MATRIX alle Hawaii – interface to fast-interpolation tools

Stefan Kallweit

(ongoing work with Simone Devoto, Tomas Ježo and Christopher Schwan – built upon the MATRIX framework)

Istituto Nazionale di Fisica Nucleare

UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

This project is supported from the European Union's Horizon 2020
research and innovation programme under grant agreement No 714788.

xFitter External meeting, CERN, Geneva, May 2–5, 2023

Outline

Precision calculations — the key to fully exploit LHC measurements

Sample case: diboson production

- important SM test ➥ trilinear couplings
- background for Higgs analyses and BSM searches
- very clean signatures in leptonic decay channels
- good statistics already with available data

All diboson processes available at NNLO QCD accuracy in the public MATRIX framework

[Grazzini, SK, Wiesemann (2018)]

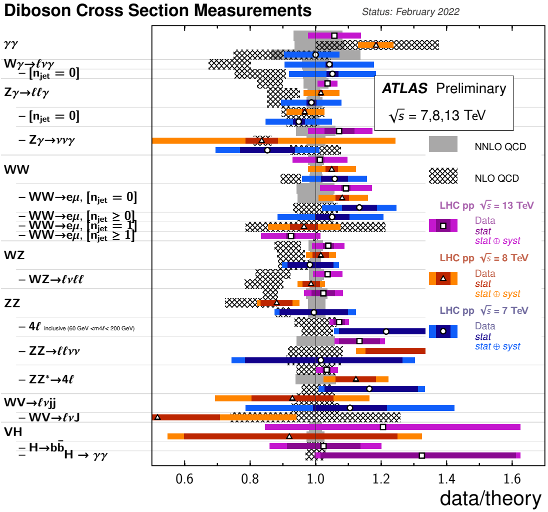
- inevitable for data–theory agreement

Mandatory steps to match experimental precision also in the future

- leading QCD corrections beyond NNLO
- EW corrections and combination with QCD

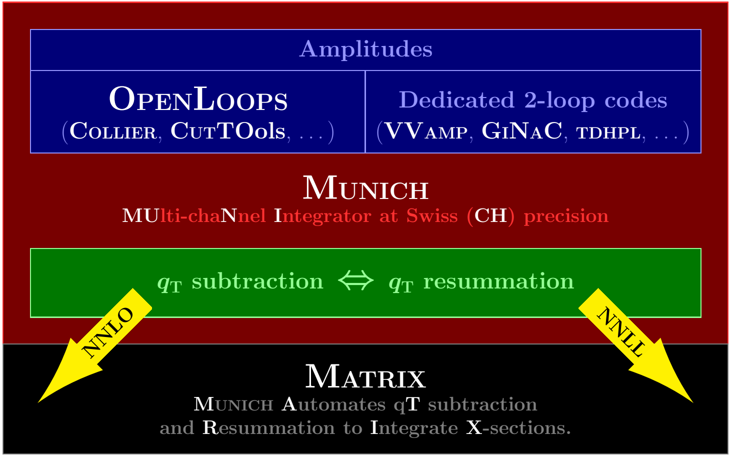
➥ **MATRIX v2** [Grazzini, SK, Wiesemann (2021)]

[ATLAS collaboration (2022)]



The MATRIX framework for automated NNLO QCD calculations (and beyond)

[Grazzini, SK, Wiesemann (2018) + Rathlev; Buonocore, Devoto, Mazzitelli, Rottoli, Sargsyan, Savoini, Yook, . . .]



Amplitudes

OPENLOOPS
(COLLIER, CUTTOOls, . . .)

Dedicated 2-loop codes
(VVAMP, GINAC, TDHPL, . . .)

MUNICH
MUlti-chaNnel Integrator at Swiss (CH) precision

q_T subtraction $\Leftrightarrow$ q_T resummation

NNLO

NNLL

MATRIX
MUNICH Automates qT subtraction
and Resummation to Integrate X-sections.

available under `https://matrix.hepforge.org/`

MATRIX v1 (fall 2017)

- H, V, $\gamma\gamma$, Vγ, VV at NNLO QCD for all leptonic decay channels

MATRIX v2 (summer 2021)

- combination with NLO EW for all leptonic V and VV processes
- loop-induced gg channel at NLO QCD for neutral VV processes

MATRIX v2.1 (spring 2023)

- bin-wise $q_{T,\mathrm{cut}} \to 0$ extrapolation also for all distributions
- recoil-driven linear power corrections (relevant for Drell–Yan)
- $\gamma\gamma\gamma$ at NNLO QCD ($2 \to 3$)
- $t\bar{t}$ at NNLO QCD (heavy-quark FS)

Motivation for having a MATRIX interface to fast-interpolation tools

Choice in **MATRIX**: Interface to **PINEAPPL** — can be converted to **APPLgrid/fastNLO** formats

- **PDF and α_S uncertainties**
  - in principle possible directly in MATRIX, but very expensive in runtime and/or disk space
  - PINEAPPL grids allow PDF uncertainties to be calculated a posteriori at basically no cost
- **Scale (regularization and factorization) variation uncertainties**
  - available in MATRIX, simulaneously for different dynamic scale choices (and variation by factors)
  - PINEAPPL requires dedicated grids for each dynamic scale, variation by arbitrary factors a posteriori
- **Splitting of results into partonic channels**
  - available in MATRIX, but needs to be specified a priori (precision goals for different channels)
  - PINEAPPL grids store information on luminosities to achieve channel splitting a posteriori
- **Performing PDF fits based on full NNLO information**
  - practically impossible directly in MATRIX since repeated expensive NNLO runs would be required
  - PINEAPPL grids store all information about results of higher-order calculation

➥ **Interface to fast-interpolation tools highly desirable in particular in context of PDFs**

➥ **Goal: make all MATRIX features available in the format of PINEAPPL grids**

The MUNICH/MATRIX framework for automated NNLO calculations

MATRIX — MUNICH Automates qT-subtraction and Resummation to Integrate X-sections
[Grazzini, SK, Wiesemann (2018)]

- public tool to perform fully differential NNLO QCD calculations for a large class of processes
- core of the framework: the C++ parton-level Monte Carlo generator

  MUNICH — MUlti-chaNnel Integrator at swiss (CH) precision [SK]

  - bookkeeping of partonic subprocesses for all contributions
  - fully automated dipole subtraction for NLO calculations (massive, QCD and EW)
    [Catani, Seymour (1997), Catani, Dittmaier, Seymour, Trocsanyi (2002), Dittmaier (2000), SK, Lindert, Maierhöfer, Pozzorini, Schönherr (2015)]
  - general amplitude interface 1-loop amplitudes 2-loop amplitudes
  - highly efficient multi-channel Monte Carlo integration with several optimization features
  - simultaneous monitoring of slicing parameter and automated extrapolation
- PYTHON script to simplify the use of MATRIX
  - installation of MUNICH and all supplementary software
  - interactive shell steering all run phases without human intervention (grid-, pre-, main-run, summary)
  - organization of parallelized running on multicore machines and commonly used clusters: SLURM, HTCONDOR, LSF, etc.

Idea of the q_T subtraction method for (N)NLO cross sections

Consider the production of a **colourless final state F** via $\mathrm{q\bar{q}} \to \mathbf{F}$ or $\mathbf{gg} \to \mathbf{F}$: $\left.\mathrm{d}\boldsymbol{\sigma}_\mathbf{F}^\mathbf{(N)NLO}\right|_{q_\mathrm{T}\neq 0} = \mathrm{d}\boldsymbol{\sigma}_\mathbf{F+jet}^\mathbf{(N)LO}$

where q_T refers to the transverse momentum of the colourless system F [Catani, Grazzini (2007)]

- $\left.\mathrm{d}\boldsymbol{\sigma}_\mathbf{F}^\mathbf{(N)NLO}\right|_{q_\mathrm{T}\neq 0}$ is singular for $q_\mathrm{T} \to 0$
  - ➥ limiting behaviour known from transverse-momentum resummation [Bozzi, Catani, de Florian, Grazzini (2006)]
- Define a universal counterterm $\boldsymbol{\Sigma}$ with the complementary $q_\mathrm{T} \to 0$ behaviour [Bozzi, Catani, de Florian, Grazzini (2006)]
  $\mathrm{d}\boldsymbol{\sigma}^\mathbf{CT} = \boldsymbol{\Sigma}(q_\mathrm{T}/q) \otimes \mathrm{d}\boldsymbol{\sigma}^\mathbf{LO}$ where q is the invariant mass of the colourless system F
- Add the $q_\mathrm{T} = 0$ piece with the hard–virtual coefficient $\boldsymbol{\mathcal{H}}_\mathbf{F}$, which contains the 1-(2-)loop amplitudes at (N)NLO and compensates for the subtraction of $\boldsymbol{\Sigma}$ [Catani, Cieri, de Florian, Ferrera, Grazzini (2013)]

➥ **Master formula for (N)NLO cross section in q_T subtraction method**

$$\mathrm{d}\boldsymbol{\sigma}_\mathbf{F}^\mathbf{(N)NLO} = \boldsymbol{\mathcal{H}}_\mathbf{F}^\mathbf{(N)NLO} \otimes \mathrm{d}\boldsymbol{\sigma}^\mathbf{LO} + \left[\mathrm{d}\boldsymbol{\sigma}_\mathbf{F+jet}^\mathbf{(N)LO} - \boldsymbol{\Sigma}^\mathbf{(N)NLO} \otimes \mathrm{d}\boldsymbol{\sigma}^\mathbf{LO}\right]_{\mathbf{cut}_{q_\mathrm{T}} \to 0}$$

- all ingredients known for extension to N^{3}LO [Luo, Yang, Zhu, Zhu (2019; 2020), Ebert, Mistlberger, Vita (2020), Cieri, Chen, Gehrmann, Glover, Huss (2019), Camarda, Cieri, Ferrera (2021), Chen, Gehrmann, Glover, Huss, Yang, Zhu (2021)]

Extension to heavy coloured particles at NNLO QCD and beyond

Extension of q_T subtraction method to production of heavy coloured particles ($Q\bar{Q}$, $Q\bar{Q}X$, etc.)

$$d\sigma^{\mathrm{NNLO}}_{Q\bar{Q}X} = \mathcal{H}^{\mathrm{NNLO}}_{Q\bar{Q}X} \otimes d\sigma_{\mathrm{LO}} + \left[d\sigma^{\mathrm{NLO}}_{Q\bar{Q}X+\mathrm{jet}} - d\sigma^{\mathrm{NNLO}}_{Q\bar{Q}X,\mathrm{CT}} \right]_{\mathrm{cut}_{q_T}\to 0}$$

- counterterm accounts for IR behaviour of real contribution, including soft singularities related to emissions from final-state quarks [Catani, Grazzini, Torre (2014), Ferroglia, Neubert, Pecjak, Yang (2009), Li, Li, Shao, Yang, Zu (2013)]
- massive NLO subtraction required for real-emission part, e.g. massive dipole subtraction [Catani, Seymour (1997), Catani, Dittmaier, Seymour, Trocsanyi (2002)]
- $\mathcal{H}^{Q\bar{Q}X}_{\mathrm{NNLO}}$ contains remainder of integrated final-state soft singularities
    - known for heavy-quark pairs [Catani, Devoto, Grazzini, Mazzitelli (2023), Angeles-Martinez, Czakon, Sapeta (2018)]
    - more involved kinematics for associated heavy-quark pair production [Devoto, Mazzitelli (to appear)]

Extension of q_T subtraction method to mixed QCD–EW corrections of $\mathcal{O}(\alpha_s^m \alpha^n)$

$$d\sigma^{(m,n)}_{\mathrm{F}} = \mathcal{H}^{(m,n)}_{\mathrm{F}} \otimes d\sigma_{\mathrm{LO}} + \left[d\sigma^{(m,n)}_{\mathrm{F,R}} - d\sigma^{(m,n)}_{\mathrm{F,CT}} \right]_{\mathrm{cut}_{q_T}\to 0}$$

- limitation: F contains no massless jets (for $m \geq 1$) and no massless charged particles (for $n \geq 1$) [Buonocore, Grazzini, Tramontano (2020), Buonocore (2020), De Florian, Der, Fabre (2018), Cieri, De Florian, Der, Mazzitelli (2020)]

Investigation of $r_{\text{cut}} = \text{cut}_{q_T/q}$ dependence — sample case $\text{pp} \to \gamma\gamma + X$

Result for $r_{\text{cut}} \to 0$ via extrapolation

- automated and simultaneous scan over reasonable range of r_{cut} values
- quadratic least-χ^2 fit with variable range

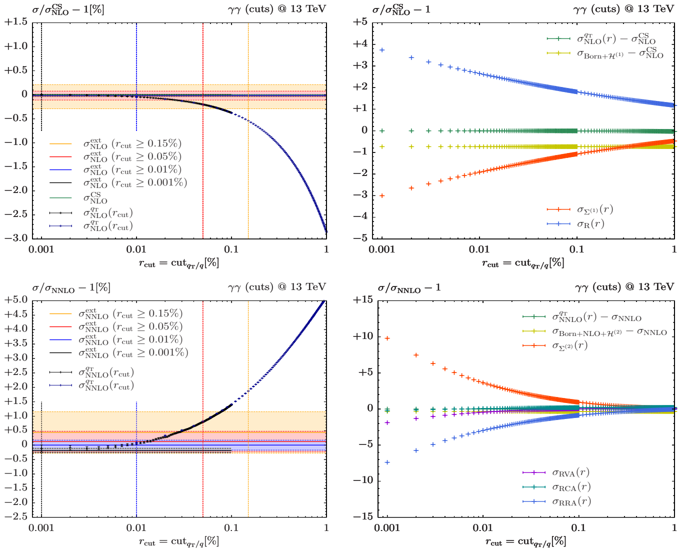
$$\boldsymbol{\sigma_{(\text{N})\text{NLO}}(r_{\text{cut}}) = A r_{\text{cut}}^2 + B r_{\text{cut}} + \sigma_{(\text{N})\text{NLO}}}$$

- error estimate based on combination of statistical error and variation of r_{cut} range

➡ **Significant r_{cut} dependence for processes involving isolated photons** (similar between NLO and NNLO QCD)

➡ good agreement of extrapolated results within errors for different start values

- $r_{\text{cut}} \geq 0.15\%$
- $r_{\text{cut}} \geq 0.05\%$
- $r_{\text{cut}} \geq 0.01\%$
- $r_{\text{cut}} \geq 0.001\%$



Investigation of $r_\mathrm{cut} = \mathrm{cut}_{q_\mathrm{T}/q}$ dependence — sample case $\mathrm{pp} \to \ell^-\ell^+\ell'^-\ell'^+ + X$

Result for $r_\mathrm{cut} \to 0$ via extrapolation

- same procedure for all processes

$$\sigma_{\mathrm{(N)NLO}}(r_\mathrm{cut}) = A r_\mathrm{cut}^2 + B r_\mathrm{cut} + \sigma_{\mathrm{(N)NLO}}$$

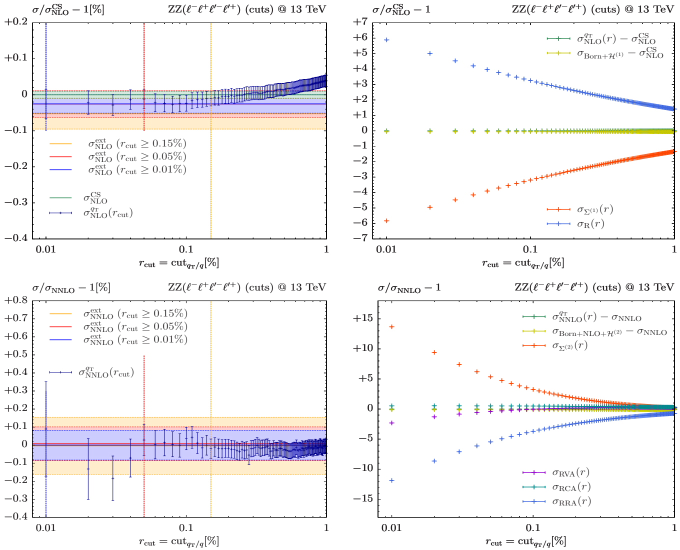
➥ **No significant r_cut dependence for processes without isolated photons** (similar between NLO and NNLO QCD)

➥ good agreement of extrapolated results within errors for different start values

- **$r_\mathrm{cut} \geq 0.15\%$**
- **$r_\mathrm{cut} \geq 0.05\%$**
- **$r_\mathrm{cut} \geq 0.01\%$**

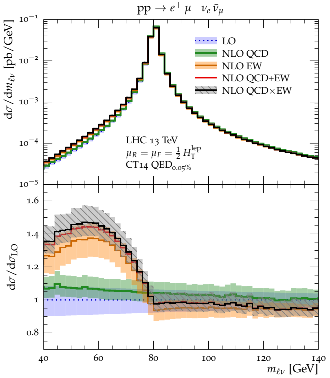
➥ larger cancellation between contributions (factor of ≈ 15 at $r_\mathrm{cut} = 0.01\%$)

➥ **Important exception:**
linear power corrections induced by particular fiducial cut configurations



Main features of EW corrections

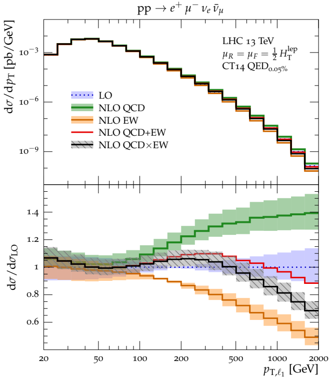
- **Shape corrections in invariant-mass distributions**



[SK, Lindert, Pozzorini, Schönherr (2017)]

Pure QED effect:
photon bremsstrahlung off decay leptons (migration effect)

- **Negative corrections in high-energy observables**



[SK, Lindert, Pozzorini, Schönherr (2017)]

Genuine EW effect:
enhancement due to large universal Sudakov logarithms

- **Photon-induced processes**
  - ➥ inclusion via LUXQED PDFs [Manohar, Nason, Salam, Zanderighi (2016; 2017)]
  - as Born processes, e.g. $\gamma\gamma \to WW$
  - as EW corrections, from IS $\gamma \to q\bar{q}^*$ splittings
- **Subdominant production modes (not maximal in α_s)**
  - e.g. $q\bar{q} \to Z^*/\gamma^* \to t\bar{t}$
  - interferences between QCD and EW production modes
  - corresponding tower of NLO contributions that cannot be uniquely qualified as QCD or EW corrections (in parts)

Performance features of the MUNICH phase space integrator

Issue of poorly populated regions

- sample case: high-energy tails
- standard phase space optimization samples points in bulk region

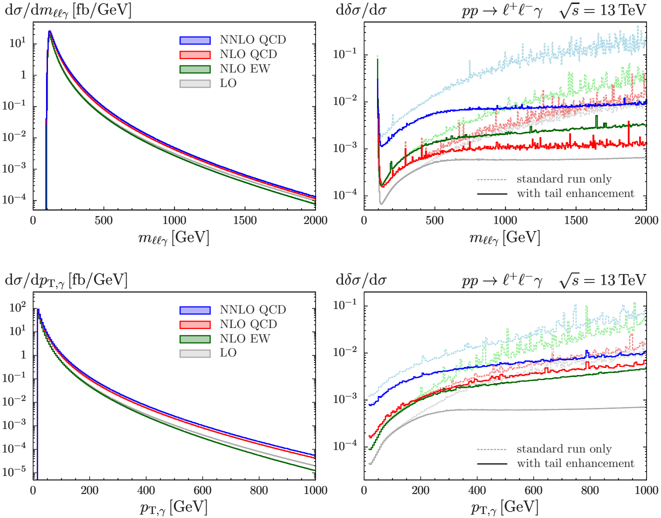
Solution in MUNICH integrator

- additional runs with optimization including a general bias factor
- sophisticated automated combination with results from standard runs

Significantly improved errors

- $\mathcal{O}(10)$ and better with doubled runtime
- simultaneous enhancement of observables

Good performance also for off-shell regions of intermediate resonances



Available processes in MATRIX v2.1 and beyond

H (HTL)		**NNLO QCD**				
Z	**$(\ell\ell/\nu\nu)$**	**NNLO QCD**	**NLO EW**	**NNLO QCD–EW**	**ggNLO QCD**	**(linPCs)**
$W^{\pm}$	**$(\ell\nu)$**	**NNLO QCD**	**NLO EW**	**NNLO QCD–EW**	**(linPCs)**	
HH (HTL, FT$_{\text{approx}}$)		**NNLO QCD**				
ZH	**$(\ell\ell H/\nu\nu H)$**	**NNLO QCD**	**NLO EW**			
$W^{\pm}H$	**$(\ell\nu H)$**	**NNLO QCD**	**NLO EW**			
$\gamma\gamma$		**NNLO QCD**	**NLO EW**			
$Z\gamma$	**$(\ell\ell\gamma/\nu\nu\gamma)$**	**NNLO QCD**	**NLO EW**			
$W^{\pm}\gamma$	**$(\ell\nu\gamma)$**	**NNLO QCD**	**NLO EW**			

Z: [Phys.Rev.Lett. 128 (2022) 1, 012002, Phys.Lett.B 829 (2022) 137118]

$W^{\pm}$: [Phys.Rev.D 103 (2021) 114012]

HH: [JHEP 09 (2016) 151, JHEP 05 (2018) 059]

$Z\gamma$: [Phys.Lett.B 731 (2014) 204-207, JHEP 07 (2015) 085]

$W^{\pm}\gamma$: [JHEP 07 (2015) 085]

Available processes in MATRIX v2.1 and beyond

W^+W^- $(2\ell 2\nu)$ NNLO QCD NLO EW ggNLO QCD

[Phys.Rev.Lett. 113 (2014) 21, 212001, JHEP 08 (2015) 154, JHEP 08 (2016) 140, Phys.Lett.B 786 (2018) 382-389, JHEP 02 (2020) 087, Phys.Lett.B 804 (2020) 135399]

ZZ $(4\ell/2\ell 2\nu/4\nu)$ NNLO QCD NLO EW ggNLO QCD

[Phys.Lett.B 735 (2014) 311-313, JHEP 08 (2015) 154, Phys.Lett.B 750 (2015) 407-410, Phys.Lett.B 786 (2018) 382-389, JHEP 03 (2019) 070, JHEP 02 (2020) 087]
[Phys.Lett.B 819 (2021) 136465]

$W^\pm Z$ $(3\ell\nu/\ell 3\nu)$ NNLO QCD NLO EW

[Phys.Lett.B 761 (2016) 179-183, JHEP 05 (2017) 139, JHEP 02 (2020) 087]

$\gamma\gamma\gamma$ NNLO QCD NLO EW ggNLO QCD

[Phys.Lett.B 812 (2021) 136013]

$t\bar{t}$ NNLO QCD NLO EW

[Phys.Rev.D 99 (2019) 5, 051501, JHEP 07 (2019) 100, JHEP 08 (2020) 08, 027]

$b\bar{b}$ NNLO QCD NLO EW

[JHEP 03 (2021) 029]

$Ht\bar{t}$ NNLO QCD NLO EW

[Eur.Phys.J.C 81 (2021) 6, 491, Phys.Rev.Lett. 130 (2023) 11, 111902]

$W^\pm b\bar{b}$ $(\ell\nu b\bar{b})$ NNLO QCD NLO EW

[Phys.Rev.D 107 (2023) 7, 074032]

Implemented features regarding PineAPPL grids

- **General interface to generate PineAPPL grids at runtime**
  - separate grids for each contribution and each subprocess according to run organisation in Matrix:
    - re-organisation of convolution with PDFs for contributions with collinear splittings
    - extraction of coefficients of $\mu_{R/F}$ logarithms to reconstruct scale variations

  ➥ parallel runs produce individual grids
  - grids stored for several r_cut values in r_cut-dependent contributions
  - individual grids for each dynamic scale choice required
  - Matrix summary routine merges all individual grids to result grids for desired orders (e.g. NNLO QCD)
- **Extrapolation $r_\mathrm{cut} \to 0$ of PineAPPL grids**
  - bin-wise quadratic least-χ^2 fits to find optimal extrapolation range

  ➥ no integration error information contained in grids, thus extracted from Matrix
  - extrapolated result grid through linear combination of fixed-r_cut grids
- **Single- and double-differential distributions supported**
  - PineAPPL grids only deal with single-differential distributions, but contain remapping information
  - extension to multi-differential distributions straightforward, but not supported by Matrix (yet)

Practical issues/to-be-completed features related to PineAPPL grids

- **Memory consumption of PineAPPL grids (and disk space)**
  - $\propto$ number of non-trivial distribution bins
  - $\propto$ number of fixed r_{cut} values (for r_{cut}-dependent contributions)
  - $\propto$ number of luminosities (i.e. groups of luminosities)
  - $\propto$ number of dynamic scales
  - $\propto$ number of required coefficients of $\mu_{R/F}$ logarithms (up to 6 at NNLO)

➥ memory becomes a limitation for some contributions (mostly the NNLO counterterm)

  ➥ several directions to avoid this problem under investigation. . .

- **Optimized integration for subdominant phase space regions (e.g. high-energy tails)**
  - feature requires information about integration errors, event numbers, etc. for each distribution bin

➥ information available in Matrix needs to be carefully transferred to PineAPPL grids

- **General interface to provide PDF uncertainties from PineAPPL grids in Matrix**

- **Complete metadata in PineAPPL grids**
  - store all information to make the grids fully reproducible (Matrix version, runcards, etc.)

- **Automated installation of PineAPPL and all prerequisites through Matrix script**

Inclusive results with uncertainties calculated through PineAPPL grids

Sample application from LHCHXSWG

Reduced mass and energy scan for $t\bar{t}H$ cross sections:

- NNLO QCD+NLO SM ($\mu_R = \mu_F = m_t + m_H/2$)
- PDF recommendation:

  `PDF4LHC21_40` for partons,
  `LUXqed17_plus_PDF4LHC15_nnlo_100` for photons

➥ can be straightforwardly achieved through PineAPPL grids, together with scale, PDF and α_s uncertainties

(theory uncertainties calculated directly in Matrix)

$\sqrt{s}$ [TeV]	m_H [GeV]	XS [fb]	± QCD Scale Unc.	± THU	± α_s Unc.	± PDF Unc.
13	124.6	532.0	± 3.1%	± 0.6%	± 1.7%	± 2.3%
13	125	528.4	± 3.2%	± 0.7%	± 1.7%	± 2.3%
13	125.09	526.6	± 3.1%	± 0.7%	± 1.7%	± 2.3%
13	125.38	522.7	± 3.1%	± 0.7%	± 1.7%	± 2.3%
13	125.6	519.9	± 3.1%	± 0.7%	± 1.7%	± 2.3%
13	126	515.4	± 3.1%	± 0.7%	± 1.7%	± 2.3%
13.6	124.6	596.6	± 3.0%	± 0.7%	± 1.6%	± 2.2%
13.6	125	589.9	± 2.9%	± 0.7%	± 1.6%	± 2.2%
13.6	125.09	589.6	± 3.0%	± 0.7%	± 1.6%	± 2.2%
13.6	125.38	586.2	± 3.0%	± 0.7%	± 1.6%	± 2.2%
13.6	125.6	583.5	± 3.0%	± 0.7%	± 1.6%	± 2.2%
13.6	126	577.9	± 3.1%	± 0.7%	± 1.6%	± 2.2%
14	124.6	639.7	± 2.9%	± 0.7%	± 1.6%	± 2.2%
14	125	636.1	± 3.0%	± 0.6%	± 1.6%	± 2.2%
14	125.09	633.3	± 2.9%	± 0.6%	± 1.6%	± 2.2%
14	125.38	632.4	± 3.1%	± 0.6%	± 1.6%	± 2.2%
14	125.6	627.9	± 3.0%	± 0.6%	± 1.6%	± 2.2%
14	126	621.2	± 3.0%	± 0.7%	± 1.6%	± 2.2%

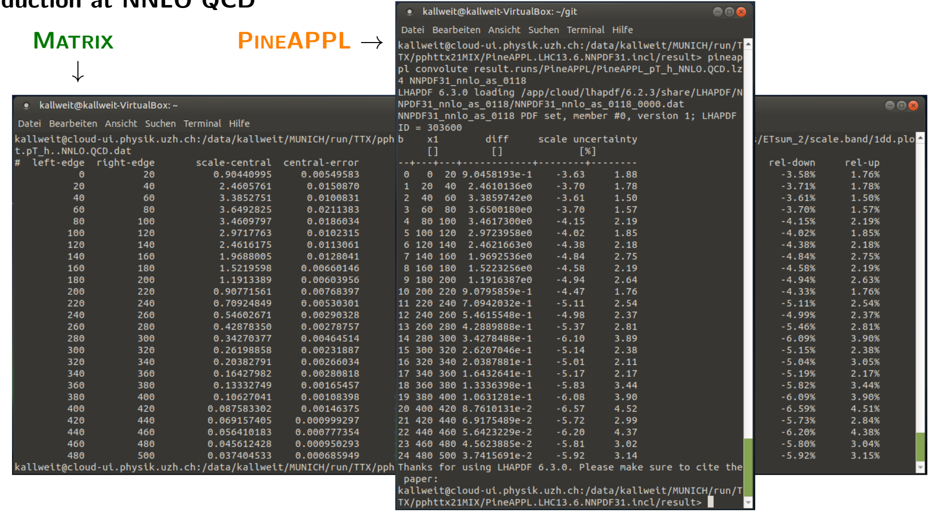
Accuracy of distributions generated through PineAPPL grids

Sample case: $t\bar{t}H$ production at NNLO QCD

- $p_{T,H}$ distribution with 25 bins in $[0\,\mathrm{GeV}; 500\,\mathrm{GeV}]$
- result of bin-wise r_{cut} extrapolation
- ➡ agreement better than $\sim 0.03\%$ in each bin
- ➡ agreement as good as for fixed r_{cut} values and for other orders
- ➡ **size of final PineAPPL grid: $\sim$ 375 MB**

Matrix ↓

PineAPPL →



```
kallweit@cloud-ui.physik.uzh.ch:/data/kallweit/MUNICH/run/TTX/pph
t.pT_h..NNLO.QCD.dat
#  left-edge  right-edge       scale-central  central-error
           0          20          0.90440995     0.00549583
          20          40           2.4605761      0.0150870
          40          60           3.3852751      0.0100831
          60          80           3.6492825      0.0211383
          80         100           3.4609797      0.0186034
         100         120           2.9717763      0.0102315
         120         140           2.4616175      0.0113061
         140         160           1.9688005      0.0128041
         160         180           1.5219598     0.00660146
         180         200           1.1913389     0.00603956
         200         220          0.90771561     0.00768397
         220         240          0.70924849     0.00530301
         240         260          0.54602671     0.00290328
         260         280          0.42878350     0.00278757
         280         300          0.34270377     0.00464514
         300         320          0.26198858     0.00231887
         320         340          0.20382791     0.00266034
         340         360          0.16427982     0.00280818
         360         380          0.13332749     0.00165457
         380         400          0.10627041     0.00108398
         400         420         0.087583302     0.00146375
         420         440         0.069157405    0.000999297
         440         460         0.056410183    0.000777354
         460         480         0.045612428    0.000950293
         480         500         0.037404533    0.000685949
kallweit@cloud-ui.physik.uzh.ch:/data/kallweit/MUNICH/run/TTX/pph
```

```
s/ETsum_2/scale.band/1dd.plo

   rel-down     rel-up
     -3.58%      1.76%
     -3.71%      1.78%
     -3.61%      1.50%
     -3.70%      1.57%
     -4.15%      2.19%
     -4.02%      1.85%
     -4.38%      2.18%
     -4.84%      2.75%
     -4.58%      2.19%
     -4.94%      2.63%
     -4.33%      1.76%
     -5.11%      2.54%
     -4.99%      2.37%
     -5.46%      2.81%
     -6.09%      3.90%
     -5.15%      2.38%
     -5.04%      3.05%
     -5.19%      2.17%
     -5.82%      3.44%
     -6.09%      3.90%
     -6.59%      4.51%
     -5.73%      2.84%
     -6.20%      4.38%
     -5.80%      3.04%
     -5.92%      3.15%
```

```
kallweit@cloud-ui.physik.uzh.ch:/data/kallweit/MUNICH/run/T
TX/pphttx21MIX/PineAPPL.LHC13.6.NNPDF31.incl/result> pineap
pl convolute result.runs/PineAPPL/PineAPPL_pT_h_NNLO.QCD.lz
4 NNPDF31_nnlo_as_0118
LHAPDF 6.3.0 loading /app/cloud/lhapdf/6.2.3/share/LHAPDF/N
NPDF31_nnlo_as_0118/NNPDF31_nnlo_as_0118_0000.dat
NNPDF31_nnlo_as_0118 PDF set, member #0, version 1; LHAPDF
ID = 303600
b    x1         diff      scale uncertainty
     []          []              [%]
--+---+---+------------+--------+--------
 0   0  20 9.0458193e-1    -3.63     1.88
 1  20  40  2.4610136e0    -3.70     1.78
 2  40  60  3.3859742e0    -3.61     1.50
 3  60  80  3.6500180e0    -3.70     1.57
 4  80 100  3.4617300e0    -4.15     2.19
 5 100 120  2.9723958e0    -4.02     1.85
 6 120 140  2.4621663e0    -4.38     2.18
 7 140 160  1.9692536e0    -4.84     2.75
 8 160 180  1.5223256e0    -4.58     2.19
 9 180 200  1.1916387e0    -4.94     2.64
10 200 220 9.0795859e-1    -4.47     1.76
11 220 240 7.0942032e-1    -5.11     2.54
12 240 260 5.4615548e-1    -4.98     2.37
13 260 280 4.2889888e-1    -5.37     2.81
14 280 300 3.4278488e-1    -6.10     3.89
15 300 320 2.6207046e-1    -5.14     2.38
16 320 340 2.0387881e-1    -5.01     2.11
17 340 360 1.6432641e-1    -5.17     2.17
18 360 380 1.3336398e-1    -5.83     3.44
19 380 400 1.0631281e-1    -6.08     3.90
20 400 420 8.7610131e-2    -6.57     4.52
21 420 440 6.9175489e-2    -5.72     2.99
22 440 460 5.6423229e-2    -6.20     4.37
23 460 480 4.5623885e-2    -5.81     3.02
24 480 500 3.7415691e-2    -5.92     3.14
Thanks for using LHAPDF 6.3.0. Please make sure to cite the
 paper:
kallweit@cloud-ui.physik.uzh.ch:/data/kallweit/MUNICH/run/T
TX/pphttx21MIX/PineAPPL.LHC13.6.NNPDF31.incl/result> 
```

Conclusions & Outlook

The MATRIX framework for NNLO QCD calculations

- based on the MUNICH integrator, amplitudes from OPENLOOPS and dedicated 2-loop amplitudes

v1:
- available processes: H, V, $\gamma\gamma$, Vγ, VV at NNLO QCD for all leptonic decay channels

v2:
- combination of NNLO QCD and NLO EW corrections for Drell–Yan and massive diboson processes
- leading N^{3}LO contributions in terms of NLO QCD corrections to loop-induced gluon fusion channels

v2.1:
- recoil-driven linear power corrections and bin-wise r_{cut} extrapolation for distributions
- new processes: $\gamma\gamma\gamma$ (first 2 → 3 application), $t\bar{t}$ (heavy-quark final state)

Interface to PINEAPPL interpolation grids in MATRIX under construction

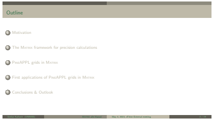
- general interface established for NNLO QCD and NLO EW computations
- extrapolation $r_{\mathrm{cut}} \to 0$ in q_T-subtraction approach implemented on level of interpolation grids
- single- and double-differential distributions available, reproducing direct results with high accuracy

➥ **New MATRIX v2.2 release with PINEAPPL interface planned as soon as remaining issues are fixed!**

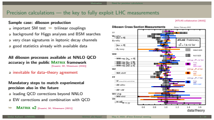
Overview


Matrix alle Hawaii – interface to fast-interpolation tools

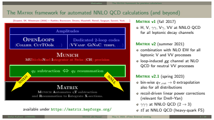
01


Outline

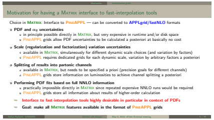
02


Precision calculations — the key to fully exploit LHC measurements

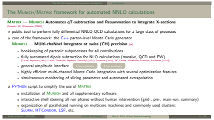
03


The Matrix framework for automated NNLO QCD calculations (and beyond)

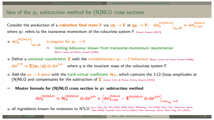
04


Motivation for having a Matrix interface to fast-interpolation tools

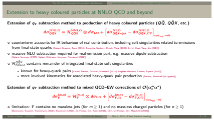
05


The Munich/Matrix framework for automated NNLO calculations

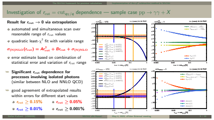
06


Idea of the q_T subtraction method for (N)NLO cross sections

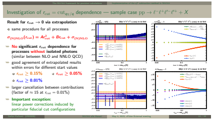
07


Extension to heavy coloured particles at NNLO QCD and beyond

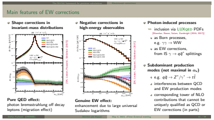
08


Investigation of $r_{\rm cut} = {\rm cut}_{q_T/q}$ dependence — sample case $pp \to \gamma\gamma + X$

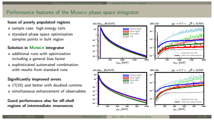
09


Investigation of $r_{\rm cut} = {\rm cut}_{q_T/q}$ dependence — sample case $pp \to \ell^-\ell'^-\ell'^+ + X$

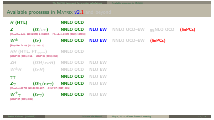
10


Main features of EW corrections

11


Performance features of the Munich phase space integrator

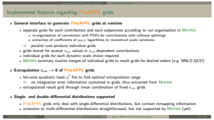
12


Available processes in Matrix v2.1

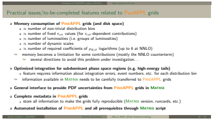
13


Available processes in Matrix v2.1

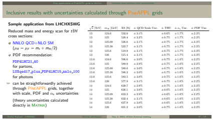
14


Implemented features regarding PineAPPL grids

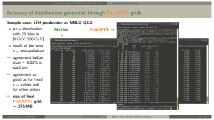
15


Practical issues/to-be-completed features related to PineAPPL grids

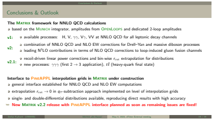
16


Inclusive results with uncertainties calculated through PineAPPL grids

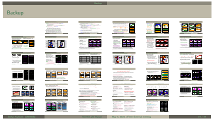
17


Accuracy of distributions generated through PineAPPL grids

18


Conclusions & Outlook

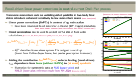
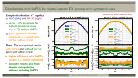
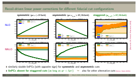
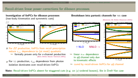
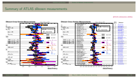
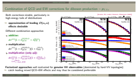
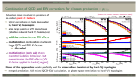
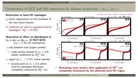
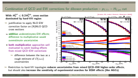
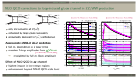
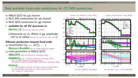
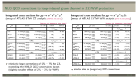
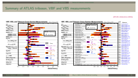
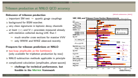
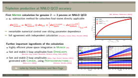
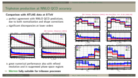
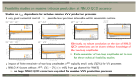
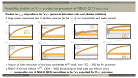
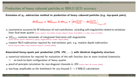
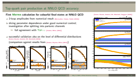
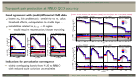
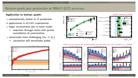
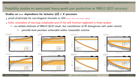
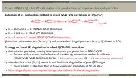
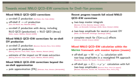
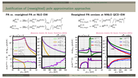
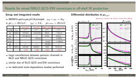
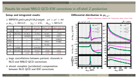
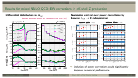
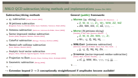
19

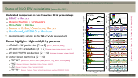

Backup

Backup

Backup





Supplying MUNICH/MATRIX with 1-loop amplitudes

Process-independent interfaces to general automated amplitude generators

- OPENLOOPS [Cascioli, Maierhöfer, Pozzorini (2012); SK, Lindert, Maierhöfer, Pozzorini, Schönherr (2015)] v2 [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)], written in FORTRAN
  - general code and process libraries
  - on-the-fly tensor reduction [Buccioni, Pozzorini, Zoller (2018)] with hybrid-precision stability system
  - scalar integrals from COLLIER [Denner, Dittmaier, Hofer (2006); Denner, Dittmaier (2011)] or ONELOOP [van Hameren (2011)]
- RECOLA [Actis, Denner, Hofer, Lang, Scharf, Uccirati (2017)] v2 [Denner, Lang, Uccirati (2017)], written in FORTRAN
  - on-the-fly generation of amplitudes
  - tensor reduction and scalar integrals via COLLIER [Denner, Dittmaier, Hofer (2006); Denner, Dittmaier (2003, 2006, 2011)]
  - different model files available, also for SMEFT and BSM applications
- modular structure of MUNICH allows other generators to be interfaced as well

Several dedicated interfaces developed in context of MATRIX applications

- loop×tree and loop×loop colour (and spin) correlators
- helicity amplitudes, colour-stripped amplitudes to construct 4-colour correlators
- imaginary parts of loop×tree amplitudes and correlators, helicity-flip amplitudes

Interfacing dedicated 2-loop amplitudes to MUNICH/MATRIX

- Higgs, Drell–Yan, **VH**, $\gamma\gamma$, **V**γ production
  - direct implementation of public analytic results, e.g. for Vγ [Gehrmann, Tandredi (2012)]
- **VV** production — qqVVAMP [Gehrmann, von Manteuffel, Tancredi (2015)] and ggVVAMP [von Manteuffel, Tancredi (2015)] libraries
  - C++ libraries using GINAC [Bauer, Frink, Kreckel (2002); Vollinga, Weinzierl (2005)] and CLN for arbitrary precision arithmetics
  - IBP approach, generated using MATHEMATICA, FORM [Vermaseren et al.], REDUZE2 [von Manteuffel, Studerus ('12)]
  - independent calculation of amplitudes in [Caola, Henn, Melnikov, Smirnov, Smirnov (2015; 2016)]
  - Higgs-mediated helicity amplitudes with full m_t dependence from [Harlander, Prausa, Usovitsch (2019; 2020)]
- $\gamma\gamma\gamma$ production — amplitudes from [Abreu, Page, Pascual, Sotnikov ('20)]
  - C++ library, generated by CARAVEL [Abreu et al. (2020)], applying PENTAGONFUNCTIONS++ [Chicherin, Sotnikov (2020)]
  - numerical unitarity and analytic reconstruction techniques [Ita (2015); Abreu et al. (2018; 2018; 2019; 2019)]
- **HH** production (full m_t dependence) — HHGRID library [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)]
  - PYTHON based numerical interpolation of amplitude grid
  - generated by 2-loop extension of GOSAM [Jones (2016)], REDUZE2 [von Manteuffel, Studerus ('12)], SECDEC3 [Borowka et al. (2015)]
- $\mathbf{Q\bar{Q}}$ production — amplitude grids from [Bärnreuther, Czakon, Fiedler (2014)]
  - FORTRAN routine for numerical interpolation of 2-dimensional grid, improved by expansions

Recoil-driven linear power corrections in neutral-current Drell–Yan process

Transverse-momentum cuts on undistinguished particles in two-body final states introduce enhanced sensitivity to low momentum scales [Salam, Slade (2021)]

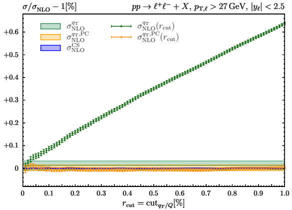
- **Linear power corrections (linPCs) in context of q_T subtraction**
  - have been resummed to all orders for s-channel (DY, Higgs) production [Ebert, Michel, Stewart, Tackmann (2021); Billis, Dehnadi, Ebert, Michel, Tackmann (2021)]
- **Recoil prescription** can be used to predict linPCs also in fixed-order calculations [Buonocore, SK, Rottoli, Wiesemann (2022), Camarda, Cieri, Ferrera ('21)] :

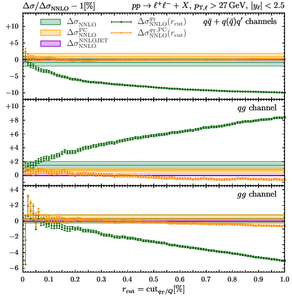
$$\Delta\sigma^{\text{linPCs}}(r_{\text{cut}}) = \int d\Phi_F \int_{\epsilon}^{r_{\text{cut}}} dr' \left(\frac{d\sigma^{\text{CT}}}{d\Phi_F\, dr'}\, \Theta_{\text{cuts}}(\Phi_F^{\text{rec}}) - \frac{d\sigma^{\text{CT}}}{d\Phi_F\, dr'}\, \Theta_{\text{cuts}}(\Phi_F) \right)$$

  - Φ_F^{rec} describes frame where system F is assigned a recoil q_T (boost from Collins–Soper frame, but precise prescription irrelevant)

- **Adding the contribution $\Delta\sigma^{\text{linPCs}}(r_{\text{cut}})$ reduces leading (recoil-driven) r_{cut} dependence from linear (without linPCs) to (at most) quadratic**
  - Illustration for **symmetric cuts** at NLO (upper plot) and NNLO (lower plot; reference result from NNLOjet [Bizon et al. (2021)])

[Buonocore, SK, Rottoli, Wiesemann (2022)]





Distributions with linPCs for neutral-current DY process with symmetric cuts

Sample distribution: ℓ^+ rapidity at NLO (left) and NNLO (right)

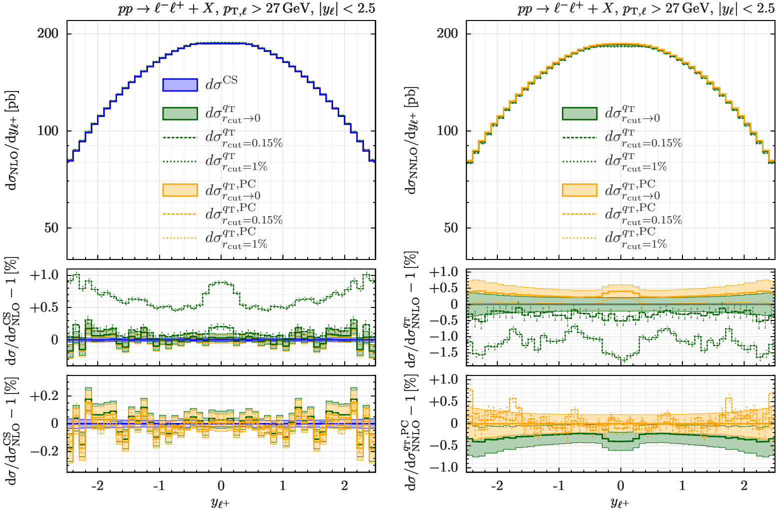
- up to $\sim 2\%$ deviations for highest considered value $r_{\text{cut}} = 1\%$ without linPCs
- good agreement between considered r_{cut} values with linPCs (within errors)

Note: The extrapolated results with linPCs and without linPCs agree well within errors!

➡ **higher efficiency with linPCs (larger r_{cut} values sufficient)**

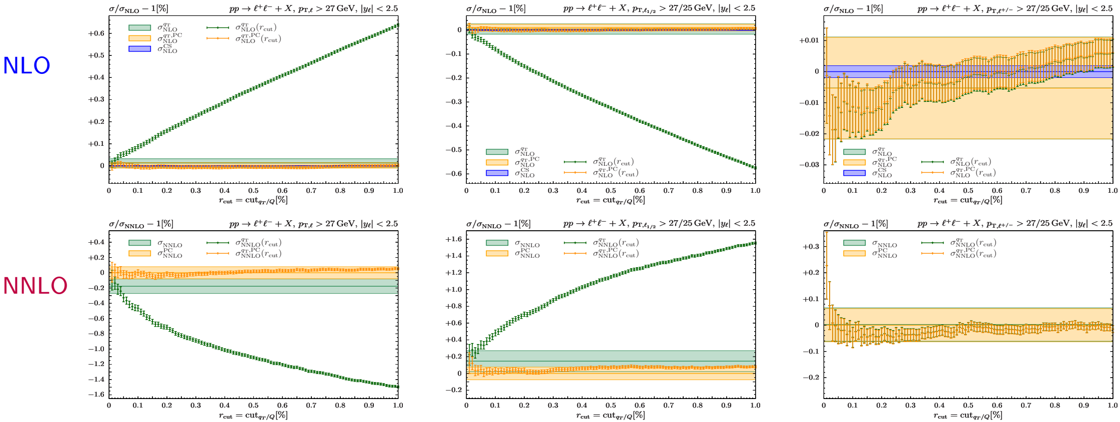
➡ **accurate results also from binwise extrapolation without including linPCs**

[Buonocore, SK, Rottoli, Wiesemann (2022)]



Recoil-driven linear power corrections for different fiducial cut configurations

symmetric $(p_{T,\ell} > 27\ \text{GeV})$ **asymmetric** $(p_{T,\ell_{1/2}} > 27/25\ \text{GeV})$ **staggered** $(p_{T,\ell^\pm} > 27/25\ \text{GeV})$



[Buonocore, SK, Rottoli, Wiesemann (2022)]

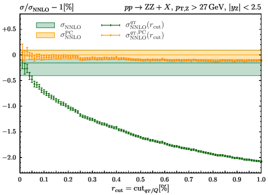
- similarly sizable linPCs (with opposite sign) for **symmetric** and **asymmetric** cuts
- **linPCs absent for staggered cuts** (as long as $q_T < \delta p_T$) ➥ also for other alternative cuts [Salam, Slade (2021)]

Recoil-driven linear power corrections for diboson processes

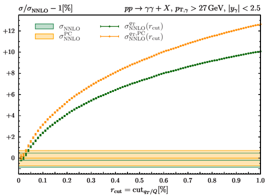
Investigation of linPCs for diboson processes

(two-body kinematics and symmetric cuts)

ZZ



γγ



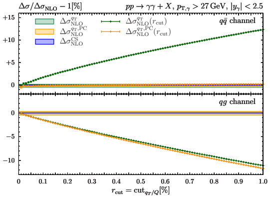
- For *ZZ* production, linPCs from recoil prescription reduce $r_{\rm cut}$ dependence to (at most) quadratic
  - ➥ formally proven only for *s*-channel production

[Ebert, Michel, Stewart, Tackmann (2021)]

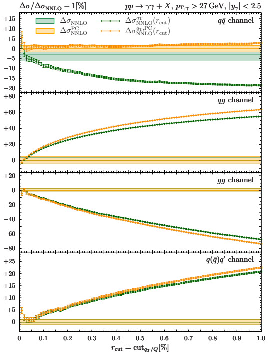
- For $\gamma\gamma$ production, $r_{\rm cut}$ dependence from photon isolation dominates over recoil-driven linPCs

Breakdown into partonic channels for $\gamma\gamma$ case

[Buonocore, SK, Rottoli, Wiesemann (2022)]



↑ NLO NNLO →



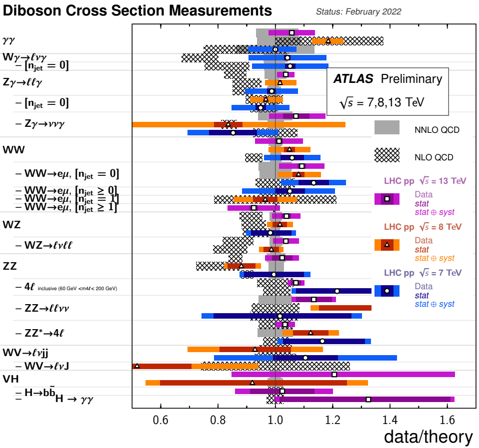
➥ linear $r_{\rm cut}$ dependence in $q\bar q$ channel only due to kinematic effects

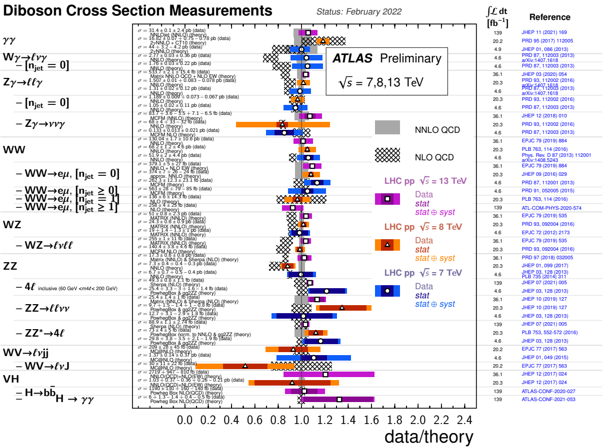
➥ cured by recoil-driven linPCs for $q\bar q$ channel

Note: Recoil-driven linPCs absent for staggered cuts (e.g. on $|y|$-ordered bosons), like in Drell–Yan case

Summary of ATLAS diboson measurements

[ATLAS collaboration (2022)]





Combination of QCD and EW corrections for diboson production – $p_{\mathrm{T,V_2}}$

Both corrections sizable, particularly in high-energy tails of distributions

➥ **approximation of leading $\mathcal{O}(\alpha_s\alpha)$ effects desirable**

Different combination approaches

- **additive:**

  $$d\sigma^{\mathrm{LO}}(1+\delta_{\mathrm{QCD}}^{\mathrm{(N)NLO}}+\delta_{\mathrm{EW}}^{\mathrm{NLO}})$$

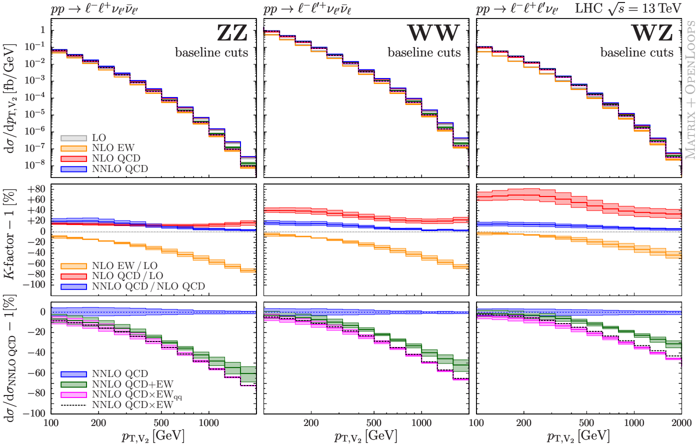
- **multiplicative:**

  $$d\sigma^{\mathrm{LO}}(1+\delta_{\mathrm{QCD}}^{\mathrm{(N)NLO}})(1+\delta_{\mathrm{EW}}^{\mathrm{NLO}})$$

- **multiplicative (only $q\bar{q}$):**

  $$d\sigma_{\mathrm{q\bar{q}}}^{\mathrm{LO}}(1+\delta_{\mathrm{q\bar{q},QCD}}^{\mathrm{(N)NLO}})(1+\delta_{\mathrm{q\bar{q},EW}}^{\mathrm{NLO}})+\sigma_{\gamma-\mathrm{ind.,EW}}^{\mathrm{NLO}}$$

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



Factorized approaches well motivated for **genuine VV observables** (dominated by hard-VV topologies)

➥ catch leading mixed QCD–EW effects and may thus be considered preferrable

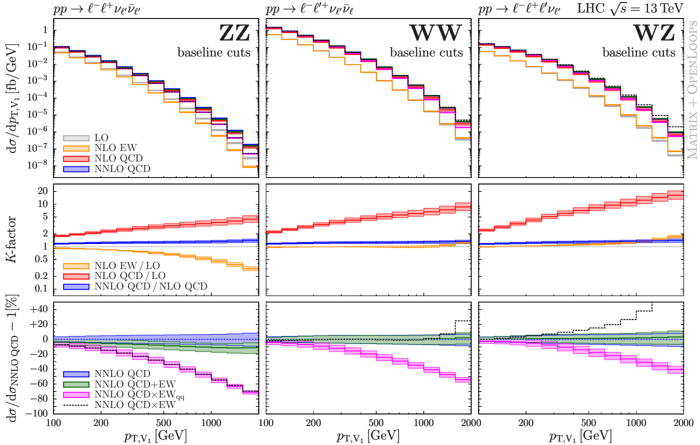
Combination of QCD and EW corrections for diboson production – $p_{\mathrm{T,V_1}}$

Situation more involved in presence of so-called **giant *K*-factors**

- ➥ QCD corrections in tails dominated by **hard-Vj topologies**
- ➥ also large positive EW corrections (photon-induced hard-Vj topologies)

- **additive** underestimates EW effects
- **multiplicative** combination multiplies large QCD and EW *K*-factors
  - ➥ **discarded**
- **multiplicative (only $q\bar{q}$)** shows expected Sudakov behaviour, but overestimates the EW effects (VV *K*-factor applied in hard-Vj region)

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



None of the approaches works perfectly well for **observables dominated by hard-Vj topologies**

➥ merged prediction, full mixed QCD–EW calculation, or phase space restriction to hard-VV topologies

Combination of QCD and EW corrections for diboson production – $p_{\mathrm{T,V_2}}/p_{\mathrm{T,V_1}}$

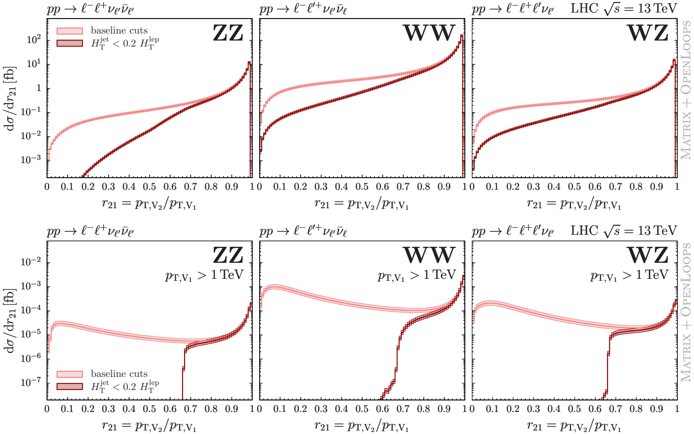
Restriction to hard-VV topologies

- direct requirement on the hardness of the two vector bosons
- moderate jet veto to suppress hard-Vj topologies: $H_{\mathrm{T}}^{\mathrm{jet}} < 0.2 H_{\mathrm{T}}^{\mathrm{lep}}$

Illustration of effect on distribution in $r_{21} = p_{\mathrm{T,V_2}}/p_{\mathrm{T,V_1}}$ at NLO QCD
($r_{21} = 1$ for LO kinematics)

- only baseline cuts (upper panels)
  - ➥ cross section peaked at $r_{21} \gtrsim 0.8$; widely unaffected by $H_{\mathrm{T}}^{\mathrm{jet}}$ cut
- region $p_{\mathrm{T,V_1}} > 1\,\mathrm{TeV}$ (lower panels)
  - ➥ second peak at $r_{21} \lesssim 0.3$ where hard-Vj topologies dominate; completely removed by $H_{\mathrm{T}}^{\mathrm{jet}}$ cut

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



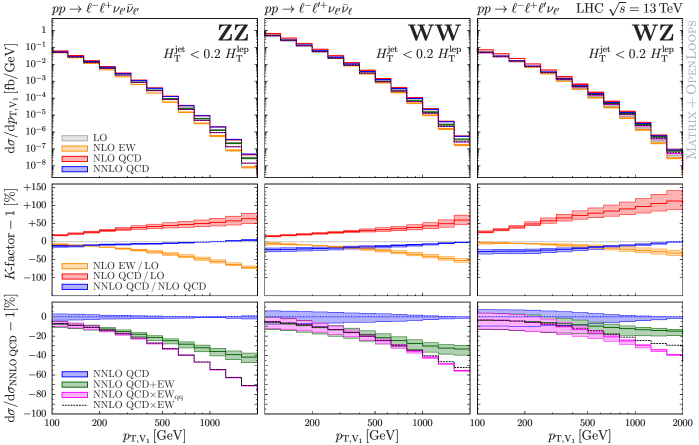
➥ **Remaining cross section after application of $H_{\mathrm{T}}^{\mathrm{jet}}$ cut completely dominated by the (desired) hard-VV region**

Combination of QCD and EW corrections for diboson production – $p_{\mathrm{T,V_1}}$ – $H_{\mathrm{T,jet}}$ cut

With $H_{\mathrm{T}}^{\mathrm{jet}} < 0.2 H_{\mathrm{T}}^{\mathrm{lep}}$, cross section dominated by hard-VV region

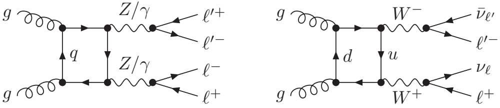
- ➥ justification to apply NLO EW correction factor on (N)NLO QCD cross sections
- **additive** underestimates EW effects; difference to multiplicative would overestimate uncertainties
- **both multiplicative** approaches well motivated to catch leading effects from mixed QCD–EW corrections
  - ➥ difference between the two as a rough estimate of $\mathcal{O}(\alpha_s\alpha)$ uncertainties

[Grazzini, SK, Lindert, Pozzorini, Wiesemann (2020)]



➥ Restriction to hard-VV topologies **reduces uncertainties from mixed QCD–EW higher-order effects**, but should also **increase the sensitivity of experimental searches for BSM effects (like AGCs)**

NLO QCD corrections to loop-induced gluon channel in ZZ/WW production



- only LO-accurate at $\mathcal{O}(\alpha_s^2)$
- enhanced by large gluon luminosity
- presumably dominant $\mathcal{O}(\alpha_s^3)$ contribution

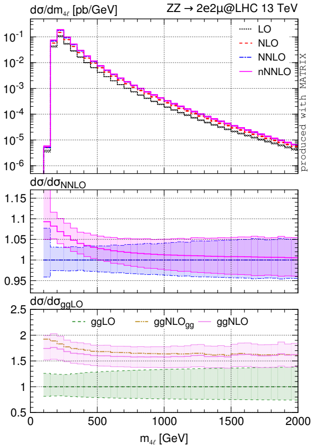
Approximate nNNLO QCD prediction

- full m_t dependence in 1-loop terms
- massless 2-loop amplitudes from ggVVAMP [von Manteuffel, Tancredi (2015)]
  - ➥ reweighted by full-m_t Born amplitude

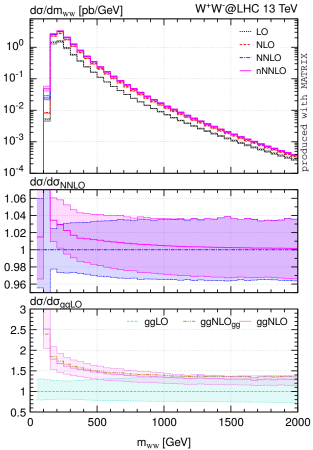
Effect of NLO QCD in *gg* channel

- highest impact in low-energy regions
- enhancement beyond NNLO QCD scale band

[Grazzini, SK, Wiesemann, Yook (2019)]



[Grazzini, SK, Wiesemann, Yook (2020)]



Best available fixed-order predictions for ZZ/WW production

- NNLO QCD for $q\bar{q}$ channel
- NLO EW combination for $q\bar{q}$ channel
- NLO QCD corrections for gg channel

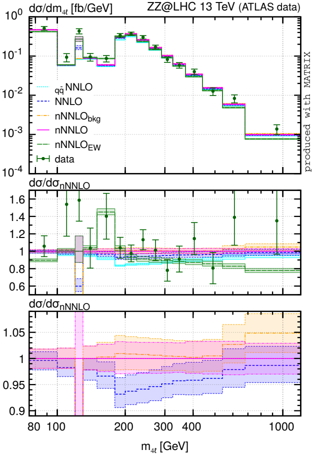
➥ **available for all VV processes in**
MATRIX v2 [Grazzini, SK, Wiesemann (2021)]

(refinements on m_t effects in gg amplitudes still to be added [Grazzini, SK, Wiesemann, Yook (2021)])

Diboson production beyond fixed order

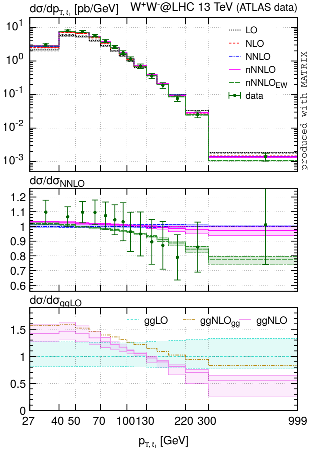
- resummation ($p_{\mathrm{T,VV}}$, $p_{\mathrm{T,jet}}^{\mathrm{veto}}$, …)
  MATRIX+RADISH [SK, Re, Rottoli, Wiesemann (2020)]
- event generation at NNLO QCD
  NNLOPS [WW: Re, Wiesemann, Zanderighi (2018)]
  MiNNLO$_{\mathrm{PS}}$ [Zγ: Lombardi, Wiesemann, Zanderighi (2021);
  WW: Lombardi, Wiesemann, Zanderighi (2021);
  ZZ: Buonocore, Koole, Lombardi, Rottoli, Wiesemann, Zanderighi (2022)]
  GENEVA
  [$\gamma\gamma$: Alioli, Broggio, Gavardi, SK, Lim, Nagar, Napoletano, Rottoli (2021)
  ZZ: Alioli, Broggio, Gavardi, SK, Lim, Nagar, Napoletano (2021);
  Wγ: Cridge, Lim, Nagar (2022)]

[Grazzini, SK, Wiesemann, Yook (2021)]



data from [JHEP 04, 048 (2019)]

[Grazzini, SK, Wiesemann, Yook (2020)]



data from [Eur. Phys. J. C 79 (2019) 884]

NLO QCD corrections to loop-induced gluon channel in ZZ/WW production

Integrated cross sections for $pp \to e^- e^+ \mu^- \mu^+$

(setup of ATLAS 8 TeV ZZ analysis [JHEP 01, 099 (2017)])

[Grazzini, SK, Wiesemann, Yook (2019)]

$\sqrt{s}$	8 TeV	13 TeV	8 TeV	13 TeV
	σ [fb]		$\sigma/\sigma_{\mathrm{NLO}} - 1$	
LO	$8.1881(8)^{+2.4\%}_{-3.2\%}$	$13.933(1)^{+5.5\%}_{-6.4\%}$	-27.5%	-29.8%
NLO	$11.2958(4)^{+2.5\%}_{-2.0\%}$	$19.8454(7)^{+2.5\%}_{-2.1\%}$	0%	0%
$q\bar{q}$NNLO	$12.09(2)^{+1.1\%}_{-1.1\%}$	$21.54(2)^{+1.1\%}_{-1.2\%}$	$+7.0\%$	$+8.6\%$
	σ [fb]		$\sigma/\sigma_{\mathrm{ggLO}} - 1$	
ggLO	$0.79355(6)^{+28.2\%}_{-20.9\%}$	$2.0052(1)^{+23.5\%}_{-17.9\%}$	0%	0%
ggNLO$_{gg}$	$1.4787(4)^{+15.9\%}_{-13.1\%}$	$3.626(1)^{+15.2\%}_{-12.7\%}$	$+86.3\%$	$+80.8\%$
ggNLO	$1.3892(4)^{+15.4\%}_{-13.6\%}$	$3.425(1)^{+13.9\%}_{-12.0\%}$	$+75.1\%$	$+70.8\%$
	σ [fb]		$\sigma/\sigma_{\mathrm{NLO}} - 1$	
NNLO	$12.88(2)^{+2.8\%}_{-2.2\%}$	$23.55(2)^{+3.0\%}_{-2.6\%}$	$+14.0\%$	$+18.7\%$
nNNLO	$13.48(2)^{+2.6\%}_{-2.3\%}$	$24.97(2)^{+2.9\%}_{-2.7\%}$	$+19.3\%$	$+25.8\%$

- relatively large corrections of 5% – 7% for ZZ, exceeding the NNLO QCD uncertainty bands (slightly smaller effect of 2% – 3% for WW)

Integrated cross sections for $pp \to e^- \mu^+ \nu_\mu \bar{\nu}_e$

(setup of ATLAS 13 TeV WW analysis [EPJC 79, 884 (2019)])

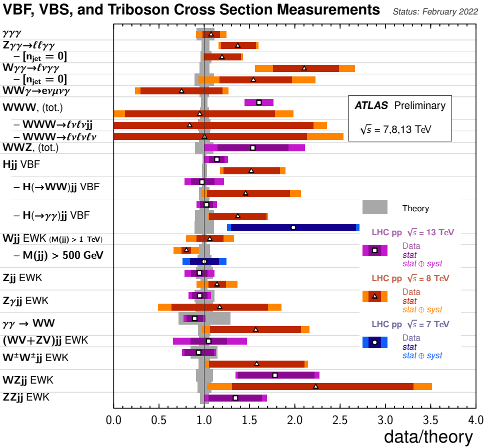
[Grazzini, SK, Wiesemann, Yook (2020)]

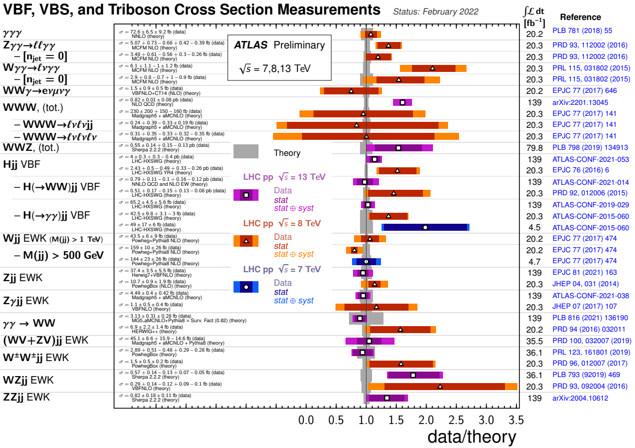
$\sqrt{s}$	jet veto	no veto	jet veto	no veto
13 TeV	σ [fb]		$\sigma/\sigma_{\mathrm{NLO}} - 1$	
LO	$284.11(1)^{+5.5\%}_{-6.5\%}$	$284.11(1)^{+5.5\%}_{-6.5\%}$	-15.5%	-43.7%
NLO	$336.42(3)^{+1.6\%}_{-2.0\%}$	$504.36(3)^{+4.1\%}_{-3.3\%}$	$+0.\%$	$+0.\%$
$q\bar{q}$NNLO	$336.8(2)^{+0.7\%}_{-0.5\%}$	$558.5(2)^{+2.1\%}_{-1.9\%}$	$+0.1\%$	$+10.7\%$
	σ [fb]		$\sigma/\sigma_{\mathrm{ggLO}} - 1$	
ggLO	$21.965(4)^{+25.7\%}_{-18.4\%}$	$21.965(4)^{+25.7\%}_{-18.4\%}$	$+0.\%$	$+0.\%$
ggNLO$_{gg}$	$31.68(6)^{+10.8\%}_{-10.6\%}$	$38.49(6)^{+15.9\%}_{-13.3\%}$	$+44.2\%$	$+75.2\%$
ggNLO	$28.79(6)^{+7.8\%}_{-9.1\%}$	$37.57(6)^{+15.3\%}_{-13.0\%}$	$+31.1\%$	$+71.0\%$
	σ [fb]		$\sigma/\sigma_{\mathrm{NLO}} - 1$	
NNLO	$358.7(2)^{+1.2\%}_{-0.9\%}$	$580.5(2)^{+2.9\%}_{-2.6\%}$	$+6.6\%$	$+15.1\%$
nNNLO	$365.6(2)^{+0.4\%}_{-0.6\%}$	$596.1(2)^{+2.8\%}_{-2.6\%}$	$+8.7\%$	$+18.2\%$
	σ [fb]		$\sigma/\sigma_{\mathrm{nNNLO}} - 1$	
nNNLO$_{\mathrm{EW}}$	$354.3(2)^{+0.5\%}_{-0.8\%}$	$580.2(2)^{+2.7\%}_{-2.6\%}$	-3.1%	-2.7%

- similar size as (negative) EW corrections

Summary of ATLAS triboson, VBF and VBS measurements

[ATLAS collaboration (2022)]





Triboson production at NNLO QCD accuracy

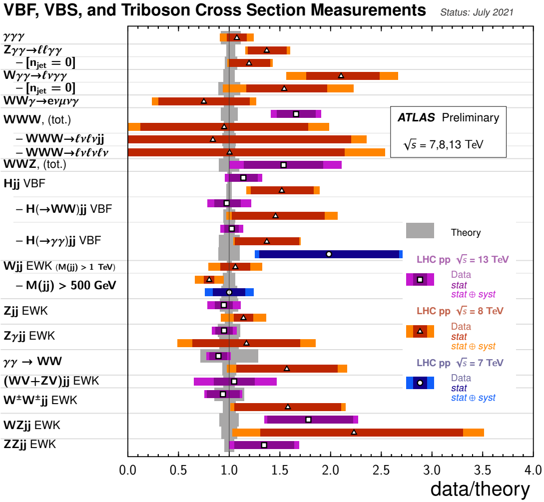
Relevance of triboson production

- important SM test ➥ quartic gauge couplings
- background for BSM searches
- very clean signatures in leptonic decay channels
- at least $\gamma\gamma\gamma$ and $V\gamma\gamma$ processes measured already with statistics collected during LHC Run I
    - ➥ much smaller cross sections for massive VVV
    - ➥ only WWW and WWZ observed recently

Prospects for triboson predictions at NNLO

- two-loop amplitudes as the bottleneck
  (only available for triphoton production by now)
- NNLO subtraction methods applicable in principle
- complicated calculation (amplitudes, phase spaces)
    - ➥ **challenge for technical performance, but feasible in the Matrix framework**

[ATLAS collaboration (2021)]



Triphoton production at NNLO QCD accuracy

First Matrix calculation for genuine 2 → 3 process at NNLO QCD

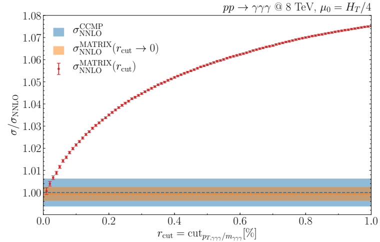
- $q_{\rm T}$ subtraction method for colourless final states directly applicable:

$$d\sigma^{\gamma\gamma\gamma}_{\rm NNLO} = \mathcal{H}^{\gamma\gamma\gamma}_{\rm NNLO} \otimes d\sigma_{\rm LO} + \left[d\sigma^{\gamma\gamma\gamma+\rm jet}_{\rm NLO} - d\sigma^{\gamma\gamma\gamma,\rm CT}_{\rm NNLO} \right]_{r_{\rm cut}\to 0}$$

➥ remarkable numerical control over slicing parameter dependence

➥ full agreement with independent calculation [Chawdhry, Czakon, Mitov, Poncelet (2020)]

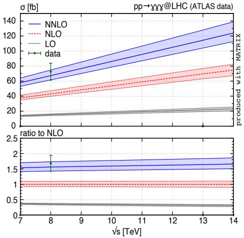
[SK, Sotnikov, Wiesemann (2021)]

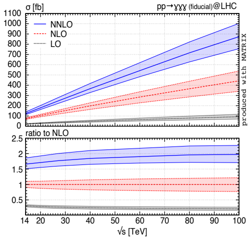


Further important ingredients of the calculation

- highly efficient phase space integration in Munich [SK]
- fast and stable 1-loop amplitudes from OpenLoops [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)]
- fast and stable 2-loop amplitudes [Abreu, Page, Pascual, Sotnikov (2021)] generated with Caravel, using PentagonFunctions++ [Abreu et al. (2020)] [Chicherin, Sotnikov (2020)]

➥ talks by Vasily Sotnikov and Ben Page



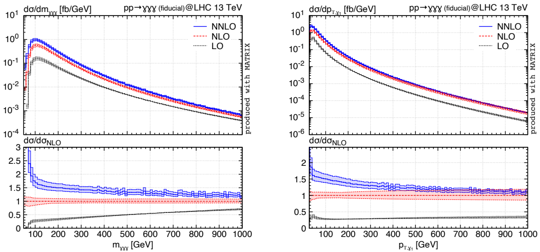


Triphoton production at NNLO QCD accuracy

Comparison with ATLAS data at 8 TeV

- perfect agreement with NNLO QCD predictions, due to both normalization and shape corrections
- significant discrepancies at lower orders

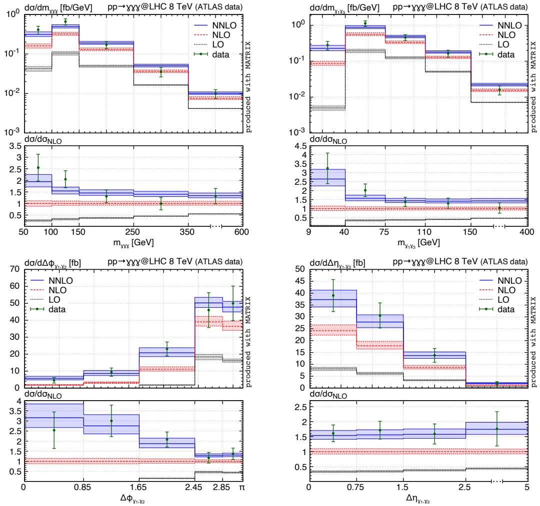
[SK, Sotnikov, Wiesemann (2021)]



- great numerical performance also with refined resolution and in suppressed phase space regions

➥ **MATRIX fully suitable for triboson processes**

[SK, Sotnikov, Wiesemann (2021)] data from [Phys. Lett. B 781 (2018) 55]

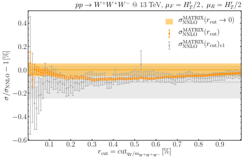


Feasibility studies on massive triboson production at NNLO QCD accuracy

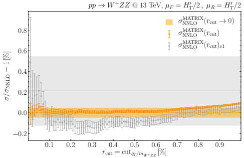
Studies on r_{cut} dependence for inclusive massive VVV production processes

- very good numerical control ➥ permille-level precision achievable within reasonable runtime

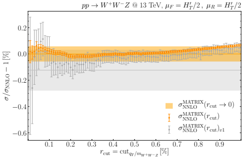
$W^+W^+W^-$



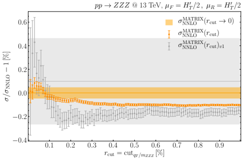
W^+ZZ



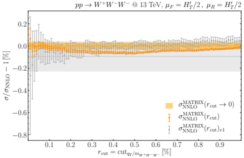
W^+W^-Z



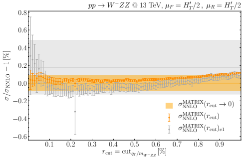
ZZZ



$W^+W^-W^-$



W^-ZZ



Obviously, no robust conclusion on the size of NNLO QCD corrections can be drawn without knowledge of the two-loop amplitudes

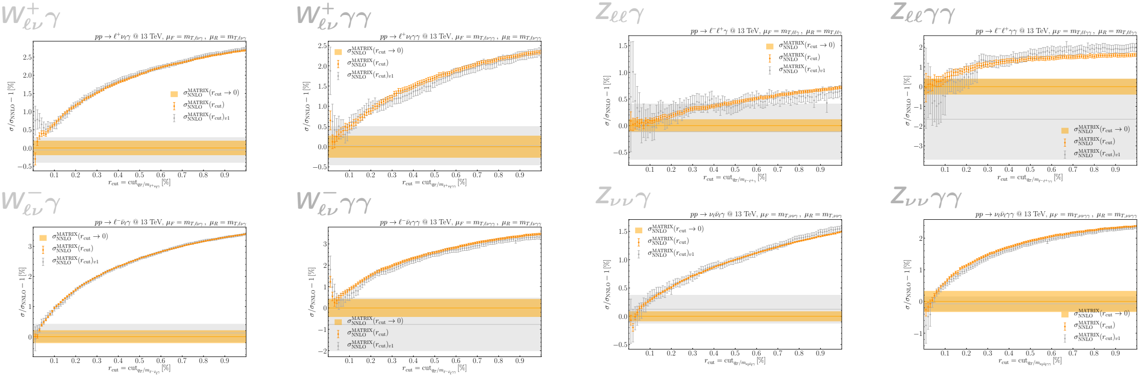
➥ Finite remainder of two-loop amplitudes set to zero for these technical feasibility studies.

- impact of finite remainder of two-loop amplitudes $H^{(2)}$ typically small, only $\mathcal{O}(2\%)$ for VV processes
- NNLO K-factors without $H^{(2)}$: $\mathcal{O}(1-2\%)$ ($\approx +6\%$ from gg channel for WWZ)
  - ➥ **no huge NNLO QCD corrections expected for massive VVV production processes**

Feasibility studies on V$\gamma\gamma$ production processes at NNLO QCD accuracy

Studies on r_{cut} dependence for V$\gamma\gamma$ processes (standard cuts and photon isolation)

- large power corrections due to photon isolation (as for $\gamma\gamma\gamma$), but numerically well under control



- impact of finite remainder of two-loop amplitudes $H^{(2)}$ small, only $\mathcal{O}(2-3\%)$ for Vγ processes
- NNLO K-factors without $H^{(2)}$: $\mathcal{O}(10-30\%)$ (depending on final state and fiducial cuts)
    - ➥ **comparable size of NNLO QCD corrections as for Vγ expected for V$\gamma\gamma$ processes**

Production of heavy coloured particles at NNLO QCD accuracy

Extension of q_{T} subtraction method to production of heavy coloured particles (e.g. top-quark pairs)

$$d\sigma^{t\bar{t}}_{\mathrm{NNLO}} = \mathcal{H}^{t\bar{t}}_{\mathrm{NNLO}} \otimes d\sigma_{\mathrm{LO}} + \left[d\sigma^{t\bar{t}+\mathrm{jet}}_{\mathrm{NLO}} - d\sigma^{t\bar{t},\mathrm{CT}}_{\mathrm{NNLO}} \right]_{r_{\mathrm{cut}}\to 0}$$

- counterterm accounts for IR behaviour of real contribution, including soft singularities related to emissions from final-state quarks [Catani, Grazzini, Torre (2014), Ferroglia, Neubert, Pecjak, Yang (2009), Li, Li, Shao, Yang, Zu (2013)]
- $\mathcal{H}^{t\bar{t}}_{\mathrm{NNLO}}$ contains remainder of integrated final-state soft singularities [Catani, Devoto, Grazzini, Mazzitelli (to appear), Angeles-Martinez, Czakon, Sapeta (2018)]
- massive NLO subtraction required for real-emission part, e.g. massive dipole subtraction [Catani, Seymour (1997), Catani, Dittmaier, Seymour, Trocsanyi (2002)]

Associated heavy-quark pair production ($t\bar{t}$H, $t\bar{t}$V, . . .) with identical singularity structure

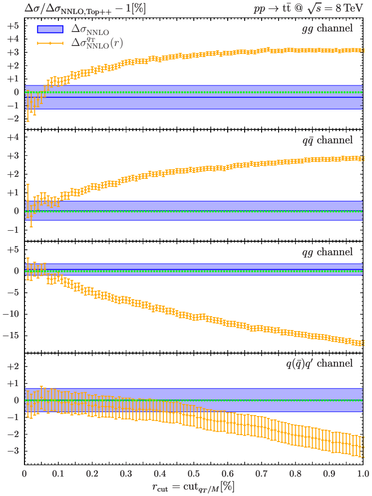
- numerical solutions for required for evaluation of the soft function due to more involved kinematics
  - ➥ no back-to-back configuration of heavy quarks
- proof-of-principle calculation for non-diagonal channels in $t\bar{t}\mathrm{H}$ [Catani, Fabre, SK, Grazzini (2021)]
- two-loop amplitudes as the bottleneck for any beyond 2 → 2 NNLO calculations

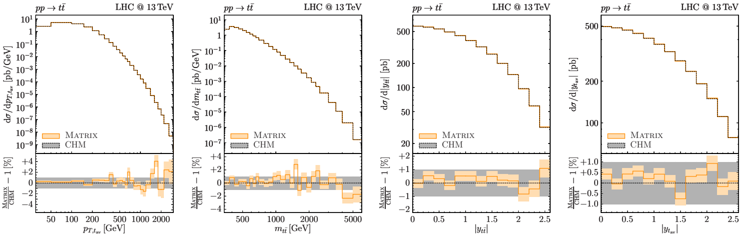
Top-quark pair production at NNLO QCD accuracy

First Matrix calculation for colourful final states at NNLO QCD

[Catani, Devoto, Grazzini, SK, Mazzitelli, Sargsyan (2019)]

- 2-loop amplitudes from numerical result [Bärnreuther, Czakon, Fiedler (2014)]
- slicing parameter dependence under good numerical control; investigation after splitting into partonic channels
  - ➥ full agreement with Top++ [Czakon, Mitov (2014)]
- successful validation also on the level of differential distributions [Catani, Devoto, Grazzini, SK, Mazzitelli (2019)] (comparison against results from [Czakon, Heymes, Mitov (2017)])



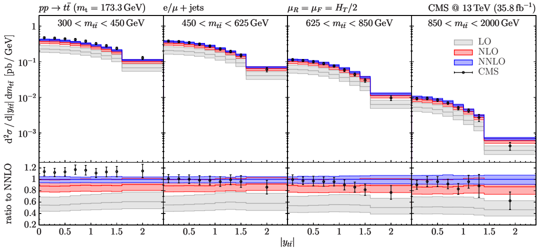


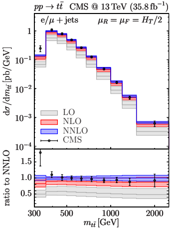
Top-quark pair production at NNLO QCD accuracy

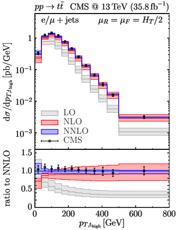
Good agreement with (multi)differential CMS data

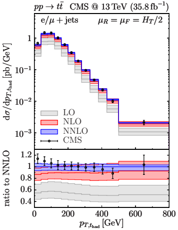
- lowest $m_{t\bar{t}}$ bin problematic: sensitivity to m_t value, threshold effects, extrapolation to stable tops, ...
- instabilities related to $p_{T,t\bar{t}} \to 0$ region
  - ➥ would require resummation/shower matching

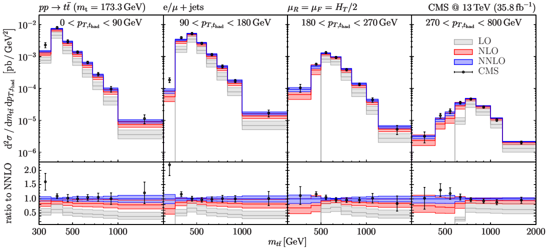
[Catani, Devoto, Grazzini, SK, Mazzitelli (2019)] data from [Phys. Rev. D 97 (2018) 112003]











Indications for perturbative convergence

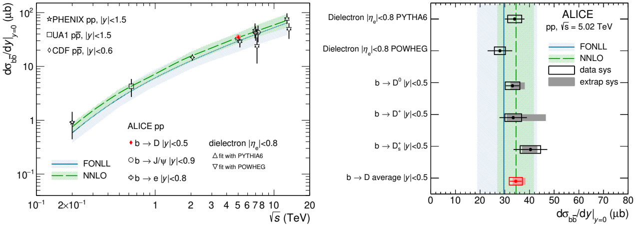
➥ widely overlapping bands from NLO to NNLO with reduced scale variation uncertainties

Bottom-quark pair production at NNLO QCD accuracy

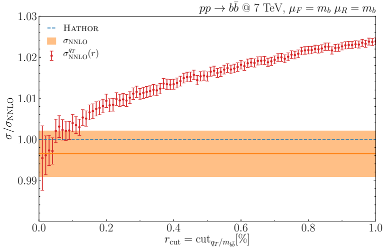
Application to bottom quarks

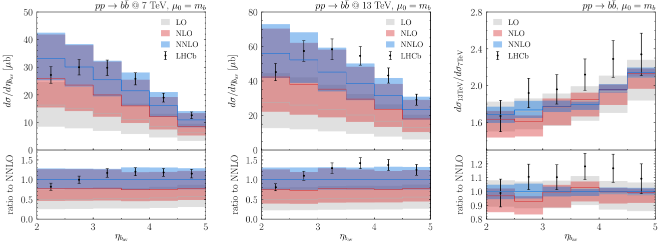
- conceptionally similar to $t\bar{t}$ production
- applications in all LHC experiments
- larger uncertainties due to lower scales
  - reduction through ratios with partial cancellation of uncertainties
- numerically more challenging ($m_b \ll m_t$)
  - calculation still remarkably stable

[ALICE Collaboration (2021)]



[Catani, Devoto, Grazzini, SK, Mazzitelli (2021)] data from [Phys. Rev. Lett. 118 (2017), no. 5 052002]



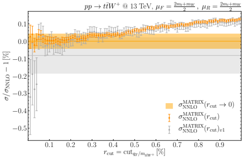


Feasibility studies on associated heavy-quark pair production at NNLO QCD accuracy

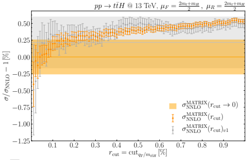
Studies on r_{cut} dependence for inclusive $Q\bar{Q}+X$ processes

- proof-of-principle for non-diagonal channels in $t\bar{t}H$ [Catani, Fabre, SK, Grazzini (2021)]
- finite remainders of two-loop amplitudes and of the soft function neglected in these studies
  - ➥ no reliable estimate of NNLO QCD result, but cancellation of IR divergences well under control
    - ➥ permille-level precision achievable within reasonable runtime

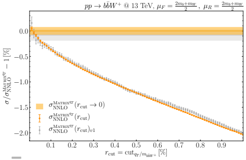
$t\bar{t}W^+$



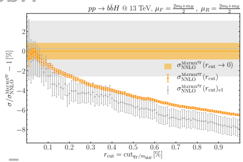
$t\bar{t}H$



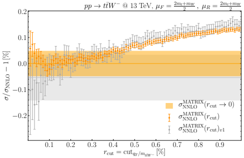
$b\bar{b}W^+$



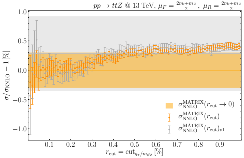
$b\bar{b}H$



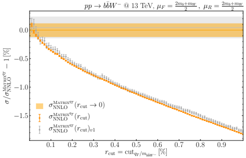
$t\bar{t}W^-$



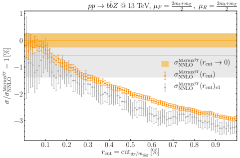
$t\bar{t}Z$



$b\bar{b}W^-$



$b\bar{b}Z$



Mixed NNLO QCD–EW calculation for production of massive charged particles

Extension of q_{T} subtraction method to mixed QCD–EW corrections of $\mathcal{O}(\alpha_s^m \alpha^n)$

$$d\sigma_{(m,n)}^{\ell\ell/\ell\nu} = \mathcal{H}_{(m,n)}^{\ell\ell/\ell\nu} \otimes d\sigma_{\mathrm{LO}} + \left[d\sigma_{(m,n)}^{\ell\ell/\ell\nu,\mathrm{R}} - d\sigma_{(m,n)}^{\ell\ell/\ell\nu,\mathrm{CT}} \right]_{r_{\mathrm{cut}} \to 0}$$

- $m = 1(2)$ and $n = 0$: (N)NLO QCD corrections
- $m = 0$ and $n = 1$: NLO EW corrections
- $m = 1$ and $n = 1$: mixed NNLO QCD–EW corrections

(limitation: no massless jets (for $m \geq 1$) and no massless charged particles (for $n \geq 1$) allowed at LO)

Strategy to cancel IR singularities in mixed QCD–EW corrections

- abelianisation procedure, starting from heavy-quark pair production at NNLO QCD
  - ➥ for neutral final states, abelianisation of standard q_{T} subtraction method is sufficient (mixed QCD–QED corrections on $pp \to \mathrm{Z}$ [De Florian, Der, Fabre (2018)], $pp \to \nu\bar{\nu}$ [Cieri, De Florian, Der, Mazzitelli (2020)])
- colourless final state ($\ell\ell/\ell\nu$) results in soft final-state singularities of pure QED origin
  - ➥ much simpler IR structure than in heavy-quark pair production at NNLO QCD
- finite charged-lepton mass required to regularize collinear final-state singularities

Towards mixed NNLO QCD–EW corrections for Drell–Yan processes

Mixed NNLO QCD–QED corrections

- on-shell Z production [De Florian, Der, Fabre (2018)]
- off-shell Z → $\nu\bar{\nu}$ production
  [Cieri, De Florian, Der, Mazzitelli (2020)]
- on-shell Z production with decay, including NLO QCD (production) × NLO QED (decay)
  [Delto, Jaquier, Melnikov, Röntsch (2019)]

Mixed NNLO QCD–EW corrections for on-shell DY production

- on-shell Z production [Bonciani, Buccioni, Rana, Vicini (2020)]
- on-shell W production
  [Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2021)]
- estimate of the impact on W mass extraction
  [Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch (2021)]

Mixed NNLO QCD–EW corrections beyond the on-shell approximation

- pole approximation (PA) [Dittmaier, Huss, Schwinn (2014 & 2015)]

Recent progress towards full mixed NNLO QCD–EW corrections

- two-loop master integrals
  [Bonciani, Di Vita, Mastrolia, Schubert (2016), Heller, von Manteuffel, Schabinger (2019), Hasan, Schubert (2020)]
- two-loop amplitude for neutral current DY
  [Heller, von Manteuffel, Schabinger, Spiesberger (2020)]
- complete result for $\mathcal{O}(n_\mathrm{F}\alpha_s\alpha)$ effects
  [Dittmaier, Schmidt, Schwarz (2020)]

Mixed NNLO QCD–EW calculation within the MATRIX framework with massive leptons (muons)

- off-shell $pp \to \mathrm{W} \to \mu^+\nu_\mu$ calculation with two-loop amplitudes in a reweighted PA approach
  [Buonocore, Grazzini, SK, Savoini, Tramontano (2021)]
- off-shell $pp \to \mathrm{Z}/\gamma \to \mu^+\mu^-$ calculation with full two-loop amplitudes [Bonciani, Rana, Vicini (to appear)]
  [Bonciani, Buonocore, Grazzini, SK, Tramontano, Rana, Vicini ('21)]

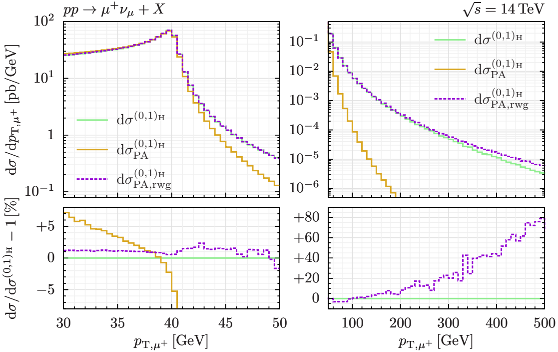
Justification of (reweighted) pole approximation approaches

PA vs. reweighted PA at NLO EW

$$H^{(0,1)}_{\mathrm{PA}} = 2\mathrm{Re}\left(\mathcal{M}^{(0,1)}_{\mathrm{fin}}\mathcal{M}^{(0,0)*}\right)_{\mathrm{PA}} \Big/ \left|\mathcal{M}^{(0,0)}\right|^2$$

$$H^{(0,1)}_{\mathrm{PA,rwg}} = 2\mathrm{Re}\left(\mathcal{M}^{(0,1)}_{\mathrm{fin}}\mathcal{M}^{(0,0)*}\right)_{\mathrm{PA}} \Big/ \left|\mathcal{M}^{(0,0)}_{\mathrm{PA}}\right|^2$$

[Buonocore, Grazzini, SK, Savoini, Tramontano (2021)]

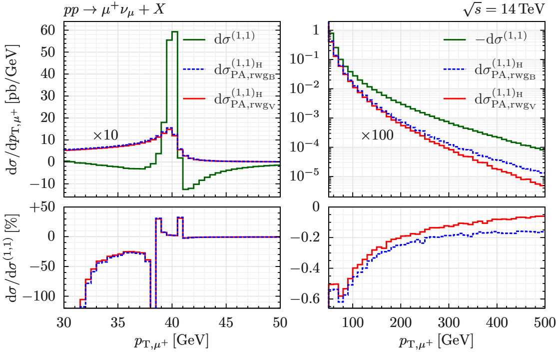


Reweighted PA versions at NNLO QCD–EW

$$H^{(1,1)}_{\mathrm{PA,rwg_B}} = H^{(1,1)}_{\mathrm{PA}} \times \left|\mathcal{M}^{(0,0)}\right|^2 \Big/ \left|\mathcal{M}^{(0,0)}_{\mathrm{PA}}\right|^2$$

$$H^{(1,1)}_{\mathrm{PA,rwg_V}} = H^{(1,1)}_{\mathrm{PA}} \times H^{(0,1)} \big/ H^{(0,1)}_{\mathrm{PA}}$$

[Buonocore, Grazzini, SK, Savoini, Tramontano (2021)]



Results for mixed NNLO QCD–EW corrections in off-shell W production

Setup and integrated results

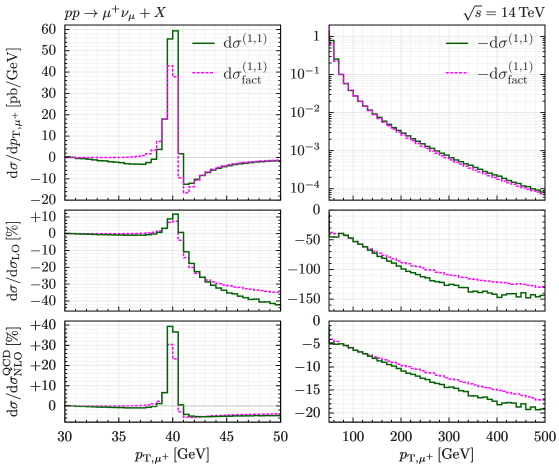
- `NNPDF31_nnlo_as_0118_luxqed` $\mu_R = \mu_F = m_W$
- $p_{T,\mu} > 25\,\text{GeV}\,, \quad |y_\mu| < 2.5\,, \quad p_{T,\text{miss}} > 25\,\text{GeV}$

		NLO QCD	NLO EW	NNLO QCD	NNLO QCD–EW
σ [pb]	σ_{LO}	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	5029.2	970.5(3)	−143.61(15)	251(4)	− 7.0(1.2)
qg	—	−1079.86(12)	—	−377(3)	39.0(4)
$q(g)\gamma$	—	—	2.823(1)	—	0.055(5)
$q(\bar{q})q'$	—	—	—	44.2(7)	1.2382(3)
gg	—	—	—	100.8(8)	—
tot	5029.2	−109.4(4)	−140.8(2)	19(5)	33.3(1.3)
		−2.2%	−2.8%	+0.4%	+0.6%

- large cancellations between partonic channels in NLO and NNLO QCD corrections
- similar size of NLO QCD and EW corrections
- no dedicated scale dependence studies performed

Differential distribution in p_{T,μ^+}

[Buonocore, Grazzini, SK, Savoini, Tramontano (2021)]



Results for mixed NNLO QCD–EW corrections in off-shell Z production

Setup and integrated results

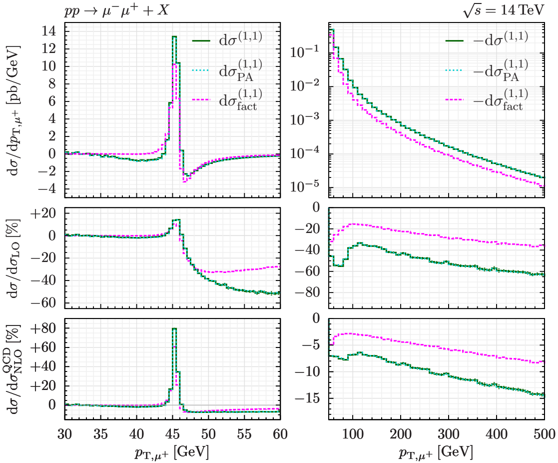
- NNPDF31_nnlo_as_0118_luxqed $\mu_R = \mu_F = m_Z$
- $p_{\mathrm{T},\mu} > 25\,\mathrm{GeV}\,, \quad |y_\mu| < 2.5\,, \quad m_{\mu\mu} > 50\,\mathrm{GeV}$

		NLO QCD	NLO EW	NNLO QCD	NNLO QCD–EW
σ [pb]	σ_{LO}	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	809.56(1)	191.85(1)	−33.76(1)	49.9(7)	−4.8(3)
qg	—	−158.08(2)	—	−74.8(5)	8.6(1)
$q(g)\gamma$	—	—	−0.839(2)	—	0.084(3)
$q(\bar{q})q'$	—	—	—	6.3(1)	0.19(0)
gg	—	—	—	18.1(2)	—
$\gamma\gamma$	1.42(0)	—	−0.0117(4)	—	—
tot	810.98(1)	33.77(2)	−34.61(1)	−0.5(9)	4.0(3)
		+4.2%	−4.3%	−0.1%	+0.5%

- large cancellations between partonic channels in NLO and NNLO QCD corrections
- almost complete (accidental) compensation between NLO QCD and EW corrections

Differential distribution in p_{T,μ^+}

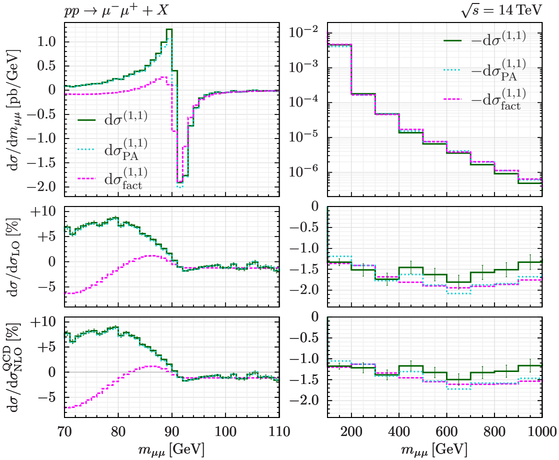
[Bonciani, Buonocore, Grazzini, SK, Tramontano, Rana, Vicini ('21)]



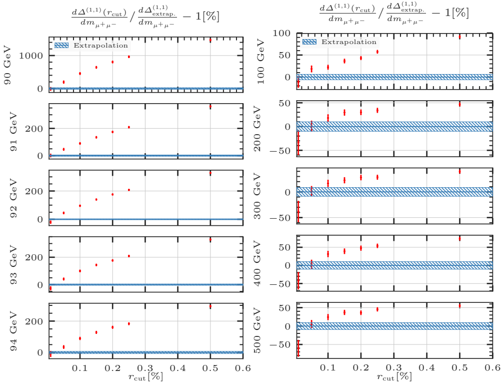
Results for mixed NNLO QCD–EW corrections in off-shell Z production

Differential distribution in $m_{\mu\mu}$

[Bonciani, Buonocore, Grazzini, SK, Tramontano, Rana, Vicini ('21)]



Numerical control over power corrections by binwise $r_{\mathrm{cut}} \to 0$ extrapolation



➥ inclusion of power corrections could significantly improve numerical performance

NNLO QCD subtraction/slicing methods and implementations (status Oct 2021)

Subtraction/slicing methods

- q_T subtraction [Catani, Grazzini (2007)]
- N-jettiness subtraction [Boughezal, Focke, Liu, Petriello (2015); Gaunt, Stahlhofen, Tackmann, Walsh (2015)]
- Antenna subtraction [Gehrmann, Gehrmann-De Ridder, Glover (2005)]
- Sector-improved residue subtraction [Czakon (2010); Boughezal, Melnikov, Petriello (2012)]
- ColorFul subtraction [Somogyi, Trocsanyi, Del Duca (2005)]
- Nested soft–collinear subtraction [Caola, Melnikov, Röntsch (2017)]
- Analytic local sector subtraction [Magnea, Maina, Pelliccioli, Signorile-Signorile, Torrielli, Uccirati (2018)]
- Projection to Born [Cacciari, Dreyer, Karlberg, Salam, Zanderighi (2015)]
- Geometric subtraction [Herzog (2018)]
- . . .

General (public) frameworks

- **MATRIX** (q_T slicing) [Grazzini, SK, Wiesemann, . . .]
  - Z, W, H, $\gamma\gamma$, Zγ, Wγ, WW, ZZ, WZ
  - ZH, WH, HH, $t\bar{t}$, $b\bar{b}$, $\gamma\gamma\gamma$, . . .
- **MCFM** (N-jettiness slicing) [Campbell, K. Ellis, Giele, Neumann, Williams, . . .]
  - Z, W, H, ZH, WH, $\gamma\gamma$, Zγ
  - Wγ, γj, Zj, Wj, Hj, . . .
- **NNLOJET** (antennna subtraction) [Gehrmann, Gehrmann–de Ridder, Glover, Huss, Chen, Gauld, . . .]
  - jj, γj, Zj, Wj, Hj, Zb, . . .
- **STRIPPER** (sector-improved residue subtraction) [Czakon, Mitov, Poncelet, Chawdhry, . . .]
  - $t\bar{t}$, jj, WW, Wc, $\gamma\gamma\gamma$, $\gamma\gamma$j, jjj, . . .
- . . .

↪ **Extension beyond 2 → 2 conceptionally straightforward if amplitudes become available!**

Recent achievements in (N)NNLO QCD calculations (status Oct 2021)

First 2 → 3 calculations at NNLO QCD

- $\gamma\gamma\gamma$ [Chawdhry, Czakon, Mitov, Poncelet (2020), SK, Sotnikov, Wiesemann (2021)]
- $\gamma\gamma$j [Chawdhry, Czakon, Mitov, Poncelet ('21)]
- jjj [Czakon, Mitov, Poncelet ('21)]

Recent achievements in 2-loop 2 → 3 amplitudes

- leading-colour jjj [Abreu, Page, Pascual, Sotnikov (2021), Abreu, Febres Cordero, Ita, Page, Sotnikov (2021)]
- $q\bar{q} \to \gamma\gamma$j [Agarwal, Buccioni, von Manteuffel, Tancredi ('21)]
- $gg \to \gamma\gamma g$ [Badger, Brønnum-Hansen, Chicherin, Gehrmann, Hartanto, Henn, Marcoli, Moodie, Peraro, Zoia ('21)]
- leading-colour W$b\bar{b}$ [Badger, Hartanto, Zoia (2021)]
- leading-colour H$b\bar{b}$ [Badger, Hartanto, Kryś, Zoia ('21)]

Heavy-quark loops for gg → diboson processes

- HH [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016), Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, Wellmann (2019)]
- $\gamma\gamma$ [Maltoni, Mandal, Zhao (2019), Chen, Heinrich, Jahn, Jones, Kerner (2020)]
- ZZ [Agarwal, Jones, von Manteuffel (2021), Brønnum-Hansen, Wang (2021)]
- WW [Brønnum-Hansen, Wang (2021)]
- ZH [Chen, Heinrich, Jones, Kerner, Klappert, Schlenk (2021)]

Inclusive 2 → 1 calculations at N^{3}LO QCD

- H [Anastasiou, Duhr, Dulat, Herzog, Mistlberger (2015), + Furlan, Gehrmann,Lazopoulos (2016), Mistlberger (2018)]
- $b\bar{b} \to$ H [Duhr, Dulat, Mistlberger (2020), + Hirschi (2020)]
- W [Duhr, Dulat, Mistlberger (2020 & 2020)]
- γ^* [Duhr, Dulat, Mistlberger (2020)]

Fully differential 2 → 1 calculations at N^{3}LO QCD

- H [Dulat, Mistlberger, Pelloni (2019), Cieri, Chen, Gehrmann, Glover, Huss (2019)]
- H ($\to \gamma\gamma$) [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni ('21)]
- Z/γ^* [Camarda, Cieri, Ferrera ('21), Chen, Gehrmann, Glover, Huss, Yang ('21)]

➥ combination of local NNLO subtraction with slicing/projection methods promoted to N^{3}LO

First 3-loop amplitudes beyond 2 → 1

- leading-colour $\gamma\gamma$ [Caola, von Manteuffel, Tancredi (2021)]
- four-quark scattering [Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi ('21)]

Status of NLO EW calculations (status Oct 2021)

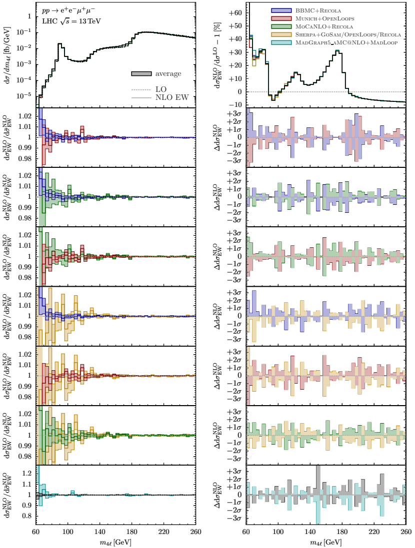
Dedicated comparison in Les Houches 2017 proceedings

- BBMC + Recola
- Munich/Matrix + OpenLoops
- MoCaNLO + Recola
- Sherpa + GoSam/ OpenLoops/ Recola
- MadGraph5_aMC@NLO + MadLoop

➥ conceptionally solved, as for NLO QCD calculations

Recent highlights: high-multiplicity processes

- off-shell $t\bar{t}W$ production $(2 \to 8)$ [Denner, Pelliccioli (2021)]
- off-shell $t\bar{t}H$ production $(2 \to 7)$ [Denner, Lang, Pellen, Uccirati (2017)]
- off-shell WWW production $(2 \to 6)$ [Dittmaier, Knippen, Schwan (2020)]
- vector boson scattering $(2 \to 6)$
  - $W^\pm W^\pm$ [Biedermann, Denner, Pellen (2017), Denner, Lang, Pellen, Uccirati (2017)]
  - WZ [Denner, Dittmaier, Maierhöfer, Pellen, Schwan (2019)]
  - ZZ [Denner, Franken, Pellen, Schmidt (2020)]
  - W^+W^- [Denner, Franken, Pellen, Schmidt (preliminary results at RADCOR 2021)]



[Les Houches 2017: Physics at TeV Colliders SM WG Report (2018)]