Waves in non-neutral plasma

- Diocotron wave
- Plasma wave (Trivelpiece-Gould)
  (EAW)
- Cyclotron wave

François Anderegg

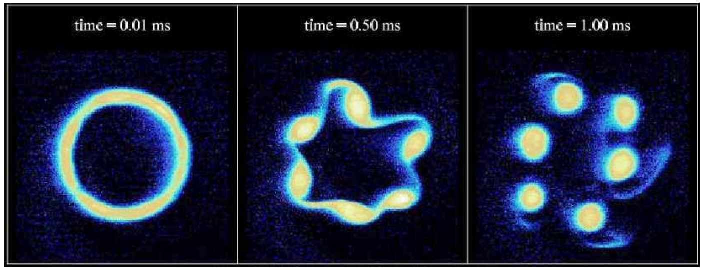
UCSD

Diocotron mode

Where does the name comes from?

Diocotron comes from Greek "pursue, chase"

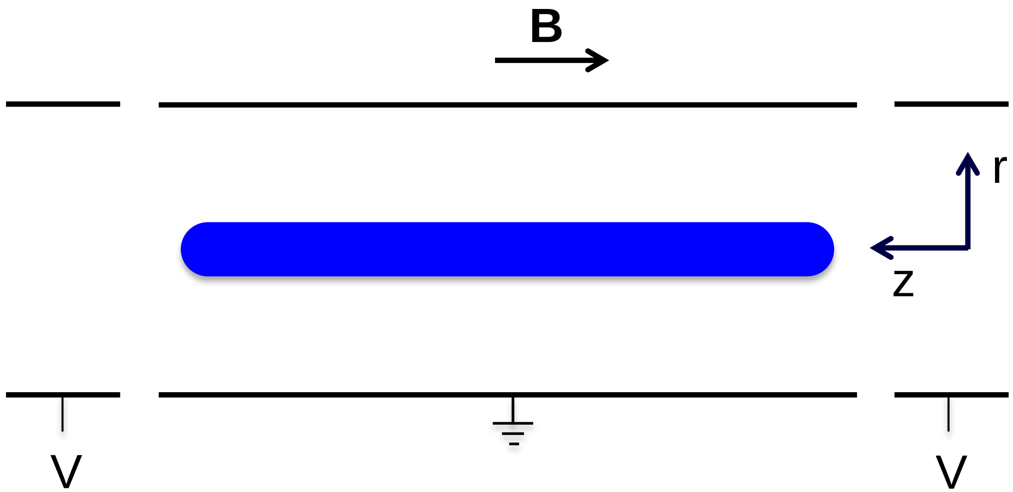
Hollow electron beam => **Diocotron instability**

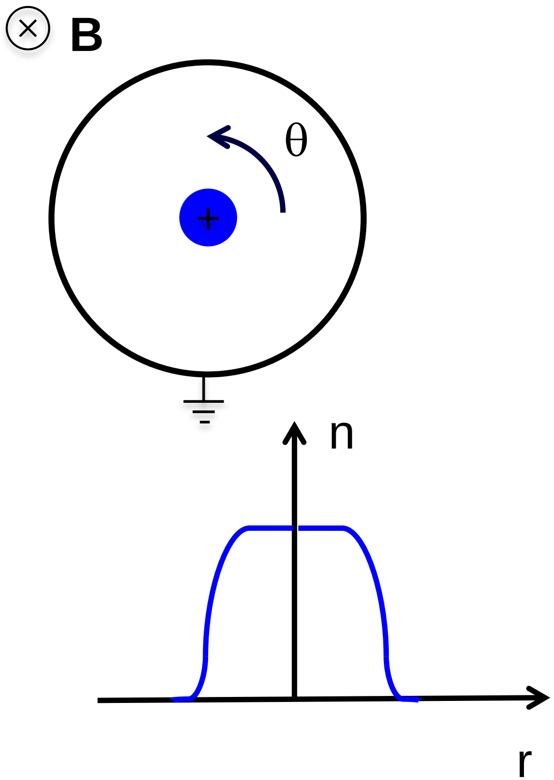


This talk is about **stable diocotron wave NOT diocotron instability** of hollow plasma

Diocotron wave

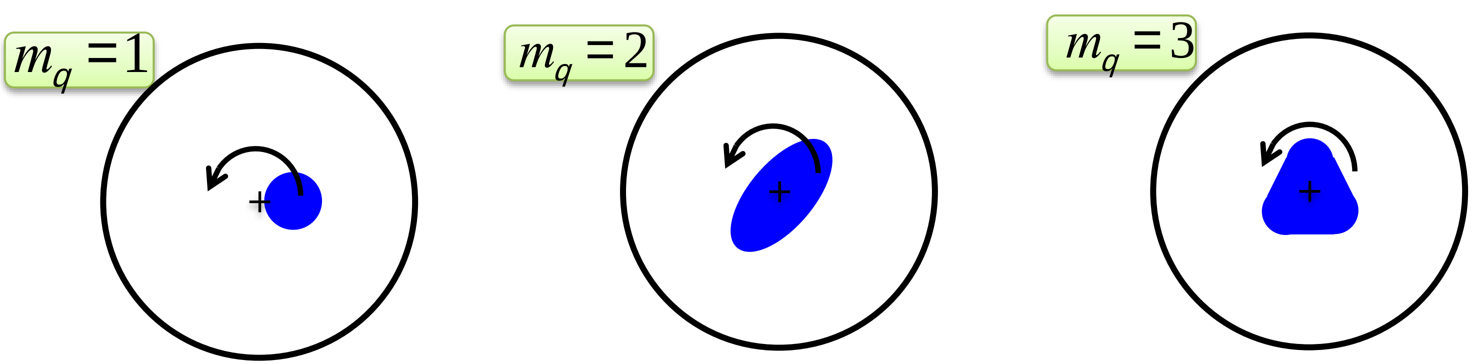
Let's consider the simplest case: monotonically decreasing density profile



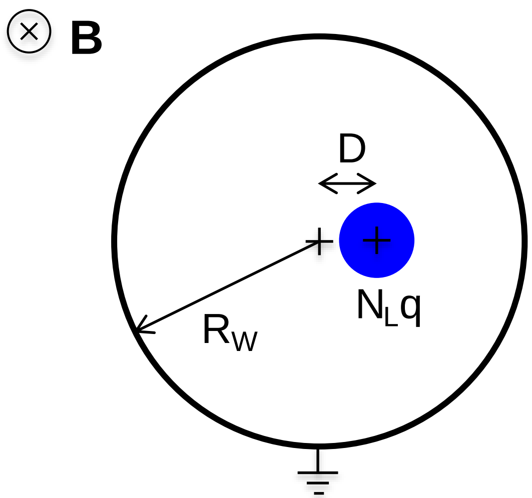


Long "rigid" plasma "Penning-Malmberg"

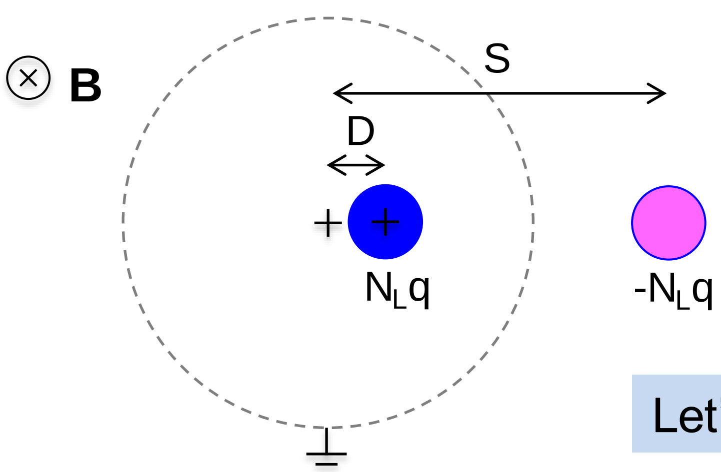
Density perturbation: $\delta n(r)\exp\{i(m_\theta \theta + k_z z - \omega t)\}$



“Infinitely” long plasma column m_θ=1



Replace the wall by an equal and opposite **image charge** such that the potential at $r=R_w$ is constant



Let’s find out S such that $\phi(R_w,\theta)$=constant

Potential of ∞ line charge

$$\phi(r,\theta) = -\frac{N_L q}{2\pi\varepsilon_0}\left[\ln\sqrt{r^2+D^2-2rD\cos\theta} - \underbrace{\ln\sqrt{r^2+S^2-2rS\cos\theta}}_{\text{image}}\right]$$

$$= \frac{N_L q}{2\pi\varepsilon_0}\ln\left[\frac{r}{S}\frac{\sqrt{1+\frac{D^2}{r^2}-\frac{2D}{r}\cos\theta}}{\sqrt{1+\frac{r^2}{S^2}-\frac{2r}{S}\cos\theta}}\right]$$

choose $\underbrace{\frac{S}{R_w} = \frac{R_w}{D}}$ then at r= R_w $\phi(R_w,\theta) = -\frac{N_L q}{2\pi\varepsilon_0}\ln\left(\frac{D}{R_w}\right)$

$$\boxed{S = \frac{R_W^2}{D}}$$

Independent of θ

Electric field from a line charge

Using Gauss' law $$E = \frac{\Sigma Q}{2\pi\varepsilon_0 r L} = \frac{N_L q}{2\pi\varepsilon_0 r}$$

The image charge electric field at r=0 is: $$E_i = \frac{-N_L q}{2\pi\varepsilon_0\, S} = \frac{-N_L q\, D}{2\pi\varepsilon_0\, R_W^2}$$

The ExB drift velocity of the (real) charge in the electric field of the image charge is:

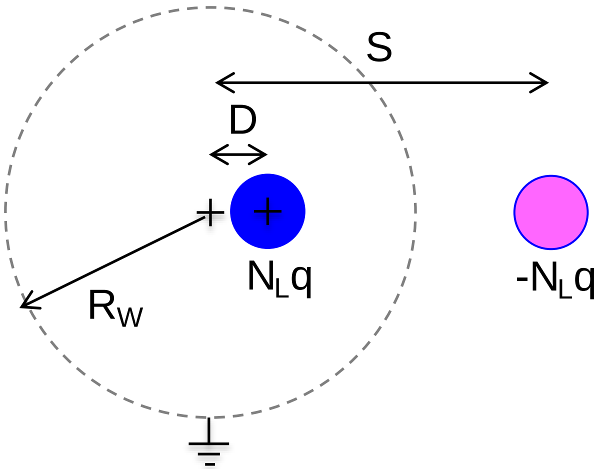
Assuming $D/R_w \ll 1$ $$v_d = \frac{E_i}{B} = \frac{-N_L q\, D}{2\pi\varepsilon_0\, B_z\, R_W^2}$$

The *infinite length small amplitude* **diocotron frequency is**:

$$\boxed{f_{dio} =} \frac{v_d}{2\pi D} = \boxed{\frac{N_L q}{4\pi^2\varepsilon_0\, B_z\, R_W^2}}$$

The diocotron plasma mode is a negative energy mode!

The image charge have opposite sign of the “real” charge



$$S = \frac{R_W^2}{D}$$

Increasing D reduces S

The plasma is attracted towards its image charge.
The electrostatic energy decreases as the mode amplitude increases.
Kinetic energy is negligible.

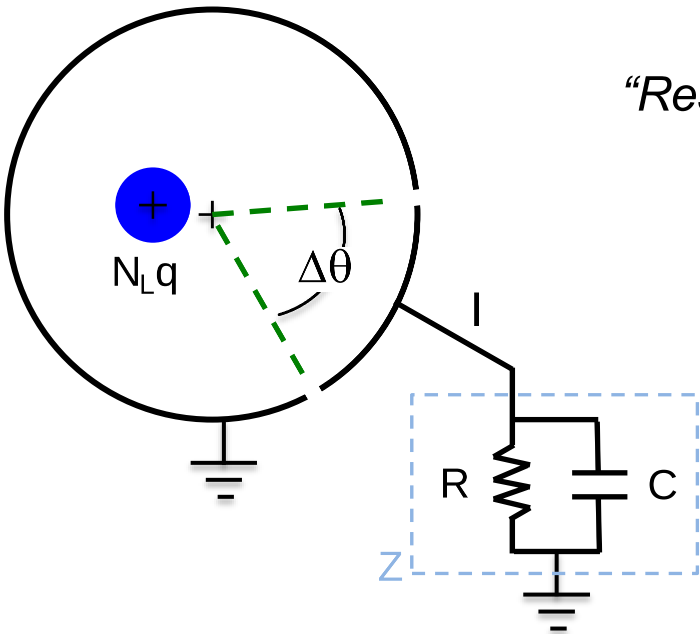
How much electrostatic energy to displace the plasma by D in the image electric field?

$$W_{ES} = \int_0^D F\,dx = \int_0^D Q \cdot E_i\,dx = \int_0^D N_L q L_P \frac{(-N_L q \cdot x)}{2\pi\varepsilon_0 R_W^2} dx = \frac{-(N_L q)^2}{4\pi\varepsilon_0} \frac{D^2}{R_W^2} L_p$$

$$E_i = \frac{-N_L q\ D}{2\pi\varepsilon_0\ R_W^2}$$

Negative energy

The diocotron mode can be destabilized by dissipation!

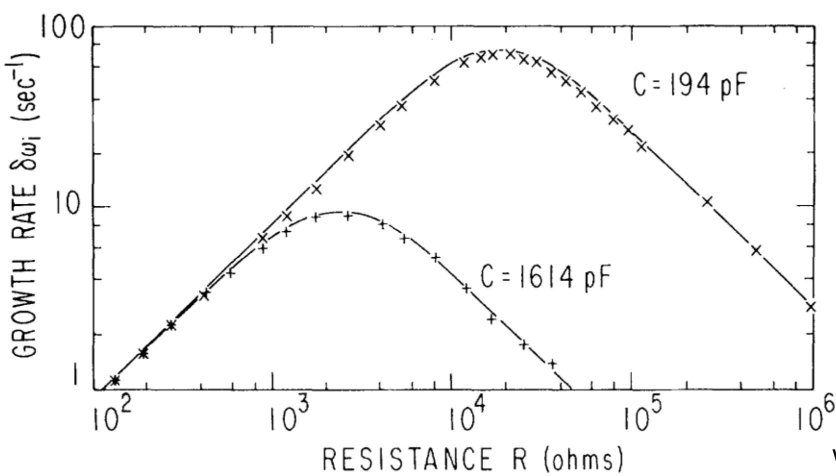


"Resistive growth"

Power dissipated in the load $P = \frac{1}{2} I^2 \,\mathrm{Re}\, Z$

Energy in the wave: W_{ES}

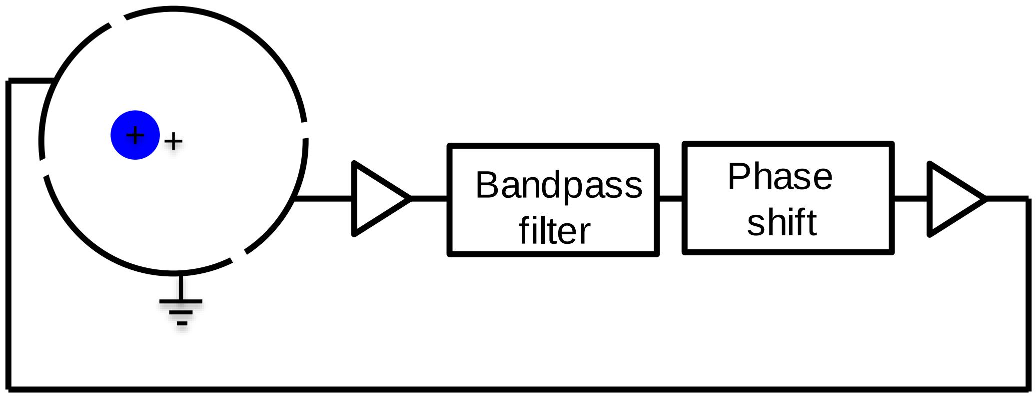
The growth rate $\gamma = \dfrac{P}{2W_{ES}} = \left(\dfrac{4\varepsilon_0}{\pi}\right) \dfrac{\omega^2 L_s^2 \sin^2 \frac{\Delta\theta}{2}}{L_p} \mathrm{Re}(Z)$



$$\mathrm{Re}(Z) = \frac{R}{1+\omega^2 R^2 C^2}$$

W.D. White, J.D. Malmberg and C.F. Driscoll PRL 49, 1822(1982)

Use feedback circuit to damp the mode



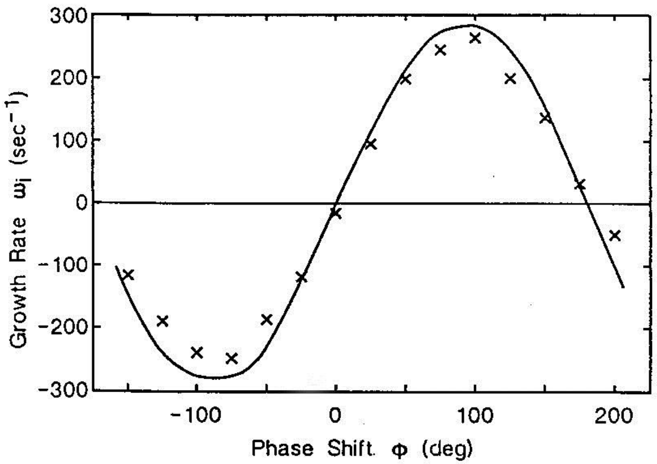


Figure 3.4: Feedback growth rate versus phase shift.

Diocotron mode is a tool to move the plasma off axis

- phaser picture (before CCD image of plasma)
- load off axis multi-trap (Surko's group)

The diocotron frequency can be used to measure the line density

We have to be careful here

$$f_{\infty}^{dio} = \frac{N_L q}{4\pi^2 \varepsilon_0 \; B_z \; R_W^2}$$

Valid for ***infinite length, small amplitude***

For a measured N_L and a measured diocotron frequency,

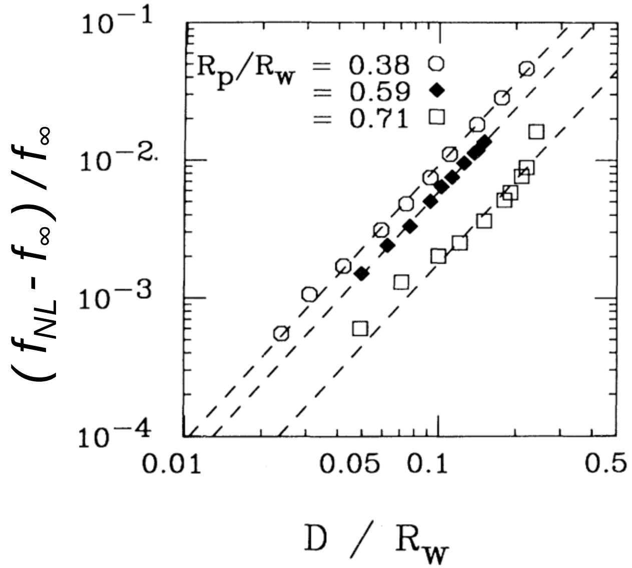
the infinite length equation gives a frequency too small by a factor of 2 or 3

Large amplitude diocotron

For large displacement, the column distorts into an elliptical cross-section

Non-linear correction

$$f_{NL} = f_\infty + f_\infty \left(\frac{1 - 2\left(R_p / R_W\right)^2}{\left[1 - \left(R_p / R_W\right)^2\right]^2} \right) \left(\frac{D}{R_W} \right)^2$$



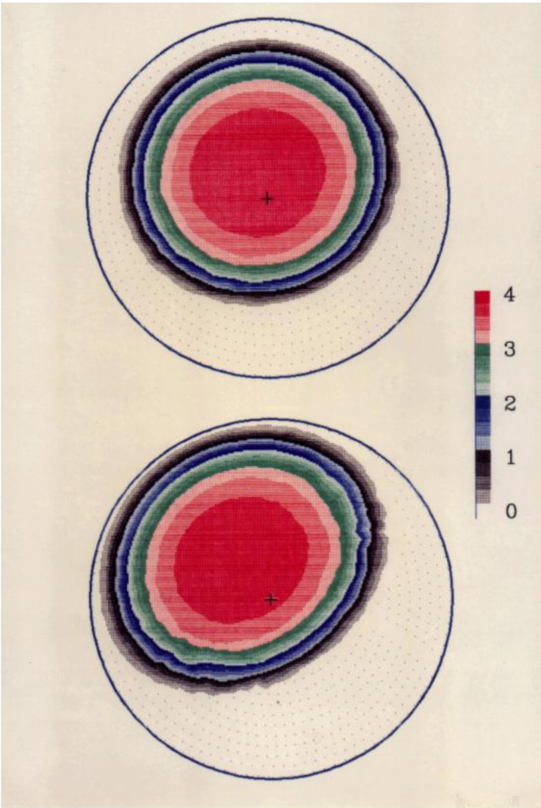


Fig 4 Measured density $n(r,\theta)$ phase coherent with a stable $l=1$ diocotron mode of 2 different amplitudes. Colors represent density on a linear scale of 10^6 cm^{-3}.

C.F. Driscoll and K.S Fine Phys. Fluids B, 2, 1359 (1990)

K.S. Fine, C.F. Driscoll and J.H. Malmberg, PRL, 63, 2232, (1989)

Finite length diocotron

Confining potential push the plasma in the **z-direction**.

This result in a **radial force on a off axis plasma**.

Confining potential add to the force due to the image charge

$$F_{tot} = \underbrace{F_i}_{\text{Image}} + \underbrace{F_c}_{\text{confinement voltage}}$$

$$\frac{f_{dio}}{f_\infty} = \frac{F_{tot}}{F_{i,\infty}} = 1 + \left[\frac{j_{01}}{2} \left(\underbrace{\frac{1}{4} + \ln\left(\frac{R_W}{R_p}\right)}_{\text{Plasma electrostatic pressure}} + \underbrace{\frac{T}{N_L e^2}}_{\text{Plasma kinetic pressure}} \right) \underbrace{- 0.671}_{\text{finite length on image charge}} \right] \left(\frac{R_W}{L_p}\right)$$

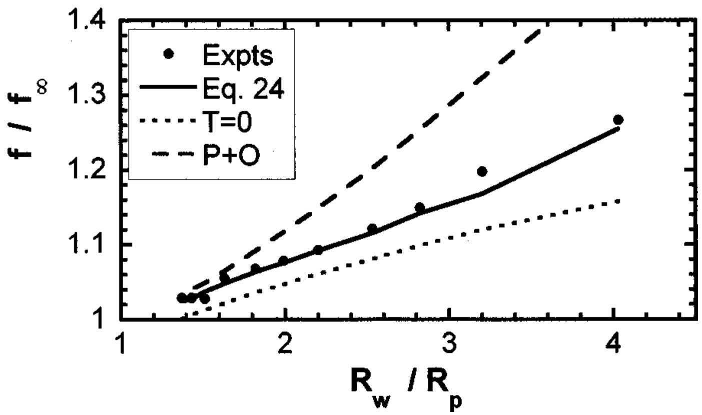
“Plasma electrostatic pressure”

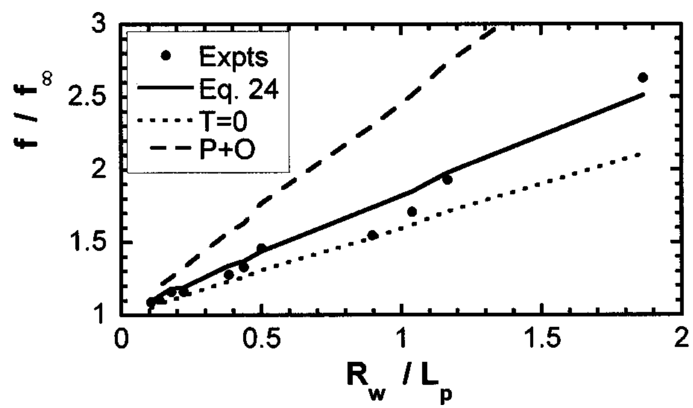
“Plasma kinetic pressure”

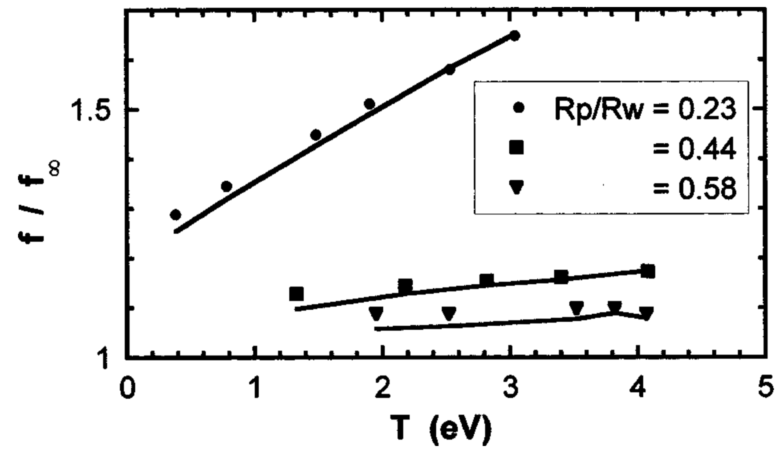
“finite length on image charge”

K.S Fine and C.F. Driscoll Phys .Pla. 5, 601,(1998)

Experimental test of finite length effects







$$\frac{4\lambda_D^2}{R_p^2}$$

"The kinetic pressure is small for plasma"
"Large for few particles"

$$\frac{f_{dio}}{f_\infty} = \frac{F_{tot}}{F_{i,\infty}} = 1 + \left[\frac{j_{01}}{2} \left(\frac{1}{4} + \ln\left(\frac{R_W}{R_p}\right) + \frac{T}{N_L e^2} \right) - 0.671 \right] \left(\frac{R_W}{L_p}\right)$$

K.S Fine and C.F. Driscoll Phys .Plas. 5, 601,(1998)

Magnetron mode

For small short plasma the confining potential dominates

"confinement"

"image potential"

$$f = -\frac{E_r}{2\pi D\ B_z} = \frac{1}{2\pi D\ B_z}\left[\frac{\partial \phi_c}{\partial r} + \frac{\partial \phi_i}{\partial r}\right]_{r=D}$$

$$= \frac{1}{2\pi\ B_z}\left[\underbrace{1.15\frac{V_c}{R_W^2}\frac{L}{R_W}}_{\substack{\text{230kHz} \\ \text{“confinement potential”}}} \underbrace{-1.0027\frac{Q}{R_W^3}}_{\substack{\text{1.4kHz} \\ \text{“Image field”}}}\right]$$

For example:

$V_c = 10V$

$R_W = 1\ cm$

$L/R_W = 0.2$

K.S Fine and C.F. Driscoll Phys .Plas. 5, 601,(1998)

Higher order diocotron mode

$$f^{dio}_{m_\theta} = f_{ExB}\left[m_\theta - 1 + \left(\frac{R_p}{R_W}\right)^{2m_\theta}\right]$$

Where $$f_{ExB} = \frac{en}{4\pi\varepsilon_0 B}$$

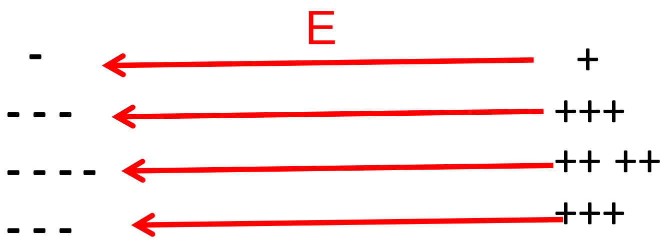
"square profile" plasma rotation frequency

$f^{dio}_{m_\theta=2} \cong f_{ExB}$ The m_θ=2 mode is close to the rotation frequency

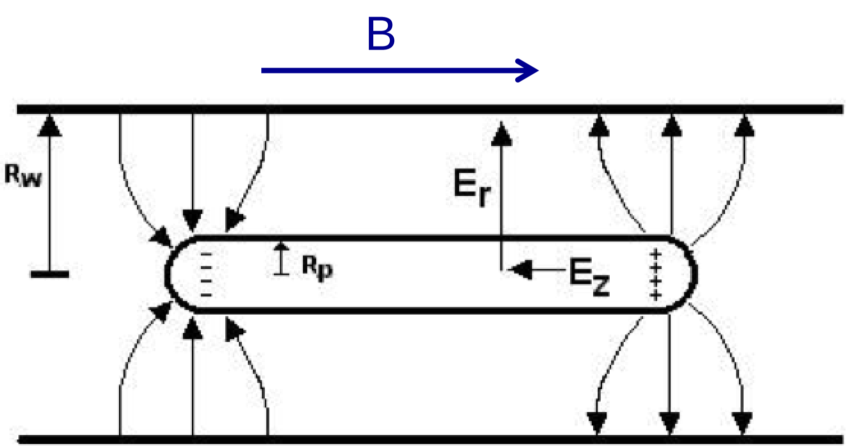
See Joel Fajans presentation

Plasma wave

Infinite plasma



Long plasma in a “Penning-Malmberg” trap **Trivelpiece Gould** mode



E_z=0 at the wall

E is radial at the wall

E_z is reduce by the conducting the wall

$f_{TG} < f_{Langmuir}$

Density perturbation:

$$\delta n(r)\exp\{i(m_\theta\theta + k_z z - \omega t)\}$$

Continuity

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z} n \cdot v_z = 0$$

$$-i\omega\ \delta n + n\ ik_z\delta v_z = 0$$

$$\delta n = \frac{n\ k_z\delta v_z}{\omega}$$

Keep only z dynamics

Newton

$$m\frac{\partial v_z}{\partial t} = qE_z\ =\ -q\frac{\partial}{\partial z}\varphi$$

$$-m\ i\omega\ \delta v_z = -q\ ik_z\delta\varphi$$

$$\delta v_z = \frac{q\ k_z\delta\varphi}{m\ \omega}$$

$$\delta n = \frac{n\ \mathrm{q} k_z^2\delta\varphi}{m\omega^2}$$

Poisson

$$\nabla^2\varphi = -4\pi q\ \delta n$$

$$-k^2\delta\varphi = -4\pi q\ \delta n$$

Keep all k in Poisson eq.

$$-k^2\delta\varphi = \frac{-4\pi q^2 n k_z^2\delta\varphi}{m\omega^2}$$

$$\omega^2 = \frac{k_z^2}{k^2}\ \frac{4\pi q^2 n}{m}$$

$$\boxed{\omega^2 = \frac{k_z^2}{k^2}\ \omega_p^2}$$

“cold Trivelpiece Gould mode”

With thermal pressure $$\boxed{\omega^2 = \frac{k_z^2}{k^2}\omega_p^2 + 3\bar{v}^2 k_z^2}$$ **"Trivelpiece Gould mode"**

$$k^2 = k_z^2 + k_\perp^2$$

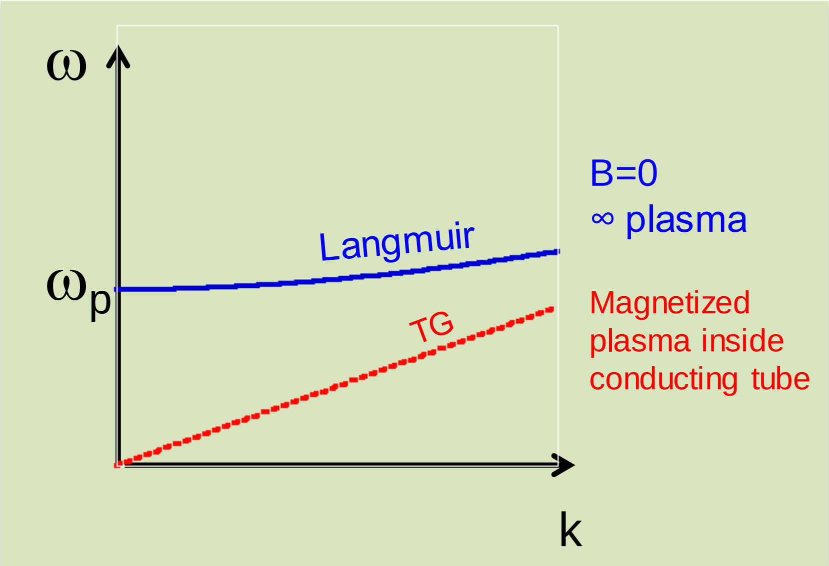
$$k_\perp = \frac{1}{R_p}\underbrace{\left(\frac{2}{\ln\left(R_W/R_p\right)}\right)^{\frac{1}{2}}}_{\text{cylindrical geometry}}$$

Valid for $R_p << R_W$

If we kept all k not only k_z, we would get

$$\boxed{\omega^2 = \omega_p^2\left(1 + \tfrac{3}{2}k^2\lambda_D^2\right)}$$

Standard plasma wave in unmagnetized, infinite plasma: **"Langmuir wave"**



Finite length TG modes

For trapped plasma the wavelength has to fit in the plasma

Standing wave

$$k_z = \frac{m_z \pi}{L_p}$$

For a long column "acoustic" dispersion relation

$$\boxed{\omega = \omega_p \left(\frac{R_p}{R_W}\right) R_W k_z \left(\frac{1}{2}\ln\left(\frac{R_W}{R_p}\right)\right)^{\frac{1}{2}} \left[1 + \frac{3}{2}\left(\frac{\bar{v}}{v_{ph}}\right)^2\right]}$$

Valid for $m_\theta=0$

$$\boxed{\omega - m_\theta \omega_E = \omega_p \left(\frac{R_p}{R_W}\right) R_W k_z \frac{1}{j_{m_\theta m_r}} \left[1 + \frac{3}{2}\left(\frac{\bar{v}}{v_{ph}}\right)^2\right]}$$

$m_\theta \neq 0$ **mode** are **Doppler shifted by the rotation frequency**

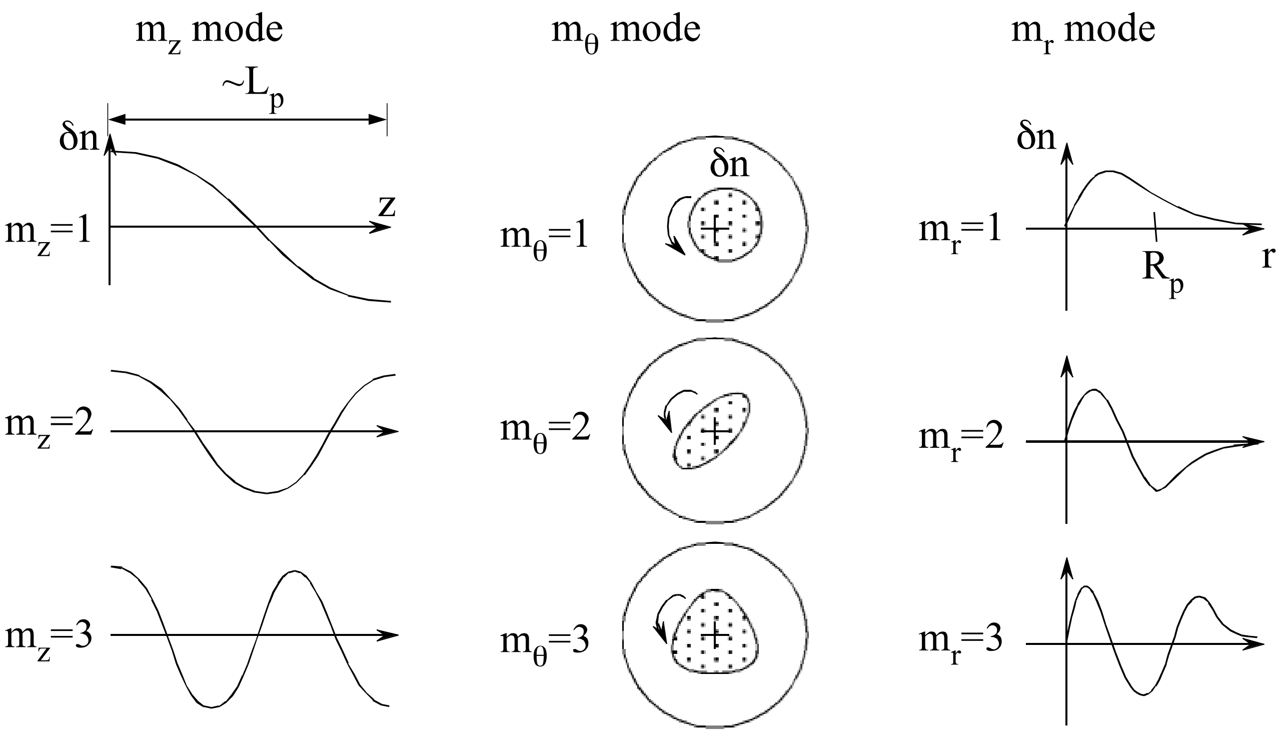
This can be useful for some rotating wall application

$$k_{eff} = k_z + \alpha_1 R_p + \alpha_2 R_W$$

k_{eff} correspond to a longer wavelength than $k_z = \frac{m_z \pi}{L_p}$

Jennings J.K.; Spencer R.L.; Hansen K.C . Phys. Plas. , 2, 2630, (1995)

All these m

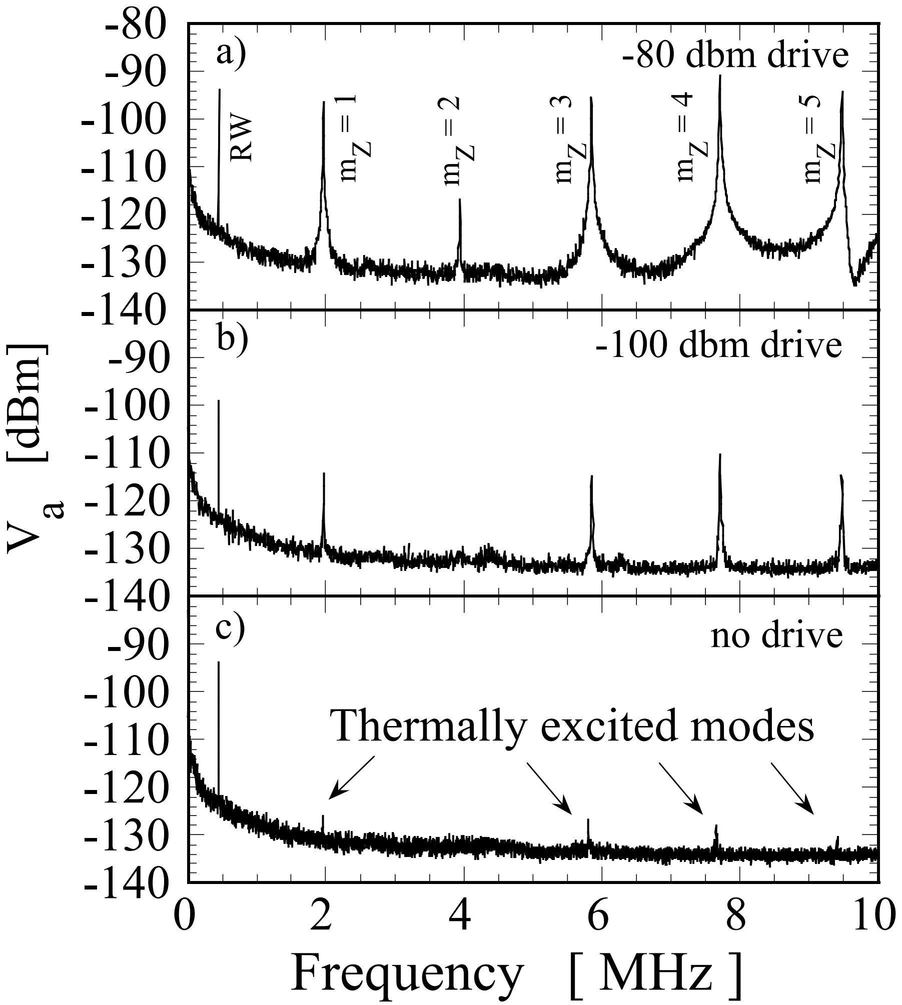


Higher m_z results in higher frequency

Higher m_θ results in lower ω - $m_\theta\omega_E$

Higher m_r results in lower frequency

TG modes are easy to excite

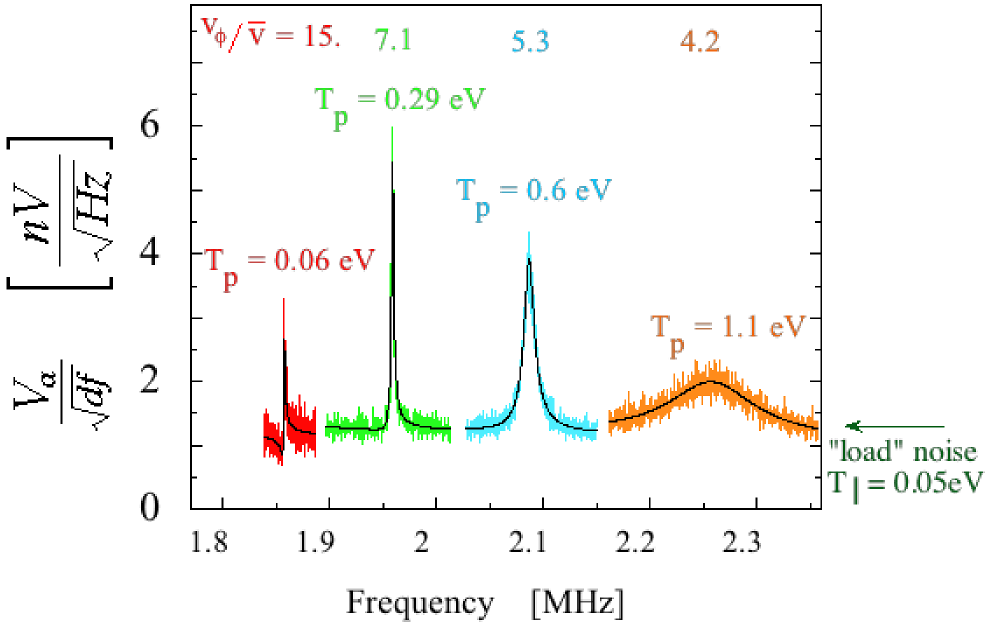


TG modes are excited at very low level by thermal fluctuations

Provide effective diagnostic tool.

F. Anderegg, N. Shiga, J.R. Danielson, D.H.E. Dubin, and C.F. Driscoll, R.W. Gould, Phys. Rev. Lett. **90**, 115001 (2003)

Thermally excited TG plasma modes



As the plasma temperature T_p increases:

- Landau damping increases

- Mode frequency increases

- "Area under the mode increases"

F. Anderegg, N. Shiga, J.R. Danielson, D.H.E. Dubin, and C.F. Driscoll, R.W. Gould PRL, **90**, 115001 (2003)

Non-Neutral plasma can relax to a state of **thermal equilibrium** in the rotating frame.

Plasma mode is

excited by

- thermal e^- motion in the plasma

"reverse Landau damping" *"Cerenkov radiation"*

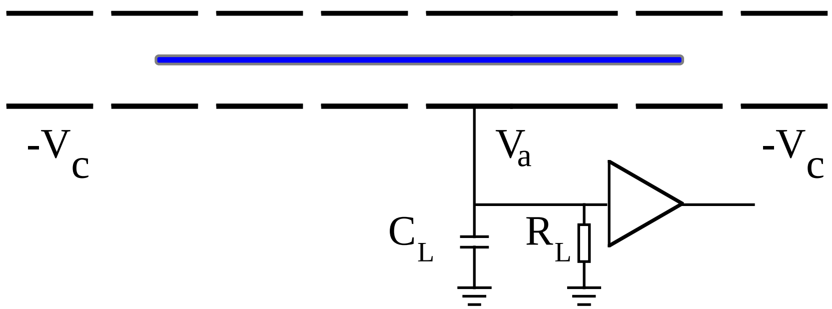
- noise in the load

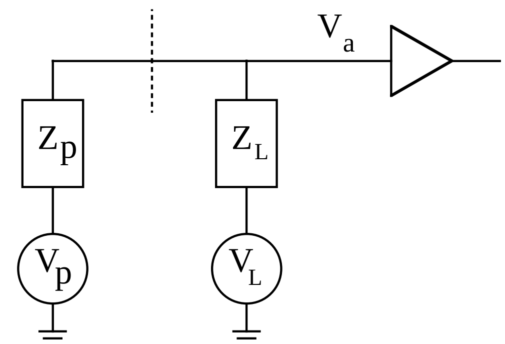
damped by

- Landau damping
- Load dissipation

Nyquist's theorem ("thermal noise in a resistor")

$$\boxed{\frac{V^2}{df} = 4k_BT \ \mathrm{Re}(Z)}$$





Nyquist's

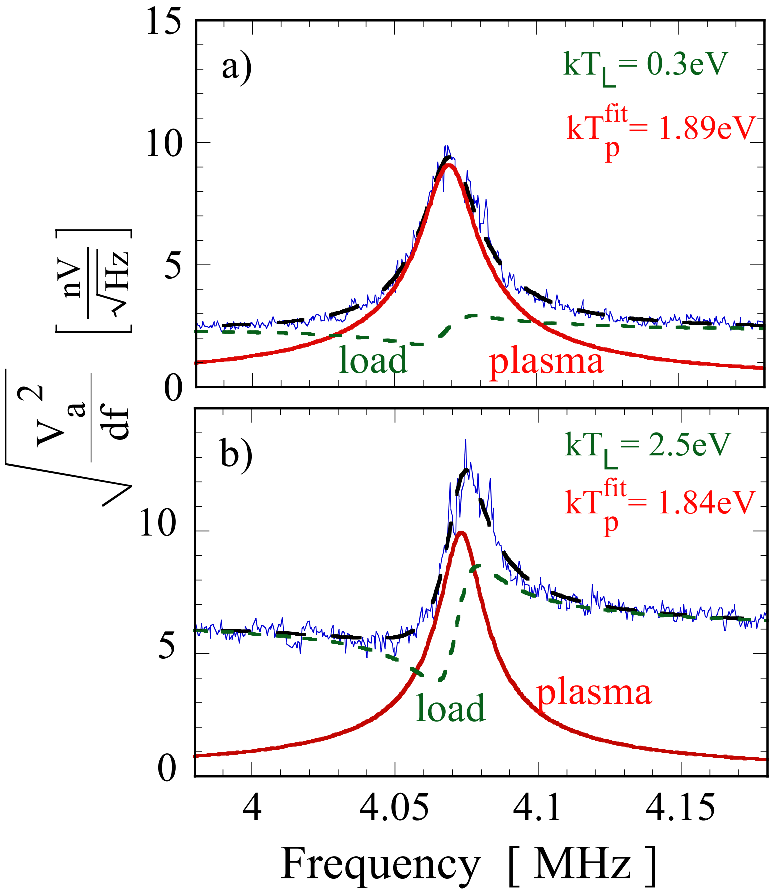
$$\boxed{\frac{V_a^2}{df} = \underbrace{4k_BT_p \ \mathrm{Re}(Z_p)\left|\frac{Z_L}{Z_p+Z_L}\right|^2}_{\text{Plasma “Lorentzian”}} + \underbrace{4k_BT_L \ \mathrm{Re}(Z_L)\left|\frac{Z_p}{Z_p+Z_L}\right|^2}_{\text{Load “base line”}}}$$

$$Z_p^{-1} = \frac{G\omega_m^2}{i\left(\omega-\omega_m\right)+\gamma_m}$$

$$Z_L^{-1} = R_L^{-1} + i\omega C_L$$

Nyquist's

$$\frac{V_a^2}{df} = \underbrace{4k_B T_p \ \mathrm{Re}(Z_p)\left|\frac{Z_L}{Z_p + Z_L}\right|^2}_{\substack{\text{Plasma} \\ \text{“Lorentzian”}}} \underbrace{+\,4k_B T_L \ \mathrm{Re}(Z_L)\left|\frac{Z_p}{Z_p + Z_L}\right|^2}_{\substack{\text{Load} \\ \text{“base line”}}}$$

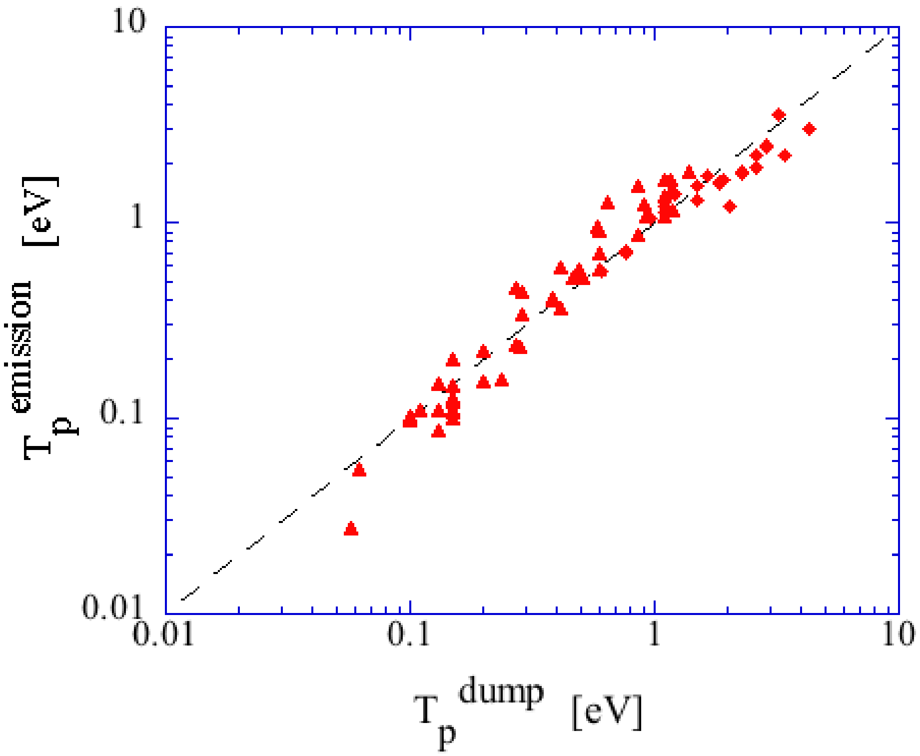


"Quiet load"

Noise added to the load

N. Shiga, F. Anderegg, D.H.E. Dubin, C.F. Driscoll and R.W. Gould, Phys.Plas, 13,022109, (2006)

Plasma temperature from thermally excited mode

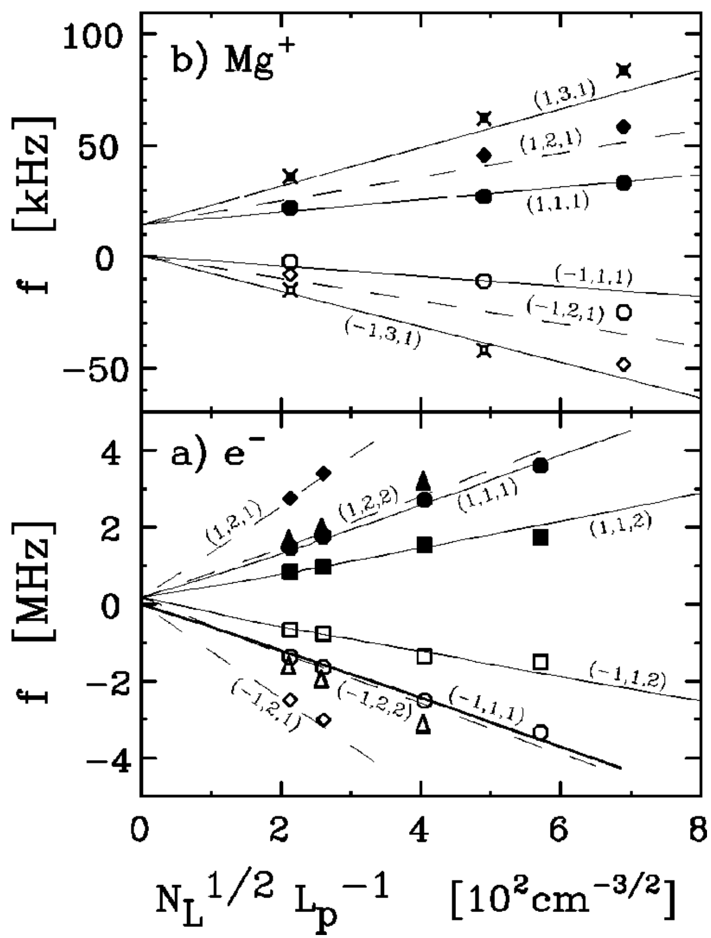


Use a room-temperature amplifier

The plasma temperature is non-destructively determined by ”listening” to plasma fluctuations.

F. Anderegg, N. Shiga, D.H.E. Dubin, and C.F. Driscoll, UCSD; and Roy Gould, Phys. Plas. 10, 1556 (2003)

Higher order TG modes



(m_θ, m_z, m_r)

TG modes travel forward or backward on the rotating plasma column

E.M. Hollmann, F. Anderegg, and C.F. Driscoll Phys. Plasmas **7**, 2776 (2000)

Plasma modes in spheroidal plasma: Dubin modes

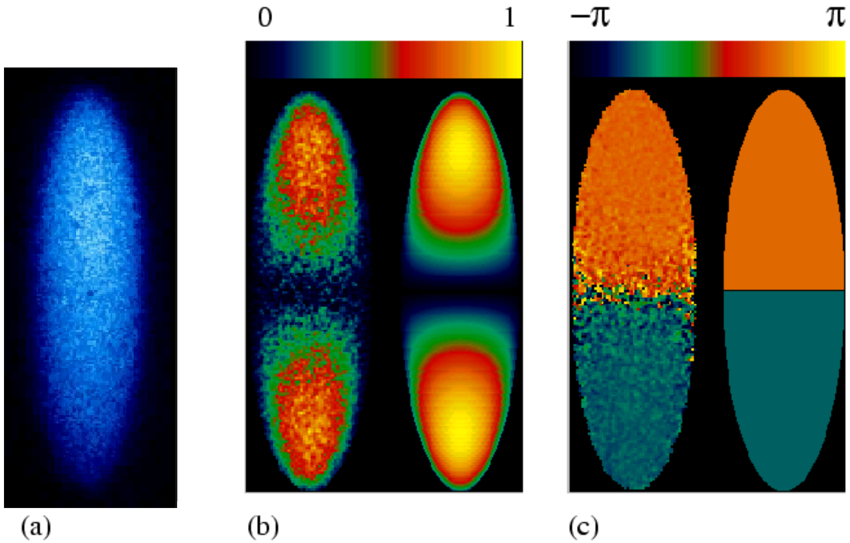


Figure 3. (a) Movie of sideview image data obtained on a plasma with $\omega_r/2\pi=$ 1.00 MHz while driving a (2,0) mode at $\omega_{2,0}/2\pi$=1.656 MHz. The magnetic field and axial laser beam point up. The ion cloud dimensions are $2z_0 = 0.76$ mm and $2r_0 = 0.24$ mm, and the density $n_0 = 2.70 \times 10^9$ cm^{-3}. Comparison of the amplitude (b) and phase (c) extracted from the (2,0) mode in (a) with the predictions of linear theory. The theory predictions are on the right.

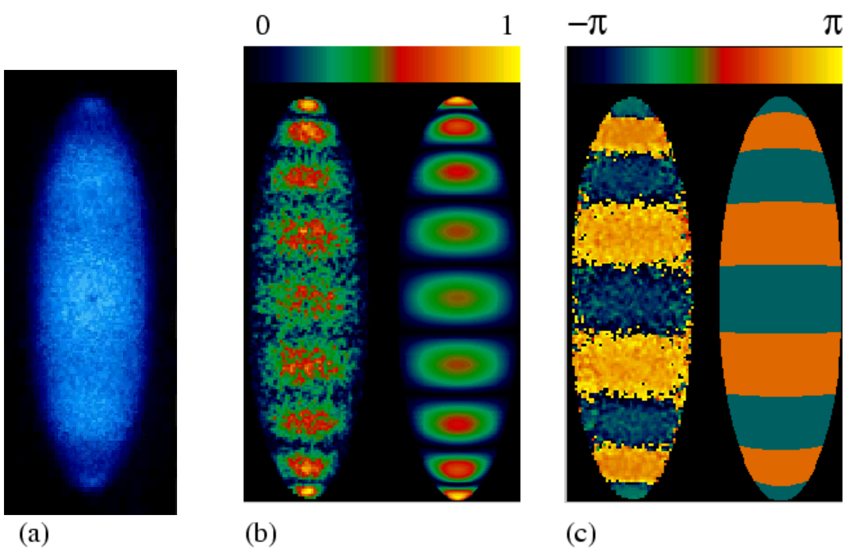


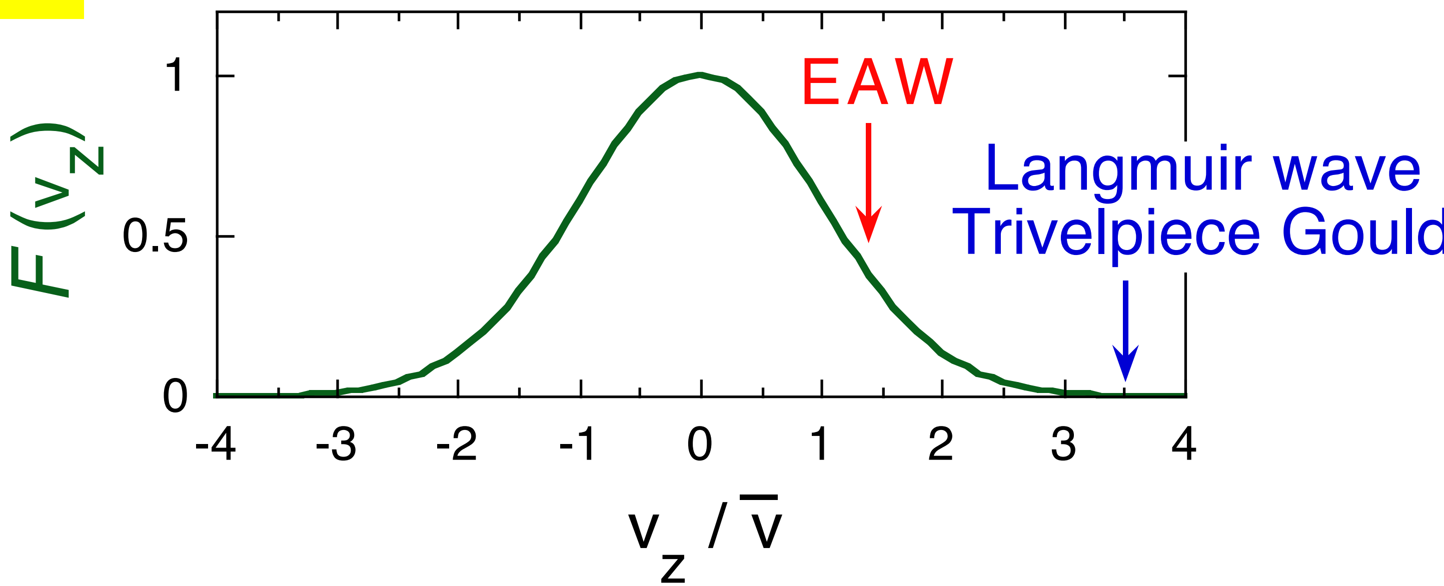
Figure 4. (a) Movie of sideview image data obtained on the plasma of Fig. 3 with $\omega_r/2\pi$= 1.00 MHz while driving a (9,0) mode at $\omega_{9,0}/2\pi$=2.952 MHz. Comparison of the amplitude (b) and phase (c) extracted from the (9,0) mode in (a) with the predictions of linear theory. The theory predictions are on the right.

Travis Mitchell, J. Bollinger, X.-P. Huang, and Wayne Itano, Opt. Express 2, 314 (1998)

Electron Acoustic Wave (EAW)

Electron **A**coustic **W**aves are plasma waves with a slow phase velocity

$$\omega \approx 1.4\, k\, \bar{v}$$



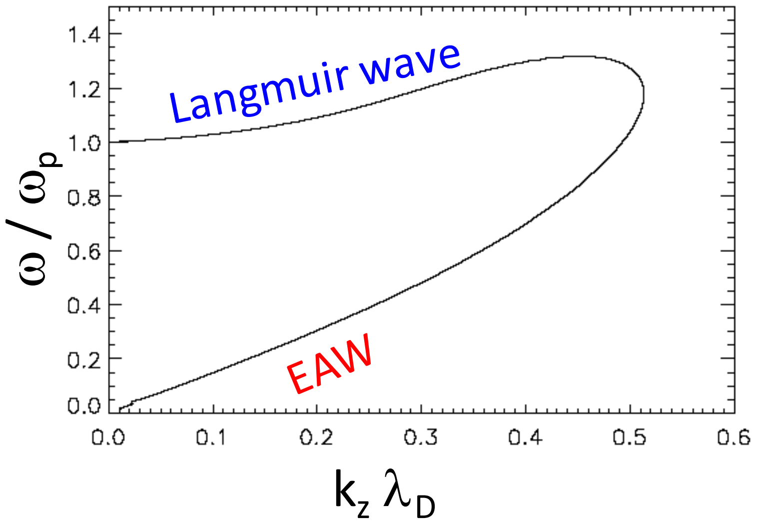
The EAW wave is nonlinear so as to flatten the particle distribution to avoid strong Landau damping.

J. P. Holloway and J. J. Dorning, Phys. Rev. A **44**, 3856 , 1991.

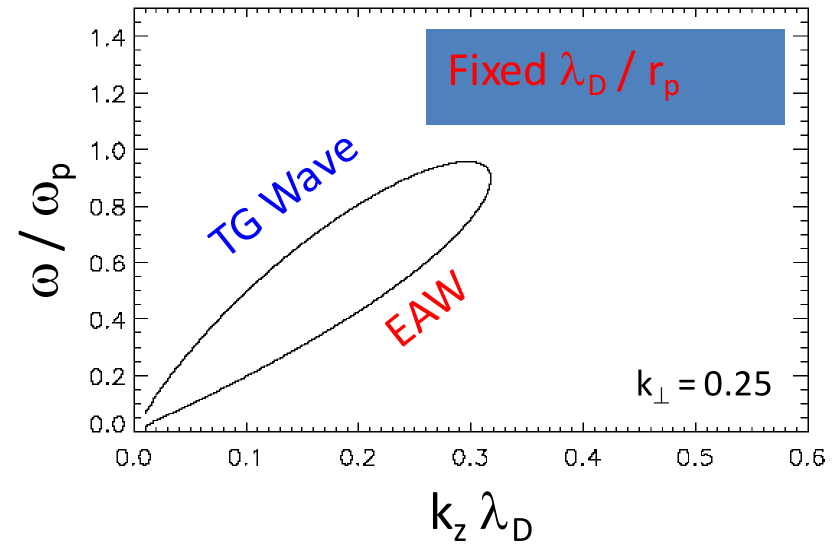
F. Valentini, T. M. O'Neil, and D. H. E. Dubin, Phys. Plasmas **13**, 052303, 2006

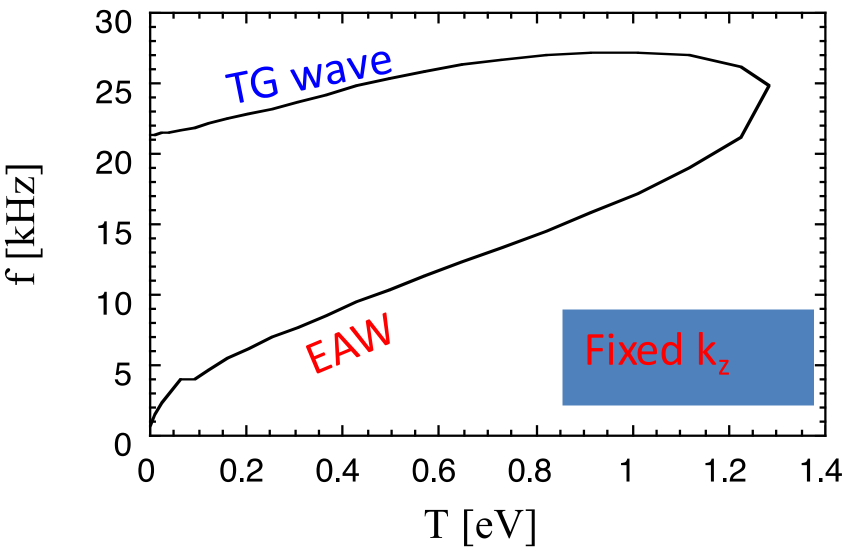
Dispersion Relation

Infinite size plasma
(homogenous)



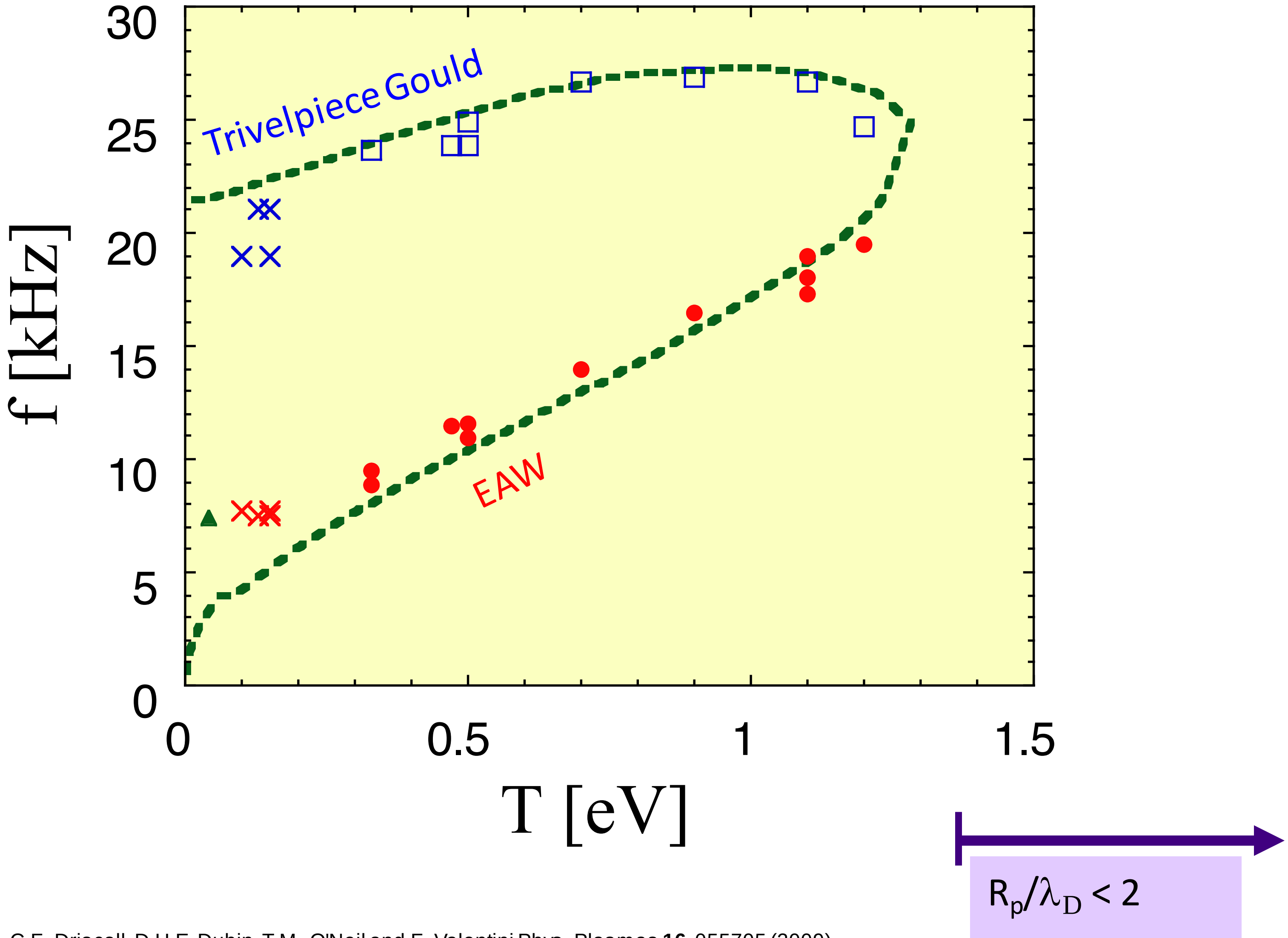
Trapped NNP
(long column finite radial size)





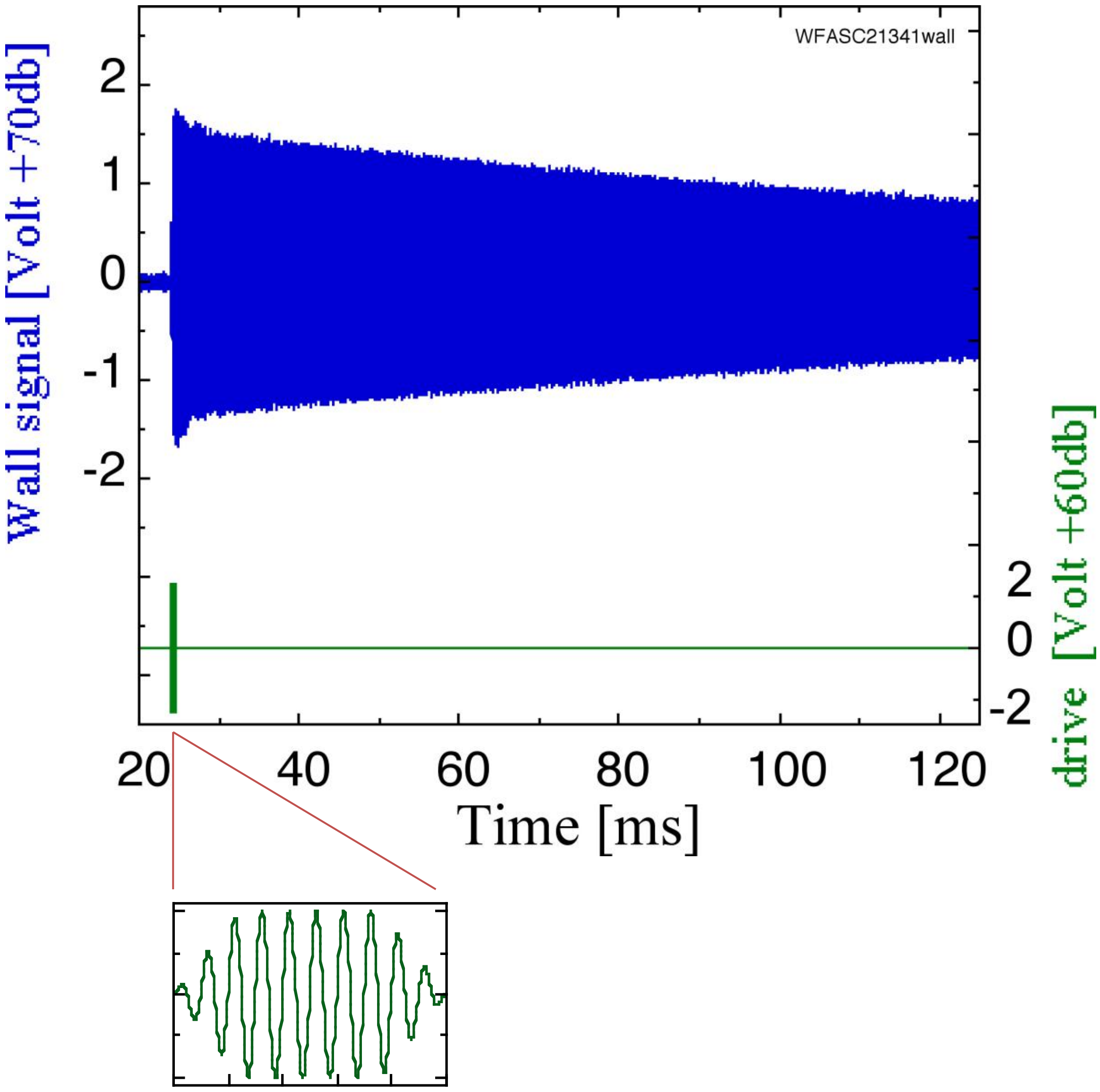
Experiment:
fixed $k_z = \pi / L_p$
$R_w = 6.4\ R_p$

Measured Wave Dispersion



F. Anderegg, C.F. Driscoll, D.H.E. Dubin, T.M. O'Neil and F. Valentini Phys. Plasmas **16**, 055705 (2009)

Received Wall Signal



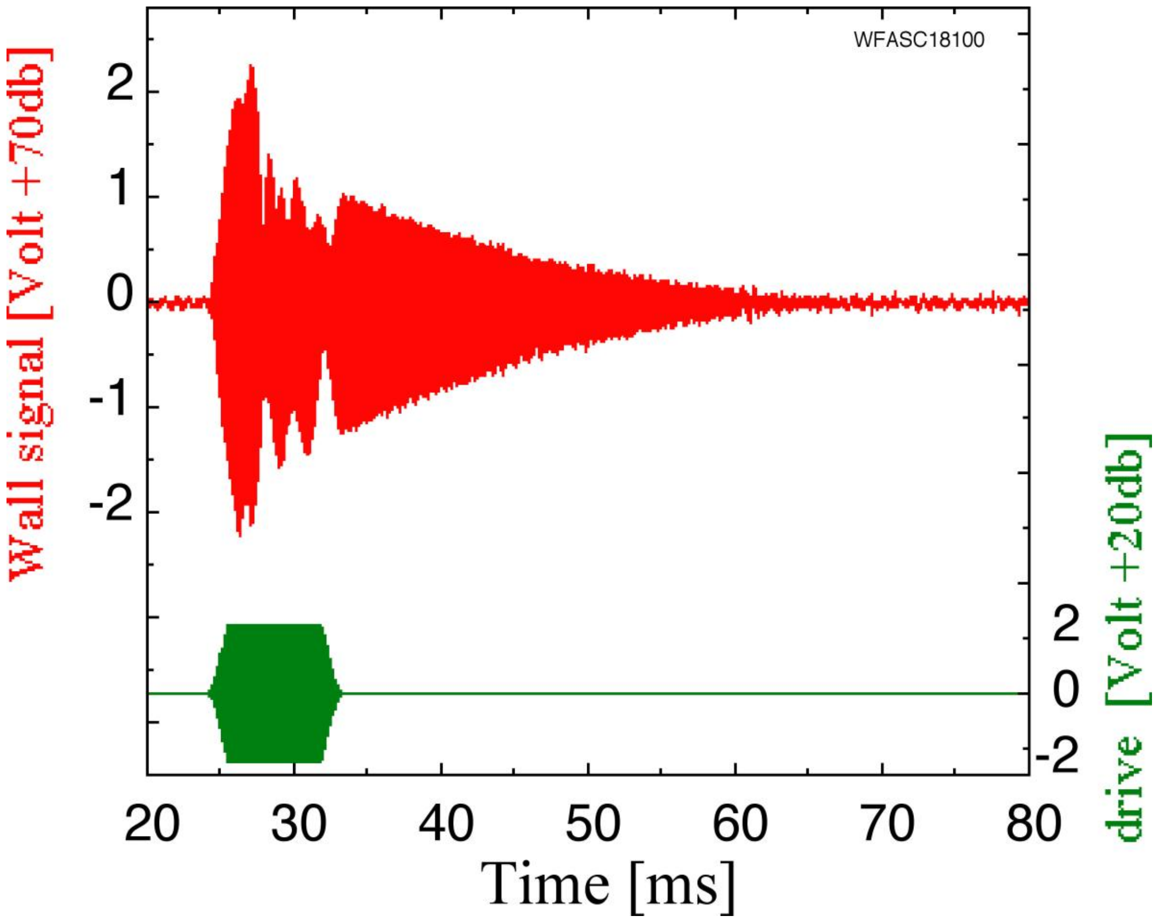
Trivelpiece Gould mode

The plasma response grows smoothly during the drive

10 cycles
21.5 kHz

Received Wall Signal

"*Electron*" Acoustic Wave



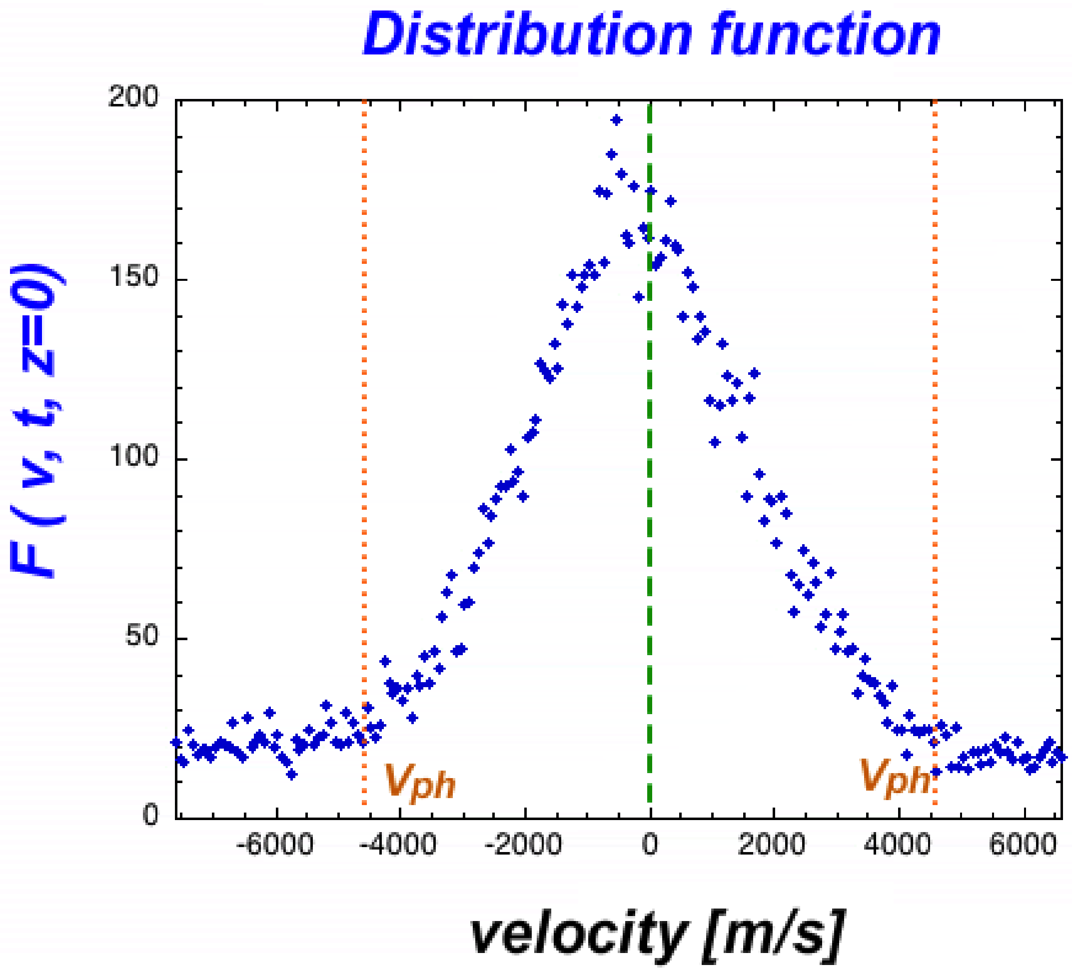
During the drive the plasma response is erratic.

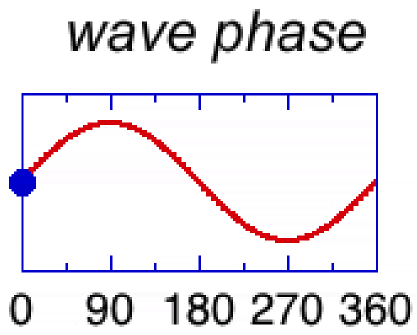
Plateau formation

100 cycles
10.7 kHz

Distribution Function versus Phase

Standing Trivelpiece Gould Mode





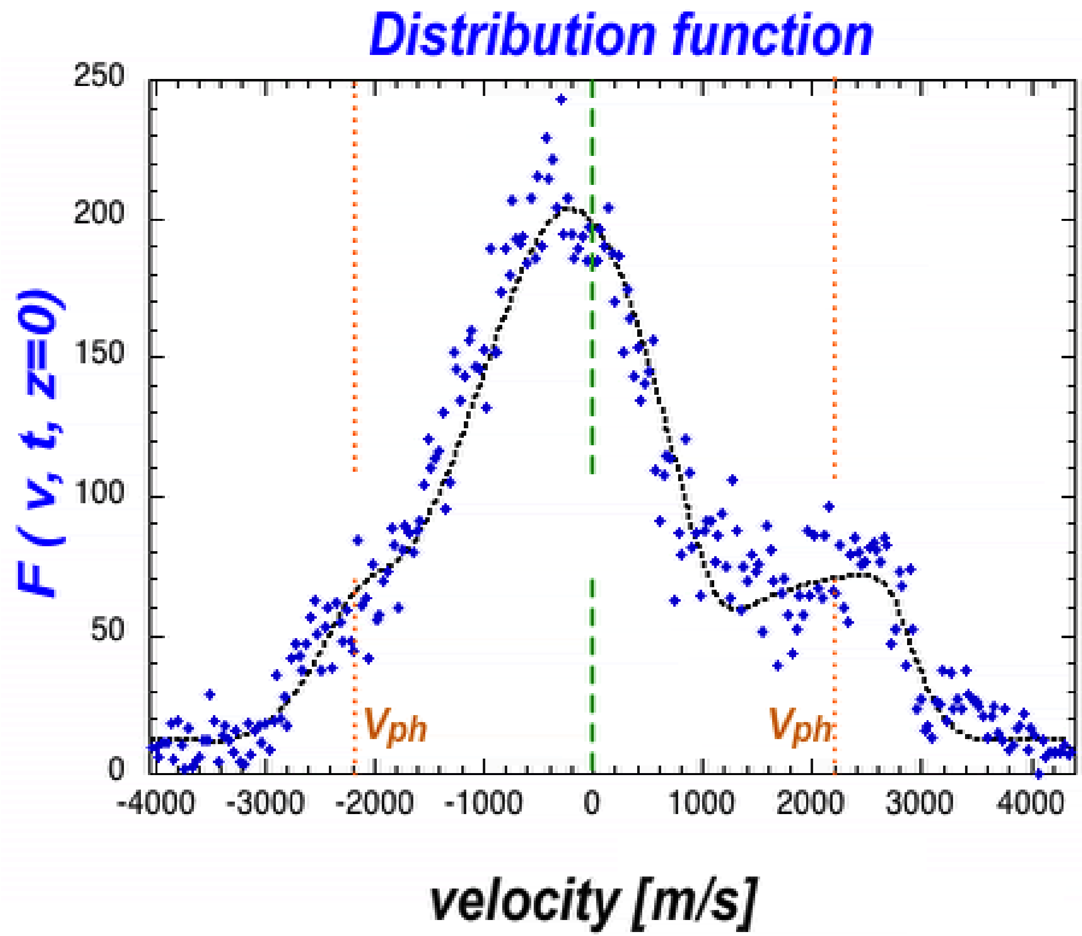
T = 0.77eV

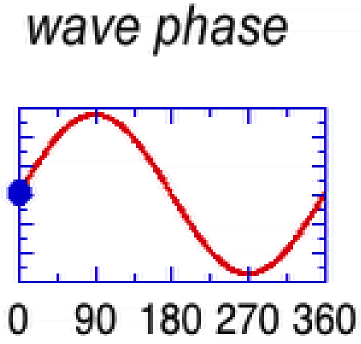
f = 21.5kHz

21341-21541

Distribution Function versus Phase

Standing "Electron" Acoustic Wave





T = 0.3eV
f = 10.7kHz

18055-18305

Cyclotron wave

For single particle the cyclotron frequency is $\boxed{\Omega_c = \frac{q\,B}{m}}$

In a plasma the cyclotron mode frequency is shifted from Ω_c

2 solutions exist to the ES dispersion relation:

slow rotation (**diocotron branch**)

$$\omega_d = \omega_E \left(\frac{R_p}{R_W}\right)^2$$

where the rotation frequency $\omega_E = \frac{qn}{2\varepsilon_0 B}$

fast rotation (**cyclotron branch**)

$$\boxed{\omega = \Omega_c + \omega_E \left\{ (m_\theta - 2) + \left[1 - \left(\frac{R_p}{R_W}\right)^{2m_\theta} \right] \right\}}$$

for m_θ=1 $\omega = \Omega_c - \overbrace{\omega_E \left(\frac{R_p}{R_W}\right)^2}^{\omega_{dio}}$

Theory works well for one species plasma

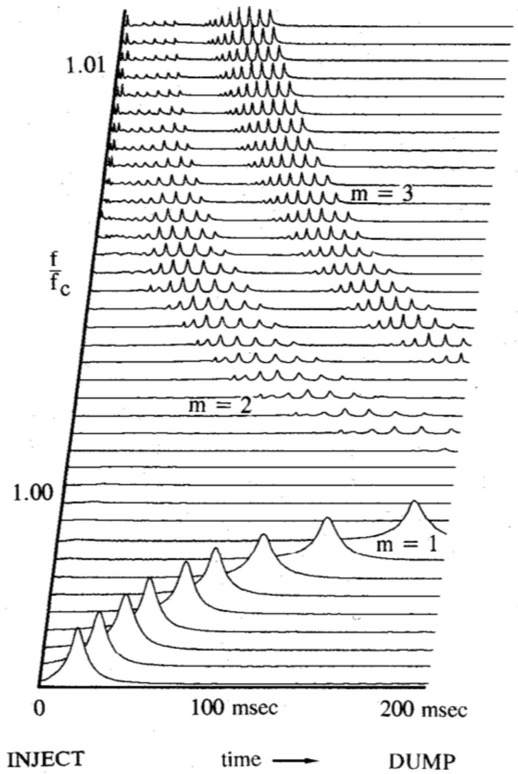
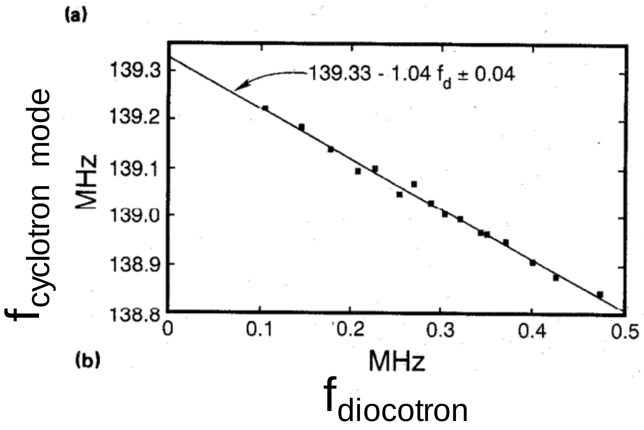


FIG. 4. Transmission between octupole sections showing normal modes of the plasma. The magnetic field is decremented on successive traces. For clarity every third trace is displayed. The $m=4$ resonances are too small to be seen in this display.



R.W. Gould and M.A. LaPointe, PRL. 67, 3685, (1991)
and Phys Fluids B, 4, 2038, (1992)

Useful tool for identification of impurities in ion plasma

Thermal cyclotron spectroscopy.

- **Heat resonantly impurity ions** at their cyclotron frequency
- **Hot impurity ions heat Mg_{24}** through collisions
- **Fluorescence of the Mg_{24}** cooling laser beam **increases**