Università degli Studi di Milano

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# Electroweak physics at present and future colliders


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# Outline of the talk

- The relevance of precision tests of the Standard Model of electroweak and strong interactions

- The processes and the observables relevant for the determination of the electroweak parameters

- The theoretical and computational challenges to extract in a significant way information from the data

- Exploit the dependence on the energy of our observables in order to extract information sensitive to new physics effects

- Revisit the analysis strategies adopted at LEP in view of the much higher FCC precision level relying on full template fit approach

- Prepare the technology to compute N3LO-EW corrections to fermion pair production

# Introductory remarks

- There are big unanswered questions like dark matter, dark energy, matter-antimatter asymmetry;
  if the answer can be formulated according to a "particle paradigm", then we can search for such particles;
  direct searches so far unsuccessful, we can formulate precision indirect tests and look for any BSM physics signs

- A model (e.g. the SM) can be tested by checking how well it describes physical observables (i.e. xsecs and asymmetries)
  To this goal, we need the best predictions for the differential distributions, in order to make more significant the comparison

- Since every model has its own specific predictions (e.g. masses and couplings), we can test it at this level →
  we must devise a procedure to extract such parameters (pseudo-observables) from the data and
  then compare with the corresponding theoretical predictions

- The possibility to parameterise our ignorance about BSM physics in the SMEFT language implies that we clarify
  how we test this model and how we determine fundamental parameters in this model

- The search for BSM signals benefits of a very precise understanding of the energy dependence of the observables
  One single deviation from the SM is not conclusive evidence of New Physics. (e.g. the CDF result for $m_W$ ) ;
  a systematic pattern of deviations from the SM, at different energies, would be a more significant signal

# Motivations

## from the Fermi theory to the current best predictions of MW and $\sin^2\theta$ and further

# From the Fermi theory of weak interactions to the discovery of W and Z

Fermi theory of β decay

muon decay $\mu^- \to \nu_\mu e^- \bar{\nu}_e$ $\qquad \frac{1}{\tau_\mu} \to \Gamma_\mu \to G_\mu$

QED corrections to $\Gamma_\mu$ necessary for precise determination of $G_\mu$
computable in the Fermi theory (Kinoshita, Sirlin, 1959)

The independence of the QED corrections of the underlying model (Fermi theory vs SM) allows

- to define $G_\mu$ and to measure its value with high precision

$$G_\mu = 1.1663787(6) \; 10^{-5} \; \text{GeV}^{-2}$$

- to establish a relation between $G_\mu$ and the SM parameters

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8m_W^2}(1+\Delta r)$$

The properties of physics at the EW scale
with sensitivity to the full SM and possibly to BSM via virtual corrections ( $\Delta r$ )
are related to a very well measured low-energy constant

# From the Fermi theory of weak interactions to the discovery of W and Z

The SM predicts the existence of a new neutral current, different than the electromagnetic one
(Glashow 1961, Weinberg 1967, Salam 1968)

The observation of weak neutral current immediately allowed the estimate of the value of the weak mixing angle in the correct range
GARGAMELLE, Phys.Lett. 46B (1973) 138-140

From the basic relation among the EW parameters it was immediately possible to estimate the order of magnitude of the mass of the weak bosons, in the 80 GeV range
(Antonelli, Maiani, 1981)

The discovery at the CERN SPPS of the W and Z bosons and the first determination of their masses allowed the planning of a new phase of precision studies accomplished with the construction of two $e^+e^-$ colliders (SLC and LEP) running at the Z resonance

The precise determination of MZ and of the couplings of the Z boson to fermions
and in particular the value of the effective weak mixing angle
allowed to establish a framework for a test of the SM at the level of its quantum corrections

There is evidence of EW corrections beyond QED with 26 $\sigma$ significance!
Full 1-loop and leading 2-loop radiative corrections are needed to describe the data
(indirect evidence of bosonic quantum effects)

# The renormalisation of the SM and a framework for precision tests

- The Standard Model is a renormalizable gauge theory based on $SU(3) \times SU(2)_L \times U(1)_Y$
- The gauge sector of the SM lagrangian is assigned specifying $(g, g', v, \lambda)$ in terms of 4 measurable inputs
- More observables can be computed and expressed in terms of the input parameters, including the available radiative corrections, at any order in perturbation theory
- The validity of the SM can be tested comparing these predictions with the corresponding experimental results

- The input choice $(g, g', v, \lambda) \leftrightarrow (\alpha, G_\mu, m_Z, m_H)$ minimises the parametric uncertainty of the predictions

$$
\begin{aligned}
\alpha(0) &= 1/137.035999139(31) \\
G_\mu &= 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-2} \\
m_Z &= 91.1876(21)\ \mathrm{GeV}/c^2 \\
m_H &= 125.09(24)\ \mathrm{GeV}/c^2
\end{aligned}
$$

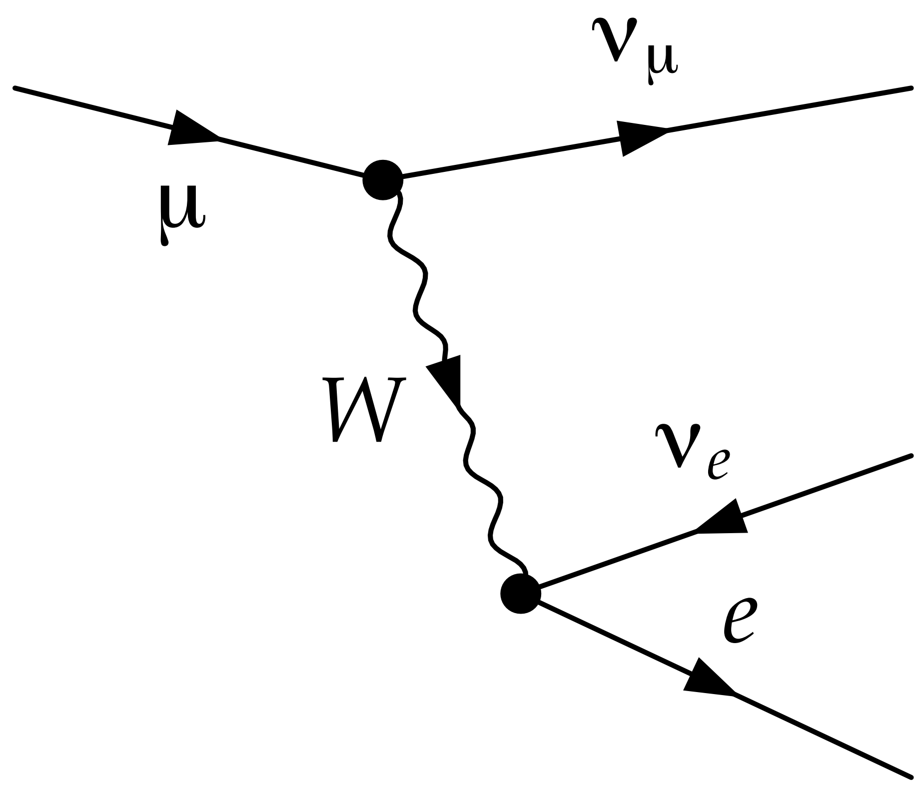
- with these inputs, $m_W$ and the weak mixing angle are predictions of the SM, to be tested against the experimental data

# The W boson mass: theoretical prediction

$$\mathcal{L}_{SM} = \mathcal{L}_{SM}(\alpha,\, G_\mu,\, m_Z; m_H; m_f; CKM)$$

$\rightarrow$ we can compute $m_W$

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8 m_W^2}\,(1+\Delta r)$$

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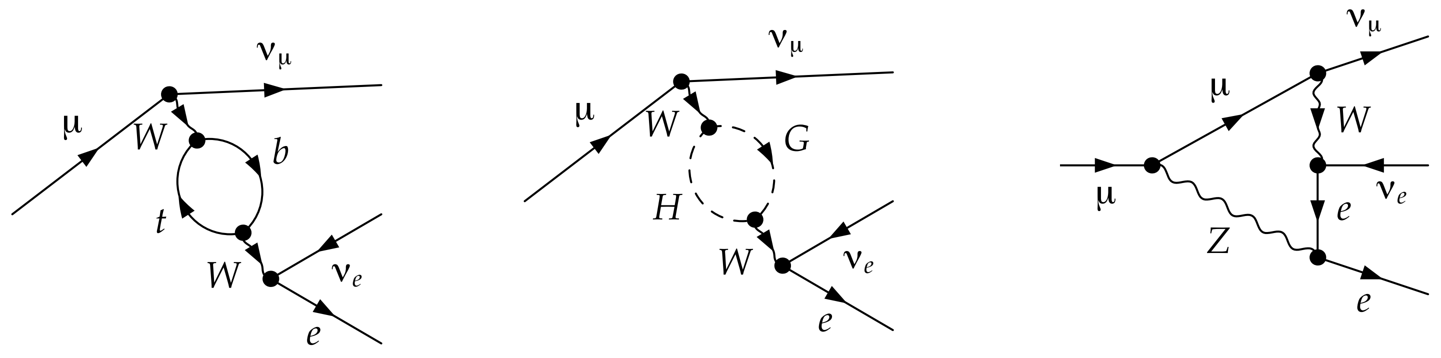



+ ....

$$\Delta r = \Delta r(\alpha, G_\mu, m_Z, m_H; m_f; CKM)$$

$$m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-\frac{4\pi\alpha}{G_\mu\sqrt{2}m_Z^2}(1+\Delta r)}\right)$$

# The W boson mass: theoretical prediction

on-shell scheme: dominant contributions to $\Delta r$

$$\Delta r = \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{\rm rem}$$

$$\Delta\alpha = \Pi^{\gamma}_{\rm ferm}(M_Z^2) - \Pi^{\gamma}_{\rm ferm}(0) \quad \rightarrow \quad \alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha}$$

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2} = 3\frac{G_F m_t^2}{8\pi^2\sqrt{2}} \quad \textit{[one-loop]} \quad \sim \frac{m_t^2}{v^2} \sim \alpha_t$$

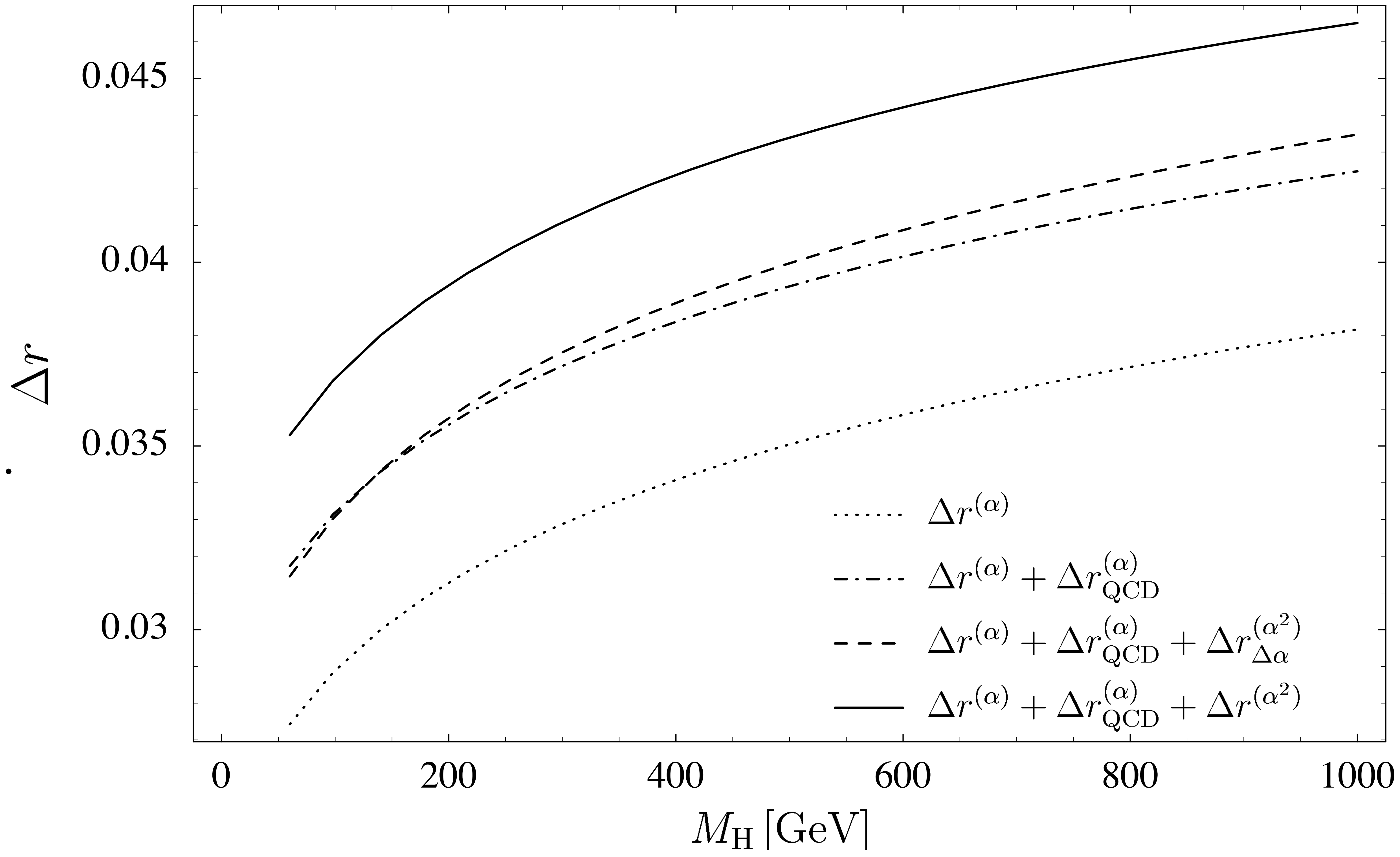
beyond one-loop order: $\sim \alpha^2,\ \alpha\alpha_t,\ \alpha_t^2,\ \alpha^2\alpha_t,\ \alpha\alpha_t^2,\ \alpha_t^3, \ldots$

reducible higher order terms from $\Delta\alpha$ and $\Delta\rho$ via

$$1 + \Delta r \ \rightarrow \ \frac{1}{(1-\Delta\alpha)(1+\frac{c_w^2}{s_w^2}\Delta\rho) + \cdots}$$

$$\rho = 1 + \Delta\rho \ \rightarrow \ \frac{1}{1-\Delta\rho}$$

effects of higher-order terms on $\Delta r$




*Consoli, WH, Jegerlehner 1989*

# The W boson mass: theoretical prediction

Sirlin, 1980, 1984; Marciano, Sirlin, 1980, 1981;
van der Bij, Veltman, 1984; Barbieri, Ciafaloni, Strumia 1993;
Djouadi, Verzegnassi 1987; Consoli, Hollik, Jegerlehner, 1989;
Chetyrkin, Kühn, Steinhauser, 1995;
Barbieri, Beccaria, Ciafaloni, Curci, Viceré, 1992, 1993; Fleischer, Tarasov, Jegerlehner, 1993;
Degrassi, Gambino, AV, 1996; Degrassi, Gambino, Sirlin, 1997;
Freitas, Hollik, Walter, Weiglein, 2000, 2003;
Awramik, Czakon, 2002; Awramik, Czakon, Onishchenko, Veretin, 2003; Onishchenko, Veretin, 2003

The best available prediction includes
the full 2-loop EW result, leading higher-order EW and QCD corrections,
resummation of reducible terms
Missing 3-loop and 4-loop terms needed to reduce the uncertainties.

$$m_W = w_0 + w_1 dH + w_2 dH^2 + w_3 dh + w_4 dt + w_5 dH dt + w_6 da_s + w_7 da^{(5)}$$

$$dt = [(M_t/173.34\,\text{GeV})^2 - 1]$$

$$da^{(5)} = [\Delta\alpha^{(5)}_{\text{had}}(m_Z^2)/0.02750 - 1]$$

$$dH = \ln\left(\frac{m_H}{125.15\,\text{GeV}}\right)$$

$$dh = [(m_H/125.15\ \text{GeV})^2 - 1].$$

$$da_s = \left(\frac{\alpha_s(m_Z)}{0.1184} - 1\right)$$

| | $124.42 \le m_H \le 125.87$ GeV | $50 \le m_H \le 450$ GeV |
|---|---|---|
| $w_0$ | 80.35712 | 80.35714 |
| $w_1$ | -0.06017 | -0.06094 |
| $w_2$ | 0.0 | -0.00971 |
| $w_3$ | 0.0 | 0.00028 |
| $w_4$ | 0.52749 | 0.52655 |
| $w_5$ | -0.00613 | -0.00646 |
| $w_6$ | -0.08178 | -0.08199 |
| $w_7$ | -0.50530 | -0.50259 |

on-shell scheme $m_W^{os} = 80.353 \pm 0.004$ GeV (Freitas, Hollik, Walter, Weiglein)
MSbar scheme. $m_W^{\overline{MS}} = 80.351 \pm 0.003$ GeV (Degrassi, Gambino, Giardino)

parametric uncertainties $\delta m_W^{par} = \pm 0.005$ GeV due to the $(\alpha, G_\mu, m_Z, m_H, m_t)$ values

# The weak mixing angle(s): theoretical prediction(s) at $q^2 = m_Z^2$

- the prediction of the weak mixing angle can be computed in different renormalisation schemes differing for the systematic inclusion of large higher-order corrections
- on-shell definition: $$\sin^2\theta_{OS} = 1 - \frac{m_W^2}{m_Z^2}$$ definition valid to all orders

  Sirlin, 1980
- MSbar definition: $$\frac{G_\mu}{\sqrt{2}} = \frac{g_0^2}{8m_{W,0}^2} \longrightarrow \hat{s}^2\hat{c}^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu m_Z^2(1-\Delta\hat{r})} \qquad \hat{s}^2 \equiv \sin^2\hat{\theta}(\mu_R = m_Z)$$

  Marciano, Sirlin, 1980; Degrassi, Sirlin, 1991

  weak dependence on top-quark corrections

- the effective leptonic weak mixing angle enters in the definition of the effective Z-f-fbar vertex at the Z resonance ( $q^2 = m_Z^2$ )

$$\mathcal{M}^{eff}_{Zl^+l^-} = \bar{u}_l\gamma_\alpha\left[\mathcal{G}_v^f(m_Z^2) - \mathcal{G}_a^f(m_Z^2)\gamma_5\right] v_l\,\varepsilon_Z^\alpha \qquad 4\,|Q_f|\sin^2\theta^f_{eff} = 1 - \frac{\mathcal{G}_v^f}{\mathcal{G}_a^f}$$

and can be computed in the SM (or in other models) in different renormalisation schemes using $(\alpha_0, G_\mu, m_Z)$ as input parameters of the calculation

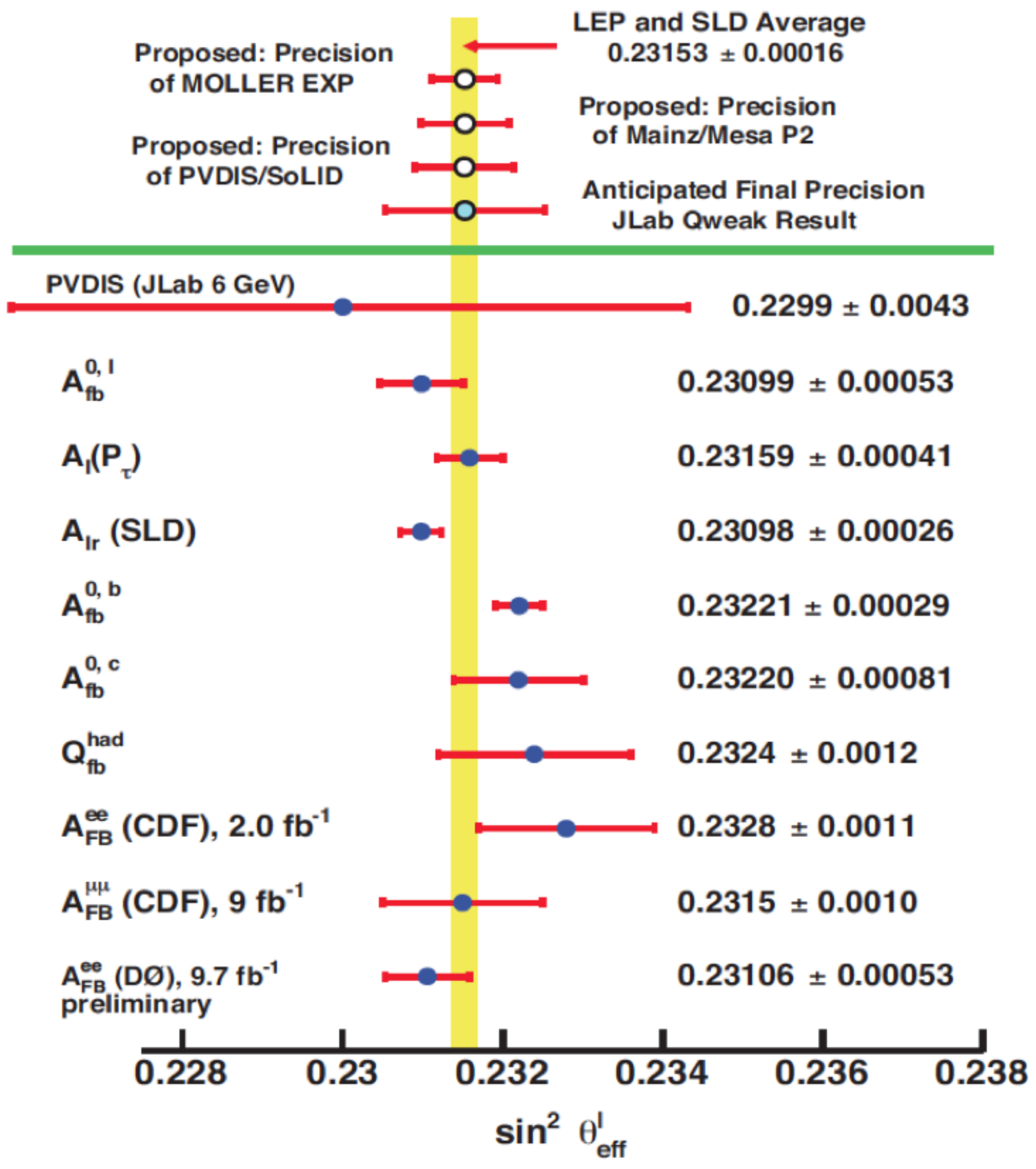
$$\sin^2\theta^{lep}_{eff} = \kappa(m_Z^2)\sin^2\theta_{OS} = \hat{\kappa}(m_Z^2)\sin^2\hat{\theta}$$

it is crucial to verify at which energy scale the predictions are defined

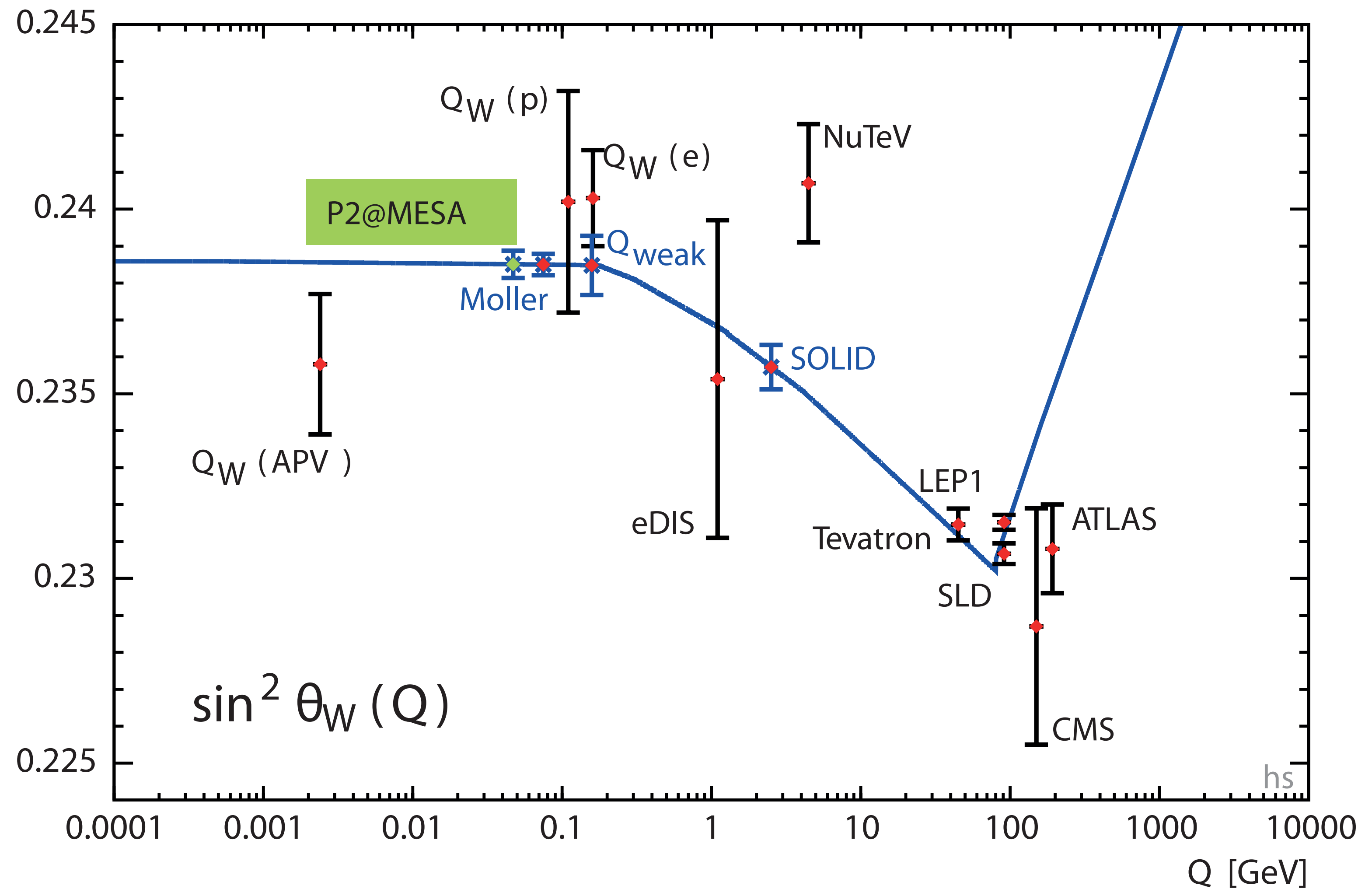
# Comparison of different weak mixing angle determinations

The sensible comparison of different determinations of $\sin^2\theta_W$ offers a test of the SM

- the values extracted at e+e- and hadron colliders are based on observables with different systematics but also use different definitions to fit the data
- for a meaningful test, it is important to compare the same weak mixing angle (different definitions appear when discussing the quantum corrections)




LEP/SLD longstanding discrepancies might be clarified



# The effective leptonic weak mixing angle: theoretical prediction

- parameterization of the full two-loop EW calculation + different sets of 3- and 4-loop corrections

I.Dubovyk, A.Freitas, J.Gluza, T.Riemann, J.Usovitsch, arXiv:1906.08815

$$\sin^2\theta^f_{\rm eff} = s_0 + d_1 L_H + d_2 L_H^2 + d_3 L_H^4 + d_4\Delta_\alpha + d_5\Delta_{\rm t} + d_6\Delta_{\rm t}^2 + d_7\Delta_{\rm t} L_H$$
$$+ d_8\Delta_{\alpha_{\rm s}} + d_9\Delta_{\alpha_{\rm s}}\Delta_{\rm t} + d_{10}\Delta_{\rm Z}$$

$$L_{\rm H} = \log\frac{M_{\rm H}}{125.7\,{\rm GeV}}, \qquad \Delta_{\rm t} = \left(\frac{m_{\rm t}}{173.2\,{\rm GeV}}\right)^2 - 1,$$
$$\Delta_{\alpha_{\rm s}} = \frac{\alpha_{\rm s}(M_{\rm Z})}{0.1184} - 1, \qquad \Delta_\alpha = \frac{\Delta\alpha}{0.059} - 1, \qquad \Delta_{\rm Z} = \frac{M_{\rm Z}}{91.1876\,{\rm GeV}} - 1$$

| Observable | $s_0$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $d_5$ |
|---|---|---|---|---|---|---|
| $\sin^2\theta^\ell_{\rm eff}\times 10^4$ | 2314.64 | 4.616 | 0.539 | −0.0737 | 206 | −25.71 |
| $\sin^2\theta^b_{\rm eff}\times 10^4$ | 2327.04 | 4.638 | 0.558 | −0.0700 | 207 | −9.554 |

| Observable | $d_6$ | $d_7$ | $d_8$ | $d_9$ | $d_{10}$ | max. dev. |
|---|---|---|---|---|---|---|
| $\sin^2\theta^\ell_{\rm eff}\times 10^4$ | 4.00 | 0.288 | 3.88 | −6.49 | −6560 | $< 0.056$ |
| $\sin^2\theta^b_{\rm eff}\times 10^4$ | 3.83 | 0.179 | 2.41 | −8.24 | −6630 | $< 0.025$ |

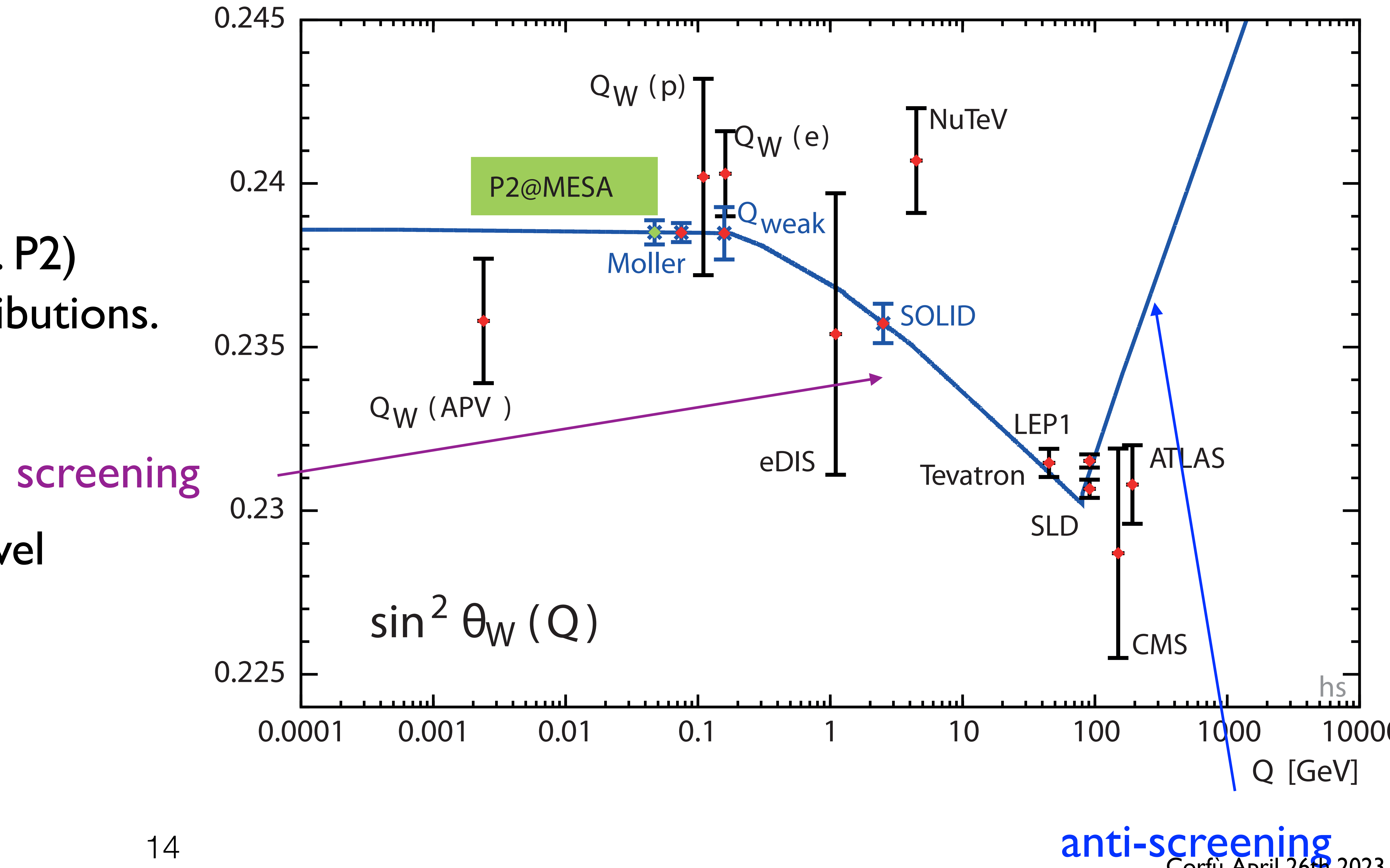
# The running of $\sin^2\hat{\theta}(\mu_R)$ at different mass scales
Erler,Ramsey-Musolf, hep-ph/0409169

The running of the MSbar parameter depends
on the particles active in the theory at a given scale $\mu^2$ and the sign of the associated beta function coefficient

$$\sin^2\theta_W(\mu)_{\overline{\rm MS}} = \frac{\alpha(\mu)_{\overline{\rm MS}}}{\alpha(\mu_0)_{\overline{\rm MS}}}\sin^2\theta_W(\mu_0)_{\overline{\rm MS}} + \lambda_1\left[1-\frac{\alpha(\mu)}{\alpha(\mu_0)}\right] + \frac{\alpha(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2}+\frac{3\lambda_3}{4}\ln\frac{\alpha(\mu)_{\overline{\rm MS}}}{\alpha(\mu_0)_{\overline{\rm MS}}}+\tilde{\sigma}(\mu_0)-\tilde{\sigma}(\mu)\right].$$

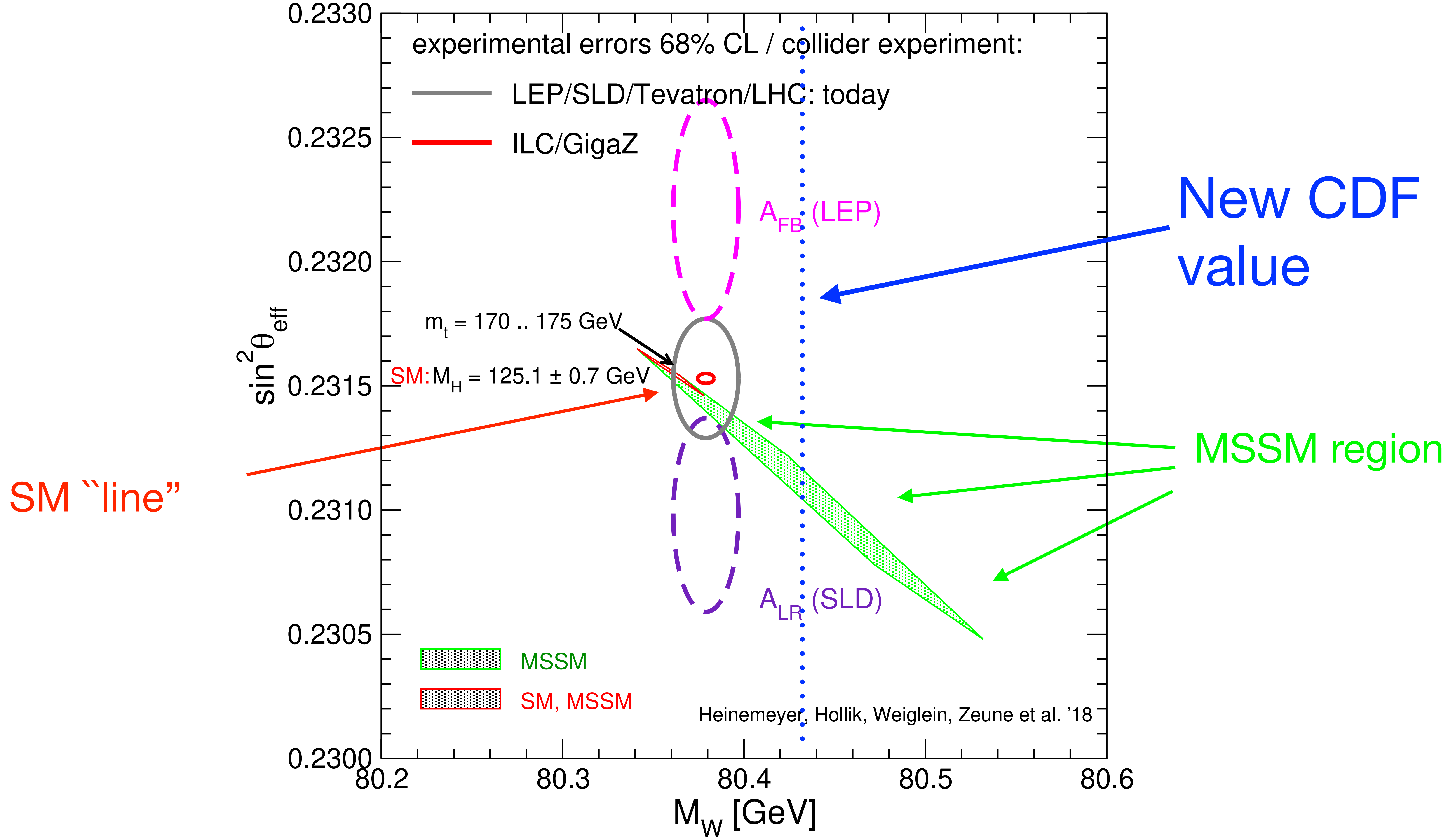
The large lever arm (3 orders of magnitude) and
the high precision of some low-energy experiments (e.g. P2)
might possibly emphasise the presence of non-SM contributions.

Alternatively ,
significant compatibility with the SM prediction
would be a striking success of the SM at the quantum level



# Relevance of a simultaneous study of $m_W$ and of the weak mixing angle

*[S. Heinemeyer, W. Hollik, G. W., L. Zeune '18]*

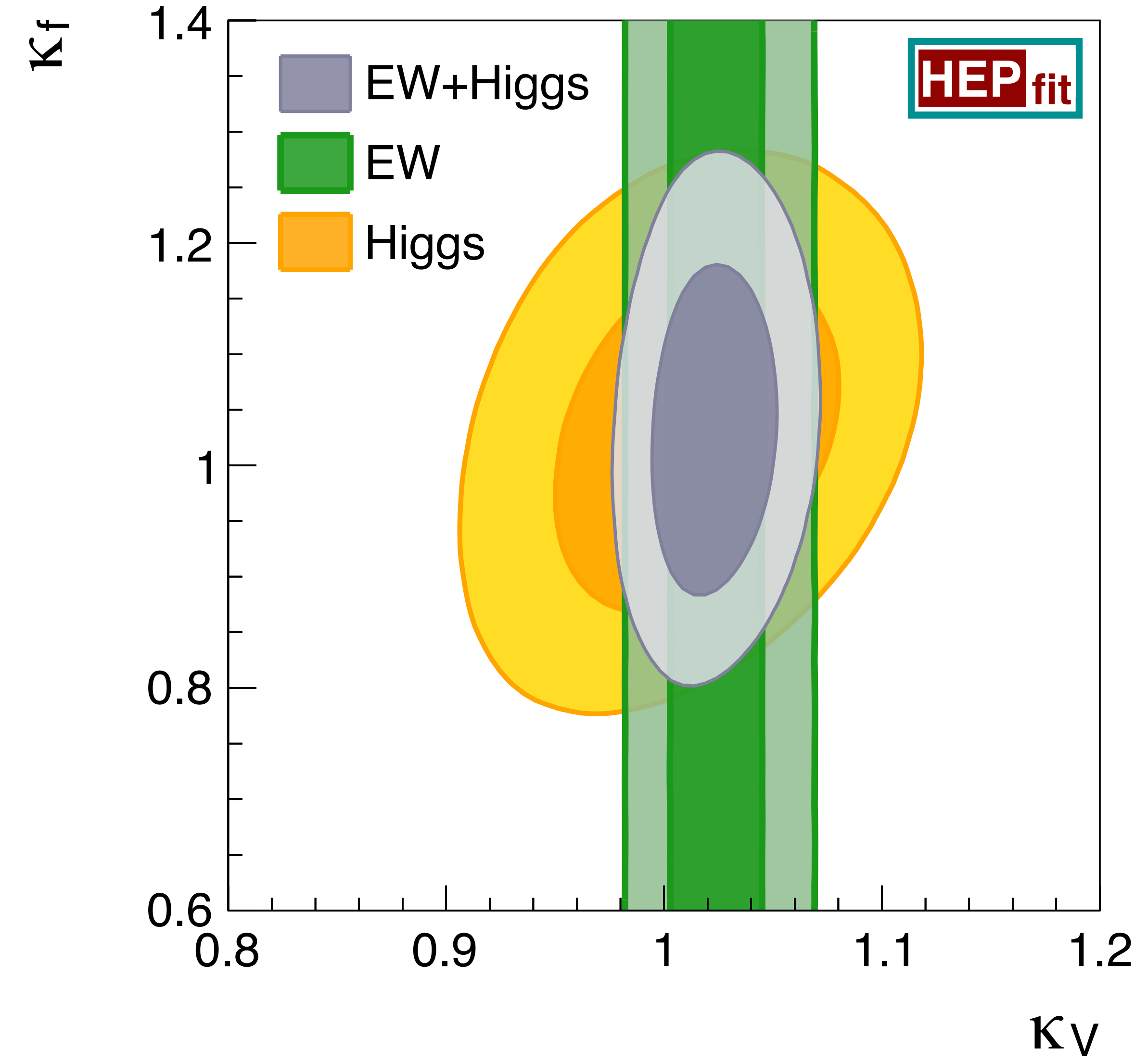



sensitivity to different sets of oblique corrections, i.e. to different combinations of gauge boson self-energies

independent determinations of these two parameters crucial for testing different New Physics alternatives

# Relevance of new high-precision measurement of EW parameters

de Blas et al, arXiv:1608.01509




$$\mathcal{L}_{\rm Eff} = \sum_{d=4}^{\infty} \frac{1}{\Lambda^{d-4}} \mathcal{L}_d = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots$$

$$\mathcal{L}_d = \sum_i C_i^d \mathcal{O}_i \qquad [\mathcal{O}_i] = d \longrightarrow \left(\frac{q}{\Lambda}\right)^{d-4}$$

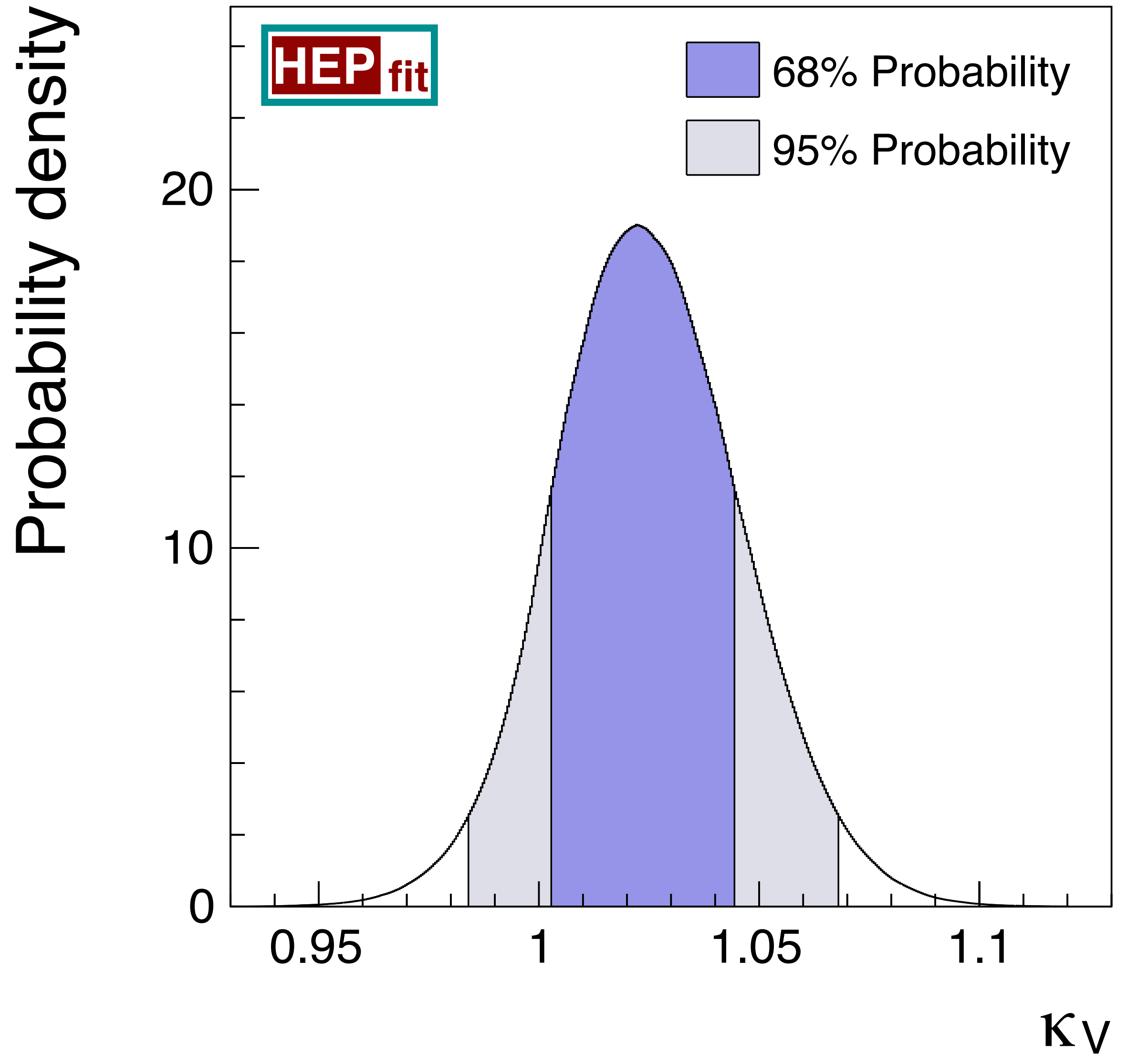
Effects suppressed by $q = v, E < \Lambda$

$\Lambda$: Cut-off of the EFT

$$\mathcal{O}_{\phi WB} = \phi^\dagger \sigma_a \phi B^{\mu\nu} W^a_{\mu\nu}$$

EWSB → $v^2 B^{\mu\nu} W^3_{\mu\nu}$ gauge boson masses

EWSB → $vhB^{\mu\nu} W^3_{\mu\nu}$ h→ZZ, γ γ




$$M_W^2 = M_Z^2 c^2 \left[1 - \frac{c^2}{c^2 - s^2}\left(\frac{1}{2}C_{\phi D} + 2\frac{s}{c}C_{\phi WB} + \frac{s^2}{c^2}\Delta_{G_\mu}\right)\frac{v^2}{\Lambda^2}\right]$$

A precise measurement of $m_W$ and $\sin^2\theta_{eff}$ constrains several dim-6 operators contributing to Higgs and gauge interaction vertices.

Today still one of the strongest constraints

# Global fits: New Physics scrutinised with S,T,U parameters

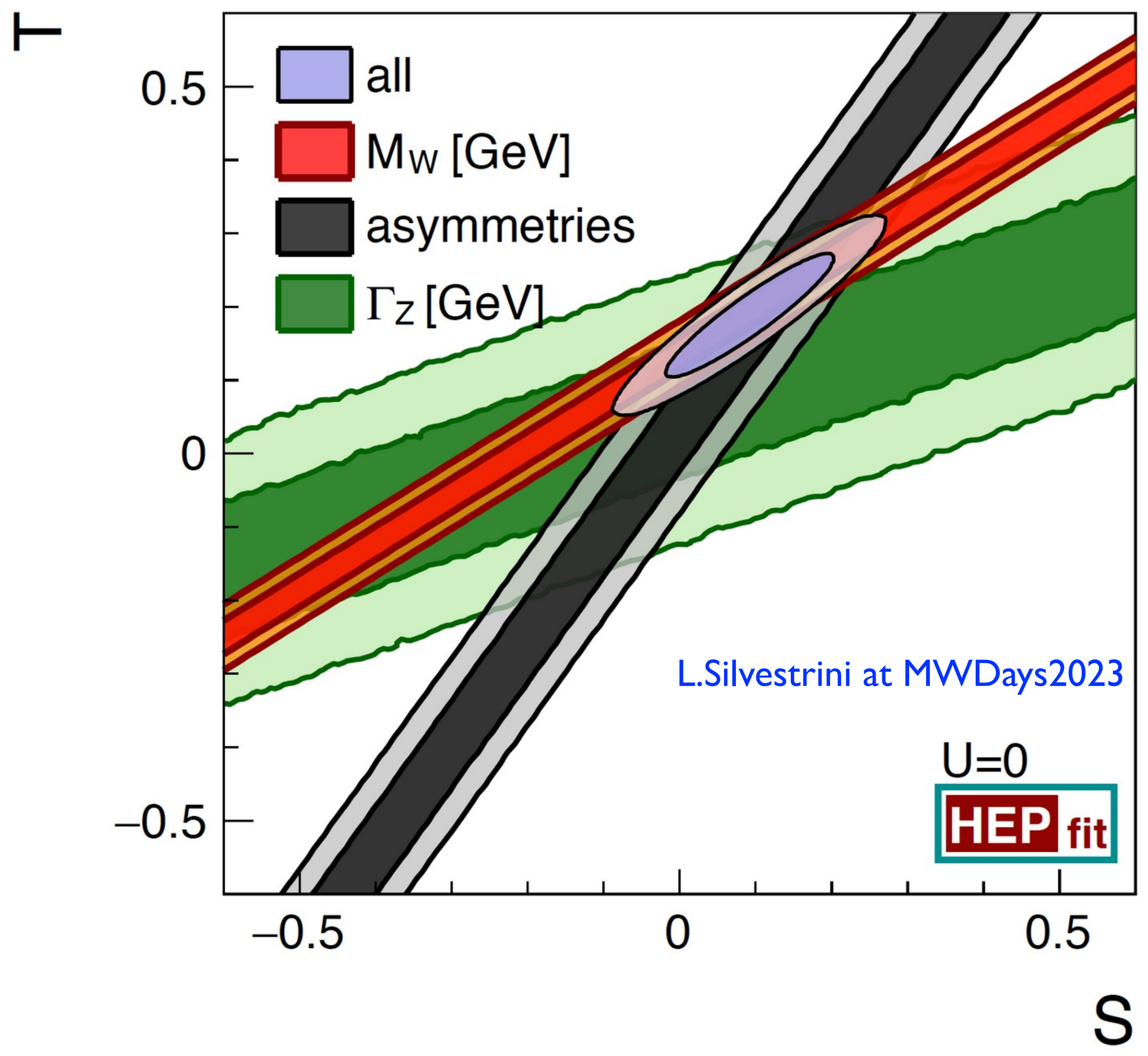
Assuming that New Physics dominant contribution is in Gauge Boson propagators

$$S = -16\pi\Pi_{30}^{NP'}(0) = 16\pi\left(\Pi_{33}^{NP'}(0) - \Pi_{3Q}^{NP'}(0)\right)$$
$$T = \frac{4\pi}{\sin^2\theta_W \cos^2\theta_W m_Z^2}\left(\Pi_{11}^{NP}(0) - \Pi_{33}^{NP}(0)\right)$$
$$U = 16\pi\left(\Pi_{11}^{NP'}(0) - \Pi_{33}^{NP'}(0)\right)$$

then the EWPO are modified as

$$\delta\Gamma_Z \propto -10\,(3 - 8\sin^2\theta_W)\,S + (63 - 126\sin^2\theta_W - 40\sin^4\theta_W)\,T$$
$$\delta m_W, \delta\Gamma_W \propto S - 2\cos^2\theta_W\,T - \frac{\cos^2\theta_W - \sin^2\theta_W}{2\sin^2\theta_W}\,U$$
other observables $\propto S - 4\cos^2\theta_W \sin^2\theta_W\,T$

de Blas et al, arXiv:2112.07274



# Physical processes, observables and parameter determination

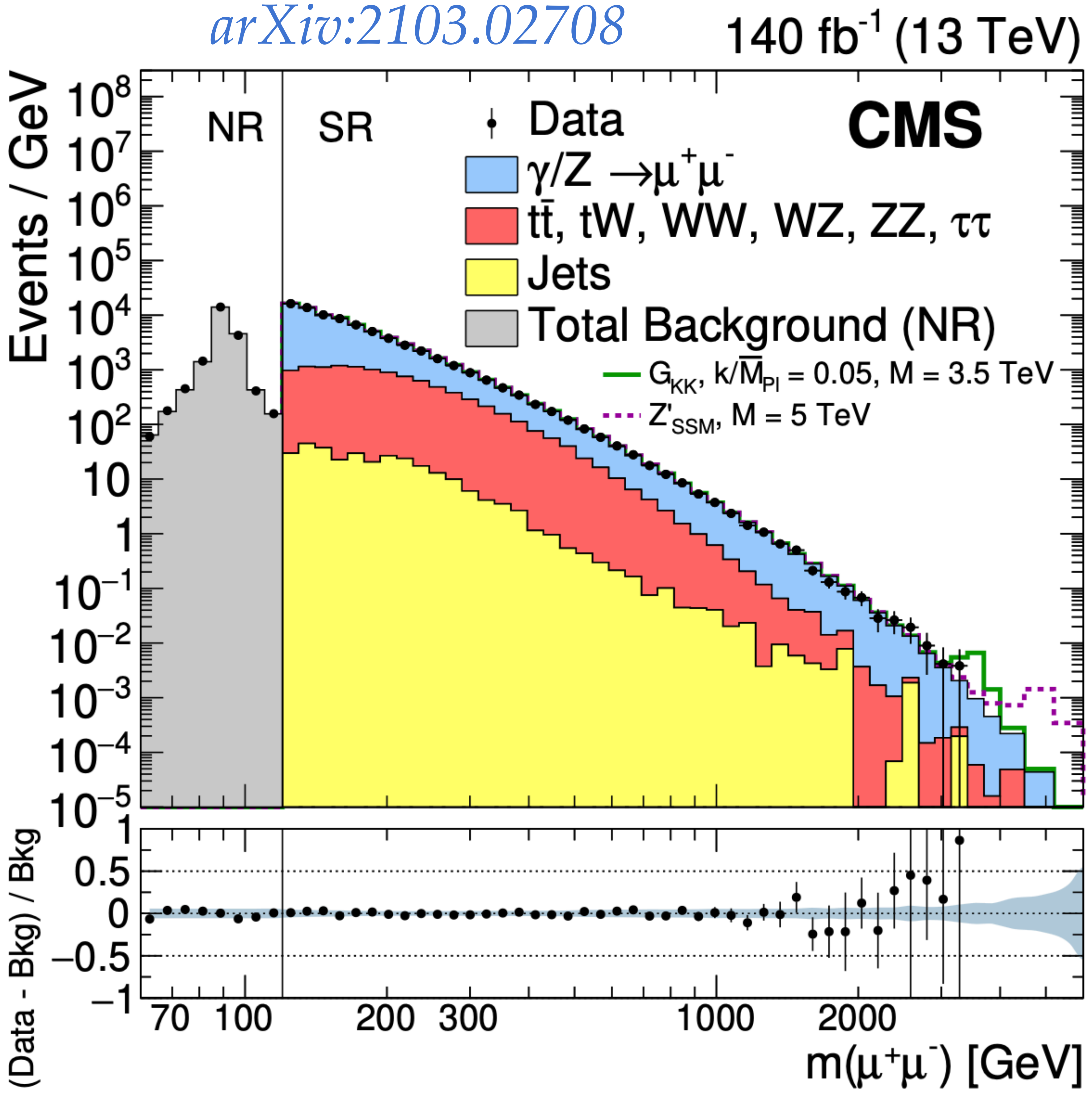
- the high-mass Drell-Yan process at the HL-LHC

  higher order SM radiative corrections and New Physics

  sensitivity to the weak mixing angle

# 1) Relevance of Neutral Current Drell-Yan measurements: searches for New Physics signals

*arXiv:2103.02708*

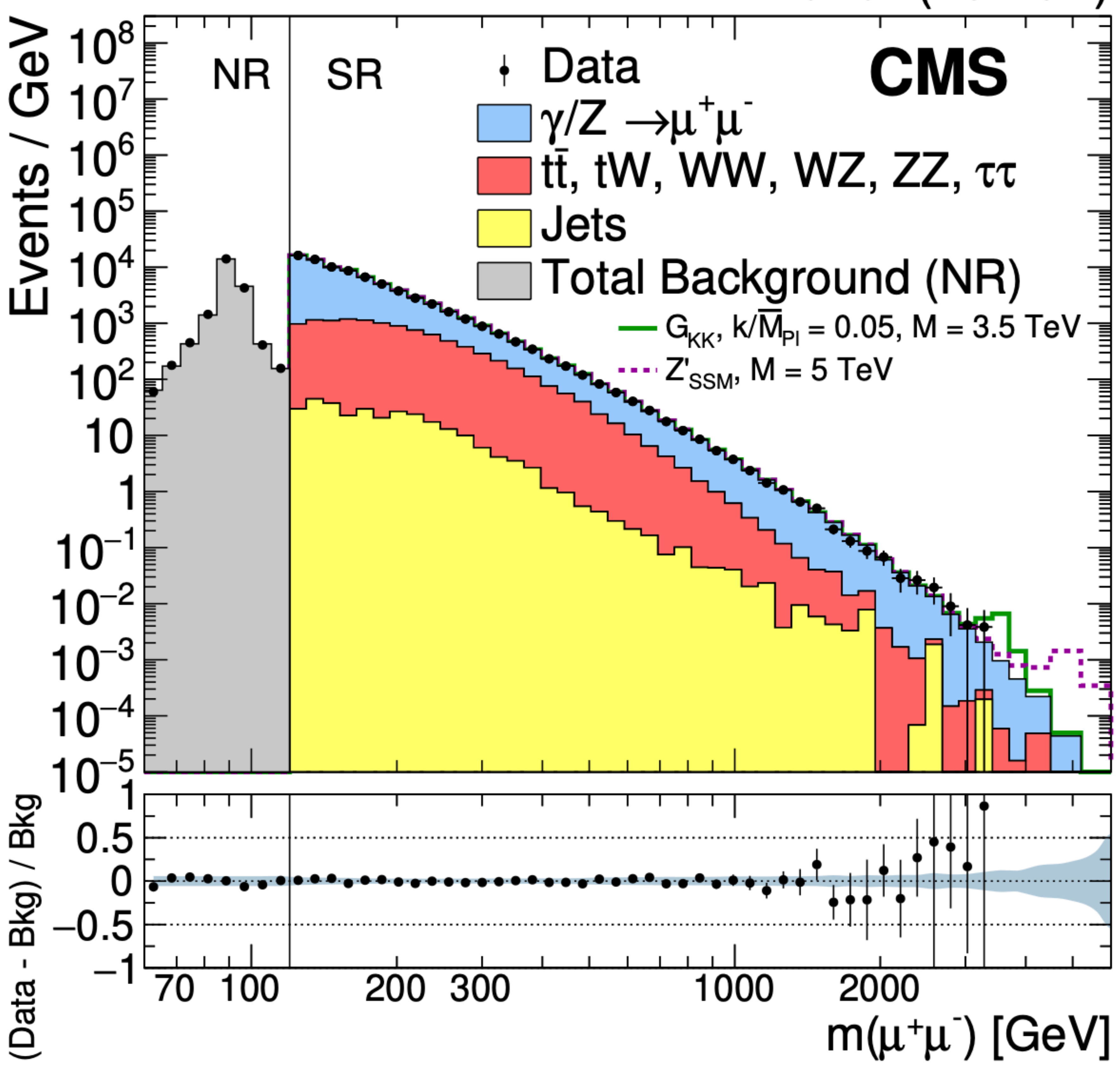



| mass window [GeV] | stat. unc. 140fb$^{-1}$ | stat. unc. 3ab$^{-1}$ |
|---|---|---|
| **$600<m_{\mu\mu}<900$** | 1.4% | 0.2% |
| **$900<m_{\mu\mu}<1300$** | 3.2% | 0.6% |

At the end of High-Luminosity LHC we will be able to test the TeV region with data at per mille level
i.e.
to test the SM at the level of its quantum corrections

# 1) Relevance of Neutral Current Drell-Yan measurements: searches for New Physics signals

*arXiv:2103.02708*

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| **600<$m_{\mu\mu}$<900** | 1.4% | 0.2% |
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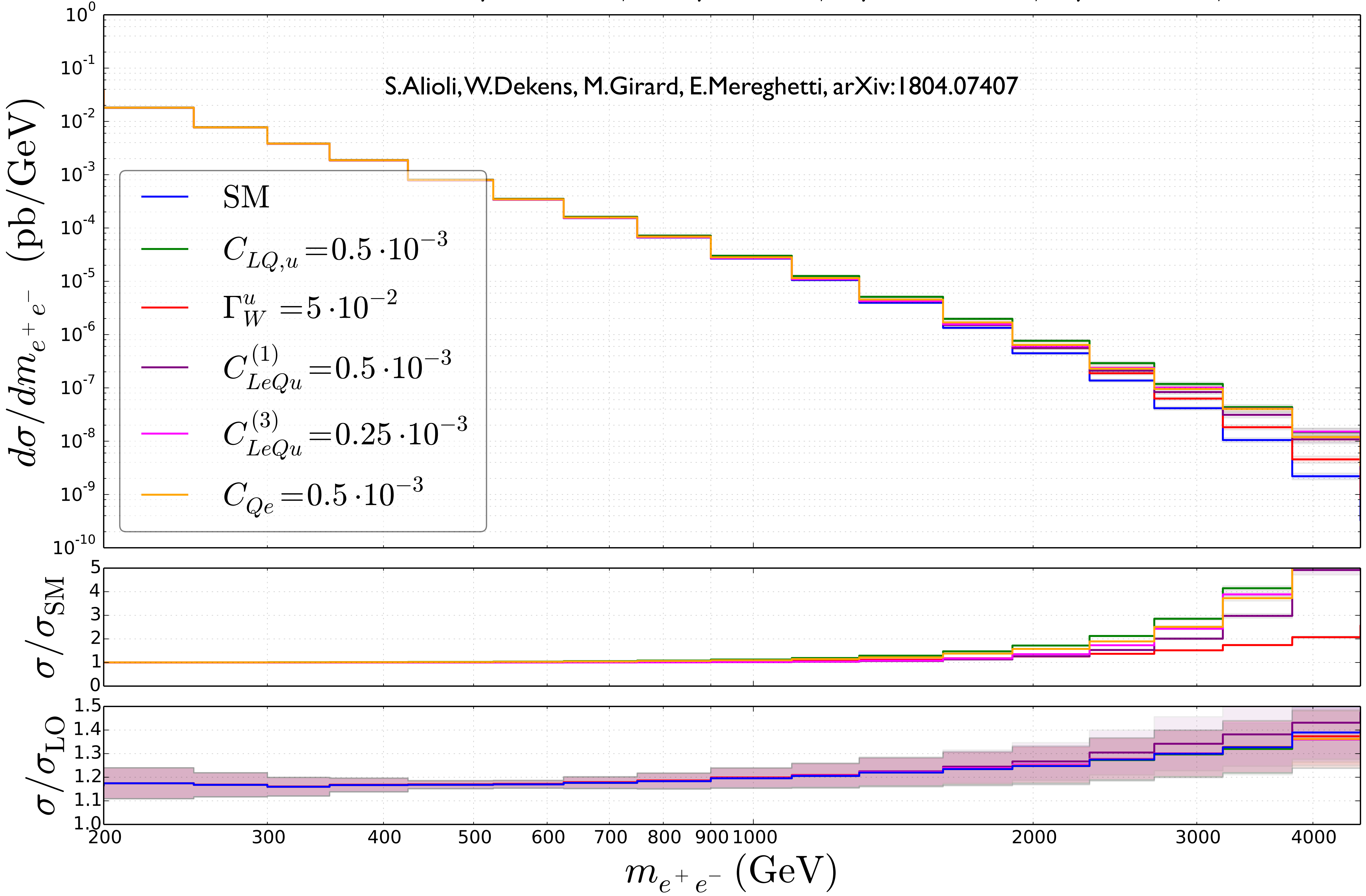
$$\mathcal{L} = \mathcal{L}_{X^2\varphi^2} + \mathcal{L}_{\psi^2 X\varphi} + \mathcal{L}_{\psi^2\varphi^2 D} + \mathcal{L}_{\psi^2\varphi^3} + \mathcal{L}_{\psi^4}$$



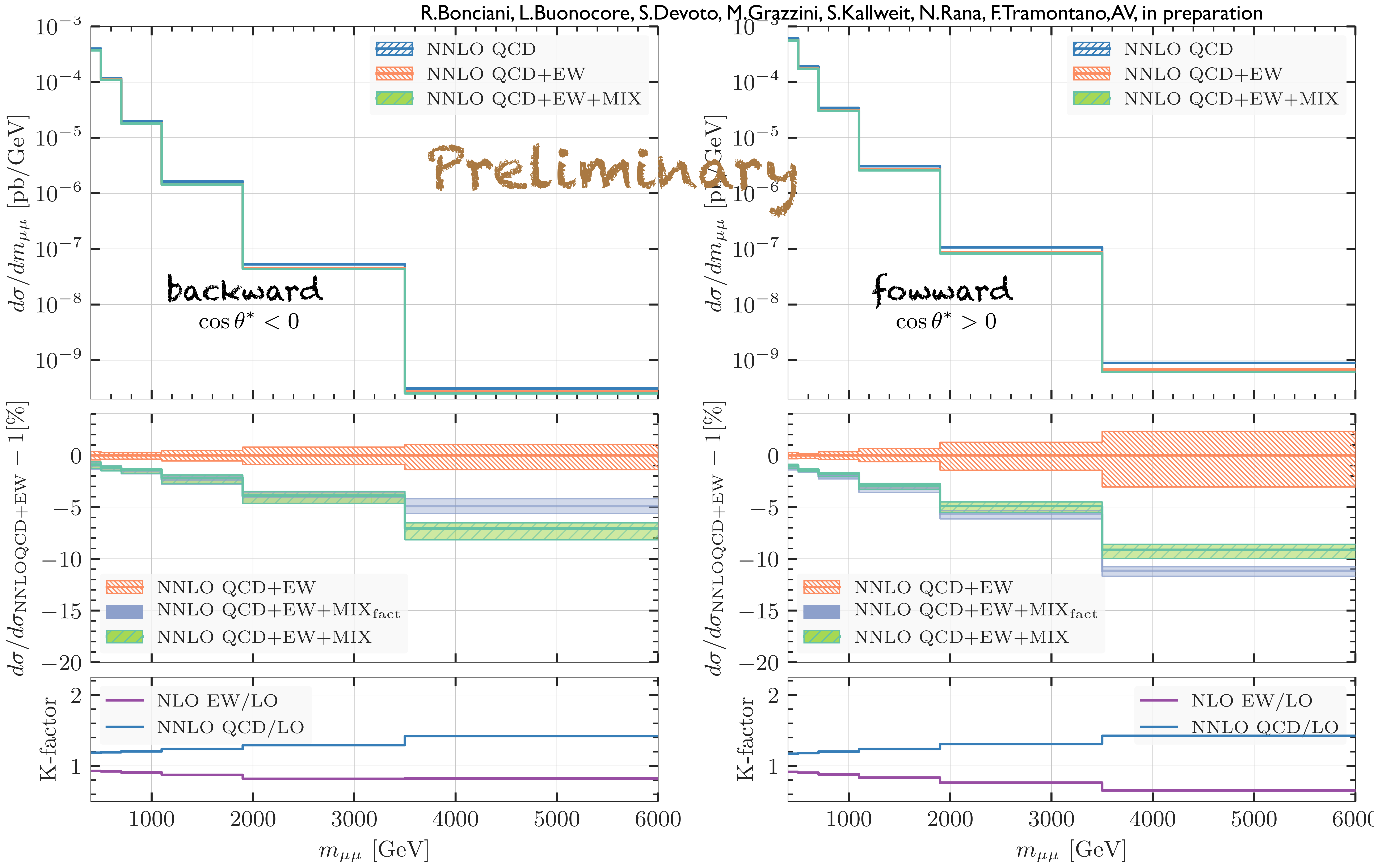

A deviation from the SM prediction can point towards New Physics

Is the SM prediction under control at the O(0.5%) level in the TeV region of the $m_{\ell\ell}$ distribution ?

# I) Precision prediction of the dilepton invariant mass distribution in NC DY

R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano,AV, arXiv:2106.11953
T.Armadillo, R.Bonciani, S.Devoto, N.Rana, ,AV, arXiv:2201.01754

R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano,AV, in preparation




**Complete NNLO QCD-EW corrections to Neutral-Current Drell-Yan**

Not negligible
mixed QCD-EW corrections

Very large cancellation of
NLO QCD and EW effects

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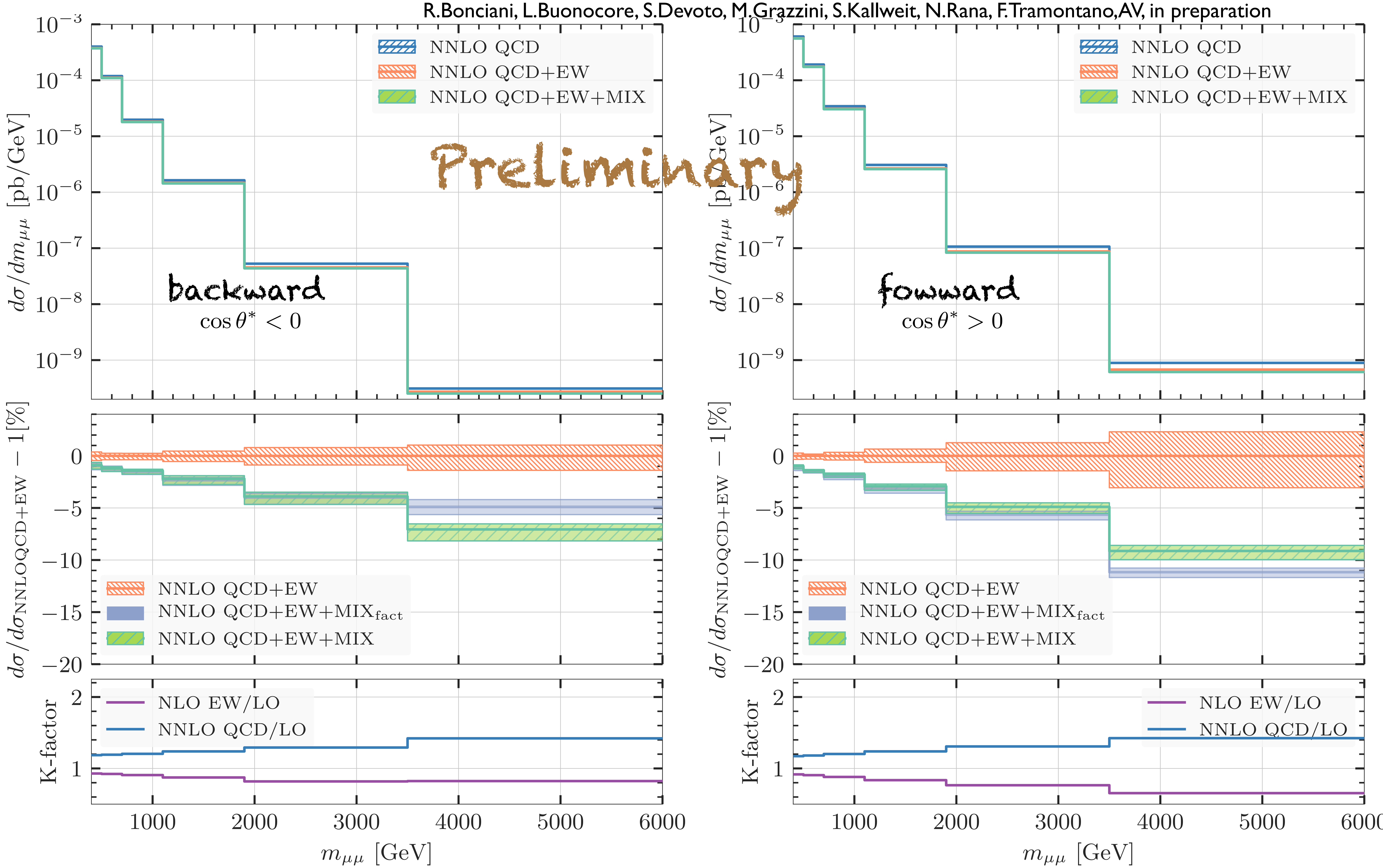


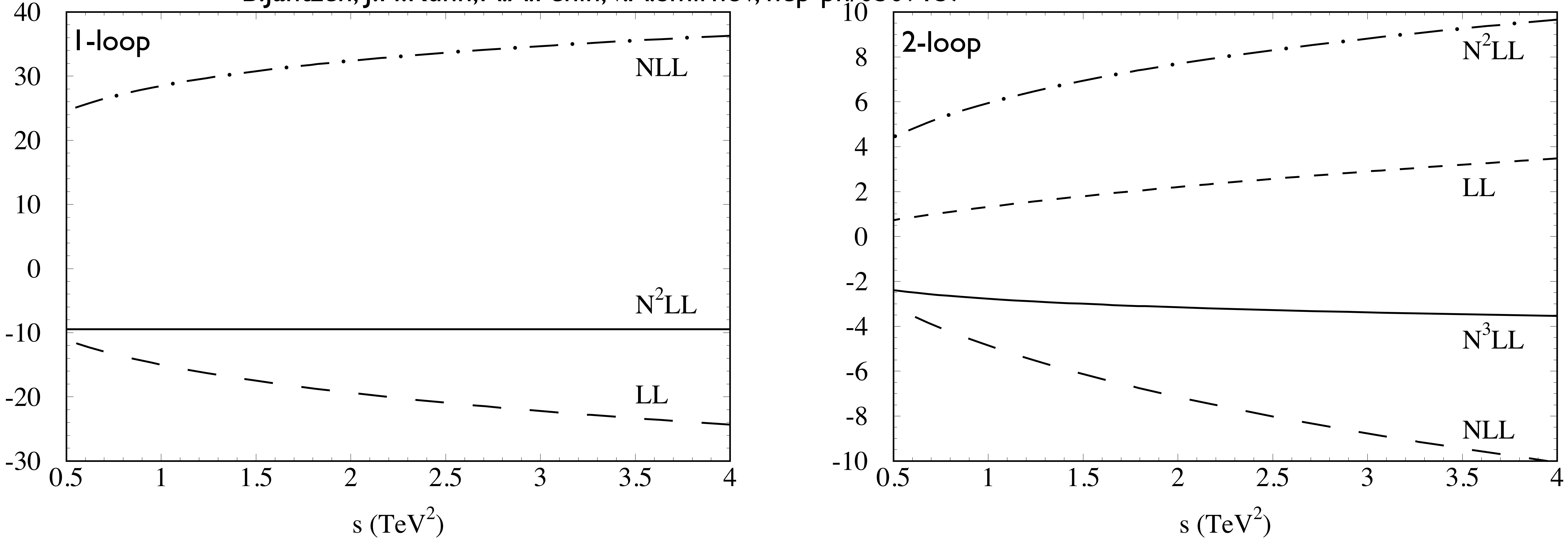

It is crucial to control the SM prediction at sub-percent level before we any SMEFT analysis
Missing higher orders can easily mimic and fake BSM signals (i.e. non-vanishing Wilson coefficients)

The SMEFT operators of the previous slide contribute also to the $m_W$ prediction
→ close interplay and constraints between precision parameters and high-energy searches

# 1) Need for a full NNLO-EW calculation to reduce the uncertainties to percent level

The NNLO-EW corrections could modify in a non-trivial way the large-mass/momentum tails of the distributions
Large logarithmic corrections (EW Sudakov logs) appear in the virtual corrections
At two-loop level, we have up to the fourth power of $\log(s/m_V^2)$,
the different corrections are comparable in size and with alternate signs
→ how can we estimate the constant term ?

B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph/0509157




corrections to $e^+e^- \to q\bar{q}$
due to EW Sudakov logs

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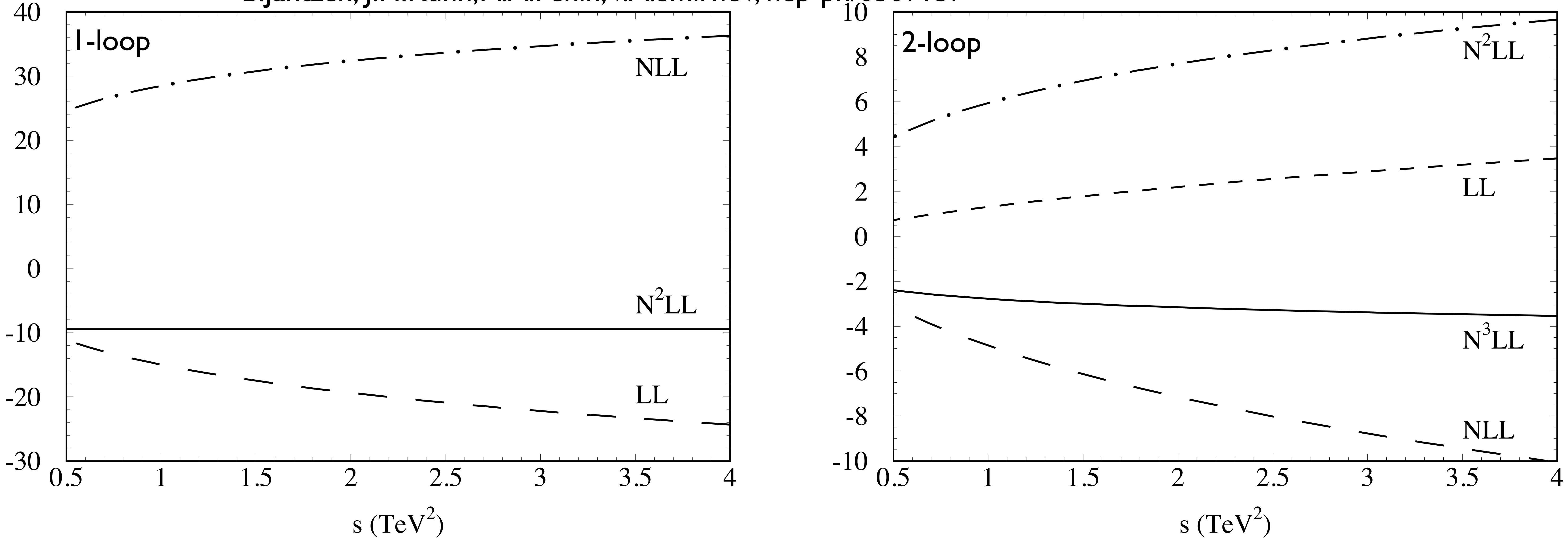
→ how can we estimate the constant term ?

B.Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph/0509157




corrections to $e^+e^- \to q\bar{q}$
due to EW Sudakov logs

The NNLO-EW corrections require an extra step compared to the mixed QCD-EW case
- for the number of additional Master Integrals (→ automation)
- for the complexity of the amplitudes ( size problems? large cancellations? )
- for the conceptual problems ($\gamma_5$?, complex-mass scheme at two-loop? )

many preliminary steps achieved/ongoing

# 2) The dilepton invariant mass distribution in NC-DY at high mass and the weak mixing angle

The triple-differential cross section at LO

$$\frac{d^3\sigma}{dm_{\ell\ell}dy_{\ell\ell}d\cos\theta_{CS}} = \frac{\pi\alpha^2}{3m_{\ell\ell}s}\Bigg((1+\cos^2\theta_{CS})\sum_q S_q[f_q(x_1,Q^2)f_{\bar{q}}(x_2,Q^2)+f_q(x_2,Q^2)f_{\bar{q}}(x_1,Q^2)] + \cos\theta_{CS}\sum_q A_q \mathrm{sign}(y_{\ell\ell})\cdot[f_q(x_1,Q^2)f_{\bar{q}}(x_2,Q^2)-f_q(x_2,Q^2)f_{\bar{q}}(x_1,Q^2)]\Bigg)$$

$$S_q = e_\ell^2 e_q^2 + P_{\gamma Z}\cdot e_\ell v_\ell e_q v_q + P_{ZZ}\cdot(v_\ell^2+a_\ell^2)(v_q^2+a_q^2)$$
$$A_q = P_{\gamma Z}\cdot 2e_\ell a_\ell e_q a_q + P_{ZZ}\cdot 8v_\ell a_\ell v_q a_q,$$

$$P_{\gamma Z}(m_{\ell\ell}) = \frac{2m_{\ell\ell}^2(m_{\ell\ell}^2-m_Z^2)}{\sin^2\theta_W\cos^2\theta_W[(m_{\ell\ell}^2-m_Z^2)^2+\Gamma_Z^2 m_Z^2]}$$
$$P_{ZZ}(m_{\ell\ell}) = \frac{m_{\ell\ell}^4}{\sin^4\theta_W\cos^4\theta_W[(m_{\ell\ell}^2-m_Z^2)^2+\Gamma_Z^2 m_Z^2]}$$

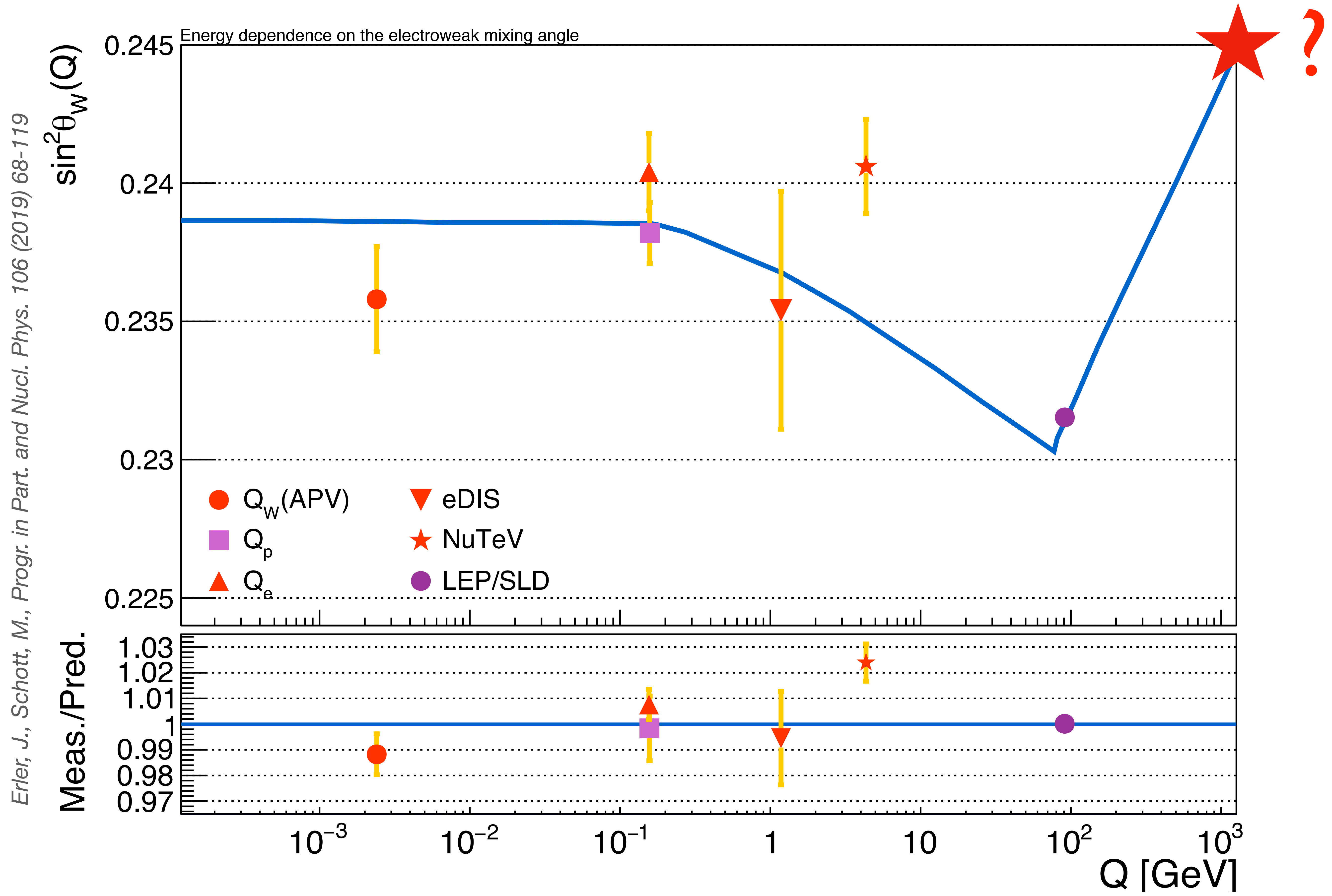
The 3D differential xsec exhibits a dependence on the specific $\sin^2\theta_W$ value, modulated by the different combinations of $\gamma$ and Z propagators.

At the Z resonance, specific sensitivity to $\sin^2\theta_W$, via the ratio of vector/axial-vector couplings, assessed from the study of $A_{FB}$ and $A_{LR}$ asymmetries

Also at large invariant masses the xsec features a sensitivity to $\sin^2\theta_W$, stemming from both normalisation and angular-dependent factors!

→ at NLO-EW we can study $\sin^2\hat{\theta}(\mu_R)$, the MSbar renormalised mixing angle
and exploit the large mass range to test the running of this quantity

# 2) The MSbar weak mixing angle $\sin^2\hat{\theta}(\mu_R)$ at large scales




The RGE evolution depends on the number of active flavours contributing to the $\beta$-function
Above $\mu = m_W$ there is an change of sign which features a positive slope.

Can we test this prediction of the SM, i.e. 1) the running and 2) the value of the slope ?
Is there enough sensitivity?

# 2) $\sin^2\hat{\theta}(\mu_R)$ determination at hadron colliders at large invariant masses

S.Amoroso, M.Chiesa, C.L Del Pio, E.Lipka, F.Piccinini, F.Vazzoler, AV, arXiv:2302.10782

The study has to be performed at least at NLO-EW.

The amplitude has at NLO-EW different groups of corrections: QED, weak.
Only a specific subset of such corrections contributes to the redefinition of the renormalised parameter,
while the rest (e.g. boxes and part of the vertices) is a genuine process dependent correction.

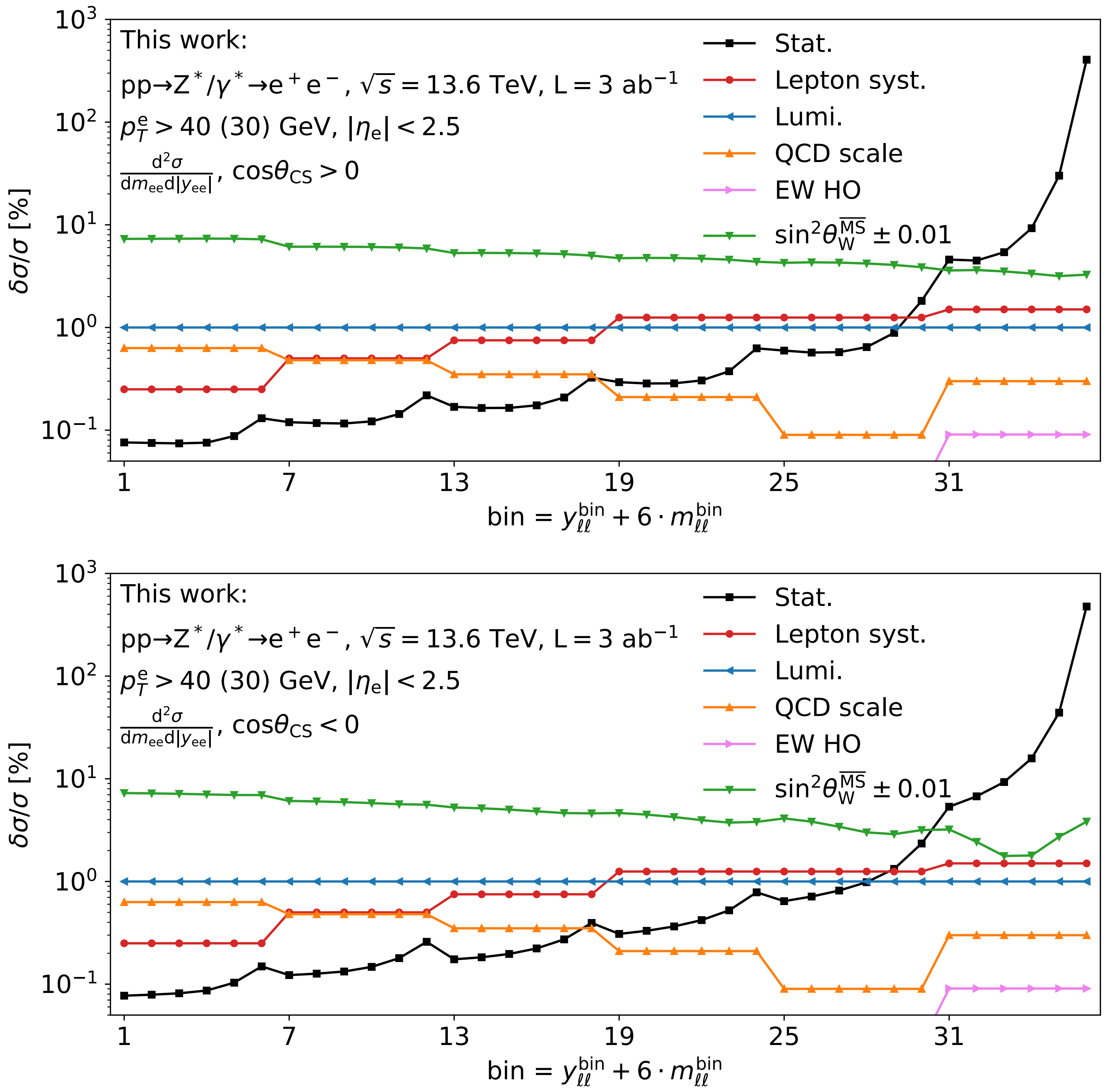
In order to claim that we are sensitive to the precise $\sin^2\hat{\theta}(\mu_R)$ value,
$\sin^2\hat{\theta}(\mu_R)$ must be among the input parameters of the renormalised lagrangian.
A new version of the POWHEG NC DY QCD+EW has been prepared,
which admits as input parameters ( $\hat{\alpha}(\mu_R), \sin\hat{\theta}(\mu_R), m_Z$ ) , renormalised at NLO-EW .

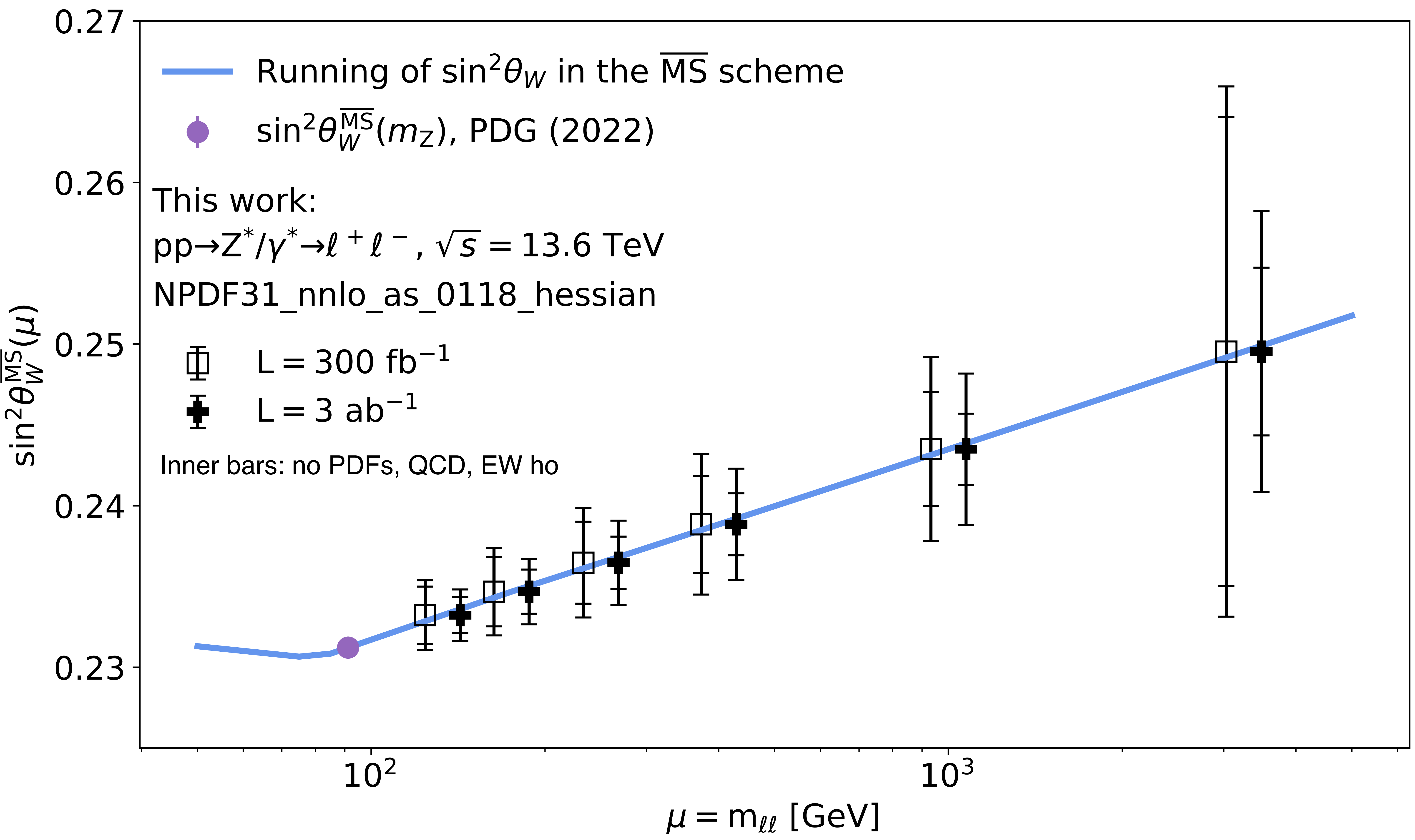
Thanks to this choice, $\sin^2\hat{\theta}(\mu_R)$ can be left as a free fit parameter, and extracted from the data.
The explicit presence of the other corrections, insensitive to $\sin^2\hat{\theta}(\mu_R)$, allows to correctly estimate
the dependence on this parameter, at each mass scale.

We need to estimate the change of the xsec, for a given $\sin^2\hat{\theta}(\mu_R)$ variation. In the sensitivity study
we identify the minimal variation which can be appreciated in the fit to the data, for given experimental errors.

# 2) $\sin^2\hat{\theta}(\mu_R)$ determination at hadron colliders at large invariant masses

S.Amoroso, M.Chiesa, C.L Del Pio, E.Lipka, F.Piccinini, F.Vazzoler, AV, arXiv:2302.10782




The running of the MSbar angle can be established at LHC in Run III and at HL-LHC with percent precision.
The remaining uncertainties do not affect the conclusion of the sensitivity study, performed at NLO.

For the actual measurement instead the best theoretical predictions will be needed, to avoid interpretation mismatches:
full NNLO (QCD, EW and mixed QCDxEW) and leading higher orders, as discussed before.

- the MSbar weak mixing angle from low-energy experiments

P2 at MESA
Møller at Jefferson lab
Qweak at Jefferson lab

# 3) The weak mixing angle at low energy scales

Goal: testing the parity-violating structure of the weak interactions at different energy scales

Problems: a) define an observable quantity, analogous to $\sin^2\theta_{eff}^{lep}$ at $q^2 = m_Z^2$ ,

now e.g. at $q^2 = 0$ for the t-channel processes like e-p or e-e- scattering

b) given the large size of the NLO corrections at $q^2 = 0$, the fixed-order result is not sufficient

we have to resum to all orders large classes of radiative corrections in the definition of a running parameter

Solution 1: introduction of $\sin^2\theta_{eff}^{e^-e^-}$ at $q^2 = 0$ to describe Møller scattering Ferroglia, Ossola, Sirlin, hep-ph/0307200

it absorbs the effect of the EW corrections to the Møller amplitude

in a new effective parameter $\sin^2\theta_{eff}^{e^-e^-}$, via a gauge-invariant form factor $\kappa(q^2 = 0)$,

in a tree-level-like structure

this parameter is a physical observable which can be i) predicted and ii) measured → comparison with $\sin^2\theta_{eff}^{lep}$

Solution 2: the definition of $\sin^2\hat{\theta}(\mu_R)$ in the MSbar scheme is strictly bound to the presence of a renormalisation scale $\mu_R$

$\sin^2\hat{\theta}(\mu_R)$ satisfies the RGE (→ it needs a boundary condition computed at one given scale $q^2$)

this quantity can be predicted in the SM using $(\alpha(0), G_\mu, m_Z)$ as basic input parameters

the scale $\mu_R$ allows to probe the size of resummed radiative correction to the couplings at different scales

# 3) The running of $\sin^2\hat\theta(\mu_R)$ and the prediction of $\sin^2\hat\theta(0)$ Erler,Ramsey-Musolf, hep-ph/0409169

given $\sin^2\hat\theta(m_Z^2)$, we want to study a process with $Q^2 \ll m_Z^2$ $\rightarrow$ the radiative corrections contain large $\log(Q^2/m_Z^2)$ factors

in the MSbar scheme, the RGE allows to compute the coupling at an arbitrary scale $\mu^2$, once the value at a given $Q^2$ is known

$$\sin^2\hat\theta(Q^2) = \hat\kappa(Q^2,\mu^2)\ \sin^2\hat\theta(\mu^2)$$

setting $\mu^2 = Q^2$ resums the large $\log(Q^2/\mu^2)$ in $\sin^2\theta(\mu^2)$
the behaviour at the physical thresholds is fixed via matching conditions

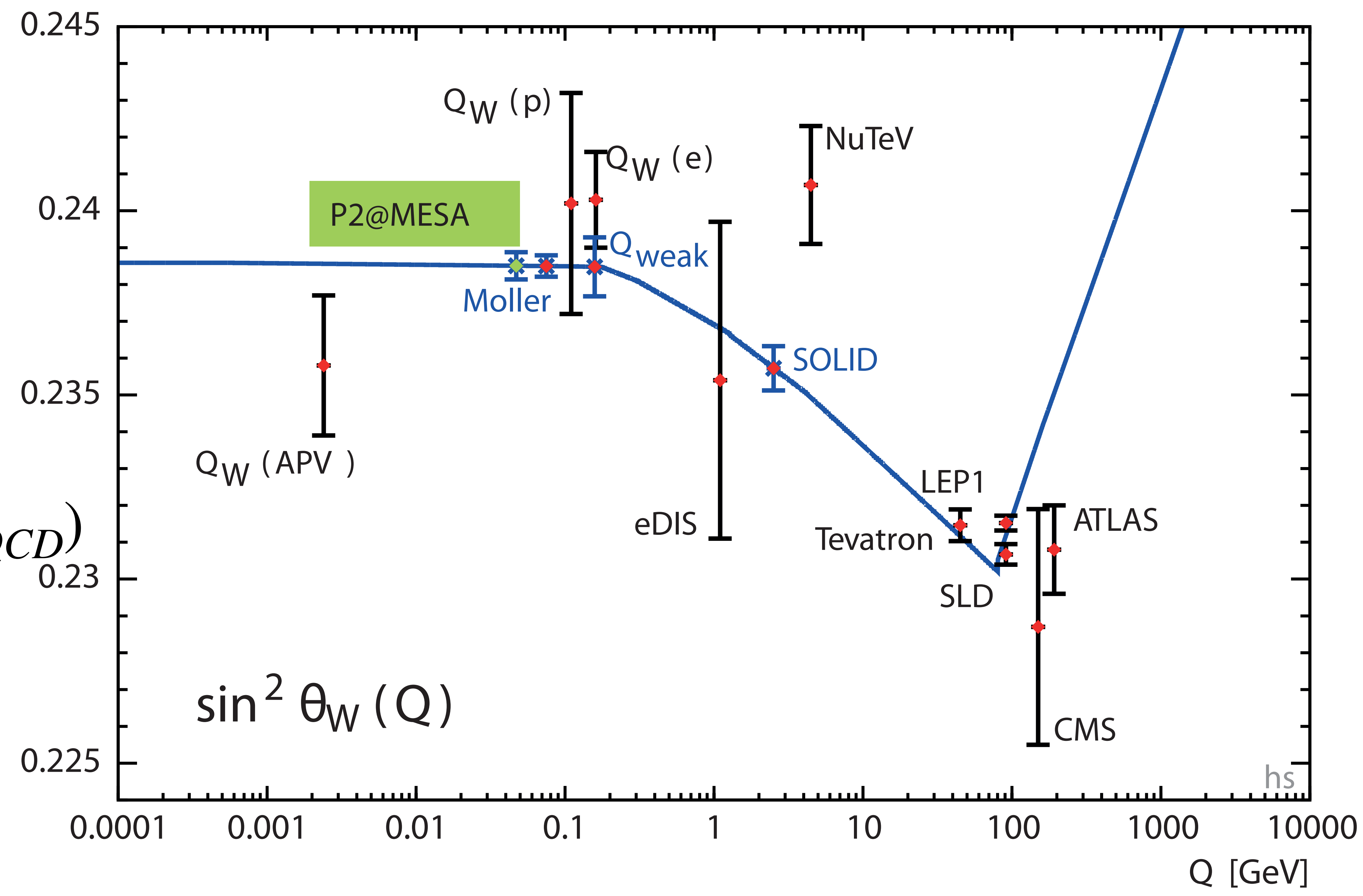
$$\sin^2\theta_W(\mu)_{\overline{\rm MS}} = \frac{\alpha(\mu)_{\overline{\rm MS}}}{\alpha(\mu_0)_{\overline{\rm MS}}}\sin^2\theta_W(\mu_0)_{\overline{\rm MS}} + \lambda_1\left[1-\frac{\alpha(\mu)}{\alpha(\mu_0)}\right] + \frac{\alpha(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4}\ln\frac{\alpha(\mu)_{\overline{\rm MS}}}{\alpha(\mu_0)_{\overline{\rm MS}}} + \tilde\sigma(\mu_0) - \tilde\sigma(\mu)\right].$$

we predict $\sin^2\hat\theta(0) = \hat\kappa(0)\ \sin^2\hat\theta(m_Z^2)$
resumming large perturbative corrections in $\hat\kappa(0)$

in ep scattering non-perturbative contributions enter via $\Sigma_{\gamma Z}(\mu\sim\Lambda_{QCD})$
and are treated along with the e.m. coupling

gauge invariance is respected in the MSbar $\hat\kappa$ factor

$\hat\kappa(0) = 1.03232 \pm 0.00029$
$\sin^2\hat\theta(m_Z^2) = 0.23124(6) \rightarrow \sin^2\hat\theta(0) = 0.23871(9)$




Kumar, Mantry, Marciano, Soudry, arXiv:1302.6263

# 3) Parity violation: what can be learned from precision e- p measurements?

The asymmetry $A_{PV} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{-G_F Q^2}{4\sqrt{2}\pi\alpha_{em}} \left( Q_W - F(E_i, Q^2) \right)$ is obtained polarising the electron beam

$$A_{PV}(P2) \sim -40 \cdot 10^{-9}$$

- $A_{PV}$ is proportional to the weak charge of the proton, accidentally suppressed in the SM: $Q_W(p) = 1 - 4\sin^2\theta_W \sim 0.09$

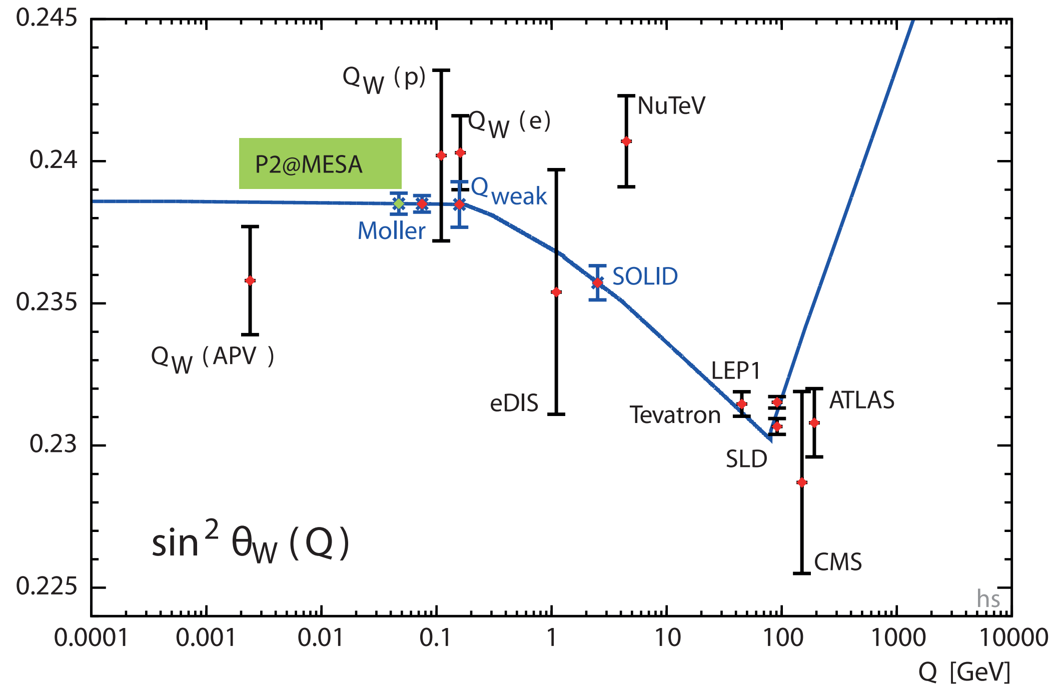
- the tree-level suppression of $Q_W(p)$ i) enhances the sensitivity to $\sin^2\theta_W$ : $\Delta Q_W / Q_W \sim 0.09\ \Delta\sin^2\theta_W / \sin^2\theta_W$

  → a measurement at the 1.4% level of $A_{PV}(P2)$ allows a determination of $\sin^2\theta_W$ with an error $\Delta\sin^2\theta_W \sim 33 \cdot 10^{-5}$ (cfr. LEP error $\Delta\sin^2\theta_W \sim 16 \cdot 10^{-5}$)

  ii) enhances the impact of the radiative corrections (e.g. -39% in Møller scattering)

- radiative corrections contribute to the precise value of the asymmetry $A_{PV}$ ( → $\sin^2\theta_W$ determination)
  may include BSM contributions (tree-level suppression of $Q_W(p)$ →enhanced sensitivity to BSM effects)

- the value of the effective weak mixing angle at $q^2 = 0$ is about 3% larger than at $q^2 = m_Z^2$
  this SM prediction has to be tested and it might reveal BSM effects





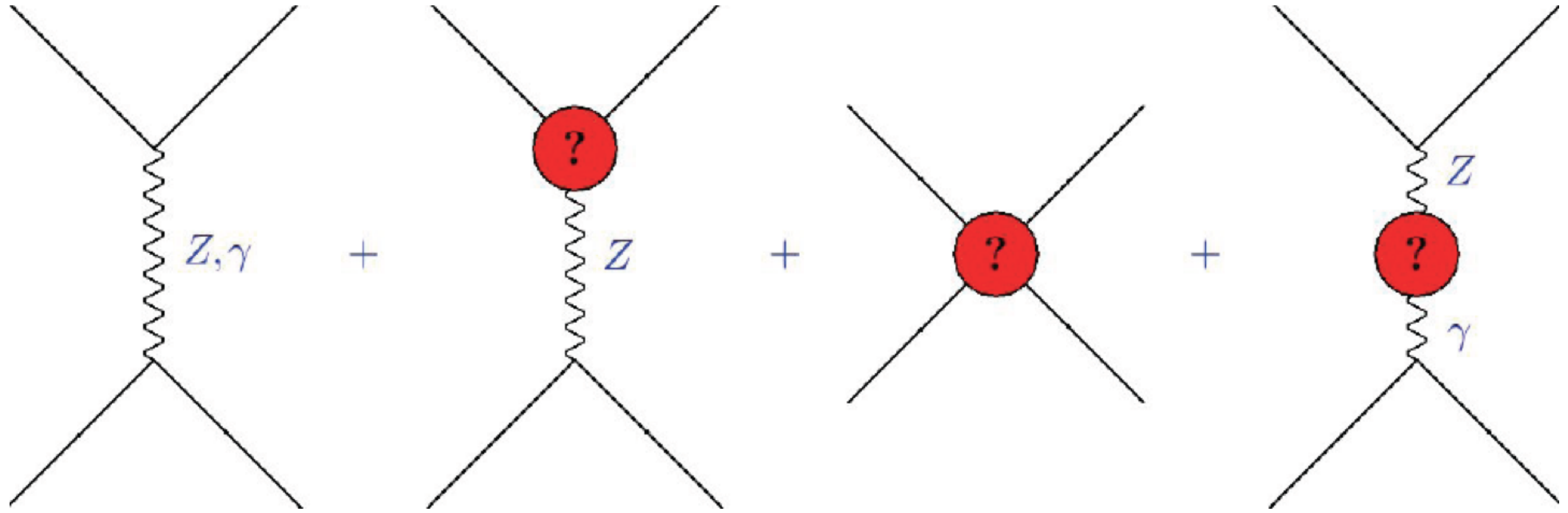

Alessandro Vicini - University of Milano

# 3) BSM searches

Any significant tension of $A_{PV}^{SM}$ with the data might be interpreted as a BSM signal

Different kinds of new interaction might yield the same observable effect:

new parity-violating contact interaction operators
new dark bosons
new additional gauge bosons (Z')




The P2 potential to discover new physics is enhanced by :

a) accidental suppression of the proton weak charge at tree level → BSM effects have stronger impact on $A_{PV}$

$$A_{PV} = \frac{-G_F Q^2}{4\sqrt{2}\pi\alpha_{em}} \left( Q_W - F(E_i, Q^2) + \Delta_{SM\, rad.corr.}(Q^2) + \Delta_{BSM}(Q^2) \right)$$

b) absence of suppression of the interferences of BSM with SM tree level amplitudes (at variance with the Z pole)
at the Z pole the SM amplitude is purely imaginary and the interference with real BSM amplitudes vanishes

The P2 high precision makes its discovery potential comparable to the one of high-energy experiments

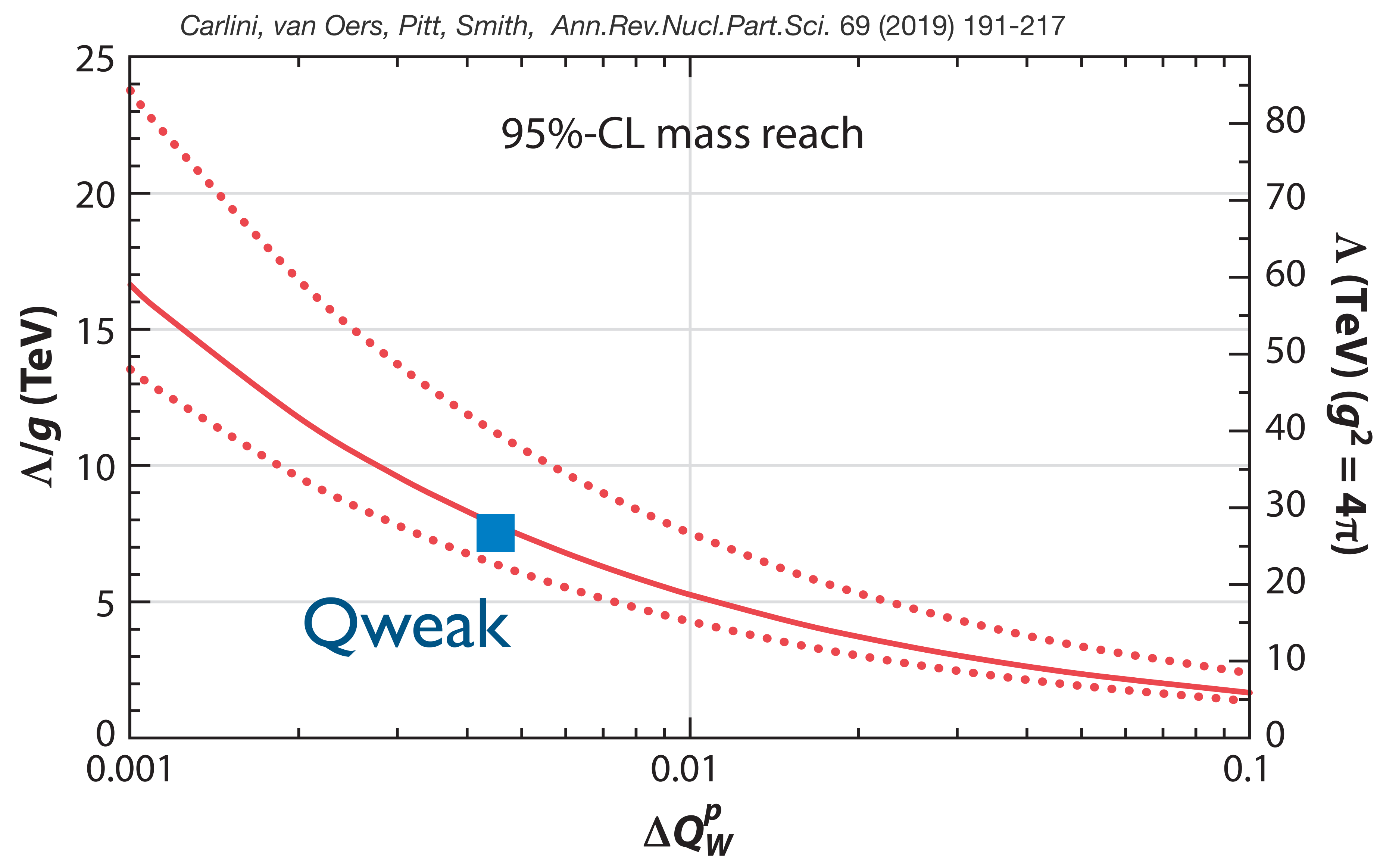
# 3) BSM searches

New contact interactions

$$\mathcal{L}_{\rm SM}^{\rm PV} = -\frac{G_F}{\sqrt{2}}\bar{e}\gamma_\mu\gamma_5 e\sum_q C_{1q}\ \bar{q}\gamma^\mu q,$$
$$\mathcal{L}_{NEW}^{\rm PV} = \frac{g^2}{4\Lambda^2}\bar{e}\gamma_\mu\gamma_5 e\sum_f h_V^q\ \bar{q}\gamma^\mu q,$$

$$\frac{\Lambda}{g} \sim \frac{1}{\sqrt{\sqrt{2}G_F|\Delta Q_W^p|}}$$

Limits on the scale of New Physics can be set in the strong coupling ($g^2 = 4\pi$) assumption or for the Wilson coefficient



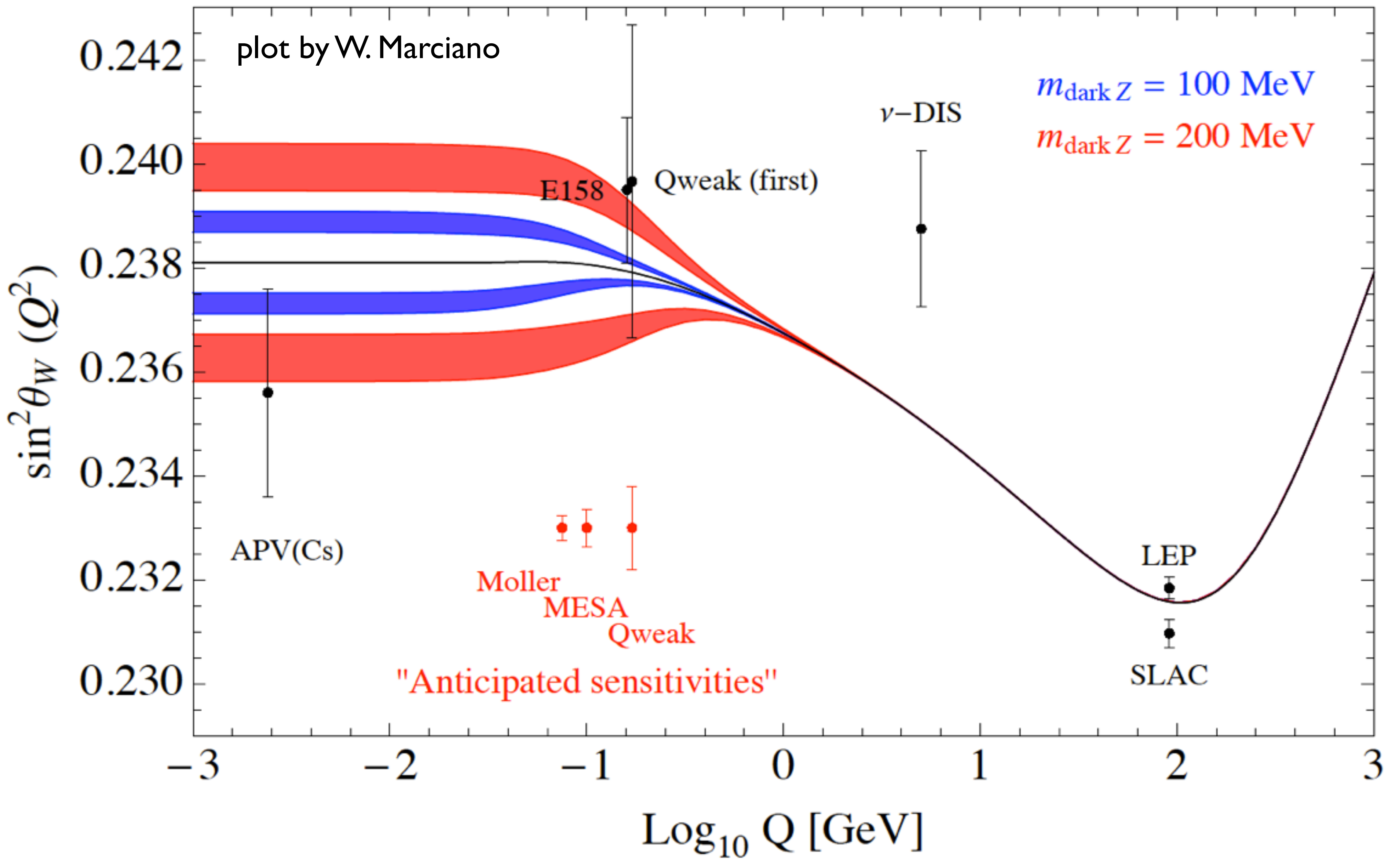

The exclusion range is computed about a SM central value hypothesis for $Q_W^p$ (solid line) with ±1σ

The expected $\Delta Q_W^p(P2) \sim 0.0011$ will push the exclusion limit up to the 80 TeV level in the strong coupling scenario and in the most favoured configuration

The limits will be stronger than at LEP2 thanks to the higher precision of the weak charge determination

# 3) BSM searches

New dark parity-violating bosons




A new dark bosons, mixing with the SM Z boson, may modify the strength of the parity-violating couplings

The effects can be completely absent at the Z resonance, where the SM amplitude is purely imaginary.

The presence of the extra boson modifies the running of $\sin^2\hat{\theta}(\mu_R)$,
with a modulation due to the assumed boson mass and couplings

The sensitivity to this kind of interaction is quite unique to the low-energy electron-scattering experiments

# Comments on the $\sin^2\theta_W$ studies at different energy scales

In these 3 examples we search for deviations from the SM → it is necessary to have the full NNLO-EW result

The possibility to interpret the results in terms of a running parameter/non-vanishing Wilson coefficient relies on a detailed knowledge of the energy dependence of the rest of the xsec

- the actual running parameter is the weak MSbar coupling $\hat{\alpha}(\mu_R)/\sin^2\hat{\theta}(\mu_R)$
- higher-order Sudakov logs have to be kept under control

→ we do not want to mismatch the SM process dependent corrections as contributions to $\sin^2\hat{\theta}(\mu_R)$

Hadron colliders predictions suffer in general from PDF uncertainties,
but,
we can consider the limiting case of a "perfect calibration" at the Z resonance,
which reabsorbs a fraction of the proton PDFs uncertainty, assuming no physics in the proton,
→ the slope of the invariant mass distribution is the relevant observable for such searches

The running of $\sin^2\hat{\theta}(\mu_R)$ depends on one single boundary condition
(matching conditions do not affect this feature, they just add extra theoretical uncertainties)
→ the possibility to include several constraints at different scales is extremely powerful
in terms of a simultaneous exclusion of different BSM models

- the Z resonance at hadron and e+e- colliders

determination of the effective leptonic weak mixing angle

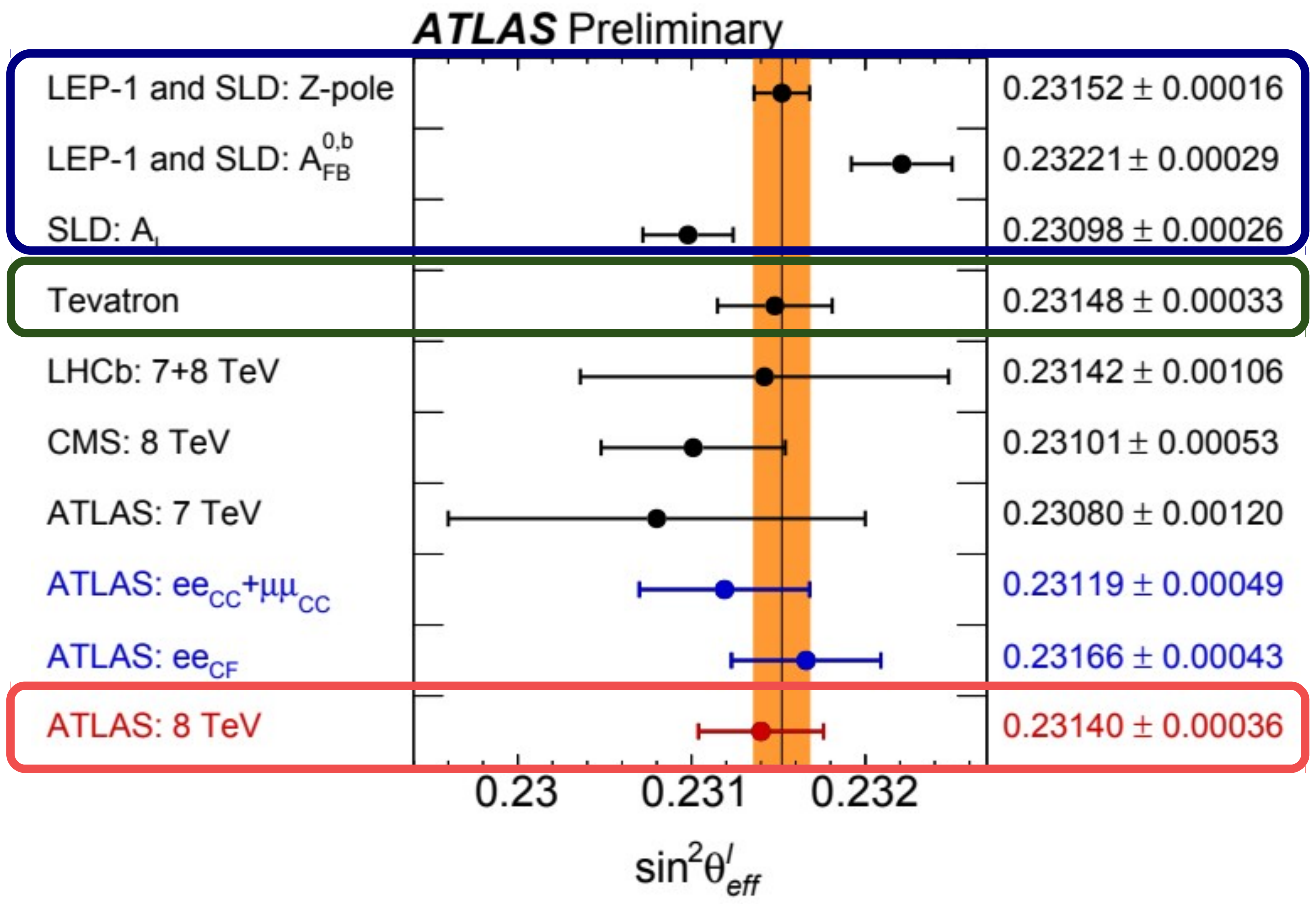
# Complementarity of different $\sin^2\theta_W$ determinations

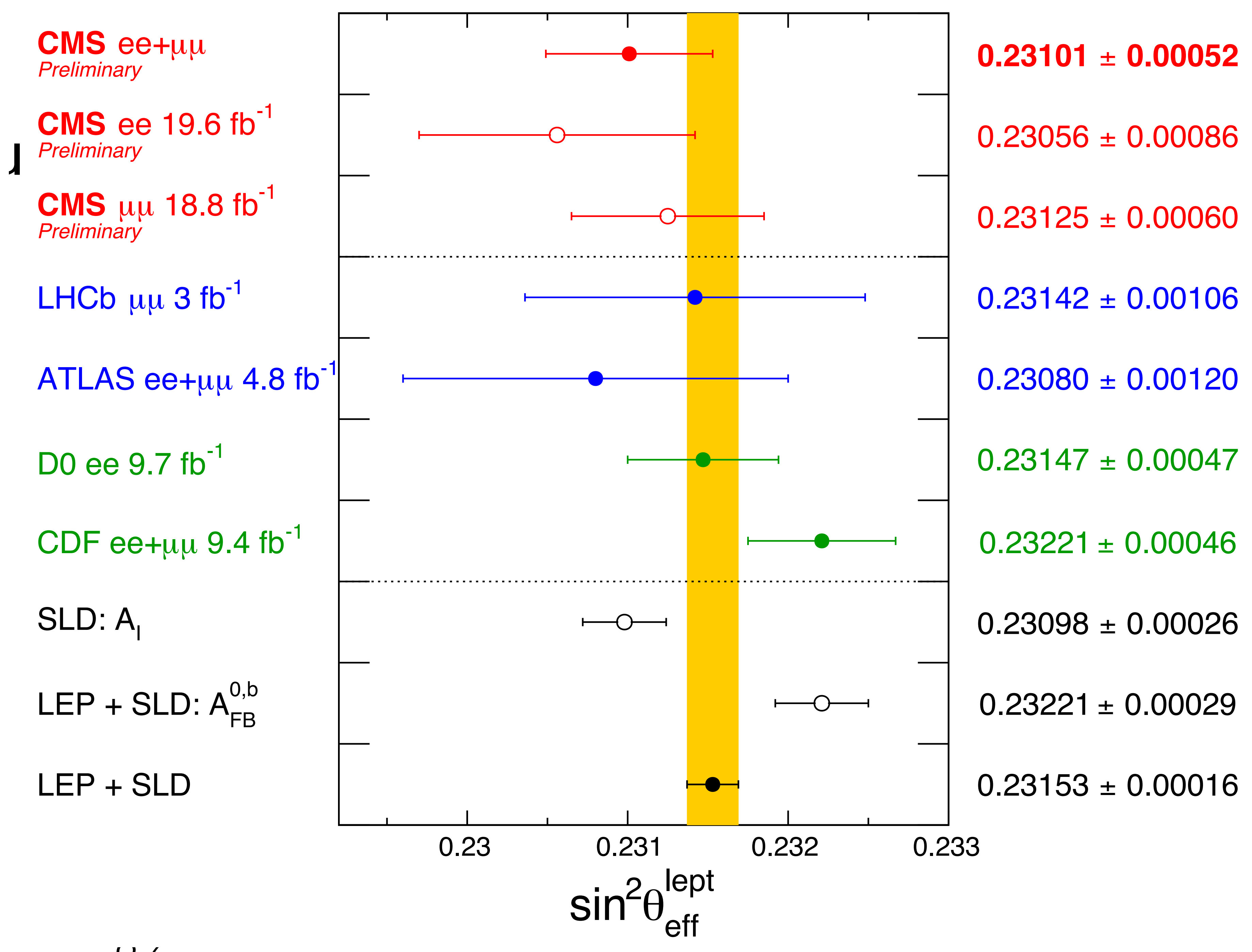
- The comparison/combination of these different results is valuable if we consider exactly the same quantity:
  a popular example is $\sin^2\theta_{eff}^{lep}$ , but in view of the current discussion it could be $\sin^2\hat{\theta}(m_Z^2)$
- for each collider/observable we have to "access" the hard scattering process (proportional to $\sin^2\theta_{eff}^{lep}$ or to $\sin^2\hat{\theta}(m_Z^2)$ )
  by deconvoluting standard QED/QCD effects, dealing with the proton (lepton) PDFs, and considering higher-order corrections

→ different strategies and input schemes are adopted in the literature; their consistency has to be checked

ATL-CONF-2018-037

cfr. the MW combination working group



# Weak mixing angle determination at hadron colliders (I)

$$\mathcal{M}^{eff}_{Zl^+l^-} = \bar{u}_l \gamma_\alpha \left[\mathcal{G}^f_v(m_Z^2) - \mathcal{G}^f_a(m_Z^2)\gamma_5\right] v_l \varepsilon^\alpha_Z$$

invariant mass Forward-Backward asymmetry
in neutral-current DY

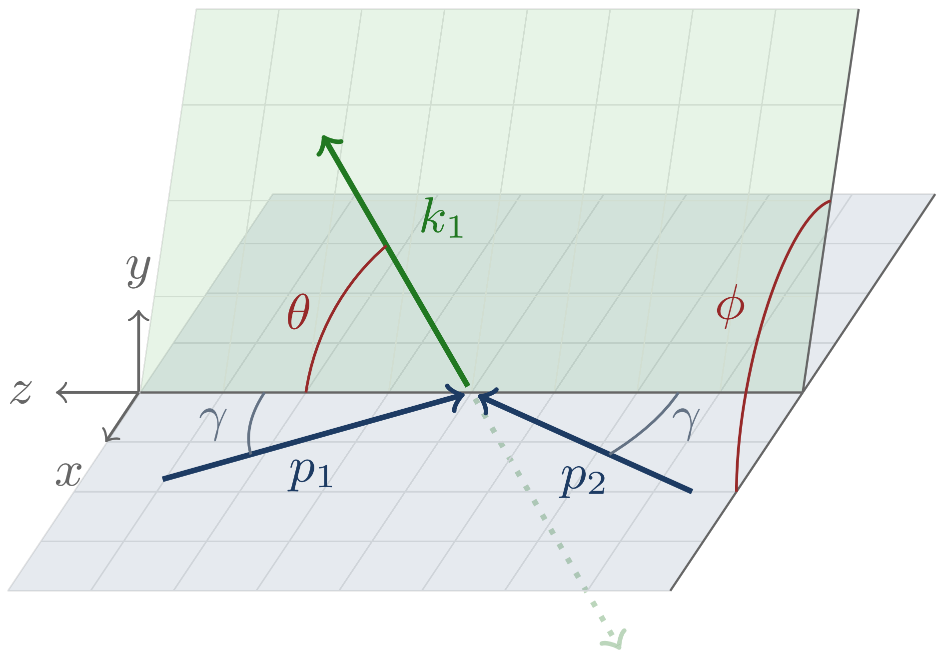
$$A_{FB}(M_{l^+l^-}) = \frac{F(M_{l^+l^-}) - B(M_{l^+l^-})}{F(M_{l^+l^-}) + B(M_{l^+l^-})}$$

$$F(M_{l^+l^-}) = \int_0^1 \frac{d\sigma}{d\cos\theta^*} d\cos\theta^* \qquad B(M_{l^+l^-}) = \int_{-1}^0 \frac{d\sigma}{d\cos\theta^*} d\cos\theta^*$$

scattering angle defined in the Collins-Soper frame → "Forward" ("Backward")

$$\cos\theta^* = f \frac{2}{M(l^+l^-)\sqrt{M^2(l^+l^-) + p_t^2(l^+l^-)}} [p^+(l^-)p^-(l+) - p^-(l^-)p^+(l^+)]$$

$$p^\pm = \frac{1}{\sqrt{2}}(E \pm p_z) \qquad f = \frac{|p_z(l^+l^-)|}{p_z(l^+l^-)}$$




we would like to appreciate parity violation like at LEP,
observing an asymmetry with respect to the direction of the incoming particle
→ it is not possible because we have both $q\bar{q}$ and $\bar{q}q$ annihilation processes
→ at the LHC the symmetry of the collider (p-p) removes one possible preferred direction
but…

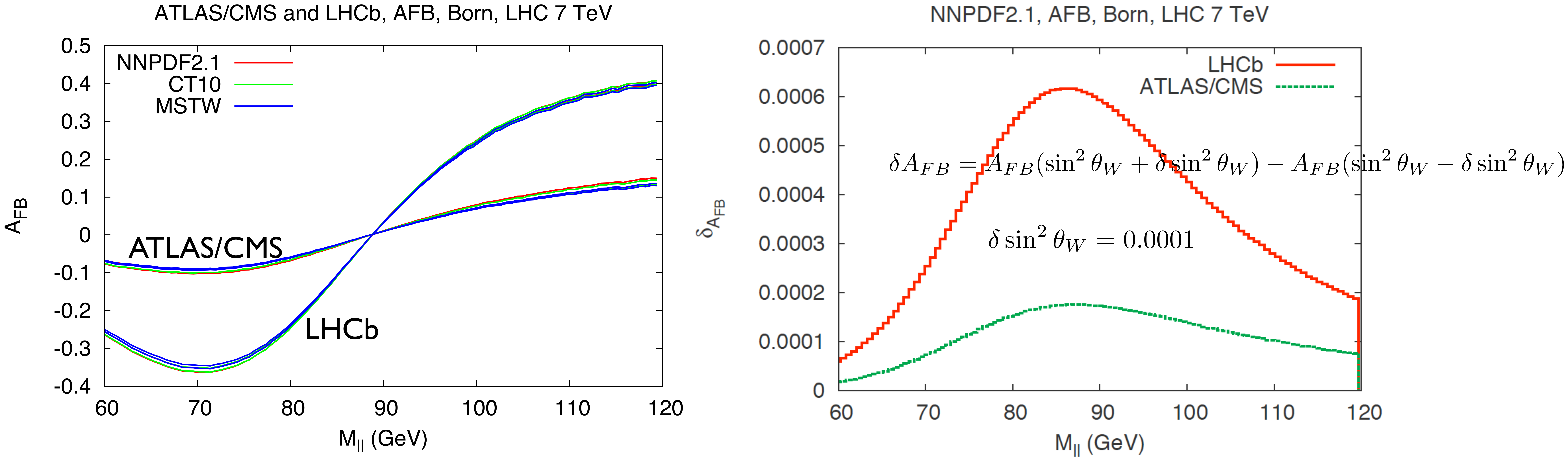
# Weak mixing angle determination at hadron colliders (II)

…but

at a given lepton-pair rapidity $Y$, $q\bar{q}$ and $\bar{q}q$ have different weight because of the PDFs $\Rightarrow$ do not cancel each other

the parton luminosity unbalance is due to the different *x* dependence of the valence and sea quarks

AFB is more pronounced at large $Y$, e.g. at LHCb




close to $m_Z$ : small AFB but good sensitivity to the weak mixing angle, large PDF uncertainties

away from $m_Z$ : large AFB, no sensitivity to the weak mixing angle, possible effects from new Z', constraining power on PDFs unc

away from $m_Z$: "model independent" parameterisation of AFB is not possible, we compute it in the SM

# Determination of $\sin^2\theta_{eff}^{lep}$ in the LHC framework

A few differences compared to the LEP measurement and analysis framework

- the initial state is a mixture, weighted by PDFs, of different quark flavours
  - → PDF uncertainty + problems to disentangle individual Z decay widths
- the precision on the Z peak cross section is lower than the one at LEP for e+e-→hadrons
  - → $\sigma_{had}$ was at LEP an important constraint of the pseudo-observable fit
- the experimental analysis involves an invariant mass window (instead of only $q^2=MZ^2$)
  - → non-factorisable contributions spoil the factorisation (initial)x(final) form factors

→ it is not possible to pursue the LEP approach in terms of pseudo-observables at LHC

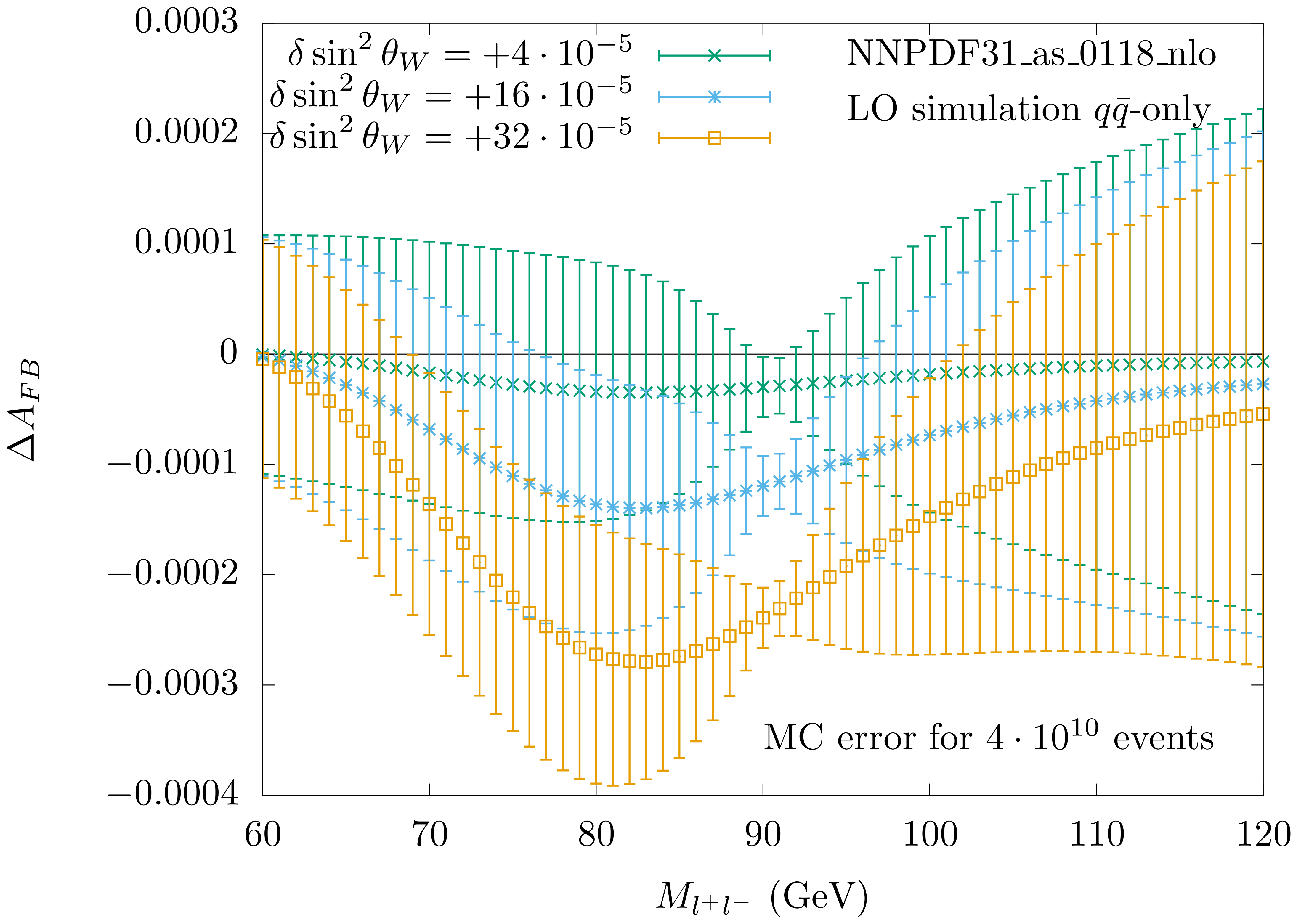
$$A_{FB}^{exp}(m_Z^2) - \mathscr{A}_{nonfact} = \frac{3}{4}\mathscr{A}_e\mathscr{A}_f$$

→ a template fit approach in the full SM is needed to analyse the AFB data and offers a well defined procedure

- to extract $\sin^2\theta_{eff}^{lep}$
- to assign the associated theoretical uncertainties

→ we need to be able to prepare templates of $A_{FB}(m_{\ell\ell}^2)$ for different values of $\sin^2\theta_{eff}^{lep}$

# Estimate of $\sin^2\theta_{eff}^{lep}$: template fit approach




$$\chi_i^2 = \sum_{j=1}^{N_{bins}} \frac{(t_j^{(i)} - d_j)^2}{(\sigma_j^{templ})^2 + (\sigma_j^{data})^2} \qquad i = 1,\ldots,N_{templ}$$

t^(i) are templates of the AFB distribution computed at LO, with NNPDF3.1 QCD-only, for different values of $\sin^2\theta_{eff}^{lep}$ labelled by i

d are (pseudo)data

Plotting $\chi^2_i$ as a function of *i* yields a parabola, whose minimum selects the preferred $\sin^2\theta_{eff}^{lep}$ value

The fit is barely sensitive to $\delta\sin^2\theta_{eff}^{lep}$ = 4 $10^{-5}$

A MC statistics 4 times larger would be needed to have clear sensitivity over the whole fitting range [80,100]

# Commonly used electroweak input schemes

$(g, g', v; \lambda)$ + 9 yukawa couplings + 4 CKM param's $\lambda \to m_H = v\sqrt{\lambda/2}$

Different possibilities to express $(g, g', v)$ in terms of measured quantities.

$(g, g', v) \to (\alpha_0, G_\mu, m_Z)$ LEP scheme: minimal parametric uncertainty in the predictions
Z and γ diagrams have their "natural" coupling
$m_W$ and $\sin^2\theta_W$ are predictions, can not be fitted

$\to (G_\mu, m_W, m_Z)$ Gmu scheme: $m_W$ is a free parameter which can be fitted
(introduced at LEP2)

independent of light-quark masses
it reabsorbs large logarithmic corrections

α and $\sin^2\theta_W$ are predictions, can not be fitted

$\to (\alpha_0, m_W, m_Z)$ $\alpha_0$ scheme: dependent on the light-quark masses
receives large logarithmic corrections

In these schemes the weak mixing angle is not an input, is predicted → is fixed → can not be measured
→ we need a scheme with $\sin^2\theta_{eff}^{lep}$ among the input param's

# An electroweak scheme with $(G_\mu, m_Z, \sin^2\theta^\ell_{eff})$ as inputs

M.Chiesa, F.Piccinini, AV, arXiv:1906.11569

The weak mixing angle is related to the left- and right-handed (vector and axial-vector) couplings of the Z boson to fermions

$$\sin^2\theta^l_{eff} = \frac{I^l_3}{2Q_l}\left(1-\frac{g^l_V}{g^l_A}\right) = \frac{I^l_3}{Q_l}\left(\frac{-g^l_R}{g^l_L-g^l_R}\right)$$

The request that the tree-level relation holds to all orders fixes the counterterm for $\sin^2\theta^{lep}_{eff}$ on-shell definition

$$\delta\sin^2\theta^\ell_{eff} = -\frac{1}{2}\frac{g^\ell_L g^\ell_R}{(g^\ell_L-g^\ell_R)^2}\,\mathrm{Re}\left(\frac{\delta g^\ell_L}{g^\ell_L}-\frac{\delta g^\ell_R}{g^\ell_R}\right)$$

The renormalised angle is identified with the LEP leptonic effective weak mixing angle

The Z mass is defined in the complex mass scheme.

$\Delta$r is evaluated with $\sin^2\theta^{lep}_{eff}$ as input and differs from the usual $(\alpha, m_W, m_Z)$ expression

See also D.C.Kennedy, B.W.Lynn, Nucl.Phys.B322, 1; F.M.Renard, C.Verzegnassi, Phys.Rev.D52,1369;
A.Ferroglia, G.Ossola, A.Sirlin, Phys.Lett.B507,147; A.Ferroglia, G.Ossola, M.Passera, A.Sirlin, Phys.Rev.D65 (2002) 113002

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The request that the tree-level relation holds to all orders fixes the counterterm for $\sin^2\theta^{lep}_{eff}$ on-shell definition

$$\delta\sin^2\theta^\ell_{eff} = -\frac{1}{2}\frac{g^\ell_L g^\ell_R}{(g^\ell_L - g^\ell_R)^2}\,\mathrm{Re}\left(\frac{\delta g^\ell_L}{g^\ell_L} - \frac{\delta g^\ell_R}{g^\ell_R}\right)$$

The renormalised angle is identified with the LEP leptonic effective weak mixing angle

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$\Delta$r is evaluated with $\sin^2\theta^{lep}_{eff}$ as input and differs from the usual $(\alpha, m_W, m_Z)$ expression

See also D.C.Kennedy, B.W.Lynn, Nucl.Phys.B322, 1; F.M.Renard, C.Verzegnassi, Phys.Rev.D52,1369;
A.Ferroglia, G.Ossola, A.Sirlin, Phys.Lett.B507,147; A.Ferroglia, G.Ossola, M.Passera, A.Sirlin, Phys.Rev.D65 (2002) 113002

This scheme allows to express any observable as $\mathcal{O} = \mathcal{O}(G_\mu, m_Z, \sin^2\theta^{lep}_{eff})$

so that templates as a function of $\sin^2\theta^{lep}_{eff}$ can be easily generated

→ direct relation between the data and the parameter of interest

→ simple estimate of all the systematic effects, theoretical and experimental

The result of the fit in this scheme can be directly combined with LEP results

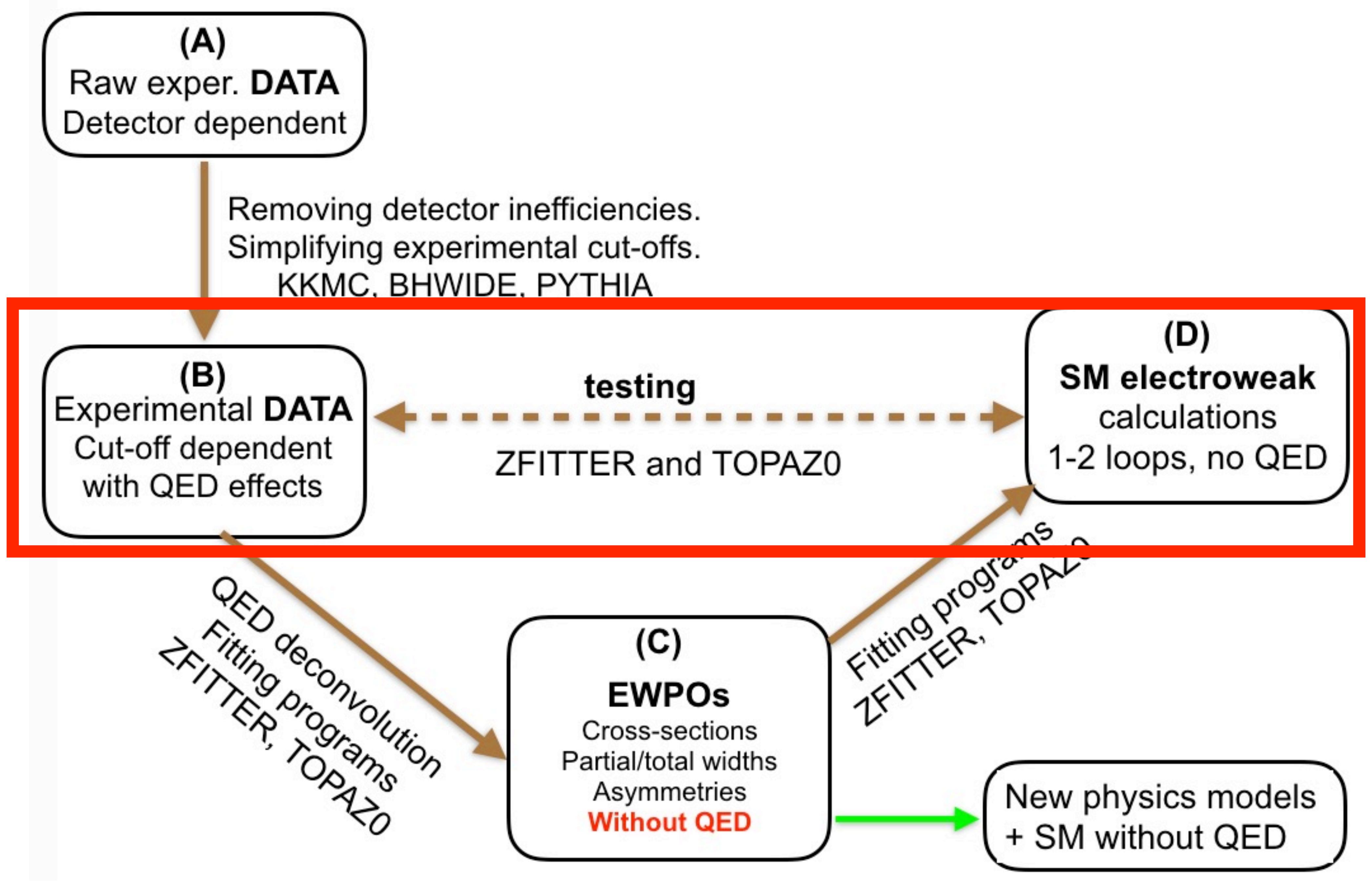
# FCC precision target

see A.Blondel, P.Janot arXiv:2106.13885

| Observable | Present value ± error | FCC-ee stat. | FCC-ee syst. | Comment and leading exp. error |
|---|---|---|---|---|
| $m_Z$ (keV) | 91186700 ± 2200 | **4** | 100 | From Z line shape scan<br>Beam energy calibration |
| $\Gamma_Z$ (keV) | 2495200 ± 2300 | **4** | 25 | From Z line shape scan<br>Beam energy calibration |
| $\sin^2\theta_W^{eff}(\times 10^6)$ | 231480 ± 160 | **2** | 2.4 | from $A_{FB}^{\mu\mu}$ at Z peak<br>Beam energy calibration |
| $1/\alpha_{QED}(m_Z^2)(\times 10^3)$ | 128952 ± 14 | **3** | Small | From $A_{FB}^{\mu\mu}$ off peak<br>QED&EW errors dominate |
| $R_\ell^Z$ $(\times 10^3)$ | 20767 ± 25 | **0.06** | 0.2–1 | Ratio of hadrons to leptons<br>Acceptance for leptons |
| $\alpha_s(m_Z^2)$ $(\times 10^4)$ | 1196 ± 30 | **0.1** | 0.4–1.6 | From $R_\ell^Z$ above |
| $\sigma_{had}^0$ $(\times 10^3)$ (nb) | 41541 ± 37 | **0.1** | 4 | Peak hadronic cross section<br>Luminosity measurement |
| $N_\nu(\times 10^3)$ | 2996 ± 7 | **0.005** | 1 | Z peak cross sections<br>Luminosity measurement |
| $R_b$ $(\times 10^6)$ | 216290 ± 660 | **0.3** | < 60 | Ratio of $b\bar{b}$ to hadrons<br>Stat. extrapol. from SLD |
| $A_{FB}^b, 0$ $(\times 10^4)$ | 992 ± 16 | **0.02** | 1–3 | b-quark asymmetry at Z pole<br>From jet charge |
| $A_{FB}^{pol,\tau}$ $(\times 10^4)$ | 1498 ± 49 | **0.15** | <2 | $\tau$ polarization asymmetry |

| Observable | Present value ± error | FCC-ee stat. | FCC-ee syst. | Comment and leading exp. error |
|---|---|---|---|---|
| | | | | $\tau$ decay physics |
| $\tau$ lifetime (fs) | 290.3 ± 0.5 | **0.001** | 0.04 | Radial alignment |
| $\tau$ mass (MeV) | 1776.86 ±0.12 | **0.004** | 0.04 | Momentum scale |
| $\tau$ leptonic $(\mu\nu_\mu\nu_\tau)$ B.R. (%) | 17.38 ±0.04 | **0.0001** | 0.003 | e/$\mu$/hadron separation |
| $m_W$ (MeV) | 80350 ± 15 | **0.25** | 0.3 | From WW threshold scan<br>Beam energy calibration |
| $\Gamma_W$ (MeV) | 2085 ± 42 | **1.2** | 0.3 | From WW threshold scan<br>Beam energy calibration |
| $\alpha_s(m_W^2)(\times 10^4)$ | 1170 ± 420 | **3** | Small | from $R_\ell^W$ |
| $N_\nu(\times 10^3)$ | 2920 ± 50 | **0.8** | Small | Ratio of invis. to leptonic<br>in radiative Z returns |
| $m_{top}$ (MeV/c$^2$) | 172740 ± 500 | **17** | Small | From $t\bar{t}$ threshold scan<br>QCD errors dominate |
| $\Gamma_{top}$ (MeV/c$^2$) | 1410 ± 190 | **45** | Small | From $t\bar{t}$ threshold scan<br>QCD errors dominate |
| $\lambda_{top}/\lambda_{top}^{SM}$ | 1.2 ± 0.3 | **0.10** | Small | From $t\bar{t}$ threshold scan<br>QCD errors dominate |
| ttZ couplings | ± 30% | **0.5–1.5%** | Small | From $\sqrt{s} = 365$ GeV run |

# The LEP/SLD legacy: $\sin^2\theta_{eff}^{lep}$ determination; two distinct approaches ($m_t, m_H$ fit)

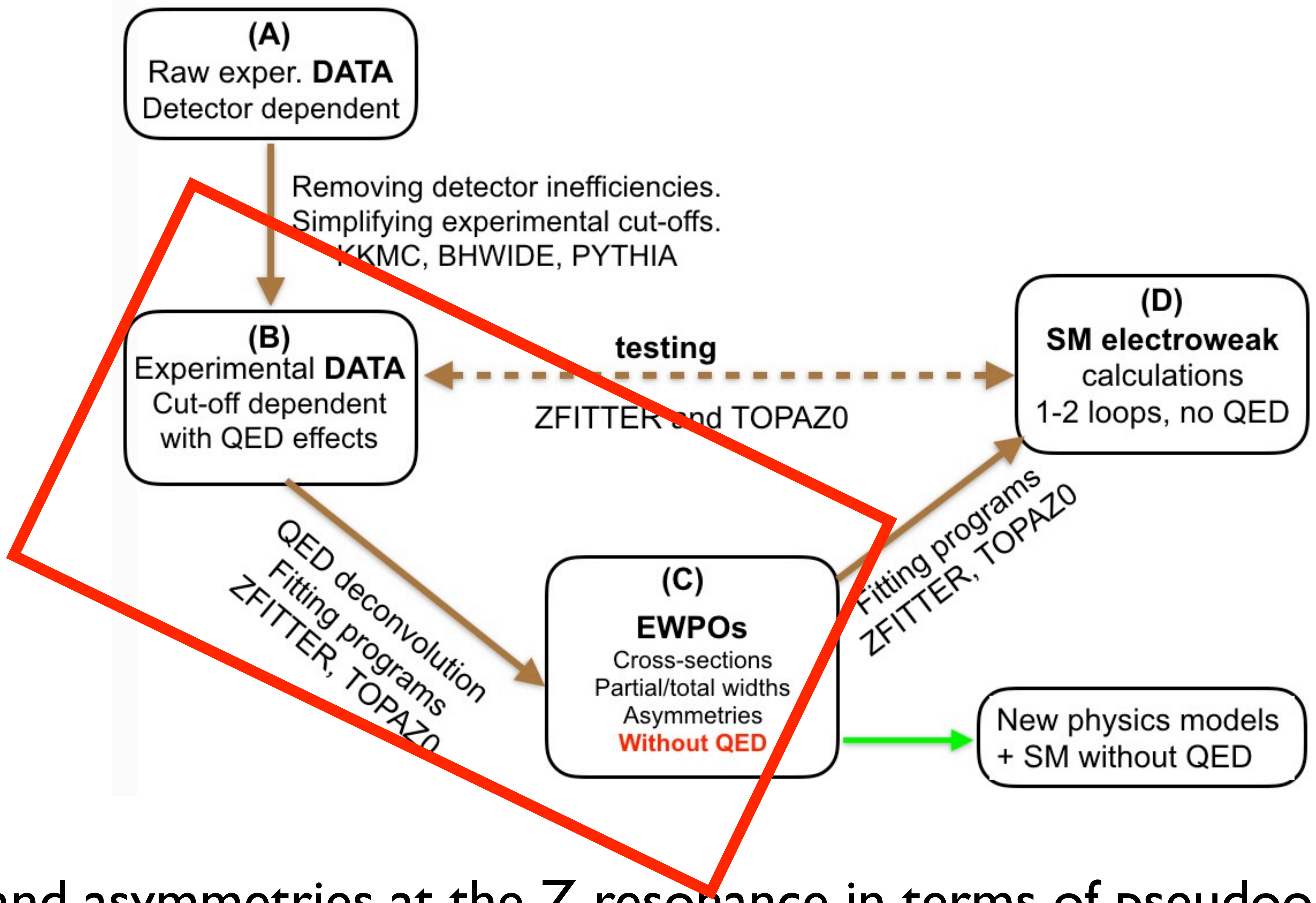



- SM prediction of xsecs and asymmetries computed as a function of $(\alpha, G_\mu, m_Z; m_t, m_H)$

- $m_t$ and $m_H$ fit to the data to maximise the agreement

- $\sin^2\theta_{eff}^{lep}$ has then been computed in the SM using Zfitter/TOPAZ0 with best $m_t$ and $m_H$ values and compared with the pseudo observable determination (next slide)

# The LEP/SLD legacy: $\sin^2\theta_{eff}^{lep}$ determination; two distinct approaches (pseudoobservables)




- parameterisation of xsecs and asymmetries at the Z resonance in terms of pseudoobservables (≠ SM observables)

$$m_Z, \Gamma_Z, \sigma^0_{had}, R^0_e, R^0_\mu, R^0_\tau, A^{0,e}_{FB}, A^{0,\mu}_{FB}, A^{0,\tau}_{FB}$$

- fit of the Z-resonance model to the data → experimental values of the pseudoobservables

- tree-level relation between the experimental Z decay widths (subtracted of QED/QCD effects). and the ratio $g_V/g_A$

  → algebraic solution for $\sin^2\theta_{eff}^{lep}$ → effective angle

# The LEP/SLD legacy: $\sin^2\theta_{eff}^{lep}$ determination

The $\sin^2\theta_{eff}^{lep}$ determination from pseudo-observables at LEP depended on:

- high precision in the measurement of the xsec e+e- → hadrons
- separation of individual flavours
- deconvolution of large universal QED/QCD corrections (Zfitter/TOPAZ0)
- subtraction of SM non-factorisable contributions (Zfitter/TOPAZ0)

  checked to be small, weakly dependent on $\sin^2\theta_{eff}^{lep}$ and precise compared to the LEP/SLD precision target

→ factorised expression (initial)x(final) form factors

$$A_{FB}^{exp}(m_Z^2) - \mathscr{A}_{nonfact} = \frac{3}{4}\mathscr{A}_e\mathscr{A}_f$$

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- The model of the Z resonance in terms of factorised pseudo observable (≠SM) contains $\sin^2\theta_{eff}^{lep}$ as extra free parameter
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At future e+e- colliders we (still) have to demonstrate that all the above hypotheses hold

we possibly need 3-loop calculation to control the subtraction terms arXiv:1901.02648, 1906.05379

and to define the pseudoobservables

All the pseudoobservables at the Z resonance known at full 2-loop EW I.Dubovyk, A.Freitas, J.Gluza, T.Riemann, J.Usovitsch, arXiv:1906.08815

# A proposal

Thanks to the impressive progress in computing and relying on a scheme where $\sin^2\theta_{eff}^{lep}$ appears among the inputs we can analyse FCC-ee data around the Z resonance using a template fit approach, as in M.Chiesa, F.Piccinini, AV, arXiv:1906.11569

Pro's

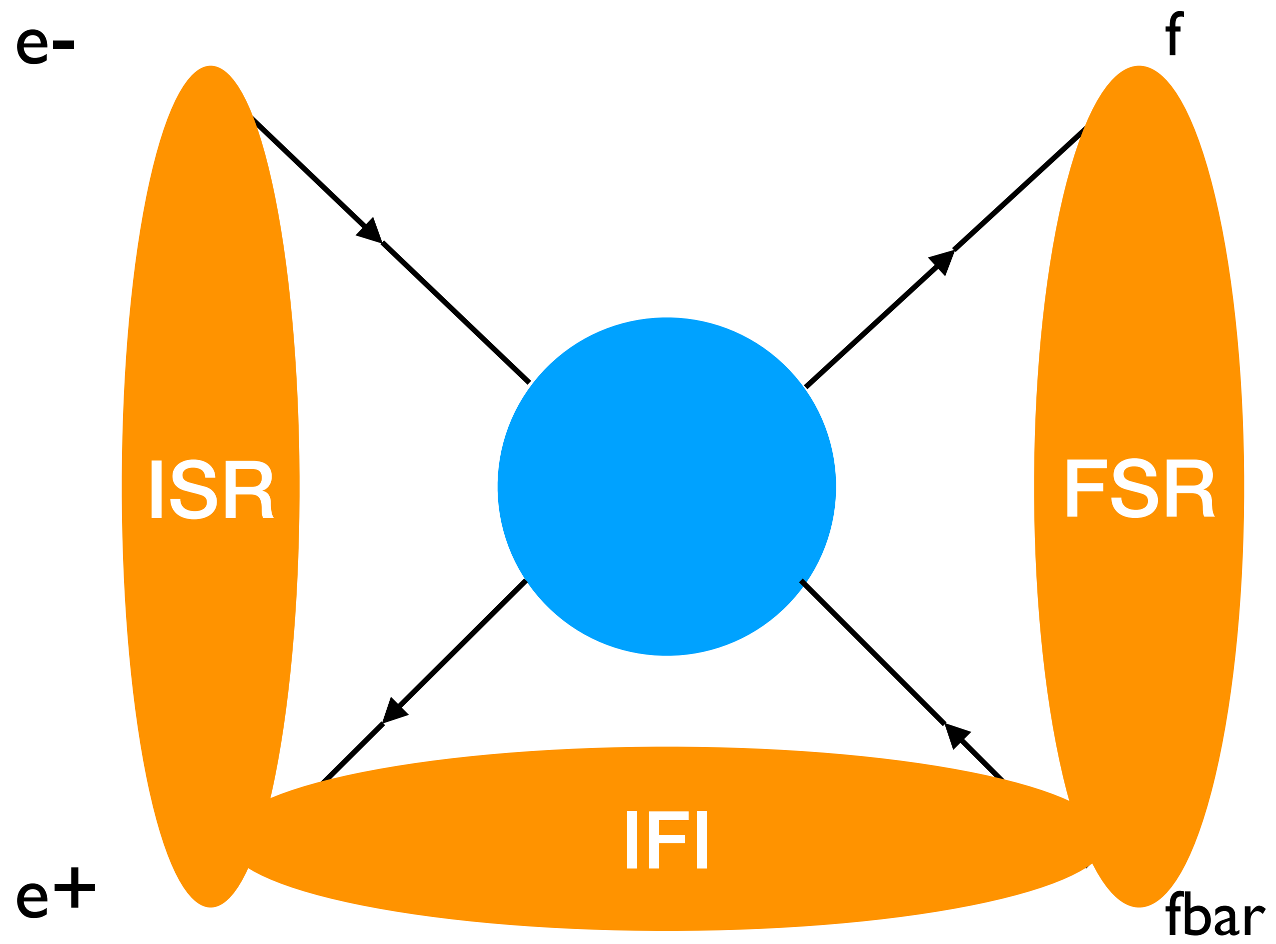
- no need to deconvolute QED effects (problematic beyond LL )
- no need to subtract non-factorizable corrections (and in any case one has to compute the difficult corrections!)
  → robust and uniquely defined SM description of the observables (xsecs and asymmetries)
- direct access to $\sin^2\theta_{eff}^{lep}$ and direct estimation of the associated uncertainties
- possibility to repeat the analysis at different energies (thanks to exact dependence on energy, no resonance expansion)

Con's or ?

- this approach provides "only" a consistency test of the SM: the best $\sin^2\theta_{eff}^{lep}$ value in that hypothesis and the associated $\chi^2$
  → need to workout a similar analysis tool in SMEFT to repeat the same study
- the precision of the templates must reach an outstanding level → reduction of MC fluctuations = very CPU intensive

# Theoretical and computational challenges

# QED factorisation in the radiative corrections to e+e- → f fbar




The largest QED corrections are associated to soft and/or collinear emissions:

L=log($s/m_e^2$)~24, $\ell$=(δE/E)

Factorisation properties of the soft and/or collinear amplitudes
allow to separate the bulk of the QED corrections from the hard scattering process

The inclusion of non-factorizable terms, potentially large, requires a complex dedicated study

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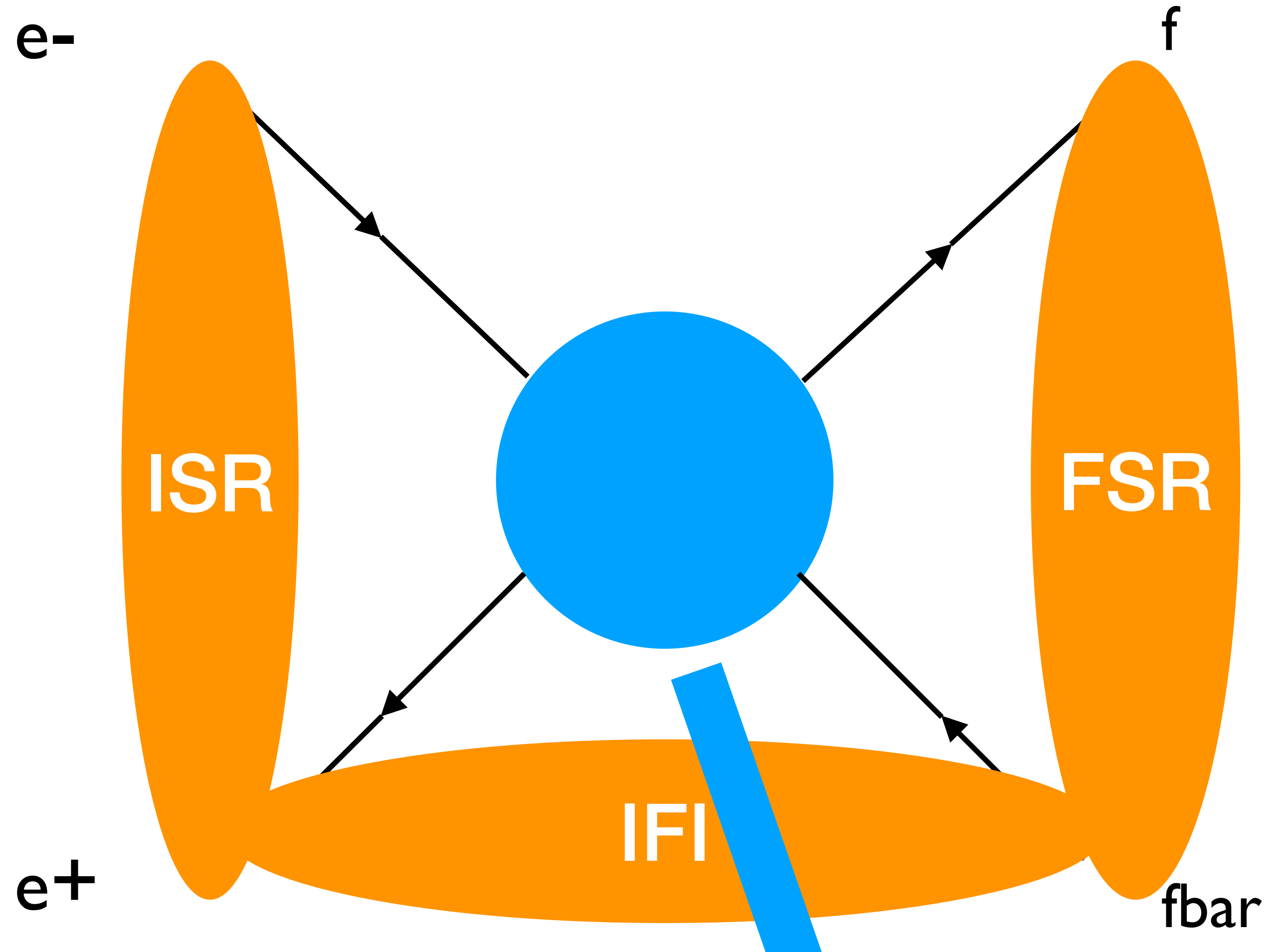


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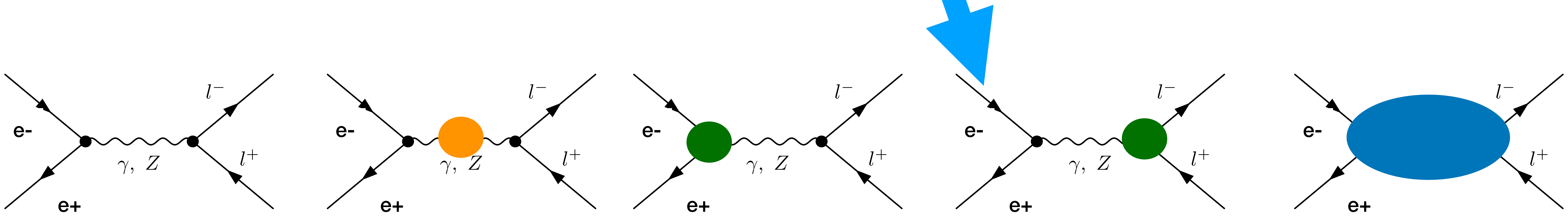



Let us discuss as a complete example the NNLO QCD-EW corrections to NC Drell-Yan, preliminary to NNLO-EW

# The Neutral Current Drell-Yan cross section in the SM: perturbative expansion

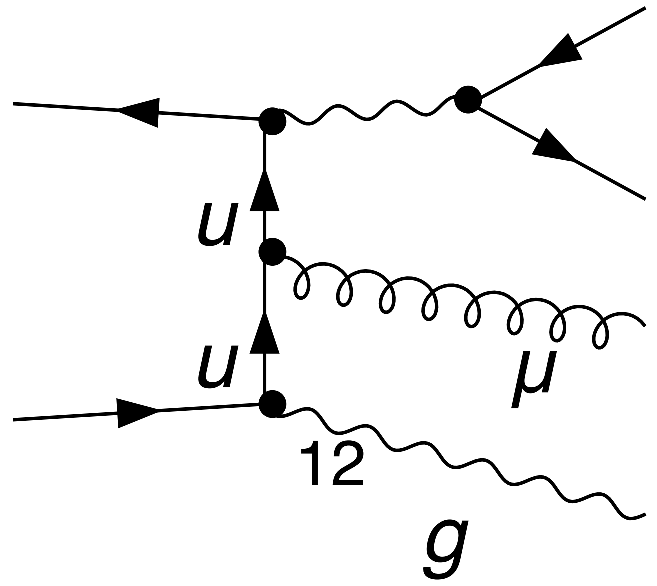
$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} + \alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} + \alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} + \alpha_s^3\,\sigma^{(3,0)} + \ldots$$

$$\sigma(h_1 h_2 \to l\bar{l} + X) = \sum_{i,j=q\bar{q},g,\gamma} \int dx_1\, dx_2\, f_i^{h_1}(x_1,\mu_F) f_j^{h_2}(x_2,\mu_F)\, \hat{\sigma}(ij \to l\bar{l} + X)$$

$\sigma^{(1,1)}$ requires the evaluation of the xsecs of the following processes, including photon-induced

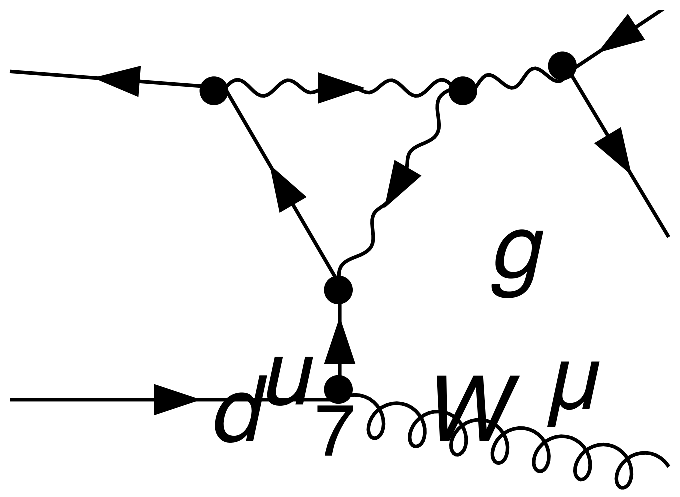
| | | |
|---|---|---|
| 0 additional partons | $q\bar{q} \to l\bar{l},\ \gamma\gamma \to l\bar{l}$ | including virtual corrections of $\mathcal{O}(\alpha_s), \mathcal{O}(\alpha), \mathcal{O}(\alpha\alpha_s)$ |
| 1 additional parton | $q\bar{q} \to l\bar{l}g,\ qg \to l\bar{l}q$ | including virtual corrections of $\mathcal{O}(\alpha)$ |
| | $q\bar{q} \to l\bar{l}\gamma,\ q\gamma \to l\bar{l}q$ | including virtual corrections of $\mathcal{O}(\alpha_s)$ |
| 2 additional partons | $q\bar{q} \to l\bar{l}g\gamma, qg \to l\bar{l}q\gamma, q\gamma \to l\bar{l}qg, g\gamma \to l\bar{l}q\bar{q}$ | |
| | $q\bar{q} \to l\bar{l}q\bar{q},\ q\bar{q} \to l\bar{l}q'\bar{q}',\ qq' \to l\bar{l}qq',\ q\bar{q}' \to l\bar{l}q\bar{q}',\ qq \to l\bar{l}qq$ | at tree level |

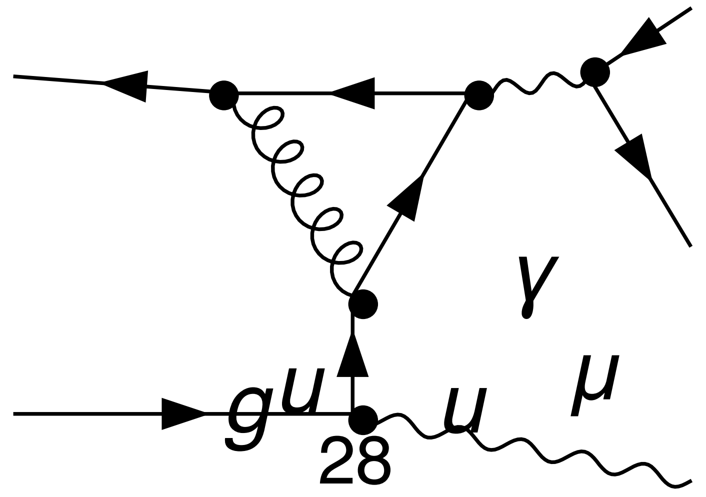
# Different kinds of contributions at $\mathcal{O}(\alpha\alpha_s)$ and corresponding problems




## double-real contributions

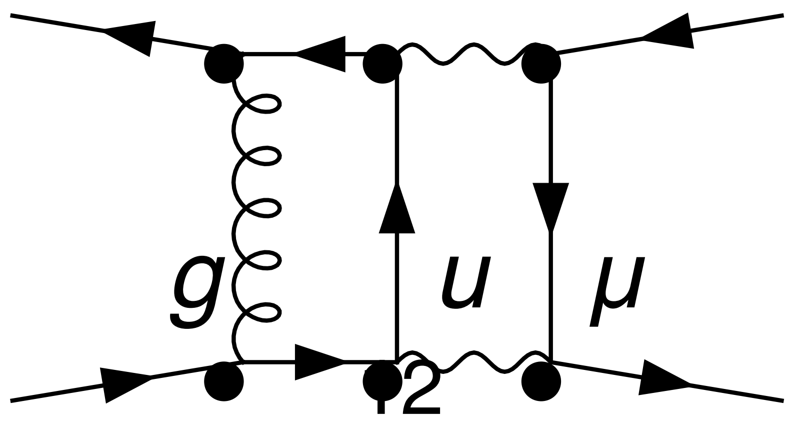
amplitudes are easily generated with OpenLoops
IR subtraction
care about the numerical convergence when aiming at 0.1% precision

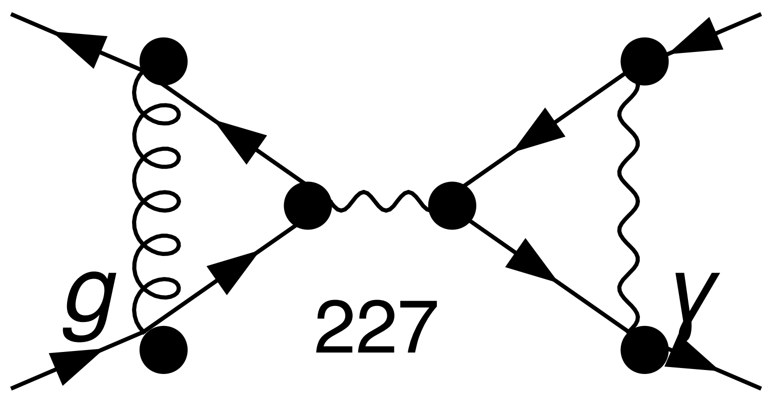



## real-virtual contributions

amplitudes are easily generated with OpenLoops or Recola
1-loop UV renormalisation and IR subtraction
care about the numerical convergence when aiming at 0.1% precision



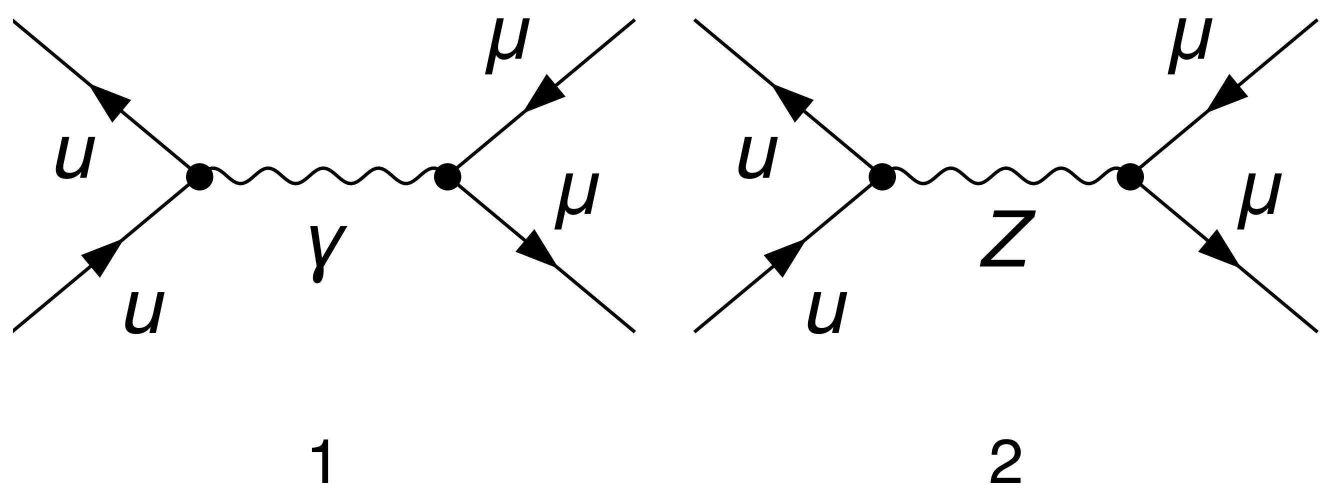




## double-virtual contributions

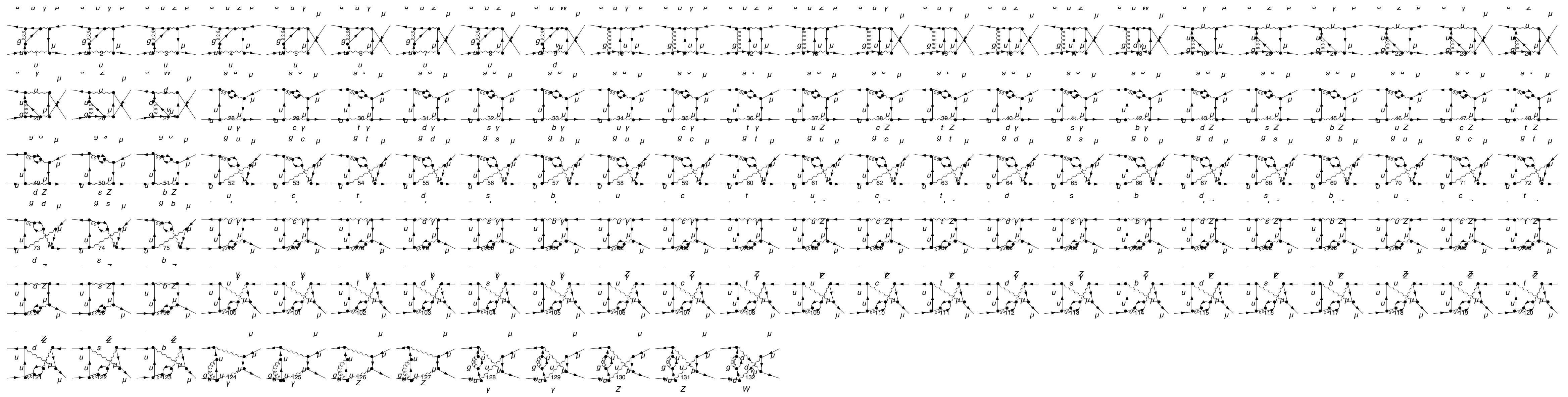
generation of the amplitudes
$\gamma_5$ treatment
2-loop UV renormalization
subtraction of the IR divergences
solution and evaluation of the Master Integrals
numerical evaluation of the squared matrix element

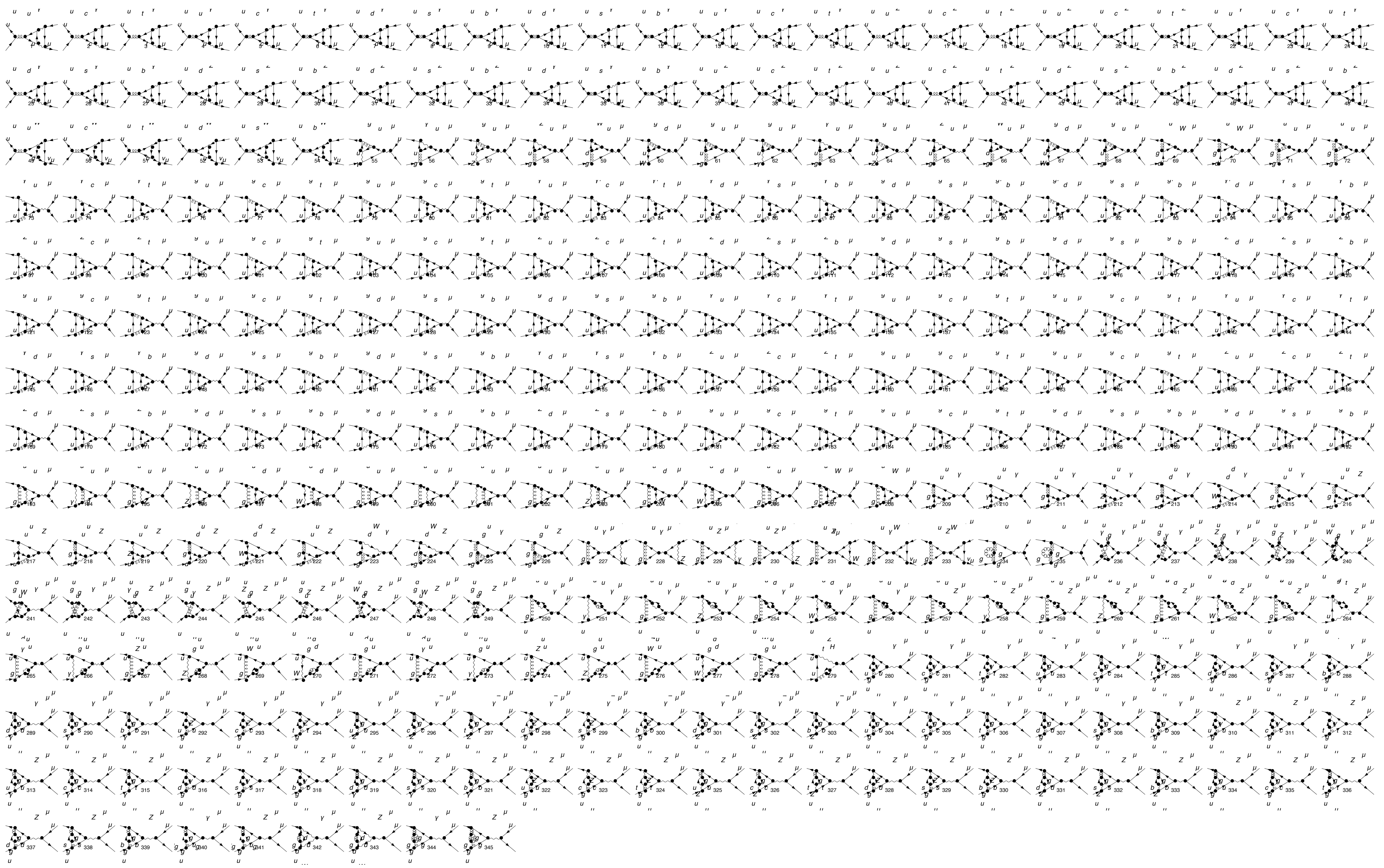
# The double virtual amplitude: generation of the amplitude

$\mathcal{M}^{(0,0)}(q\bar{q} \to l\bar{l}) =$




$\mathcal{M}^{(1,1)}(q\bar{q} \to l\bar{l}) =$ O(1000) self-energies + O(300) vertex corrections +O(130) box corrections + 1loop x 1loop
(before discarding all those vanishing for colour conservation, e.g. no fermonic triangles)





# The double virtual amplitude: reduction to Master Integrals

- The thousands of Feynman integrals present in the amplitude can be reduced to a smaller set of "Master Integrals"

$$2\ \mathrm{Re}\left(\mathcal{M}^{(1,1)}(\mathcal{M}^{(0,0)})^{\dagger}\right) = \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\ \mathcal{T}_i(s,t,m;\varepsilon)$$

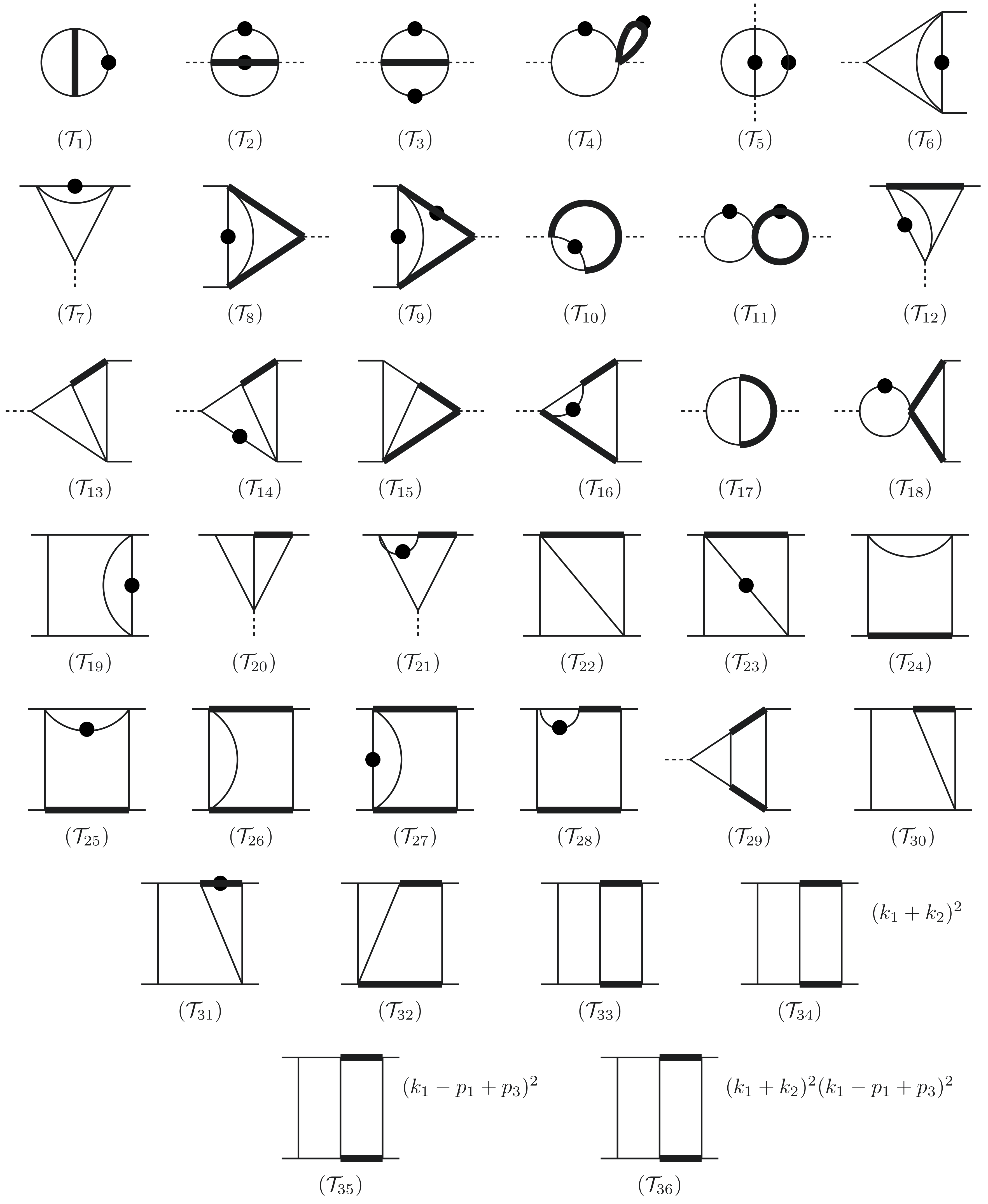
- The coefficients $c_i$ are rational functions of the invariants, masses and of $\varepsilon$

  The size of the individual expressions can rapidly "explode" to O(1GB)
  → careful work to identify the patterns of recurring subexpressions keeping the total size in the O(1 MB) range

- The complexity of the MIs depends on the number of energy scales

  In NC DY
  - at NNLO QCD-EW at most 2 internal massive lines with the same mass value
  - at NNLO-EW we may have up to 7 internal massive lines + 2 external massive lines

- Since W and Z are unstable, we must deal with complex-valued masses in the integrals




2-masses MIs

# Evaluation of the Master Integrals by series expansions

T.Armadillo, R.Bonciani, S.Devoto, N.Rana, AV, 2205.03345

The Master Integrals satisfy a system of differential equations. → solution by series expansion.

The package DiffExp by M.Hidding, arXiv:2006.05510 implements this idea, for real valued masses, with real kinematical vars.

But we need complex-valued masses of W and Z bosons (unstable particles) → we wrote a new package (SeaSyde)

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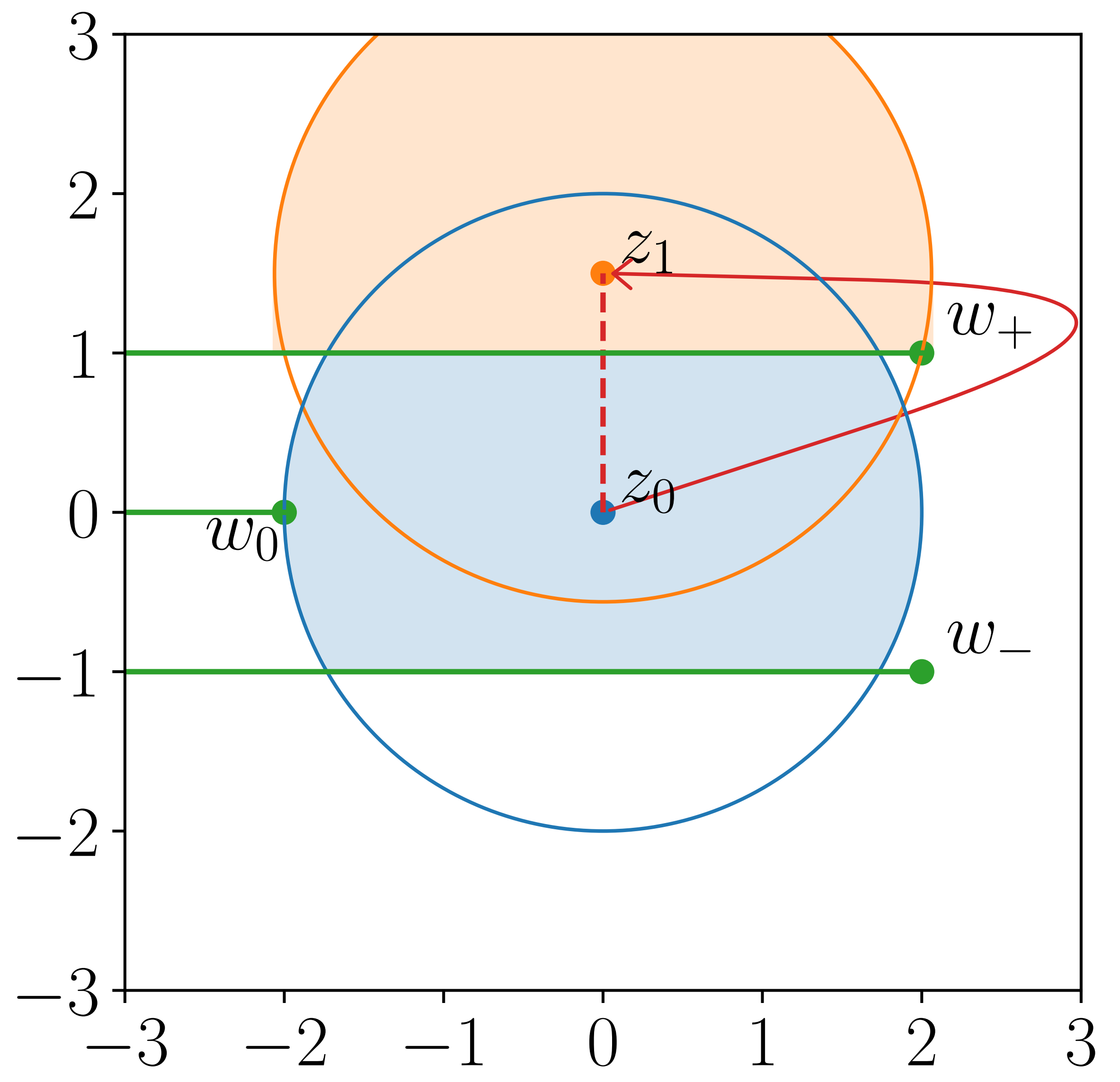
We implemented the series expansion approach, for arbitrary complex-valued masses, working in the complex plane of each kinematical variable, one variable at a time

Complete knowledge about the singular structure of the MI can be read directly from the differential equation matrix

The solution can be computed with an arbitrary number of significant digits, but not in closed form → semi-analytical

Applicable to an arbitrary integrals with any number of internal/external masses → ready for NNLO-EW applications



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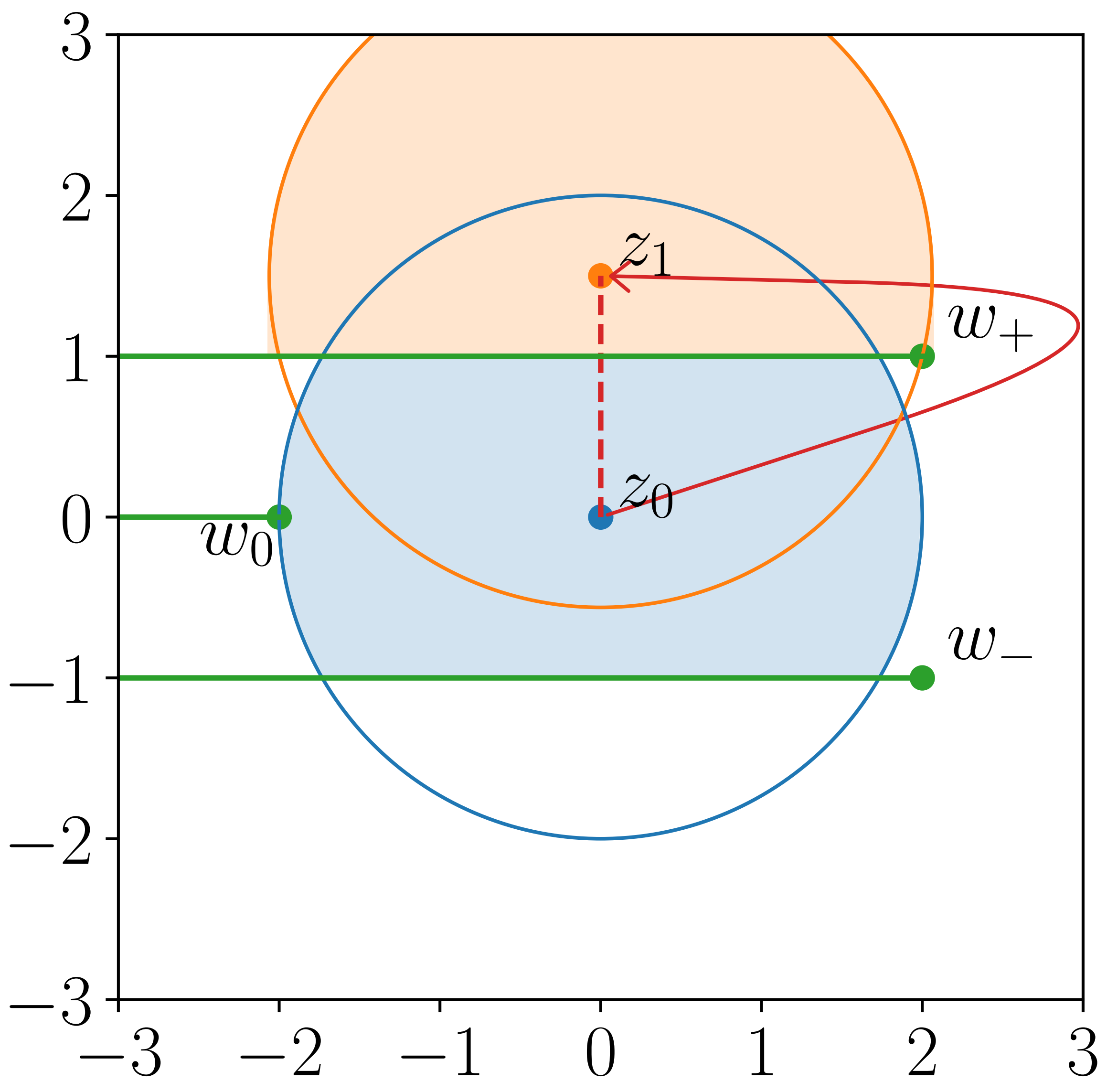


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# Open questions: mass renormalisation scheme at 2-loop EW

resonances require the treatment of the particle decay-width

pole expansions (Laurent expansion of the amplitude) are valid only in the vicinity of the resonances

the complex-mass renormalisation scheme A. Denner, S.Dittmaier, arXiv:hep-ph/0605312
provides a general, gauge invariant, definition of mass:
a complex quantity identified as the pole of the propagator in the complex $q^2$ plane

$$\mu_W^2 = M_W^2 - iM_W\Gamma_W \qquad \mu_Z^2 = M_Z^2 - iM_Z\Gamma_Z$$

$$\delta\mu_V^2 = \Sigma_{VV}(\mu_V^2) \qquad \delta\mathcal{Z}_V = -\Sigma'_{VV}(\mu_V^2)$$

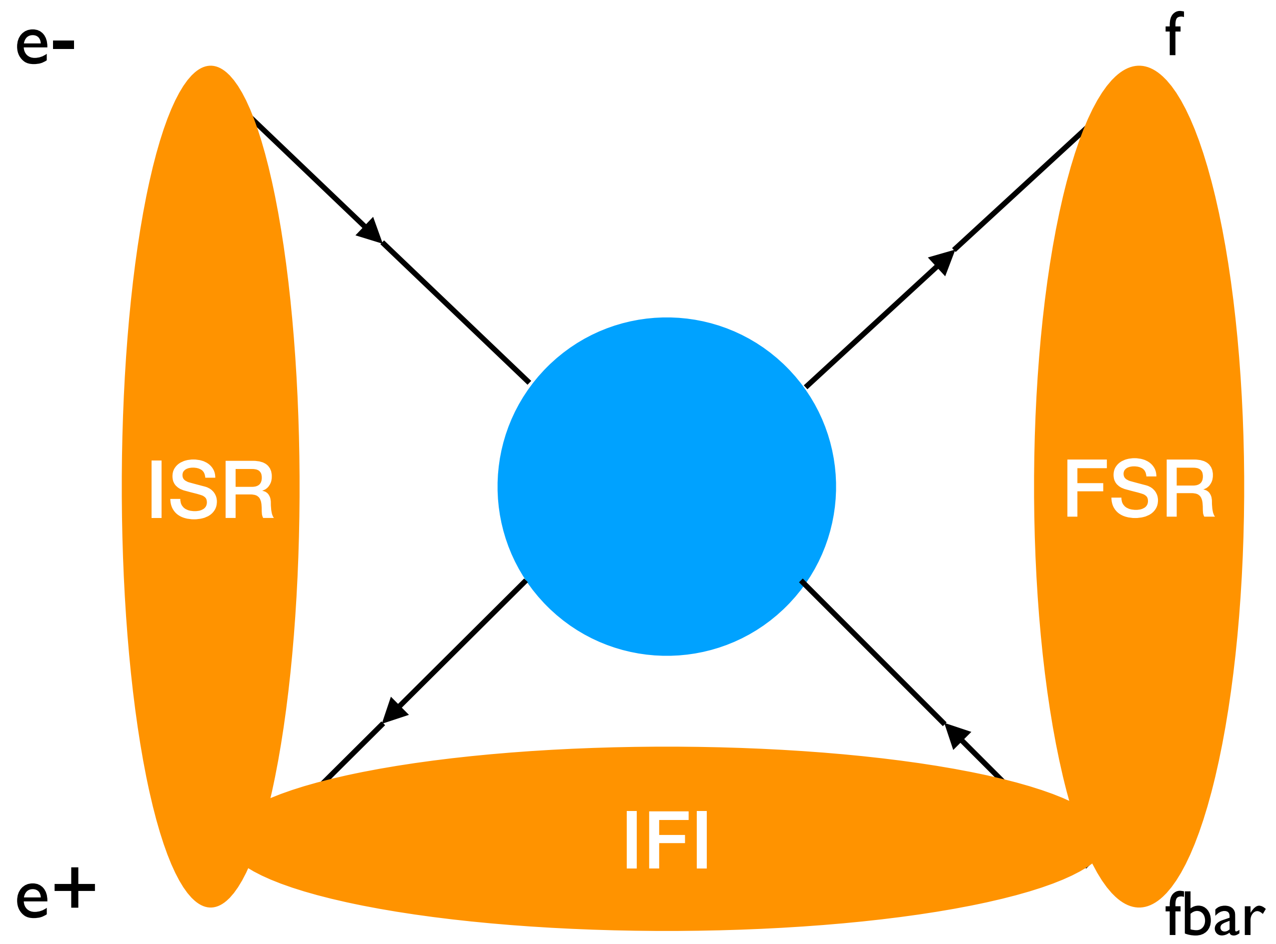
it is formally proven in general (Ward identities satisfied by the Green's functions)
but it requires a careful handling
of all the imaginary parts of the amplitudes and of the renormalised parameters
(e.g. evaluation of the self-energies at complex $q^2$
avoid double counting of self-energy and vertex terms already present in the complex mass)

not yet systematically explored beyond NLO-EW

need to evaluate the remaining theoretical ambiguities in the mass definition

# QED factorisation in the radiative corrections to e+e- → f fbar

cfr. Snowmass 2021 S.Frixione, E.Laenen et al., 2203.12557




Different approaches to
the evaluation to all orders of QED corrections and for the matching with fixed-order calculations:

1) flux functions (ZFITTER)
2) QED Parton Shower solution of DGLAP equations matched at NLO-EW (BabaYaga/HORACE)
3) CEEX
4) MC@NLO

# Leptonic Parton Distribution Functions

S.Frixione, 1909.03886,
V.Bertone, M.Cacciari, S.Frixione, G.Stagnitto, arXiv:1911.12040
V.Bertone, M.Cacciari, S.Frixione, G.Stagnitto, M.Zaro, arXiv:2207.03265

$$\sigma(l^+l^- \to f\bar{f} + X) = \sum_{i,j=e^-,e^+,\gamma,q} \int dx_1\, dx_2\, f_i^{l^+}(x_1,\mu_F) f_j^{l^-}(x_2,\mu_F)\, \hat{\sigma}(ij \to f\bar{f} + X)$$

· Parton Distribution Functions for the leptons

→ allow to introduce the collinear factorisation formalism in the description of e+e- collisions

→ contrary to the proton case, the initial conditions of the DGLAP equations can be computed from first principles

→ every lepton has a partonic content in terms of (electron, positron, photon, quarks )

→ the resummation to all orders of the initial state collinear logs is available at NLL via DGLAP
(NNLL, N3LL yet to come, possible thanks to the corresponding results in QCD)

· Questions:

- which resummation (soft vs collinear) has the largest impact on the ultimate precision for the Z lineshape prediction ?

cfr. Snowmass 2021 S.Frixione, E.Laenen et al., 2203.12557

- is the matching between all-orders QED and fixed-order EW understood, in presence of unstable particles ?

# Conclusions

The precision determination of one EW parameter like $\sin^2\theta_{eff}^{lep}$ or $\sin^2\hat{\theta}(\mu_R)$ is a useful illustration of the problems arising when we consider the ultimate combination of the results obtained at different experiments

- importance of a unique definition → need for a scheme which includes the very same weak mixing angle as input

- SM corrections can fake a contribution → best SM predictions (N3LO-EW ? ) can remove the mismatch

- the $\sin^2\hat{\theta}(\mu_R)$ running can be exploited for a powerful test of the SM
  → relevance of low- and high-mass determinations
  → an additional possibility to exploit the FCC-ee precision at all available energies

to be done:
Completion of some the most challenging calculations in the EW SM and in QFT in general

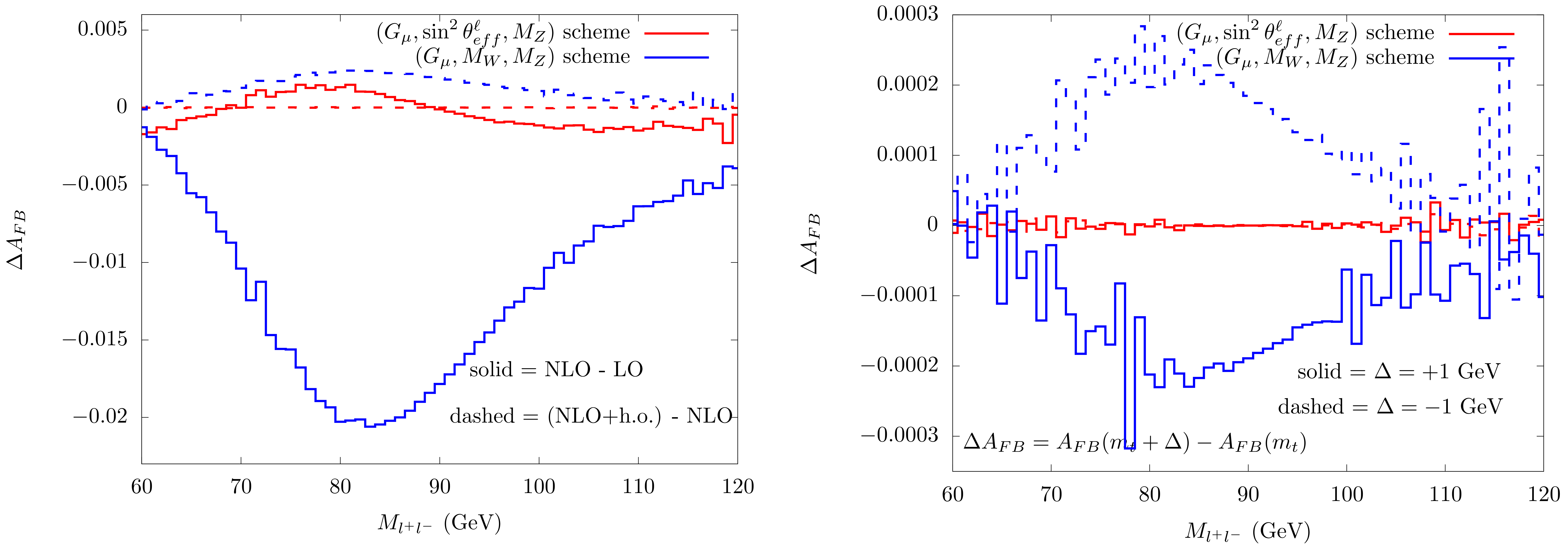
Development of a framework for the description of multiple QED and QCD radiation and matching with fixed-order results

Preparation of efficient tools for the generation of O(10^10) events needed for a precise fit

# Thank you

# $A_{FB}$ $m_t$ parametric uncertainty and perturbative convergence

M.Chiesa, F.Piccinini, AV, arXiv:1906.11569




prediction for $A_{FB}$ at the LHC in the $(G_\mu, m_Z, \sin^2\theta^\ell_{eff})$ input scheme (red), comparison with $(G_\mu, m_W, m_Z)$ (blue)

faster perturbative convergence → good control over the systematic uncertainties of the templates used to fit the data

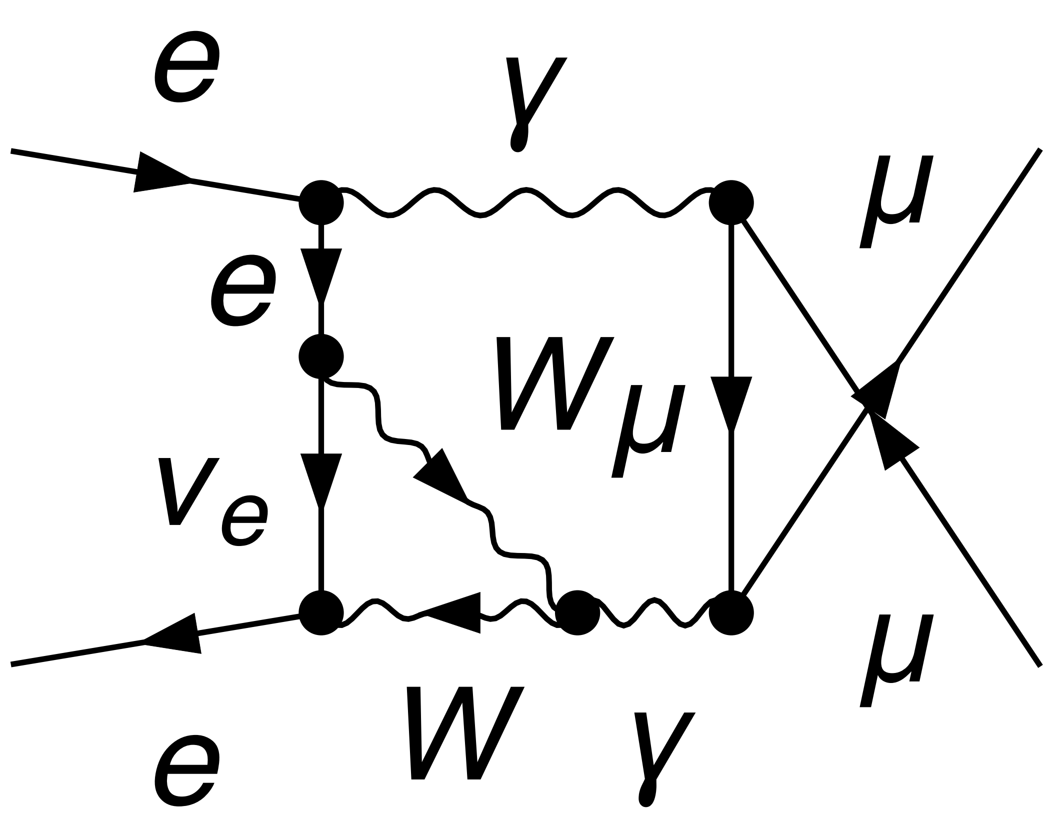
very weak parametric $m_t$ dependence

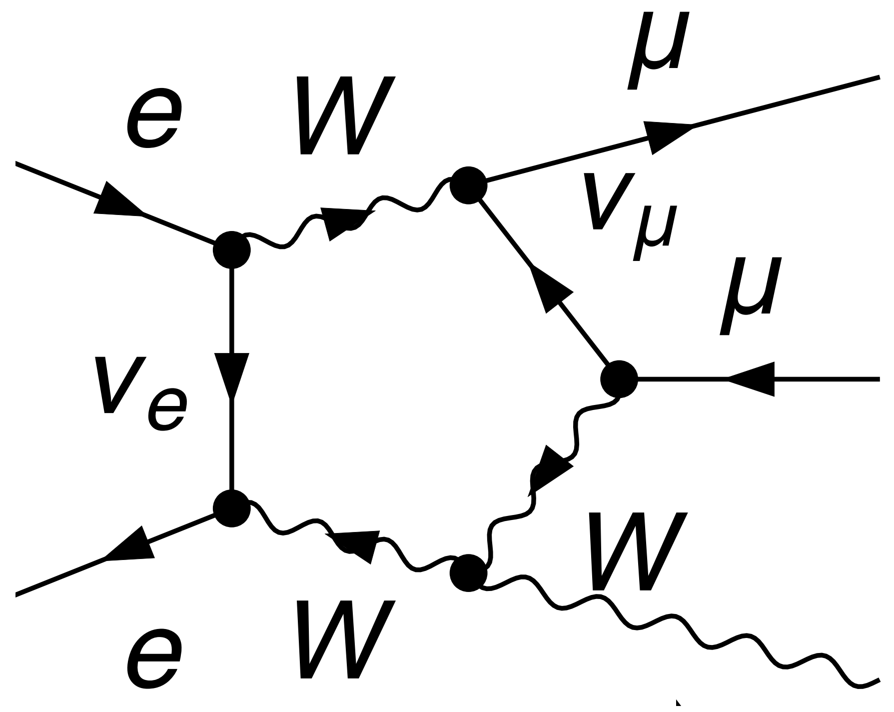
$(G_\mu, m_Z, \sin^2\theta^\ell_{eff})$ offer a very effective parameterisation of the Z resonance in terms of normalisation, position, shape

# Open questions: matching NNLO-EW with QED resummation

in the CEEX matching approach, we need to
identify the matching coefficients $\hat{\beta}_n^{(r)}$ between the full calculation and the soft-exponentiated xsec
→ identification of the relevant gauge invariant subsets of the amplitude

The coupling of photons and Ws must be handled with care
(respect gauge invariance and avoid double counting of imaginary parts
when the virtual corrections are included)




recipes devised at NLO-EW level must be extended at NNLO-EW level, in the complex mass scheme

# Matching schemes in the EW sector

ZFITTER flux functions, radiator functions

The complete scattering is described (LEP approach in ZFITTER) as
the convolution of a hard scattering cross section with flux functions

$$\sigma(s) = \int ds' \frac{1}{s} \rho(\frac{s'}{s}) \sigma(s') \qquad \rho = \rho_{ISR} + \rho_{FSR} + \rho_{IFI}$$

The flux functions encode the angular dependence of the final state recoiling against radiation.
have been computed at exact O(α) with soft photon exponentiation,
for ISR/FSR/IFI, inclusive or with cuts

The formulation naturally arises in the construction and dressing of a Born-improved approximation
→ Are the best available flux functions sufficiently precise and flexible?

# Matching schemes in the EW sector

HORACE / BabaYaga matching scheme

$$d\sigma^{\infty}_{matched} = \Pi_S(Q^2) F_{SV} \sum_{n=0}^{\infty} d\hat{\sigma}_0 \frac{1}{n!} \prod_{i=0}^{n} \left( \frac{\alpha}{2\pi} P(x_i)\, I(k_i)\, dx_i\, d\cos\theta_i\, F_{H,i} \right)$$

$$F_{SV} = 1 + \frac{d\sigma^{\alpha,ex}_{SV} - d\sigma^{\alpha,PS}_{SV}}{d\sigma_0}$$

$$F_{H,i} = 1 + \frac{d\sigma^{\alpha,ex}_{H,i} - d\sigma^{\alpha,PS}_{H,i}}{d\sigma^{\alpha,PS}_{H,i}}$$

Monte Carlo event generators for ffbar → f'f'bar production with EW corrections

- multiple photon radiation implemented via QED Parton Shower algorithm
- resummation to all orders of leading logarithms of collinear and soft origin
- matching with exact O(α) matrix elements;
- matrix element corrections applied to all emitted photons (improvement towards O(α²) accuracy)

→ is it possible to formulate a matching at NNLO level ?

# Matching schemes in the EW sector

CEEX (Coherent Exclusive EXponentiation)

$$\sigma^{(r)} = \sum_{n=0}^{\infty} \frac{1}{n!} \int \mathrm{d}\tau_n(p_1+p_2;\ p_3,p_4,\ k_1,\dots,k_n)\ \mathrm{e}^{2\alpha \Re B_4(p_a,\dots,p_d)} \frac{1}{4} \sum_{\text{spin}} \left| \mathfrak{M}_n^{(r)}\,(p,k_1,k_2,\dots k_n) \right|^2$$

$$\mathfrak{M}_n^{(r)}(p,k_1,k_2,k_3,\dots,k_n) = \prod_{s=1}^{n} \mathfrak{s}(k_s) \left\{ \hat{\beta}_0^{(r)}(p) + \sum_{j=1}^{n} \frac{\hat{\beta}_1^{(r)}(p,k_j)}{\mathfrak{s}(k_j)} + \sum_{j_1<j_2} \frac{\hat{\beta}_2^{(r)}(p,k_{j_1},k_{j_2})}{\mathfrak{s}(k_{j_1})\mathfrak{s}(k_{j_2})} + \cdots \right\}$$

· amplitude level exponentiation of the soft-photon emissions

· soft photon contributions exponentiated on top of any amplitude

· collinear contributions and hard process dependent corrections are systematically included order by order in perturbation theory

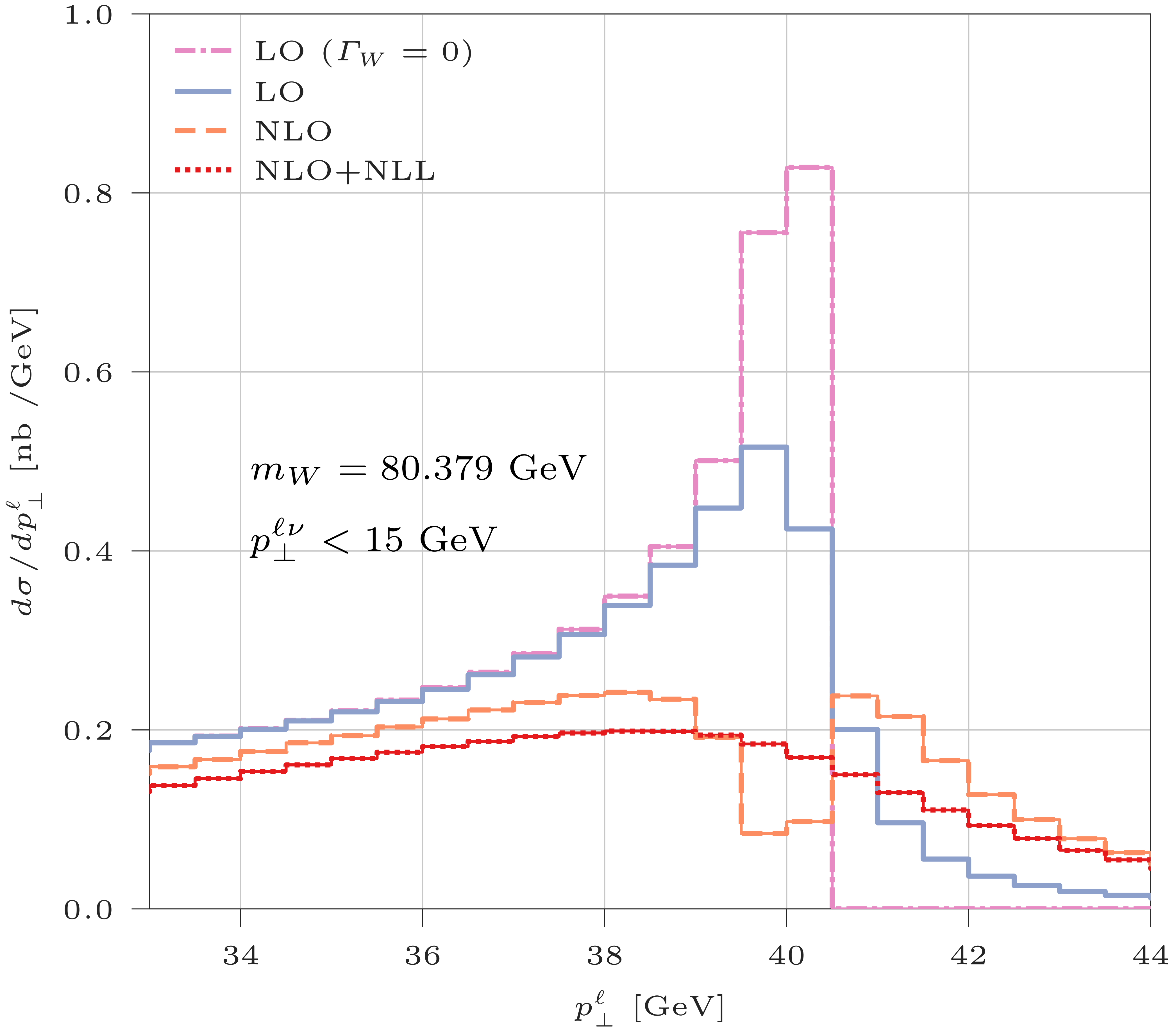
· resummation of ISR mass logarithms not possible in this formalism

KKMC Monte Carlo code for the simulation of fermion-pair production in e+e- annihilation
it includes the full O(α) EW, from DIZET (2→2 process)
exact matrix elements for one- and two-photon emissions in QED,
properly matched with soft-photon exponentiation à la YFS

· Recent developments for the electron mass dependence of second order corrections arXiv:1910.05759

· Discussion about the matching in a full EW calculation (determination of $\hat{\beta}_n^{(r)}$ coefficients)

# MW and the lepton transverse momentum distribution in charged-current Drell-Yan




The lepton transverse momentum distribution has a jacobian peak induced by the factor $1/\sqrt{1-\frac{s}{4p_\perp^2}}$.

When studying the W resonance region, the peak appears at $p_\perp \sim \frac{m_W}{2}$

Kinematical end point at $\frac{m_W}{2}$ at LO

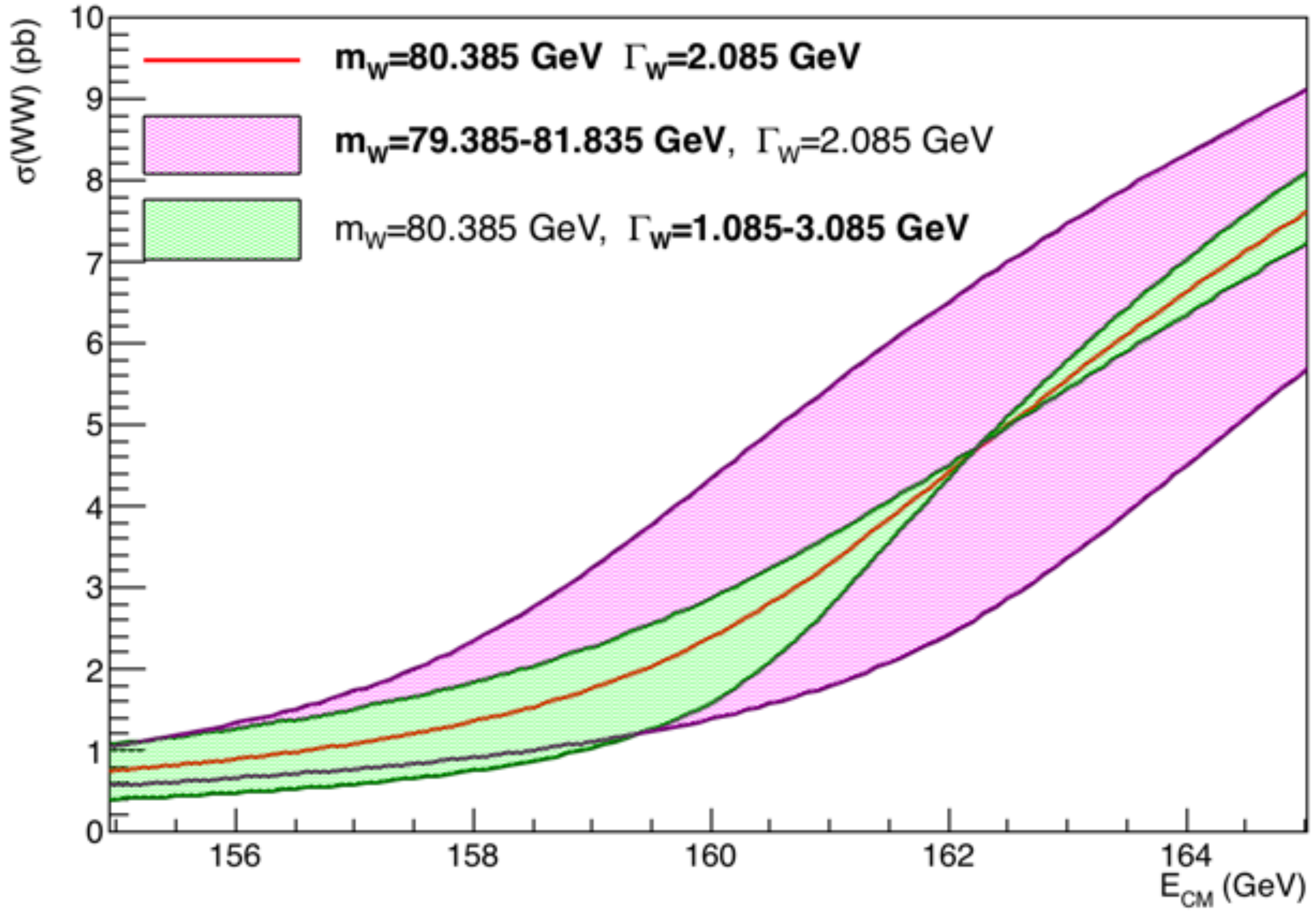
The decay width allows to populate the upper tail of the distribution

Sensitivity to soft radiation → double peak at NLO-QCD

The QCD-ISR next-to-leading-log resummation broadens the distribution and cures the sensitivity to soft radiation at the jacobian peak.

In the $p_\perp^\ell$ spectrum the sensitivity to $m_W$ and important QCD features are closely intertwined

# MW determination from the WW threshold scan




As the cross section at the WW production threshold is very sensitive to the $m_W$ value it is natural to compute the theoretical cross sections in the $(G_\mu, m_W, m_Z)$ input scheme

At threshold in lowest order $$\sigma_0(s) \approx \frac{\pi\alpha^2}{s}\frac{1}{4s_W^4}4\beta + O(\beta^3)$$

As long as $\beta \ll 1$, with low-precision requests,
MW can be determined in model independent way, based on kinematics alone

For a determination at the sub-MeV level, many details have to be considered,
with the preparation of precise SM templates

# MW determination from the WW threshold scan

see arXiv:1903.09895, 1906.05379

With a single point measurement it possible to translate the precision on the xsec into a ΔMW value

$$\Delta\sigma = 0.1\,\% \quad \longrightarrow \quad \Delta M_W = 1.5\ \mathrm{MeV}$$

An experimental precision at the $\Delta\sigma = 0.02\,\%$ is foreseen

Theoretical goal: precision of the theoretical prediction $\Delta\sigma = 0.01\,\%$

The current tools available for these analyses allow the simulation of $e^+e^- \to W^+W^- \to 4f$
at full NLO-EW + higher order Coulomb effects computed in EFT
yielding an uncertainty estimated to be $\Delta M_W \sim 3\ \mathrm{MeV}$

A reduction of $\Delta\sigma$ by one order of magnitude will require
the full NNLO-EW calculation (2→4 process!) matched with 3-loop Coulomb enhanced terms
computable in the EFT contribution

3-loop contributions without enhancement factors are estimated to be negligible

Full 2-loop QCD corrections to hadronic final states will be needed

The mass definition in the CMS and a gauge invariant handling of the imaginary parts at NNLO-EW
will be theoretical / technical points to be discussed
Matching with soft QED exponentiation at NNLO level should also be discussed