

PDFs at $N^3\text{LO}$ in APFEL++

Valerio Bertone

IRFU, CEA, Université Paris-Saclay

université
PARIS-SACLAY



May 3, 2023, xFitter external meeting, CERN

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 824093

Evolution for N³LO

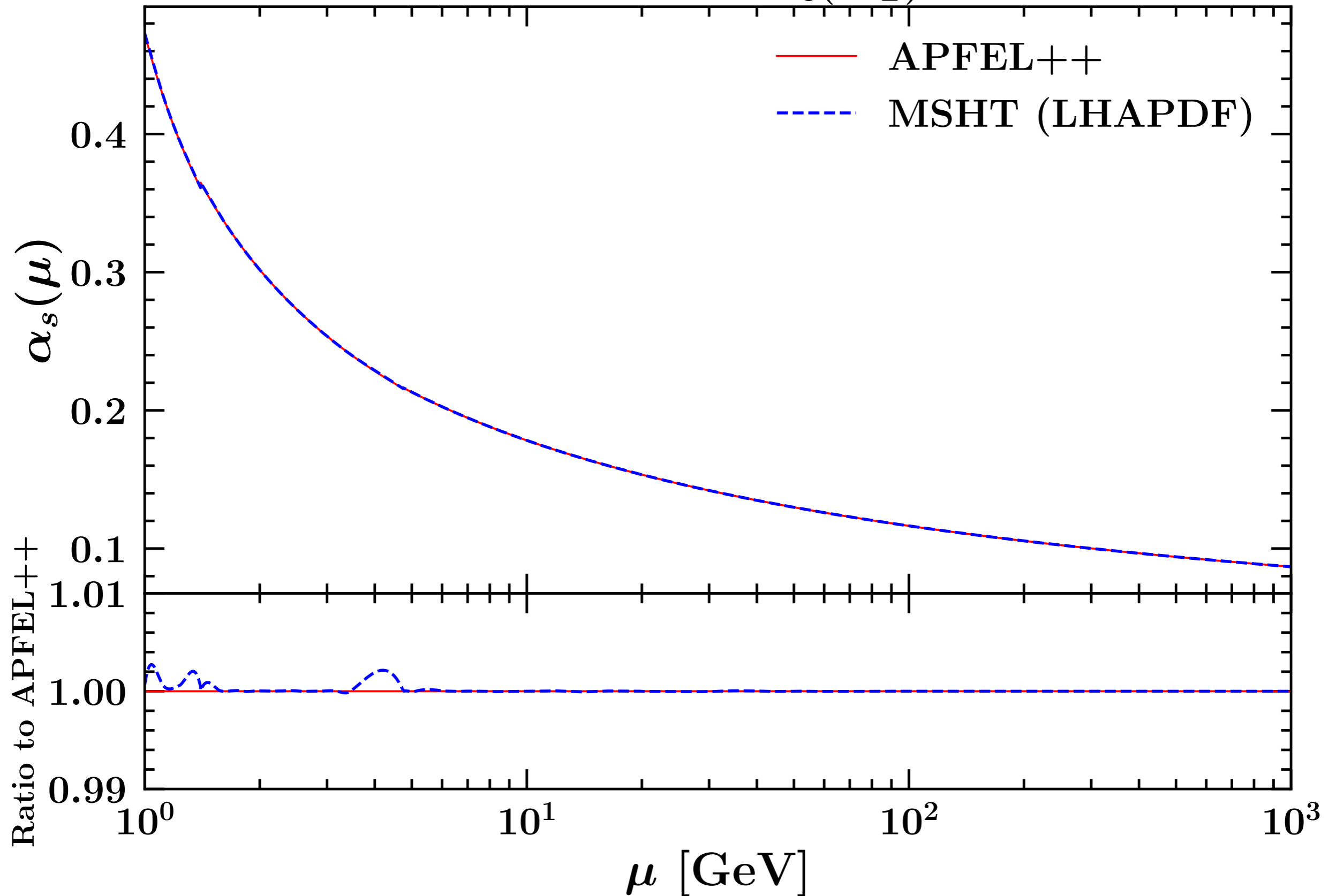
- 🍏 A fundamental ingredient to use N³LO computations in extractions of PDFs is the **evolution** accurate to the same order.
- 🍏 The main ingredients to achieve N³LO accuracy in PDF evolution are:
 - 🍏 the $O(\alpha_s^4)$ contribution to the anomalous dimensions, *i.e.* $\beta_3(n_f)$ and $P^{(3)}(x, n_f)$,
 - 🍏 $\beta_3(n_f)$ was computed long ago [[van Ritbergen, Vermaseren, Larin, hep-ph/9701390](#)].
 - 🍏 The **non-singlet** component of $P^{(3)}(x, n_f)$ exact in the planar limit has been computed relatively recently [[Moch et al., arXiv:1707.08315](#)].
- 🍏 When a variable-flavour number scheme is used, **matching conditions** for the evolution of α_s and PDFs accurate to $O(\alpha_s^3)$ are also necessary.
 - 🍏 $O(\alpha_s^3)$ matching conditions for α_s are known (see *e.g.* [[Chetyrkin et al., hep-ph/0004189](#)]).
 - 🍏 Matching conditions for PDFs fully known only up to $O(\alpha_s^2)$ (in fact, matching conditions involving a heavy quark in the initial state are known to $O(\alpha_s)$).

Evolution for N³LO

- 🍏 Recently the MSHT group has carried out a determination of PDFs at *approximated* N³LO [arXiv:2207.04739]
- 🍏 The authors also released the relevant missing ingredients to perform approximated N³LO in the VFNS:
 - 🍏 The **singlet** components of $P^{(3)}(x, n_f)$ and $O(\alpha_s^3)$ **matching functions** parameterised and fitted to the first known Mellin moments.
 - 🍏 **Uncertainty** to gauge the accuracy of the parameterisations also provided.
 - 🍏 A fortran code with the expressions released at: https://github.com/MSHTPDF/N3LO_additions
- 🍏 All the *currently known* ingredients necessary for PDF evolution at N³LO are implemented in **APFEL++**. [https://github.com/vbertone/apfelxx]

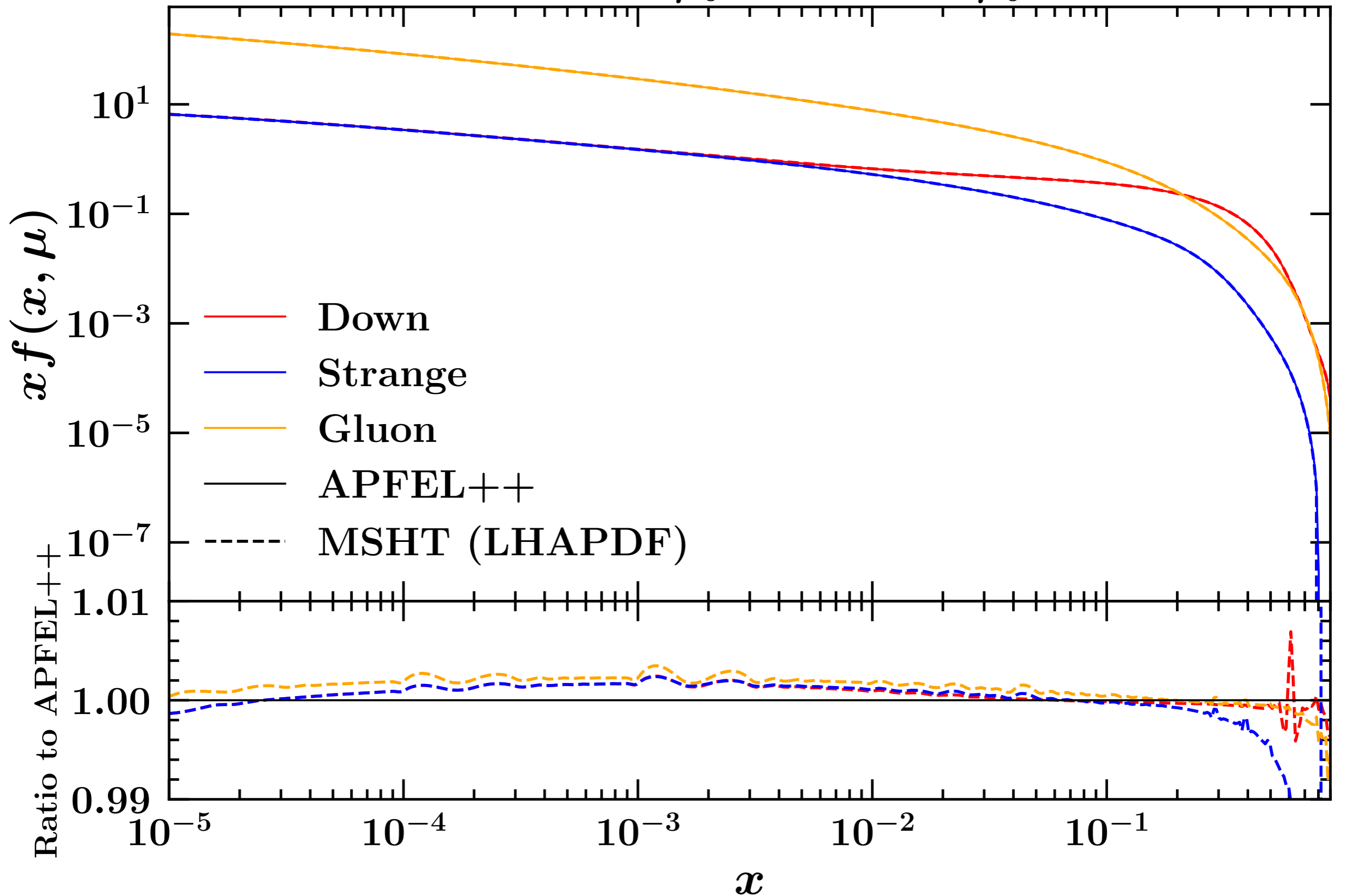
The strong coupling

$N^3\text{LO}$ evolution with $\alpha_s(M_Z) = 0.118$

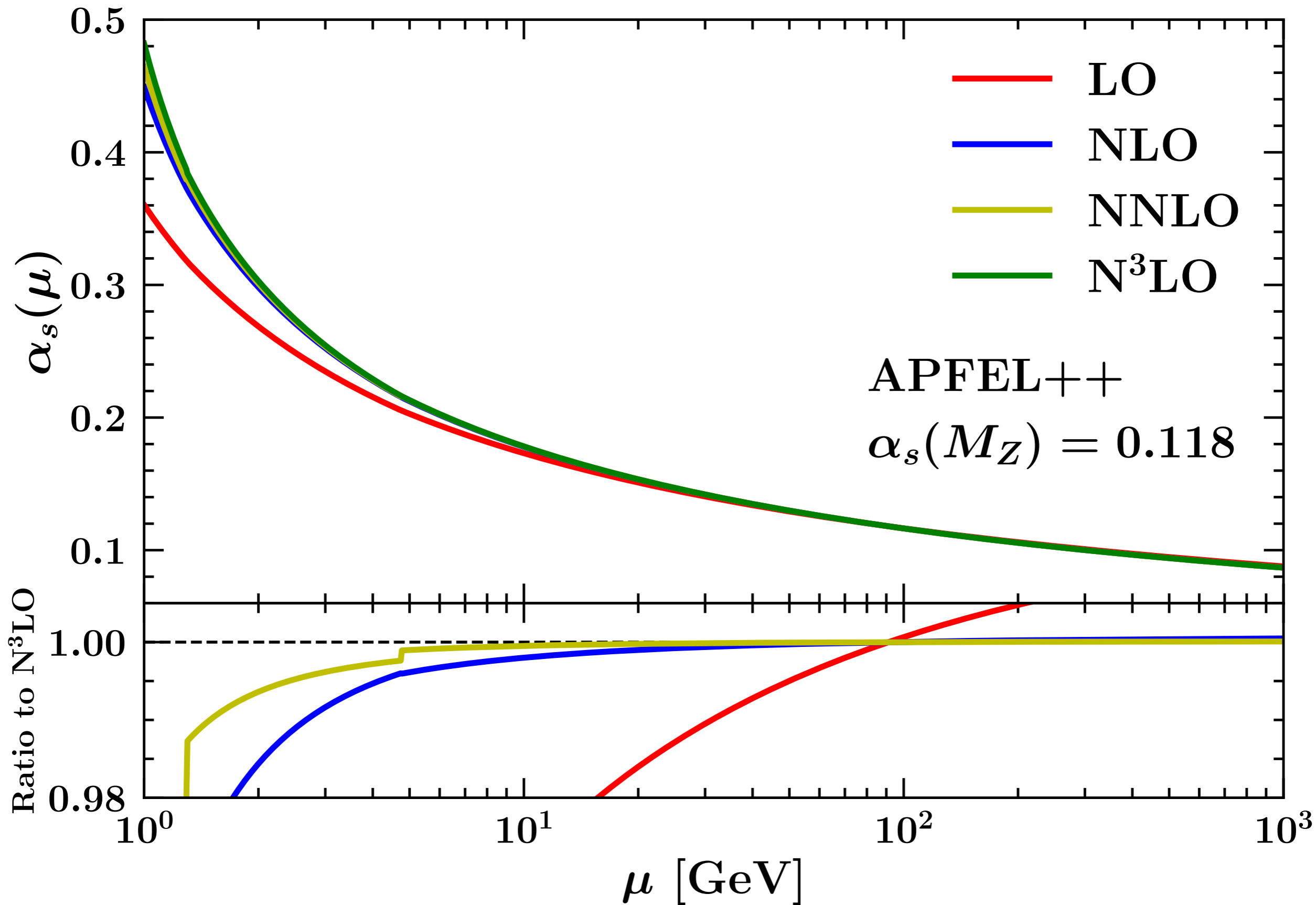


The PDFs

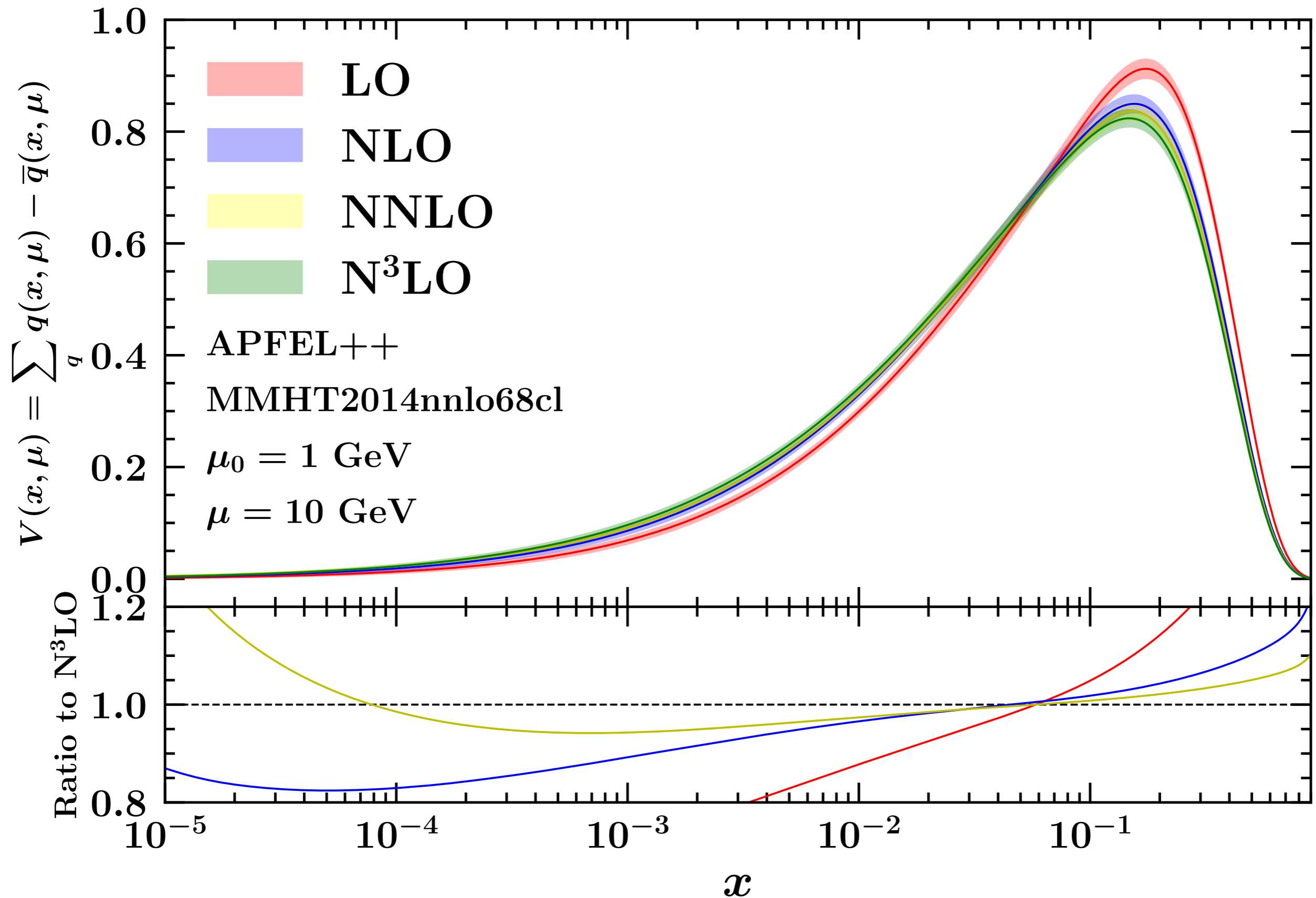
N^3 LO evolution from $\mu_0 = 1$ GeV to $\mu_0 = 100$ GeV



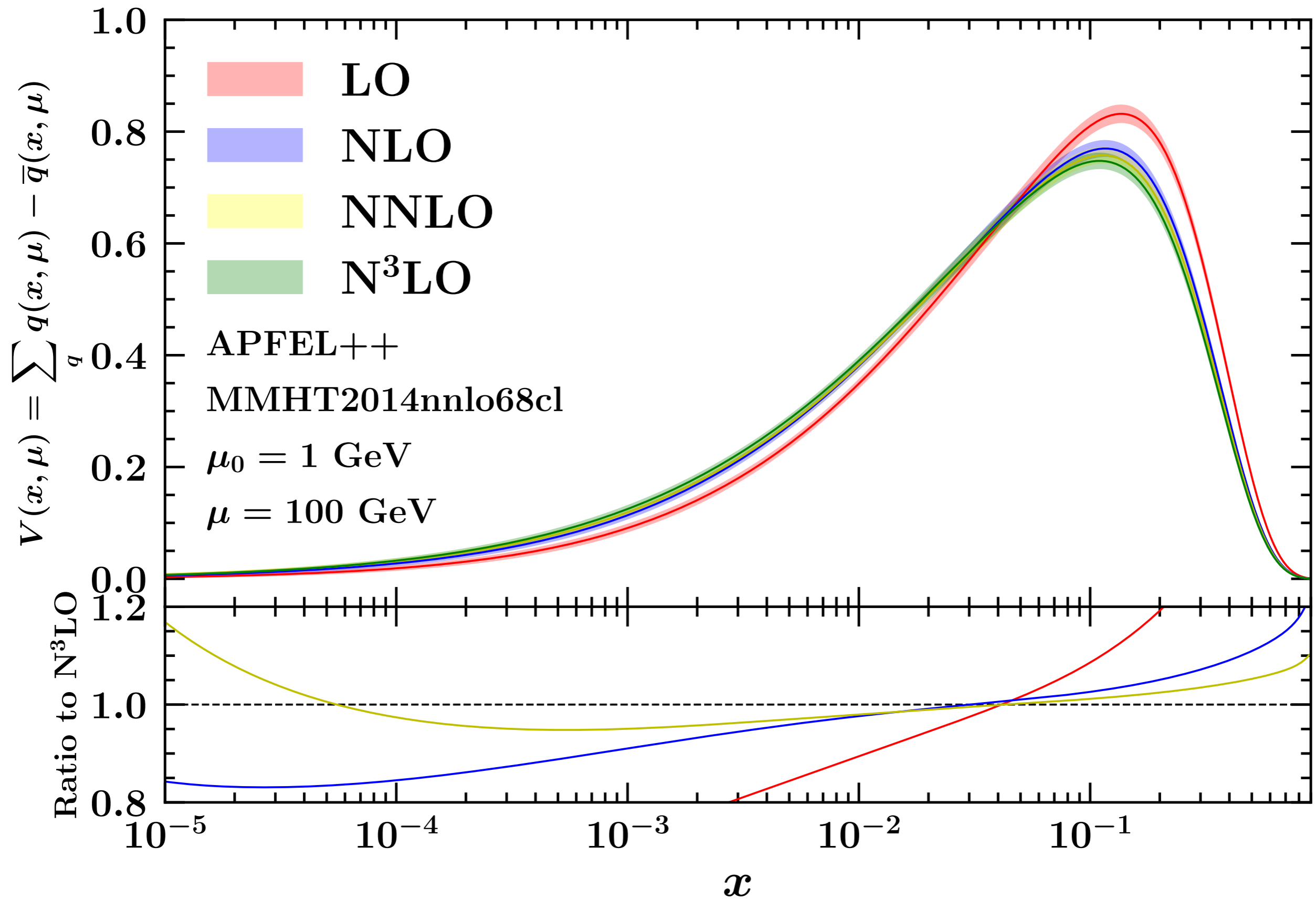
The strong coupling



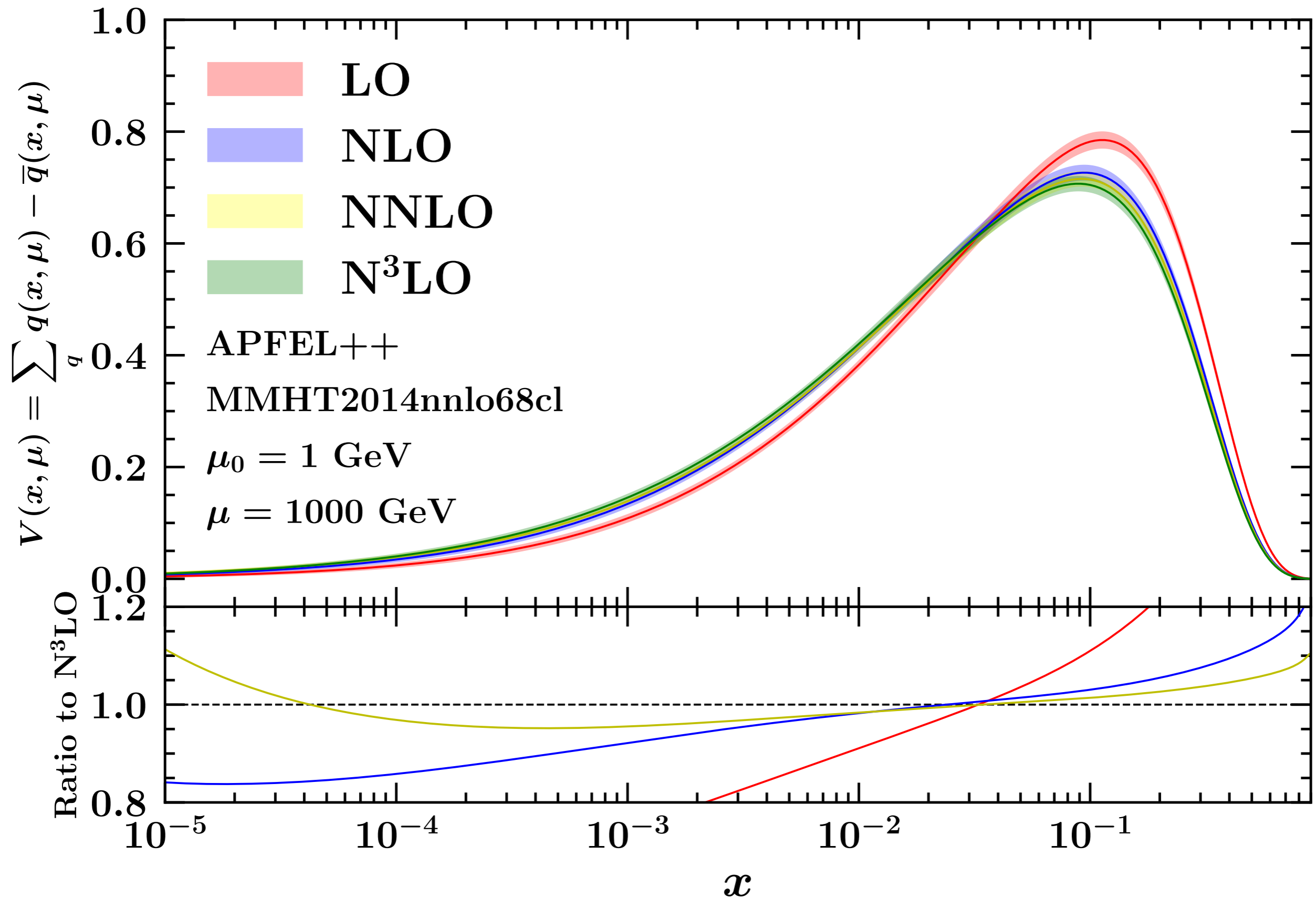
The (valence) PDFs



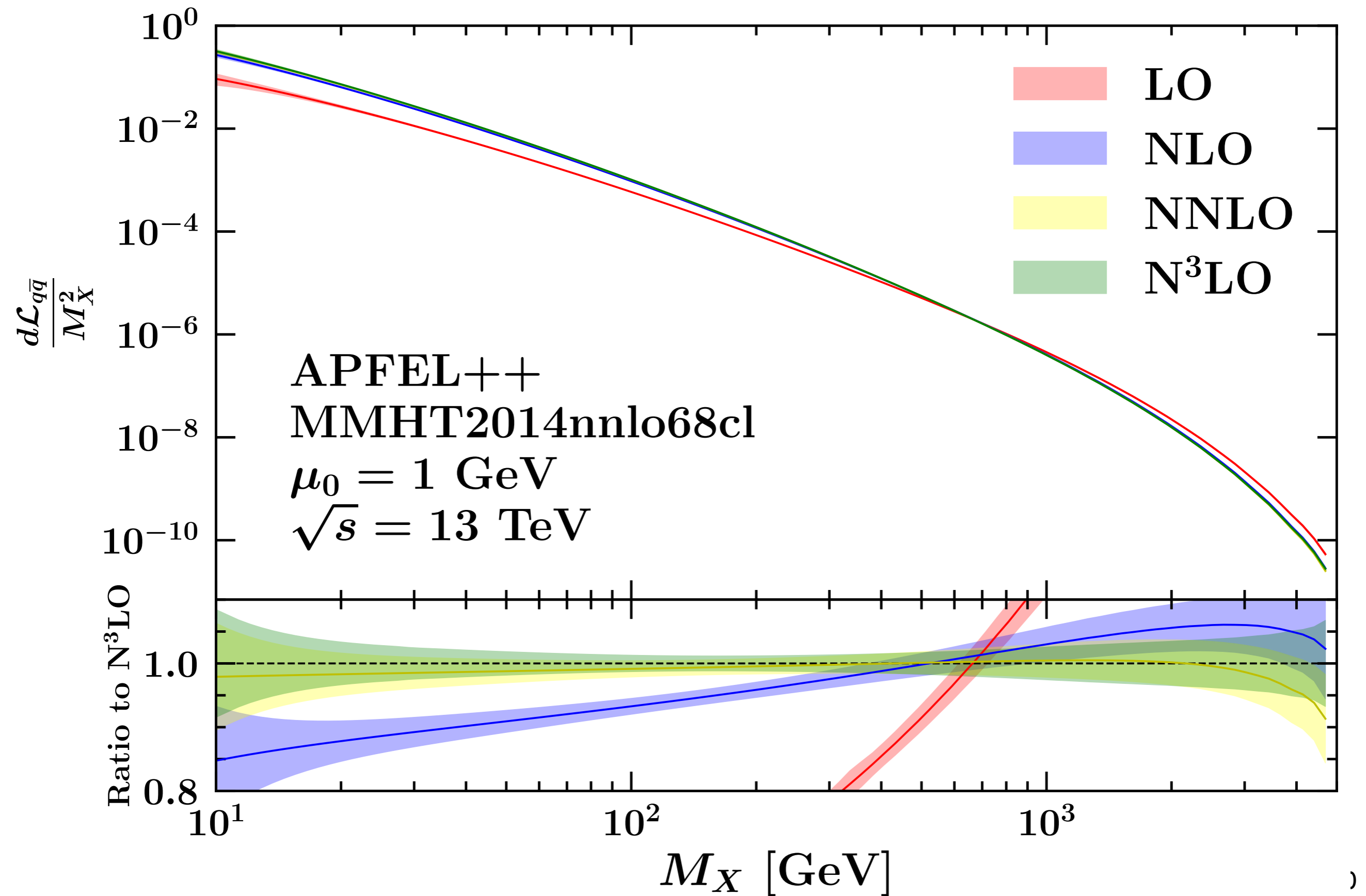
The (valence) PDFs



The (valence) PDFs

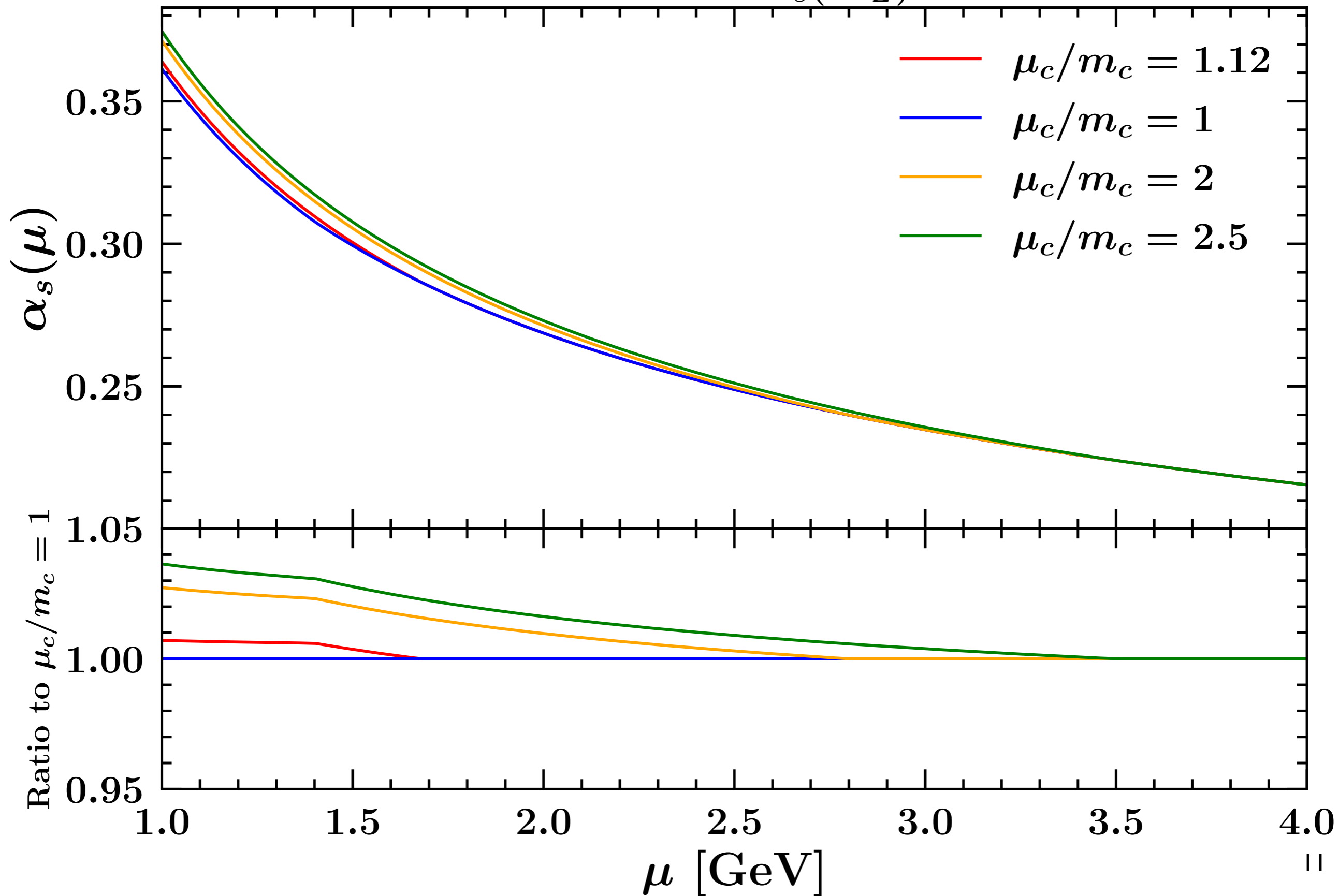


The parton luminosities



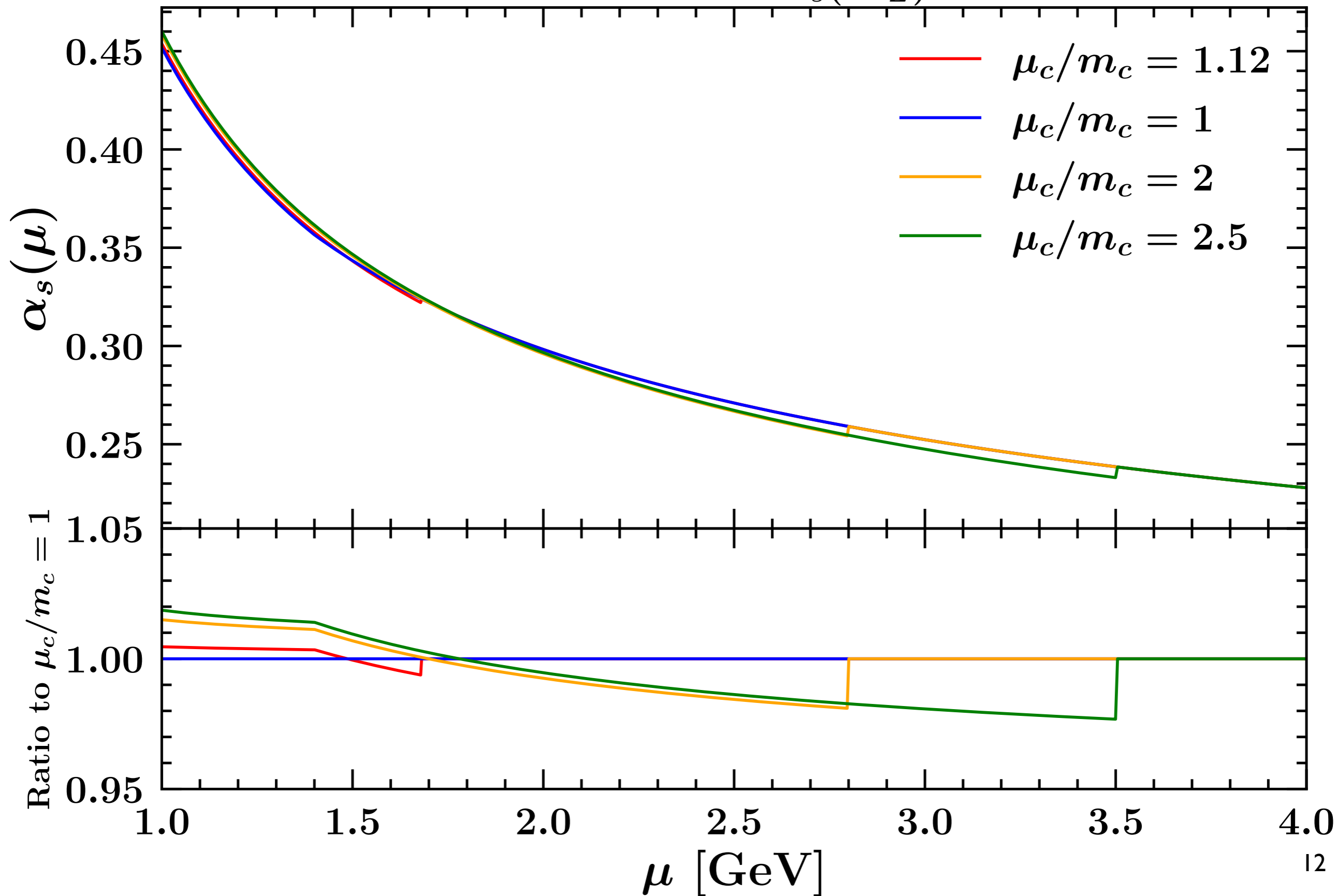
The matching conditions

LO evolution with $\alpha_s(M_Z) = 0.118$



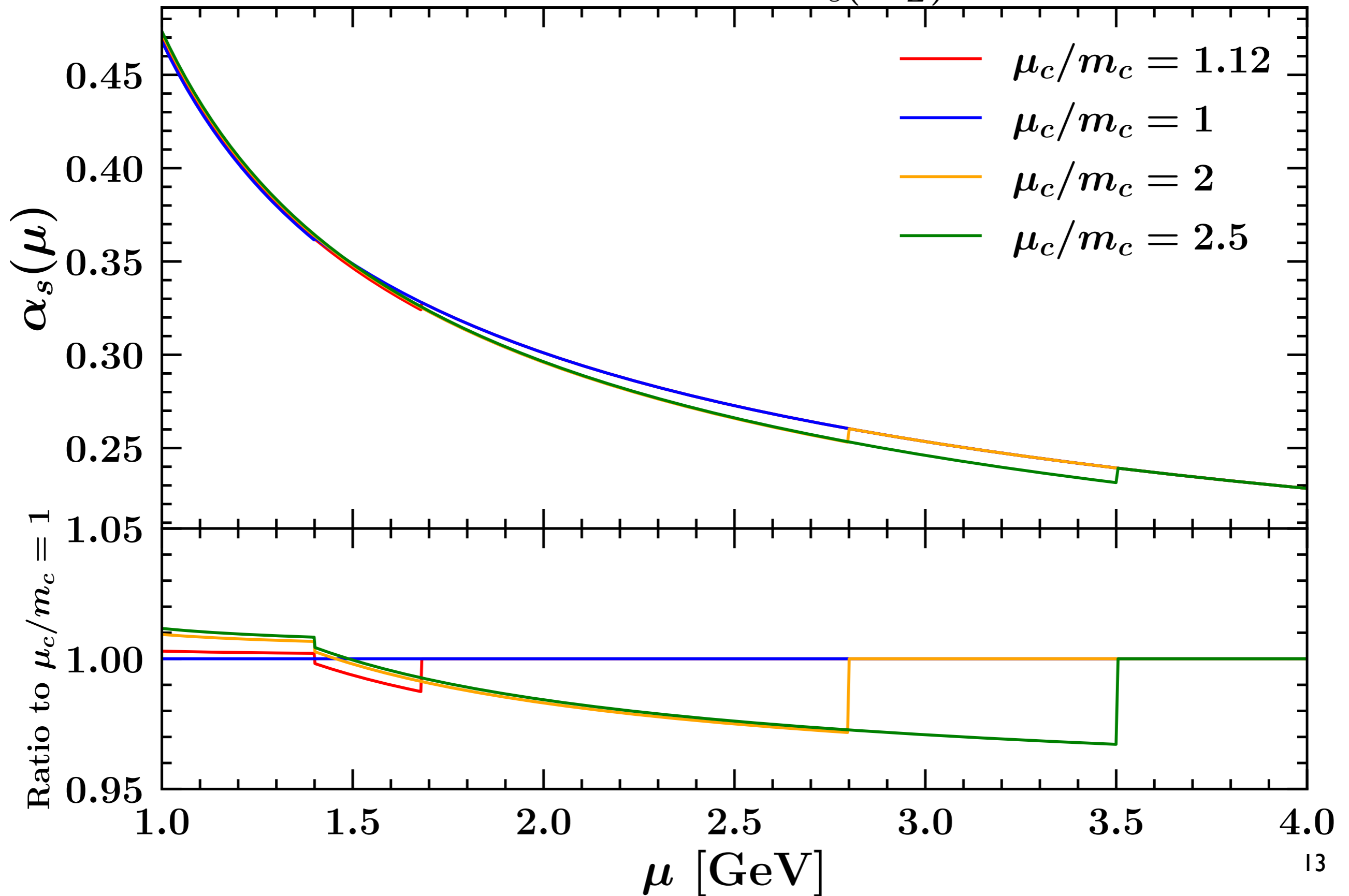
The matching conditions

NLO evolution with $\alpha_s(M_Z) = 0.118$



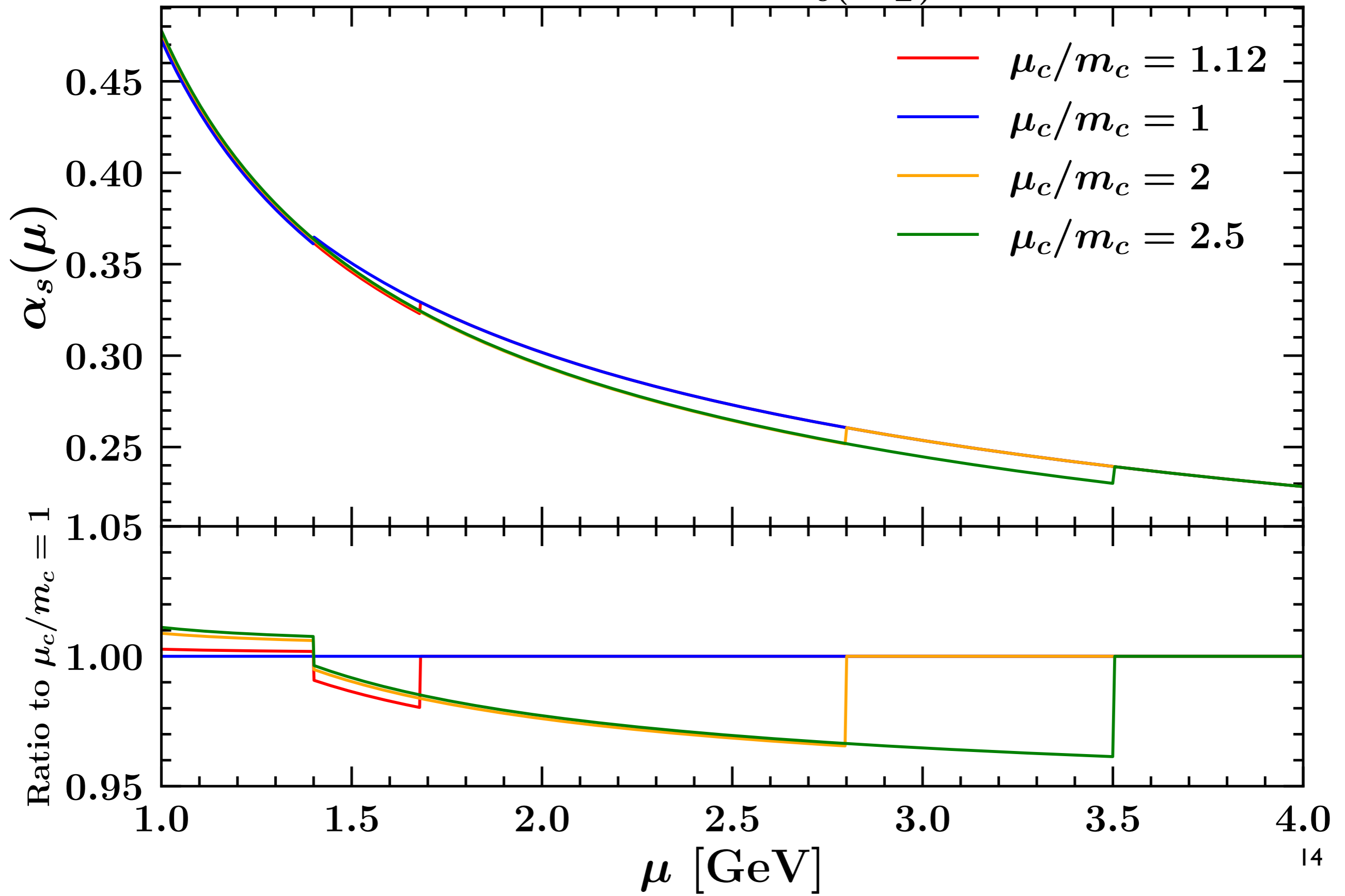
The matching conditions

NNLO evolution with $\alpha_s(M_Z) = 0.118$



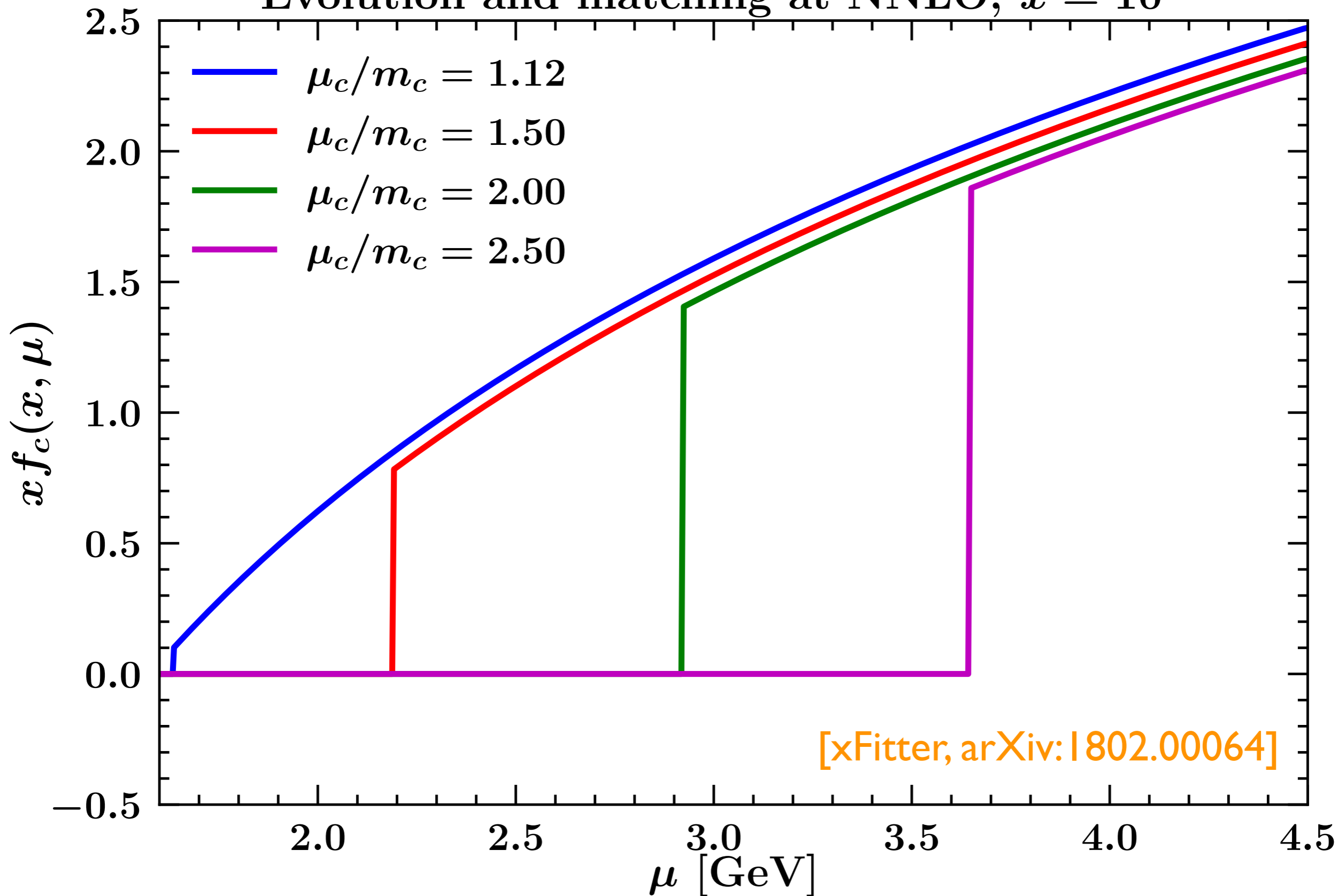
The matching conditions

N³LO evolution with $\alpha_s(M_Z) = 0.118$

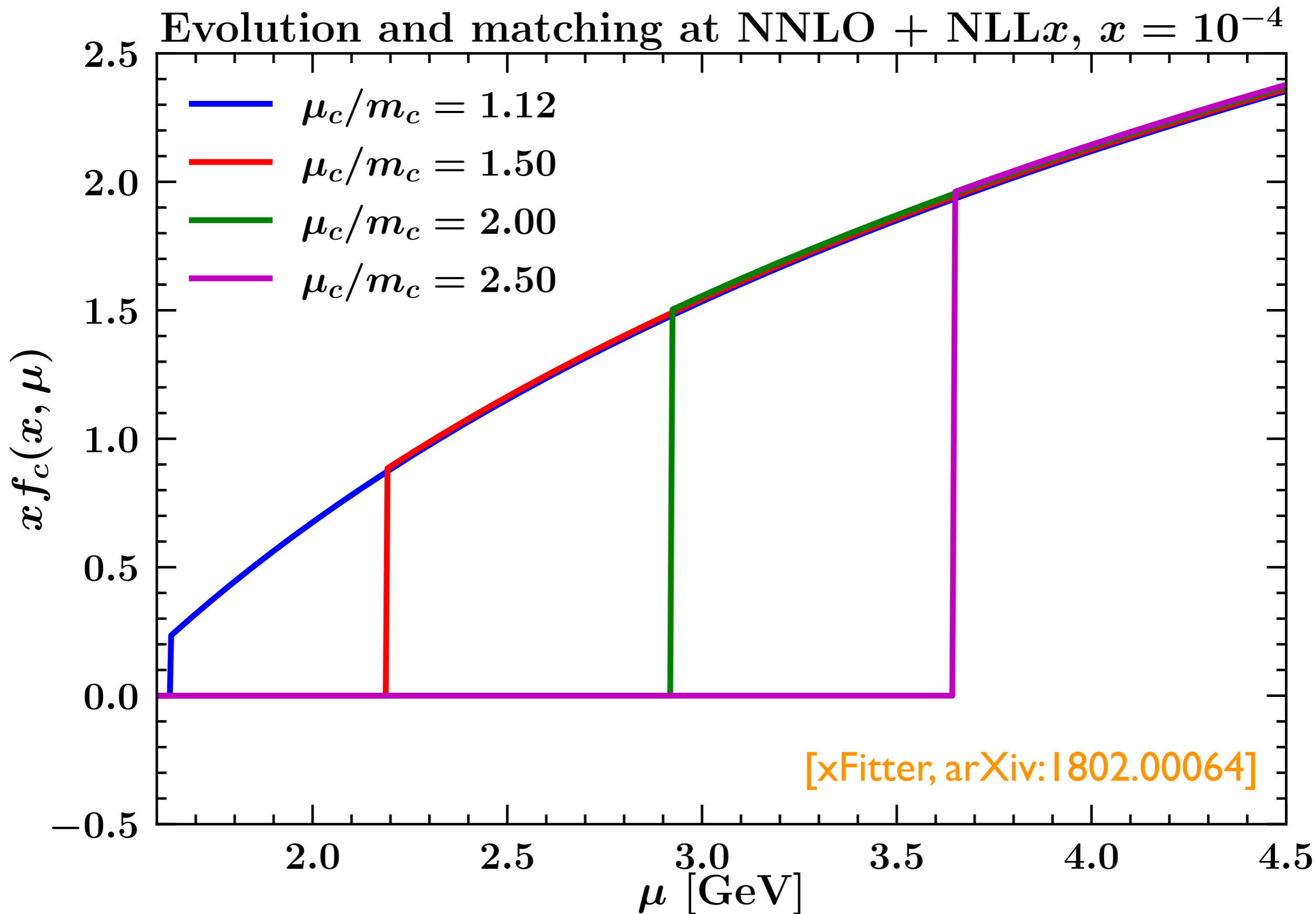


The matching conditions

Evolution and matching at NNLO, $x = 10^{-4}$



The matching conditions

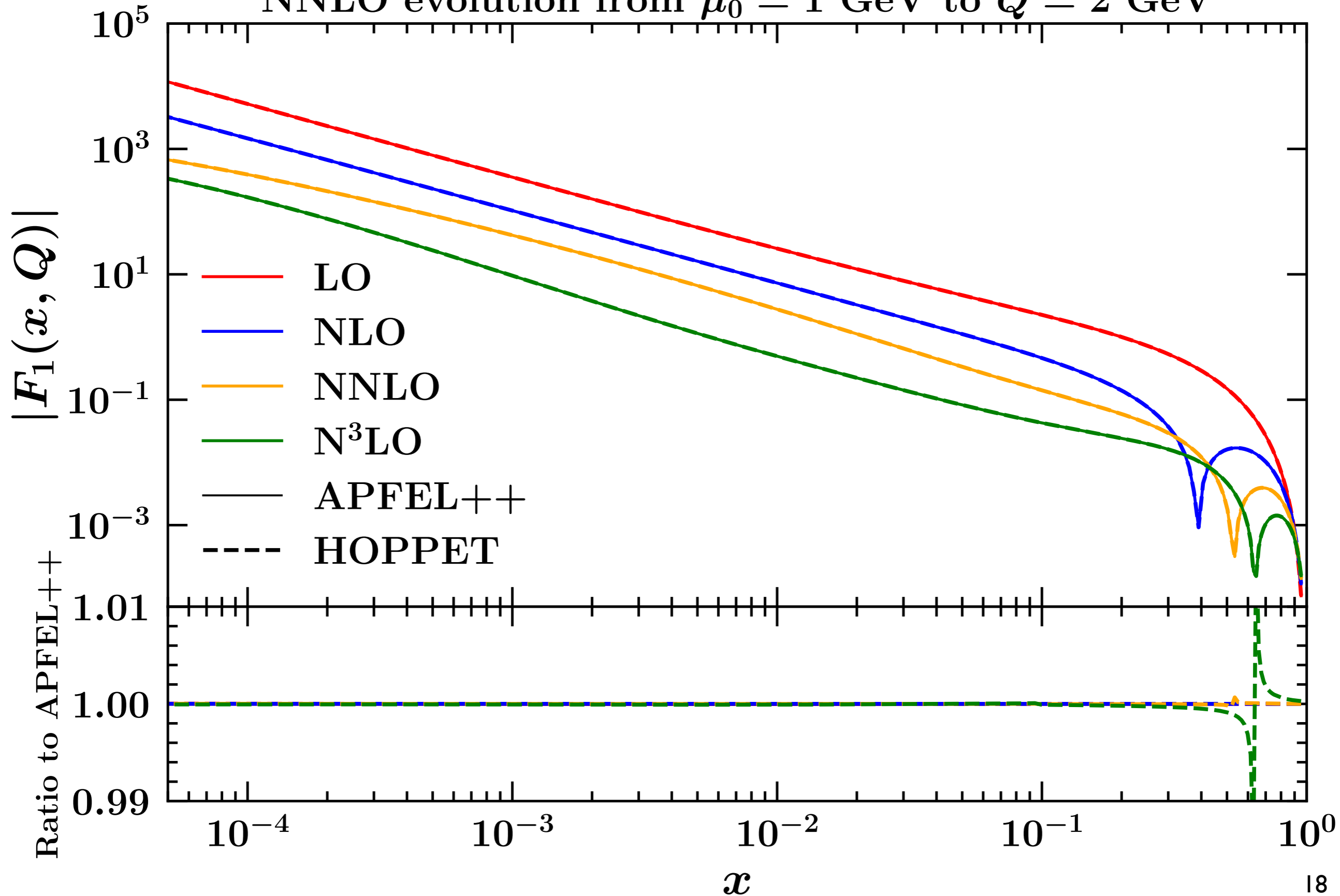


Structure functions

- 🍏 N³LO corrections to the DIS structure functions in the zero-mass scheme are known since quite long:
 - 🍏 hep-ph/0209100,
 - 🍏 hep-ph/0504242,
 - 🍏 hep-ph/0411112,
 - 🍏 hep-ph/0608307.
- 🍏 Again the *currently known* ingredients necessary for computing structure functions to N³LO are implemented in **APFEL++**.
- 🍏 Presently, with the help of Alexander Karlberg, we are carrying out a benchmark of APFEL++ and HOPPET:
 - 🍏 so far, neutral current F_2 and F_L (or F_1 and F_2) are in perfect agreement,
 - 🍏 still working of F_3 to fix a small difference.
 - 🍏 Also working on the charged-current structure functions.

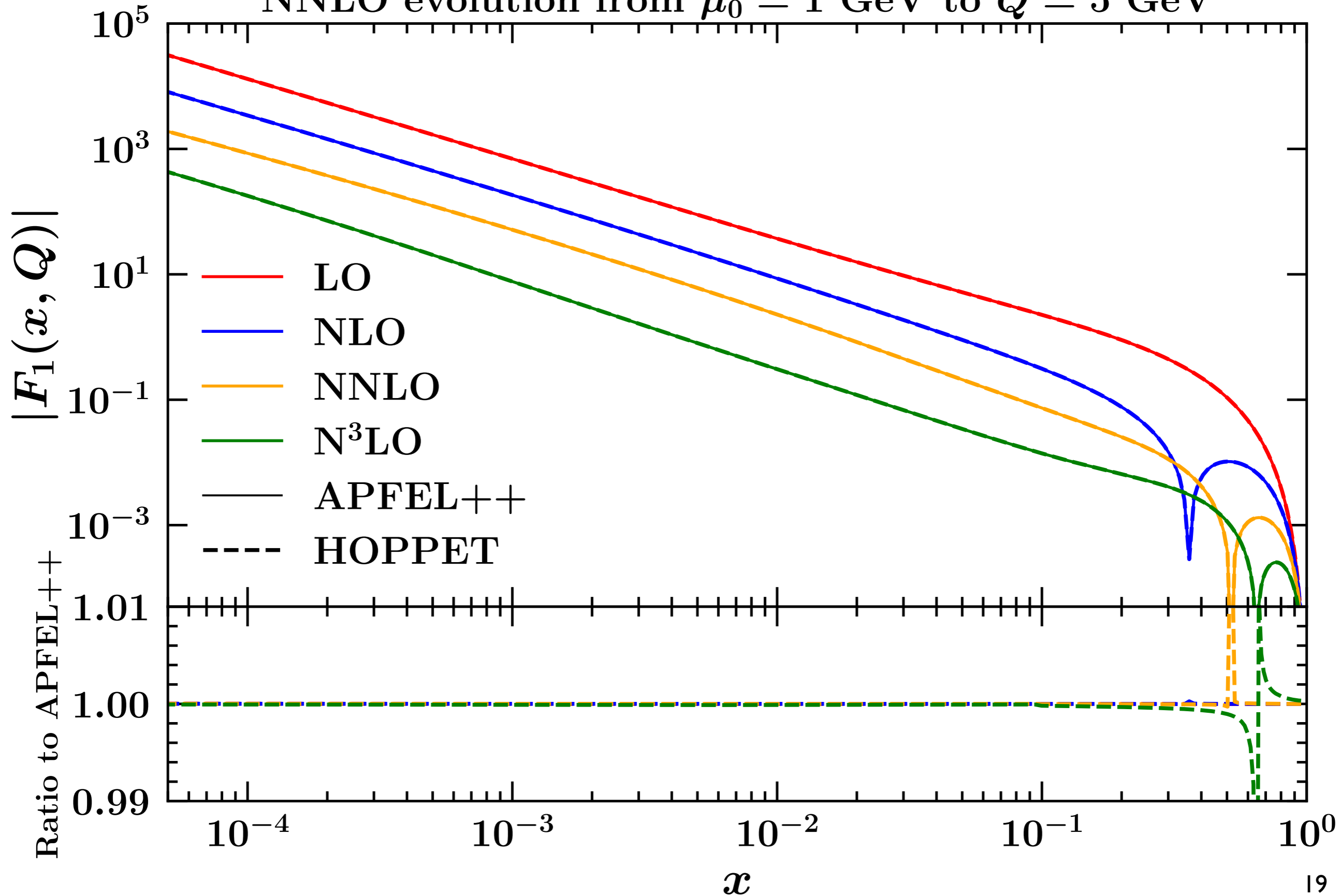
Structure functions

NNLO evolution from $\mu_0 = 1$ GeV to $Q = 2$ GeV



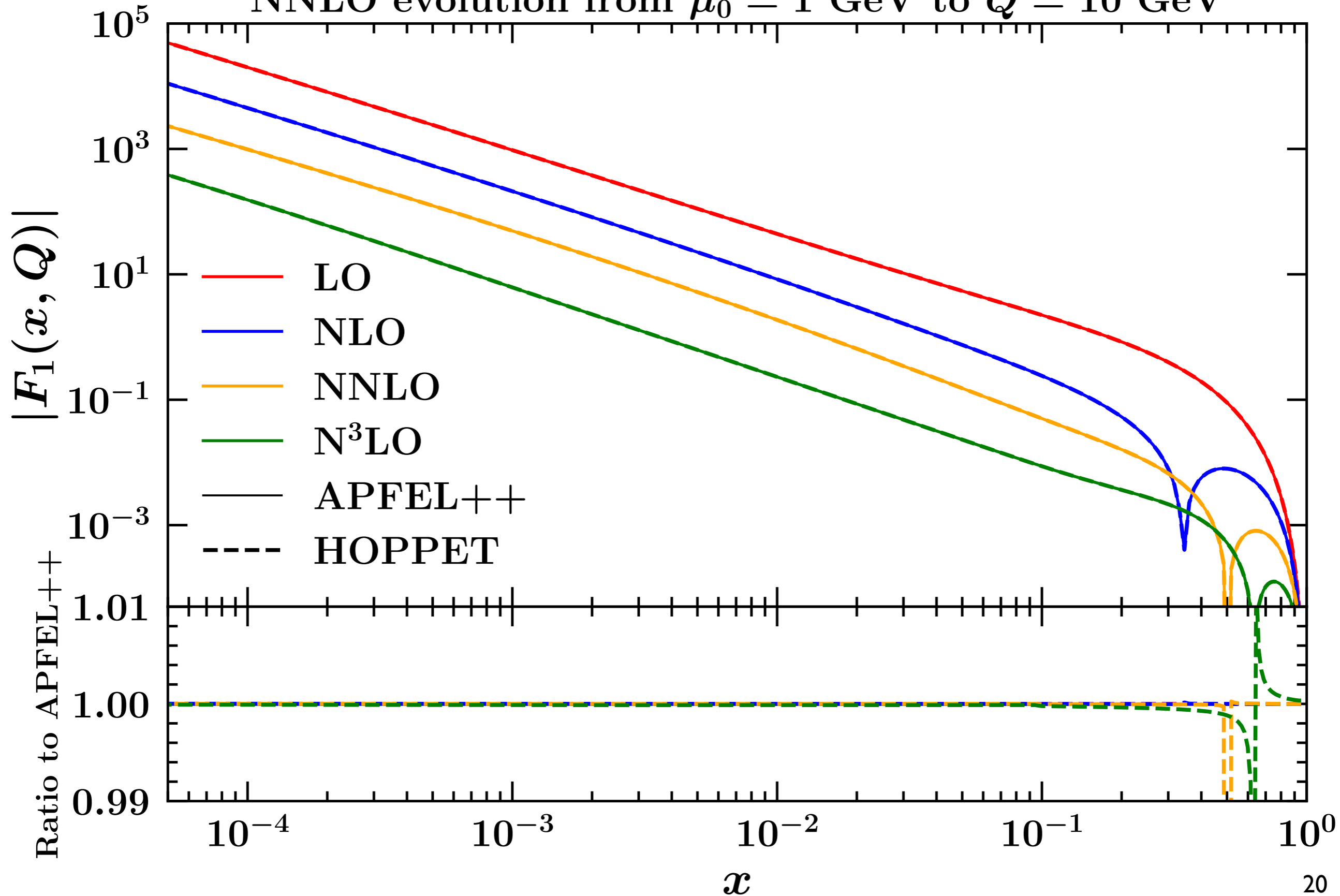
Structure functions

NNLO evolution from $\mu_0 = 1$ GeV to $Q = 5$ GeV



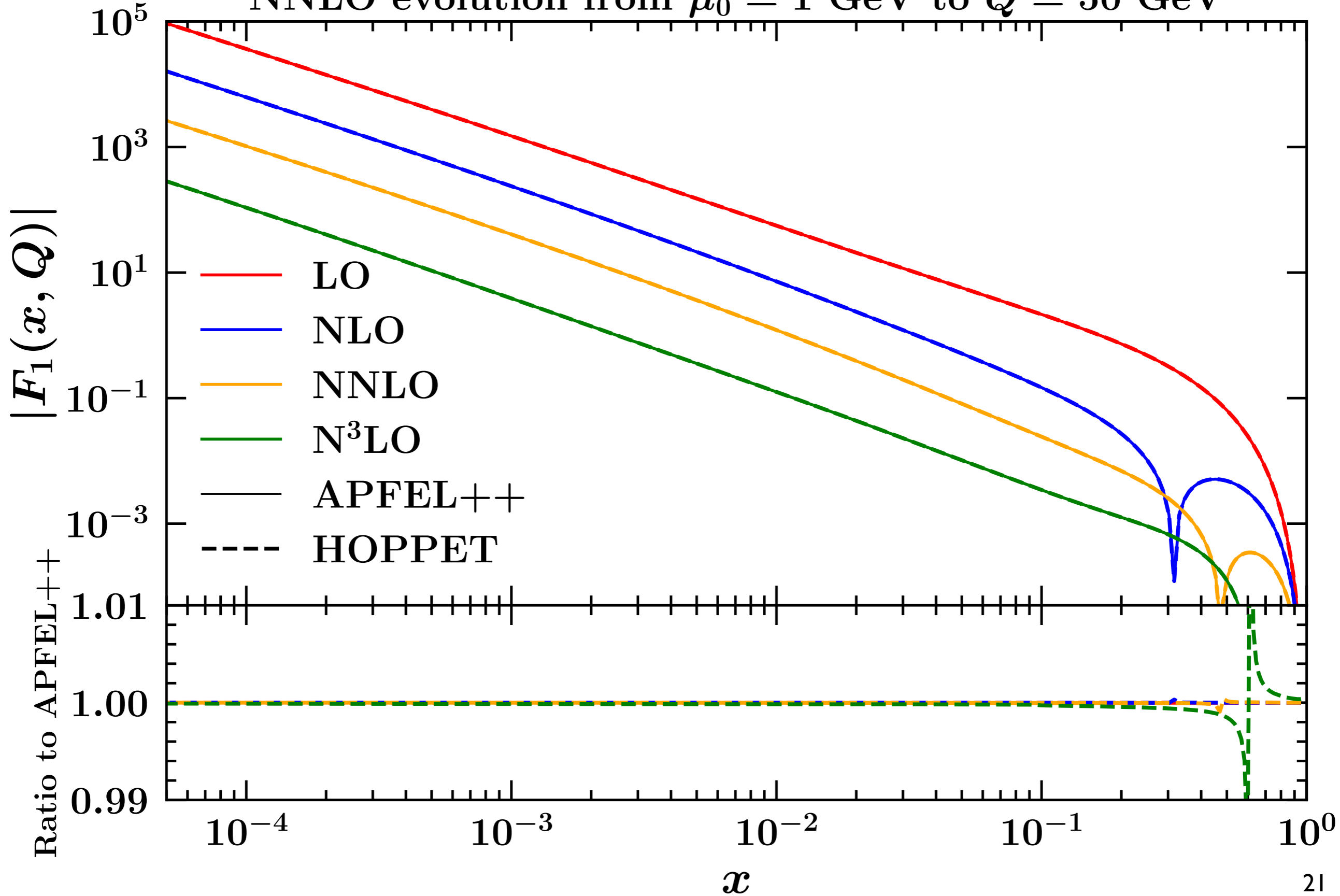
Structure functions

NNLO evolution from $\mu_0 = 1$ GeV to $Q = 10$ GeV



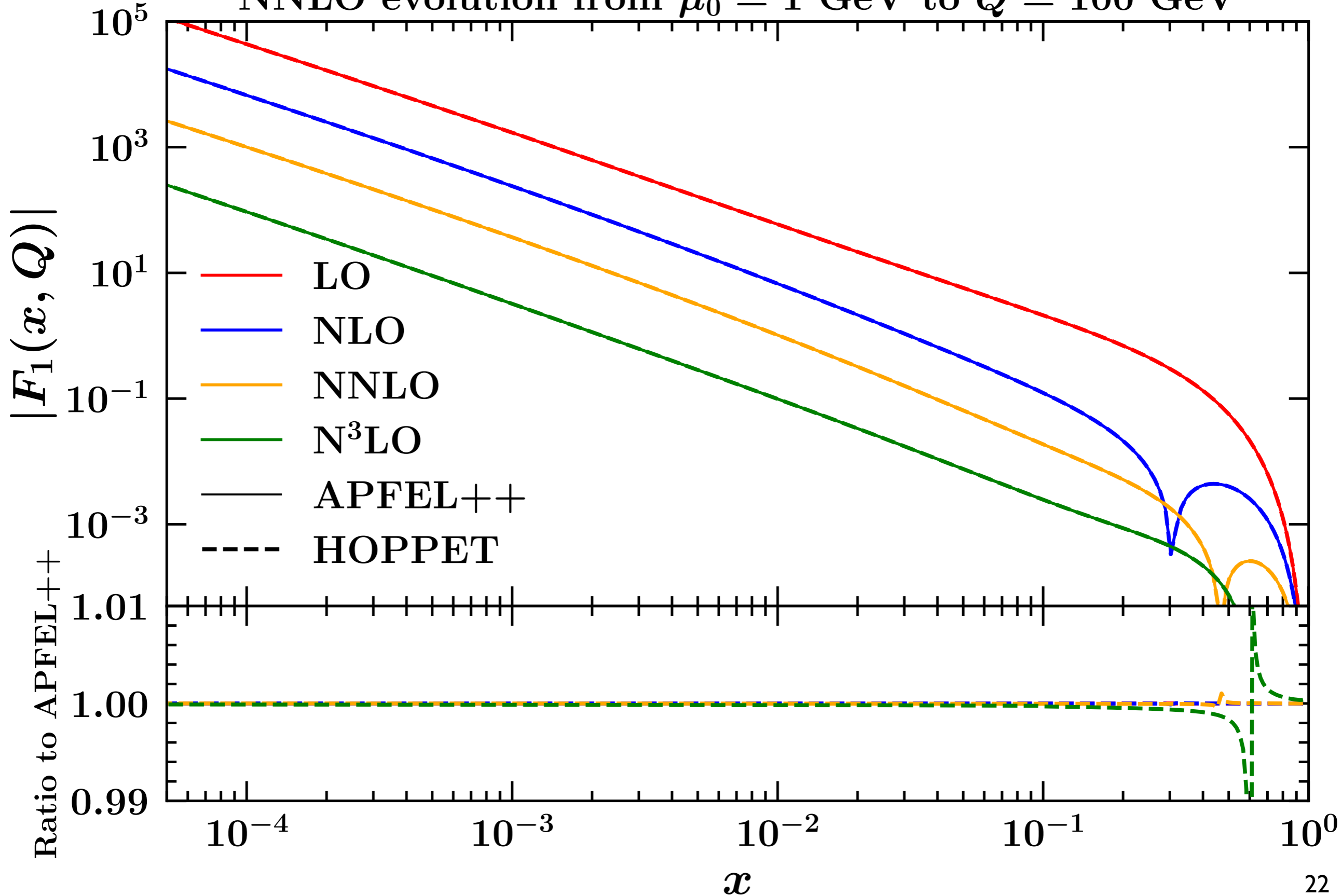
Structure functions

NNLO evolution from $\mu_0 = 1$ GeV to $Q = 50$ GeV



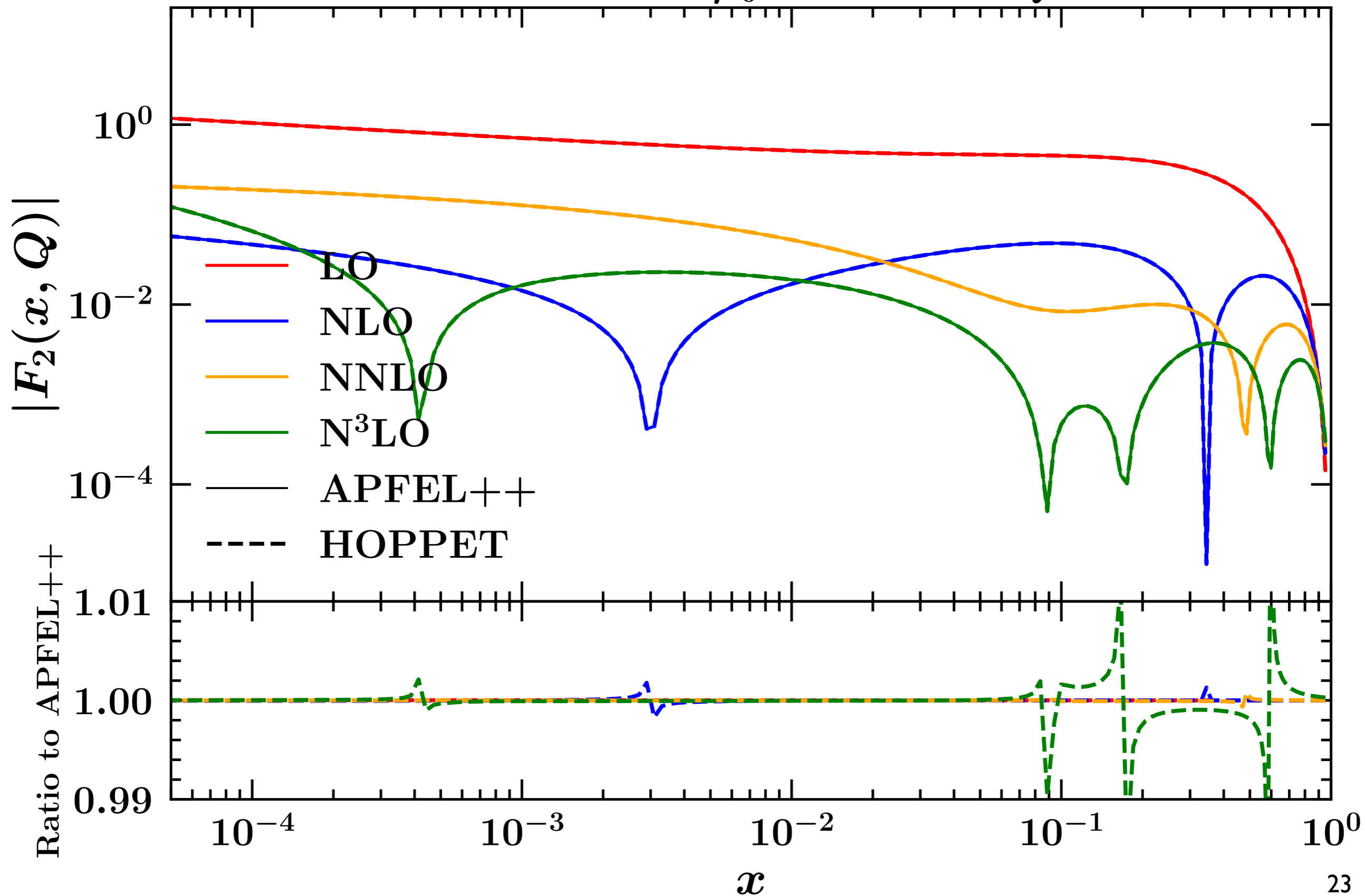
Structure functions

NNLO evolution from $\mu_0 = 1$ GeV to $Q = 100$ GeV



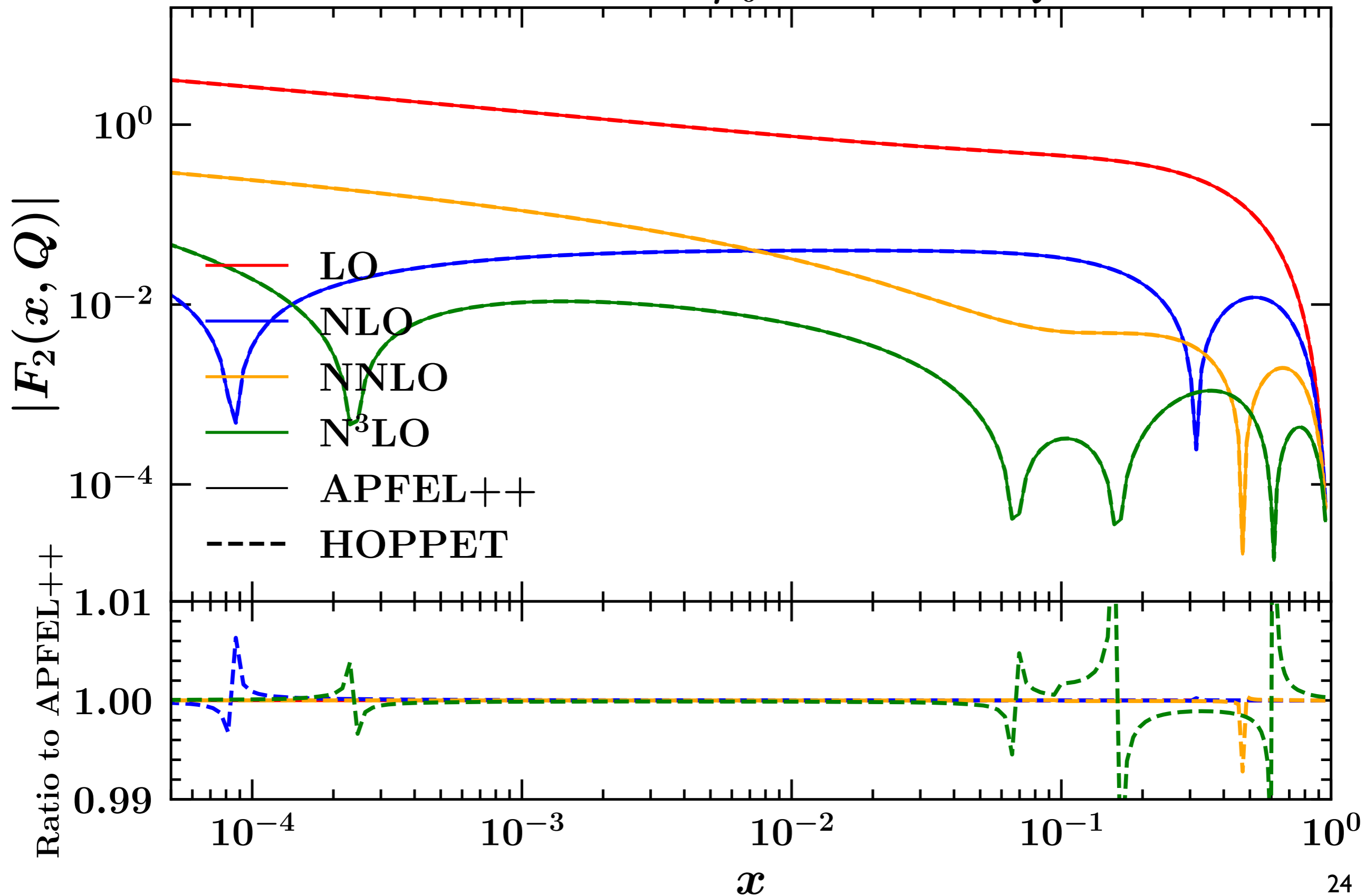
Structure functions

NNLO evolution from $\mu_0 = 1$ GeV to $Q = 2$ GeV



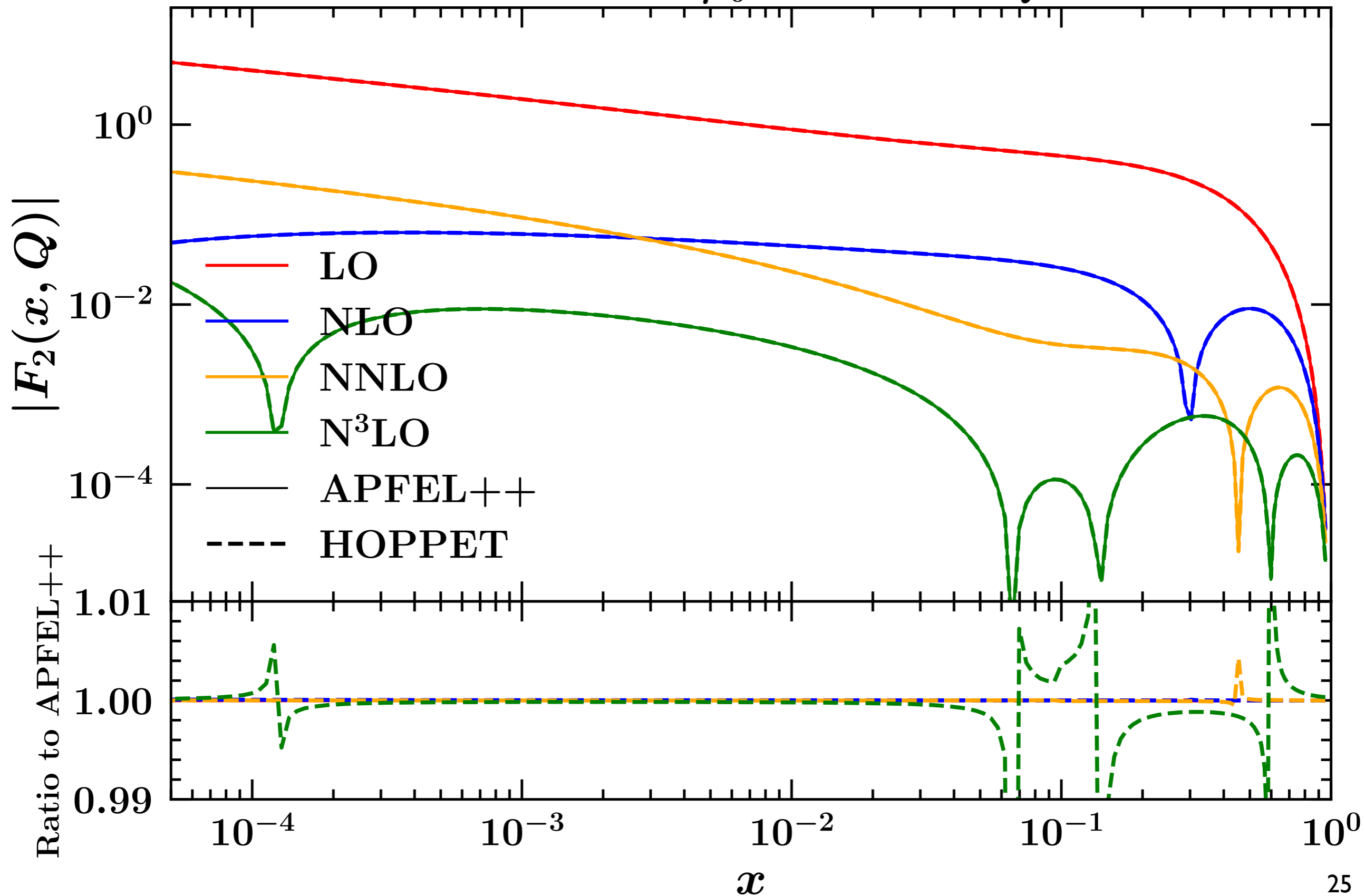
Structure functions

NNLO evolution from $\mu_0 = 1$ GeV to $Q = 5$ GeV



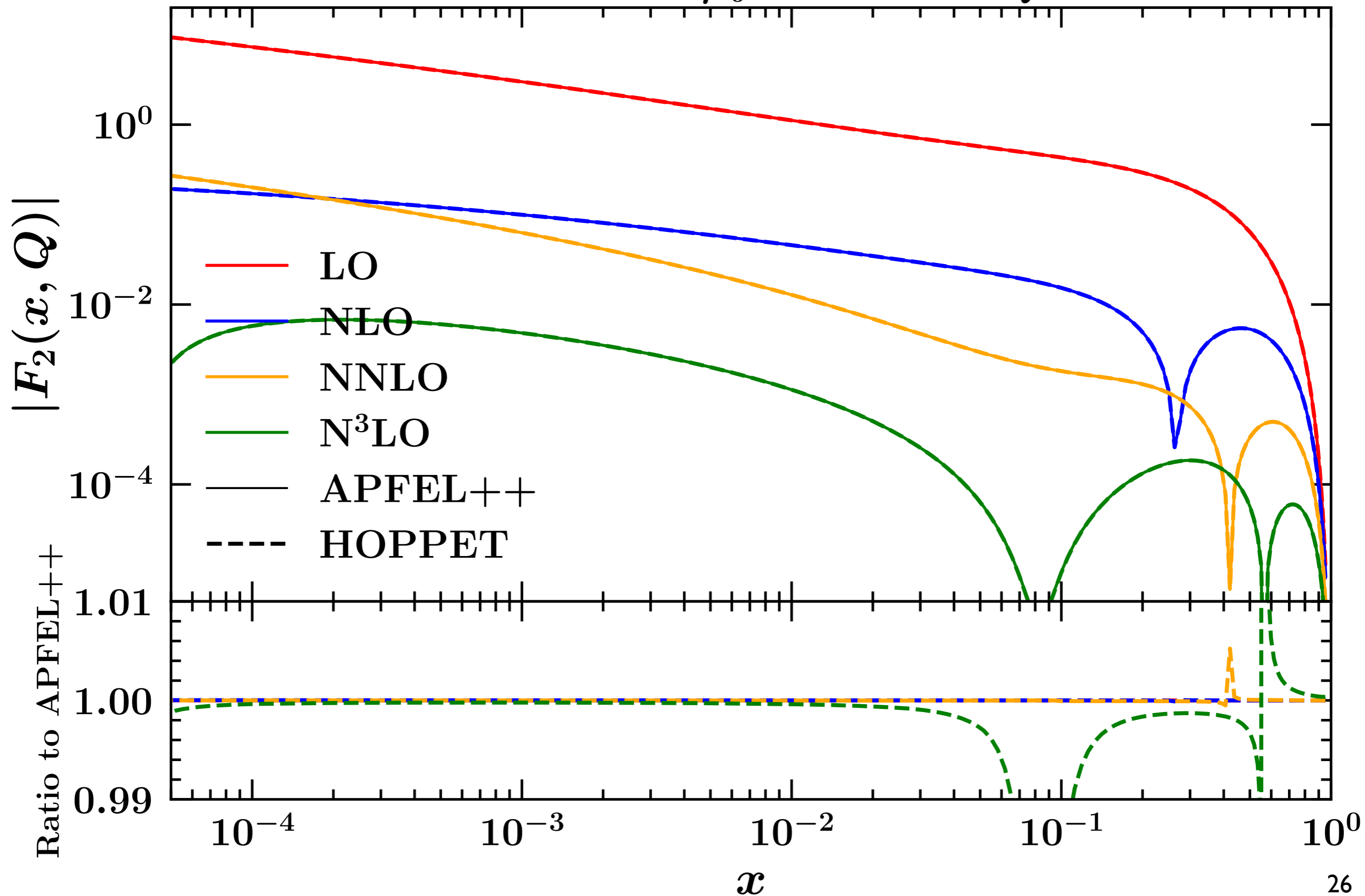
Structure functions

NNLO evolution from $\mu_0 = 1$ GeV to $Q = 10$ GeV



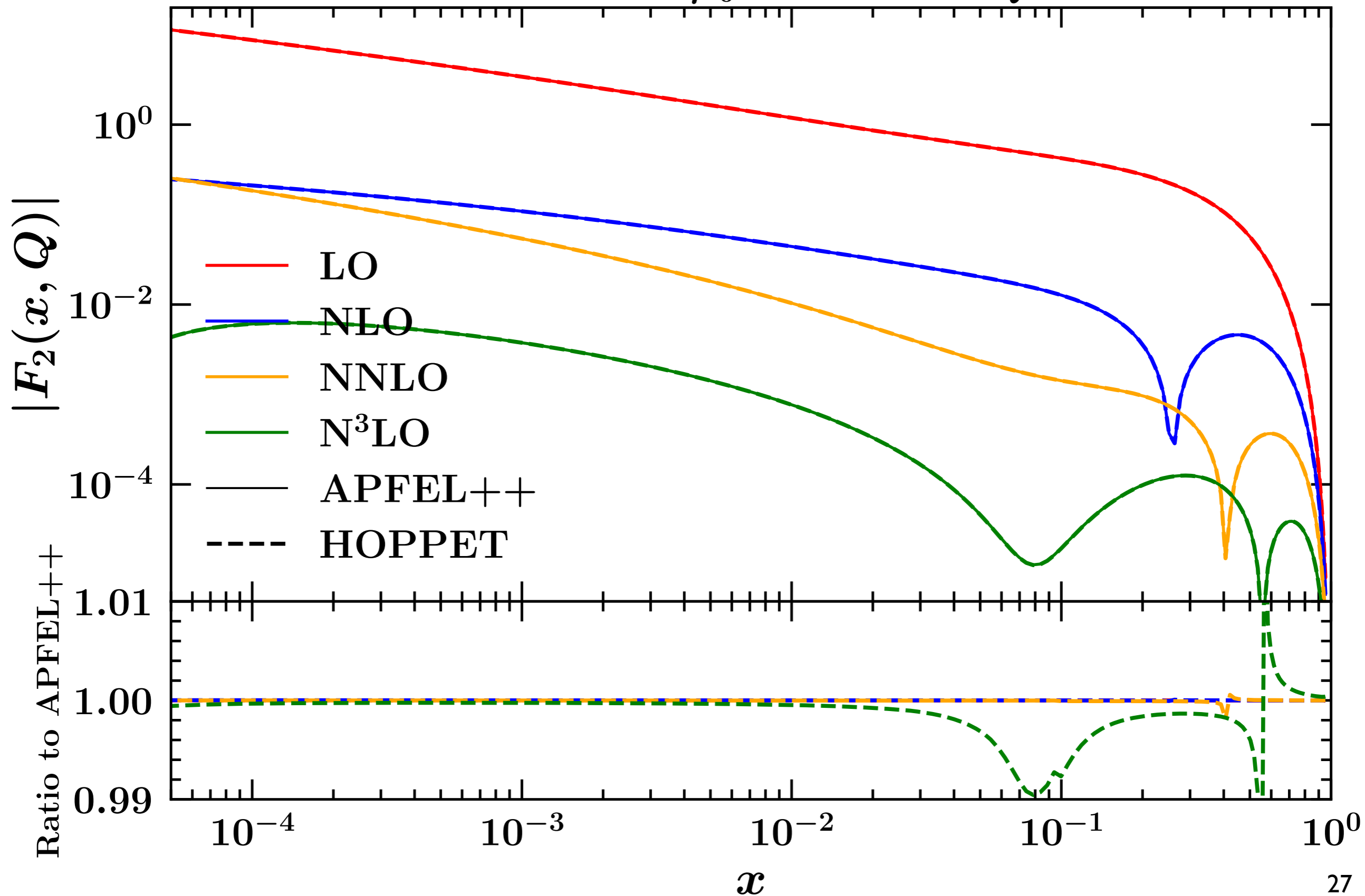
Structure functions

NNLO evolution from $\mu_0 = 1$ GeV to $Q = 50$ GeV



Structure functions

NNLO evolution from $\mu_0 = 1$ GeV to $Q = 100$ GeV



Structure functions

NNLO evolution from $\mu_0 = 1$ GeV to $Q = 500$ GeV

