

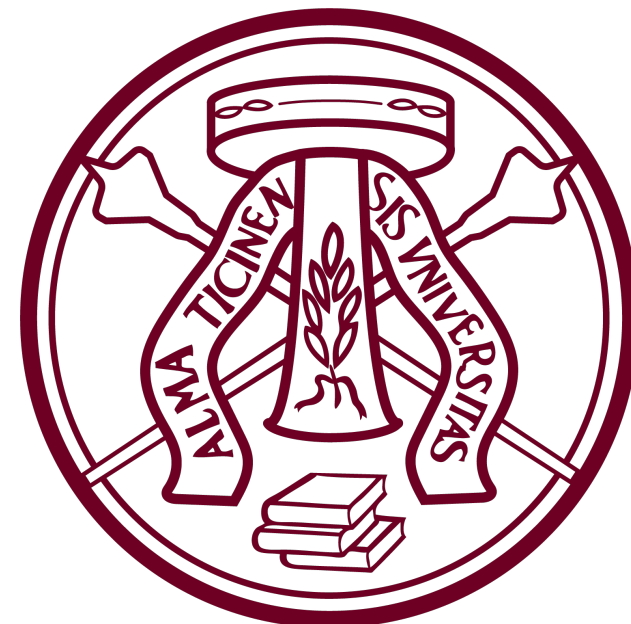
A model for pion collinear parton distribution function and form factor

xFitter External meeting

May, 3, 2023

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Based on: B. Pasquini, S. Rodini, S. Venturini, 2303.01789



Outline

- ❄ Light-front formalism
- ❄ Our model
- ❄ Results and Comments

Light-front formalism

$$|\psi\rangle = \sum_n \sum_{c_i} \sum_{\mu_i} \int [Dx]_n \psi_n^\Lambda(r) |n, w_1, w_2, \dots, w_n\rangle$$

$$w_i = (p_i^+, \vec{p}_{\perp i}, \mu_i, c_i)$$

$$|n, w_1, w_2, \dots, w_n\rangle = \prod_{j=1}^{n_q} b_{q_j}^\dagger(w_j) \prod_{l=1}^{n_{\bar{q}}} d_{q_l}^\dagger(w_l) \prod_{m=1}^{n_{gl}} a^\dagger(w_m) |0\rangle$$

$$\psi_n^\Lambda(r) = \langle \psi | n, w_1^{c_1}, w_2^{c_2}, \dots, w_n^{c_n} \rangle$$



Light-Front Wave Functions

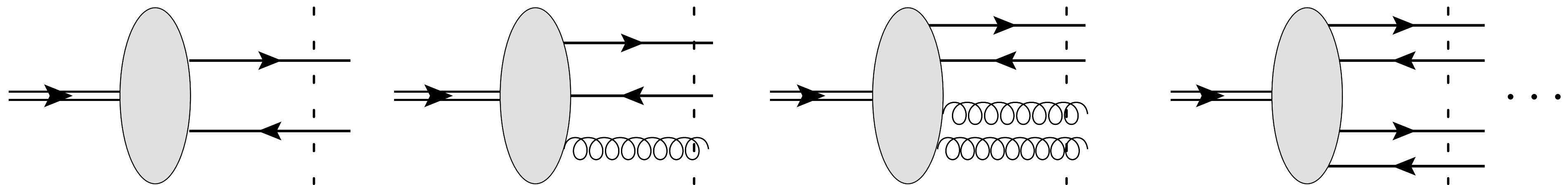
Light-front formalism

LFWF overlap representation

$$\Phi^{[\gamma^+]}(x; p) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ikz} \langle \pi(p) | \bar{q}(0) \mathcal{U}_{(0,z)} \gamma^+ q(z) | \pi(p) \rangle \Big|_{z^+=0, \vec{z}_\perp = \vec{0}_\perp}$$

$$\Phi^{[\gamma^+]}(\Delta; p) = \frac{1}{2p^+} \langle \pi(p') | \bar{q}(0) \mathcal{U}_{(0,z)} \gamma^+ q(z) | \pi(p) \rangle$$

$$| \pi(p) \rangle = \underbrace{\psi_{q\bar{q}}}_{\text{quark-antiquark}} | \pi(p)_{q\bar{q}} \rangle + \underbrace{\psi_{q\bar{q}g}}_{\text{quark-antiquark-gluon}} | \pi(p)_{q\bar{q}g} \rangle + \underbrace{\psi_{q\bar{q}gg}}_{\text{quark-antiquark-gluon-gluon}} | \pi(p)_{q\bar{q}gg} \rangle + \underbrace{\psi_{q\bar{q}s\bar{s}}}_{\text{quark-antiquark-strange-antistrange}} | \pi(p)_{q\bar{q}s\bar{s}} \rangle + \dots$$



Our model

LFWF overlap representation

Pion PDF

$$f_{1,\pi}(x) = \sum_{\substack{N=q\bar{q}, \\ q\bar{q}g,\dots}} \int [dx]_n [d^2\vec{k}_\perp]_n \left| \psi_N \left(\{x_i\}, \{\vec{k}_{\perp i}\} \right) \right|^2 \delta(x - x_i) \delta_{iq}$$

Pion e.m. FF

$$F_1(\Delta) = \sum_{\substack{N=q\bar{q}, \\ q\bar{q}g,\dots}} \sum_a \sum_j \sum_{\beta=\beta'} e_a \delta_{s_j a} \int [dx]_N [d^2\vec{k}_\perp]_N \psi_{N,\beta'}^*(r') \psi_{N,\beta}(r)$$

↑
active parton

Our model

LFWF overlap representation

$$\underline{\psi_{q\bar{q}}(1, 2)} = \phi_{q\bar{q}}(x_1, x_2) \Omega_2(x_1, x_2, \vec{k}_{\perp 1}, \vec{k}_{\perp 2})$$

$$\underline{\psi_{q\bar{q}g}(1, 2, 3)} = \phi_{q\bar{q}g}(x_1, x_2, x_3) \Omega_3(x_1, x_2, x_3, \vec{k}_{\perp 1}, \vec{k}_{\perp 2}, \vec{k}_{\perp 3})$$

$$\underline{\psi_{q\bar{q}gg}(1, 2, 3, 4)} = \phi_{q\bar{q}gg}(x_1, x_2, x_3, x_4) \Omega_4(x_1, x_2, x_3, x_4, \vec{k}_{\perp 1}, \vec{k}_{\perp 2}, \vec{k}_{\perp 3}, \vec{k}_{\perp 4})$$

$$\underline{\psi_{q\bar{q}s\bar{s}}(1, 2, 3, 4)} = \phi_{q\bar{q}s\bar{s}}(x_1, x_2, x_3, x_4) \Omega_4(x_1, x_2, x_3, x_4, \vec{k}_{\perp 1}, \vec{k}_{\perp 2}, \vec{k}_{\perp 3}, \vec{k}_{\perp 4})$$

$\phi_{q\bar{q}}, \phi_{q\bar{q}g}, \phi_{q\bar{q}gg}, \phi_{q\bar{q}s\bar{s}}$ \longleftrightarrow **Pion Distribution Amplitudes**

$$\langle 0 | \bar{u}(z) \Gamma \mathcal{U}_{(z, -z)} d(-z) | \pi^-(p) \rangle$$

Our model

Pion Distribution Amplitudes

$$\prod_{i=1}^N x_i^{2j_i-1}, \quad j_i = \begin{cases} 1, & i = q, \bar{q} \\ 3/2, & i = g \end{cases}$$

V. M. Braun and I. E. Filyanov, *Z. Phys. C* 48,239 (1990)

$$\phi_{q\bar{q}}(x_1, x_2) = \mathcal{N}_{q\bar{q}}(x_1, x_2)^{\gamma_q} \sum_n C_{q_n} \left(\mathcal{G}_n^{\gamma_q + \frac{1}{2}} (2x_1 - 1) + \mathcal{G}_n^{\gamma_q + \frac{1}{2}} (2x_2 - 1) \right) \quad 2 (+1) \text{ parameters}$$

$$\phi_{q\bar{q}g}(x_1, x_2, x_3) = \mathcal{N}_{q\bar{q}g} x_1 x_2 x_3^2 \sum_N \sum_{n \leq N} C_n^N \left((1 - x_3)^n \mathcal{J}_{N-n}^{(2,6)}(1 - 2x_3) \mathcal{J}_n^{(1,1)}\left(\frac{x_2 - x_1}{1 - x_3}\right) \right. \\ \left. + (1 - x_3)^n \mathcal{J}_{N-n}^{(2,6)}(1 - 2x_3) \mathcal{J}_n^{(1,1)}\left(\frac{x_1 - x_2}{1 - x_3}\right) \right) \quad 1 + (1) \text{ parameters}$$

$$\phi_{q\bar{q}s\bar{s}}(x_1, x_2, x_3, x_4) = \mathcal{N}_{q\bar{q}s\bar{s}} x_1 x_2 x_3 x_4$$

$$\phi_{q\bar{q}gg}(x_1, x_2, x_3, x_4) = \mathcal{N}_{q\bar{q}gg} x_1 x_2 (x_3 x_4)^2$$

(1) parameter

Our model

Ω -functions

S. J. Brodsky, T. Huang, P. Lepage (1983)

$$\Omega_{N,\beta}(x_1, \mathbf{k}_{\perp 1}, x_2, \mathbf{k}_{\perp 2}, \dots, x_N, \mathbf{k}_{\perp N}) = \frac{(16\pi^2 a_\beta^2)^{N-1}}{\prod_{i=1}^N x_i} \exp\left(-a_\beta^2 \sum_{i=1}^N \frac{\mathbf{k}_{\perp i}^2}{x_i}\right)$$

$$\text{DAs} \longleftrightarrow \int [d^2 \mathbf{k}_{\perp}]_N \Omega_{N,\beta} = 1$$

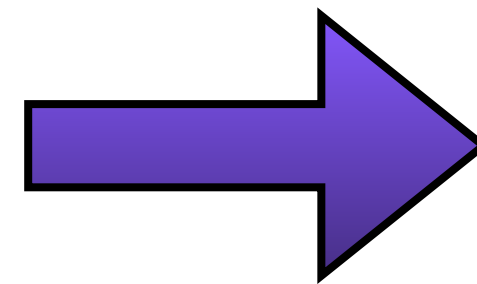
$$\text{PDF} \longleftrightarrow \int [d^2 \mathbf{k}_{\perp}]_N \Omega_{N,\beta}^2 = \frac{(8\pi^2 a_\beta^2)^{N-1}}{\prod_{i=1}^N x_i}$$

$$\text{DAs} \longleftrightarrow \int [d^2 \mathbf{k}_{\perp}]_N \Omega_{N,\beta} = \frac{1}{(2\sqrt{2}\pi a_\beta)^{N-1}}$$

$$\text{PDF} \longleftrightarrow \int [d^2 \mathbf{k}_{\perp}]_N \Omega_{N,\beta}^2 = \frac{1}{\prod_{i=1}^N x_i}$$

Results and Comments

PDF fit



FF fit

N_{points}	N_{par}	$\hat{\chi}^2/N_{\text{d.o.f.}}$
260	6	0.884

N_{points}	N_{par}	$\hat{\chi}^2/N_{\text{d.o.f.}}$
100	3 (4 - 1)	1.194



Phys.Rev.D 102 (2020) 1, 014040



CI 68% = [0.890, 1.204]

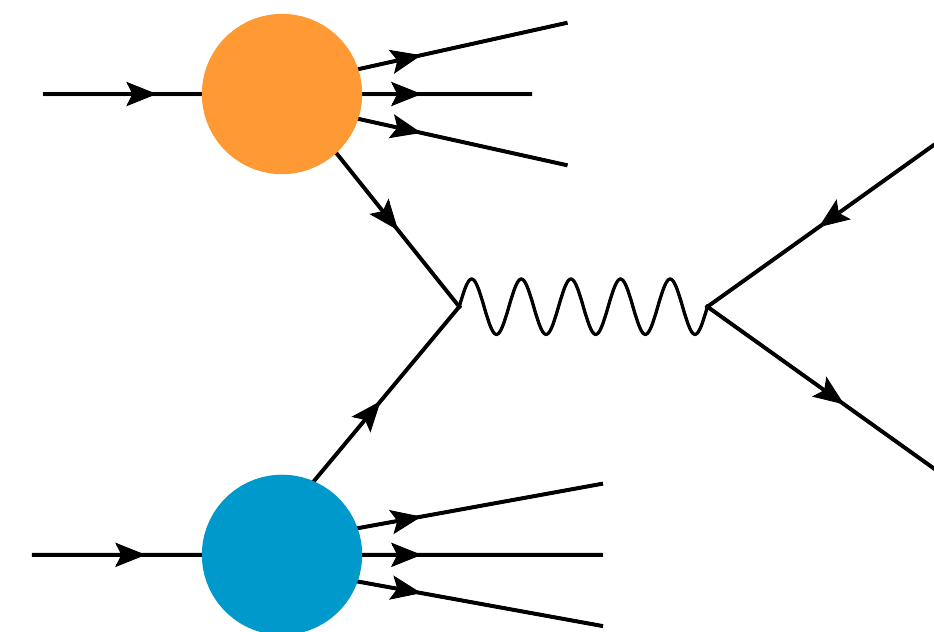
Results and Comments

PDF fit - data selection

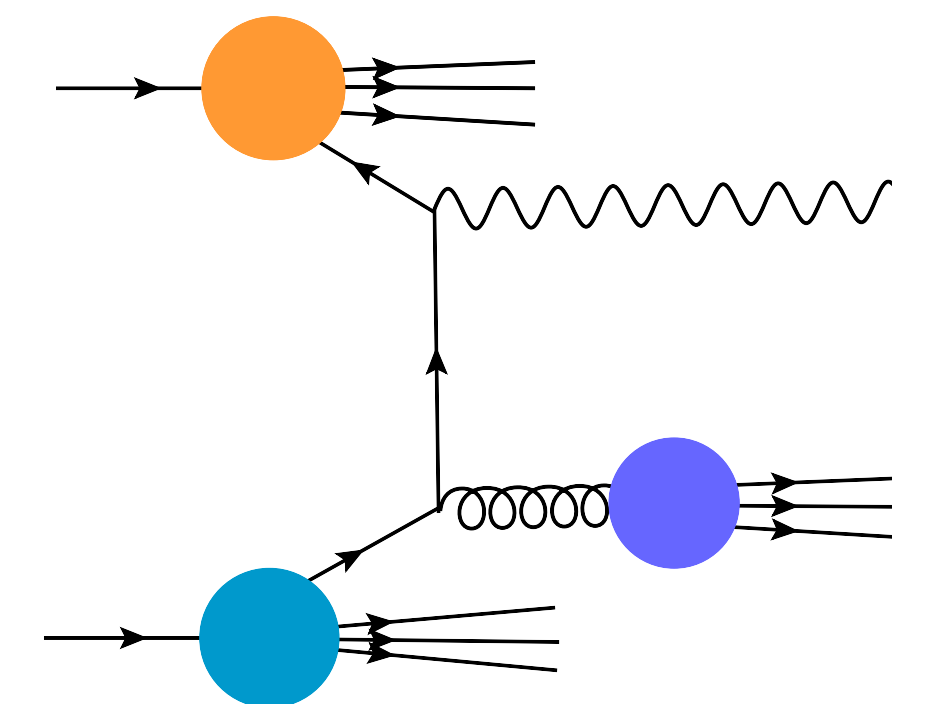
N_{points}	N_{par}	$\hat{\chi}^2 / N_{\text{d.o.f.}}$
260	6	0.884



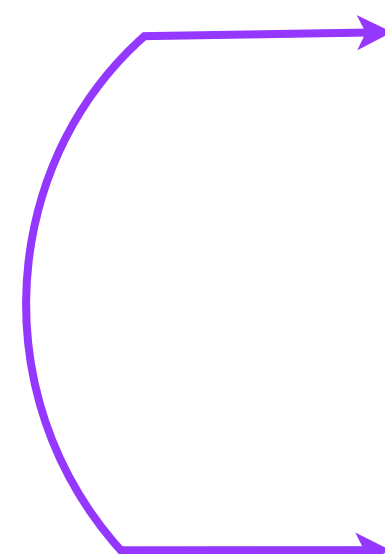
NA10	E615	WA70
70	91	99



Drell Yan with dimuon production
E615, NA10



Prompt photon production
WA70



$$x_F \geq 0$$

$$4.16 \text{ GeV} < Q < 7.68 \text{ GeV}$$

Results and Comments

PDF fit - chi-squared definition

N_{points}	N_{par}	$\hat{\chi}^2 / N_{\text{d.o.f.}}$
260	6	0.884



$$\chi^2 = \sum_i \frac{(d_i - \tilde{t}_i)^2}{(\delta_i^{\text{syst}})^2 + \left(\sqrt{\frac{\tilde{t}_i}{d_i}} \delta_i^{\text{stat}} \right)^2} + \sum_{\alpha} b_{\alpha}^2$$

0.884 is the minimum of the chi-squared, for the **single minimization**

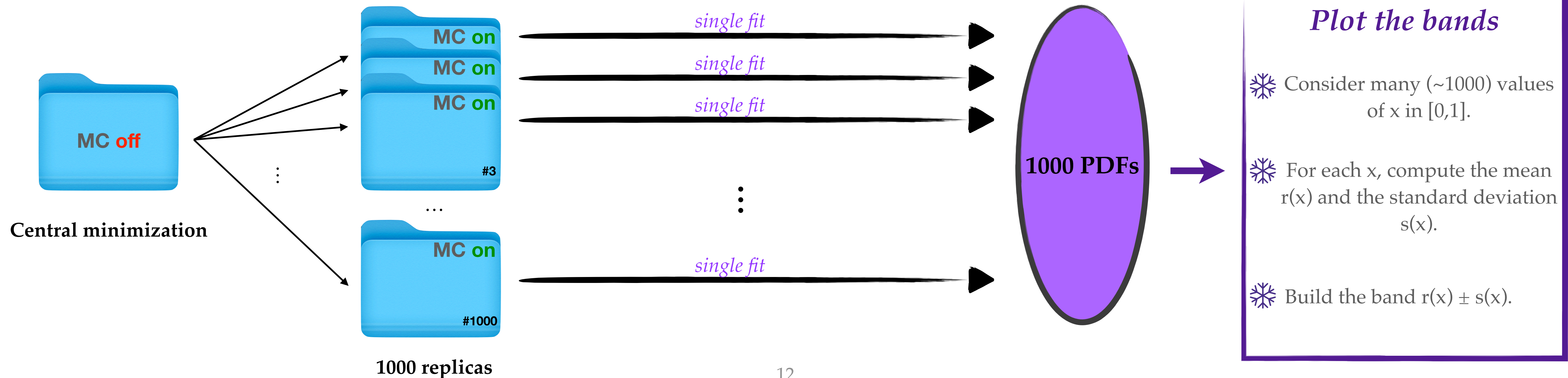
Results and Comments

PDF fit - error bands

N_{points}	N_{par}	$\hat{\chi}^2 / N_{\text{d.o.f.}}$
260	6	0.884



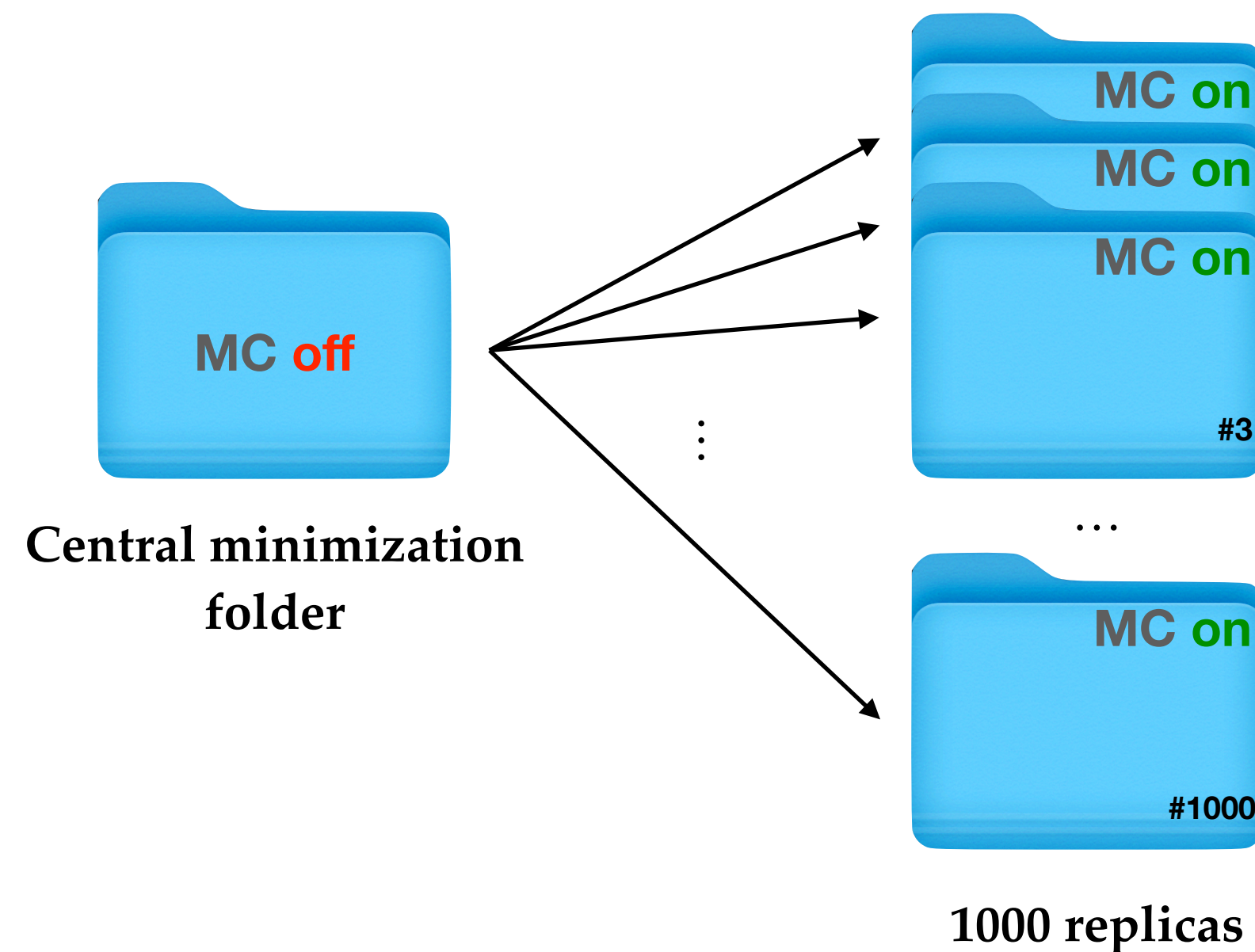
Replica method



Results and Comments

PDF fit - scale variation

N_{points}	N_{par}	$\hat{\chi}^2 / N_{\text{d.o.f.}}$
260	6	0.884

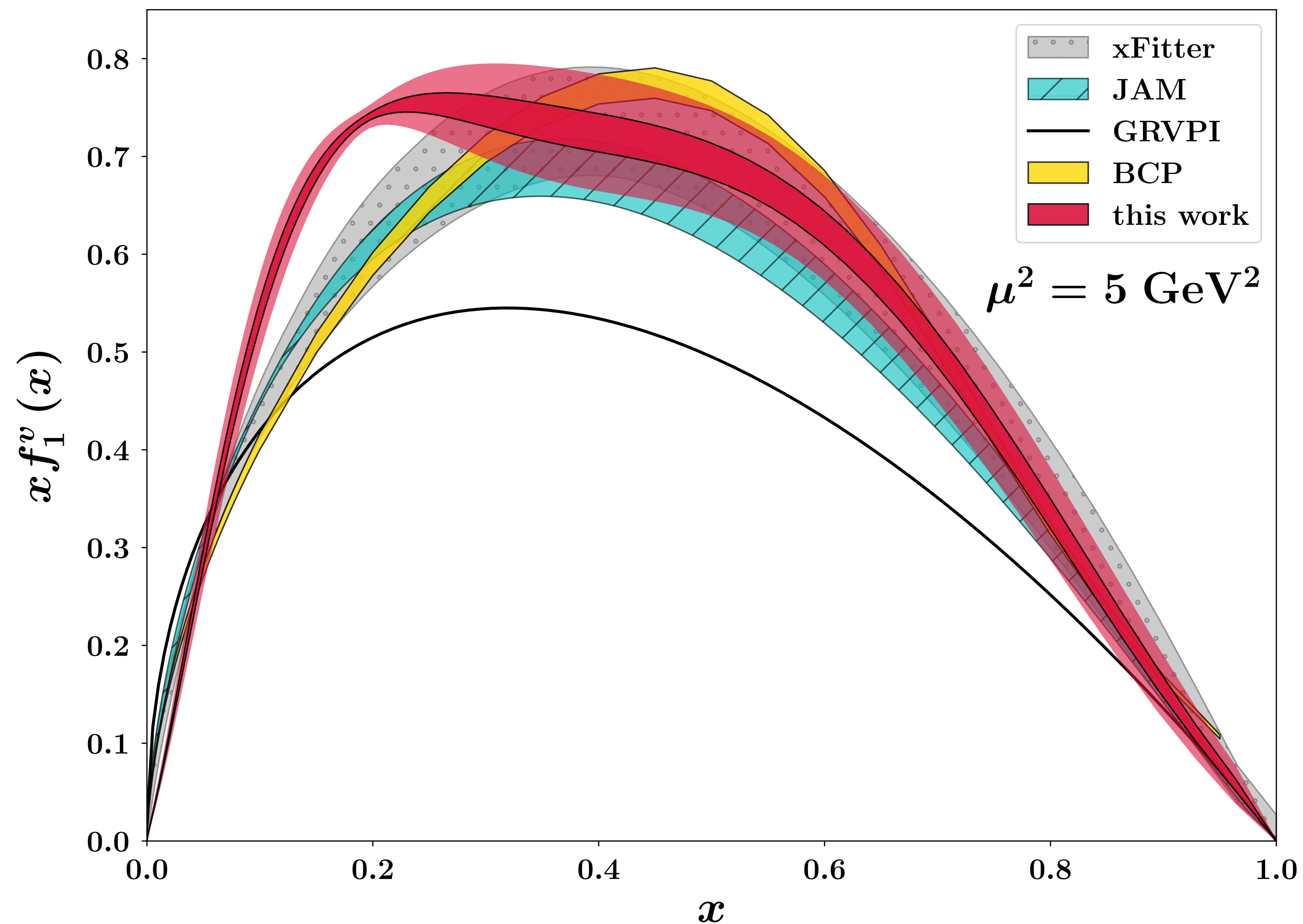


for each single replica:

$$\left\{ \begin{array}{l} \ast \text{ generate } \frac{\mu_0}{2} < \mu_F < \mu_0 \\ \ast \text{ generate } \mu_F \leq \mu_R < 2\mu_0 \end{array} \right.$$

Results and Comments

Pion PDFs



I. Novikov et al.,
Phys.Rev.D102,014040 (2020)

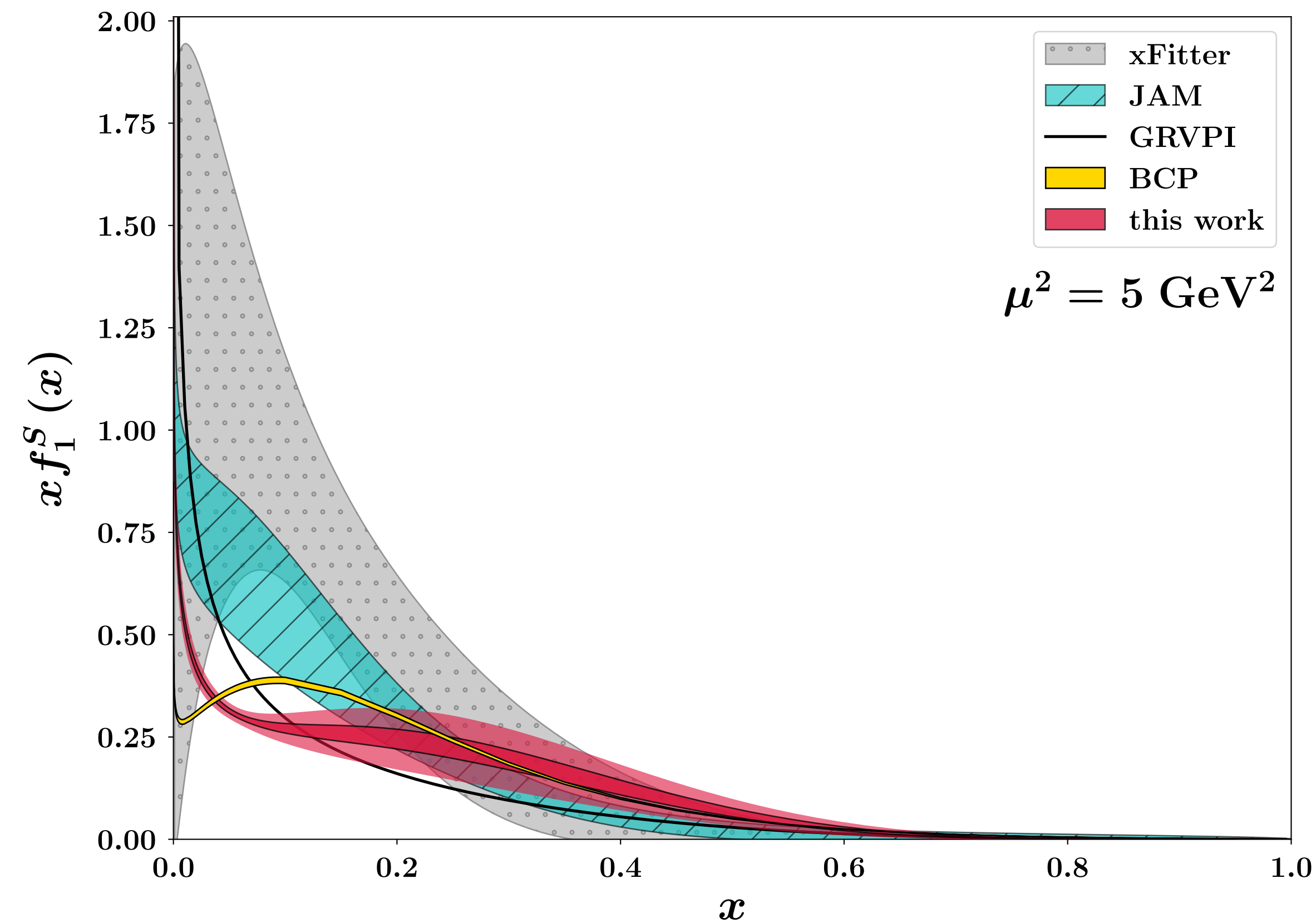
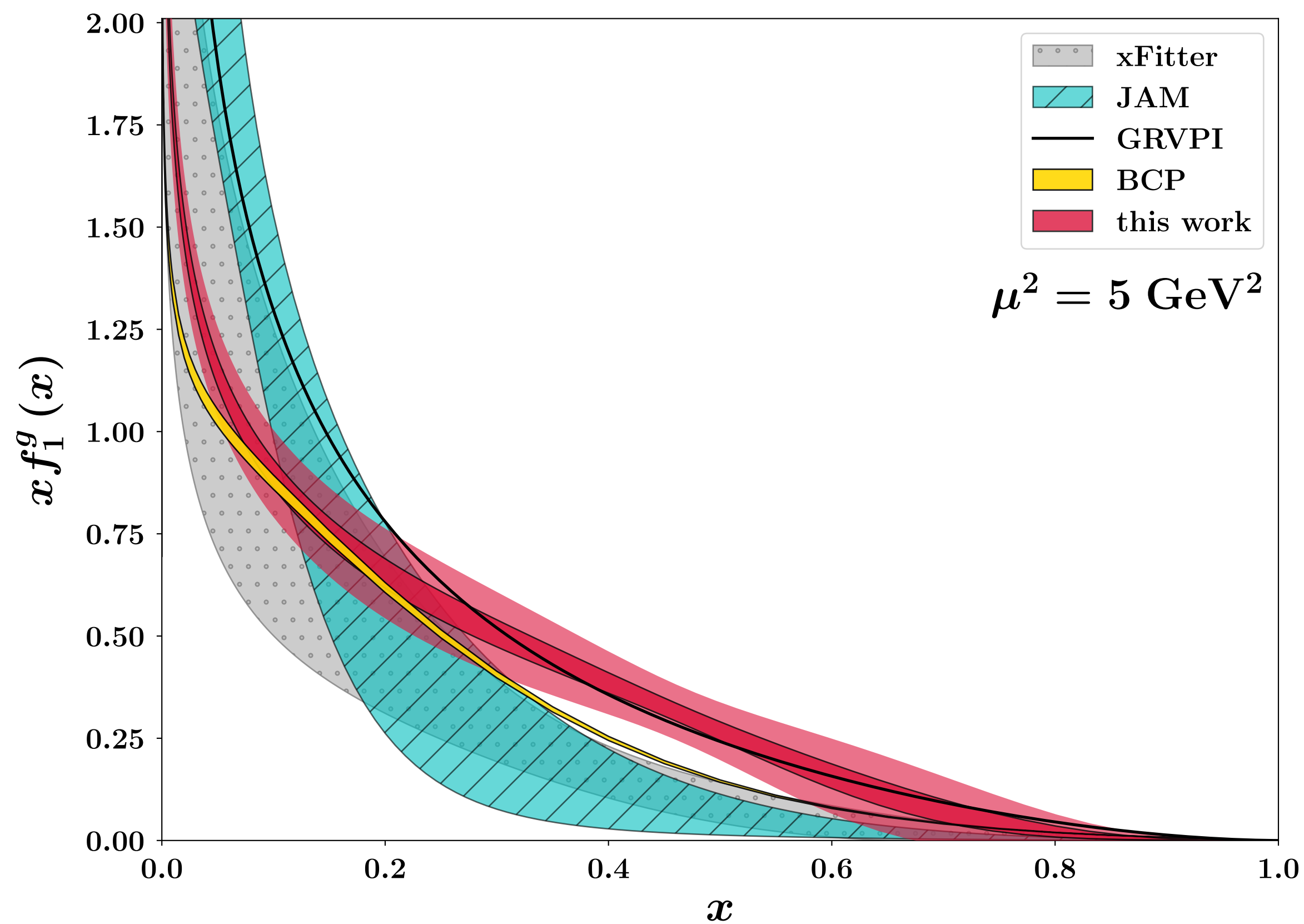
P. C. Barry, C.-R. Ji, N. Sato, W.
Melnitchok (JAM), *Phys.Rev.Lett.127,*
232001 (2021)

M. Gluck, E. Reya, A. Vogt, *Z.Phys.C*
53, 651 (1992)

C. Bourrelly, W.-C. Chang, J.-C. Peng,
Phys.Rev.D 105,076018 (2022)

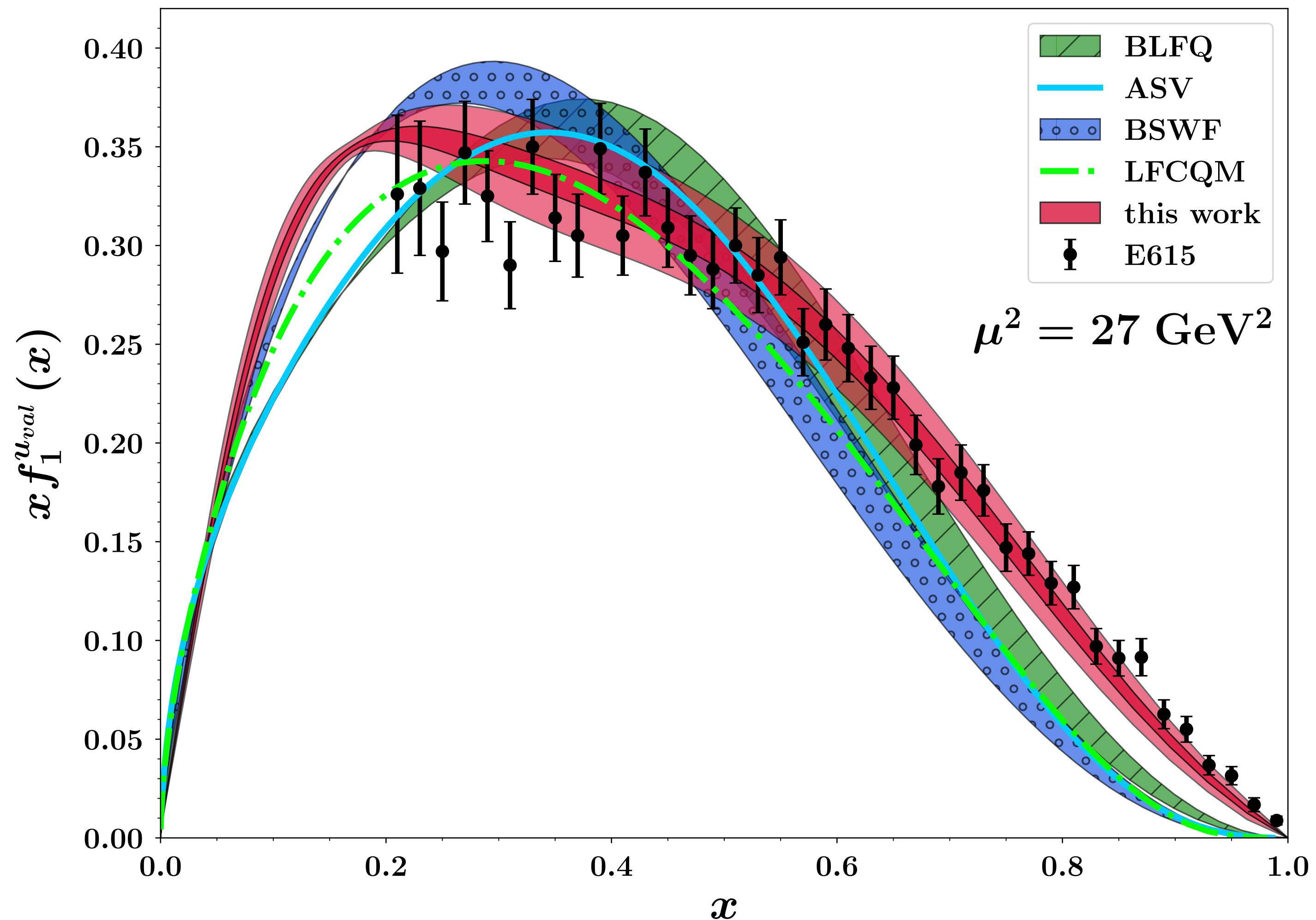
Results and Comments

Pion PDFs



Results and Comments

Pion PDFs



J. Lan, K. Fu, C. Mondal, X. Zhab, P. Vary (BFLQ), *Phys.Lett.B* 825, 136890

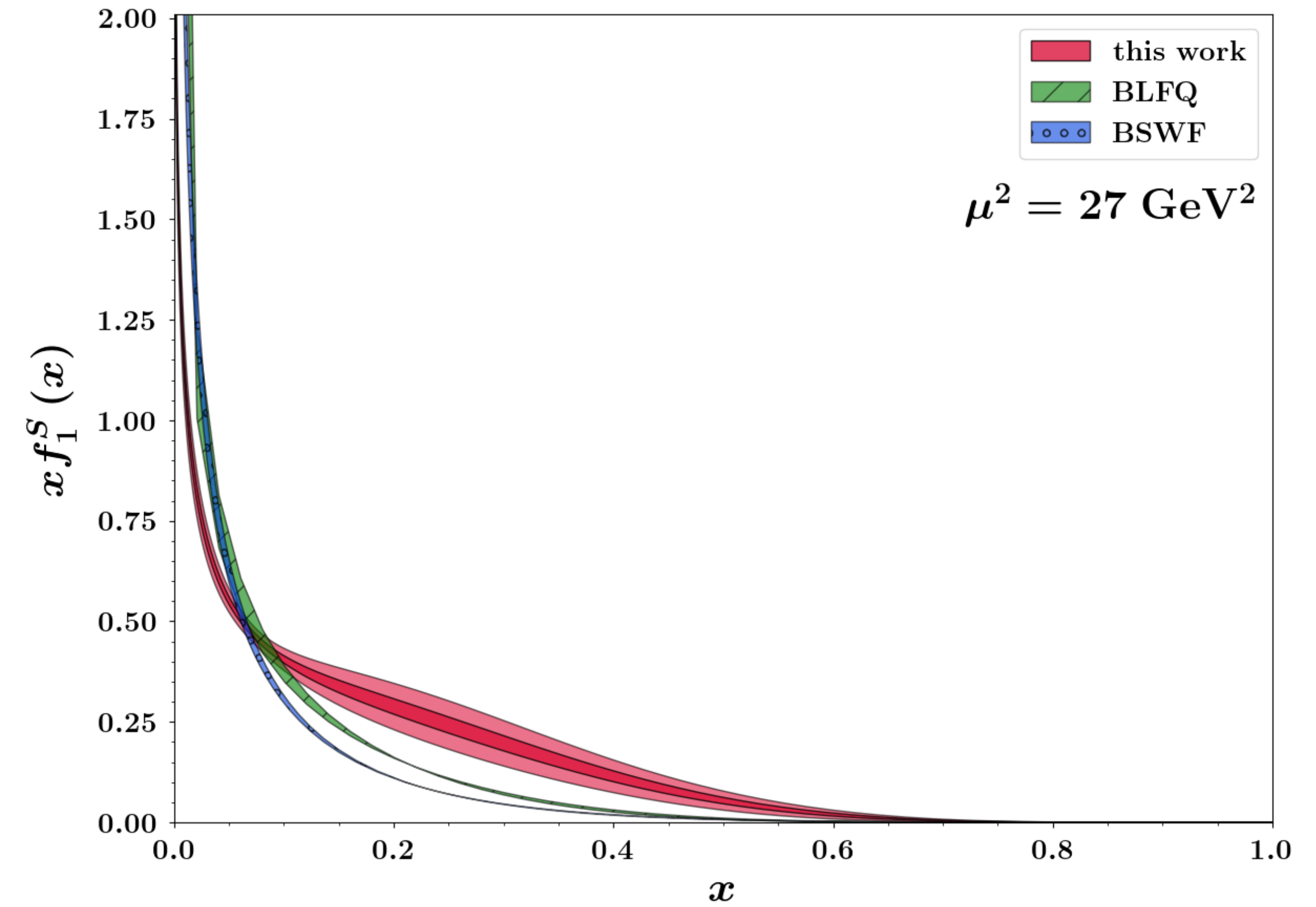
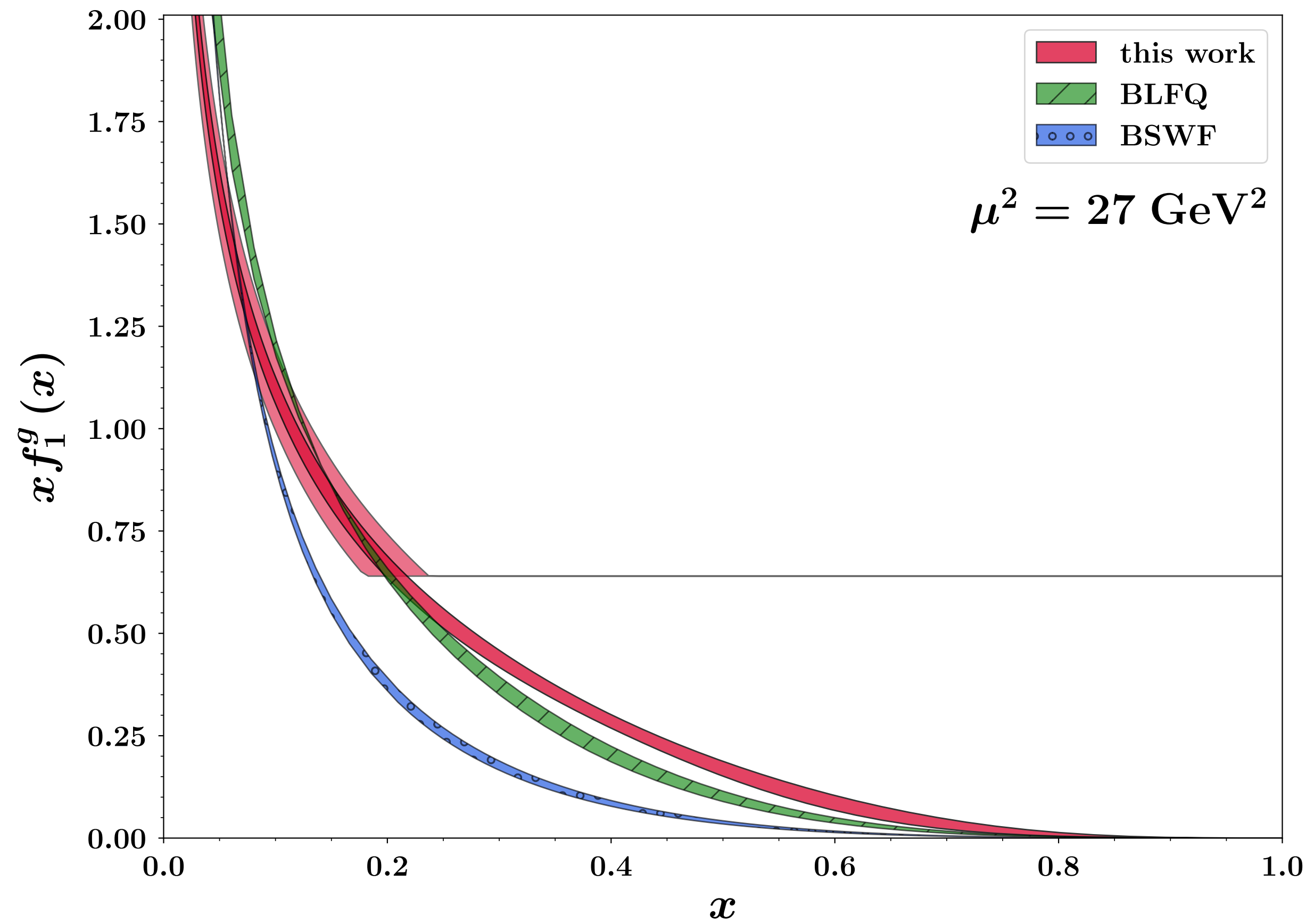
M. Aicher, A. Schafer, W. Vogelsang, *Phys.Rev.Lett.* 105, 252003 (2010)

Z.-F. Cui, M. Ding, F. Gao, K. Raya, D. Binosi, L. Chang, C. D. Roberts, J. Rodríguez-Quintero, S.M. Schmidt, *Rur.Phys.J.C* 8'0, 1064 (2020)

B. Pasquini, P. Schweitzer, *Phys.Rev.D* 90, 014050 (2014)

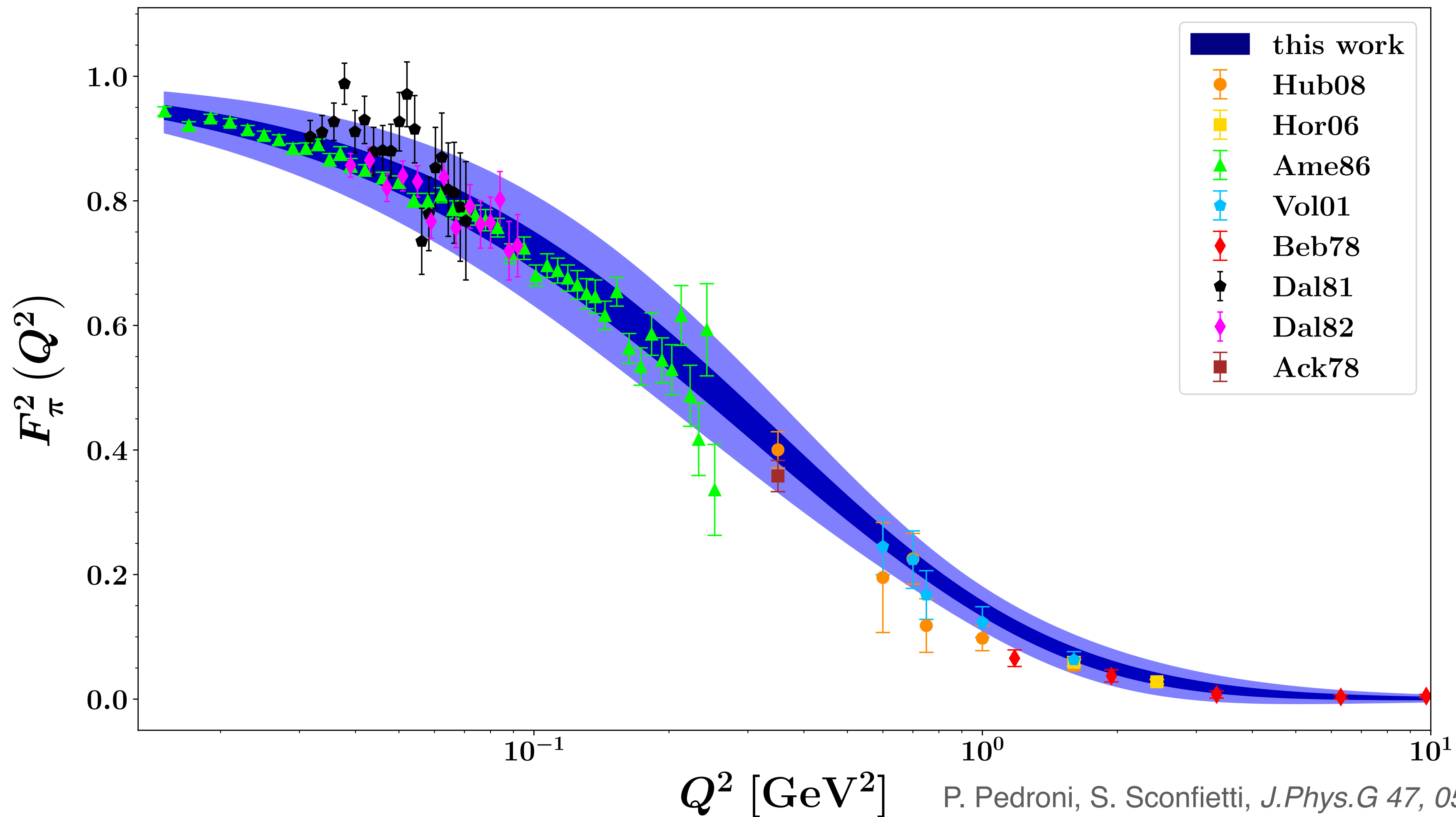
Results and Comments

Pion PDFs



Results and Comments

Pion FF fit



Results and Comments

“State of the art of the model”

Collinear PDF **fitted**

$$f_{1,\pi}(x)$$

Electromagnetic Form Factor **fitted**

$$F_1(\Delta)$$

B. Pasquini, S. Rodini, S. Venturini, 2303.01789

Results and Comments

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$$f_{1,\pi}(x, \vec{k}_\perp)$$

Transverse Momentum Dependent PDF
computed

$$H^q(x, \xi, \vec{\Delta}_\perp)$$

Generalized Parton Distribution
computed

Results and Comments

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