

# Quantum information processing with trapped ions

- Introduction to Quantum Information Processing
- Trapped-ion QIP
  - Qubits, preparation + measurement
  - Coherent operations
  - Entangled states: creation + detection

Les Houches, January 18, 2012

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Institute for Quantum Optics and Quantum Information  
Innsbruck, Austria

# Computation and Physics

1960

## Limitations and foundations

Are there physical limitations to the process of computation?

1970

Can computation be understood in terms of quantum mechanics?

## Is quantum mechanics useful?

1980

Can quantum-physical computation be more efficient than models of computation based on classical physics?

1990

Quantum algorithms, quantum error correction

2000

Physical implementations of quantum computation?

Demonstration experiments + applications

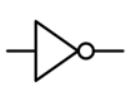
# Classical vs. quantum information processing

## Bit:

Physical system with two distinct states 0 or 1

## Logic gates

Boolean logic operation



$$\begin{aligned} 0 &\rightarrow 1 \\ 1 &\rightarrow 0 \end{aligned}$$



$$(\epsilon_1, \epsilon_2) \rightarrow \epsilon_1 \oplus \epsilon_2$$

## XOR truth table

$$(0, 0) \rightarrow 0$$

$$(0, 1) \rightarrow 1$$

$$(1, 0) \rightarrow 1$$

$$(1, 1) \rightarrow 0$$

## Quantum bit:

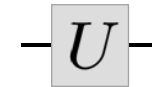
Two-level quantum system with state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

## Quantum logic gate

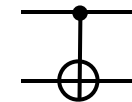
Unitary transformation

single qubit gate



$$|\psi\rangle \rightarrow U|\psi\rangle$$

two-qubit gate



$$|\epsilon_1\rangle|\epsilon_2\rangle \rightarrow |\epsilon_1\rangle|\epsilon_1 \oplus \epsilon_2\rangle$$

## CNOT truth table

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$

$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$

$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$

$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

# Classical vs. quantum information processing

## Bit:

Physical system with two distinct states 0 or 1

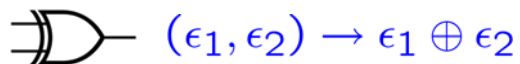
## Quantum bit:

Two-level quantum system with state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

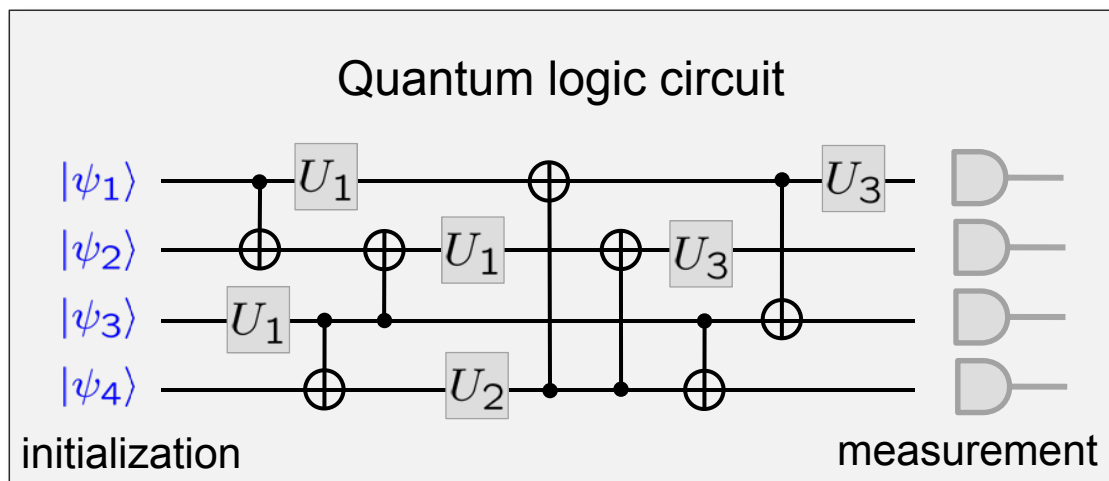
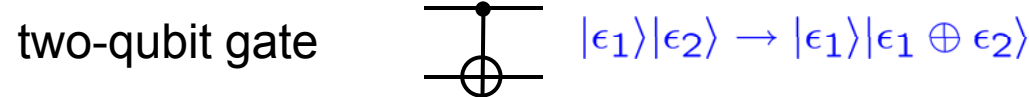
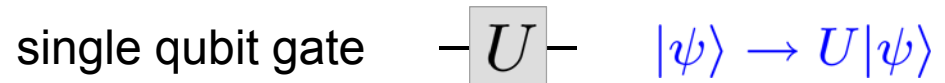
## Logic gates

Boolean logic operation



## Quantum logic gate

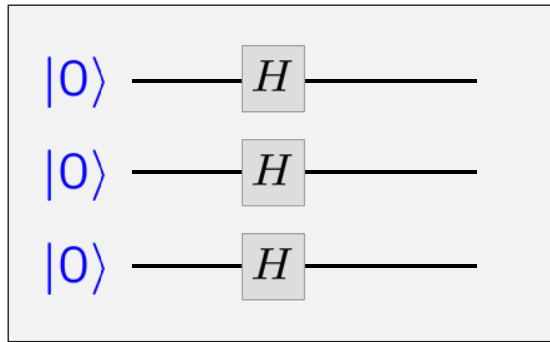
Unitary transformation



# Superpositions and entanglement

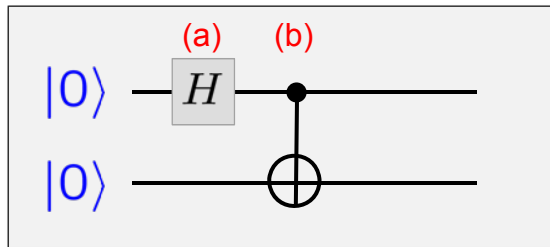
## Putting a quantum register in a superposition

$$\begin{aligned}
 |\psi\rangle = |0\rangle|0\rangle|0\rangle &\longrightarrow (|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle) \\
 &= |000\rangle + |001\rangle + |010\rangle + |011\rangle \\
 &\quad + |100\rangle + |101\rangle + |110\rangle + |111\rangle
 \end{aligned}$$



If applied at the start of a quantum algorithm, this operation enables parallel processing on all possible input states.

## Entangling quantum bits in a quantum register



$$\begin{aligned}
 |\psi\rangle = |0\rangle|0\rangle &\xrightarrow{(a)} (|0\rangle + |1\rangle)|0\rangle \\
 &\xrightarrow{(b)} |00\rangle + |11\rangle
 \end{aligned}$$

The computation creates quantum correlations between the qubits.

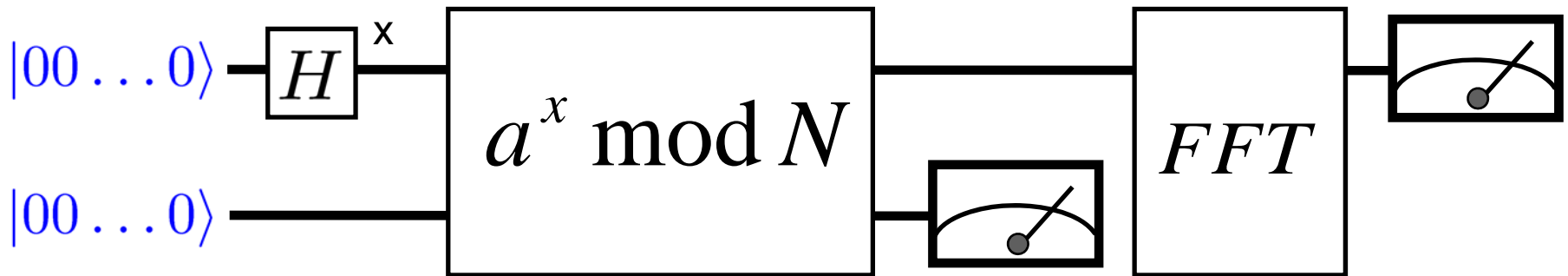
controlled-NOT gate

$$\begin{array}{c} \text{---} \\ | \\ \oplus \end{array} = |\epsilon_1\rangle|\epsilon_2\rangle \rightarrow |\epsilon_1\rangle|\epsilon_1 \oplus \epsilon_2\rangle$$

# Example: Quantum factoring algorithm

Peter Shor: A quantum computer can efficiently find prime factors of large numbers.

Theoretical Algorithm (Shor, 1994)



**Three main steps:**

1. Input superposition preparation
2. Modular exponentiation (multi-qubit gates required)
3. Quantum Fourier Transform
4. Classical pre- and post-processing

# Quantum information processing

## Quantum computing

Quantum algorithms for efficient computing

## Quantum communication

Secure communication certified by quantum physics

## Quantum foundations

Quantum theory and its interpretations; exp. tests

## Quantum simulation

Investigating many-body Hamiltonians using well-controlled quantum systems

→ tomorrow's lecture

## Quantum metrology

Entanglement-enhanced measurements

→ Piet Schmidt's lecture



# Trapped ions for quantum information processing

VOLUME 74, NUMBER 20

PHYSICAL REVIEW LETTERS

15 MAY 1995

## Quantum Computations with Cold Trapped Ions

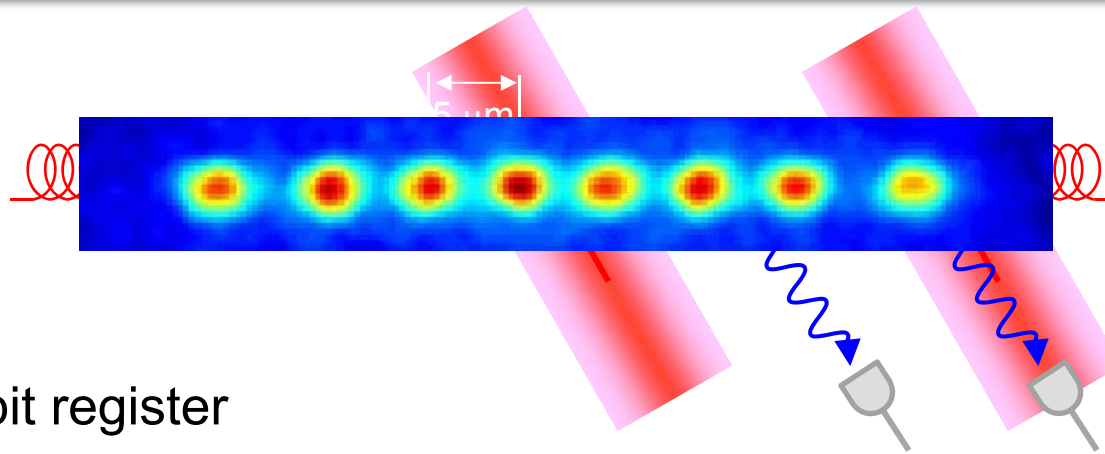
J. I. Cirac and P. Zoller\*

*Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria*

(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

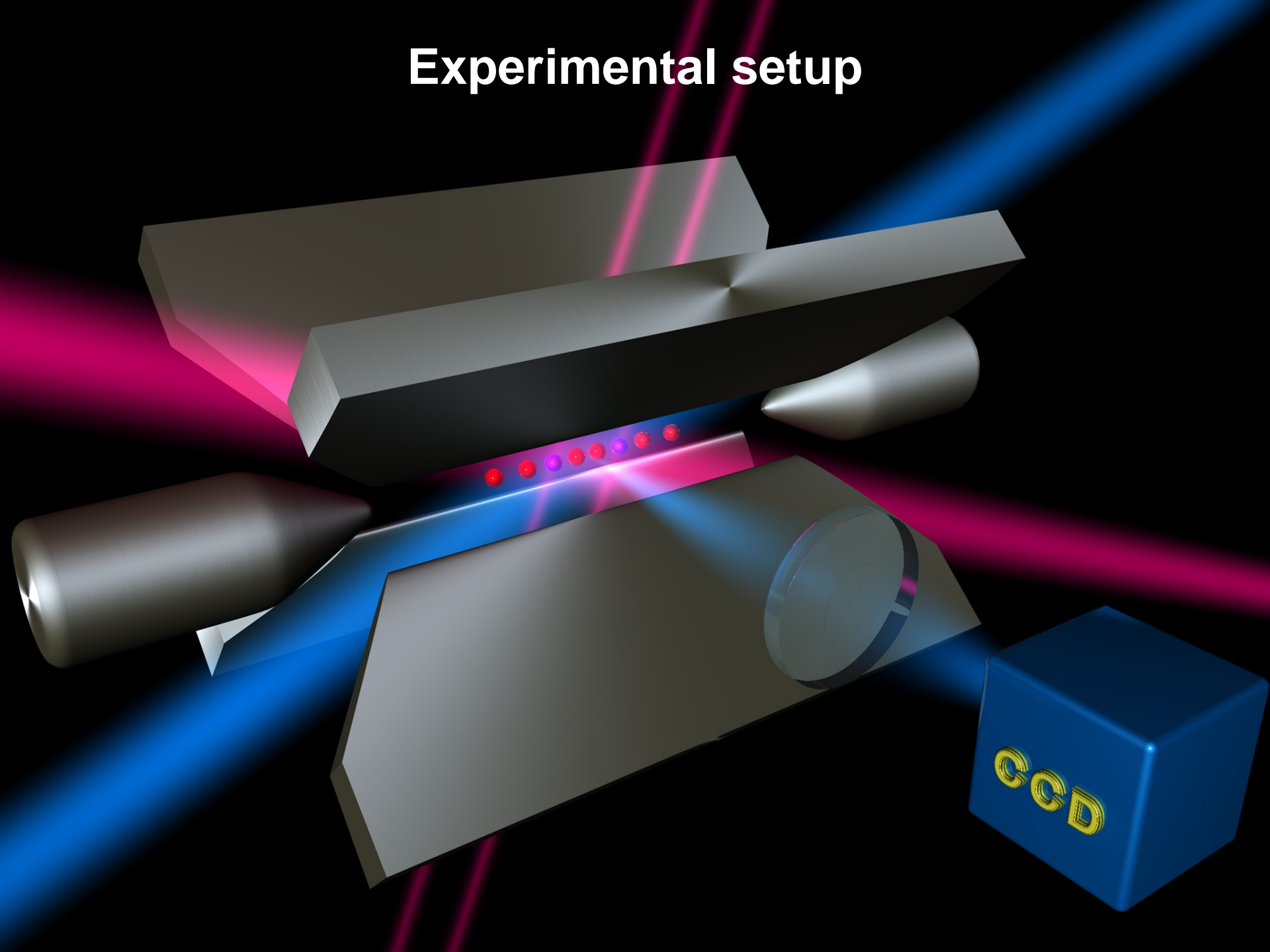
Ion string



- Qubit register
- State detection
- Single qubit gates
- Entangling gates

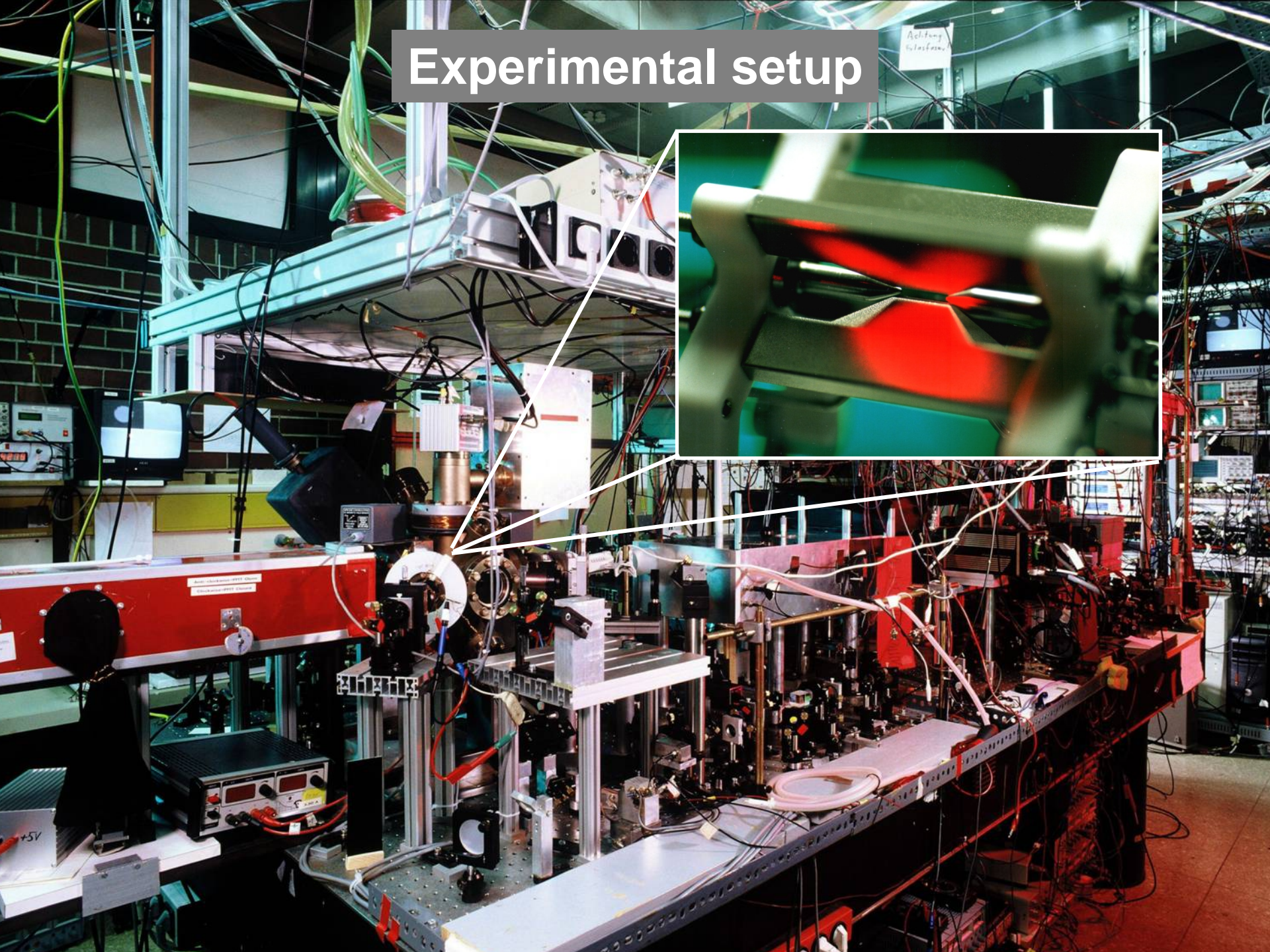


# Experimental setup





# Experimental setup



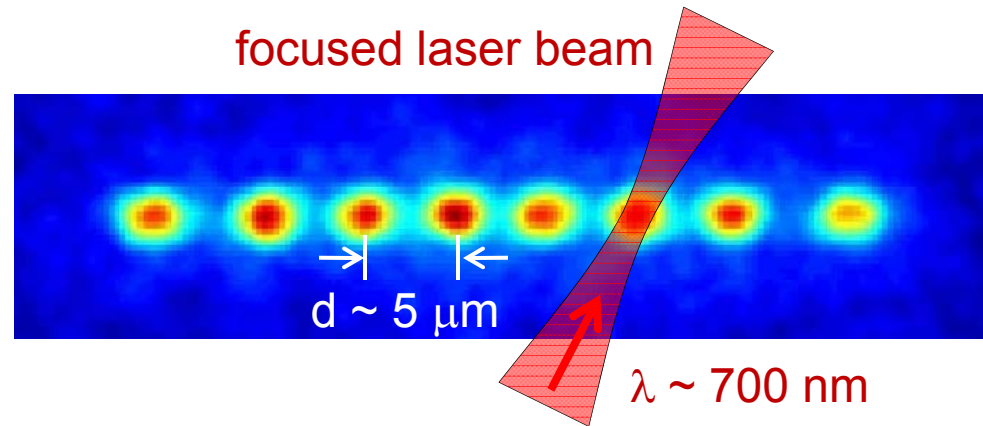


# Quantum physics with linear ion strings

Trap frequencies:

$$\nu_z \propto 1 \text{ MHz}$$

$$\nu_{x,y} \propto 5 \text{ MHz}$$



Length scales

ion distance		laser wavelength		ion localisation		Bohr radius
$d$	$>$	$\lambda$	$\gg$	$z_0$	$\gg$	$a_0$
$5 \mu\text{m}$		$700 \text{ nm}$		$10 \text{ nm}$		$50 \text{ pm}$

A red double-headed arrow is drawn below the table, spanning the width of the first two columns (ion distance and laser wavelength).

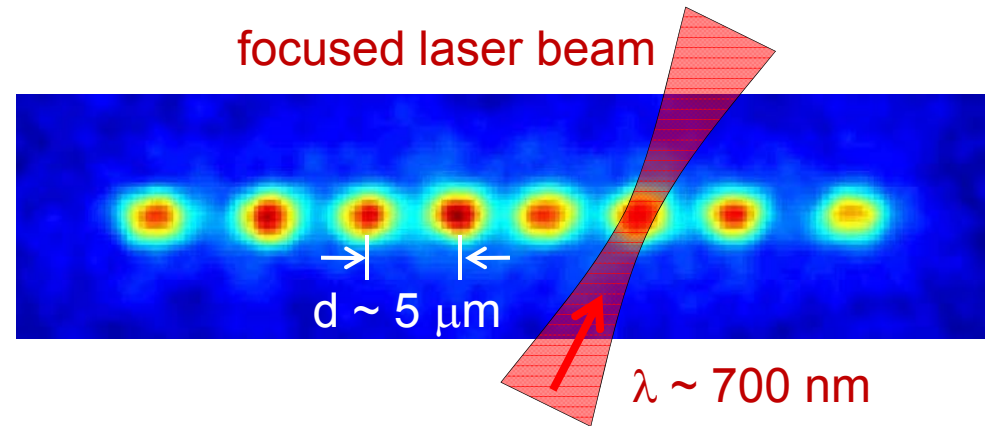
- individual addressing, spatially resolved fluorescence

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Red arrows point from the Bohr radius and ion localisation values up to the laser wavelength value.

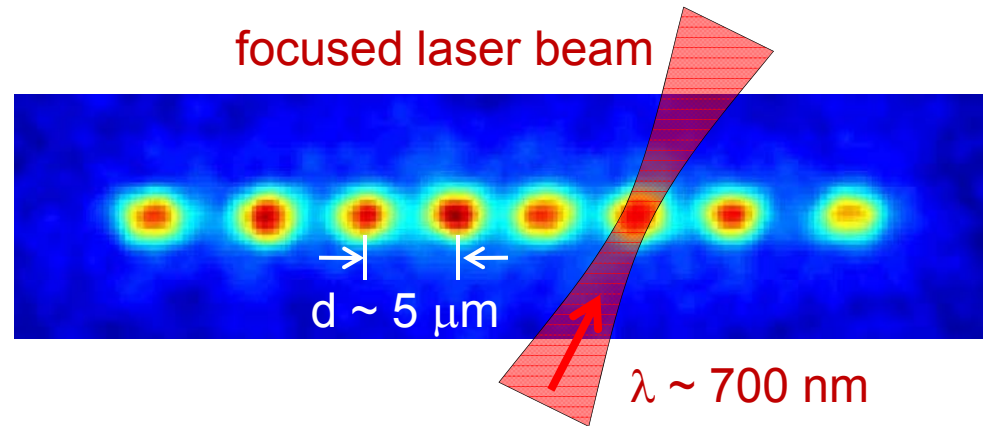
- individual addressing, spatially resolved fluorescence
- coupling internal and motional states by laser takes on simple form

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A red double-headed arrow spans the width of the table, indicating the relative scale of the ion distance and Bohr radius.

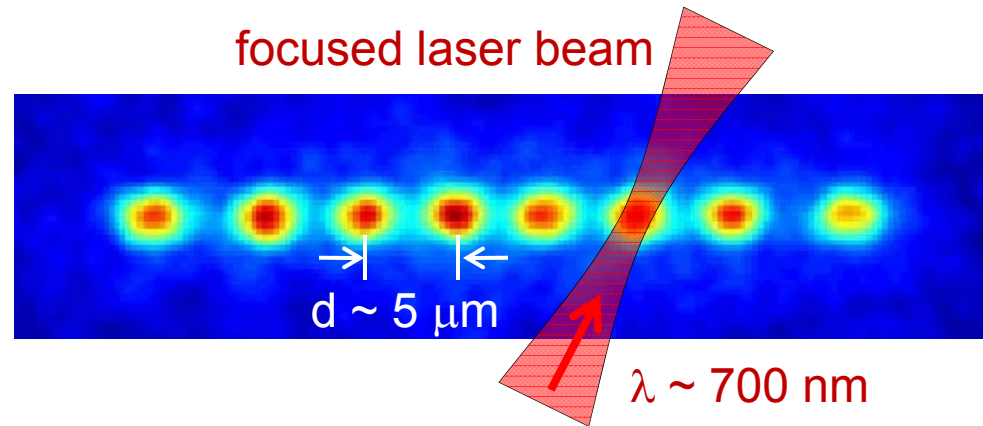
- individual addressing, spatially resolved fluorescence
- coupling internal and motional states by laser takes on simple form
- no direct state-dependent interactions between ions

# Quantum physics with linear ion strings

Trap frequencies:

$$\nu_z \propto 1 \text{ MHz}$$

$$\nu_{x,y} \propto 5 \text{ MHz}$$



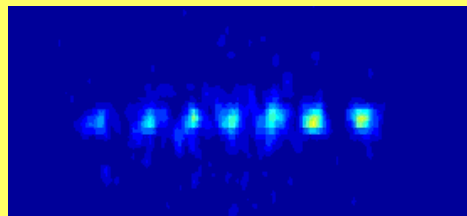
Length scales

ion distance		laser wavelength		ion localisation		Bohr radius
$d$	$>$	$\lambda$	$\gg$	$z_0$	$\gg$	$a_0$
$5 \mu\text{m}$		$700 \text{ nm}$		$10 \text{ nm}$		$50 \text{ pm}$

Vibrational modes

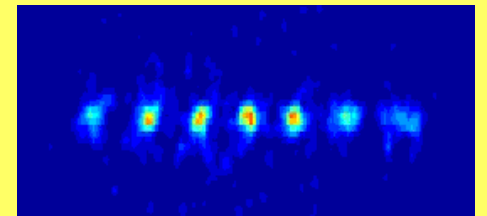
centre-of-mass mode

$$\nu = \nu_z$$



breathing mode

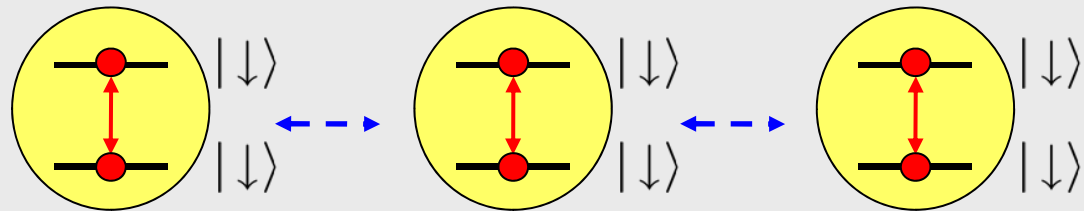
$$\nu = \sqrt{3} \nu_z$$



# Trapped ions as a quantum system

Internal degrees of freedom:

Quantum bits

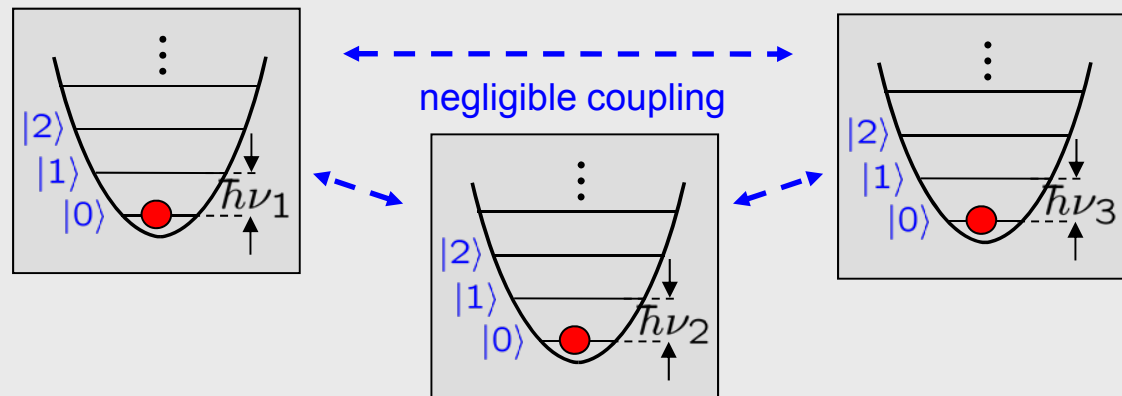


negligible coupling

laser-ion interactions !

Motional degrees of freedom:

Harmonic oscillators



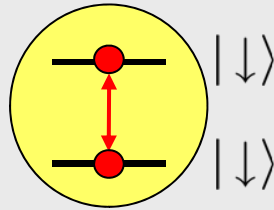
negligible coupling



# Trapped ions as a quantum system

Internal degrees  
of freedoms:

Quantum  
bits



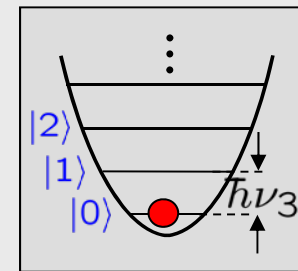
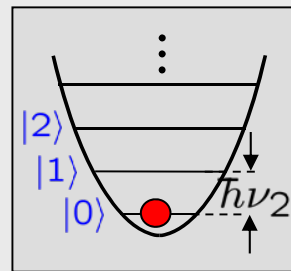
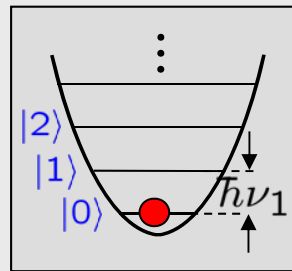
ground state  $\approx 10$  nm  $\ll$  optical wavelengths

Measurements of motional quantum  
states via coupling to internal states

Entanglement between spin and motion

Motional degrees  
of freedoms:

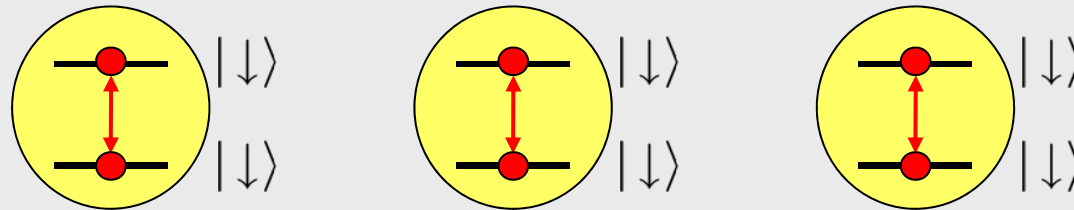
Harmonic  
oscillators



# Trapped ions as a quantum system

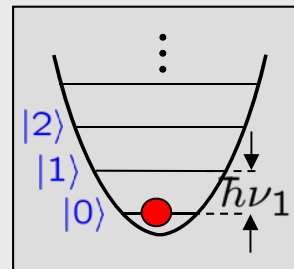
Internal degrees  
of freedoms:

Quantum  
bits



Motional degrees  
of freedoms:

Harmonic  
oscillators

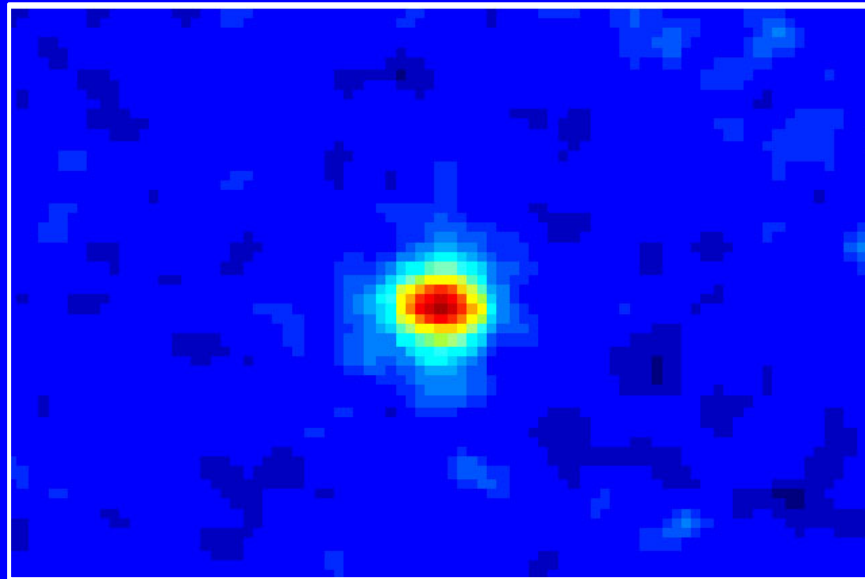


Entanglement between qubits:

Interactions mediated by coupling to  
vibrational modes

# Trapped-ion quantum bits

Encoding, manipulation and measurement



# PERIODIC TABLE

## Atomic Properties of the Elements

U.S. DEPARTMENT OF COMMERCE  
 Technology Administration  
 National Institute of Standards and Technology

**Frequently used fundamental physical constants**

For the most accurate values of these and other constants, visit [physics.nist.gov/constants](http://physics.nist.gov/constants)

1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of <sup>133</sup>Cs

speed of light in vacuum  $c$  299 792 458 m s<sup>-1</sup> (exact)

Planck constant  $h$  6.6261 × 10<sup>-34</sup> J s ( $h = h/2\pi$ )

### Singly-charged ions appropriate for quantum information processing

Group IA	1 <sup>1</sup> H Hydrogen 1.00794 1s 13.5984	2 <sup>2</sup> He Helium 4.00260 1s <sup>2</sup> 24.5874
IIA	3 <sup>3</sup> Li Lithium 6.941 1s <sup>2</sup> 2s 5.3917	4 <sup>4</sup> Be Beryllium 9.01218 1s <sup>2</sup> 2s <sup>2</sup> 9.3227
IIIA	11 <sup>11</sup> Na Sodium 22.98977 [Ne]3s 5.1391	12 <sup>12</sup> Mg Magnesium 24.3050 [Ne]3s <sup>2</sup> 7.6462
IIIA	19 <sup>19</sup> K Potassium 39.0983 [Ar]4s 4.3407	20 <sup>20</sup> Ca Calcium 40.078 [Ar]4s 6.1132
IIIA	37 <sup>37</sup> Rb Rubidium 85.4678 [Kr]5s 4.1771	38 <sup>38</sup> Sr Strontium 87.62 [Kr]5s 5.6949
IIIA	55 <sup>55</sup> Cs Cesium 132.90545 [Xe]6s 3.8939	56 <sup>56</sup> Ba Barium 137.327 [Xe]6s 5.2117
IIIA	87 <sup>87</sup> Fr Francium (223) [Rn]7s 4.0727	88 <sup>88</sup> Ra Radium (226) [Rn]7s 5.2784

IIIA	IVA	VA	VIA	VIIA	VIIIA	IB	IIB
21 <sup>21</sup> Sc Scandium 44.95591 [Ar]3d4s 6.5615	22 <sup>22</sup> Ti Titanium 47.867 [Ar]3d <sup>2</sup> 4s 6.8281	23 <sup>23</sup> V Vanadium 50.9415 [Ar]3d <sup>3</sup> 4s 6.7462	24 <sup>24</sup> Cr Chromium 51.9961 [Ar]3d <sup>5</sup> 4s 6.7665	25 <sup>25</sup> Mn Manganese 54.93805 [Ar]3d <sup>5</sup> 4s 7.4340	26 <sup>26</sup> Fe Iron 55.845 [Ar]3d <sup>6</sup> 4s 7.9024	27 <sup>27</sup> Co Cobalt 58.93320 [Ar]3d <sup>7</sup> 4s 7.8810	28 <sup>28</sup> Ni Nickel 58.6934 [Ar]3d <sup>8</sup> 4s 7.6398
39 <sup>39</sup> Y Yttrium 88.90585 [Kr]4d5s 6.2171	40 <sup>40</sup> Zr Zirconium 91.224 [Kr]4d <sup>2</sup> 5s 6.6339	41 <sup>41</sup> Nb Niobium 92.90638 [Kr]4d <sup>4</sup> 5s 6.7589	42 <sup>42</sup> Mo Molybdenum 95.94 [Kr]4d <sup>5</sup> 5s 7.0924	43 <sup>43</sup> Tc Technetium (98) [Kr]4d <sup>5</sup> 5s 7.28	44 <sup>44</sup> Ru Ruthenium 101.07 [Kr]4d <sup>7</sup> 5s 7.3605	45 <sup>45</sup> Rh Rhodium 102.90550 [Kr]4d <sup>8</sup> 5s 7.4589	46 <sup>46</sup> Pd Palladium 106.42 [Kr]4d <sup>10</sup> 8.3369
72 <sup>72</sup> Hf Hafnium 178.49 [Xe]4f <sup>14</sup> 5d <sup>2</sup> 6s 6.8251	73 <sup>73</sup> Ta Tantalum 180.9479 [Xe]4f <sup>14</sup> 5d <sup>3</sup> 6s 7.5496	74 <sup>74</sup> W Tungsten 183.84 [Xe]4f <sup>14</sup> 5d <sup>4</sup> 6s 7.8640	75 <sup>75</sup> Re Rhenium 186.207 [Xe]4f <sup>14</sup> 5d <sup>5</sup> 6s 7.8335	76 <sup>76</sup> Os Osmium 190.23 [Xe]4f <sup>14</sup> 5d <sup>6</sup> 6s 8.4382	77 <sup>77</sup> Ir Iridium 192.217 [Xe]4f <sup>14</sup> 5d <sup>7</sup> 6s 8.9670	78 <sup>78</sup> Pt Platinum 195.078 [Xe]4f <sup>14</sup> 5d <sup>9</sup> 6s 8.9587	79 <sup>79</sup> Au Gold 196.96655 200.59 [Xe]4f <sup>14</sup> 5d <sup>10</sup> 6s 9.2255
104 <sup>104</sup> Rf Rutherfordium (261) [Rn]5f <sup>14</sup> 6d <sup>2</sup> 7s 6.0 ?	105 <sup>105</sup> Db Dubnium (262)	106 <sup>106</sup> Sg Seaborgium (263)	107 <sup>107</sup> Bh Bohrium (264)	108 <sup>108</sup> Hs Hassium (265)	109 <sup>109</sup> Mt Meitnerium (268)	110 <sup>110</sup> Uun Ununillium (269)	111 <sup>111</sup> Uuu Unununium (272)

5 <sup>5</sup> B Boron 10.811 1s <sup>2</sup> 2s <sup>2</sup> 2p 8.2980	6 <sup>6</sup> C Carbon 12.0107 1s <sup>2</sup> 2s <sup>2</sup> 2p <sup>2</sup> 11.2603	7 <sup>7</sup> N Nitrogen 14.00674 1s <sup>2</sup> 2s <sup>2</sup> 2p <sup>3</sup> 14.5341	8 <sup>8</sup> O Oxygen 15.9994 1s <sup>2</sup> 2s <sup>2</sup> 2p <sup>4</sup> 13.6181	9 <sup>9</sup> F Fluorine 18.99840 1s <sup>2</sup> 2s <sup>2</sup> 2p <sup>5</sup> 17.4228	10 <sup>10</sup> Ne Neon 20.1797 1s <sup>2</sup> 2s <sup>2</sup> 2p <sup>6</sup> 21.5646
13 <sup>13</sup> Al Aluminum 26.98154 [Ne]3s <sup>2</sup> 3p 5.9858	14 <sup>14</sup> Si Silicon 28.0855 [Ne]3s <sup>2</sup> 3p <sup>2</sup> 8.1517	15 <sup>15</sup> P Phosphorus 30.97376 [Ne]3s <sup>2</sup> 3p <sup>3</sup> 10.4867	16 <sup>16</sup> S Sulfur 32.066 [Ne]3s <sup>2</sup> 3p <sup>4</sup> 10.3600	17 <sup>17</sup> Cl Chlorine 35.4527 [Ne]3s <sup>2</sup> 3p <sup>5</sup> 12.9676	18 <sup>18</sup> Ar Argon 39.948 [Ne]3s <sup>2</sup> 3p <sup>6</sup> 15.7596
31 <sup>31</sup> Ga Gallium 69.723 [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p 5.9993	32 <sup>32</sup> Ge Germanium 72.61 [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>2</sup> 7.8994	33 <sup>33</sup> As Arsenic 74.92160 [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>3</sup> 9.7886	34 <sup>34</sup> Se Selenium 78.96 [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>4</sup> 9.7524	35 <sup>35</sup> Br Bromine 79.904 [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>5</sup> 11.8138	36 <sup>36</sup> Kr Krypton 83.80 [Ar]3d <sup>10</sup> 4s <sup>2</sup> 4p <sup>6</sup> 13.9996
49 <sup>49</sup> In Indium 114.818 [Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p 5.7864	50 <sup>50</sup> Sn Tin 118.710 [Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>2</sup> 7.3439	51 <sup>51</sup> Sb Antimony 121.760 [Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>3</sup> 8.6084	52 <sup>52</sup> Te Tellurium 127.60 [Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>4</sup> 9.0096	53 <sup>53</sup> I Iodine 126.90447 10.4513 [Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>5</sup> 10.4513	54 <sup>54</sup> Xe Xenon 131.29 12.1298 [Kr]4d <sup>10</sup> 5s <sup>2</sup> 5p <sup>6</sup> 12.1298
81 <sup>81</sup> Tl Thallium 204.3833 [Hg]6p 6.1082	82 <sup>82</sup> Pb Lead 207.2 [Hg]6p <sup>2</sup> 7.4167	83 <sup>83</sup> Bi Bismuth 208.98038 [Hg]6p <sup>3</sup> 7.2856	84 <sup>84</sup> Po Polonium (209) [Hg]6p <sup>4</sup> 8.417 ?	85 <sup>85</sup> At Astatine (210) [Hg]6p <sup>5</sup> 10.7485	86 <sup>86</sup> Rn Radon (222) [Hg]6p <sup>6</sup> 10.7485

Solids  
 Liquids  
 Gases  
 Artificially Prepared

For a description of the atomic data, visit [physics.nist.gov/atomic](http://physics.nist.gov/atomic)

Atomic Number: 58

Ground-state Level: 1G<sub>4</sub>

Symbol: **Ce**

Name: Cerium

Atomic Weight: 140.116

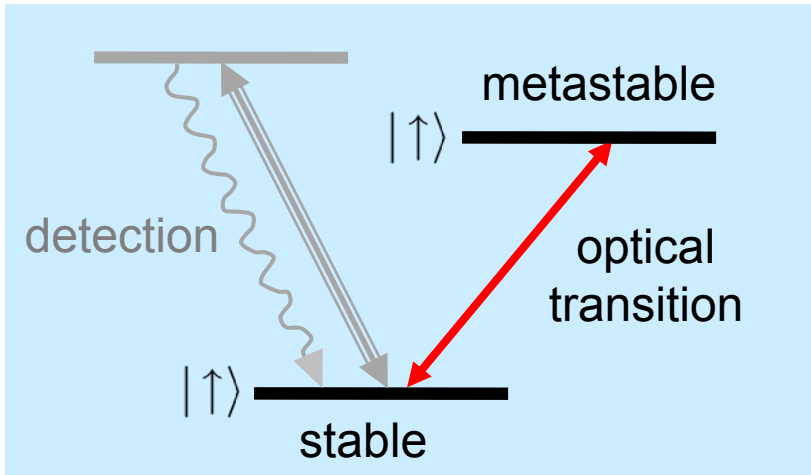
Ground-state Configuration: [Xe]4f5d6s<sup>2</sup>

Ionization Energy (eV): 5.5387

57 <sup>57</sup> La Lanthanum 138.9055 [Xe]5d6s 5.5769	58 <sup>58</sup> Ce Cerium 140.116 [Xe]4f5d6s <sup>2</sup> 5.5387	59 <sup>59</sup> Pr Praseodymium 140.90765 [Xe]4f <sup>3</sup> 6s <sup>2</sup> 5.473	60 <sup>60</sup> Nd Neodymium 144.24 [Xe]4f <sup>4</sup> 6s <sup>2</sup> 5.5250	61 <sup>61</sup> Pm Promethium (145) [Xe]4f <sup>5</sup> 6s <sup>2</sup> 5.6436	62 <sup>62</sup> Sm Samarium 150.36 [Xe]4f <sup>6</sup> 6s <sup>2</sup> 5.6704	63 <sup>63</sup> Eu Europium 151.964 [Xe]4f <sup>7</sup> 6s <sup>2</sup> 6.1501	64 <sup>64</sup> Gd Gadolinium 157.25 [Xe]4f <sup>7</sup> 5d6s <sup>2</sup> 6.1501	65 <sup>65</sup> Tb Terbium 158.92534 [Xe]4f <sup>9</sup> 6s <sup>2</sup> 5.8638	66 <sup>66</sup> Dy Dysprosium 162.50 [Xe]4f <sup>10</sup> 6s <sup>2</sup> 5.9389	67 <sup>67</sup> Ho Holmium 164.93032 [Xe]4f <sup>11</sup> 6s <sup>2</sup> 6.0215	68 <sup>68</sup> Er Erbium 167.26 [Xe]4f <sup>12</sup> 6s <sup>2</sup> 6.1077	69 <sup>69</sup> Tm Thulium 168.93421 [Xe]4f <sup>13</sup> 6s <sup>2</sup> 6.1843	70 <sup>70</sup> Yb Ytterbium 173.04 [Xe]4f <sup>14</sup> 6s <sup>2</sup> 6.2542	71 <sup>71</sup> Lu Lutetium 174.967 [Xe]4f <sup>14</sup> 5d6s 5.4259
89 <sup>89</sup> Ac Actinium (227) [Rn]6d <sup>1</sup> 7s <sup>2</sup> 5.17	90 <sup>90</sup> Th Thorium 232.0381 [Rn]6d <sup>2</sup> 7s <sup>2</sup> 6.3067	91 <sup>91</sup> Pa Protactinium 231.03588 [Rn]5f <sup>2</sup> 6d7s <sup>2</sup> 5.89	92 <sup>92</sup> U Uranium 238.0289 [Rn]5f <sup>3</sup> 6d7s <sup>2</sup> 6.1941	93 <sup>93</sup> Np Neptunium (237) [Rn]5f <sup>4</sup> 7s <sup>2</sup> 6.2657	94 <sup>94</sup> Pu Plutonium (244) [Rn]5f <sup>6</sup> 7s <sup>2</sup> 6.0262	95 <sup>95</sup> Am Americium (243) [Rn]5f <sup>7</sup> 7s <sup>2</sup> 5.9738	96 <sup>96</sup> Cm Curium (247) [Rn]5f <sup>8</sup> 6d7s <sup>2</sup> 5.9915	97 <sup>97</sup> Bk Berkelium (247) [Rn]5f <sup>9</sup> 7s <sup>2</sup> 6.1979	98 <sup>98</sup> Cf Californium (251) [Rn]5f <sup>10</sup> 7s <sup>2</sup> 6.2817	99 <sup>99</sup> Es Einsteinium (252) [Rn]5f <sup>11</sup> 7s <sup>2</sup> 6.42	100 <sup>100</sup> Fm Fermium (257) [Rn]5f <sup>12</sup> 7s <sup>2</sup> 6.50	101 <sup>101</sup> Md Mendelevium (258) [Rn]5f <sup>13</sup> 7s <sup>2</sup> 6.58	102 <sup>102</sup> No Nobelium (259) [Rn]5f <sup>14</sup> 7s <sup>2</sup> 6.65	103 <sup>103</sup> Lr Lawrencium (262) [Rn]5f <sup>14</sup> 7s <sup>2</sup> 7p 4.9 ?

# Trapped ion quantum bits

Ions with optical transition to metastable level:  $^{40}\text{Ca}^+$ ,  $^{88}\text{Sr}^+$ ,  $^{172}\text{Yb}^+$

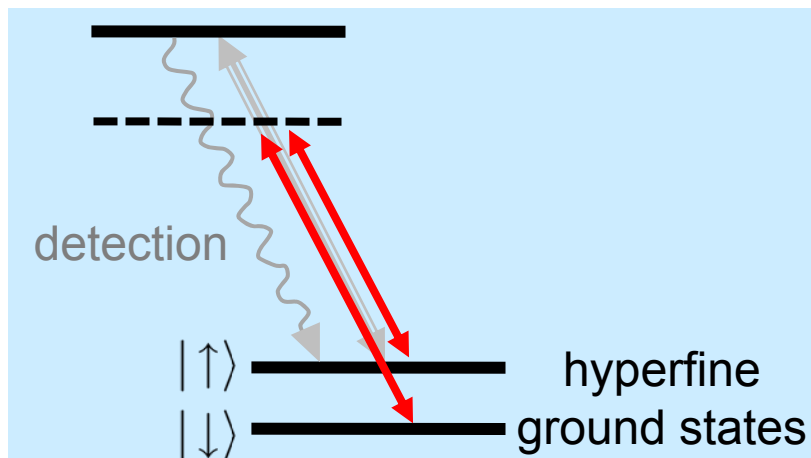


„optical qubit“

qubit manipulation requires  
ultrastable laser

$$\Psi = \alpha|\downarrow\rangle + \beta|\uparrow\rangle$$

Ions with hyperfine structure:  $^9\text{Be}^+$ ,  $^{25}\text{Mg}^+$ ,  $^{43}\text{Ca}^+$ ,  $^{111}\text{Cd}^+$ ,  $^{171}\text{Yb}^+$ ...

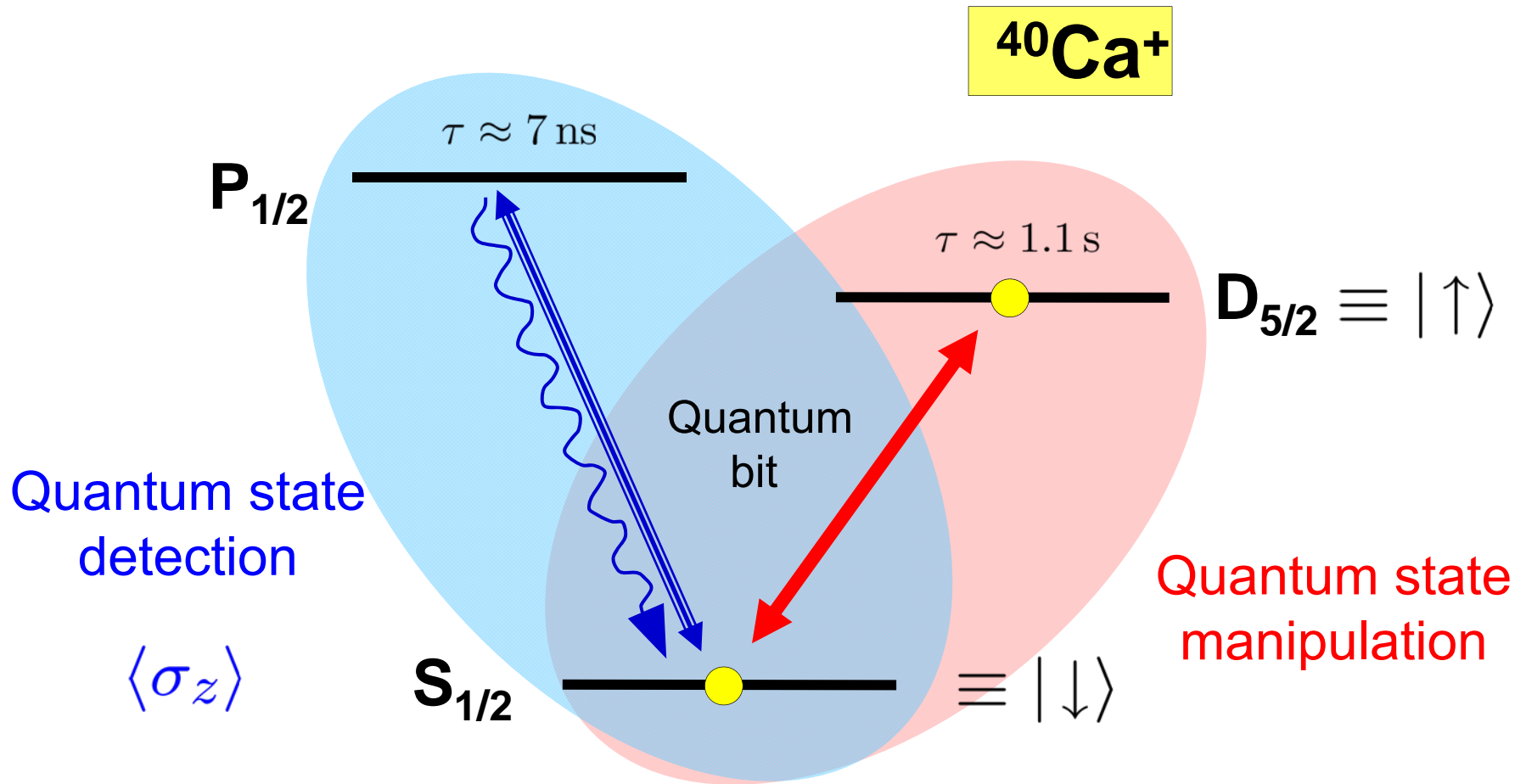


„hyperfine qubit“

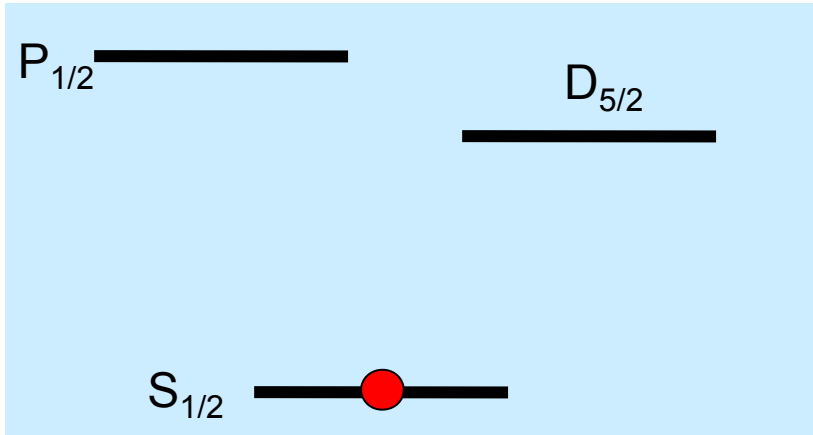
qubit manipulation with  
microwaves or lasers (Raman transitions)



# Qubit manipulation and measurement



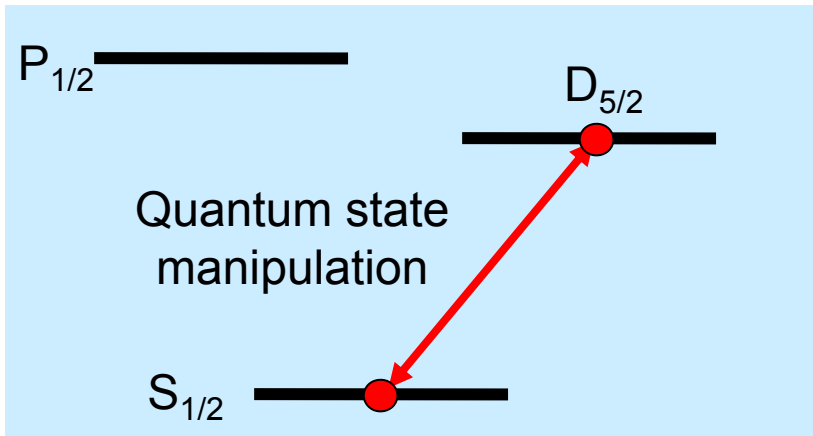
# Experimental sequence



1. Initialization in a pure quantum state



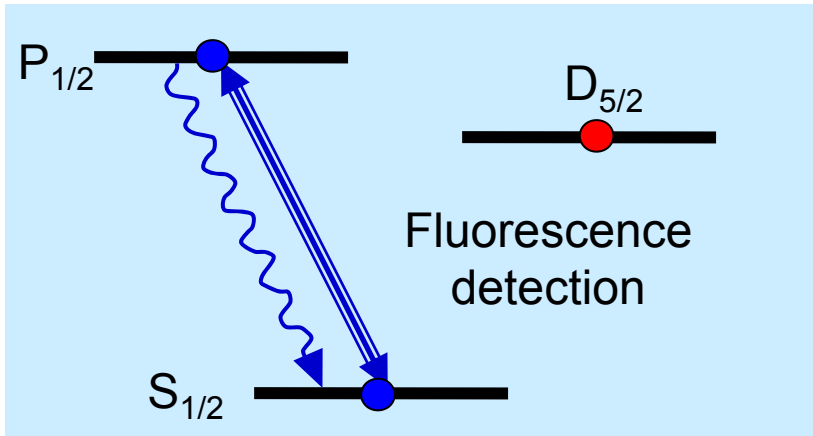
# Experimental sequence



1. Initialization in a pure quantum state

2. Quantum state manipulation on  $S_{1/2} - D_{5/2}$  transition

# Experimental sequence



1. Initialization in a pure quantum state

1-10 ms

2. Quantum state manipulation on  $S_{1/2} - D_{5/2}$  transition

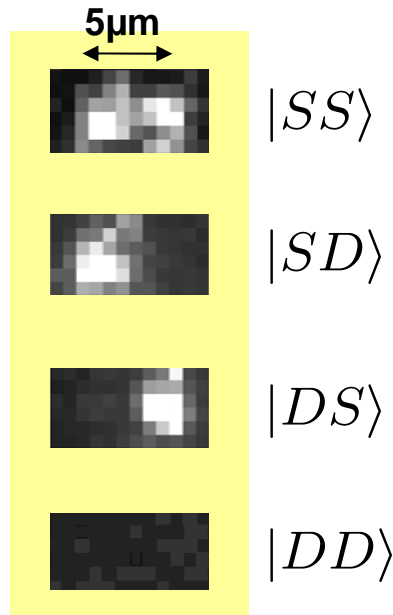
0.01- 5 ms

3. Quantum state measurement by fluorescence detection

0.2- 5 ms

Two ions:

Spatially resolved detection with CCD camera:



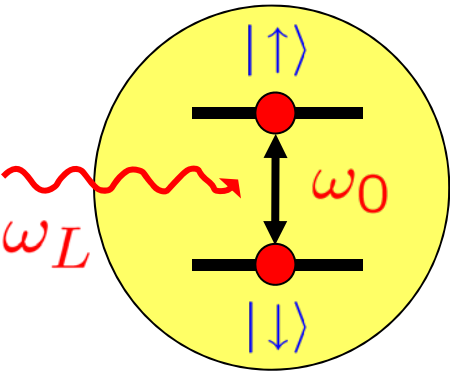
50 experiments / s

Repeat experiments  
100 - 1000 times

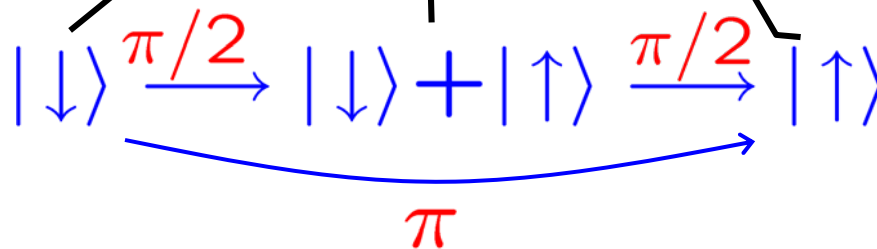
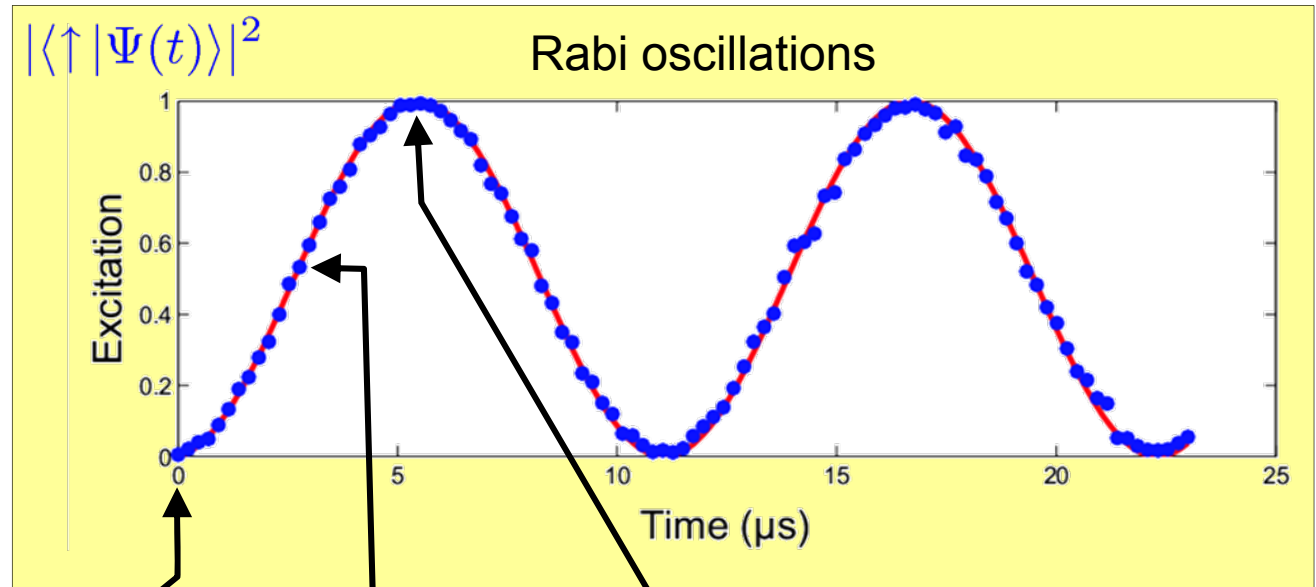
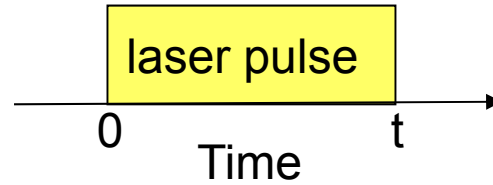
2- 20 s

# Ion-laser interaction

Resonant coherent excitation:



$$\omega_L = \omega_0$$



# Qubit superposition states

**Schrödinger picture:**

$$|\psi(t=0)\rangle \propto |\downarrow\rangle + |\uparrow\rangle \longrightarrow |\psi(t)\rangle \propto |\downarrow\rangle + e^{-i\omega_0 t} |\uparrow\rangle$$

Phase evolution: for optical qubits  $\omega_0 \sim 10^{15} \text{ s}^{-1}$

**Interaction picture:**

$$|\psi(t)\rangle \propto |\downarrow\rangle + |\uparrow\rangle \quad \text{independent of time}$$

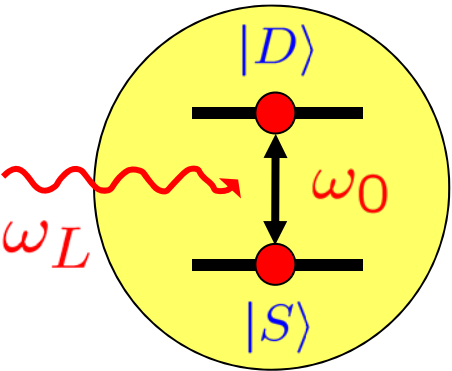
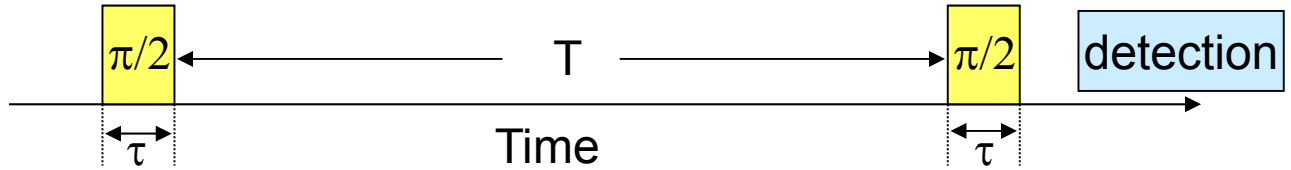
$$|\psi(t)\rangle = \cos(\theta/2) |\downarrow\rangle + e^{i\phi} \sin(\theta/2) |\uparrow\rangle$$

The phase  $\phi$  of the superposition compares two oscillatory phenomena:

- Evolution of the Bloch vector in time
- Evolution of the electromagnetic field of the laser exciting the qubit

# Ramsey spectroscopy for phase estimation

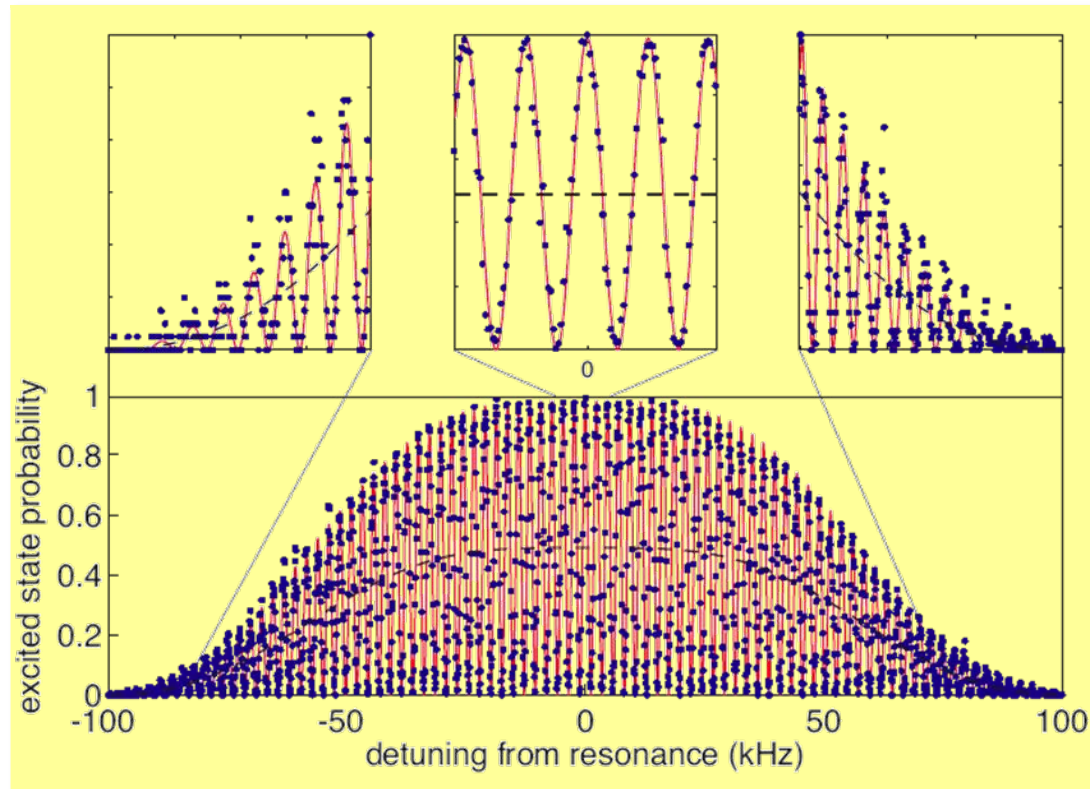
Two-pulse excitation:



$$\Delta = \omega_L - \omega_0 \approx 0$$

$$|S\rangle \xrightarrow{\pi/2} |S\rangle + |D\rangle \xrightarrow{\text{wait}} |S\rangle + e^{-i\Delta T} |D\rangle \xrightarrow{\pi/2} |D\rangle$$

$$\xrightarrow{\pi/2} |S\rangle$$

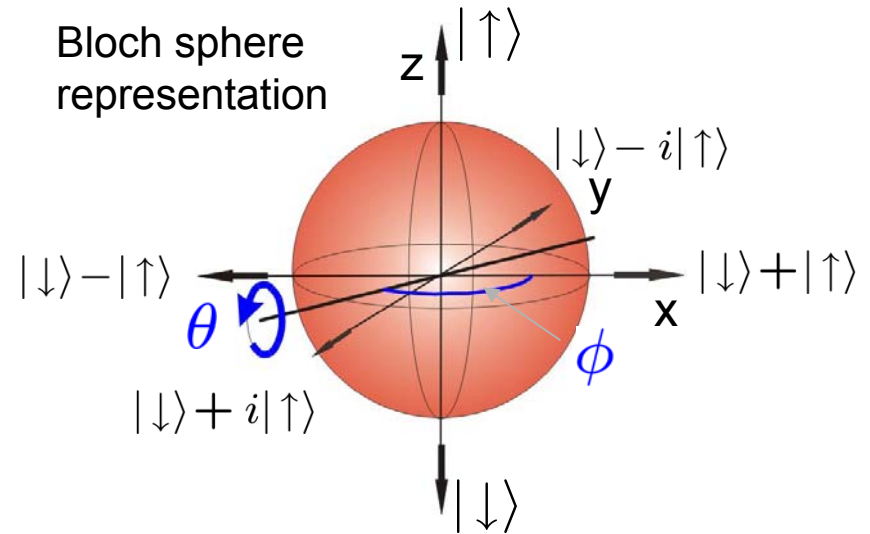


# Resonant excitation in Bloch sphere picture

$$H = \hbar \frac{\Omega}{2} (\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi})$$

$\sim$  Laser intensity

Laser phase



Example:  $\phi = 0 \longrightarrow H = \hbar \frac{\Omega}{2} \sigma_x$

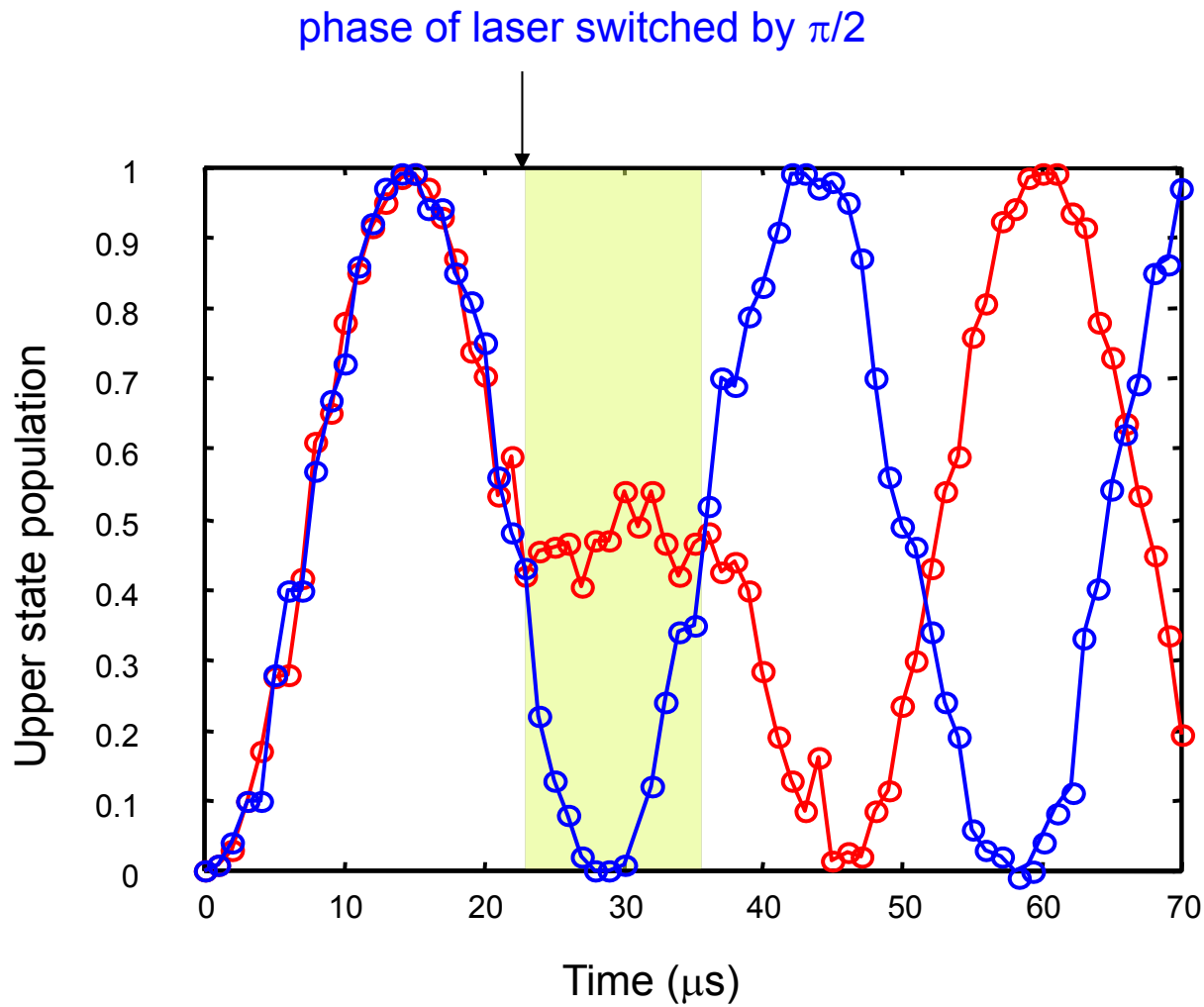
Time evolution operator:

$$U = \exp\left(-\frac{i}{\hbar} H t\right) = \exp\left(-i \frac{\Omega t}{2} \sigma_x\right) = \cos\left(\frac{\Omega t}{2}\right) - i \sin\left(\frac{\Omega t}{2}\right) \sigma_x$$

For  $\theta = \Omega t = \pi/2$

$$U|\downarrow\rangle = \frac{1}{\sqrt{2}}(I - i\sigma_x)|\downarrow\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle - i|\uparrow\rangle)$$

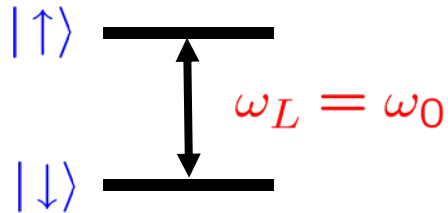
# Resonant qubit excitation





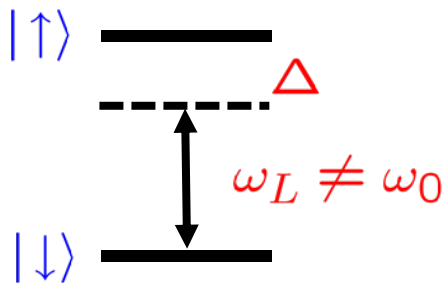
# Qubit manipulation

## Resonant excitation



$$H \propto \sigma_x \quad \text{or} \quad H \propto \sigma_y$$

## Off-resonant excitation



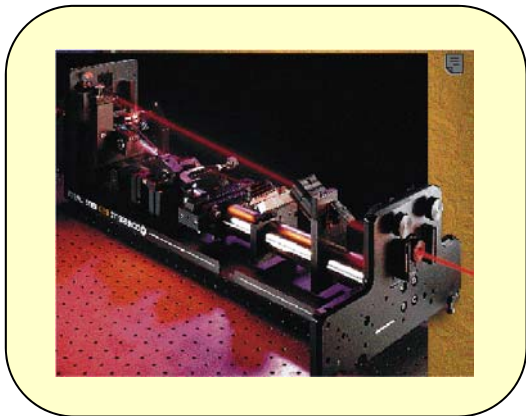
$$H \propto \sigma_z$$

ac-Stark shifts shift qubit transition frequency

Arbitrary Bloch sphere rotations can be synthesized by a combination of laser pulses.

# Laser setup for manipulating the qubit

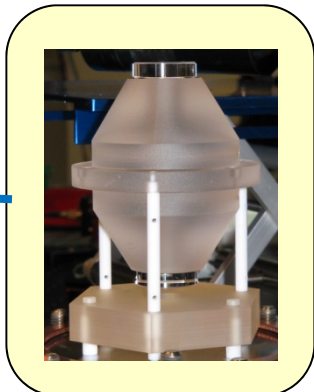
Ti:Sa laser @ 729nm



$\Delta\nu \approx 500\text{kHz}$

feedback on laser frequency

$\rightarrow \Delta\nu \approx 1\text{Hz}$

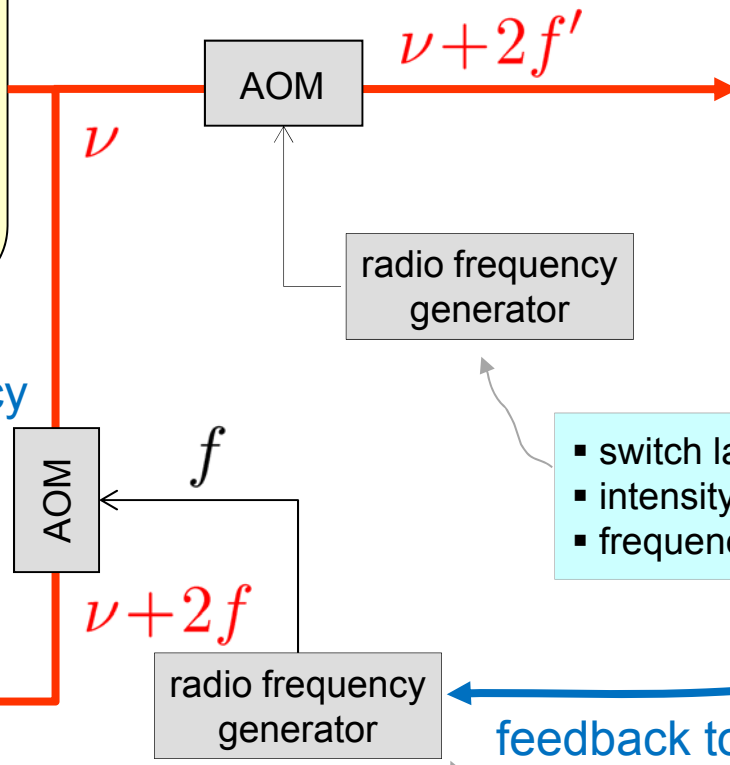
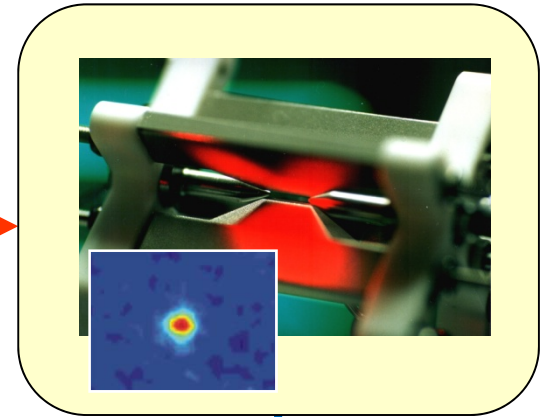


Fabry-Perot resonator

fineness  $F = 400000$ ,  
line width  $\approx 5\text{kHz}$

drift rate  $< 0.2\text{ Hz/s}$

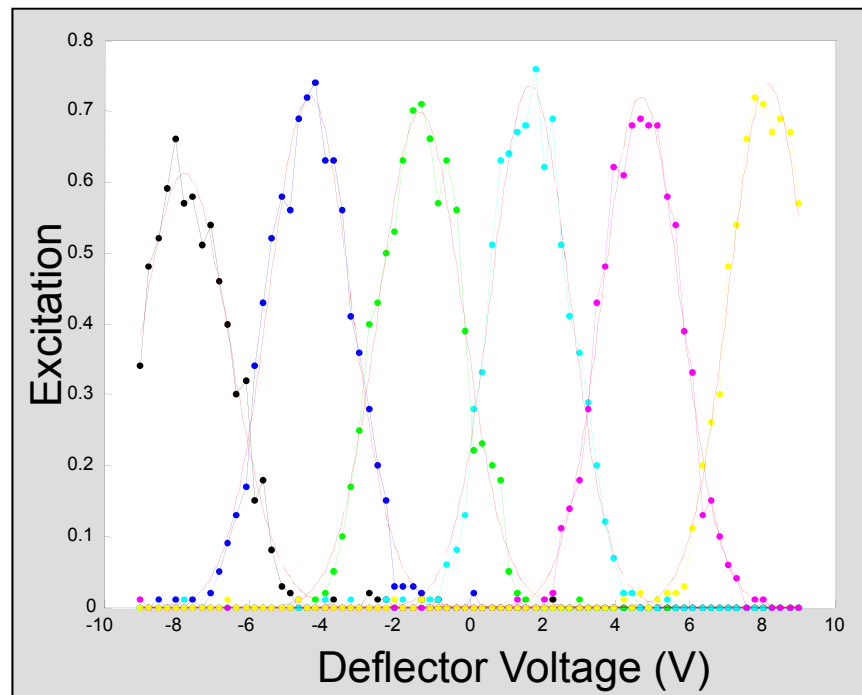
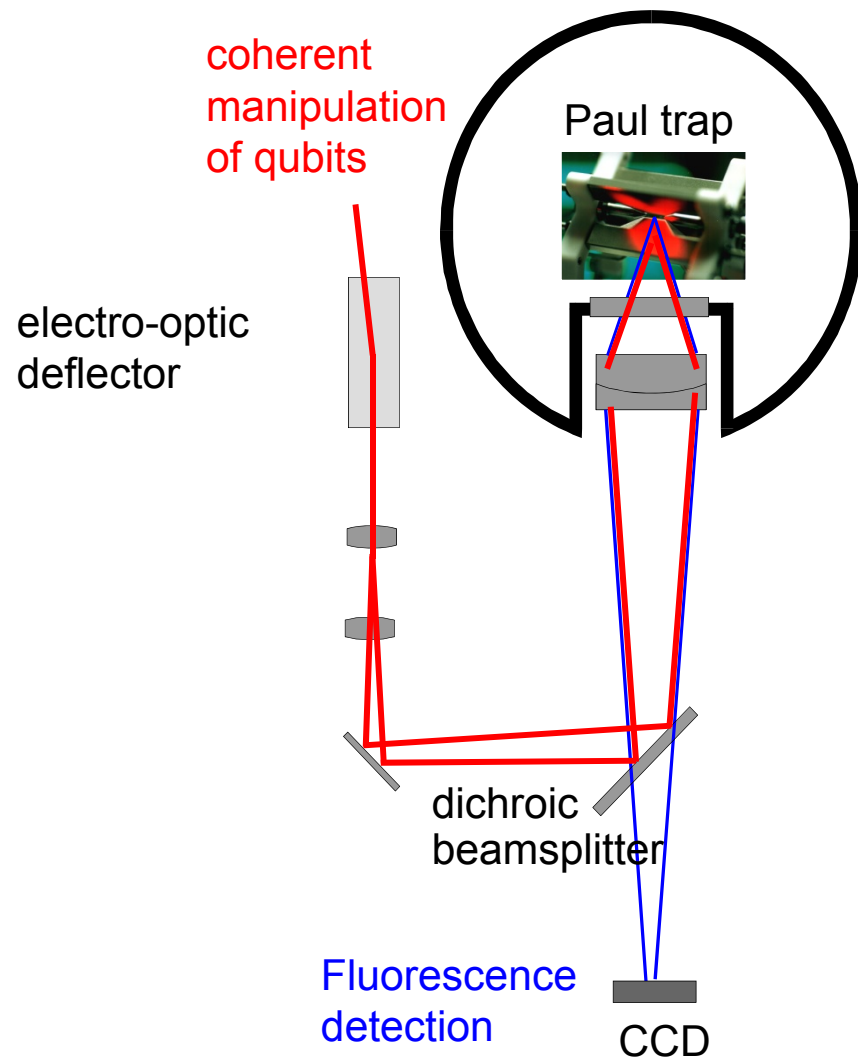
Trapped ion



- switch laser on/off
- intensity control
- frequency control

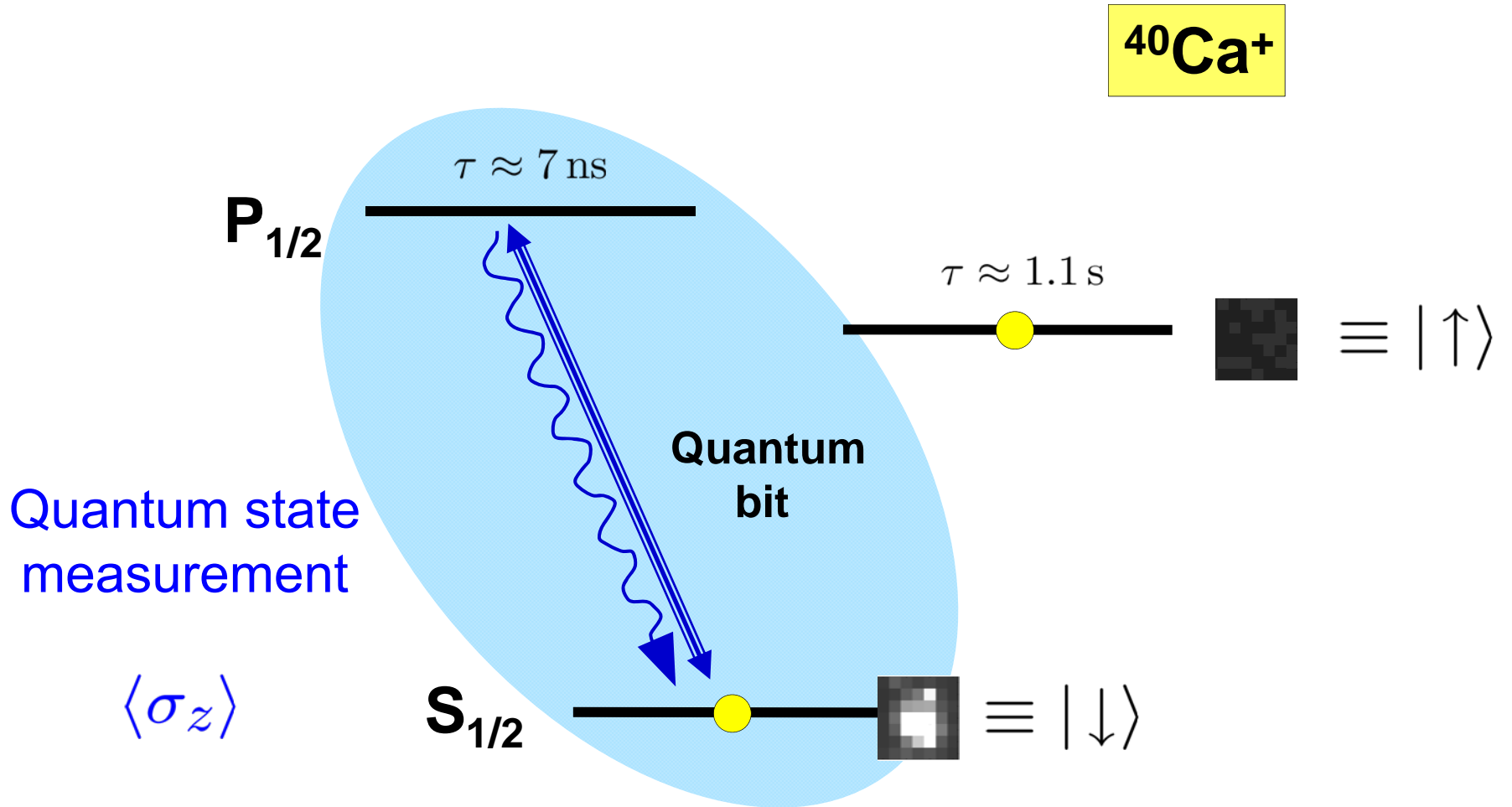
tune laser frequency into resonance with resonator frequency

# Addressing of individual ions with a focussed laser beam



- inter ion distance:  $\sim 4 \mu\text{m}$
- addressing waist:  $\sim 2 \mu\text{m}$
- $< 0.1\%$  intensity on neighbouring ions

# Measuring qubits



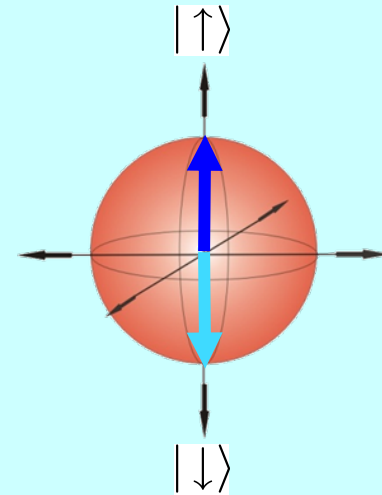
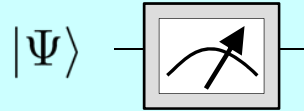
detection errors  $\sim 0.1\%$

# Further quantum measurements

## Measurement of $\sigma_z$

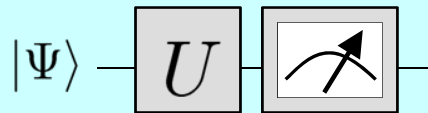
Fluorescence measurements:

$$\langle \Psi | \sigma_z | \Psi \rangle$$



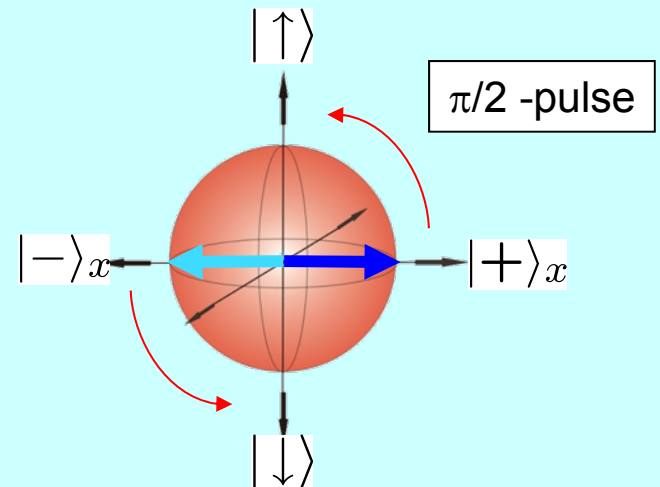
## Measurement of $\sigma_x$

Unitary transformation + fluorescence measurements



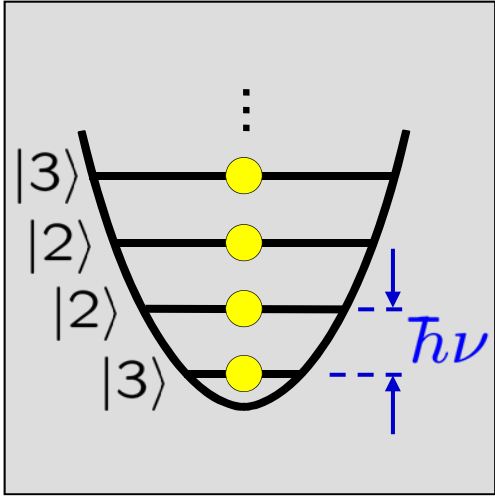
$$\langle U\Psi | \sigma_z | U\Psi \rangle = \langle \Psi | U^\dagger \sigma_z U | \Psi \rangle$$

$$A = U^\dagger \sigma_z U$$



# Coupling internal and vibrational degrees of freedom

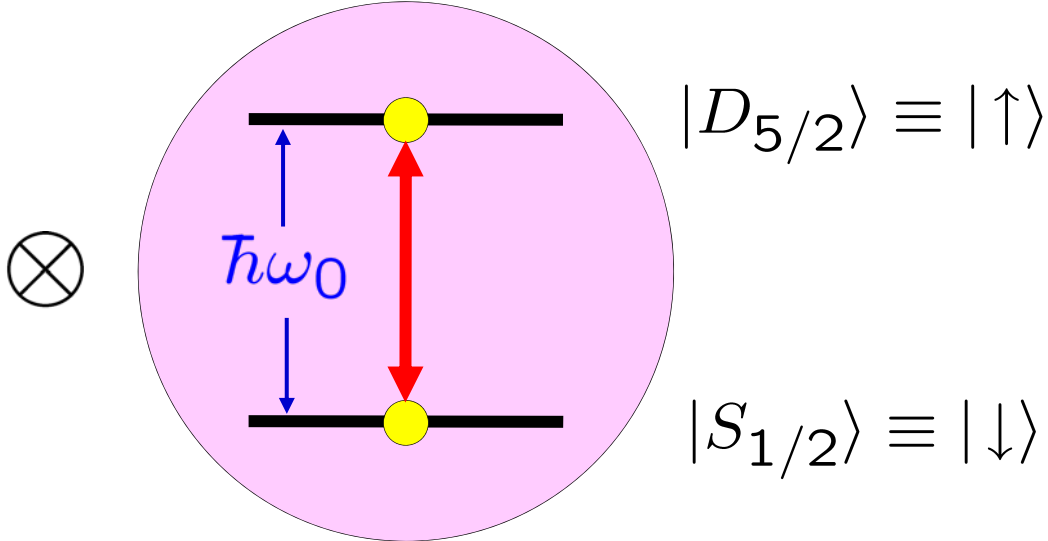
Harmonic oscillator



motional states

$|0\rangle, |1\rangle, |2\rangle, |3\rangle, \dots$

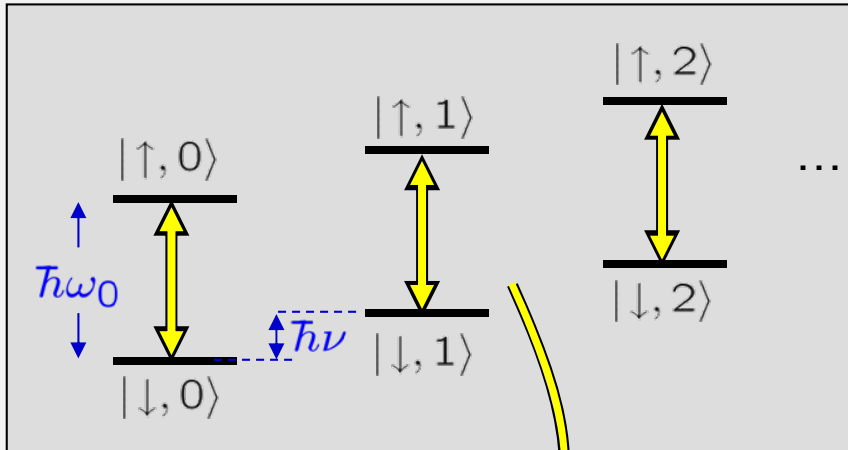
Quantum bit



internal states

$|\uparrow\rangle, |\downarrow\rangle$

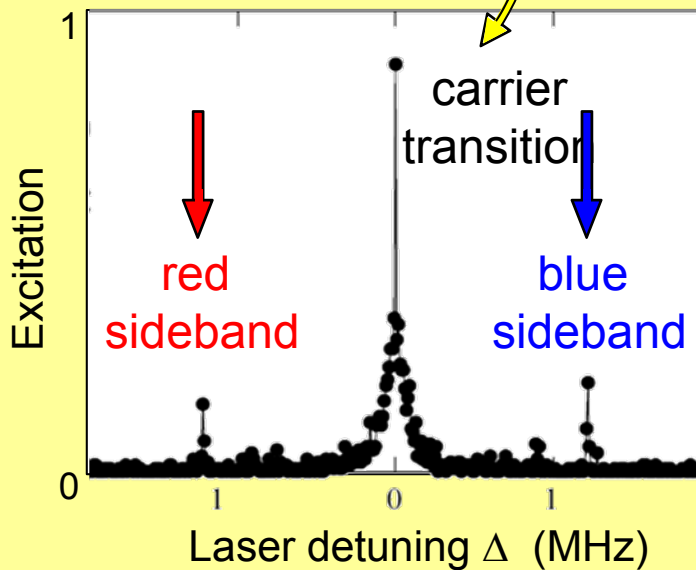
# Trapped-ion laser interactions



qubit manipulation

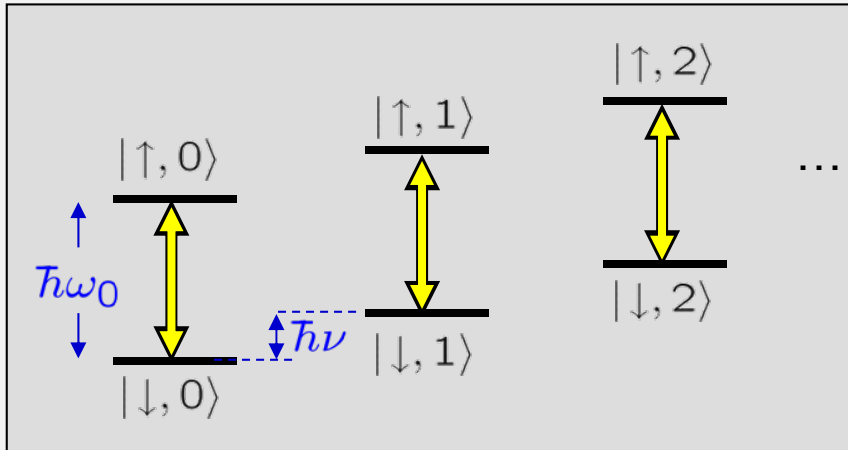
$$\omega_{laser} = \omega_0$$

$$H \propto \sigma_x, H \propto \sigma_y$$





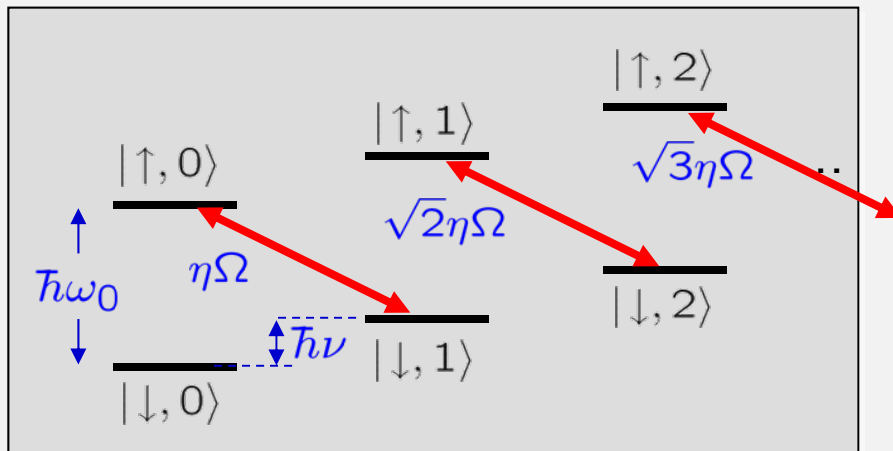
# Trapped-ion laser interactions



qubit manipulation

$$\omega_{laser} = \omega_0$$

$$H \propto \sigma_x, H \propto \sigma_y$$

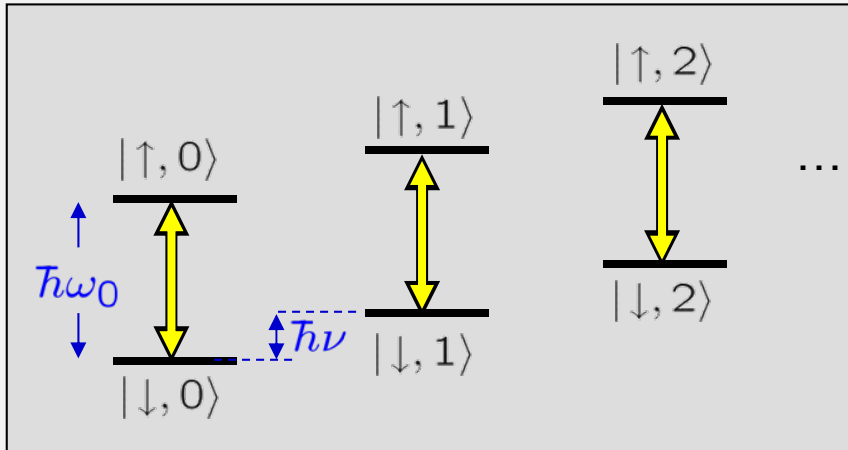


qubit-motion coupling

$$\omega_{laser} = \omega_0 - \nu$$

$$H \propto \sigma_+ a + \sigma_- a^\dagger$$

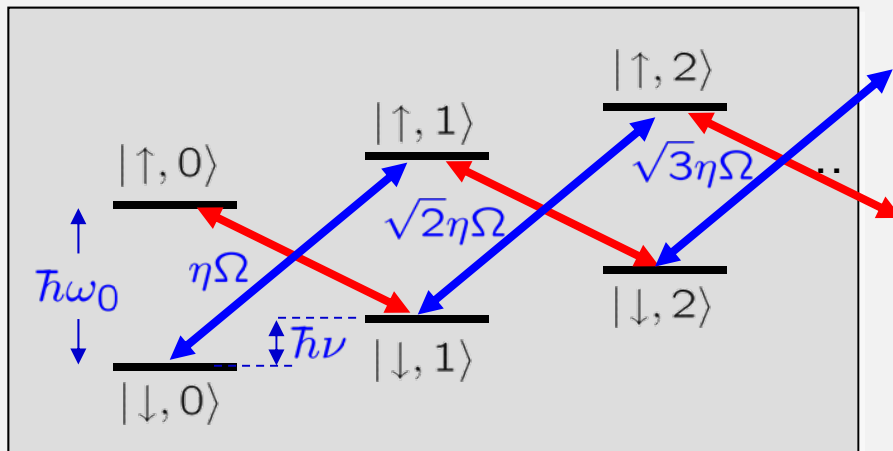
# Trapped-ion laser interactions



qubit manipulation

$$\omega_{laser} = \omega_0$$

$$H \propto \sigma_x, H \propto \sigma_y$$



qubit-motion coupling

$$\omega_{laser} = \omega_0 - \nu$$

$$H \propto \sigma_+ a + \sigma_- a^\dagger$$

$$\omega_{laser} = \omega_0 + \nu$$

$$H \propto \sigma_+ a^\dagger + \sigma_- a$$

# Sideband excitation

$$H^{(i)} = \frac{\hbar\Omega}{2} \sigma_+ e^{-i\delta t + i\phi} (I + i\eta(a^\dagger e^{i\nu t} + a e^{-i\nu t}) + \mathcal{O}(\eta^2)) + \text{h.c.}$$

Red sideband:  $\delta = -\nu$

$$H_{int} = \frac{\hbar\Omega}{2} i\eta \{ \sigma_+ a e^{+i\phi} - \sigma_- a^\dagger e^{-i\phi} \}$$

$$|g, n\rangle \longleftrightarrow |e, n - 1\rangle$$

‘Jaynes-Cummings-Hamiltonian’

- Coupling strength dependent on n

Blue sideband:  $\delta = +\nu$

$$H_{int} = \frac{\hbar\Omega}{2} i\eta \{ \sigma_+ a^\dagger e^{+i\phi} - \sigma_- a e^{-i\phi} \}$$

$$|g, n\rangle \longleftrightarrow |e, n + 1\rangle$$

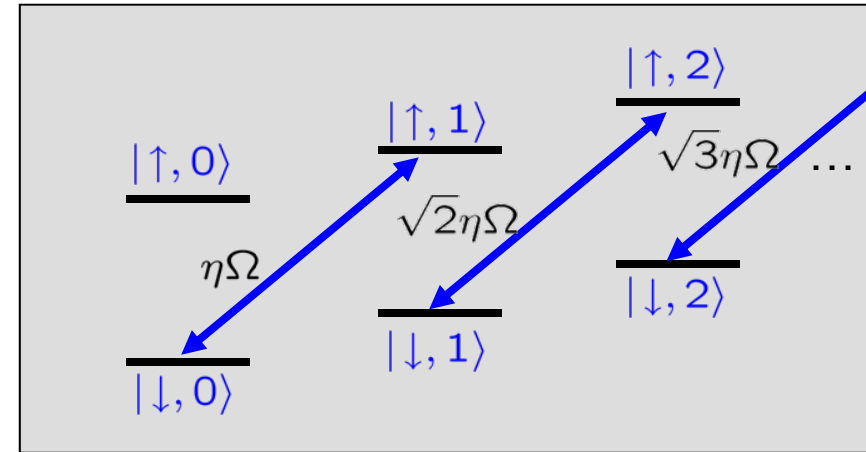
‘anti-Jaynes-Cummings-Hamiltonian’

- Coupling strength dependent on n

# Coherent excitation on the sideband

„Blue sideband“ pulses:

$$|\downarrow\rangle|n\rangle \longleftrightarrow |\uparrow\rangle|n+1\rangle$$

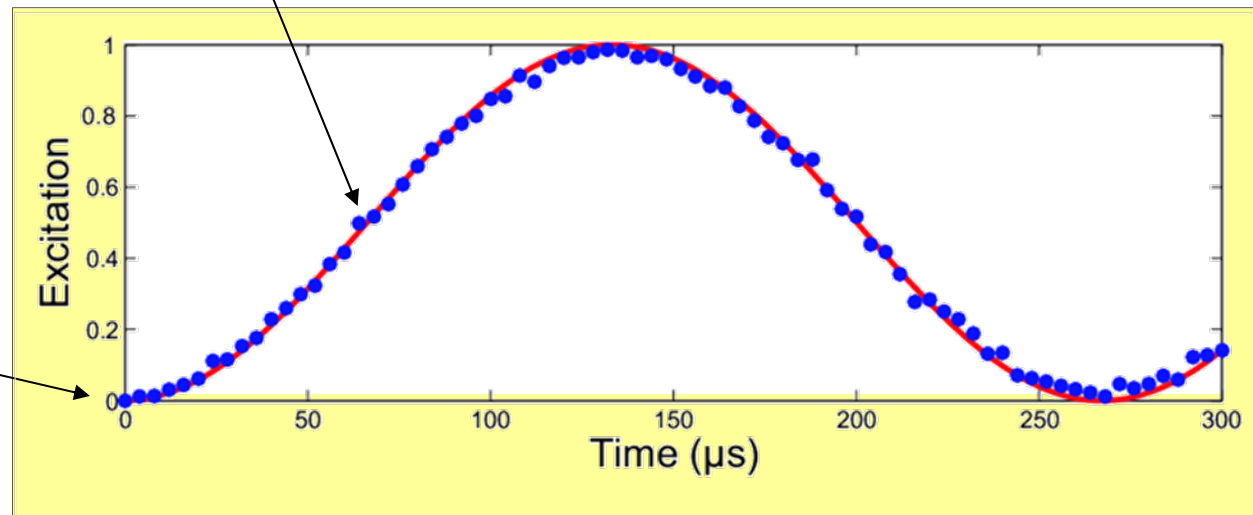


$\theta = \pi/2$  : Entanglement between internal and motional state !

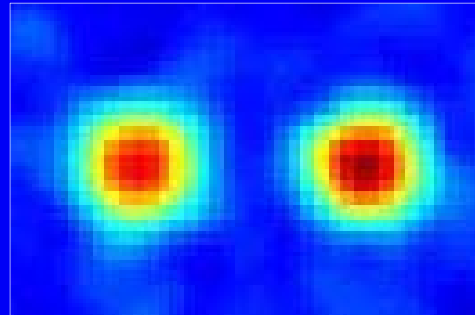
$$\frac{1}{\sqrt{2}} (|\downarrow, n=0\rangle + |\uparrow, n=1\rangle)$$

upper state population

$$|\downarrow, n=0\rangle$$

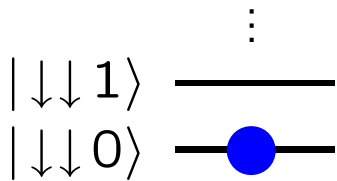
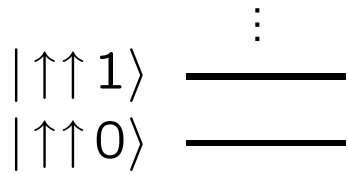


**Entangling a pair of trapped ions  
+  
detecting the entanglement**



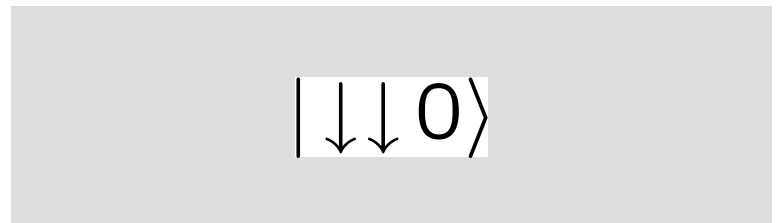
# Generation of Bell states

Two-ion energy levels

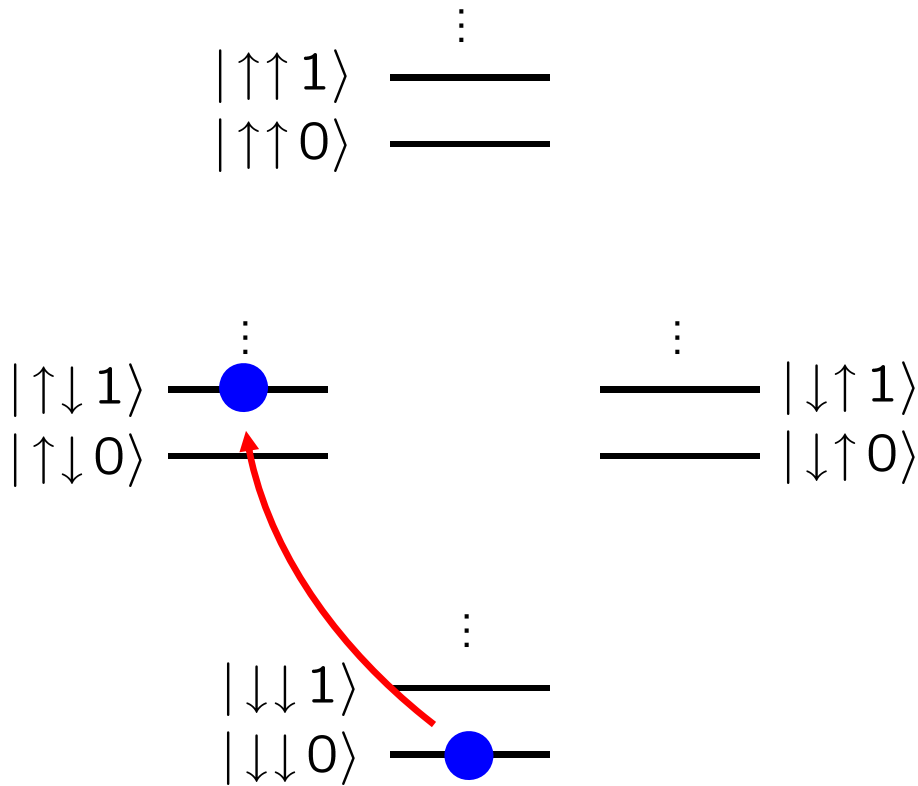


Pulse sequence:

Ion	Pulse length	Transition



# Generation of Bell states

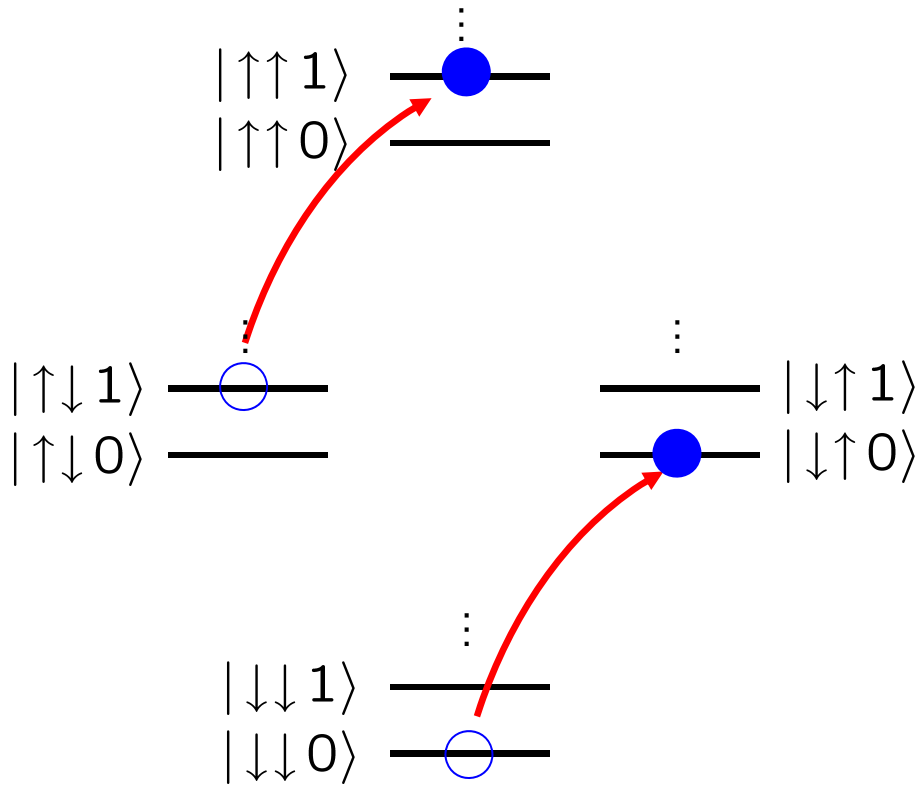


Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband

$$|\downarrow\downarrow 0\rangle + |\uparrow\downarrow 1\rangle$$

# Generation of Bell states



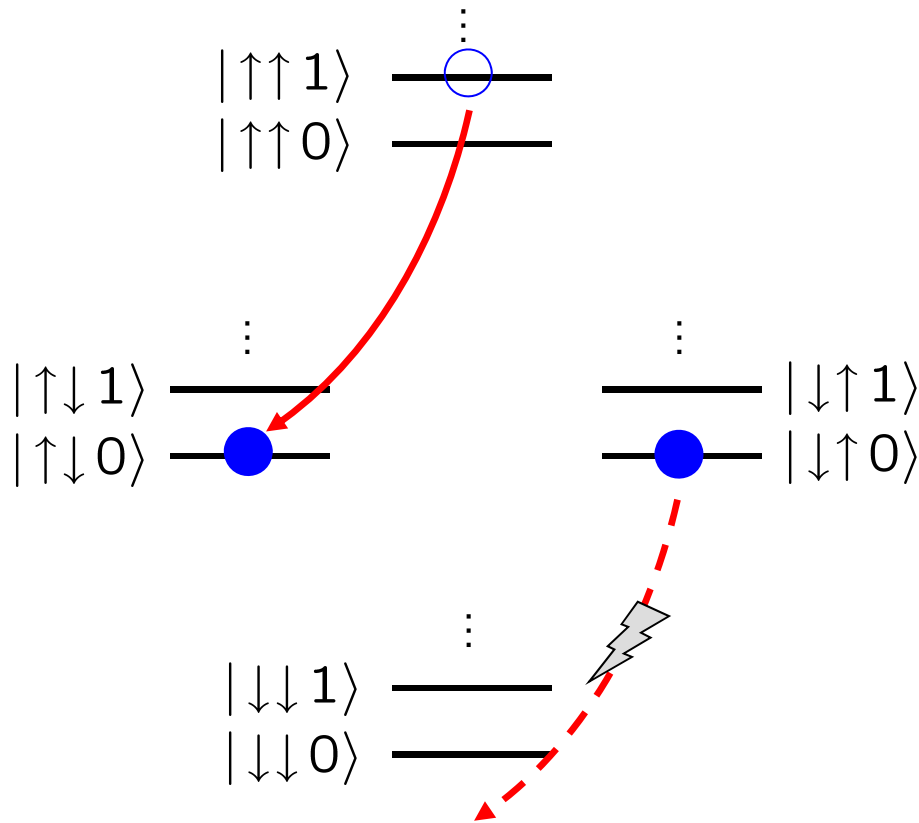
Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband
2	$\pi$	carrier

$$|\downarrow\uparrow 0\rangle + |\uparrow\uparrow 1\rangle$$



# Generation of Bell states



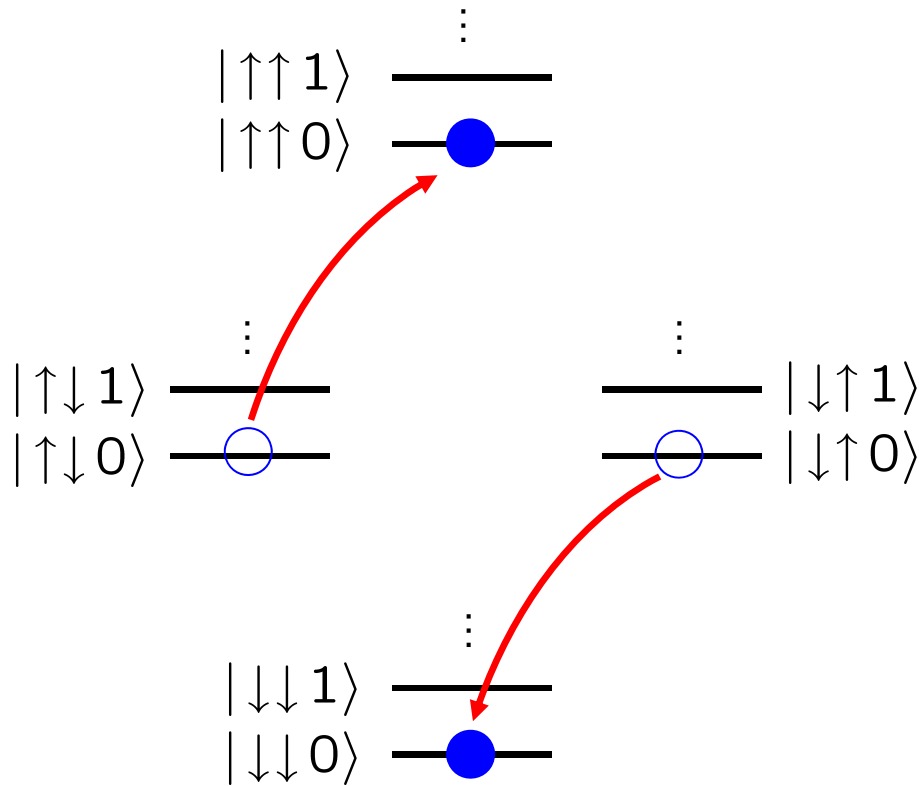
Entanglement !

Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband
2	$\pi$	carrier
2	$\pi$	blue sideband

$$(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)|0\rangle$$

# Generation of Bell states



Entanglement !

Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband
2	$\pi$	carrier
2	$\pi$	blue sideband
2	$\pi$	carrier

$$(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)|0\rangle$$

# Measuring the entangled state

We hope to create the state  $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$

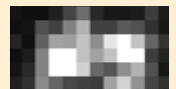
There are no pure states in experimental physics!

The state created in the experiment has to be described by a density matrix  $\rho_{exp}$ .

How can we analyze the state  $\rho_{exp}$  we created?

Fluorescence  
detection with  
CCD camera:

$|\downarrow\downarrow\rangle$



$|\downarrow\uparrow\rangle$



$|\downarrow\downarrow\rangle$



$|\uparrow\uparrow\rangle$

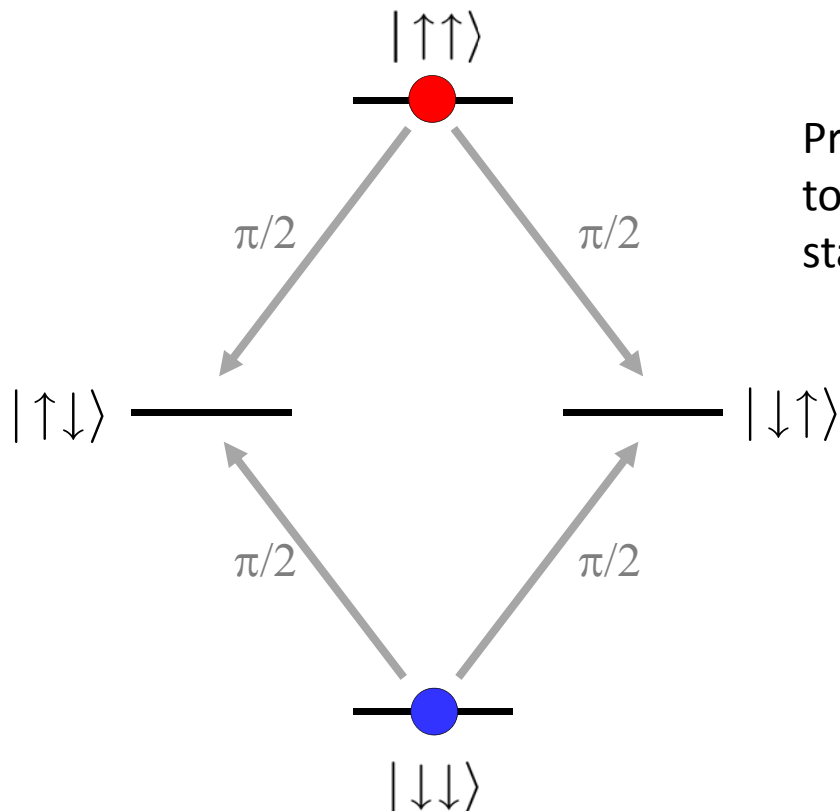


Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

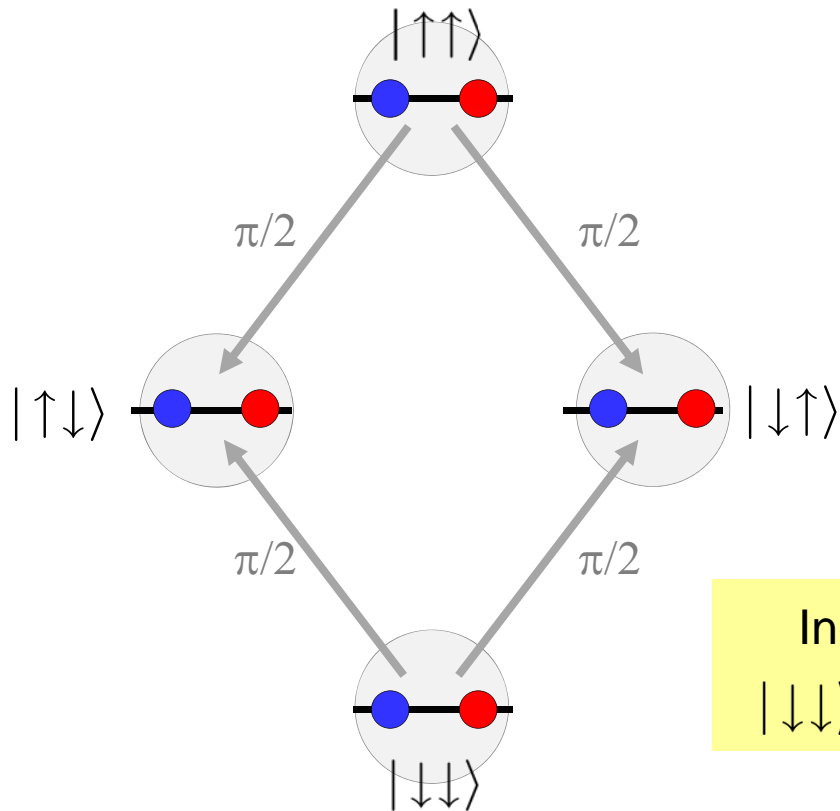
# Entanglement check : interference

To check whether the two amplitudes are phase-coherent, we have to make an interference experiment:



Prior to the state measurement, we apply  $\pi/2$  to both ions which couple the  $|\downarrow\downarrow\rangle$  and the  $|\downarrow\downarrow\rangle$  states to the intermediate states  $|\uparrow\downarrow\rangle$  and  $|\downarrow\uparrow\rangle$ .

# Entanglement check : interference

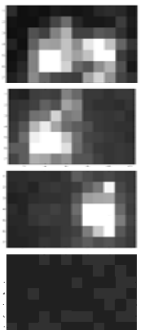


Incoherent mixture:

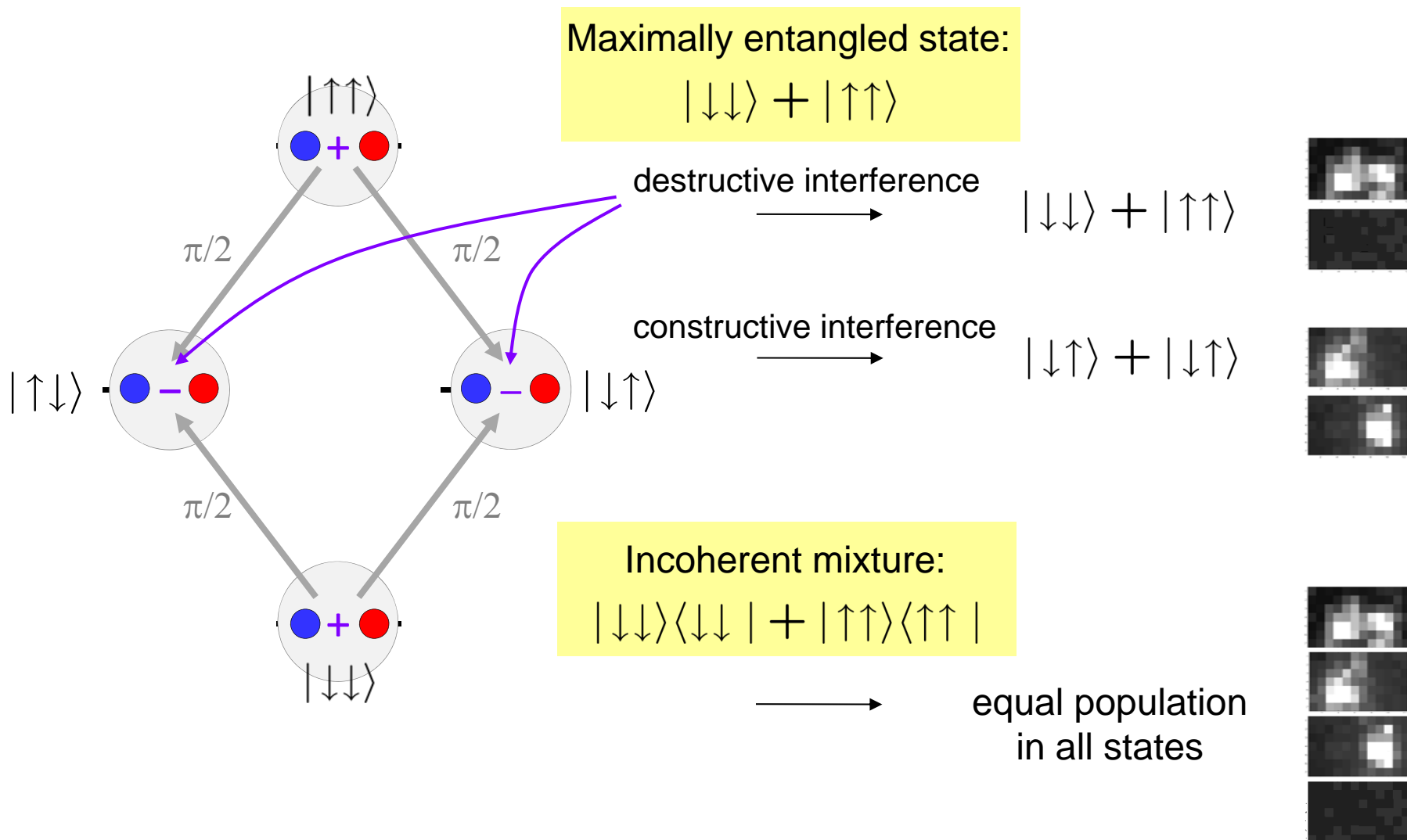
$$|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + |\uparrow\uparrow\rangle\langle\uparrow\uparrow|$$



equal population  
in all states

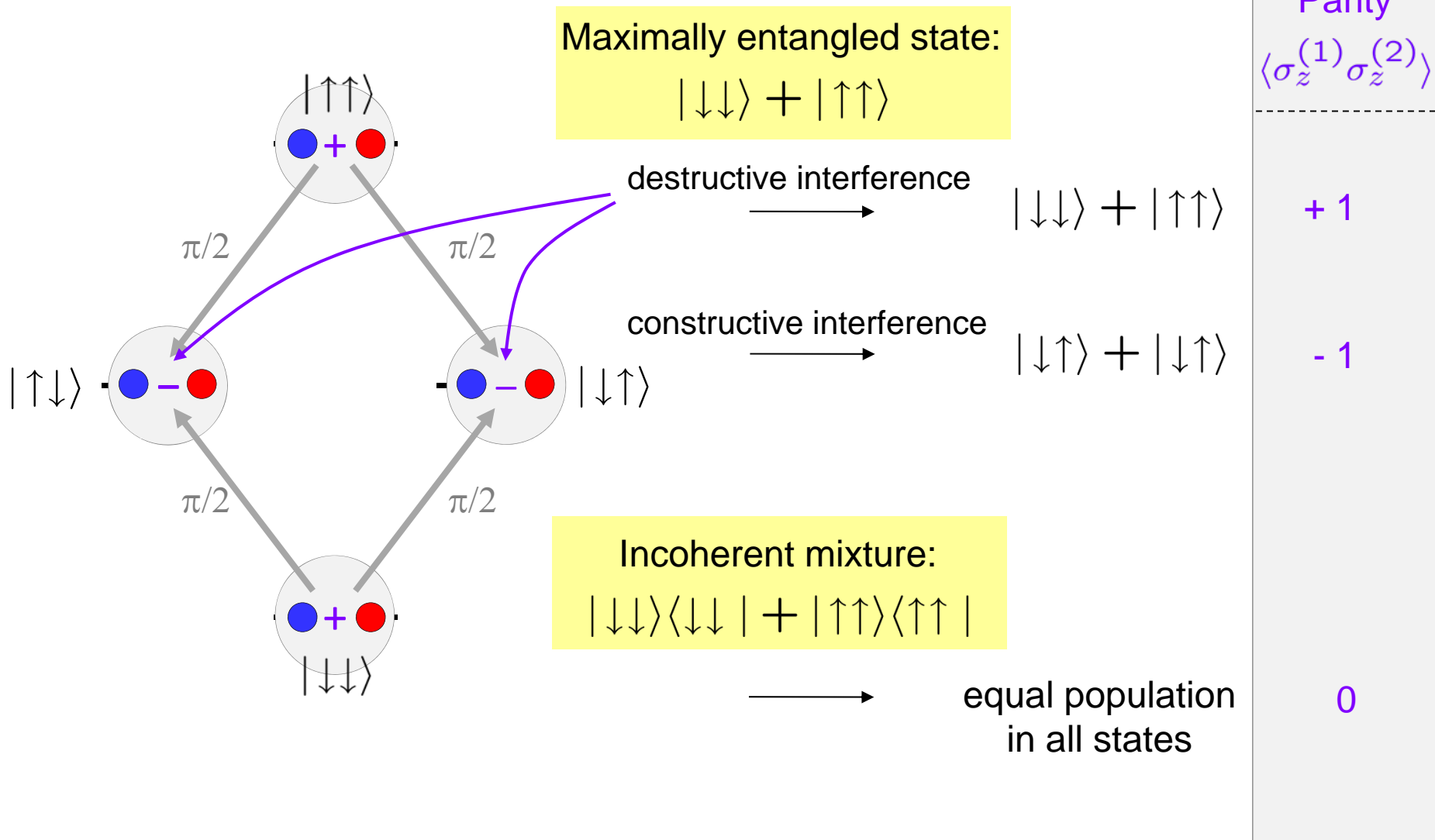


# Entanglement check : interference



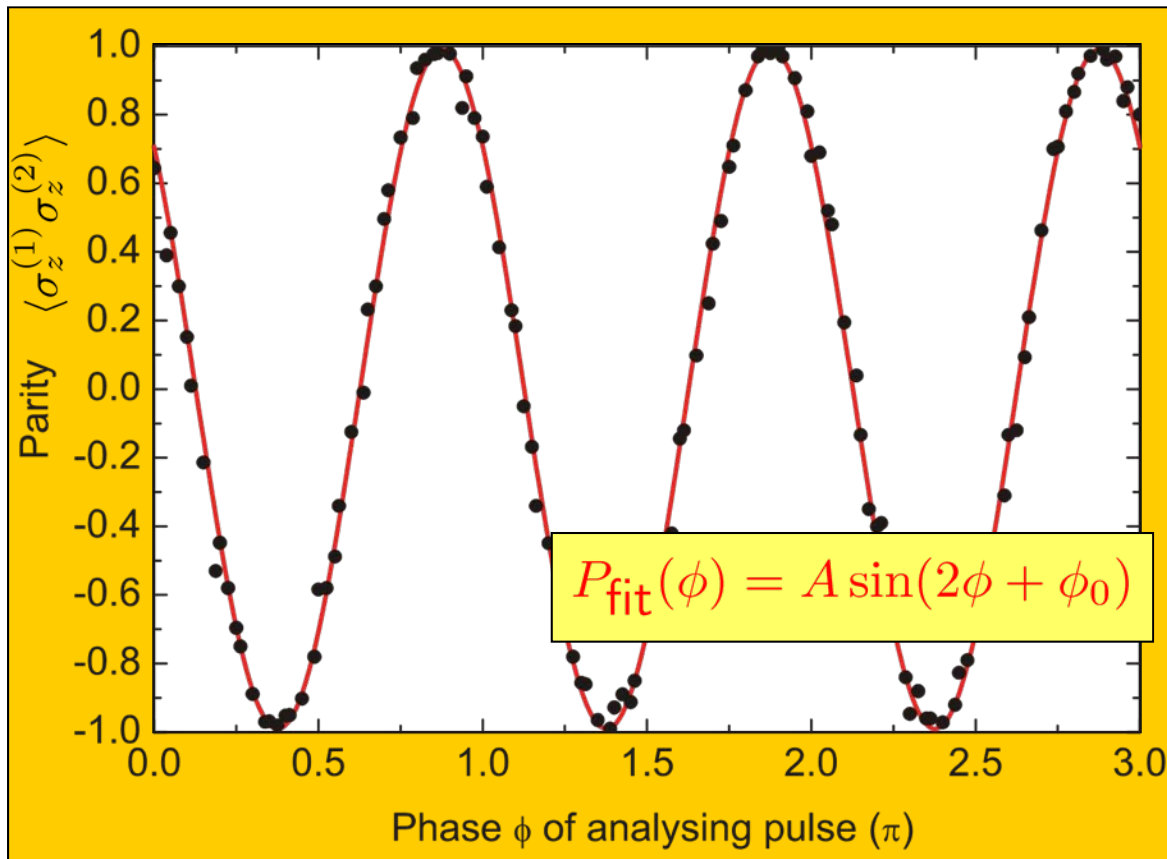
Entanglement check: Scan laser phase  $\phi$  and measure parity

# Entanglement check : interference



Entanglement check: Scan laser phase  $\phi$  and measure parity

# Mølmer-Sørensen gate: parity oscillations



Bell state:  
$$\Psi = |\downarrow\downarrow\rangle + i|\uparrow\uparrow\rangle$$

different entanglement  
creation in this experiment

$A = 0.990(1)$  29,400 measurements

$p_{\downarrow\downarrow} + p_{\uparrow\uparrow} = 0.9965(4)$  13,000 measurements

Bell state fidelity

$F = 99.3(1)\%$



# Creating entanglement vs. entangling quantum gates

## Bell state generation:

This sequence of laser pulses transforms a product state into an entangled state.

$$|\downarrow\downarrow\rangle|0\rangle \rightarrow (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)|0\rangle$$

Pulse sequence:

Ion	Pulse length	Transition
1	$\pi/2$	blue sideband
2	$\pi$	carrier
2	$\pi$	blue sideband
2	$\pi$	carrier

**But it is not a two-qubit quantum gate!!**

Why not?

Check what the sequence does to the initial state  $|\uparrow\uparrow\rangle|0\rangle$  !

Does the motional state return to the ground state at the end of the sequence?