Quantum information processing with trapped ions

- Introduction to Quantum Information Processing
- Trapped-ion QIP
  - Qubits, preparation + measurement
  - Coherent operations
  - Entangled states: creation + detection

Les Houches, January 18, 2012

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Computation and Physics

1960

Limitations and foundations

Are there physical limitations to the process of computation?

1970

Can computation be understood in terms of quantum mechanics?

Is quantum mechanics useful?

1980

Can quantum-physical computation be more efficient than models of computation based on classical physics?

1990

Quantum algorithms, quantum error correction

2000

Physical implementations of quantum computation?

Demonstration experiments + applications

Landauer, Bennett, Benioff, Feynman, Deutsch, Josza, Lloyd,…

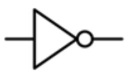
Classical vs. quantum information processing

Bit:

Physical system with two distinct states 0 or 1

Logic gates

Boolean logic operation



$0 \rightarrow 1$
$1 \rightarrow 0$

$(\epsilon_1, \epsilon_2) \rightarrow \epsilon_1 \oplus \epsilon_2$

XOR truth table

$$(0,0) \longrightarrow 0$$
$$(0,1) \longrightarrow 1$$
$$(1,0) \longrightarrow 1$$
$$(1,1) \longrightarrow 0$$

Quantum bit:

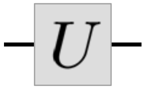
Two-level quantum system with state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Quantum logic gate

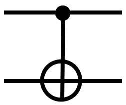
Unitary transformation

single qubit gate



$|\psi\rangle \rightarrow U|\psi\rangle$

two-qubit gate



$|\epsilon_1\rangle|\epsilon_2\rangle \rightarrow |\epsilon_1\rangle|\epsilon_1 \oplus \epsilon_2\rangle$

CNOT truth table

$$|0\rangle|0\rangle \longrightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \longrightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \longrightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \longrightarrow |1\rangle|0\rangle$$

Classical vs. quantum information processing

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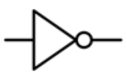

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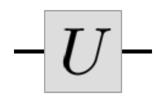

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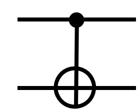
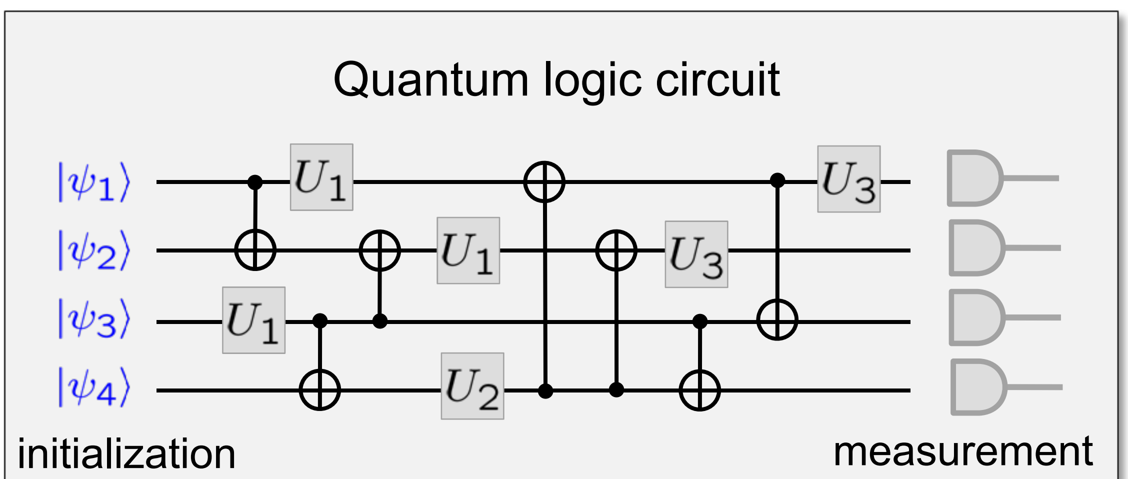
two-qubit gate



$|\epsilon_1\rangle|\epsilon_2\rangle \rightarrow |\epsilon_1\rangle|\epsilon_1 \oplus \epsilon_2\rangle$

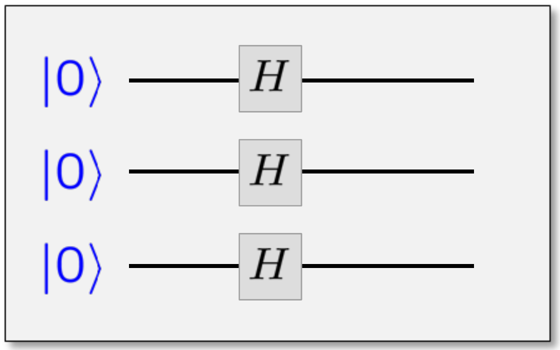


Superpositions and entanglement

Putting a quantum register in a superposition

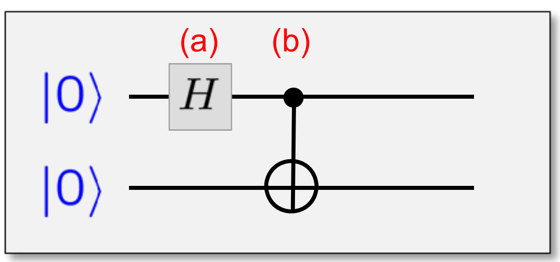
$$|\psi\rangle = |0\rangle|0\rangle|0\rangle \longrightarrow (|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$= |000\rangle + |001\rangle + |010\rangle + |011\rangle$$
$$+|100\rangle + |101\rangle + |110\rangle + |111\rangle$$



If applied at the start of a quantum algorithm, this operation enables parallel processing on all possible input states.

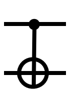
Entangling quantum bits in a quantum register



$$|\psi\rangle = |0\rangle|0\rangle \xrightarrow{(a)} (|0\rangle + |1\rangle)|0\rangle$$
$$\xrightarrow{(b)} |00\rangle + |11\rangle$$

The computation creates quantum correlations between the qubits.

controlled-NOT gate

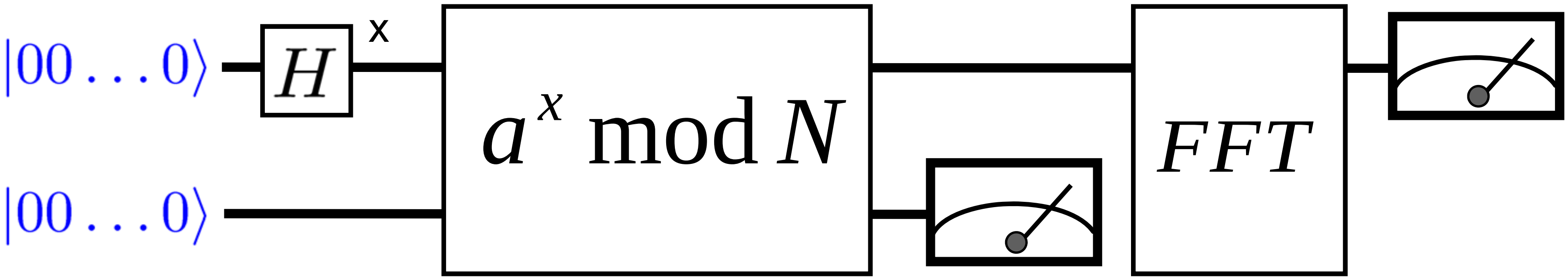


$= |\epsilon_1\rangle|\epsilon_2\rangle \rightarrow |\epsilon_1\rangle|\epsilon_1 \oplus \epsilon_2\rangle$

Example: Quantum factoring algorithm

Peter Shor: A quantum computer can efficiently find prime factors of large numbers.

Theoretical Algorithm (Shor, 1994)



Three main steps:

1. Input superposition preparation
2. Modular exponentiation (multi-qubit gates required)
3. Quantum Fourier Transform
4. Classical pre- and post-processing

Quantum information processing

Quantum computing

Quantum algorithms for
efficient computing

Quantum simulation

Investigating many-body
Hamiltonians
using well-controlled
quantum systems

→ tomorrow‘s lecture

Quantum communication

Secure communication
certified by quantum physics

Quantum metrology

Entanglement-enhanced
measurements

→ Piet Schmidt‘s lecture

Quantum foundations

Quantum theory and its
interpretations; exp. tests

Trapped ions for quantum information processing

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Quantum Computations with Cold Trapped Ions

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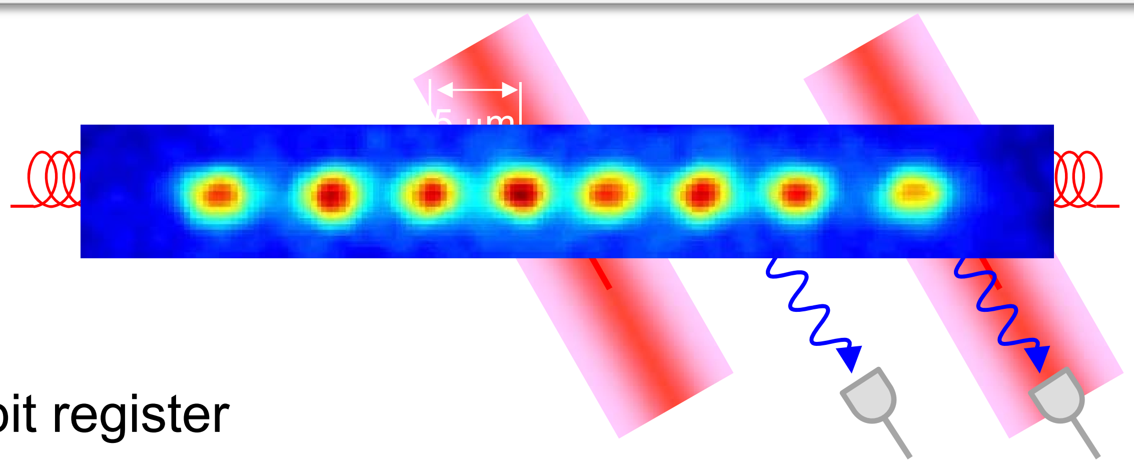
(Received 30 November 1994)

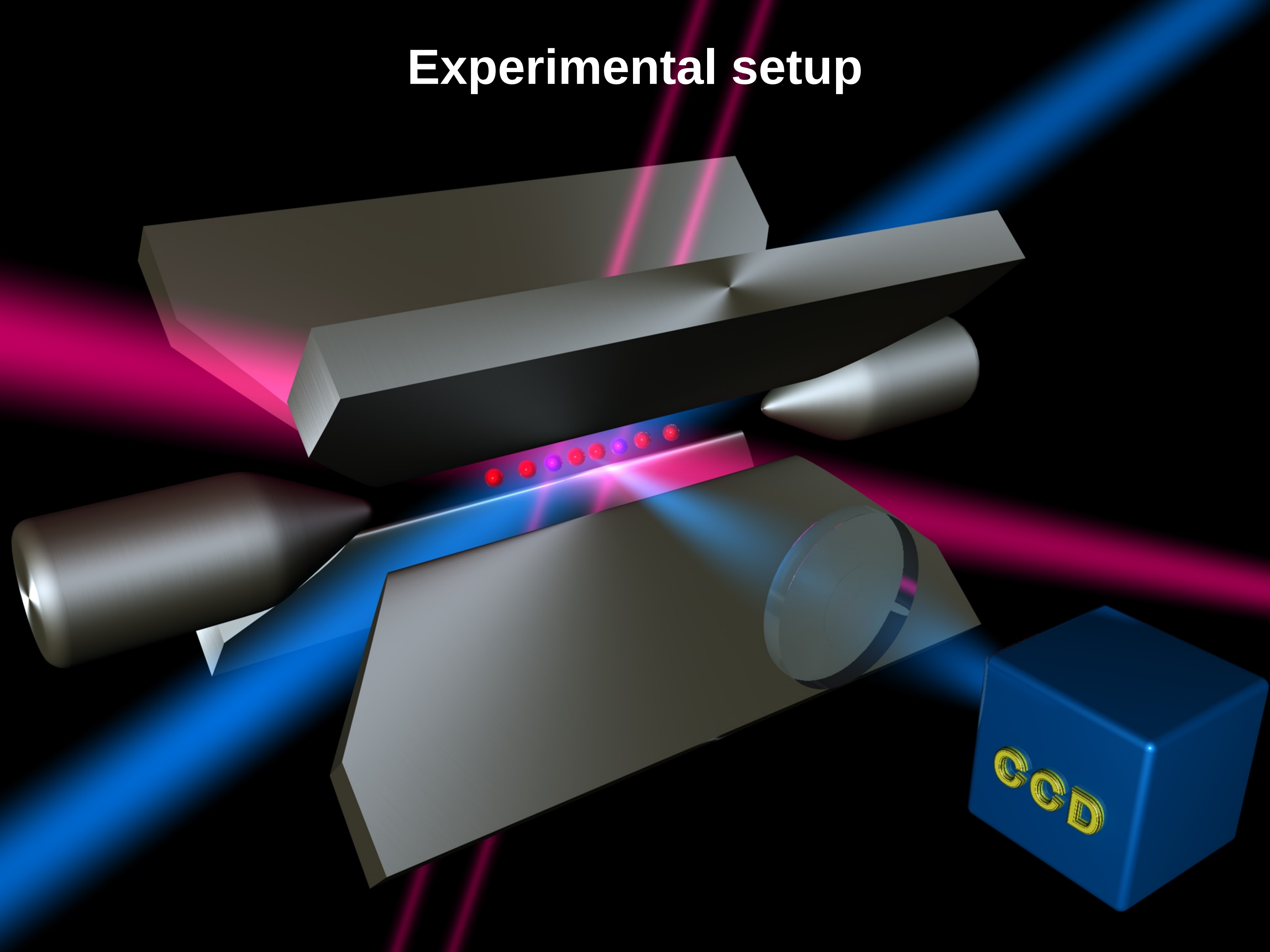
A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

Ion string



- Qubit register
- State detection
- Single qubit gates
- Entangling gates

Experimental setup



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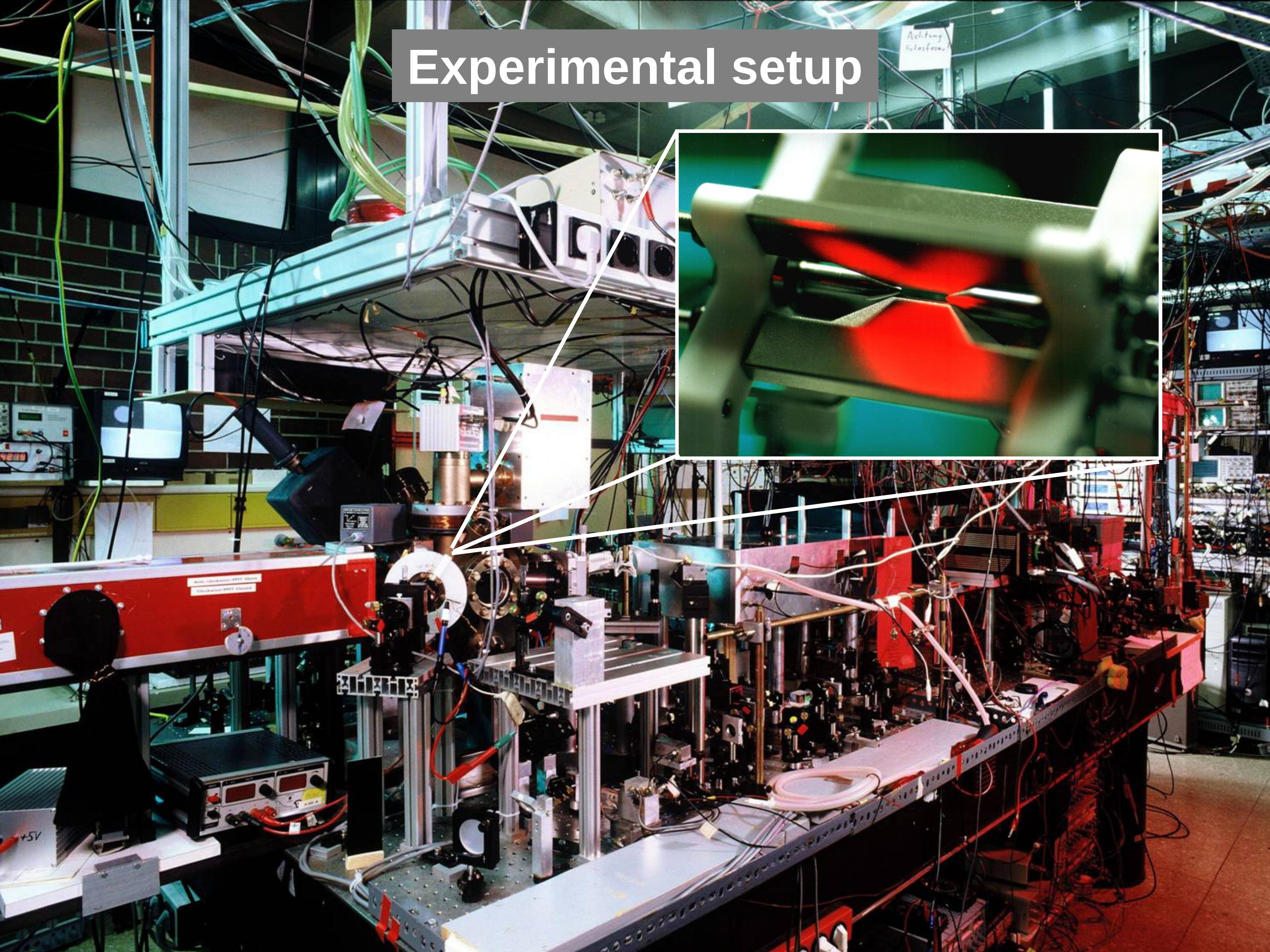
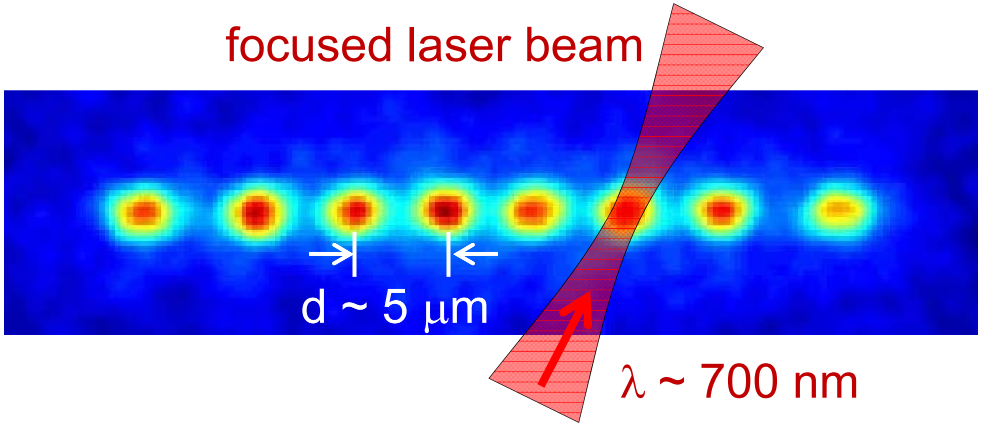


Quantum physics with linear ion strings

Trap frequencies:

$$\nu_z \quad \propto 1 \text{ MHz}$$
$$\nu_{x,y} \propto 5 \text{ MHz}$$



Length scales

ion distance		laser wavelength		ion localisation		Bohr radius
d	$>$	λ	$>>$	z_0	$>>$	a_0
5 μm		700 nm		10 nm		50 pm

- individual addressing, spatially resolved fluorescence

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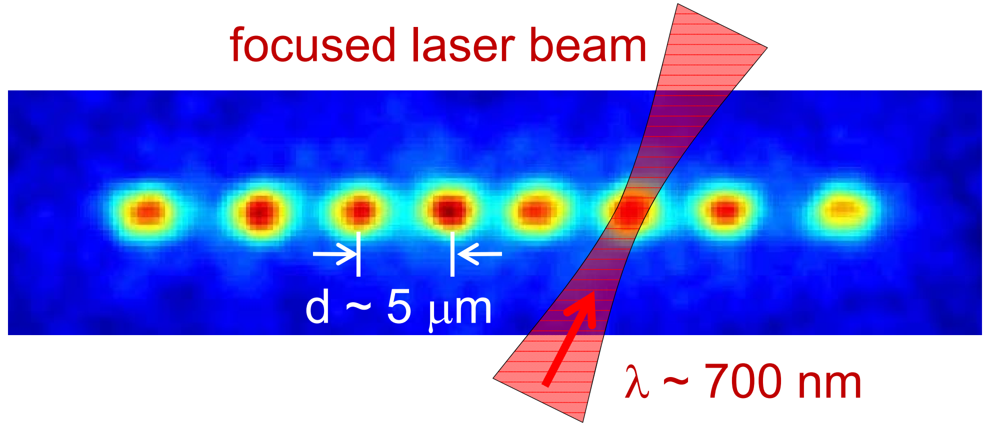

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- coupling internal and motional states by laser takes on simple form

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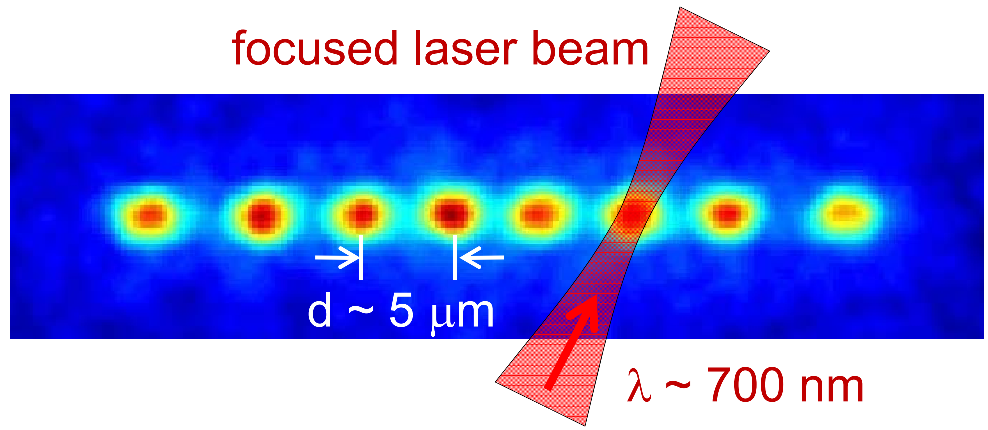

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- individual addressing, spatially resolved fluorescence
- coupling internal and motional states by laser takes on simple form
- no direct state-dependent interactions between ions

Quantum physics with linear ion strings

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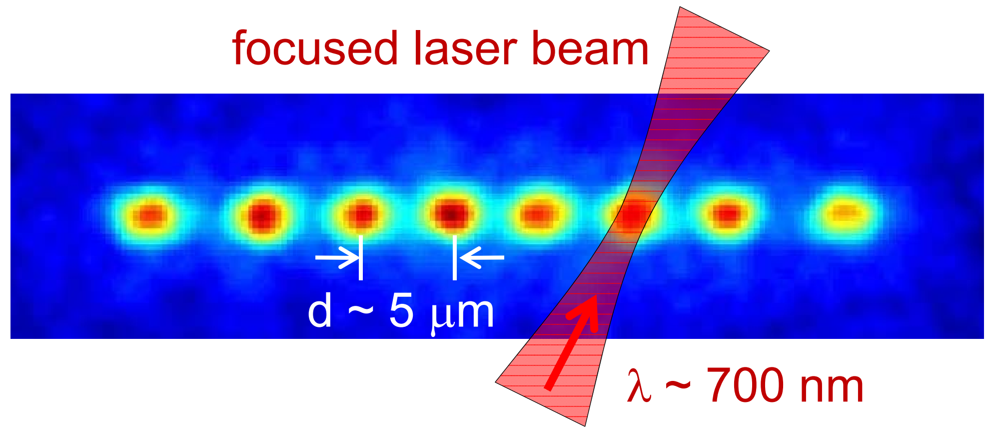

Length scales

ion distance		laser wavelength		ion localisation		Bohr radius
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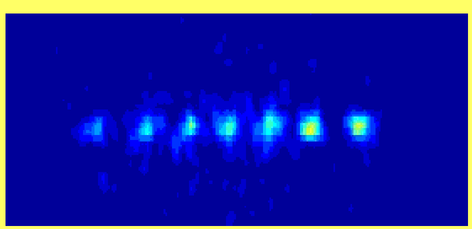
Vibrational modes

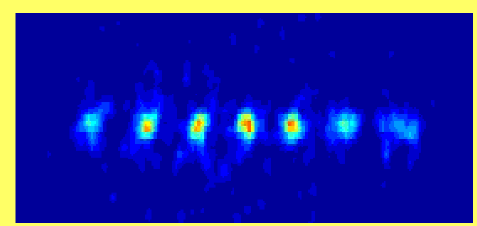
centre-of-mass mode

$$\nu = \nu_z$$



breathing mode

$$\nu = \sqrt{3}\,\nu_z$$



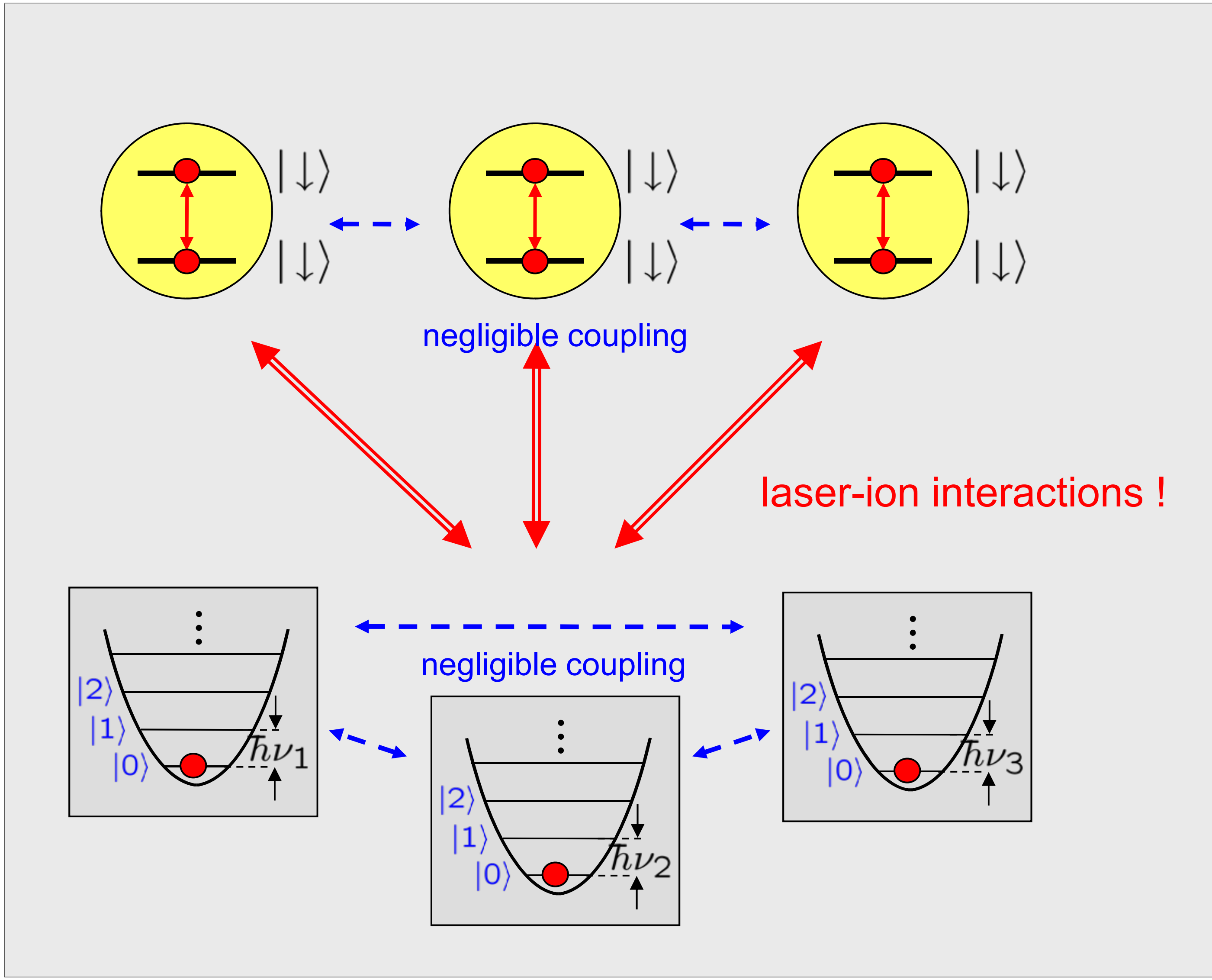
Trapped ions as a quantum system

Internal degrees of freedoms:

Quantum bits

Motional degrees of freedoms:

Harmonic oscillators



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Internal degrees of freedoms:

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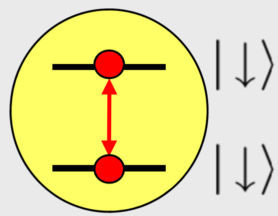


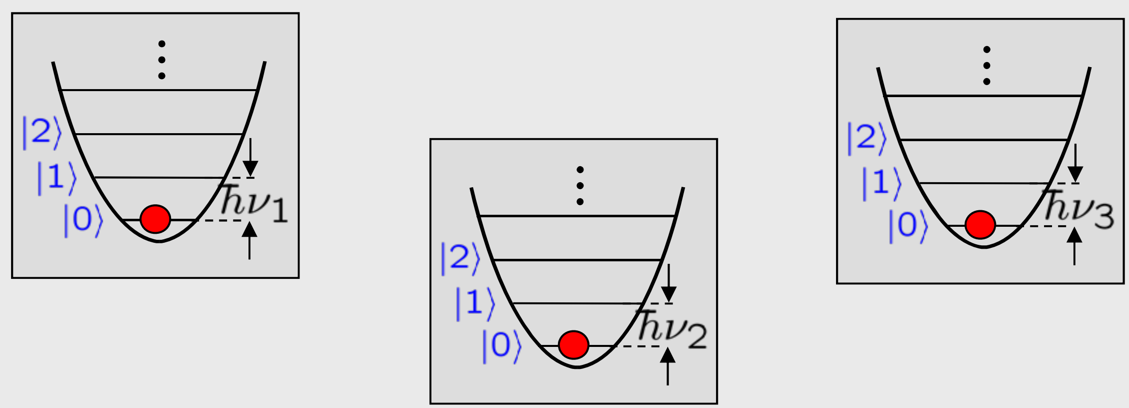
ground state ≈ 10 nm << optical wavelengths

→ Measurements of motional quantum states via coupling to internal states

Entanglement between spin and motion

Motional degrees of freedoms:

Harmonic oscillators



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Internal degrees of freedoms:

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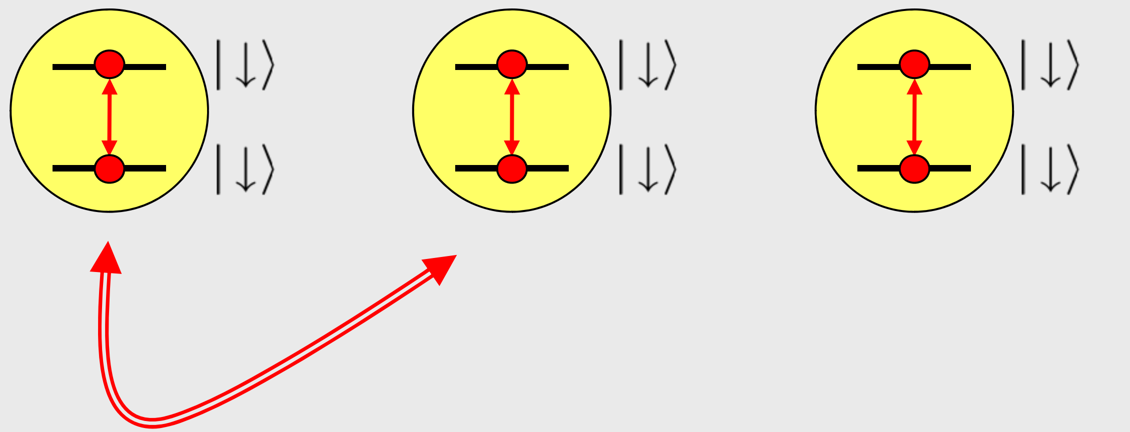
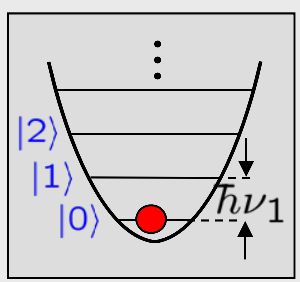


Motional degrees of freedoms:

Harmonic oscillators

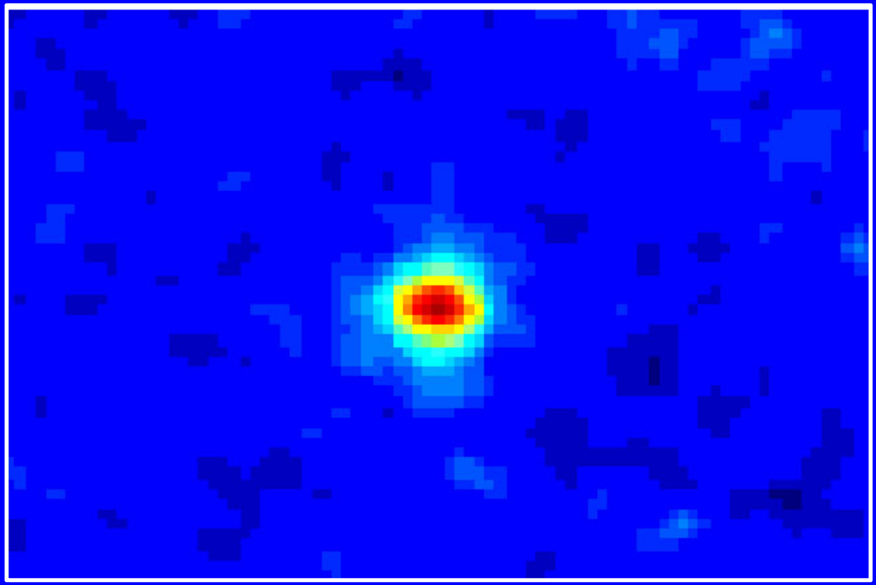


Entanglement between qubits:

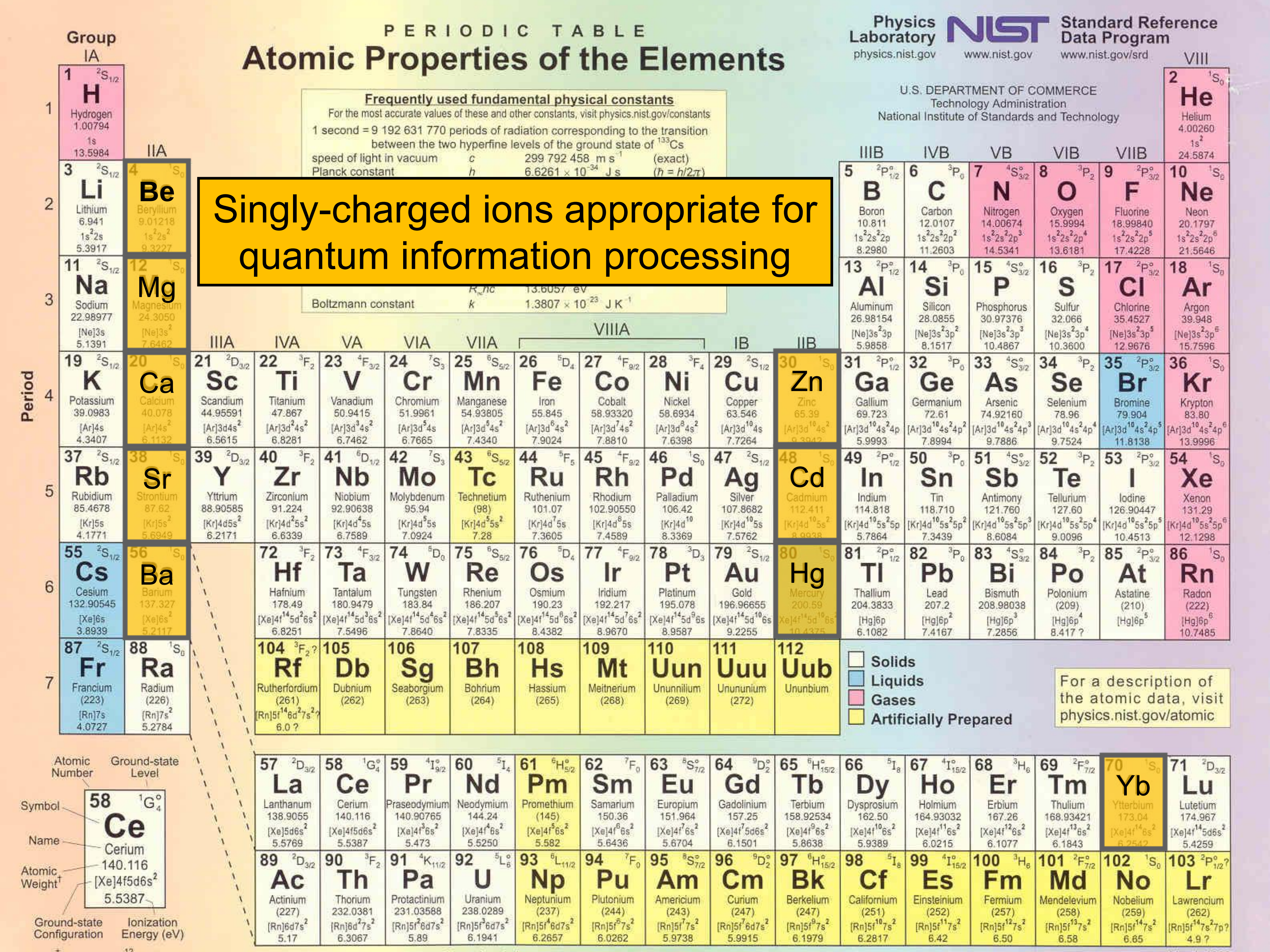
Interactions mediated by coupling to vibrational modes

Trapped-ion quantum bits

Encoding, manipulation and measurement

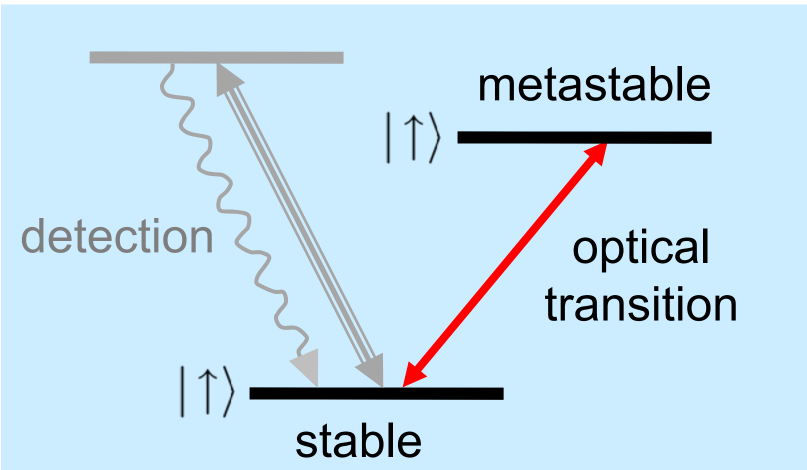


Singly-charged ions appropriate for quantum information processing



Trapped ion quantum bits

Ions with optical transition to metastable level: $^{40}Ca^{+}$, $^{88}Sr^{+}$, $^{172}Yb^{+}$

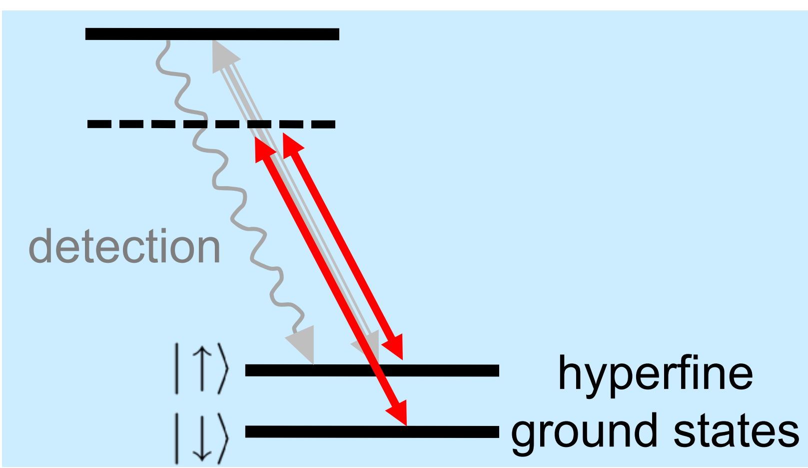


„optical qubit“

qubit manipulation requires ultrastable laser

$$\Psi = \alpha|\downarrow\rangle + \beta|\uparrow\rangle$$

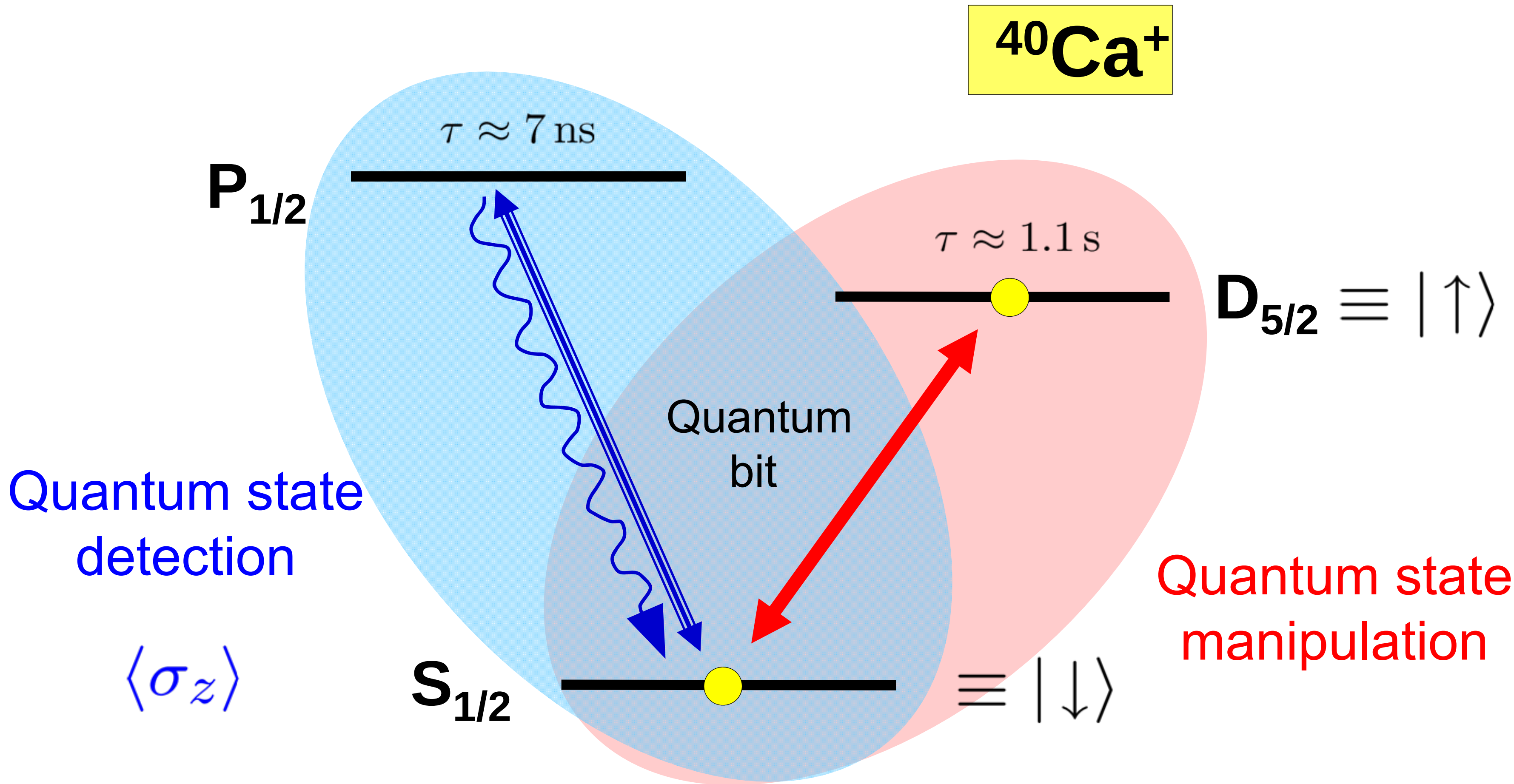
Ions with hyperfine structure: $^{9}Be^{+}$, $^{25}Mg^{+}$, $^{43}Ca^{+}$, $^{111}Cd^{+}$, $^{171}Yb^{+}$…



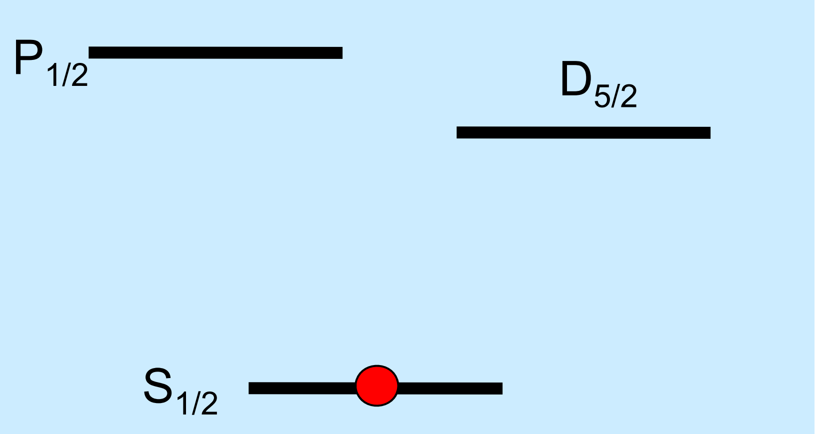
„hyperfine qubit“

qubit manipulation with microwaves or lasers (Raman transitions)

Qubit manipulation and measurement

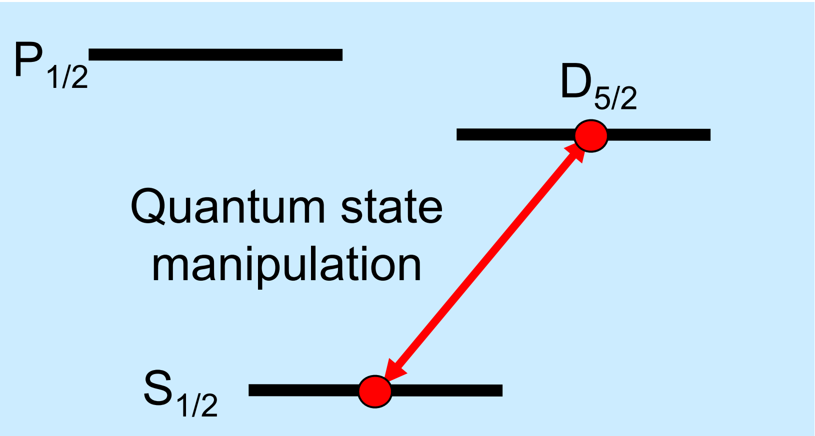


Experimental sequence



1. Initialization in a pure quantum state

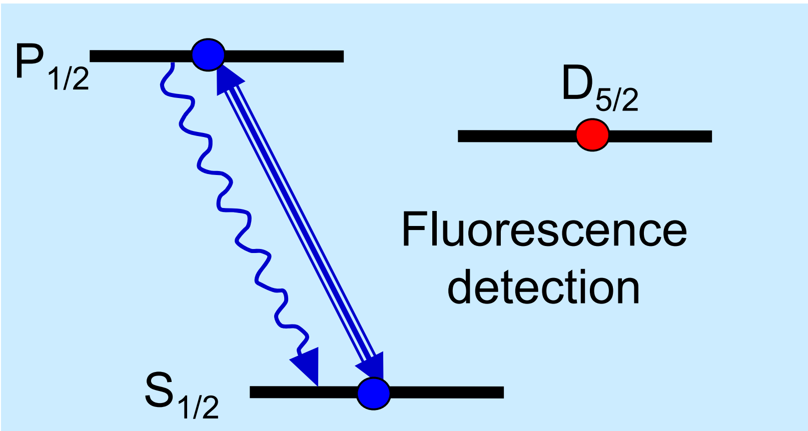
Experimental sequence



1. Initialization in a pure quantum state

2. Quantum state manipulation on $S_{1/2} – D_{5/2}$ transition

Experimental sequence



1. Initialization in a pure quantum state
   1-10 ms

2. Quantum state manipulation on $S_{1/2} – D_{5/2}$ transition
   0.01- 5 ms

3. Quantum state measurement by fluorescence detection
   0.2- 5 ms

Two ions:

Spatially resolved detection with CCD camera:

$|SS\rangle$

$|SD\rangle$

$|DS\rangle$

$|DD\rangle$

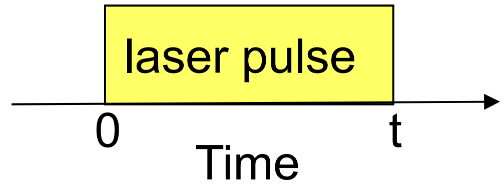
50 experiments / s

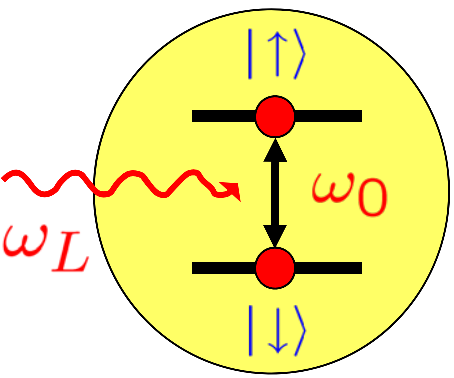
Repeat experiments 100 - 1000 times

2- 20 s

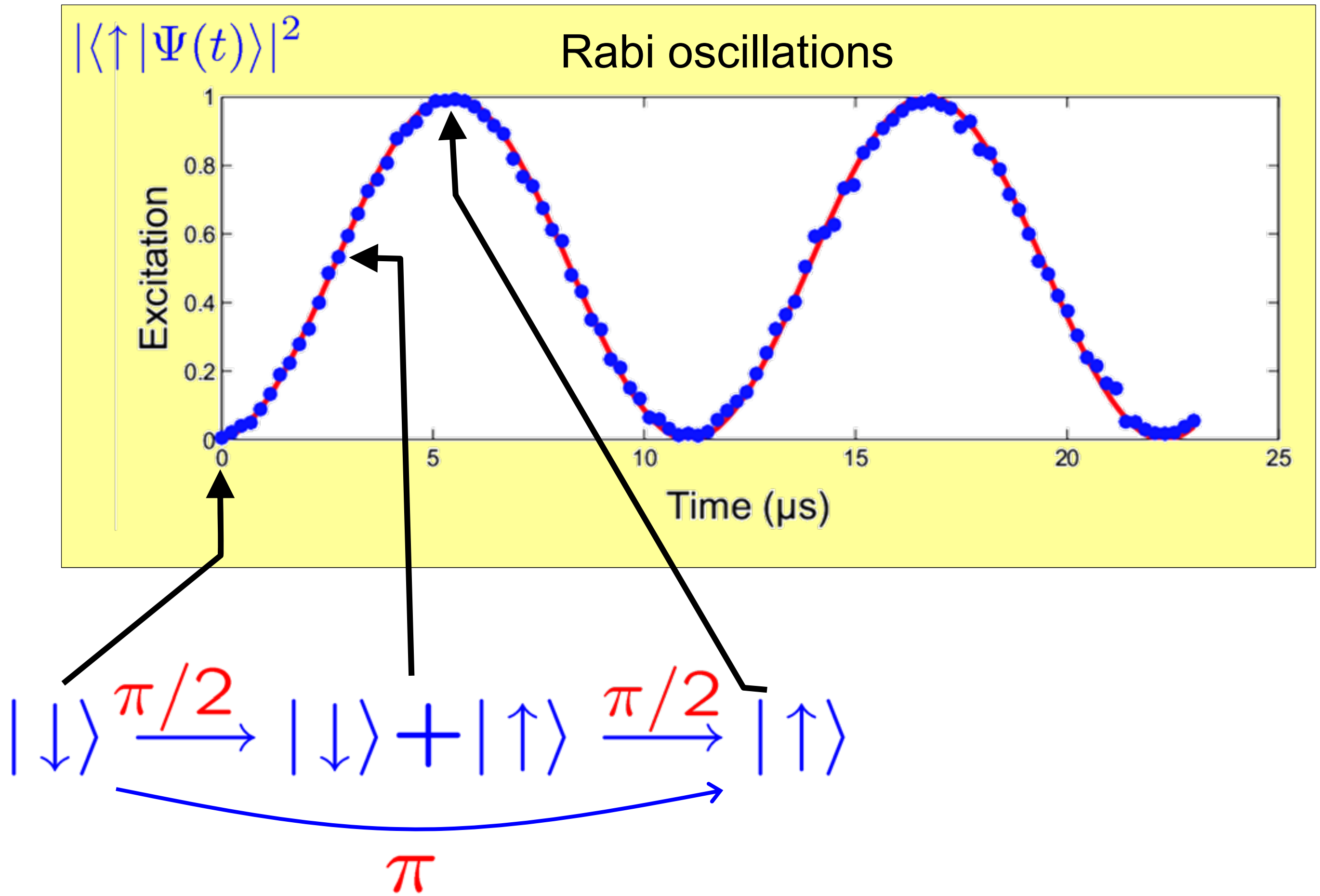
Ion-laser interaction

Resonant coherent excitation:





$$\omega_L = \omega_0$$



Qubit superposition states

Schrödinger picture:

$$|\psi(t=0)\rangle \propto |\downarrow\rangle + |\uparrow\rangle \quad \longrightarrow \quad |\psi(t)\rangle \propto |\downarrow\rangle + e^{-i\omega_0 t}|\uparrow\rangle$$

Phase evolution: for optical qubits $\omega_0 \sim 10^{15}\ \mathrm{s}^{-1}$

Interaction picture:

$$|\psi(t)\rangle \propto |\downarrow\rangle + |\uparrow\rangle$$ independent of time

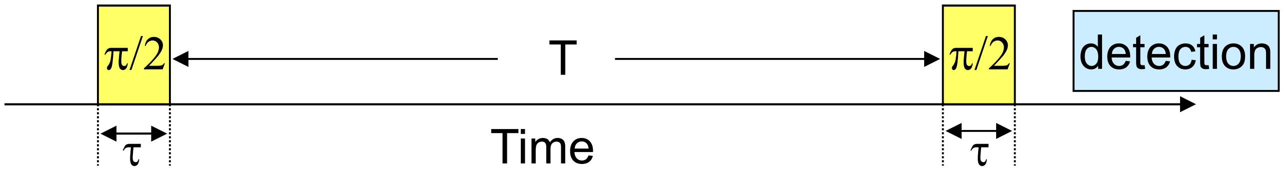
$$|\psi(t)\rangle = \cos(\theta/2)|\downarrow\rangle + e^{i\phi}\sin(\theta/2)|\uparrow\rangle$$

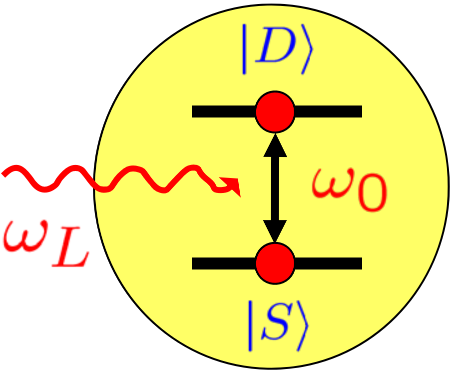
The phase ϕ of the superposition compares two oscillatory phenomena:

- Evolution of the Bloch vector in time
- Evolution of the electromagnetic field of the laser exciting the qubit

Ramsey spectroscopy for phase estimation

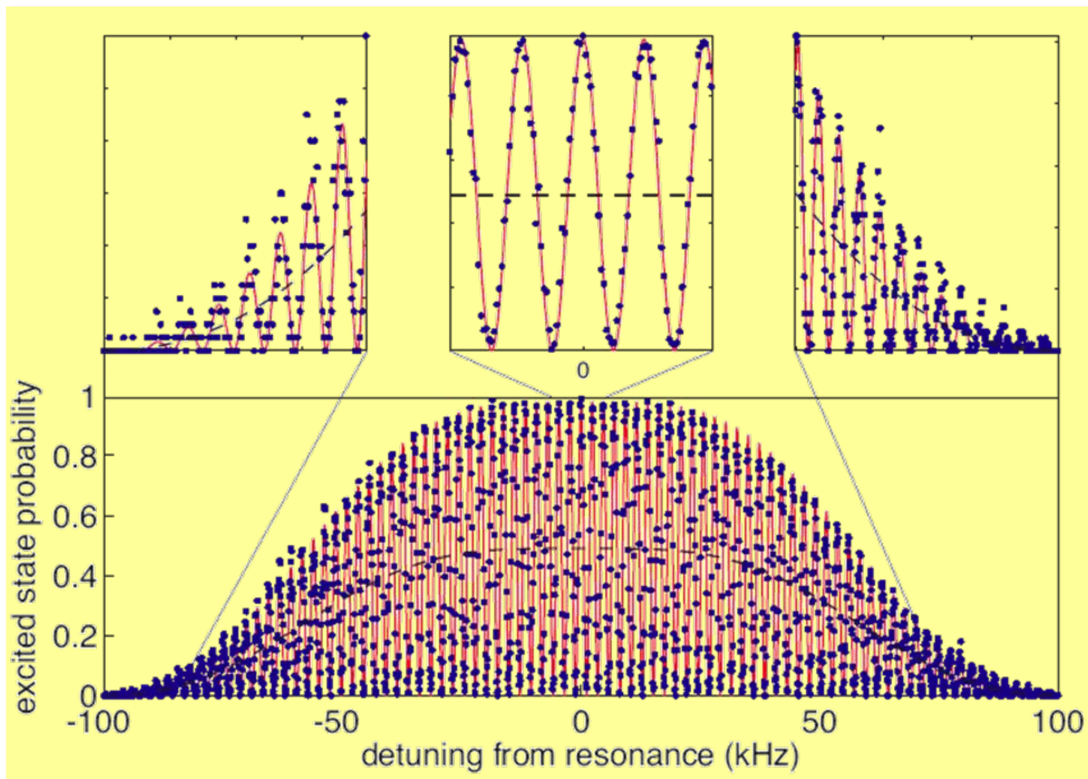
Two-pulse excitation:





$$|S\rangle \xrightarrow{\pi/2} |S\rangle + |D\rangle \xrightarrow{\text{wait}} |S\rangle + e^{-i\Delta T}|D\rangle \begin{cases} \xrightarrow{\pi/2} |D\rangle \\ \xrightarrow{\pi/2} |S\rangle \end{cases}$$

$$\Delta = \omega_L - \omega_0 \approx 0$$

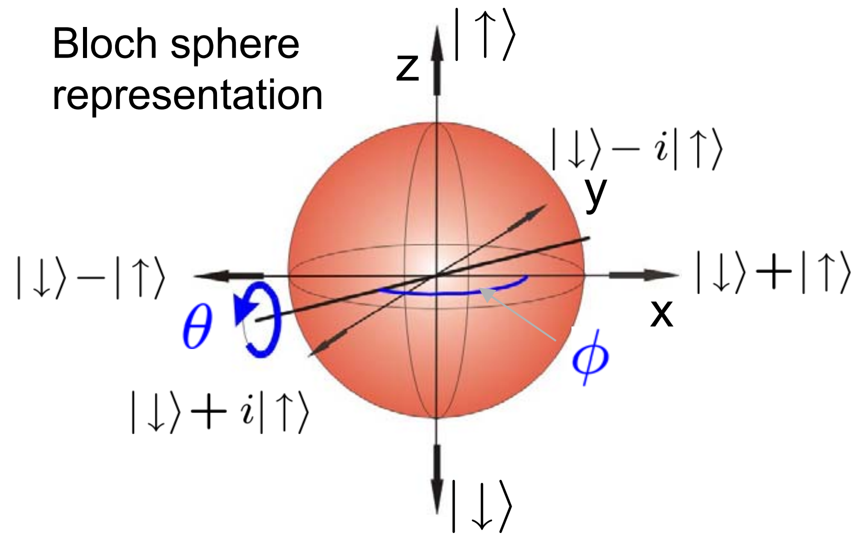


Resonant excitation in Bloch sphere picture

$$H = \hbar \frac{\Omega}{2} (\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi})$$

~ Laser intensity

Laser phase



Example: $\phi = 0 \longrightarrow H = \hbar \frac{\Omega}{2} \sigma_x$

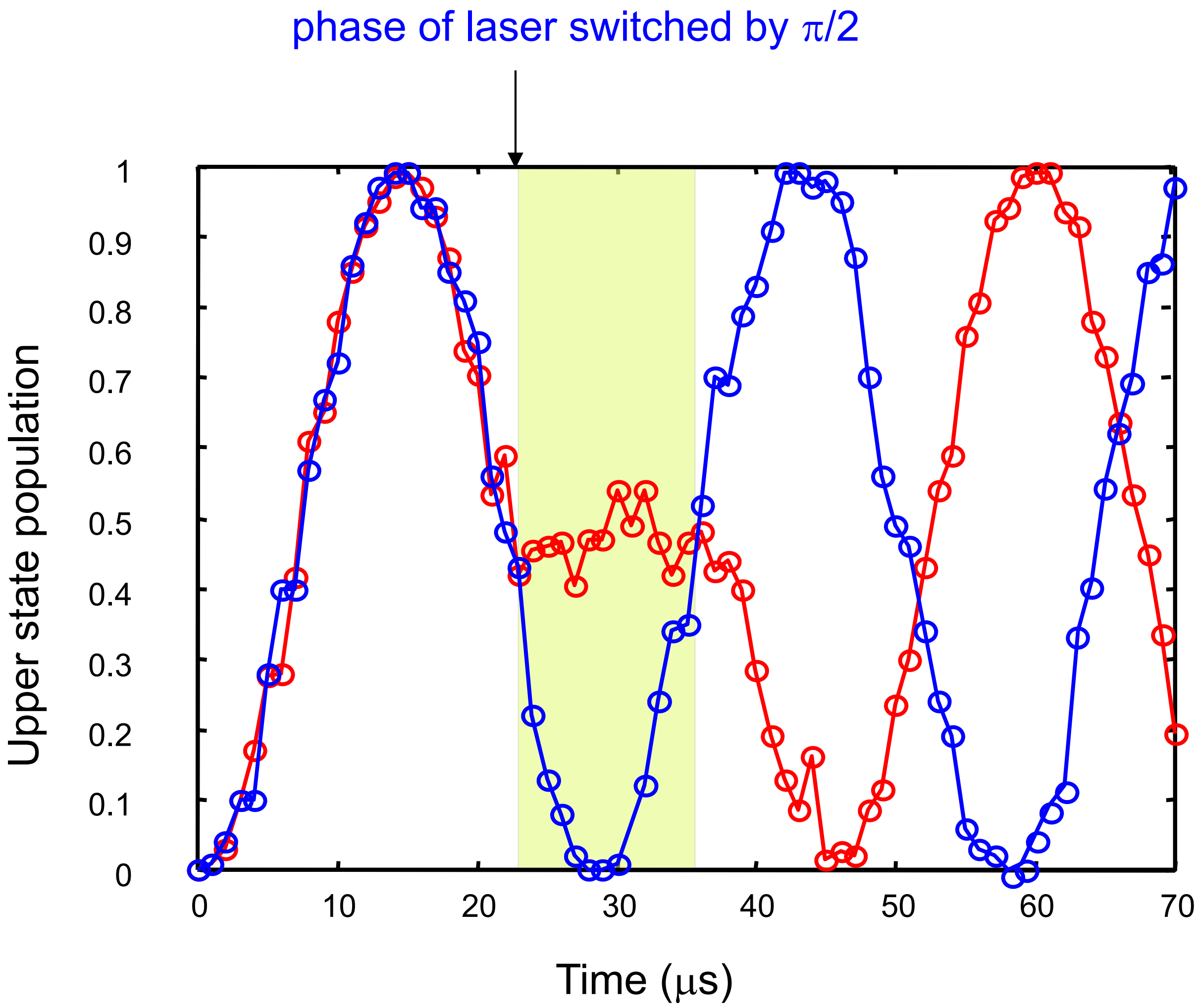
Time evolution operator:

$$U = \exp(-\frac{i}{\hbar} H t) = \exp(-i \frac{\Omega t}{2} \sigma_x) = \cos(\frac{\Omega t}{2}) - i \sin(\frac{\Omega t}{2}) \sigma_x$$

For $\theta = \Omega t = \pi/2$

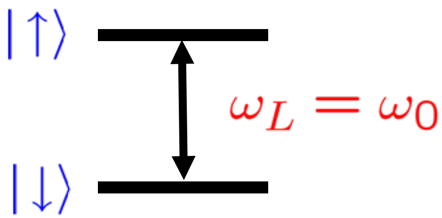
$$U|\downarrow\rangle = \frac{1}{\sqrt{2}} (I - i\sigma_x)|\downarrow\rangle = \frac{1}{\sqrt{2}} (|\downarrow\rangle - i|\uparrow\rangle)$$

Resonant qubit excitation



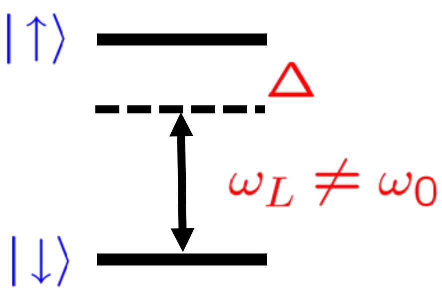
Qubit manipulation

Resonant excitation



$H \propto \sigma_x$ or $H \propto \sigma_y$

Off-resonant excitation

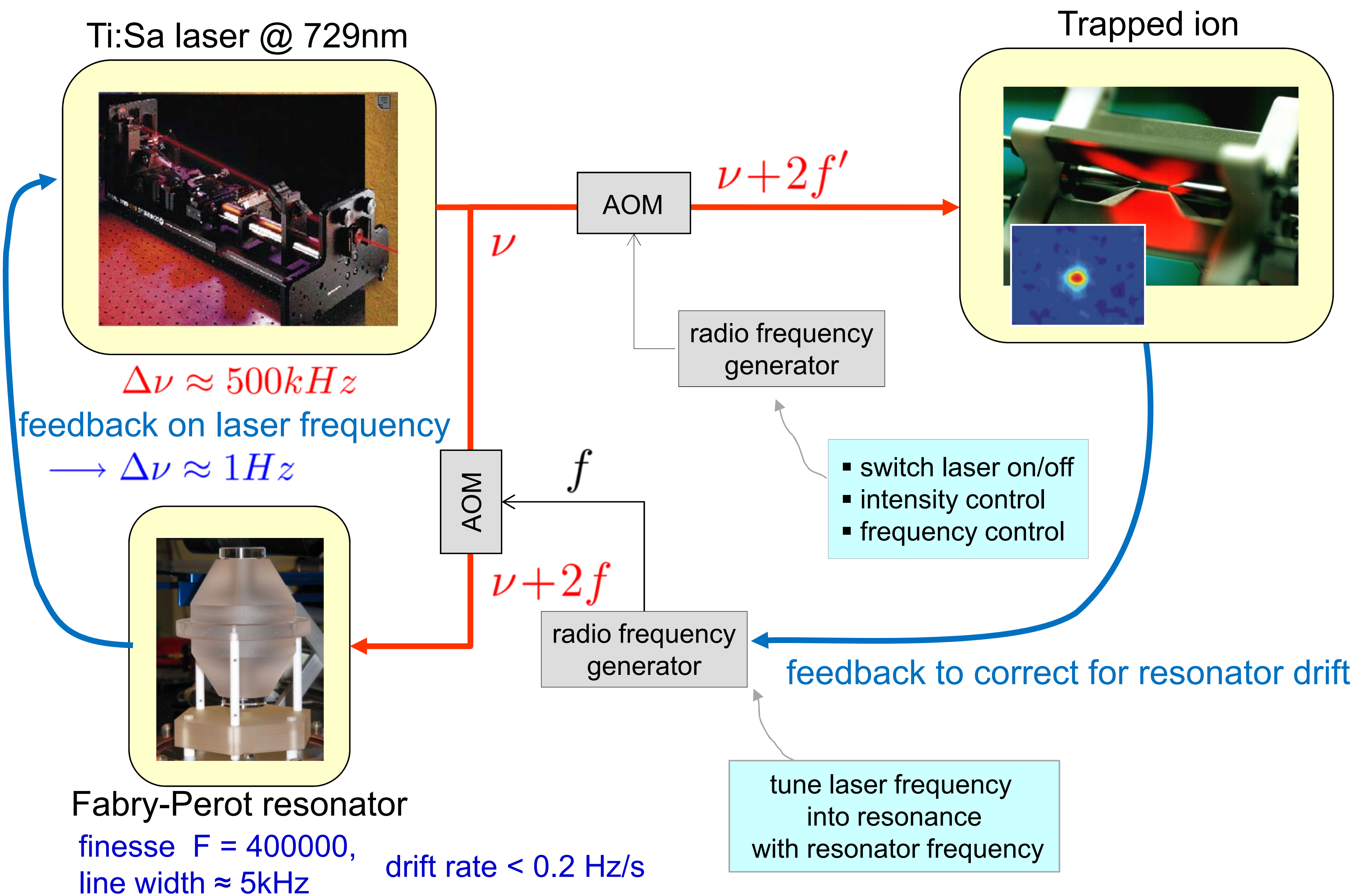


$H \propto \sigma_z$

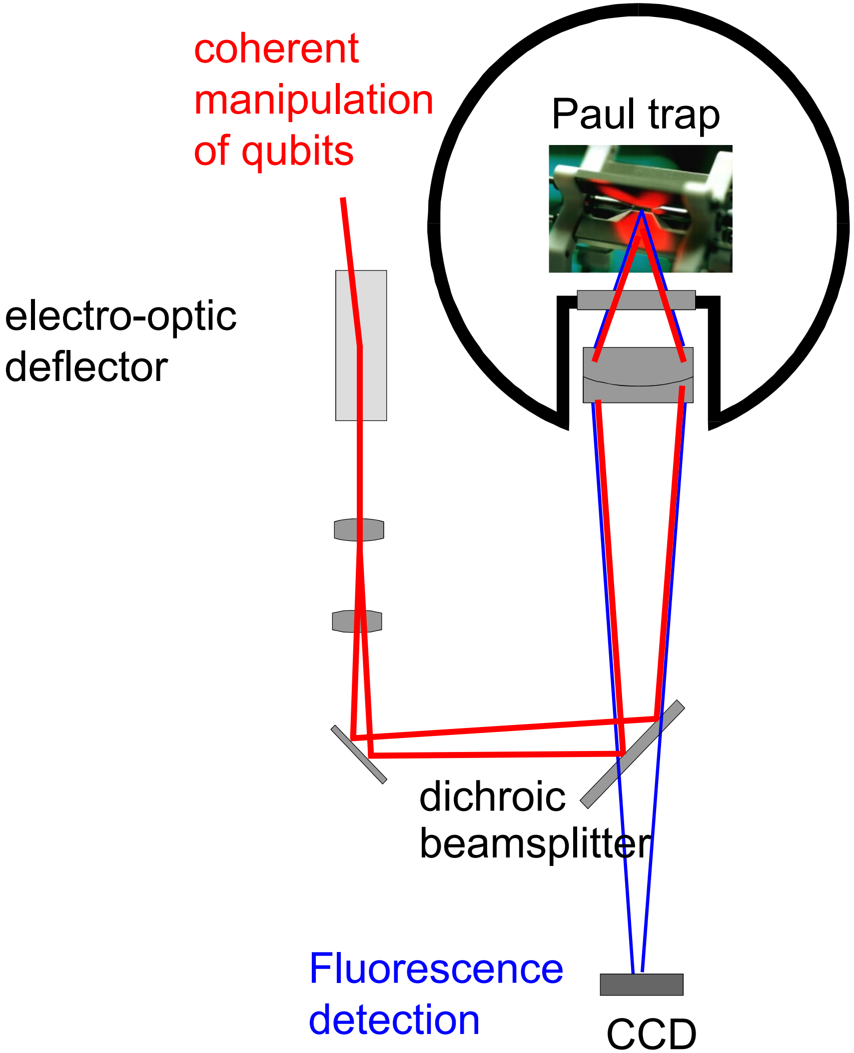
ac-Stark shifts shift qubit transition frequency

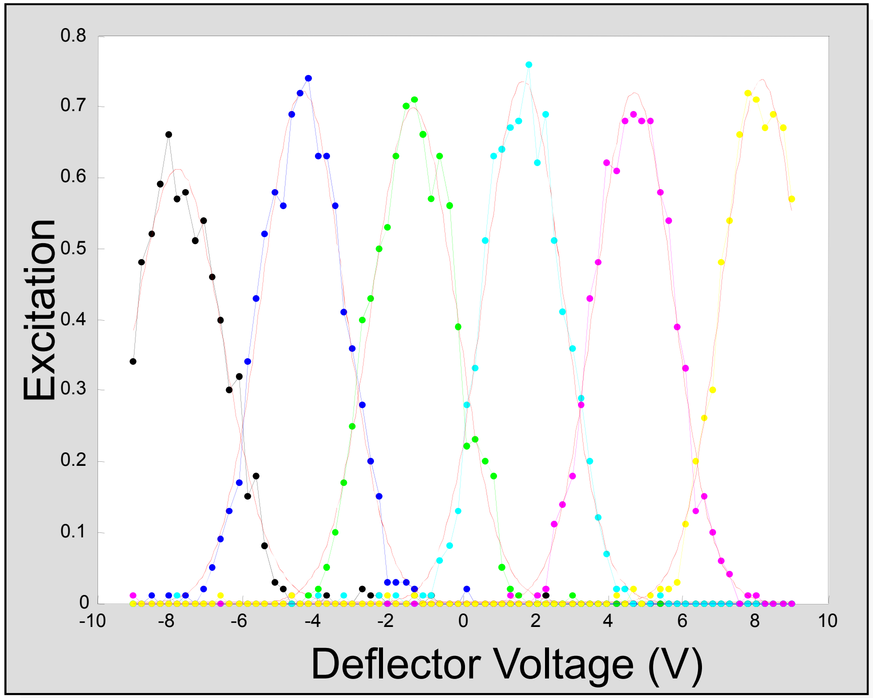
Arbitrary Bloch sphere rotations can be synthesized by a combination of laser pulses.

Laser setup for manipulating the qubit



Addressing of individual ions with a focussed laser beam

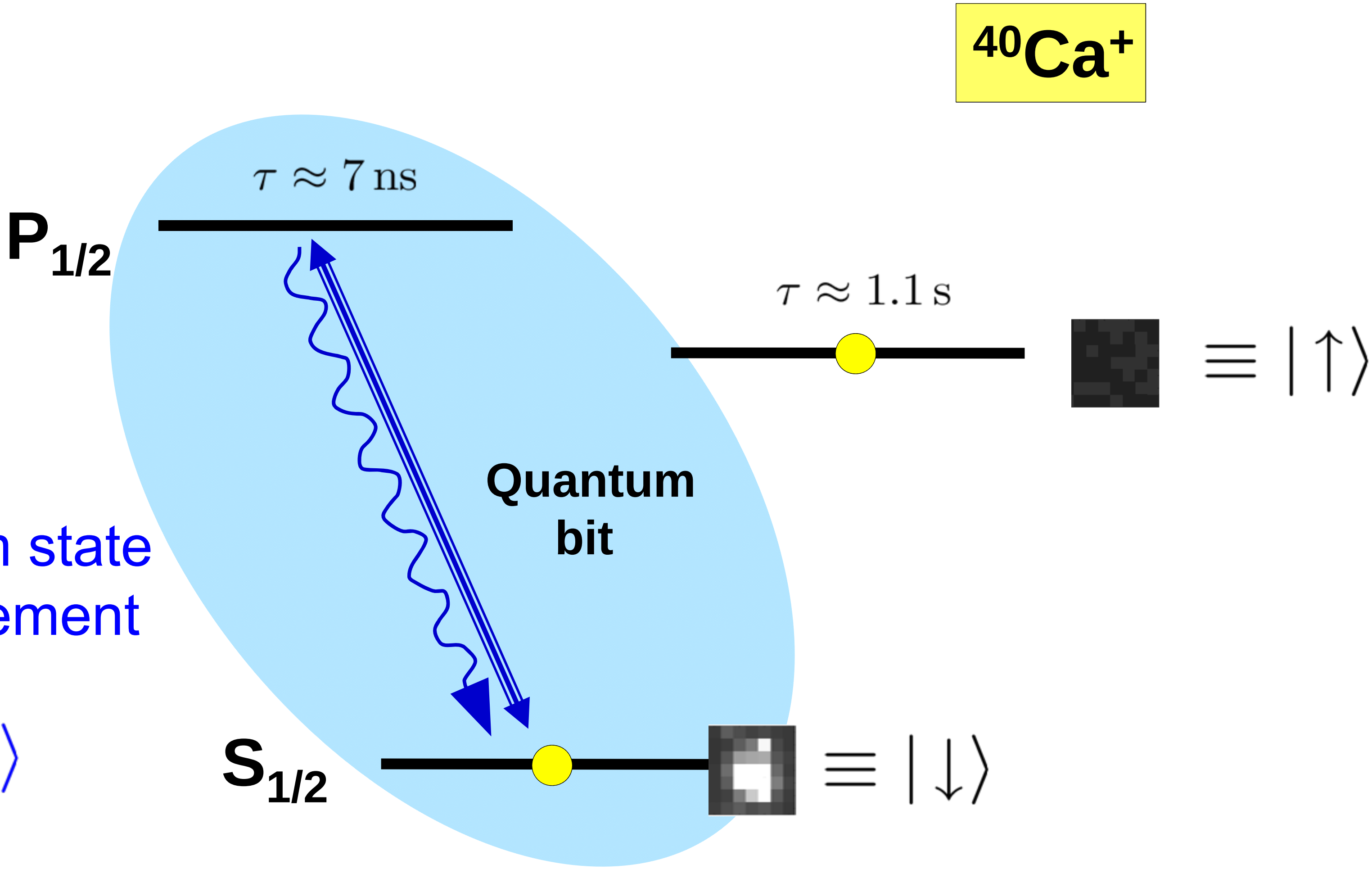




- inter ion distance: ~ 4 μm
- addressing waist: ~ 2 μm

< 0.1% intensity on neighbouring ions

Measuring qubits



Quantum state measurement

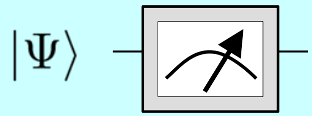
$\langle \sigma_z \rangle$

detection errors ~ 0.1%

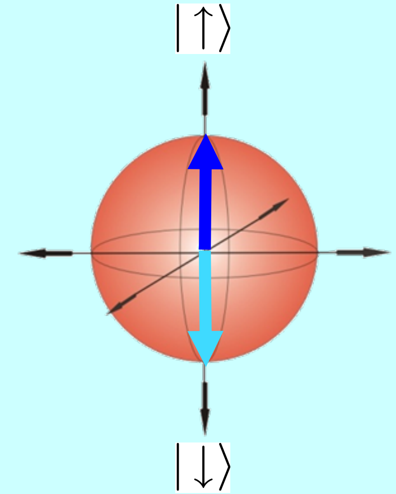
Further quantum measurements

Measurement of σ_z

Fluorescence measurements:

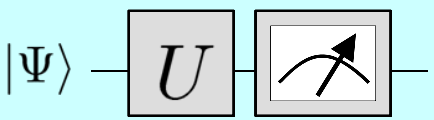


$$\langle \Psi | \sigma_z | \Psi \rangle$$



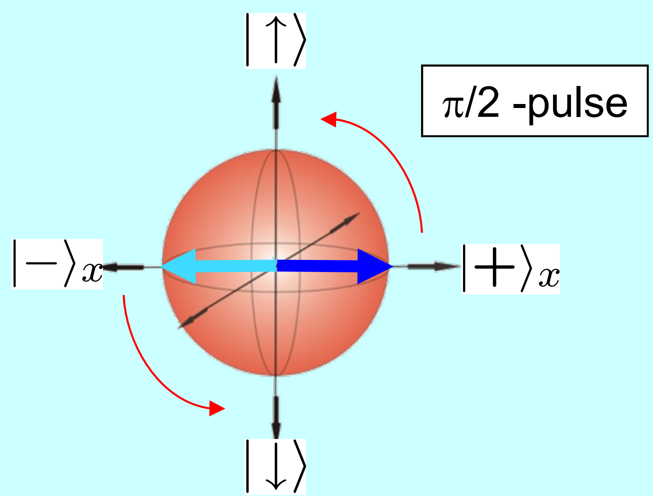
Measurement of σ_x

Unitary transformation +
fluorescence measurements



$$\langle U\Psi | \sigma_z | U\Psi \rangle = \langle \Psi | U^\dagger \sigma_z U | \Psi \rangle$$

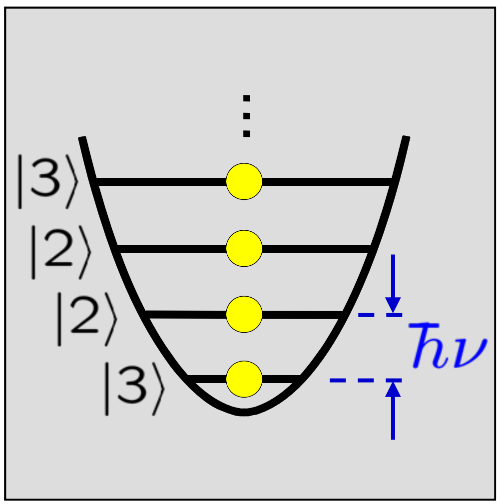
$$A = U^\dagger \sigma_z U$$



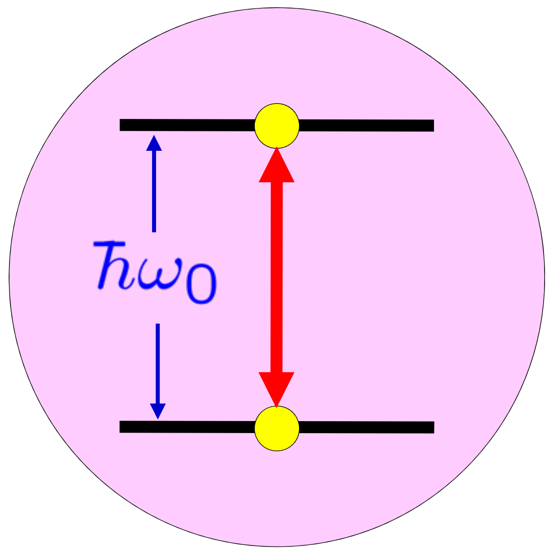
Coupling internal and vibrational degrees of freedom

Harmonic oscillator

Quantum bit



$\otimes$



$|D_{5/2}\rangle \equiv |\uparrow\rangle$

$|S_{1/2}\rangle \equiv |\downarrow\rangle$

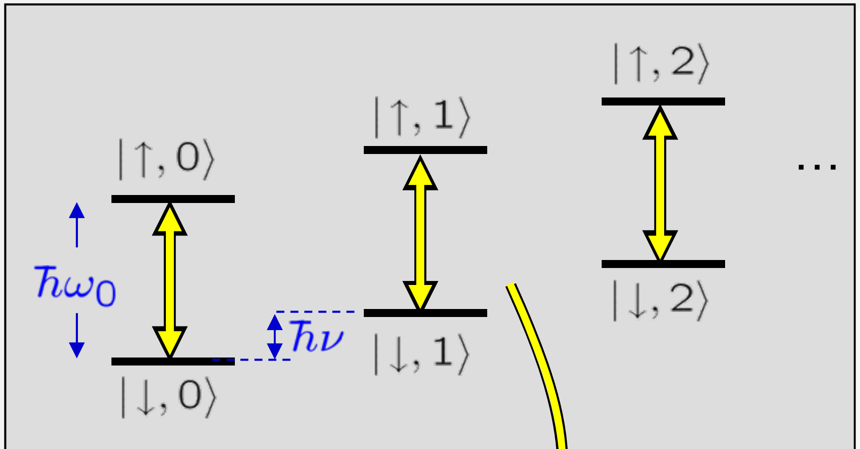
motional states

$|0\rangle, |1\rangle, |2\rangle, |3\rangle, \ldots$

internal states

$|\uparrow\rangle, |\downarrow\rangle$

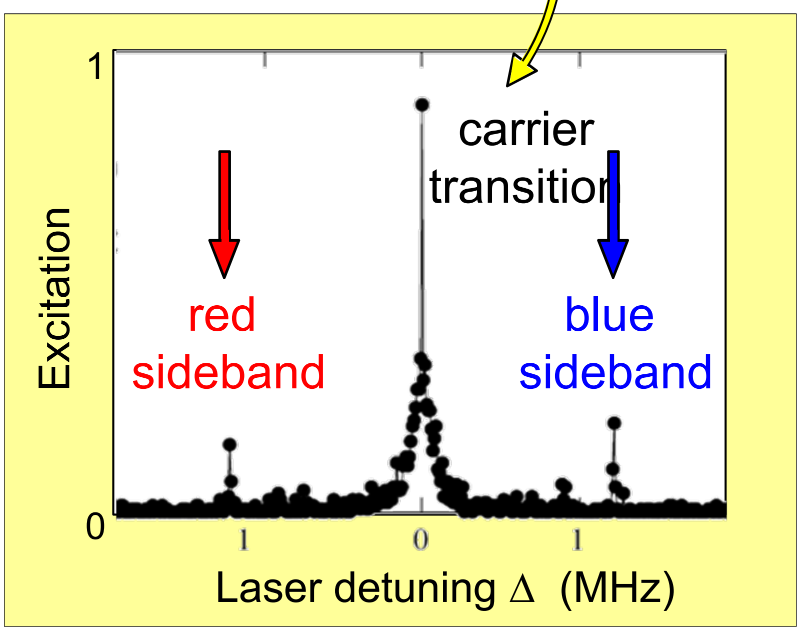
Trapped-ion laser interactions



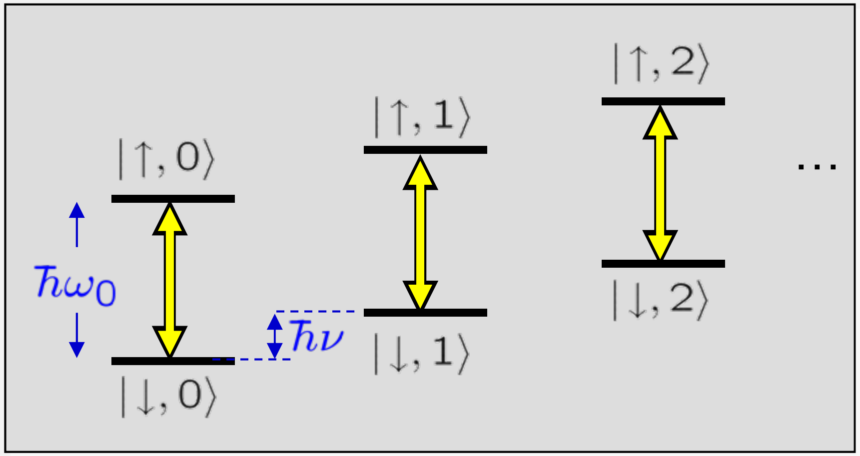
qubit manipulation

$$\omega_{laser} = \omega_0$$

$$H \propto \sigma_x, \; H \propto \sigma_y$$



Trapped-ion laser interactions



qubit manipulation

$$\omega_{laser} = \omega_0$$

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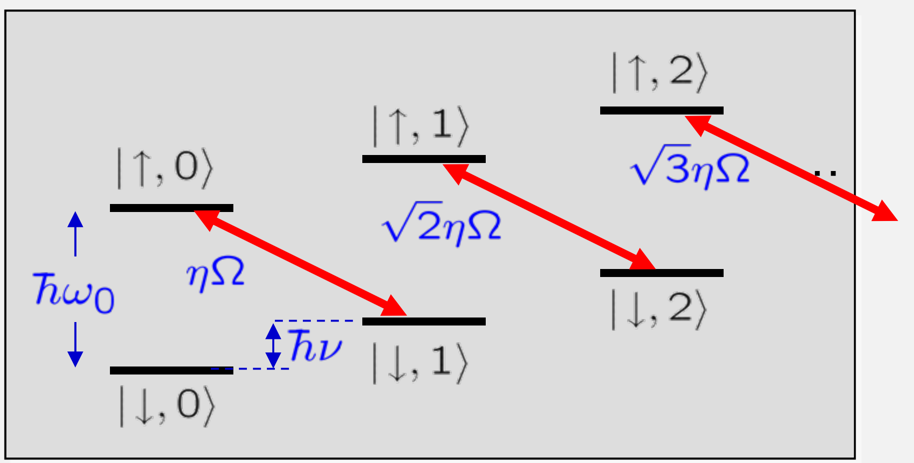
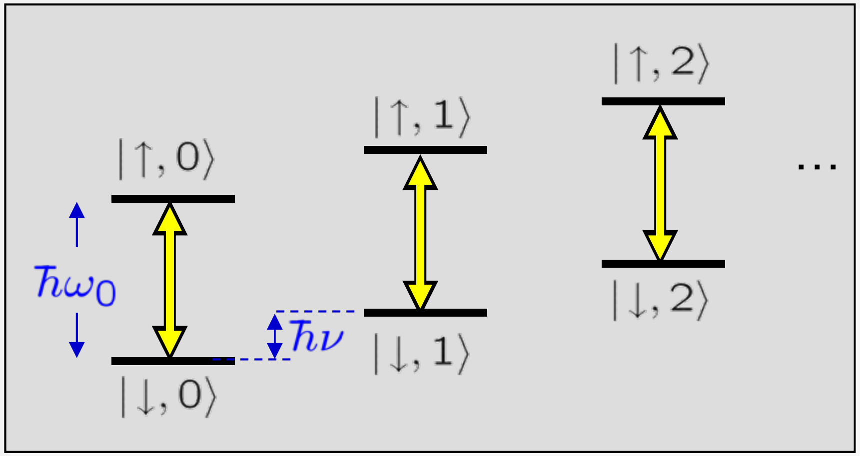


qubit-motion coupling

$$\omega_{laser} = \omega_0 - \nu$$

$$H \propto \sigma_+ a + \sigma_- a^\dagger$$

Trapped-ion laser interactions



qubit manipulation

$$\omega_{laser} = \omega_0$$

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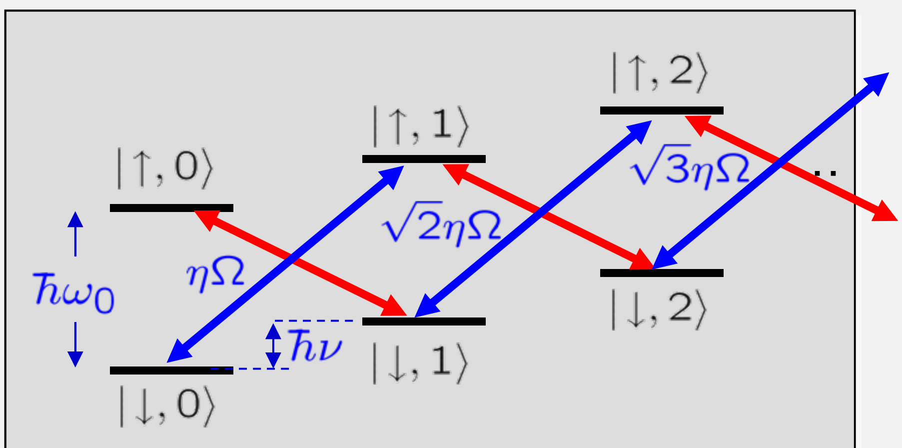


qubit-motion coupling

$$\omega_{laser} = \omega_0 - \nu$$

$$H \propto \sigma_+ a + \sigma_- a^\dagger$$

$$\omega_{laser} = \omega_0 + \nu$$

$$H \propto \sigma_+ a^\dagger + \sigma_- a$$

Sideband excitation

$$H^{(i)} = \frac{\hbar\Omega}{2}\sigma_+ e^{-i\delta t + i\phi}\left(I + i\eta(a^\dagger e^{i\nu t} + a e^{-i\nu t}) + \mathcal{O}(\eta^2)\right) + \text{h.c.}$$

Red sideband: $\delta = -\nu$

$$H_{int} = \frac{\hbar\Omega}{2} i\eta\{\sigma_+ a\, e^{+i\phi} - \sigma_- a^\dagger e^{-i\phi}\}$$

$$|g, n\rangle \longleftrightarrow |e, n-1\rangle$$

'Jaynes-Cummings-Hamiltonian'

- Coupling strength dependent on n

Blue sideband: $\delta = +\nu$

$$H_{int} = \frac{\hbar\Omega}{2} i\eta\{\sigma_+ a^\dagger e^{+i\phi} - \sigma_- a\, e^{-i\phi}\}$$

$$|g, n\rangle \longleftrightarrow |e, n+1\rangle$$

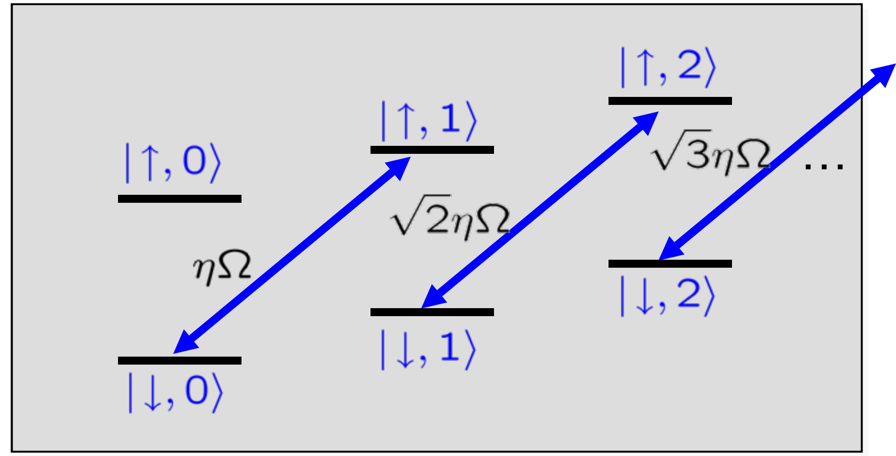
'anti-Jaynes-Cummings-Hamiltonian'

- Coupling strength dependent on n

Coherent excitation on the sideband

„Blue sideband“ pulses:

$$|\downarrow\rangle|n\rangle \longleftrightarrow |\uparrow\rangle|n+1\rangle$$

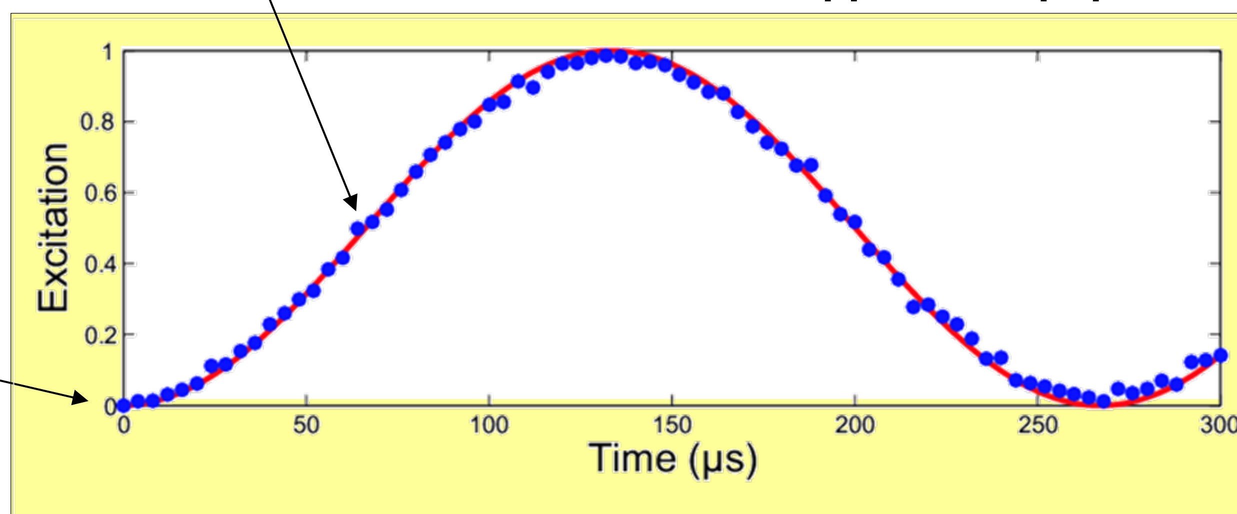


$\theta = \pi/2$: Entanglement between internal and motional state !

$$\frac{1}{\sqrt{2}}\left(|\downarrow, n=0\rangle + |\uparrow, n=1\rangle\right)$$

upper state population

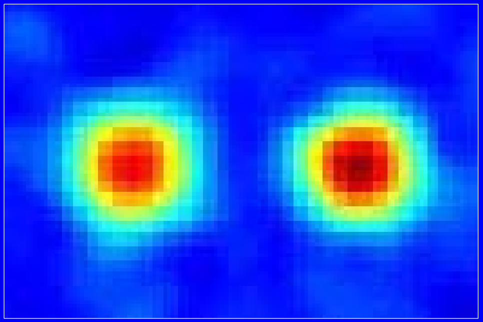
$$|\downarrow, n=0\rangle$$



Entangling a pair of trapped ions

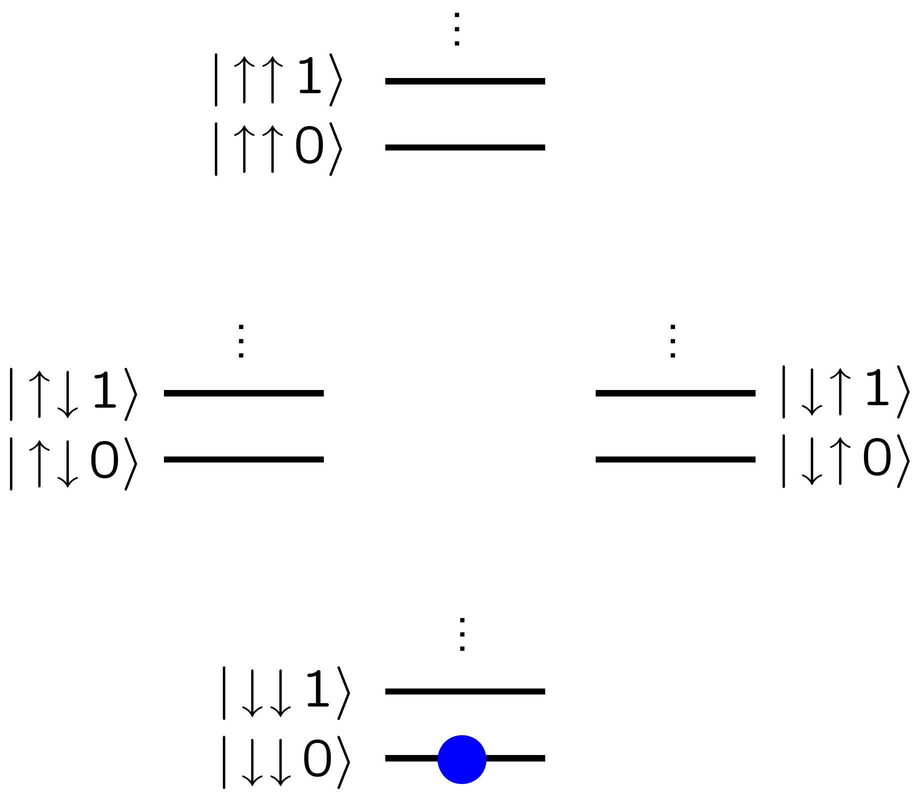
+

detecting the entanglement



Generation of Bell states

Two-ion energy levels

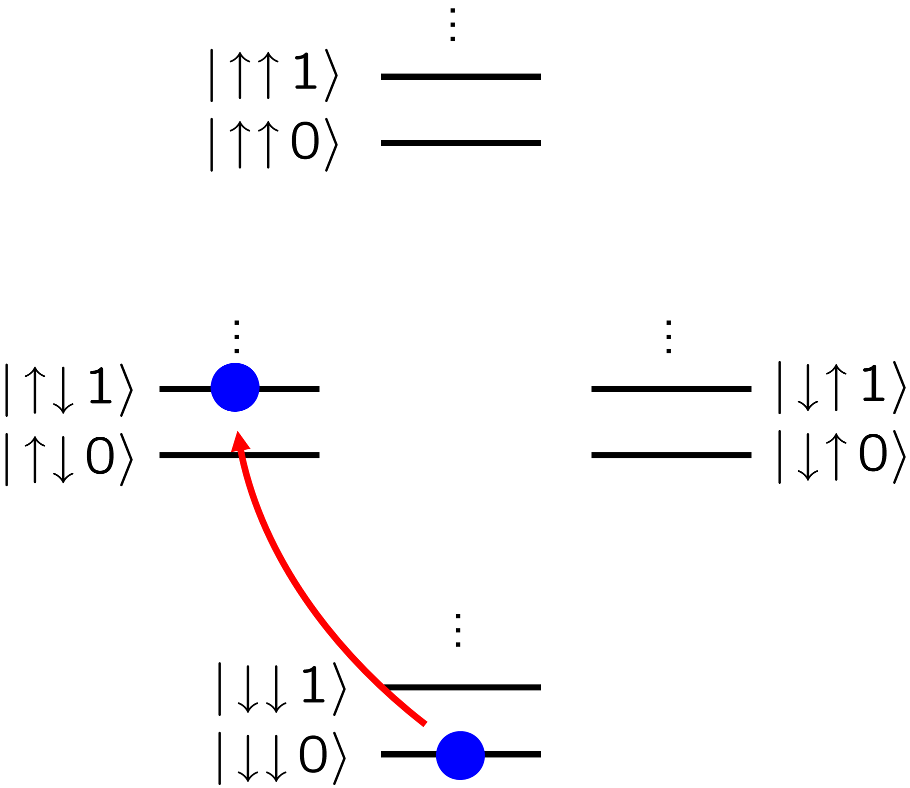


Pulse sequence:

Ion	Pulse length	Transition

$|\downarrow\downarrow 0\rangle$

Generation of Bell states

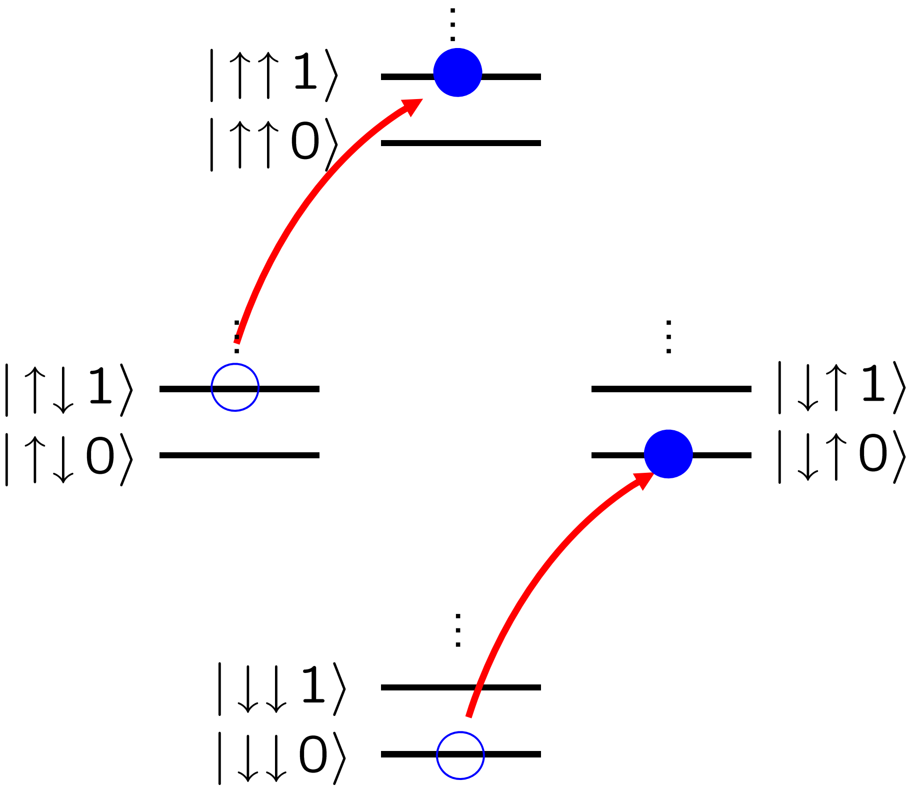


Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband

$$|\downarrow\downarrow 0\rangle + |\uparrow\downarrow 1\rangle$$

Generation of Bell states

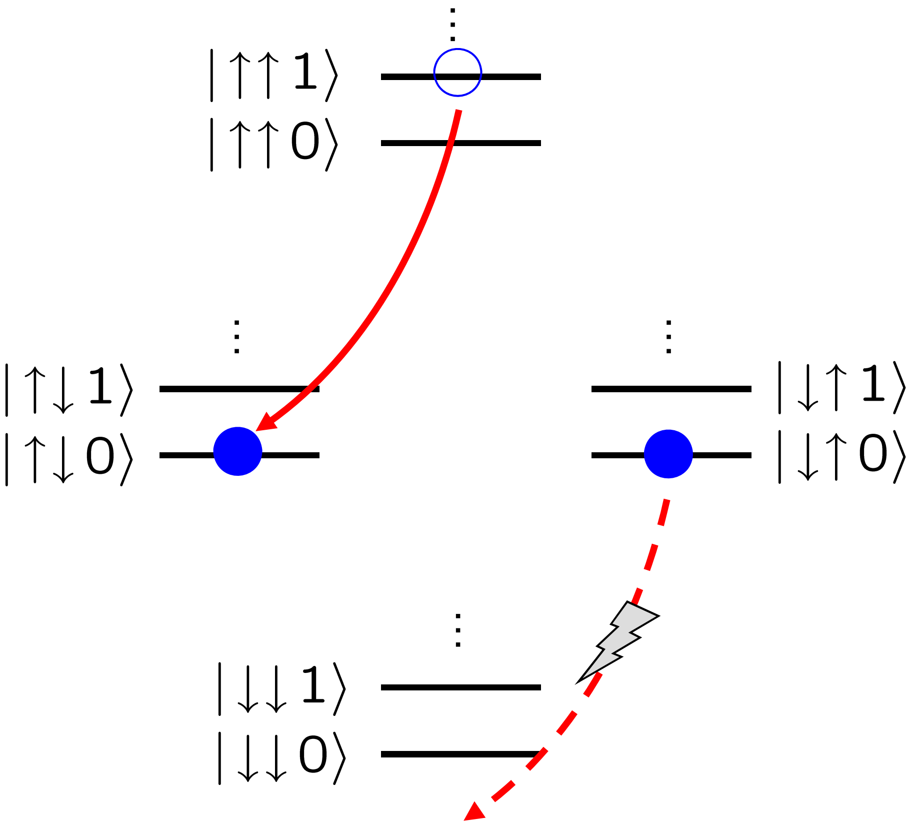


Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband
2	π	carrier

$$|\downarrow\uparrow 0\rangle + |\uparrow\uparrow 1\rangle$$

Generation of Bell states



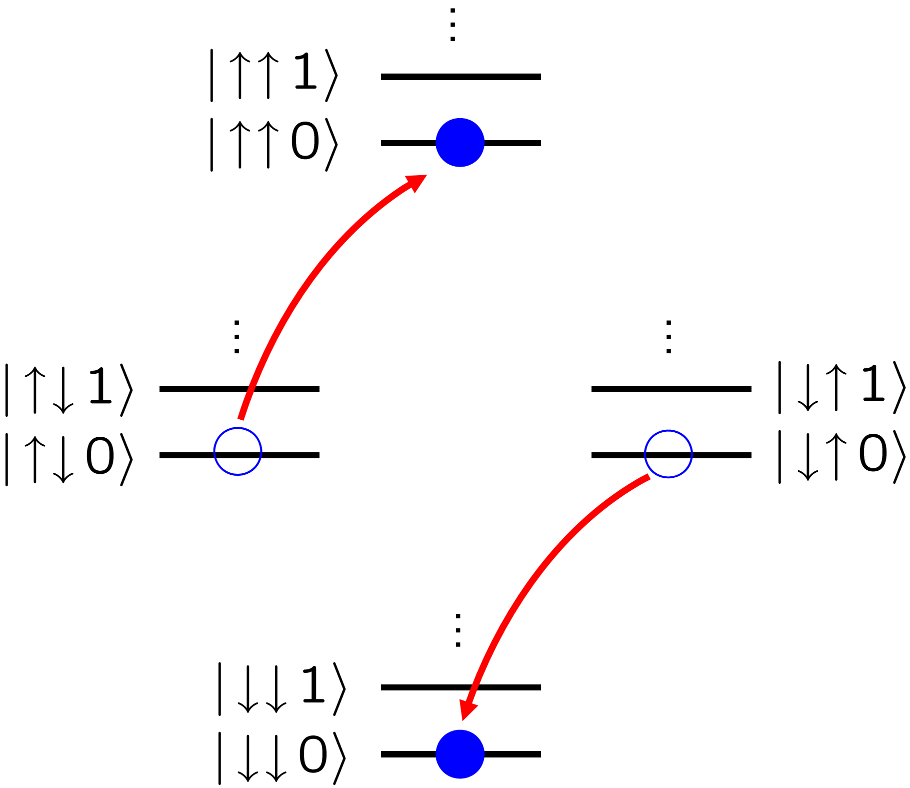
Pulse sequence:

Ion	Pulse length	Transition
1	π/2	blue sideband
2	π	carrier
2	π	blue sideband

Entanglement !

$$(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)|0\rangle$$

Generation of Bell states



Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband
2	π	carrier
2	π	blue sideband
2	π	carrier

Entanglement !

$$(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)|0\rangle$$

Measuring the entangled state

We hope to create the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$

There are no pure states in experimental physics!

The state created in the experiment has to be described by a density matrix ρ_{exp} .

How can we analyze the state ρ_{exp} we created?

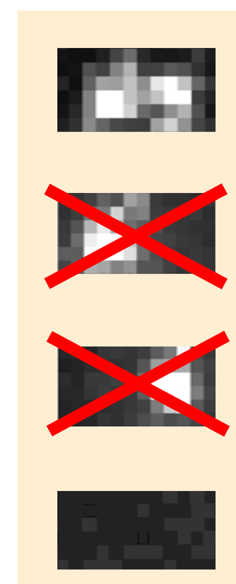
Fluorescence detection with CCD camera:

$|\downarrow\downarrow\rangle$

$|\downarrow\uparrow\rangle$

$|\downarrow\downarrow\rangle$

$|\uparrow\uparrow\rangle$

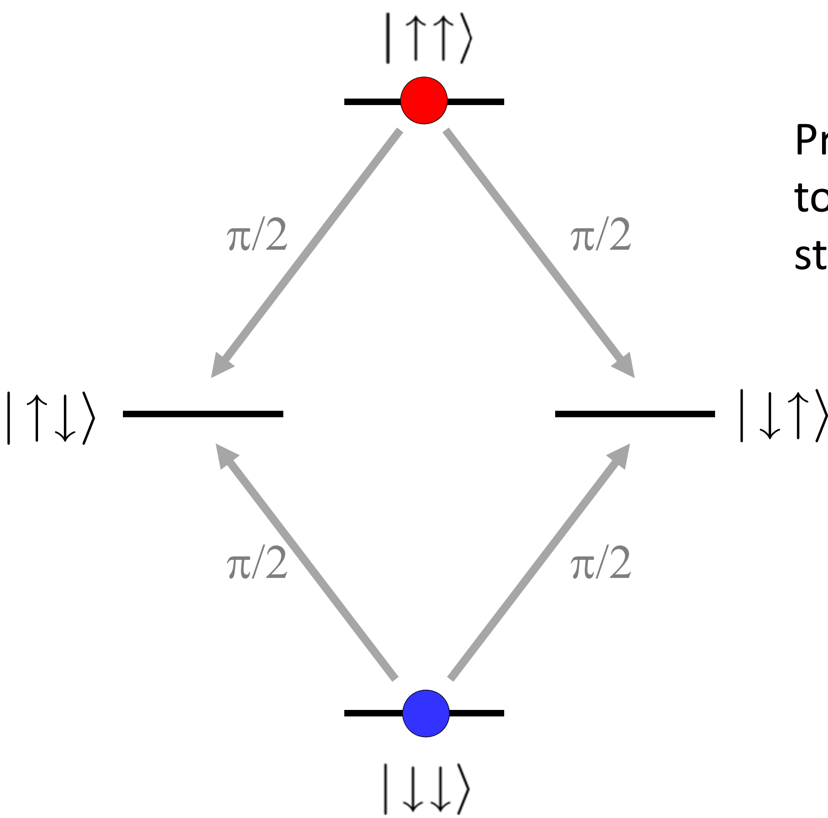


Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

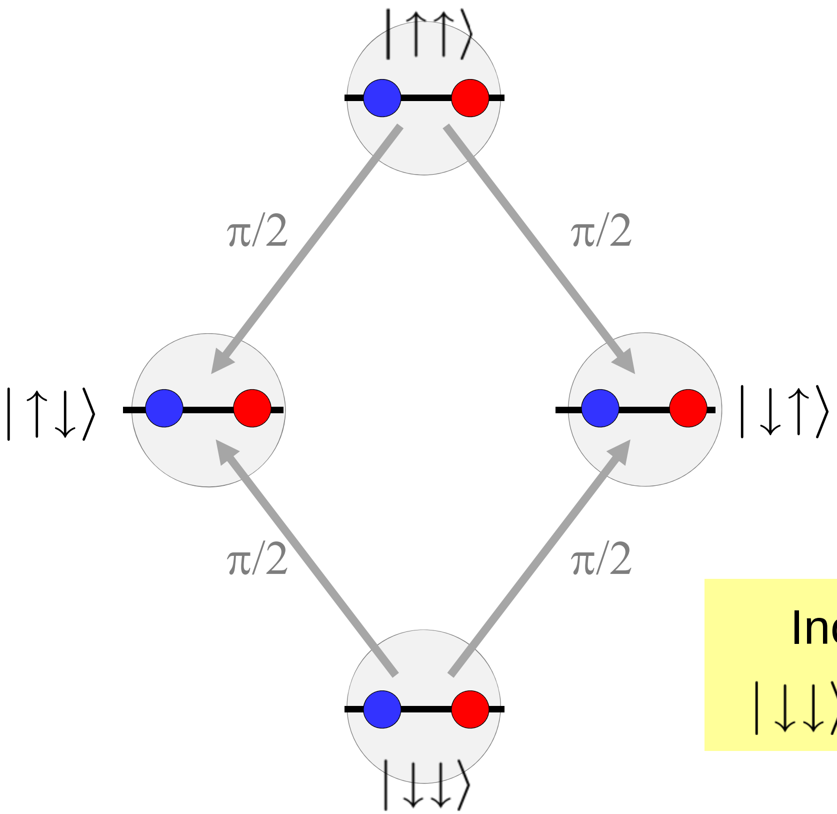
Entanglement check : interference

To check whether the two amplitudes are phase-coherent, we have to make an interference experiment:



Prior to the state measurement, we apply $\pi/2$ to both ions which couple the $|\downarrow\downarrow\rangle$ and the $|\downarrow\downarrow\rangle$ states to the intermediate states $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$.

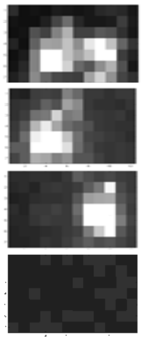
Entanglement check : interference



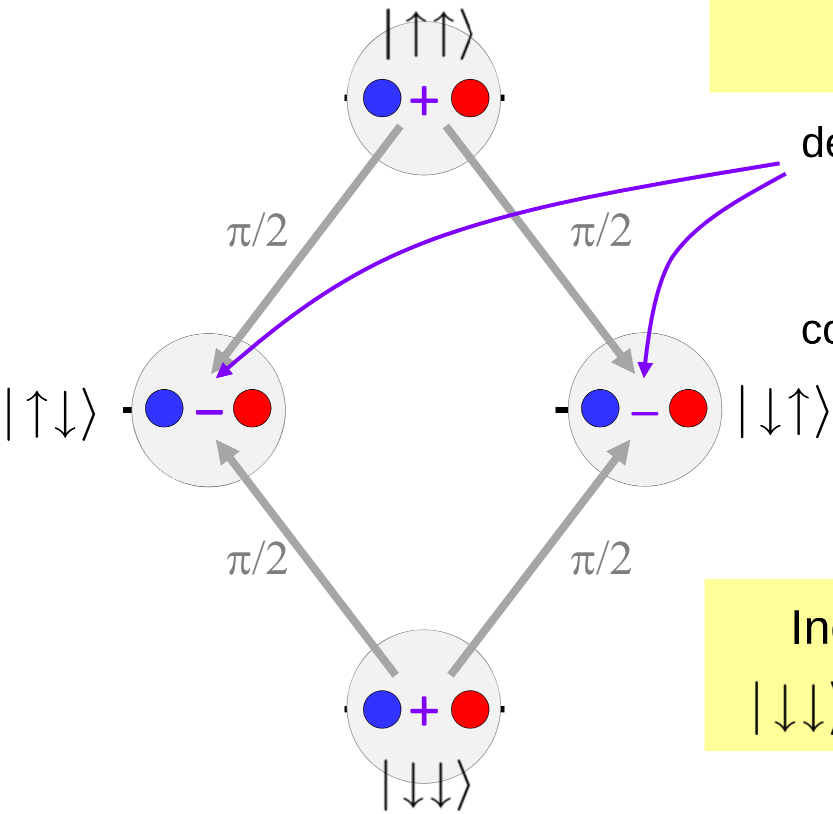
Incoherent mixture:

$$|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + |\uparrow\uparrow\rangle\langle\uparrow\uparrow|$$

→ equal population in all states



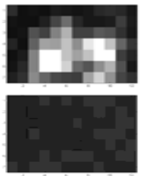
Entanglement check : interference



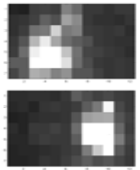
Maximally entangled state:

$|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$

destructive interference $\longrightarrow$ $|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$



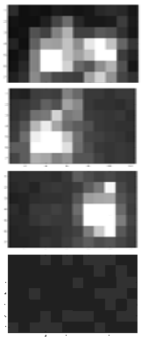
constructive interference $\longrightarrow$ $|\downarrow\uparrow\rangle + |\downarrow\uparrow\rangle$



Incoherent mixture:

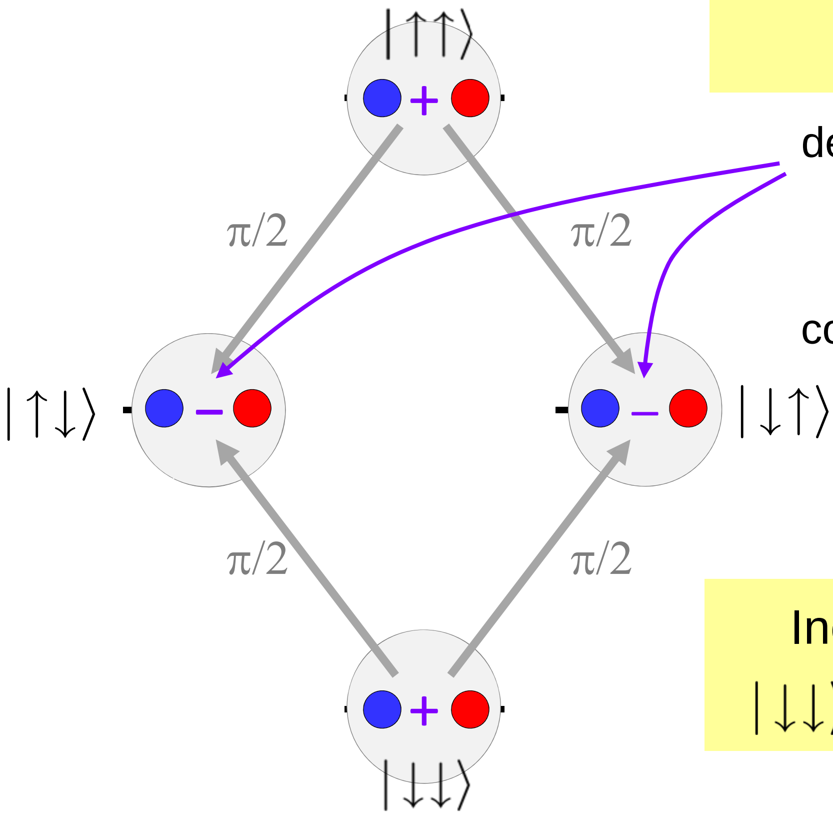
$|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + |\uparrow\uparrow\rangle\langle\uparrow\uparrow|$

$\longrightarrow$ equal population in all states



Entanglement check: Scan laser phase ϕ and measure parity

Entanglement check : interference



Maximally entangled state:

$$|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$$

		Parity $\langle \sigma_z^{(1)} \sigma_z^{(2)} \rangle$
destructive interference →	$|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$	+ 1
constructive interference →	$|\downarrow\uparrow\rangle + |\downarrow\uparrow\rangle$	- 1

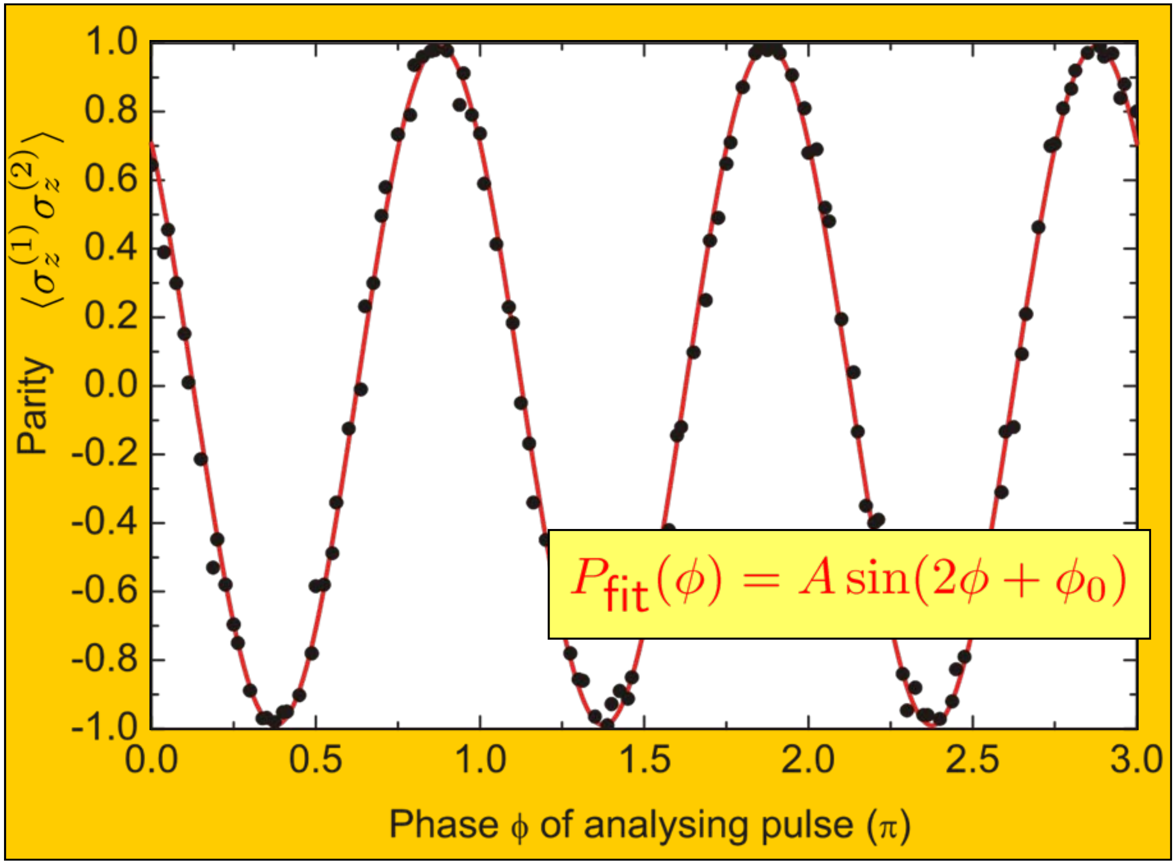
Incoherent mixture:

$$|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + |\uparrow\uparrow\rangle\langle\uparrow\uparrow|$$

→	equal population in all states	0

Entanglement check: Scan laser phase ϕ and measure parity

Mølmer-Sørensen gate: parity oscillations



Bell state:

$$\Psi = |\downarrow\downarrow\rangle + i|\uparrow\uparrow\rangle$$

different entanglement creation in this experiment

A = 0.990(1) 29,400 measurements

$p_{\downarrow\downarrow}+p_{\uparrow\uparrow}$ = 0.9965(4) 13,000 measurements

Bell state fidelity

F = 99.3(1)%

J. Benhelm *et al.*, Nature Physics **4**, 463 (2008)

Creating entanglement vs. entangling quantum gates

Bell state generation: $|\downarrow\downarrow\rangle|0\rangle \rightarrow (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)|0\rangle$

This sequence of laser pulses transforms a product state into an entangled state.

Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband
2	π	carrier
2	π	blue sideband
2	π	carrier

But it is not a two-qubit quantum gate!!

Why not?

Check what the sequence does to the initial state $|\uparrow\uparrow\rangle|0\rangle$!

Does the motional state return to the ground state at the end of the sequence?