L₂ sensitivity paper update

J. Huston Michigan State University for L2 sensitivity authors

Sensitivities of fits of parton distributions to experimental constraints

Xiaoxian Jing,¹ Amanda Cooper-Sarkar,² Aurore Courtoy,³ Thomas Cridge,⁴ Francesco Giuli,⁵ Lucian Harland-Lang,⁶ T. J. Hobbs,⁷ Joey Huston,⁸ Pavel Nadolsky,^{1,*} Keping Xie,⁹ Robert S. Thorne,⁶ and C.-P. Yuan⁸ ¹Department of Physics, Southern Methodist University, Dallas, TX 75275-0181, USA ²Department of Physics, University of Oxford, Oxford, OX1 3RH, UK ³Instituto de Física, Universidad Nacional Autónoma de México, Apartado Postal 20-364, 01000 Ciudad de México, Mexico ⁴Deutsches Elektronen-Synchrotron DESY, Notkestr. 85, Hamburg, 22607, Germany ⁵CERN, CH-1211 Geneva, Switzerland ⁶Department of Physics and Astronomy, University College, London, WC1E 6BT, UK ⁷High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA ⁸Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA ⁹Pittsburgh Particle Physics, Astrophysics and Cosmology Center, Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA (Dated: April 28, 2023) ... in progress

Prelude: uncertainties

- PDF uncertainties depend first on the experimental uncertainties of the data
- Data from two measurements, or even from within the same measurement, can both be very precise, but the result of adding both to the PDF fit can be an increase in the PDF uncertainty (or more likely) a smaller decrease in uncertainty than expected) if the data are in tension with each other
- The resultant PDF uncertainty relies on the definition of a tolerance, i.e. what is a significant increase from the global minimum χ², i.e. PDF uncertainty can be adjusted by changing the tolerance
- $\Delta \chi^2 = 1$ is not applicable for ~4000 data points from different experiments
- NB: CT (Tier 2) and MSHT (dynamic tolerance) have introduced criteria to restrict the pull of data sets that disagree with global fit

What is the L₂ sensitivity?

- For data to influence the PDF fit in a particular region of x and Q², two conditions must be met
 - the parton-level dynamics must depend on a particular PDF (say that of the gluon), as manifested in a statistical correlation
 - the data must have sufficient resolving power to contribute to the PDF likelihood analysis
- The L₂ sensitivity incorporates both of these features
- The L₂ sensitivity is a way of viewing the pulls of all of the experiments used in a global PDF fit, for a particular parton flavor, as a function of a kinematic variable, such as parton x
 - or, when plotted for a PDF luminosity, as a function of the mass
- The fit value for a particular PDF(x,Q) is determined by the sum of these pulls

What is the L₂ sensitivity...continued?

- The L₂ sensitivity provides a visualization of what is happening inside the PDF fit
- It can be considered as a faster version of Lagrange Multiplier scans (but dependent on the Gaussian approximation)
- The L₂ sensitivity streamlines comparisons among independent analyses, using the log-likelihood (χ²) values for the fitted experiments and the error PDFs
- Both the L₂ and LM methods explore the parametric dependence of the χ^2 function in the vicinity of the global minimum
- The L₂ sensitivity has been used internally by CT (in CT18), by the PDF4LHC21 benchmarking group (to determine which data sets should be in the reduced PDF fit used for benchmarking), and now by CT, MSHT and ATLASpdf in this upcoming paper

Error PDFs

- ATLAS, CT and MSHT groups adopt the Hessian format for their PDF error sets
- D error PDFs are used to determine the PDF uncertainty (assuming the probability distribution is approximately Gaussian)
- Consider an expansion of a function X of the parameters R in the vicinity of the global χ^2 minimum X_o

$$X(\vec{R}) = X_0 + \sum_{i=1}^{D} \left. \frac{\partial X}{\partial R_i} \right|_{\vec{R}=\vec{0}} R_i + \frac{1}{2} \sum_{i,j=1}^{D} \left. \frac{\partial^2 X}{\partial R_i \partial R_j} \right|_{\vec{R}=\vec{0}} R_i R_j + \dots$$

$$\left. \frac{\partial X}{\partial R_i} \right|_{\vec{R}=\vec{0}} \approx \frac{X_{+i} - X_{-i}}{2}$$
 use symmetrized form for first order derivative

$$\Delta^{\mathrm{H}} X = \left| \vec{\nabla} X \right| = \frac{1}{2} \sqrt{\sum_{i=1}^{D} \left[X_{+i} - X_{-i} \right]^2} \quad \text{define 68\% CL hypersphere}$$
$$C^{\mathrm{H}}(X,Y) = \frac{1}{4\Delta^{\mathrm{H}} X \Delta^{\mathrm{H}} Y} \sum_{i=1}^{D} \left(X_{+i} - X_{-i} \right) \left(Y_{+i} - Y_{-i} \right) \quad \begin{array}{l} \text{define correlation} \\ \text{between 2 variables} \\ \text{X and Y} \end{array}$$

L₂ sensitivity

$$S_{f,L2}^{\rm H}(E) \equiv \frac{\vec{\nabla}\chi_E^2 \cdot \vec{\nabla}f}{\Delta^{\rm H}f} \\ = \left(\Delta^{\rm H}\chi_E^2\right) \ C^{\rm H}(f,\chi_E^2)$$

2nd Lagrangian technique

 C^H represents the cosine of the correlation angle between PDF flavor f (or any defined quantity) and experimental χ²



The importance of an experiment for a particular PDF depends not only on the correlation of the cross section with that PDF, but the degree to which the cross section can determine that PDF.

• Can also be defined for the MC PDF approach

Estimated χ^2 pulls from experiments (L_2 sensitivity, T. Hobbs et al., arXiv:1904.00222, v. 2)

CT18 NNLO, g(x, 100 GeV)

Plots of L_2 sensitivities to various PDFs:



CT18 NNLO, gluon at Q=100 GeV **15 core-minutes** ----253---- ATL8ZpTbT ----109---- cdhswf3

Most sensitive experiments

542 CMS7jtR7y6T	110 ccfrf2.mi
544 ATL7jtR6uT	147 Hn1X0c
545 CMS8jtR7T	204 e866ppxf
160 HERAIpII	504 cdf2jtCor2
101 BcdF2pCor	
102 BcdF2dCor	
108 cdhswf2	

Experiments with large $\Delta \chi^2 > 0$ [$\Delta \chi^2 < 0$] pull g(x, Q) in the negative [positive] direction at the shown x

P. Nadolsky, PDF4LHC benchm. mtg

Estimated χ^2 pulls from experiments (L₂ sensitivity, T. Hobbs et al., arXiv:1904.00222, v. 2)

CT18 NNLO, g(x, 100 GeV)



Plots of L_2 sensitivities to various PDFs:

https://ct.hepforge.org/PDFs/ct18/figures/L2Sensitivity/

CT18 NNLO, gluon at Q=100 GeV

15 core-minutes

Most sensitive experiments

253	ATL8ZpTbT	109	cdhswf3
542	CMS7jtR7y6T	110	ccfrf2.mi
544	ATL7jtR6uT	147	Hn1X0c
545	CMS8jtR7T	204	e866ppxf
160	HERAIpII	504	cdf2jtCor2
101	BcdF2pCor		
102	BcdF2dCor		

----108---- cdhswf2

Note opposite pulls (tensions) in some x ranges between HERA I+II DIS (ID=160); CDF (504), ATLAS 7 (544), CMS 7 (542), CMS 8 jet (545) production; E866pp DY (204); ATLAS 8 Z pT (253) production; BCDMS and CDHSW DIS

5

CT18 NNLO g(x, <u>100 GeV</u>)



Can also look at L₂ for 2 GeV









TABLE I. Datasets included in the CT18(Z) NNLO global analyses. Here we directly compare the quality of fit found for CT18 NNLO vs CT18Z NNLO on the basis of χ_E^2 , $\chi_E^2/N_{pt,E}$, and S_E , in which $N_{pt,E}$, χ_E^2 are the number of points and value of χ^2 for experiment *E* at the global minimum. S_E is the effective Gaussian parameter [38,42,56] quantifying agreement with each experiment. The ATLAS 7 TeV 35 pb⁻¹ W/Z dataset, marked by \ddagger , is replaced by the updated one (4.6 fb⁻¹) in the CT18Z fits. The CDHSW data, labeled by \ddagger , are not included in the CT18Z fit. The numbers in parentheses are for the CT18Z NNLO fit.

Exp. ID#	Experimental dataset		$N_{pt,E}$	χ^2_E	$\chi_E^2/N_{pt,E}$	S_E
160	HERAI + II 1 fb ^{-1} , H1 and ZEUS NC and	[30]	1120	1408 (1378)	1.3 (1.2)	5.7 (5.1)
	CC $e^{\pm}p$ reduced cross sec. comb.					
101	BCDMS F_2^p	[57]	337	374 (384)	1.1(1.1)	1.4 (1.8)
102	BCDMS $F_2^{\tilde{d}}$	[58]	250	280 (287)	1.1 (1.1)	1.3 (1.6)
104	NMC F_2^d/\tilde{F}_2^p	[59]	123	126 (116)	1.0 (0.9)	0.2 (-0.4)
108^{\dagger}	$CDHSW F_2^{\tilde{p}}$	[60]	85	85.6 (86.8)	1.0 (1.0)	0.1 (0.2)
109†	CDHSW $x_B \tilde{F}_3^p$	[60]	96	86.5 (85.6)	0.9 (0.9)	-0.7(-0.7)
110	CCFR F_2^p	[61]	69	78.8 (76.0)	1.1 (1.1)	0.9 (0.6)
111	CCFR $x_B \tilde{F}_3^p$	[62]	86	33.8 (31.4)	0.4 (0.4)	-5.2 (-5.6
124	NuTeV $\nu\mu\mu$ SIDIS	[63]	38	18.5 (30.3)	0.5 (0.8)	-2.7(-0.9)
125	NuTeV $\bar{\nu}\mu\mu$ SIDIS	[63]	33	38.5 (56.7)	1.2 (1.7)	0.7 (2.5)
126	CCFR $\nu\mu\mu$ SIDIS	[64]	40	29.9 (35.0)	0.7 (0.9)	-1.1(-0.5)
127	CCFR $\bar{\nu}\mu\mu$ SIDIS	[64]	38	19.8 (18.7)	0.5 (0.5)	-2.5(-2.7)
145	H1 σ_r^b	[65]	10	6.8 (7.0)	0.7 (0.7)	-0.6 (-0.6
147	Combined HERA charm production	[66]	47	58.3 (56.4)	1.2 (1.2)	1.1 (1.0)
169	H1 F_L	[33]	9	17.0 (15.4)	1.9 (1.7)	1.7 (1.4)
201	E605 Drell-Yan process	[67]	119	103.4 (102.4)	0.9 (0.9)	-1.0(-1.1)
203	E866 Drell-Yan process $\sigma_{pd}/(2\sigma_{pp})$	[68]	15	16.1 (17.9)	1.1 (1.2)	0.3 (0.6)
204	E866 Drell-Yan process $Q^3 d^2 \sigma_{pp} / (dQ dx_F)$	[69]	184	244 (240)	1.3 (1.3)	2.9 (2.7)
225	CDF run-1 lepton A_{ch} , $p_{T\ell} > 25$ GeV	[70]	11	9.0 (9.3)	0.8 (0.8)	-0.3(-0.2)
227	CDF run-2 electron A_{ch} , $p_{T\ell} > 25$ GeV	[71]	11	13.5 (13.4)	1.2 (1.2)	0.6 (0.6)
234	DØ run-2 muon A_{ch} , $p_{T\ell} > 20 \text{ GeV}$	[72]	9	9.1 (9.0)	1.0 (1.0)	0.2 (0.1)
260	DØ run-2 Z rapidity	[73]	28	16.9 (18.7)	0.6 (0.7)	-1.7 (-1.3
261	CDF run-2 Z rapidity	[74]	29	48.7 (61.1)	1.7 (2.1)	2.2 (3.3)
266	CMS 7 TeV 4.7 fb ⁻¹ , muon A_{ch} , $p_{T\ell} > 35$ GeV	[75]	11	7.9 (12.2)	0.7 (1.1)	-0.6(0.4)
267	CMS 7 TeV 840 pb ⁻¹ , electron A_{ch} , $p_{T\ell} > 35$ GeV	[76]	11	4.6 (5.5)	0.4 (0.5)	-1.6 (-1.3
268 ^{‡‡}	ATLAS 7 TeV 35 pb ⁻¹ W/Z cross sec., A_{ch}	[77]	41	44.4 (50.6)	1.1 (1.2)	0.4 (1.1)
281	DØ run-2 9.7 fb ⁻¹ electron A_{ch} , $p_{T\ell} > 25$ GeV	[78]	13	22.8 (20.5)	1.8 (1.6)	1.7 (1.4)
504	CDF run-2 inclusive jet production	[79]	72	122 (117)	1.7 (1.6)	3.5 (3.2)
514	DØ run-2 inclusive jet production	[80]	110	113.8 (115.2)	1.0 (1.0)	0.3 (0.4)

since first derivative of χ^2
vanishes at the global
minimum, the sum of the
L ₂ sensitivities must be zero
within uncertainties

 $0 < \sum_{E} S_{f,L_2} \ll T^2 < \sum_{E} |S_{f,L_2}|$

TABLE II.	Like Table I, for newly included LHC measurements. The ATLAS 7 TeV W/Z data (4.6 fb ⁻¹), labeled by \ddagger ,	are included i
the CT18A	and CT18Z global fits, but not in CT18 and CT18X.	

Exp. ID#	Experimental dataset		$N_{pt,E}$	χ^2_E	$\chi_E^2/N_{pt,E}$	S_E
245	LHCb 7 TeV 1.0 fb ⁻¹ W/Z forward rapidity cross sec.	[81]	33	53.8 (39.9)	1.6 (1.2)	2.2 (0.9)
246	LHCb 8 TeV 2.0 fb ⁻¹ $Z \rightarrow e^-e^+$ forward rapidity cross sec.	[82]	17	17.7 (18.0)	1.0 (1.1)	0.2 (0.3)
248 [‡]	ATLAS 7 TeV 4.6 fb ⁻¹ , W/Z combined cross sec.	[39]	34	287.3 (88.7)	8.4 (2.6)	13.7 (4.8)
249	CMS 8 TeV 18.8 fb ⁻¹ muon charge asymmetry A_{ch}	[83]	11	11.4 (12.1)	1.0 (1.1)	0.2 (0.4)
250	LHCb 8 TeV 2.0 fb ⁻¹ W/Z cross sec.	[84]	34	73.7 (59.4)	2.1 (1.7)	3.7 (2.6)
253	ATLAS 8 TeV 20.3 fb ⁻¹ , Z p_T cross sec.	[85]	27	30.2 (28.3)	1.1 (1.0)	0.5 (0.3)
542	CMS 7 TeV 5 fb ⁻¹ , single incl. jet cross sec., $R = 0.7$ (extended in y)	[86]	158	194.7 (188.6)	1.2 (1.2)	2.0 (1.7)
544	ATLAS 7 TeV 4.5 fb ⁻¹ , single incl. jet cross sec., $R = 0.6$	[9]	140	202.7 (203.0)	1.4 (1.5)	3.3 (3.4)
545	CMS 8 TeV 19.7 fb ⁻¹ , single incl. jet cross sec., $R = 0.7$, (extended in y)	[87]	185	210.3 (207.6)	1.1 (1.1)	1.3 (1.2)
573	CMS 8 TeV 19.7 fb ⁻¹ , $t\bar{t}$ norm. double-diff. top p_T and y cross sec.	[88]	16	18.9 (19.1)	1.2 (1.2)	0.6 (0.6)
580	ATLAS 8 TeV 20.3 fb ⁻¹ , $t\bar{t} p_T^t$ and $m_{t\bar{t}}$ abs. spectrum	[89]	15	9.4 (10.7)	0.6 (0.7)	-1.1 (-0.8)

Examine the impact of each experiment on the different PDFs



MSHT20 NNLO gluon



MSHT20 and CT18

Note importance of ATLAS Z p_T data (also, $Z p_T$ data poorly fit at NNLO; dynamic tolerance?) CT18 NNLO MSHT20 NNLO g(x, 100 GeV) g(x, 100 GeV) 10 $T^2 = 10$ $T^2 = 10$ 20 5 — 545: CMS 8 TeV jets 10 - 71: ATLAS 8 TeV Ζ p_T sensitivity) — 160: HERA DIS combined $\Delta \chi^2(L_2 \text{ sensitivity})$ 160: HERA DIS Combined — 102: BCDMS F₂^d 4: NMC F₂^d — 504: CDF Run–2 jets 26: HERA e⁺ p NC 920 Ge — 108: CDHSW F₂ 73: CMS 8 TeV jets — 542: CMS 7 TeV jets 66: ATLAS 7 TeV jets - 10 -10 - 20 0.5 0.7 0.01 0.02 0.05 0. 10⁻⁴ 10⁻³ 10^{-4} 10⁻³ 0.01 0.02 0.05 0.1 0.2 0.5 0.7 0.2 х х ATLAS Z p_T not one of 6 most CMS 8 TeV jet data play a similar

important experiments (more restrictive kinematic region)

role as in CT18

MSHT20 NNLO and aN3LO



MSHT20 NNLO and aN3LO



160 fell out of the top 6

ATLASpdf21

-10

 10^{-3}

 10^{-4}

0.01 0.02 0.05 0.1

х

ATLAS PDF fits are based on a more limited set of data, with HERA inclusive as the backbone

series of sequential PDF fits adding W/Z data,ttbar, W/Z+jets, inclusive jets and photon ratio data

full information on correlated systematic sources of uncertainty used (not available to to other PDF fits)

ID	Data set	\sqrt{s} [TeV]	Luminosity [fb ⁻¹]	Decay channel	Observables entering the fit
160	HERA inclusive DIS [26]	Varied	Varied		Reduced cross sections
68	Inclusive $W, Z/\gamma^*$ [27]	7	4.6	e, μ combined	$\eta_\ell \; (W), y_Z \; (Z)$
89	Inclusive Z/γ^* [28]	8	20.2	e, μ combined	$\cos \theta^*$ in bins of $y_{\ell\ell}, m_{\ell\ell}$
86	Inclusive W [29]	8	20.2	μ	η_{μ}
56	$W^{\pm} + \text{jets} \ [30]$	8	20.2	e	p_{T}^W
56	Z + jets [31]	8	20.2	e	$p_{\mathrm{T}}^{\mathrm{jet}}$ in bins of $ y^{\mathrm{jet}} $
$\overline{7}$	$t\bar{t}$ [32, 33]	8	20.2	lepton + jets, dilepton	$m_{tar{t}},\ p_{\mathrm{T}}^t,\ y_{tar{t}}$
8	$t\overline{t}$ [34]	13	36	lepton + jets	$m_{tar{t}},p_{\mathrm{T}}^t,y_t,y_{tar{t}}^{\mathrm{b}}$
9	Inclusive isolated γ [35]	8, 13	$20.2, \ 3.2$	-	E_{T}^{γ} in bins of η^{γ}
10	Inclusive jets [36–38]	7,8,13	$4.5,\ 20.2,\ 3.2$	-	$p_{\mathrm{T}}^{\mathrm{jet}}$ in bins of $ y^{\mathrm{jet}} $



0.2

0.5 0.7

ATLASpdf21:Impact of addition of W/Z, ttbar data to HERA inclusive



Reduced fits (PDF4LHC21);arXiV:2203.05506

Dataset	$N_{ m pt}$	$\chi^2/N_{ m pt}$			
		CT18	MSHT20	NNPDF3.1	fitting CT18'.
BCDMS F_2^p	$329/163^{\dagger\dagger}/325^{\dagger}$	1.06	1.00	1.21	MSHT20' and
BCDMS F_2^d	$246/151^{\dagger\dagger}/244^{\dagger}$	1.06	0.88	1.10	NNPDF3.1'
NMC F_2^d/F_2^p	$118/117^\dagger$	0.93	0.93	0.90	to a common
NuTeV dimuon $\nu + \bar{\nu}$	38 + 33	0.79	0.83	1.22	data set, as
HERAI+II	1120	1.23	1.20	1.22	prelude to the
E866 $\sigma_{pd}/(2\sigma_{pp})$	15	1.24	0.80	0.43	PDF4LHC21
LHCb 7 TeV & 8TeV W,Z	29 + 30	1.15	1.17	1.44	combination
LHCb 8 TeV $Z \rightarrow ee$	17	1.35	1.43	1.57	
ATLAS 7 TeV W,Z (2016)	34	1.96	1.79	2.33	
D0 Z rapidity	28	0.56	0.58	0.62	
CMS 7 TeV electron $A_{\rm ch}$	11	1.47	1.52	0.76	
ATLAS 7 TeV $W, Z(2011)$	30	1.03	0.93	1.01	
CMS 8TeV incl. jet	$185/174^{\dagger\dagger}$	1.03	1.39	1.30	
Total $N_{\rm pt}$		2263	1991	2256	
Total $\chi^2/N_{\rm pt}$		1.14	1.15	1.20	

Table 3.3. Same as Table 3.2, now displaying the results obtained after each group has carried out the corresponding fits to this reduced dataset. That is, the input PDF is now the best-fit value obtained for each group to the reduced dataset rather than the common PDF4LHC15 PDF input used in Table 3.2. ^{††}MSHT [†]NNPDF.



Summary

- L₂ sensitivity is a useful variable to understand what is happening/has happened inside a PDF fit, any tensions that may exist between data sets, and how the fit may be improved in the future
- Paper should be coming out shortly
- Meanwhile, two websites can be used to explore L₂ sensitivity for CT18, MSHT20 (NNLO and aN3LO) and ATLASpdf21
 - https://www.physics.smu.edu/nadolsky/work/pdf4lhc21/L2 sens/index2.html
 - https://www.physics.smu.edu/nadolsky/work/pdf4lhc21/L2 sens/index3.html



I have been in the Remove Trump business so long, now that it's over, I don't know what to do with the rest of my life

Have you ever considered using L_2 sensitivity?



Reduced fits

Reasonable agreement for the most part.



Uncertainties

Uncertainties increased with respect to full global fits



PDF Iuminosities

