

New developments in the APPLfast project

xFitter External meeting | CERN, Geneva, Switzerland

Lucas Kunz (with Fazila Ahmadova, Daniel Britzger, Xuan Chen, Claire Gwenlan, Gudrun Heinrich, Alexander Yohei Huss, João Ramalho Pires, Klaus Rabbertz, Mark R. Sutton) | 04/05/2023

KARLSRUHE INSTITUTE OF TECHNOLOGY

APPLfast



fastNLO

Motivation

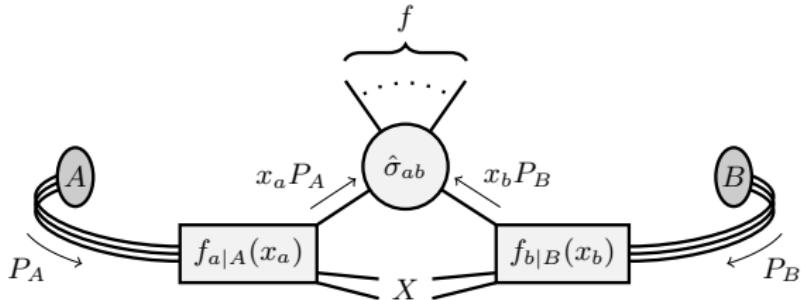


Figure by A. Huss

$$\begin{aligned} d\sigma_{pp \rightarrow X} = & \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b f_a(x_a, \alpha_s(\mu_R), \mu_F) f_b(x_b, \alpha_s(\mu_R), \mu_F) \\ & \times d\hat{\sigma}_{ab \rightarrow X}(x_a, x_b, \alpha_s(\mu_R), \mu_R, \mu_F) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)^p \end{aligned}$$

Motivation

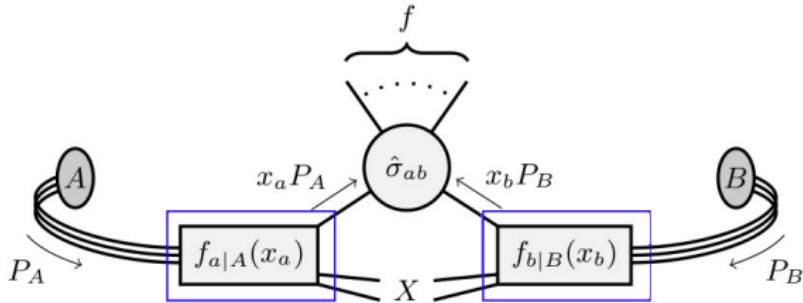


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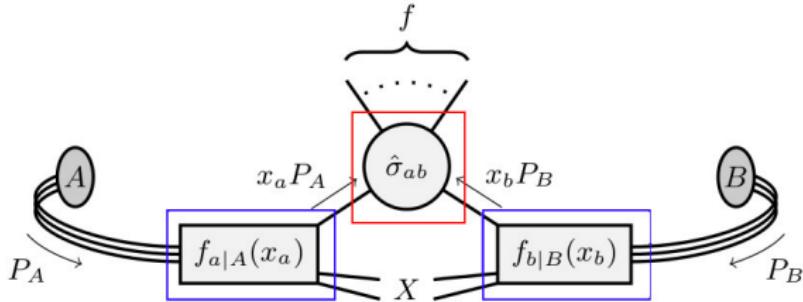


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$$\begin{aligned} d\sigma_{pp \rightarrow X} = & \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \, [f_a(x_a, \alpha_s(\mu_R), \mu_F) f_b(x_b, \alpha_s(\mu_R), \mu_F)] \\ & \times [d\hat{\sigma}_{ab \rightarrow X}(x_a, x_b, \alpha_s(\mu_R), \mu_R, \mu_F)] + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)^p \end{aligned}$$

Motivation

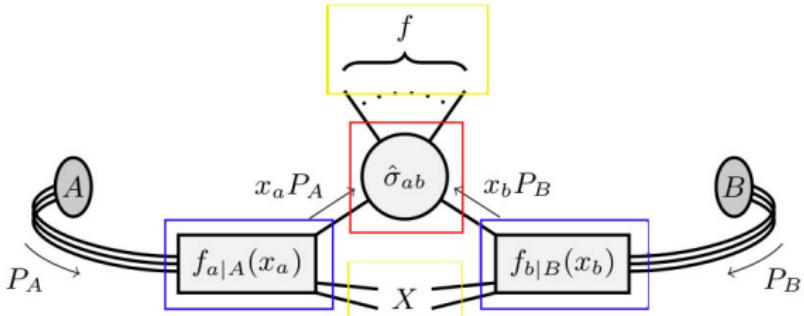
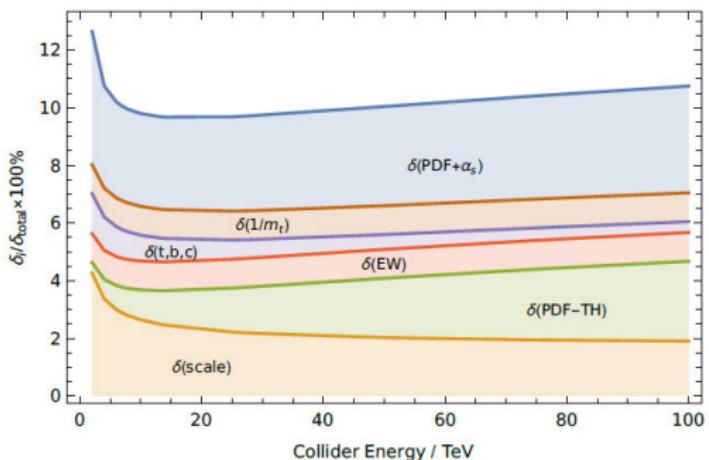


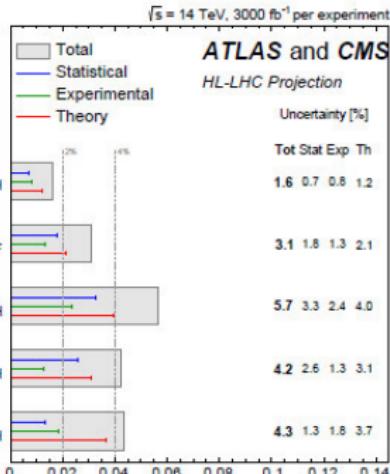
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Motivation



Relative uncertainty of the Higgs boson production cross section
[Dulat, Lazopoulos, Mistlberger '18]



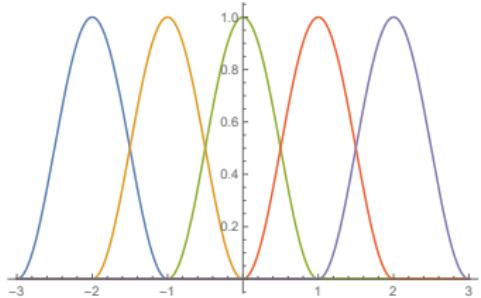
Higgs production uncertainty estimates for the HL-LHC
[HL-LHC Working Group 2 '19]

Grid Technique - Logic

- NNLOJET: fixed order Monte Carlo calculations
- fastNLO/APPLgrid: grid libraries
- APPLfast: interface connecting the two sides



Grid Technique - Interpolation



- Split interval $I = [a, b]$ into $N + 1$ nodes, $a = x^{[0]}$, $b = x^{[N]}$
- Partition of unity into a set of functions:
 - $1 = \sum_{i=0}^N E_i(x) \quad \forall x \in I$
 - $E_i(x^{[i]}) = 1 \quad \forall i \in \{0, \dots, N\}$

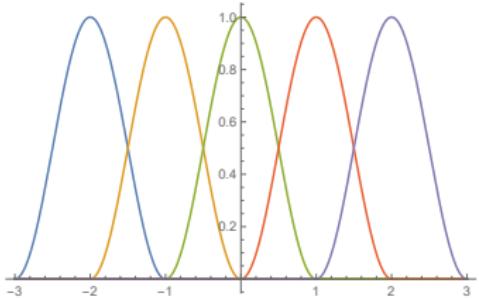
- \Rightarrow Functions on the interval can be approximated:

$$f(x) \simeq \sum_{i=0}^N f^{[i]} E_i(x) \text{ where } f^{[i]} = f(x^{[i]})$$

- \Rightarrow Integrals can also be approximated:

$$\int_a^b f(x)g(x) dx \simeq \sum_{i=0}^N f^{[i]} g_{[i]} \text{ with } g_{[i]} := \int_a^b E_i(x)g(x) dx$$

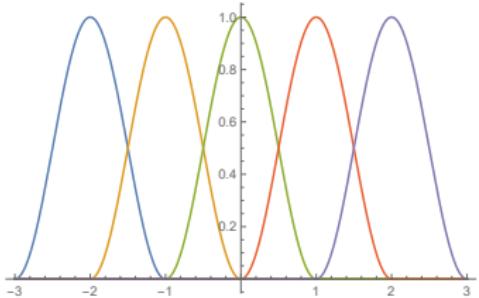
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Grid Technique - Formulae

$$d\hat{\sigma}_{ab \rightarrow X}(x, \alpha_s, \mu) = \sum_k \left(\frac{\alpha_s(\mu_R)}{2\pi} \right)^{k+r} d\hat{\sigma}_{ab \rightarrow X}^{(k)}(x, \alpha_s, \mu)$$

Evaluation with Monte Carlo event generator:

- Fixed-order ($k = 0, 1, \dots$) parton level calculations
- Phase-space samples (x_m, Φ_m) with weights $w_{ab \rightarrow X, m}^{(k)}$

$$\Rightarrow \sigma_{pp \rightarrow X}(x, \alpha_s, \mu) = \sum_{a,b} \sum_k \sum_{m=1}^{M_p} \left(\frac{\alpha_s(\mu_{R,m})}{2\pi} \right)^{k+r} \hat{\sigma}_{ab \rightarrow X, m}^{(k)} \\ \times w_{ab \rightarrow X, m}^{(k)} f_a(x_{a,m}, \mu_{F,m}) f_b(x_{b,m}, \mu_{F,m})$$

Grid Technique - Formulae

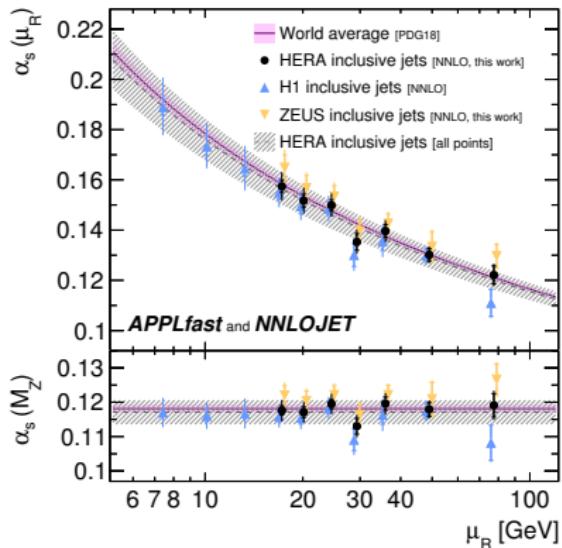
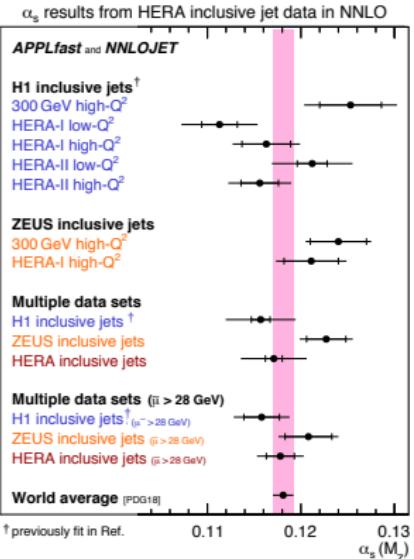
For pp collisions: 4 functions $E_i(x_a)$, $E_j(x_b)$, $E_v(\mu_R)$, $E_w(\mu_F)$

$$\Rightarrow \sigma_{pp \rightarrow X}(x, \alpha_s, \mu) = \sum_{i,j,v,w=0}^N \sum_{a,b} \sum_k \left(\frac{\alpha_s^{[v]}}{2\pi} \right)^{k+r} f_a^{[i,w]} f_b^{[j,w]} \\ \times \hat{\sigma}_{ab \rightarrow X}^{(k)} [i,j,v,w]$$

with

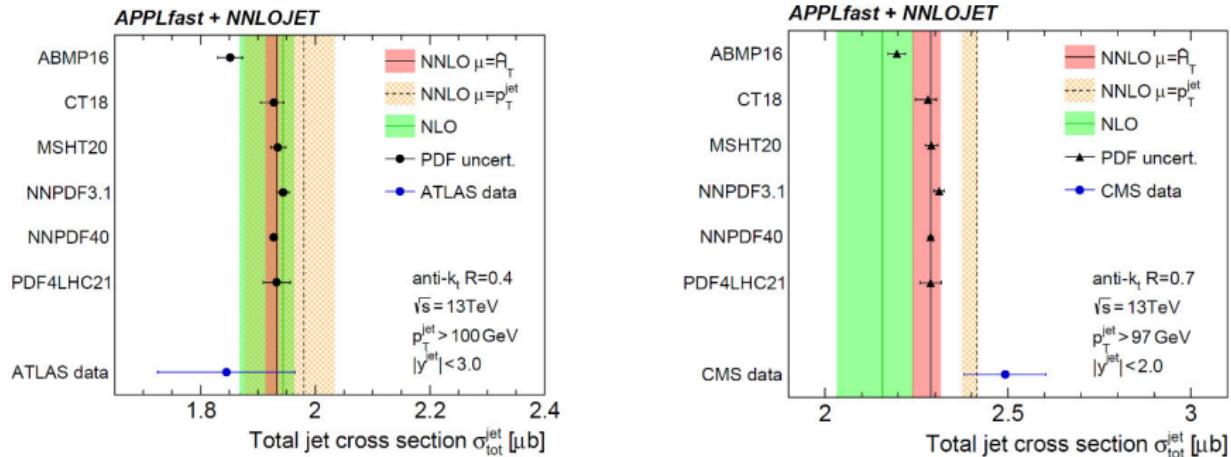
$$\hat{\sigma}_{ab \rightarrow X}^{(k)} [i,j,v,w] := \sum_{m=1}^{M_p} E_i(x_{a,m}) E_j(x_{b,m}) E_v(\mu_{R,m}) E_w(\mu_{F,m}) \\ \times w_{ab \rightarrow X, m}^{(k)} \hat{\sigma}_{ab \rightarrow X, m}^{(k)}$$

Phenomenology modules 1



Determination of the strong coupling constant from HERA data
[Britzger, Gehrmann, Huss, Rabbertz, et al. '19]

Phenomenology modules 1



Comparison of the total jet cross section using different PDFs
[Britzger, Gehrmann, Huss, Rabbertz, et al. '22]

New Interface

- Interface adapted to use modules 2 of NNLOJET
 - better colour sampling
 - full colour dijet code
 - printout of intermediate results during production step
⇒ workflow can better detect problematic phase space points
 - more flexible decomposition of logarithmic scale coefficients
⇒ no need for „magical numbers“ in scale setup any more

```
muf = 1.0 * mll      mur = 1.0 * mll
muf = 0.5 * mll      mur = 0.5 * mll
muf = 2.0 * mll      mur = 2.0 * mll
muf = 1.0 * mll      mur = 0.5 * mll
muf = 0.5 * mll      mur = 1.0 * mll
muf = 1.0 * mll      mur = 2.0 * mll
muf = 2.0 * mll      mur = 1.0 * mll

!
!           mll      mur =          mll
!           muf = 90.0171313005  mur = 90.0171313005
!           muf = 54.5981500331  mur = 54.5981500331
!           muf = 148.4131591026  mur = 148.4131591026
!           muf = 54.5981500331  mur = 90.0171313005
!           muf = 90.0171313005  mur = 54.5981500331
!           muf = 148.4131591026  mur = 90.0171313005
```

Phenomenology modules 2 - dijet fc

- Two different dijet full colour data sets produced (so far):
 - CMS at 7 TeV, anti-kt, R=0.6
 - double differential in $m_{12} \in [260.0, 5040.0]$ and $y^* \in [0.0, 3.0]$
 - PDF set: NNPDF31 nnlo as 0118
- ATLAS at 13 TeV, anti-kt, R=0.4
- double differential in $m_{12} \in [260.0, 9066.0]$ and $y^* \in [0.0, 3.0]$
- PDF set: NNPDF31 nnlo as 0118
- ⇒ plots shown on the following slides

Phenomenology modules 2 - dijet fc

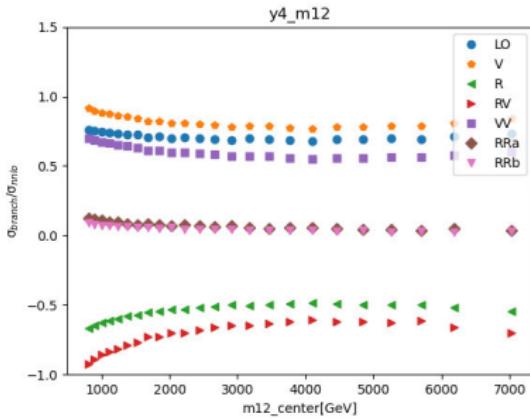
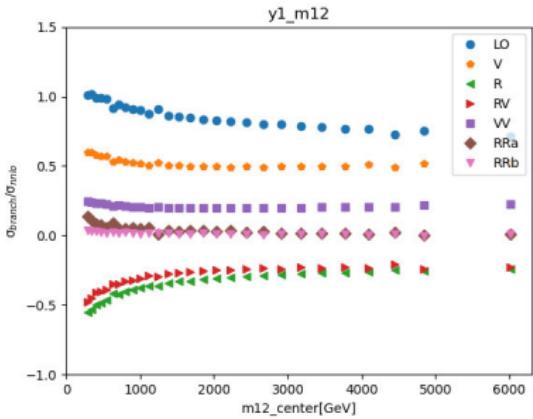
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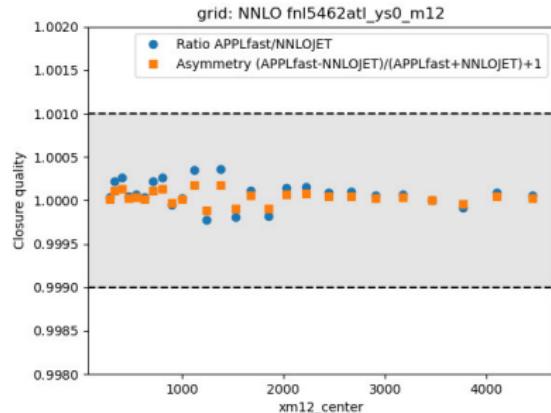
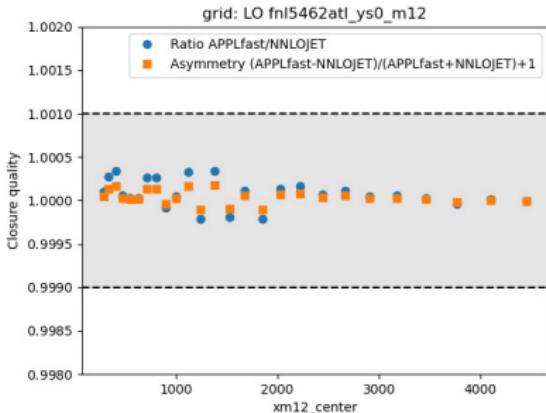
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Phenomenology modules 2 - channels



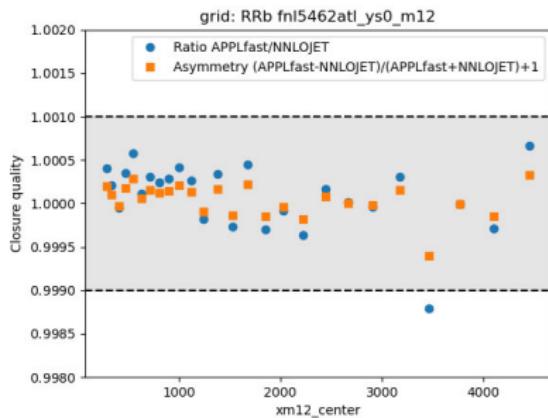
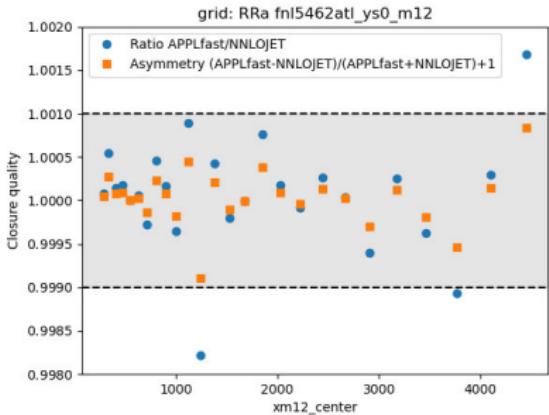
Plot of relative contributions of different channels shows large cancellations between real and virtual parts for higher y^*

Phenomenology modules 2 - closure



Overall we find good closure at sub-permille accuracy

Phenomenology modules 2 - closure



Even the most problematic channels (double real) show nice behaviour

Phenomenology modules 2 - runtimes

	Event	Jobs	neval	Tot Time	Cross section	Error
LO	$0.5 \cdot 10^9$	$27 \cdot 8$	$108 \cdot 10^9$	$4.7 \cdot 10^3$ h	$5.249331E+08$	$1.315478E+04$
V	$8 \cdot 10^6$	$28 \cdot 8$	$1.792 \cdot 10^9$	$3.6 \cdot 10^3$ h	$4.089646E+08$	$1.072727E+05$
R	$4 \cdot 10^6$	$84 \cdot 8$	$5.088 \cdot 10^9$	$22.9 \cdot 10^3$ h	$-3.296991E+08$	$2.205647E+05$
	$10 \cdot 10^6$					
VV	$15 \cdot 10^6$	$55 \cdot 8$	$6.006 \cdot 10^9$	$4.8 \cdot 10^3$ h	$2.200059E+08$	$7.435571E+04$
RV	$0.67 \cdot 10^6$	$100 \cdot 8$	$1.1952 \cdot 10^9$	$41.6 \cdot 10^3$ h	$-3.385389E+08$	$8.122503E+05$
	$1.7 \cdot 10^6$					
RRa	$0.69 \cdot 10^6$	$300 \cdot 8$	$1.656 \cdot 10^9$	$116 \cdot 10^3$ h	$5.278204E+07$	$2.521830E+06$
RRb	$3.75 \cdot 10^6$	$81 \cdot 8$	$3.436 \cdot 10^9$	$19 \cdot 10^3$ h	$2.386325E+07$	$1.117482E+06$
	$11.2 \cdot 10^6$					

	LO	NLO	NLO_only	NNLO	NNLO_only
Number of evaluations	$108 \cdot 10^9$	$\sim 114.9 \cdot 10^9$	$\sim 6.9 \cdot 10^9$	$\sim 127.2 \cdot 10^9$	$\sim 12.3 \cdot 10^9$
Cross-section	$5.249331E+08$	$6.041986E+08$	$7.926550E+07$	$5.623109E+08$	$-4.188771E+07$
error	$1.315478E+04$	$4.835966E+05$	$2.452676E+05$	$2.886867E+06$	$2.876399E+06$

Next steps

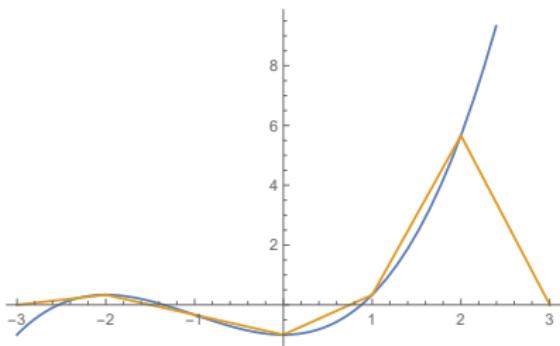
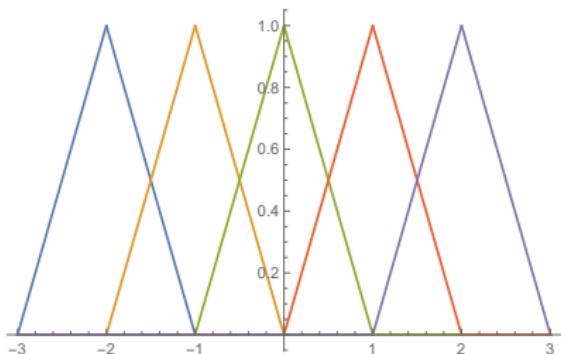
- finalize validation of NNLO code
 - optimize workflow and runtime
 - make sure closure works in all channels
 - reproduce results in [\[Britzger, Gehrmann, Huss, Rabbertz, et al. '22\]](#)
- calculate di-jet differential distributions at full colour
- $\alpha_s(M_Z)$ determination from LHC data
- provide setup for further developments and calculations

Thank you for your attention!

Backup - Interpolation

Example: $f(x) = \frac{1}{3}x^3 + x^2 - 1$ on the Interval $I = [-2, 2]$

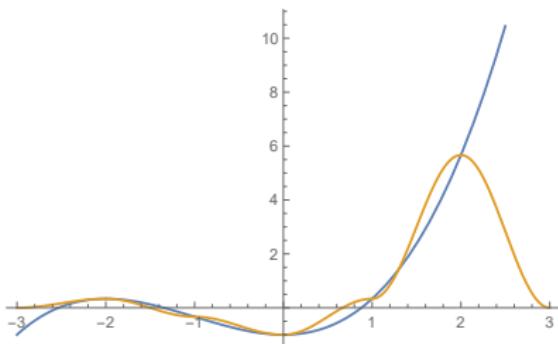
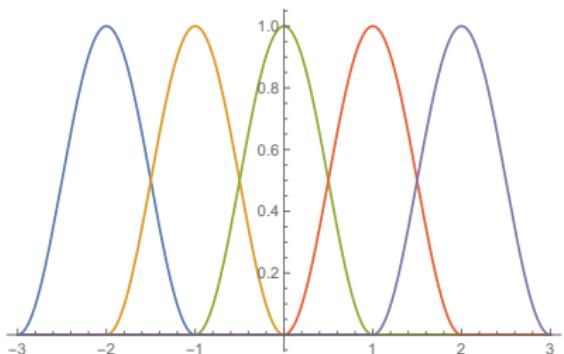
Five nodes $\{-2, -1, 0, 1, 2\}$



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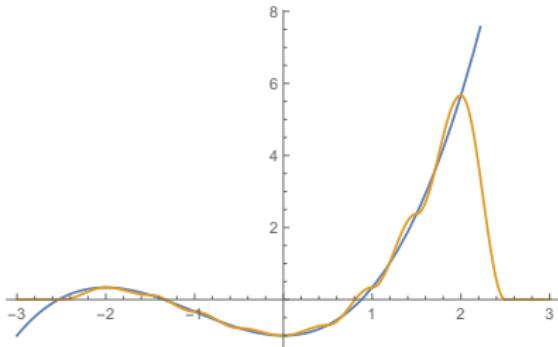
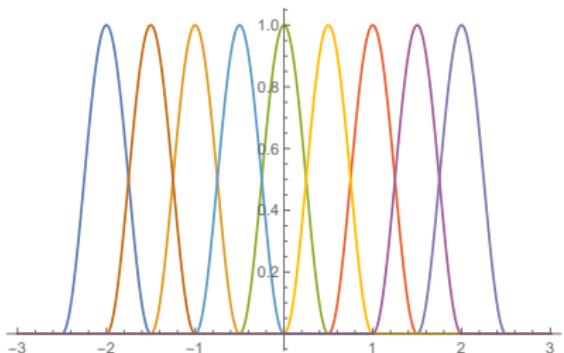
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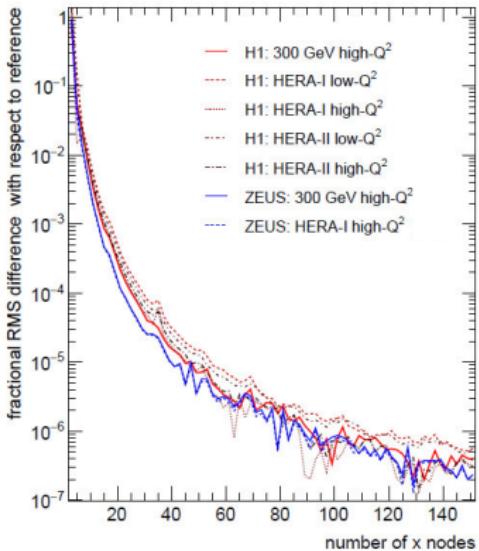
Backup - Interpolation

Example: $f(x) = \frac{1}{3}x^3 + x^2 - 1$ on the Interval $I = [-2, 2]$

Nine nodes $\{-2, -1.5, \dots, 1.5, 2\}$



Backup - Interpolation



Fractional root mean square difference between interpolation and reference
[\[Britzger, Gehrman, Huss, Rabbertz, et al. '19\]](#)

Backup - Logarithm decomposition

$$\sigma_{pp \rightarrow X}(x, \alpha_s, \mu) = \sum_{i,j,v,w=0}^N \sum_{a,b} \sum_k \left(\frac{\alpha_s^{[v]}}{2\pi} \right)^{k+r} f_a^{[i,w]} f_b^{[j,w]} \hat{\sigma}_{ab \rightarrow X}^{(k)} [i,j,v,w]$$

$$d\hat{\sigma}_{ab \rightarrow X}^{(k)} [i,j,v,w] (\mu_R^2, \mu_F^2) = \sum_{\alpha+\beta \leq k} d\hat{\sigma}_{ab \rightarrow X}^{(k|\alpha,\beta)} [i,j,v,w] \ln^\alpha \left(\frac{\mu_R^2}{\mu_0^2} \right) \ln^\beta \left(\frac{\mu_F^2}{\mu_0^2} \right)$$

$$\hat{\sigma}_{ab \rightarrow X}^{(k|\alpha,\beta)} [i,j,v,w] = \sum_{m=1}^{M_p} E_i(x_{a,m}) E_j(x_{b,m}) E_v(\mu_{R,m}) E_w(\mu_{F,m}) w_{ab \rightarrow X, m}^{(k)} \hat{\sigma}_{ab \rightarrow X, m}^{(k|\alpha,\beta)}$$