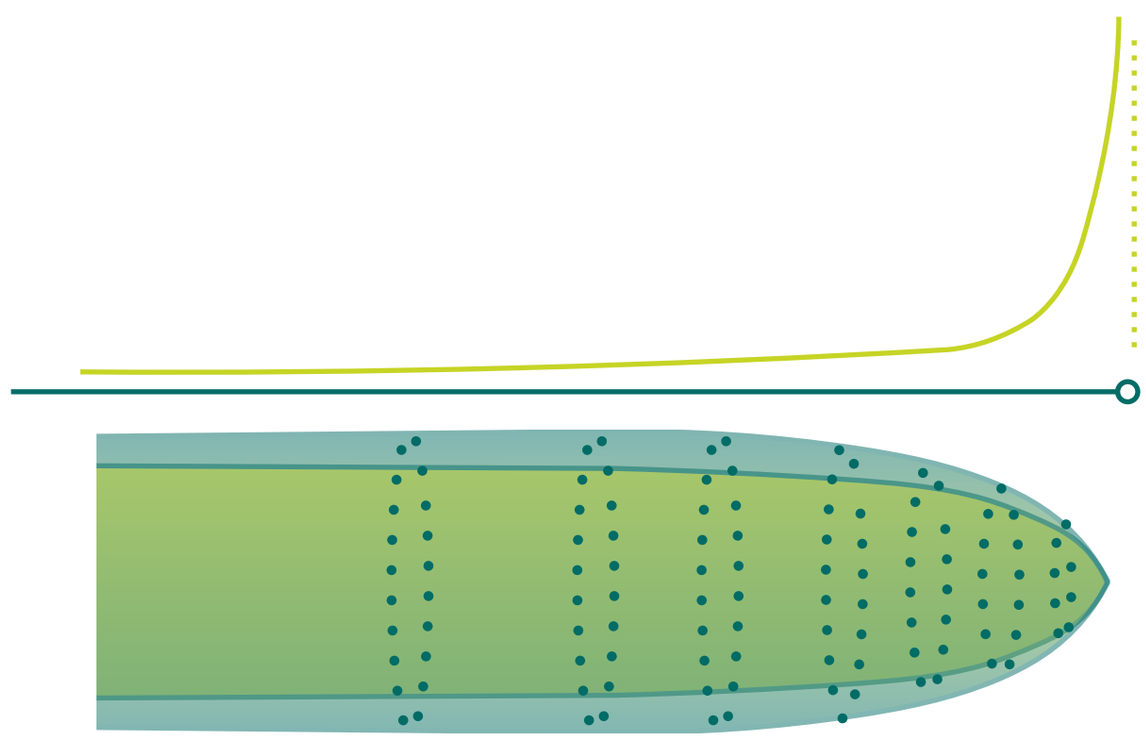

9-dimensional Domain Wall

[Blumenhagen, Cribiori, Kneissl, AM '22]

Goal: Backreaction of gauge neutral, non-supersymmetric 9-dimensional object w/ brane-like dilaton coupling

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Physical realisation: non-BPS $\widehat{D8}$ -brane, non-SUSY stack of $16 \times \overline{D8} + O8^{++}$

Action: $$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left(\mathscr{R} - \frac{1}{2} (\partial \Phi)^2 \right) - T \int d^{10} \sqrt{-g} e^{\frac{5}{4}\Phi} \delta(r) \, .$$

transverse direction

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transverse direction

Solution Ansatz: $$ds^2 = e^{2\mathscr{A}(r,y)}ds_8^2 + e^{2\mathscr{B}(r,y)}(dr^2 + dy^2)\,.$$

$$\mathscr{A} = A(r) + U(y) \qquad \mathscr{B} = B(r) + V(y) \qquad \Phi = \chi(r) + \psi(y)$$

Solutions $I, II^{\pm}$: Both periodic in r - trigonometric dependence [Blumenhagen, Font '00]

$\rightarrow$ spontaneous compactification on S^1

Qualitative behaviour

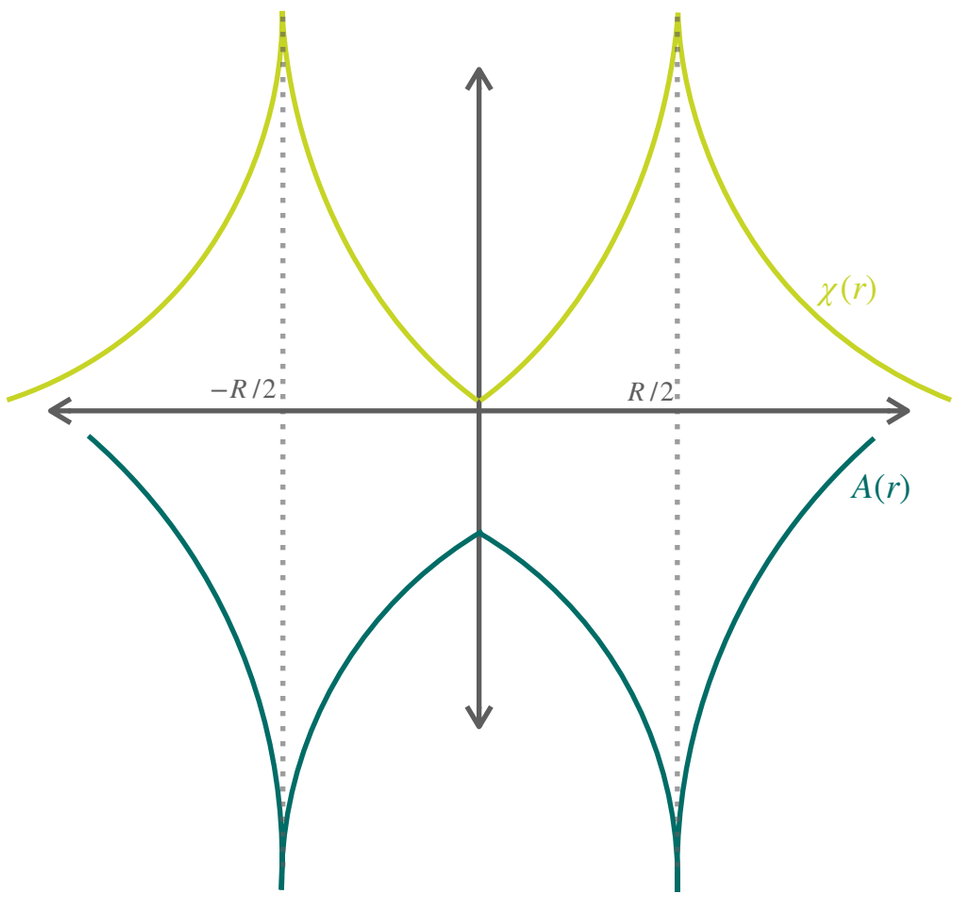
[Blumenhagen, Cribiori, Kneissl, AM '22]

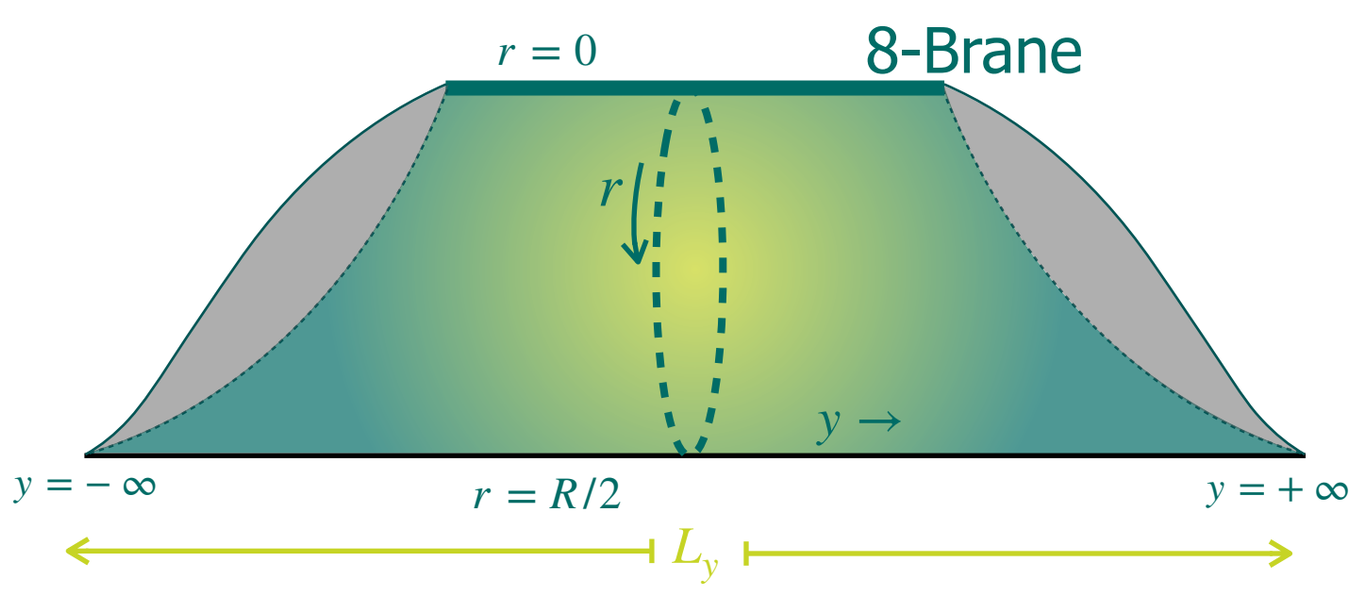
r-direction spontaneously compactified on S^1 with radius R: $e^{\frac{5}{4}\phi_0} \sim \frac{1}{\lambda R}$

$\uparrow$ $\kappa_{10}^2 T$

Logarithmic singularities at $r = \pm \frac{R}{2}$, string coupling diverges

y-direction: infinite length in sols I, II^-, becomes finite interval in sol II^+





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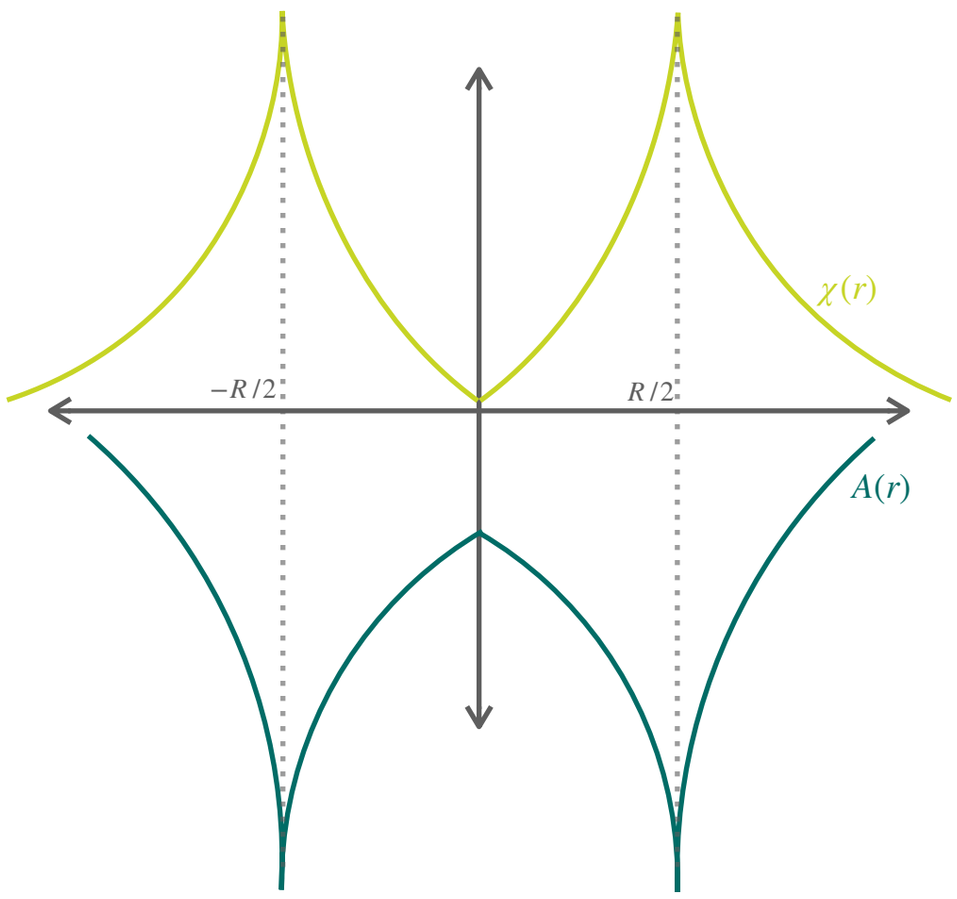

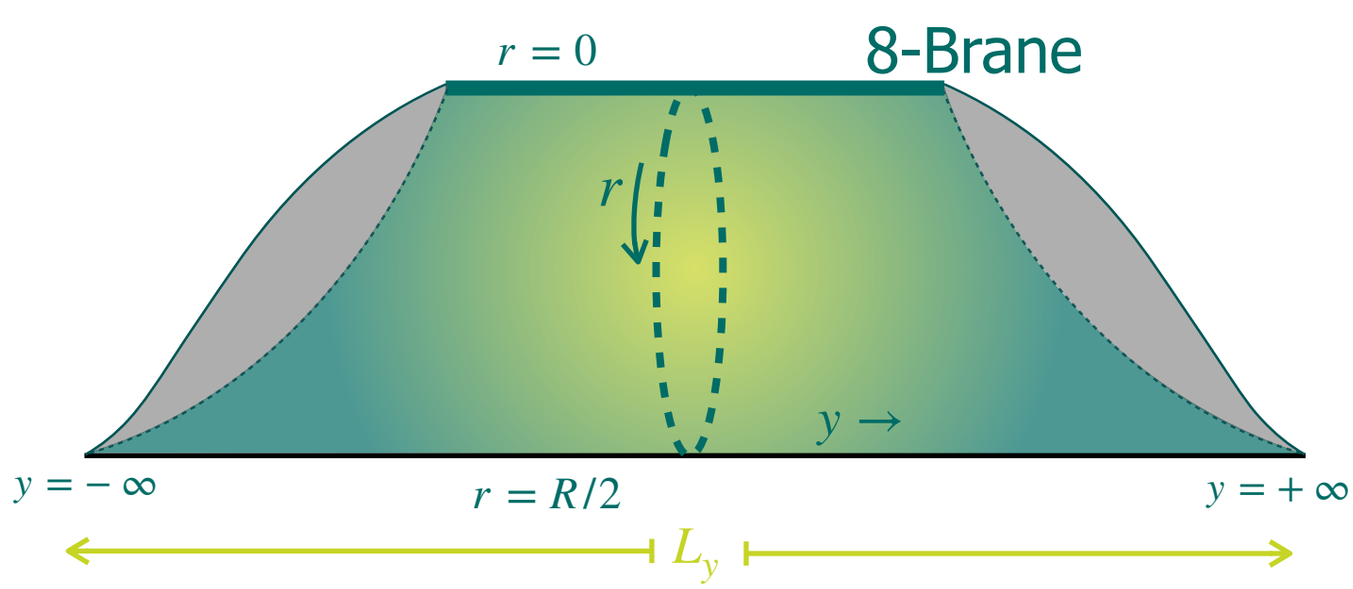

$$\Delta \sim L_y \sim \mathcal{T}^{-1}$$

$$\Delta \sim e^{-\sqrt{2}D}$$

$$|\mathcal{R}| \sim e^{2\sqrt{2}D}$$

ETW Defects

[Blumenhagen, Cribiori, Kneissl, AM '22]

Input: 8-dimensional defect : log-singularity, S^1 direction capped off

Poincaré symmetry along the "brane" preserved

2d transversal rotational symmetry broken

Non-Isotropic Solution Ansatz: $$ds^2 = e^{2\hat{\mathscr{A}}(\rho,\phi)} ds_8^2 + e^{2\hat{\mathscr{B}}(\rho,\phi)} (d\rho^2 + \rho^2 d\phi^2)\,.$$

Separation of variables

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Separation of variables

Solutions: $$\hat{A}(\rho) = \frac{1}{8}\log\left|\cosh\left(8\hat{K}\log(\frac{\rho}{\rho_0})\right)\right| \qquad \hat{B}(\rho) = -\log\left(\frac{\rho}{\rho_0}\right) + \left(\frac{\hat{\alpha}^2}{32} - \frac{7}{2}\right)\hat{A}(\rho)$$

$$\hat{\chi}(\rho) = \hat{\alpha}\hat{A}(\rho)$$

$$\vdots$$

$$\hat{\psi}(\phi) = \frac{\hat{\alpha}}{8}\log\left|\cos(8\hat{K}\phi)\right| \pm 2\log\left|\tan(4\hat{K}\phi + \frac{\pi}{4})\right|$$

$ETW\ 7^{\pm}$ solutions

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Separation of variables

Solutions $ETW7^{\pm}$:

Logarithmic singularities at $\rho = 0$, string coupling diverges

For appropriate parameter selection same scaling as 9d defect

Dynamical Cobordism scaling satisfied: $\Delta \sim e^{-\sqrt{2}D}$, $|\mathscr{R}| \sim e^{2\sqrt{2}D}$

ETW Defects

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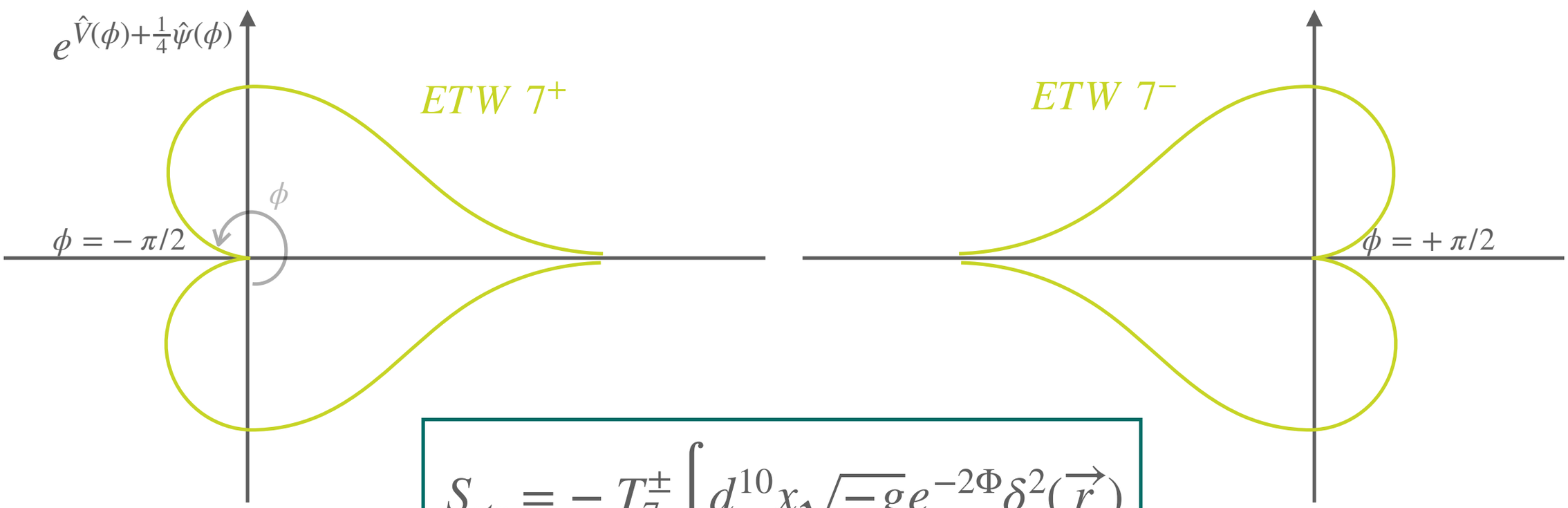

$$S_{str} = -T_7^{\pm} \int d^{10}x \sqrt{-g}\, e^{-2\Phi} \delta^2(\vec{r})$$

with $\kappa_{10}^2 T_7 = 2\pi$

Cobordism Interpretation

[Blumenhagen, Cribiori, Kneissl, AM '22]

Backreaction of 9d Domain Wall

↓

Spontaneous compactification on S^1

↓

$\boxed{\text{Cobordism Group: } \Omega_1^{\xi} \neq 0}$

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Solution II^+

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↓

Solutions $ETW\ 7^{\pm}$

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Spacetime capped off

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$\Omega^{\xi + ETW7^{\pm}} = 0$

Solutions I, II^-

Infinite L_y, defects not in EFT, gauging

↓

non-BPS $\widehat{D8}$ brane

↓

$\Omega_1^{\text{Spin}} \to KO^{-1}(pt)$

$\mathbb{Z}_2$ charge cancellation

↓

Single $\widehat{D8}$ inconsistent

Summary and Outlook

The Cobordism Conjecture holds more generally: $\Omega_n^{\mathrm{QG}}(X) = 0$.

Cobordism and K-theory charges combine, giving geometrical and localised contributions respectively to tadpole cancellation conditions.

The Cobordism Conjecture is consistent with dimensional reduction.

Tadpole conditions can be systematically constructed from bottom up.

Dynamical cobordism successfully predicts new defects and gives physical meaning to known singularities in known solutions.

Eom for defect solved → new 7-brane defect, predicted by cobordism breaking.

generalization recently explicitly shown in [Blumenhagen, Kneissl, Wang ’23]

Summary and Outlook

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Independent verification of cobordism-predicted defects

Clarification of cases with torsion

Extension to more complicated cobordism groups

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Thank you!