# Constraints on the Higgs decay to a photon and a dark photon 

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Phys.Rev.Lett. 130 (2023) 14, 141801, arXiv:2205.10976
Long version: arXiv:2304.04165

National Center for Theoretical Sciences, Taipei, Taiwan

LHC DM WG spring meeting
May 16th, 2023

## General context I

The dark photon $A^{\prime}$ is an hypothetical Abelian gauge boson that can mix with the photon and has been the subject of extensive theoretical and experimental studies.

A potential discovery channel that has received considerable attention is the decay

$$
h \rightarrow A A^{\prime}
$$

where $A^{\prime}$ is assumed to be (effectively) massless and invisible.

## General context II

An extensive phenomenological literature on the subject

- Gabrielli '14 [Phys.Rev.D 90 no. 5, (2014) 055032]
- Biswas '15 [JHEP 06 (2015) 102]
- Biswas '16 [Phys.Rev.D 93 no. 9, (2016) 093011]
- Biswas '17 [Phys.Rev.D 96 no. 5, (2017) 055012]
which predicted an upper limit $\mathrm{BR}\left(h \rightarrow A A^{\prime}\right)<5 \%$.

Motivations:

- Predicted by certain BSM models (flavour)
- Potentially observable
- Interesting experimental channel


## General context III

This motivated four experimental searches

- CMS '19 [JHEP 10 (2019) 139]

Z-associated channel

- CMS '20 [JHEP 03 (2021) 011]

VBF channel

- ATLAS '21 [Eur.Phys.J.C 82 no. 2, (2022) 105] VBF channel
- ATLAS '22 [2212.09649]

Z-associated channel
The strongest limit obtained is $\mathrm{BR}\left(h \rightarrow A A^{\prime}\right)<1.8 \%$ at $95 \% \mathrm{CL}$ by ATLAS '21.

## Overview

Goal:
Investigate experimental and theoretical constraints on $h \rightarrow A A^{\prime}$

Improvements:

- Additional constraints
- More rigorous treatment
- More recent data

Constraints:

- Higgs signal strengths
- Oblique parameters
- Electric dipole moment (EDM) of the electron
- Unitarity


## General form of the amplitude and what it tells us

Gauge invariance imposes

$$
M^{h \rightarrow A A^{\prime}}=\left[S^{h \rightarrow A A^{\prime}}\left(p_{1} \cdot p_{2} g_{\mu \nu}-p_{1 \mu} p_{2 \nu}\right)+i \tilde{S}^{h \rightarrow A A^{\prime}} \epsilon_{\mu \nu \alpha \beta} p_{1}^{\alpha} p_{2}^{\beta}\right] \epsilon_{p_{1}}^{\nu} \epsilon_{p_{2}}^{\mu} .
$$

This amplitude cannot be generated at tree-level with a renormalizable Lagrangian.

The process must take place at loop-level.

## SM particles in the loop?

- There could be kinetic mixing between $A$ and $A^{\prime}$

$$
\epsilon F^{\mu \nu} F_{\mu \nu}^{\prime}
$$

- SM particles in the loop could then contribute to $h \rightarrow A A^{\prime}$.
- However, $\epsilon$ is constrained to be very small for a light $A^{\prime}$.
- $\mathrm{BR}\left(h \rightarrow A A^{\prime}\right)$ would be too small for the LHC.


Fabbrichesi '20

## Mediators I

Solution:

New mediators that:

- Interact with the Higgs
- Are charged under EM
- Are charged under a new $U(1)^{\prime}$

Comments:

- It's not possible to be completely model independent.
- We will consider a very large set of mediators and explain why the bounds would be difficult to avoid.
- The constraints on these mediators are what gives the constraints on $\mathrm{BR}\left(h \rightarrow A A^{\prime}\right)$.


## Mediators II

We will consider models that:

1. Have a renormalizable Lagrangian that preserves all gauge symmetries
2. Lead to the $h \rightarrow A A^{\prime}$ decay at one loop
3. Contain no mediators charged under QCD
4. Contain only mediators that are complex scalars or vector-like fermions
5. Contain no more than two new fields
6. Contain no mediators that mix with SM fields or have a non-zero expectation value

## Mediators III

All models must include an interaction term between the Higgs boson and the mediators.

These terms fall into a finite number of categories:
Fermion: $\quad \bar{\psi}_{1}\left(A_{L} P_{L}+A_{R} P_{R}\right) \psi_{2} H+$ h.c.,

$$
\begin{array}{lll}
\text { Scalar: } & \text { I: } \mu \phi_{1}^{\dagger} \phi_{2} H+\text { h.c., } & \text { II: } \lambda H^{\dagger} H \phi^{\dagger} \phi, \\
& \text { III: } \lambda H^{\dagger} H \phi_{1}^{\dagger} \phi_{2}+\text { h.c., } & \text { IV: } \lambda H H \phi_{1}^{\dagger} \phi_{2}+\text { h.c. } .
\end{array}
$$

Each possible form corresponds to a category of models.

## Example: Fermion mediators

Consider the Lagrangian

$$
\mathcal{L}_{m}=-\left[\sum_{a, b, c} \hat{d}_{a b c}^{p n} \bar{\psi}_{1}^{a}\left(A_{L} P_{L}+A_{R} P_{R}\right) \psi_{2}^{b} H^{c}+\text { h.c. }\right]-m_{1} \bar{\psi}_{1} \psi_{1}-m_{2} \bar{\psi}_{2} \psi_{2} .
$$

where $a, b$ and $c$ are $S U(2)_{L}$ indices and are summed from 1 to the size of the corresponding multiplet and with

$$
\hat{d}_{a b c}^{p n}=C_{j_{1} m_{1} j_{2} m_{2}}^{J M M}=\left\langle j_{1} j_{2} m_{1} m_{2} \mid J M\right\rangle,
$$

where

$$
\begin{aligned}
& J=\frac{p-1}{2}, \\
& j_{1}=\frac{n-1}{2}, \\
& j_{2}=\frac{1}{2}, \\
& M=\frac{p+1-2 a}{2}, \quad m_{1}=\frac{n+1-2 b}{2}, \quad m_{2}=\frac{3-2 c}{2} \text {. }
\end{aligned}
$$

There is a phase that cannot be reabsorbed.

## Higgs decay I



+ arrows reversed

The amplitudes are

$$
\begin{aligned}
M^{h \rightarrow A A} & =\left[S^{h \rightarrow A A}\left(p_{1} \cdot p_{2} g_{\mu \nu}-p_{1 \mu} p_{2 \nu}\right)+i \tilde{S}^{h \rightarrow A A} \epsilon_{\mu \nu \alpha \beta} p_{1}^{\alpha} p_{2}^{\beta}\right] \epsilon_{p_{1}}^{\nu} \epsilon_{p_{2}}^{\mu} \\
M^{h \rightarrow A A^{\prime}} & =\left[S^{h \rightarrow A A^{\prime}}\left(p_{1} \cdot p_{2} g_{\mu \nu}-p_{1 \mu} p_{2 \nu}\right)+i \tilde{S}^{h \rightarrow A A^{\prime}} \epsilon_{\mu \nu \alpha \beta} p_{1}^{\alpha} p_{2}^{\beta}\right] \epsilon_{p_{1}}^{\nu} \epsilon_{p_{2}}^{\mu} \\
M^{h \rightarrow A^{\prime} A^{\prime}} & =\left[S^{h \rightarrow A^{\prime} A^{\prime}}\left(p_{1} \cdot p_{2} g_{\mu \nu}-p_{1 \mu} p_{2 \nu}\right)+i \tilde{S}^{h \rightarrow A^{\prime} A^{\prime}} \epsilon_{\mu \nu \alpha \beta} p_{1}^{\alpha} p_{2}^{\beta}\right] \epsilon_{p_{1}}^{\nu} \epsilon_{p_{2}}^{\mu}
\end{aligned}
$$

## Higgs decay II

With

$$
\begin{aligned}
& S^{h \rightarrow A A}=e^{2} \sum_{a} \operatorname{Re}\left(\Omega_{a a}\right) \tilde{Q}_{a a}^{2} S_{a}+S_{S M}^{h \rightarrow A A}, \quad \tilde{S}^{h \rightarrow A A}=e^{2} \sum_{a} \operatorname{lm}\left(\Omega_{a a}\right) \tilde{Q}_{a a}^{2} \tilde{S}_{a}+\tilde{S}_{S M}^{h \rightarrow A A}, \\
& S^{h \rightarrow A A^{\prime}}=e e^{\prime} \sum_{a} \operatorname{Re}\left(\Omega_{a z}\right) \tilde{Q}_{a \mathrm{a}} Q^{\prime} S_{a}, \\
& \tilde{S}^{h \rightarrow A A^{\prime}}=e e^{\prime} \sum_{a} \operatorname{lm}\left(\Omega_{a z}\right) \tilde{Q}_{a \mathrm{a}} Q^{\prime} \tilde{S}_{a}, \\
& S^{h \rightarrow A^{\prime} A^{\prime}}=e^{\prime 2} \sum_{a} \operatorname{Re}\left(\Omega_{a \mathrm{ab}}\right) Q^{\prime 2} S_{a}, \\
& \tilde{S}^{h \rightarrow A^{\prime} A^{\prime}}=e^{\prime 2} \sum_{a} \operatorname{lm}\left(\Omega_{a \mathrm{a}}\right) Q^{\prime 2} \tilde{S}_{a},
\end{aligned}
$$

where in the example of the fermion case

$$
\begin{aligned}
& S_{a}=-\frac{m_{a}}{2 \pi^{2} m_{h}^{2}}\left(2+\left(4 m_{a}^{2}-m_{H}^{2}\right) C_{0}\left(0,0, m_{H}^{2} ; m_{a}, m_{a}, m_{a}\right)\right), \\
& \tilde{S}_{a}=-i \frac{m_{a}}{2 \pi^{2}} C_{0}\left(0,0, m_{H}^{2} ; m_{a}, m_{a}, m_{a}\right),
\end{aligned}
$$

and

$$
\begin{gathered}
\Gamma^{h \rightarrow A A}=\frac{\left|S^{h \rightarrow A A}\right|^{2}+\left|\tilde{S}^{h \rightarrow A A}\right|^{2}}{64 \pi} m_{h}^{3}, \quad \Gamma^{h \rightarrow A A^{\prime}}=\frac{\left|S^{h \rightarrow A A^{\prime}}\right|^{2}+\left|\tilde{S}^{h \rightarrow A A^{\prime}}\right|^{2}}{32 \pi} m_{h}^{3}, \\
\Gamma^{h \rightarrow A^{\prime} A^{\prime}}=\frac{\left|S^{h \rightarrow A^{\prime} A^{\prime}}\right|^{2}+\left|\tilde{S}^{h \rightarrow A^{\prime} A^{\prime}}\right|^{2}}{64 \pi} m_{h .}^{3} .
\end{gathered}
$$

## Higgs decay III

Comments:

- The presence of the Levi-Civita symbols comes from $\gamma^{5}$ in the Higgs/fermion vertex.
- The amplitudes are correlated. The constraints on $h \rightarrow A A$ and $h \rightarrow A^{\prime} A^{\prime}$ will constrain $h \rightarrow A A^{\prime}$.
- The Higgs signal strengths have two potential blind spots:
- Purely imaginary contribution to $S^{h \rightarrow A A}$
- LEP forces $S^{h \rightarrow A A}$ to be purely real.
- Contibution only to $\tilde{S}^{h \rightarrow A A}$
- Will be constrained by the electron EDM


## Higgs signal strengths

$\kappa$ formalism

$$
\kappa_{i}^{2}=\frac{\sigma_{i}}{\sigma_{i}^{S M}} \quad \text { or } \quad \kappa_{i}^{2}=\frac{\Gamma_{i}}{\Gamma_{i}^{S M}},
$$

and

$$
\kappa_{A A}^{2}=\frac{\left|S^{h \rightarrow A A}\right|^{2}+\left|\tilde{S}^{h \rightarrow A A}\right|^{2}}{\left|S_{S M}^{h \rightarrow A A}\right|^{2}+\left|\tilde{S}_{S M}^{h \rightarrow A A}\right|^{2}}, \quad \kappa_{Z A}^{2}=\frac{\left|S^{h \rightarrow Z A}\right|^{2}+\left|\tilde{S}^{h \rightarrow Z A}\right|^{2}}{\left|S_{S M}^{h \rightarrow Z A}\right|^{2}+\left|\tilde{S}_{S M}^{h}{ }^{Z A}\right|^{2}}
$$

Do $\chi^{2}$ fit using results from

- CMS-PAS-HIG-19-005
- ATLAS-CONF-2021-053


## EDM


$h A$ diagrams are dominant near limits $h A$ contribution $\propto \operatorname{Im}\left(\Omega_{a a}\right)$

Effectively forces $\Omega_{a a}$ to be purely real or tiny

## Oblique parameters

The Peskin-Takeuchi parameters are defined as

$$
\begin{aligned}
& \alpha S=4 s_{W}^{2} c_{W}^{2}\left[\Pi_{Z Z}^{\prime}(0)-\frac{c_{W}^{2}-s_{W}^{2}}{s_{W} c_{W}} \Pi_{Z A}^{\prime}(0)-\Pi_{A A}^{\prime}(0)\right] \\
& \alpha T=\frac{\Pi_{W W}(0)}{m_{W}^{2}}-\frac{\Pi_{Z Z}(0)}{m_{Z}^{2}}
\end{aligned}
$$

Unsurprising constraint


## Unitarity

An amplitude can be expanded as

$$
\mathcal{M}=16 \pi \sum_{l}(2 l+1) a_{l} P_{l}(\cos \theta), \quad \max \left(\left|\operatorname{Re}\left(a_{0}^{\text {eig }}\right)\right|\right)<\frac{1}{2} .
$$

Different cases:

$$
\begin{array}{ll}
\text { Fermion: }\left|A_{R}\right|^{2}+\left|A_{L}\right|^{2} & <\frac{32 \pi}{p} \\
\text { Scalar II: } \frac{1}{16 \sqrt{2} \pi}\left[\sum_{i, j}\left|\sum_{r} \lambda^{\prime} \hat{d}_{22 i j}^{n}\right|^{2}\right]^{\frac{1}{2}} & <\frac{1}{2}, \\
\text { Scalar III: } \frac{1}{16 \sqrt{2} \pi}\left[\sum_{i, j}\left|\sum_{r} \lambda^{\prime} \hat{d}_{22 i j}^{p r}\right|^{2}\right]^{\frac{1}{2}} & <\frac{1}{2}, \\
\text { Scalar IV: } \frac{|\lambda|}{16 \sqrt{2} \pi}\left[\left.\sum_{i, j}| |_{22 i j}^{p n}\right|^{2}\right]^{\frac{1}{2}} & <\frac{1}{2} .
\end{array}
$$

## Assembling everything

- Consider multiple benchmark models for each category
- Scan entire parameter space with Markov chain using Metropolis-Hasting algorithm
- Impose $\left|Q^{\prime} e^{\prime}\right|<\sqrt{4 \pi}$ for fermions or $\left|Q^{\prime} e^{\prime}\right|<\frac{\sqrt{4 \pi}}{q^{1 / 4}}$ for scalars.


## Constraints: Fermion mediators



## Constraints: Scalar mediators






## Caveats

Three caveats:

- There should be a lower limit on the mass of the charged mediators, but there does not exist sufficiently general searches. The limits would probably be stronger.
- It could have been possible that $S_{B S M}^{h \rightarrow A A} \approx-2 S_{S M}^{h \rightarrow A A}$. That would avoid most of the Higgs signal strength constraints, but it does not happen for the fermion cases considered. It can technically happen for some scalar cases, but only for rare cases and extreme fine-tuning.
- The bounds are model dependent.


## Conclusion

Goal:
Investigate experimental and theoretical constraints on $h \rightarrow A A^{\prime}$

Conclusions:

- $\mathrm{BR}\left(h \rightarrow A A^{\prime}\right) \lesssim 0.4 \%$ at best*
- Difficult to even get this large
- Far stronger constraints than previous phenomenology papers (5\%) and experimental searches (1.8\%)
- Seems like it would be very challenging to find something in this channel at the LHC


## Thanks!

## Higgs decay IV

- If the loops are dominated by one particle, $S^{h \rightarrow A A}$ is purely real and $\tilde{S}^{h \rightarrow A A}=0$, we have

$$
\mathrm{BR}\left(h \rightarrow A A^{\prime}\right) \approx \sqrt{\mathrm{BR}\left(h \rightarrow A^{\prime} A^{\prime}\right) \mathrm{BR}(h \rightarrow A A)}\left|\frac{\Delta \mathrm{BR}(h \rightarrow A A)}{\mathrm{BR}(h \rightarrow A A)}\right|,
$$

where $\mathrm{BR}(h \rightarrow A A)=\mathrm{BR}(h \rightarrow A A)_{\mathrm{SM}}+\Delta \mathrm{BR}(h \rightarrow A A)$.
Considering
$\mathrm{BR}(h \rightarrow A A) \sim 0.23 \%,\left|\frac{\Delta \mathrm{BR}(h \rightarrow A A)}{\mathrm{BR}(h \rightarrow A A)}\right| \lesssim 25 \%$, $\mathrm{BR}\left(h \rightarrow A^{\prime} A^{\prime}\right) \lesssim 10 \%$.
we get a rough estimate of $\mathrm{BR}\left(h \rightarrow A A^{\prime}\right) \lesssim 0.4 \%$.

## EDM I

$$
\frac{d_{e}^{A h}}{e}=-\sum_{a} \frac{\alpha \tilde{Q}_{a \mathrm{a}}^{2} m_{a} m_{e}}{16 \pi^{3} m_{h}^{2} v} \operatorname{Im}\left(\Omega_{a \mathrm{a}}\right) \int_{0}^{1} d x \frac{1}{x(1-x)^{j}} j\left(0, \frac{m_{a}^{2}}{x(1-x) m_{h}^{2}}\right),
$$

where

$$
j(r, s)=\frac{1}{r-s}\left(\frac{r \ln r}{r-1}-\frac{s \ln s}{s-1}\right) .
$$

## EDM II

$$
\frac{d_{e}^{Z h}}{e}=\sum_{a, b} \frac{\tilde{Q}_{b b}}{32 \pi^{4} m_{h}^{2}} g_{e e}^{V} g_{e e}^{S}\left(m_{a} C_{a b}^{1} f_{1}\left(m_{a}, m_{b}\right)+m_{b} C_{a b}^{2} f_{2}\left(m_{a}, m_{b}\right)\right)
$$

where

$$
C_{a b}^{1}=\operatorname{Re}\left(i g_{b a}^{S} g_{a b}^{A *}-g_{b a}^{P} g_{a b}^{V *}\right), \quad C_{a b}^{2}=-\operatorname{Re}\left(i g_{b a}^{S} g_{a b}^{A *}+g_{b a}^{P} g_{a b}^{V *}\right)
$$

with

$$
\begin{array}{ll}
g_{e e}^{S}=-\frac{m_{e}}{v}, & g_{e e}^{V}=-\frac{\sqrt{g^{2}+g^{\prime 2}}}{2}\left(-\frac{1}{2}+2 s_{W}^{2}\right), \\
g_{a b}^{S}=-\frac{\left(\Omega_{b a}+\Omega_{a b}^{*}\right)}{2}, & g_{a b}^{P}=-i \frac{\left(\Omega_{b a}-\Omega_{a b}^{*}\right)}{2}, \\
g_{a b}^{V}=-\frac{\sqrt{g^{2}+g^{\prime 2}}}{2}\left(B_{R b a}+B_{L b a}\right), & g_{a b}^{A}=-\frac{\sqrt{g^{2}+g^{\prime 2}}}{2}\left(B_{R b a}-B_{L b a}\right),
\end{array}
$$

and
$f_{1}\left(m_{a}, m_{b}\right)=\int_{0}^{1} d x j\left(\frac{m_{Z}^{2}}{m_{h}^{2}}, \frac{\tilde{\Delta}_{a b}}{m_{h}^{2}}\right), \quad f_{2}\left(m_{a}, m_{b}\right)=\int_{0}^{1} d x j\left(\frac{m_{Z}^{2}}{m_{h}^{2}}, \frac{\tilde{\Delta}_{a b}}{m_{h}^{2}}\right) \frac{(1-x)}{x}$,

## EDM III

$\frac{d_{e}^{W W}}{e}=-\frac{\alpha^{2} m_{e}}{8 \pi^{2} s_{W}^{4} m_{W}^{2}} \sum_{a, b} \frac{m_{a} m_{b}}{m_{W}^{2}} \operatorname{lm}\left(\hat{A}_{L b a} \hat{A}_{R b a}^{*}\right)\left[\tilde{Q}_{b b} \mathcal{G}\left(r_{a}, r_{b}, 0\right)+\tilde{Q}_{a a} \mathcal{G}\left(r_{b}, r_{a}, 0\right)\right]$,
where $r_{a}=m_{a}^{2} / m_{W}^{2}, r_{b}=m_{b}^{2} / m_{W}^{2}$ and
$\mathcal{G}\left(r_{a}, r_{b}, r_{c}\right)=\int_{0}^{1} \frac{d \gamma}{\gamma} \int_{0}^{1} d y y\left[\frac{\left(R-3 K_{a b}\right) R+2\left(K_{a b}+R\right) y}{4 R\left(K_{a b}-R\right)^{2}}+\frac{K_{a b}\left(K_{a b}-2 y\right)}{2\left(K_{a b}-R\right)^{3}} \ln \frac{K_{a b}}{R}\right]$
where

$$
R=y+(1-y) r_{c} \quad K_{a b},=\frac{r_{a}}{1-\gamma}+\frac{r_{b}}{\gamma} .
$$

## Oblique parameters fermions

$$
\begin{aligned}
S= & \frac{1}{2 \pi} \sum_{a, b}\left\{\left(\left|\hat{A}_{L a b}\right|^{2}+\left|\hat{A}_{R a b}\right|^{2}\right) \psi_{+}\left(y_{a}, y_{b}\right)+2 \operatorname{Re}\left(\hat{A}_{L a b} \hat{A}_{R a b}^{*}\right) \psi_{-}\left(y_{a}, y_{b}\right)\right. \\
& \left.-\frac{1}{2}\left[\left(\left|X_{a b}\right|^{2}+\left|X_{R a b}\right|^{2}\right) \chi_{+}\left(y_{a}, y_{b}\right)+2 \operatorname{Re}\left(X_{L a b} X_{R a b}^{*}\right) \chi_{-}\left(y_{a}, y_{b}\right)\right]\right\}, \\
T= & \frac{1}{16 \pi s_{W}^{2} c_{W}^{2}} \sum_{a, b}\left\{\left(\left|\hat{A}_{L a b}\right|^{2}+\left|\hat{A}_{R a b}\right|^{2}\right) \theta_{+}\left(y_{a}, y_{b}\right)+2 \operatorname{Re}\left(\hat{A}_{L a b} \hat{A}_{R a b}^{*}\right) \theta_{-}\left(y_{a}, y_{b}\right)\right. \\
& \left.-\frac{1}{2}\left[\left(\left|X_{L a b}\right|^{2}+\left|X_{R a b}\right|^{2}\right) \theta_{+}\left(y_{a}, y_{b}\right)+2 \operatorname{Re}\left(X_{L a b} X_{R a b}^{*}\right) \theta_{-}\left(y_{a}, y_{b}\right)\right]\right\},
\end{aligned}
$$

with $y_{a}=m_{a}^{2} / m_{Z}^{2}, X_{L / R}=-2 B_{L / R}+2 \tilde{Q} s_{W}^{2}$ and

$$
\begin{aligned}
& \psi_{+}\left(y_{1}, y_{2}\right)=\frac{1}{3}-\frac{1}{9} \ln \frac{y_{1}}{y_{2}}, \\
& \psi_{-}\left(y_{1}, y_{2}\right)=-\frac{y_{1}+y_{2}}{6 \sqrt{y_{1} y_{2}}}, \\
& \chi_{+}\left(y_{1}, y_{2}\right)=\frac{5\left(y_{1}^{2}+y_{2}^{2}\right)-22 y_{1} y_{2}}{9\left(y_{1}-y_{2}\right)^{2}}+\frac{3 y_{1} y_{2}\left(y_{1}+y_{2}\right)-y_{1}^{3}-y_{2}^{3}}{3\left(y_{1}-y_{2}\right)^{3}} \ln \frac{y_{1}}{y_{2}}, \\
& \chi_{-}\left(y_{1}, y_{2}\right)=-\sqrt{y_{1} y_{2}}\left[\frac{y_{1}+y_{2}}{6 y_{1} y_{2}}-\frac{y_{1}+y_{2}}{\left(y_{1}-y_{2}\right)^{2}}+\frac{2 y_{1} y_{2}}{\left(y_{1}-y_{2}\right)^{3}} \ln \frac{y_{1}}{y_{2}}\right] \\
& \theta_{+}\left(y_{1}, y_{2}\right)=y_{1}+y_{2}-\frac{2 y_{1} y_{2}}{y_{1}-y_{2}} \ln \frac{y_{1}}{y_{2}}, \\
& \theta_{-}\left(y_{1}, y_{2}\right)=2 \sqrt{y_{1} y_{2}}\left[\frac{y_{1}+y_{2}}{y_{1}-y_{2}} \ln \frac{y_{1}}{y_{2}}-2\right] .
\end{aligned}
$$

## Oblique parameters scalars

$$
\begin{aligned}
& S=\frac{1}{2 \pi} \sum_{a, b}\left[\left|B_{a b}\right|^{2}-\left(c_{W}^{2}-s_{W}^{2}\right) B_{a b} \tilde{Q}_{a b}-c_{W}^{2} s_{W}^{2} \tilde{Q}_{a b}^{2}\right] F_{1}\left(y_{a}, y_{b}\right) \\
& T=\frac{1}{16 \pi c_{W}^{2} s_{W}^{2}}\left[\sum_{a, b}\left|\hat{A}_{a b}\right|^{2} F_{2}\left(y_{a}, y_{b}\right)-\sum_{a}\left[\hat{A} \hat{A}^{\dagger}+\hat{A}^{\dagger} \hat{A}\right]_{a a} F_{3}\left(y_{a}\right)\right. \\
& \left.\quad-2 \sum_{a, b}\left|B_{a b}\right|^{2} F_{2}\left(y_{a}, y_{b}\right)+4 \sum_{a}\left[B^{2}\right]_{a a} F_{3}\left(y_{a}\right)\right]
\end{aligned}
$$

where $y_{a}=m_{a}^{2} / m_{Z}^{2}$ and

$$
\begin{aligned}
F_{1}\left(y_{1}, y_{2}\right) & =-\frac{5 y_{1}^{2}-22 y_{1} y_{2}+5 y_{2}^{2}}{9\left(y_{1}-y_{2}\right)^{2}}+\frac{2\left(y_{1}^{2}\left(y_{1}-3 y_{2}\right) \ln y_{1}-y_{2}^{2}\left(y_{2}-3 y_{1}\right) \ln y_{2}\right)}{3\left(y_{1}-y_{2}\right)^{3}}, \\
F_{2}\left(y_{1}, y_{2}\right) & =3\left(y_{1}+y_{2}\right)-\frac{2\left(y_{1}^{2} \ln y_{1}-y_{2}^{2} \ln y_{2}\right)}{y_{1}-y_{2}}, \\
F_{3}\left(y_{1}\right) & =2 y_{1}-2 y_{1} \ln y_{1} .
\end{aligned}
$$

## Example: Fermion mediators II

Once the Higgs is replaced by its expectation value
$\mathcal{L}_{m} \supset-\sum_{a, b} \frac{A_{L V}}{\sqrt{2}} \hat{d}_{a b 2}^{p n} \bar{\psi}_{1}^{a} P_{L} \psi_{2}^{b}-\sum_{a, b} \frac{A_{R}^{*} V}{\sqrt{2}} \hat{d}_{b a 2}^{p n} \bar{\psi}_{2}^{a} P_{L} \psi_{1}^{b}-m_{1} \bar{\psi}_{1} P_{L} \psi_{1}-m_{2} \bar{\psi}_{2} P_{L} \psi_{2}+$ h.c.
Introduce the convenient notation

$$
\hat{\psi}=\binom{\psi_{1}}{\psi_{2}}
$$

and

$$
d_{a b}^{p n}= \begin{cases}\hat{d}_{a(b-p) 2}^{p n}, & \text { if } a \in[1, p] \text { and } b \in[p+1, n+p] \\ 0, & \text { otherwise }\end{cases}
$$

## Fermion mediators: Higgs interactions and mass IV

The mass Lagrangian can be written as

$$
\mathcal{L}_{m} \supset-\sum_{a, b} M_{a b} \overline{\hat{\psi}}^{a} P_{L} \hat{\psi}^{b}+\text { h.c. }
$$

where the mass matrix is
$M=m_{1}\left(\begin{array}{ll}\mathbb{I}_{p \times p} & 0_{p \times n} \\ 0_{n \times p} & 0_{n \times n}\end{array}\right)+m_{2}\left(\begin{array}{ll}0_{p \times p} & 0_{p \times n} \\ 0_{n \times p} & \mathbb{I}_{n \times n}\end{array}\right)+\frac{A_{L} v}{\sqrt{2}} d^{p n}+\frac{A_{R}^{*} v}{\sqrt{2}} d^{p n T}$.
The mass matrix can then be diagonalized by introducing $\tilde{\psi}^{a}$

$$
P_{L} \hat{\psi}=R_{L} P_{L} \tilde{\psi}, \quad P_{R} \hat{\psi}=R_{R} P_{R} \tilde{\psi}
$$

where $R_{L}$ and $R_{R}$ are unitary matrices that diagonalize $M^{\dagger} M$ and $M M^{\dagger}$ respectively.

## Fermion mediators: Higgs interactions and mass V

The interactions of the Higgs with the mass eigenstates are

$$
\mathcal{L}_{m} \supset-\sum_{a, b} \Omega_{a b} h \overline{\tilde{\psi}}^{a} P_{L} \tilde{\psi}^{b}+\text { h.c. }
$$

where $\Omega$ is

$$
\Omega=\frac{A_{L}}{\sqrt{2}} R_{R}^{\dagger} d^{p n} R_{L}+\frac{A_{R}^{*}}{\sqrt{2}} R_{R}^{\dagger} d^{p n T} R_{L}
$$

## Fermion mediators: Gauge interactions I

The interactions of the $A / A^{\prime}$ with $\tilde{\psi}^{a}$ are

$$
\mathcal{L}_{g} \supset-e A_{\mu} \overline{\tilde{\psi}} \gamma^{\mu} \tilde{Q} \tilde{\psi}-Q^{\prime} e^{\prime} A_{\mu}^{\prime} \overline{\tilde{\psi}} \gamma^{\mu} \tilde{\psi}
$$

where $e^{\prime}$ is the $U(1)^{\prime}$ gauge coupling and

$$
\tilde{Q}=R_{L}^{\dagger} \hat{Q} R_{L}=R_{R}^{\dagger} \hat{Q} R_{R}
$$

where

$$
\hat{Q}=\left(\begin{array}{cc}
Y^{p}+T_{3}^{p} & 0_{p \times n} \\
0_{n \times p} & Y^{n}+T_{3}^{n}
\end{array}\right)
$$

where $\left(T_{3}^{p}\right)_{a b}=(p+1-2 a) \delta_{a b} / 2$ and similarly for $T_{3}^{n}$.

## Fermion mediators: Gauge interactions II

The interactions between the $Z$ boson and $\tilde{\psi}^{a}$ are

$$
\mathcal{L}_{g} \supset-\sqrt{g^{2}+g^{\prime 2}} Z_{\mu} \overline{\tilde{\psi}} \gamma^{\mu}\left(B_{L} P_{L}+B_{R} P_{R}\right) \tilde{\psi}
$$

where $B_{L}$ and $B_{R}$ are

$$
\begin{aligned}
& B_{L}=R_{L}^{\dagger}\left(\begin{array}{cc}
-s_{W}^{2} Y^{p}+c_{W}^{2} T_{3}^{p} & 0_{p \times n} \\
0_{n \times p} & -s_{W}^{2} Y^{n}+c_{W}^{2} T_{3}^{n}
\end{array}\right) R_{L}, \\
& B_{R}=R_{R}^{\dagger}\left(\begin{array}{cc}
-s_{W}^{2} Y^{p}+c_{W}^{2} T_{3}^{p} & 0_{p \times n} \\
0_{n \times p} & -s_{W}^{2} Y^{n}+c_{W}^{2} T_{3}^{n}
\end{array}\right) R_{R} .
\end{aligned}
$$

## Fermion mediators: Gauge interactions III

The interactions between the $W$ boson and $\tilde{\psi}^{a}$ are

$$
\mathcal{L}_{g} \supset-\frac{g}{\sqrt{2}} \overline{\tilde{\psi}} \gamma^{\mu}\left(\hat{A}_{L} P_{L} W_{\mu}^{+}+\hat{A}_{R} P_{R} W_{\mu}^{+}\right) \tilde{\psi}+\text { h.c. }
$$

where

$$
\hat{A}_{L}=R_{L}^{\dagger}\left(\begin{array}{cc}
T_{+}^{p} & 0_{p \times n} \\
0_{n \times p} & T_{+}^{n}
\end{array}\right) R_{L}, \quad \hat{A}_{R}=R_{R}^{\dagger}\left(\begin{array}{cc}
T_{+}^{p} & 0_{p \times n} \\
0_{n \times p} & T_{+}^{n}
\end{array}\right) R_{R},
$$

with $\left(T_{+}^{p}\right)_{a b}=\sqrt{a(p-a)} \delta_{a, b-1}$ and similarly for $T_{+}^{n}$.

## Fermion mediators: Unitarity I

The top loop has a mass of $\sim 173 \mathrm{GeV}, y_{t} \sim 1$ and $N_{c}=3$, yet gives a tiny contribution to $h \rightarrow A A$.

Mediators must have either large Yukawa couplings or dark electric charge.

An amplitude can be expanded as

$$
\mathcal{M}=16 \pi \sum_{l}(2 l+1) a_{l} P_{l}(\cos \theta) .
$$

Consider the basis of $\bar{\psi}_{1}^{a} \psi_{2}^{b}$ pairs given by
$\bar{\psi}_{1}^{1} \psi_{2}^{1}, \bar{\psi}_{1}^{1} \psi_{2}^{2}, \ldots, \bar{\psi}_{1}^{1} \psi_{2}^{n}, \bar{\psi}_{1}^{2} \psi_{2}^{1}, \bar{\psi}_{1}^{2} \psi_{2}^{2}, \ldots, \bar{\psi}_{1}^{2} \psi_{2}^{n}, \ldots, \bar{\psi}_{1}^{p} \psi_{2}^{1}, \bar{\psi}_{1}^{p} \psi_{2}^{2}, \ldots, \bar{\psi}_{1}^{p} \psi_{2}^{n}$.

## Fermion mediators: Unitarity II

Then, the matrix of $a_{0}$ for the scattering $\bar{\psi}_{1}^{a} \psi_{2}^{b} \rightarrow \bar{\psi}_{1}^{c} \psi_{2}^{d}$ is
$a_{0}^{m a t}=\left(\begin{array}{ccccccccccccc}F_{11}^{11} & F_{11}^{12} & \ldots & F_{11}^{1 n} & F_{11}^{21} & F_{11}^{22} & \ldots & F_{11}^{2 n} & \ldots & F_{11}^{p 1} & F_{11}^{p 2} & \ldots & F_{11}^{p n} \\ F_{12}^{11} & F_{12}^{12} & \ldots & F_{12}^{1 n} & F_{12}^{21} & F_{12}^{22} & \ldots & F_{12}^{2 n} & \ldots & F_{12}^{p 1} & F_{12}^{p 2} & \ldots & F_{12}^{p n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ F_{1 n}^{11} & F_{1 n}^{12} & \ldots & F_{1 n}^{1 n} & F_{1 n}^{21} & F_{1 n}^{22} & \ldots & F_{1 n}^{2 n} & \ldots & F_{1 n}^{p 1} & F_{1 n}^{p 2} & \ldots & F_{1 n}^{p n} \\ F_{21}^{11} & F_{21}^{12} & \ldots & F_{21}^{1 n} & F_{21}^{21} & F_{21}^{22} & \ldots & F_{21}^{2 n} & \ldots & F_{21}^{p 1} & F_{21}^{p 2} & \ldots & F_{21}^{p n} \\ F_{22}^{11} & F_{22}^{12} & \ldots & F_{22}^{1 n} & F_{22}^{21} & F_{22}^{22} & \ldots & F_{22}^{2 n} & \ldots & F_{22}^{p 1} & F_{22}^{p 2} & \ldots & F_{22}^{p n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ F_{2 n}^{11} & F_{2 n}^{12} & \ldots & F_{2 n}^{1 n} & F_{2 n}^{21} & F_{2 n}^{22} & \ldots & F_{2 n}^{2 n} & \ldots & F_{2 n}^{p 1} & F_{2 n}^{p 2} & \ldots & F_{2 n}^{p n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ F_{p 1}^{11} & F_{p 1}^{12} & \ldots & F_{p 1}^{1 n} & F_{p 1}^{21} & F_{p 1}^{22} & \ldots & F_{p 1}^{2 n} & \ldots & F_{p 1}^{p 1} & F_{p 1}^{p 2} & \ldots & F_{p 1}^{p n} \\ F_{p 2}^{11} & F_{p 2}^{12} & \ldots & F_{p 2}^{1 n} & F_{p 2}^{21} & F_{p 2}^{22} & \ldots & F_{p 2}^{2 n} & \ldots & F_{p 2}^{p 1} & F_{p 2}^{p 2} & \ldots & F_{p 2}^{p n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ F_{p n}^{11} & F_{p n}^{12} & \ldots & F_{p n}^{1 n} & F_{p n}^{21} & F_{p n}^{22} & \ldots & F_{p n}^{2 n} & \ldots & F_{p n}^{p 1} & F_{p n}^{p 2} & \ldots & F_{p n}^{p n}\end{array}\right)$

## Fermion mediators: Unitarity III

Where $F_{a b}^{c d}$ is

$$
F_{a b}^{c d}=\frac{d_{a b 2}^{p n} d_{c d 2}^{p n}}{32 \pi}\left(\begin{array}{cc}
-\left|A_{R}\right|^{2} & A_{R} A_{L}^{*} \\
A_{L} A_{R}^{*} & -\left|A_{L}\right|^{2}
\end{array}\right)
$$

and corresponds to $a_{0}$ for different combinations of helicity in the basis $(\uparrow \uparrow, \downarrow \downarrow)$. Call $a_{0}^{\text {eig }}$ the set of eigenvalues of $a_{0}^{\text {mat }}$. Unitarity requires

$$
\max \left(\left|\operatorname{Re}\left(a_{0}^{\text {eig }}\right)\right|\right)<\frac{1}{2} .
$$

This simplifies to

$$
\left|A_{R}\right|^{2}+\left|A_{L}\right|^{2}<\frac{32 \pi}{p}
$$

## Scalar mediators: Scalar case I A

Consider:

- A complex scalar $\phi_{1}$ with the properties:
- Representation of $S U(2)_{L}$ of dimension $p=n \pm 1$
- Weak hypercharge of $Y^{p}=Y^{n}+1 / 2$
- Charge of $Q^{\prime}$ under $U(1)^{\prime}$
- A complex scalar $\phi_{2}$ with the properties:
- Representation of $S U(2)_{L}$ of dimension $n$
- Weak hypercharge of $Y^{n}$
- Charge of $Q^{\prime}$ under $U(1)^{\prime}$


## Scalar mediators: Scalar case I B

This allows the Lagrangian

$$
\mathcal{L}_{m}^{1}=-\left[\sum_{a, b, c} \mu \hat{d}_{a b c}^{p n} \phi_{1}^{a \dagger} \phi_{2}^{b} H^{c}+\text { h.c. }\right]-m_{1}^{2}\left|\phi_{1}\right|^{2}-m_{2}^{2}\left|\phi_{2}\right|^{2},
$$

where

$$
\hat{d}_{a b c}^{p n}=C_{j_{1} m_{1} j_{2} m_{2}}^{J M},
$$

with

$$
\begin{aligned}
& J=\frac{p-1}{2}, \quad j_{1}=\frac{n-1}{2}, \quad j_{2}=\frac{1}{2}, \\
& M=\frac{p+1-2 a}{2}, \quad m_{1}=\frac{n+1-2 b}{2}, \quad m_{2}=\frac{3-2 c}{2} \text {. }
\end{aligned}
$$

$\mu$ can be made real.

## Scalar mediators: Scalar case II A

Consider:

- A complex scalar $\phi$ with the properties:
- Representation of $S U(2)_{L}$ of dimension $n$
- Weak hypercharge of $Y^{n}$
- Charge of $Q^{\prime}$ under $U(1)^{\prime}$


## Scalar mediators: Scalar case II B

This allows the Lagrangian

$$
\mathcal{L}_{m}^{2}=-\sum_{r \in\{n-1, n+1\}} \sum_{a, b, c, d} \lambda^{r} \hat{d}_{a b c d}^{n r} H^{a \dagger} H^{b} \phi^{c \dagger} \phi^{d}-m^{2}|\phi|^{2} .
$$

The $S U(2)_{L}$ tensor $\hat{d}_{a b c d}^{n r}$ is given by

$$
\hat{d}_{a b c d}^{n r}=\sum_{M} C_{j_{1} m_{1} j_{2} m_{2}}^{J M} C_{j_{3} m_{3} j_{4} m_{4}}^{J M},
$$

where $M$ is summed over $\{-J,-J+1,-J+2, \ldots,+J\}$ and

$$
\begin{aligned}
& j_{1}=\frac{1}{2}, \\
& j_{2}=\frac{n-1}{2}, \\
& j_{3}=\frac{1}{2}, \\
& j_{4}=\frac{n-1}{2}, \\
& J=\frac{r-1}{2}, \\
& m_{1}=\frac{3-2 a}{2}, \quad m_{2}=\frac{n+1-2 c}{2}, \quad m_{3}=\frac{3-2 b}{2}, \quad m_{4}=\frac{n+1-2 d}{2} \text {. }
\end{aligned}
$$

Two possible coefficients if $n \neq 1$.

## Scalar mediators: Scalar case III A

Consider:

- A complex scalar $\phi_{1}$ with the properties:
- Representation of $S U(2)_{L}$ of dimension $p \in\{n-2, n, n+2\}$
- Weak hypercharge of $Y^{p}=Y^{n}$
- Charge of $Q^{\prime}$ under $U(1)^{\prime}$
- A complex scalar $\phi_{2}$ with the properties:
- Representation of $S U(2)_{L}$ of dimension $n$
- Weak hypercharge of $Y^{n}$
- Charge of $Q^{\prime}$ under $U(1)^{\prime}$


## Scalar mediators: Scalar case III B

This allows the Lagrangian:
$\mathcal{L}_{m}^{3}=-\left[\sum_{r \in \mathcal{R}} \sum_{a, b, c, d} \lambda^{r} \hat{d}_{a b c d}^{p n r} H^{a \dagger} H^{b} \phi_{1}^{c \dagger} \phi_{2}^{d}+\right.$ h.c. $]-m_{1}^{2}\left|\phi_{1}\right|^{2}-m_{2}^{2}\left|\phi_{2}\right|^{2}$,
where $\mathcal{R}=\{n-1, n+1\} \cap\{p-1, p+1\}$ and $\hat{d}_{a b c d}^{p n r}$

$$
\hat{d}_{a b c d}^{p n r}=\sum_{M} C_{j_{1} m_{1} j_{2} m_{2}}^{J M} C_{j_{3} m_{3} j_{4} m_{4}}^{J M},
$$

where $M$ is summed over $\{-J,-J+1,-J+2, \ldots,+J\}$ and

$$
\begin{array}{rlrlrl}
j_{1} & =\frac{1}{2}, & j_{2} & =\frac{p-1}{2}, & j_{3}=\frac{1}{2}, & j_{4}=\frac{n-1}{2}, \\
m_{1} & =\frac{3-2 a}{2}, & m_{2}=\frac{p+1-2 c}{2} & m_{3}=\frac{3-2 b}{2}, & m_{4}=\frac{n+1-2 d}{2} .
\end{array}
$$

If $p=n \neq 1$, there are two coefficients and 1 otherwise.

## Scalar mediators: Scalar case IV A

Consider:

- A complex scalar $\phi_{1}$ with the properties:
- Representation of $S U(2)_{L}$ of dimension $p \in\{n-2, n, n+2\}$
- Weak hypercharge of $Y^{p}=Y^{n}+1$
- Charge of $Q^{\prime}$ under $U(1)^{\prime}$
- A complex scalar $\phi_{2}$ with the properties:
- Representation of $S U(2)_{L}$ of dimension $n$
- Weak hypercharge of $Y^{n}$
- Charge of $Q^{\prime}$ under $U(1)^{\prime}$
- $p$ and $n$ are not both 1 .


## Scalar mediators: Scalar case IV B

Consider:

$$
\mathcal{L}_{m}^{4}=-\left[\lambda \hat{d}_{a b c d}^{p n} H^{a} H^{b} \phi_{1}^{c \dagger} \phi_{2}^{d}+\text { h.c. }\right]-m_{1}^{2}\left|\phi_{1}\right|^{2}-m_{2}^{2}\left|\phi_{2}\right|^{2},
$$

where

$$
\hat{d}_{a b c d}^{p n}=\sum_{M_{1}} C_{j_{1} m_{1} j_{2} m_{2}}^{J_{1} M_{1}} C_{J_{1} M_{1} j_{3} m_{3}}^{J_{2} M_{2}},
$$

where $M_{1}$ is summed over $\{-1,0,1\}$ and

$$
\begin{array}{rlrll}
j_{1} & =\frac{1}{2}, & j_{2} & =\frac{1}{2}, & j_{3}=\frac{n-1}{2},
\end{array} J_{1}=1, \quad J_{2}=\frac{p-1}{2},
$$

$\lambda$ can be made real.

## Scalar mediators: Constraints I

- Treatment of gauge and Higgs interactions is similar to the fermion case.
- Higgs signal strengths is similar to the fermion case.
- We computed the oblique parameters ourselves.
- EDM constraints are not needed (no $\gamma^{5}$ ).


## Scalar mediators: Constraints III

For case IV, this can be simplified to:

$$
|\lambda|<8 \pi \sqrt{\frac{6}{p}} .
$$

Also,

$$
\left|Q^{\prime} e^{\prime}\right|<\frac{\sqrt{4 \pi}}{q^{1 / 4}}
$$

where $q=n+p$ for cases I, III, IV and $q=n$ for case II.

