The Higgs self-coupling and Gegenbauer naturalness

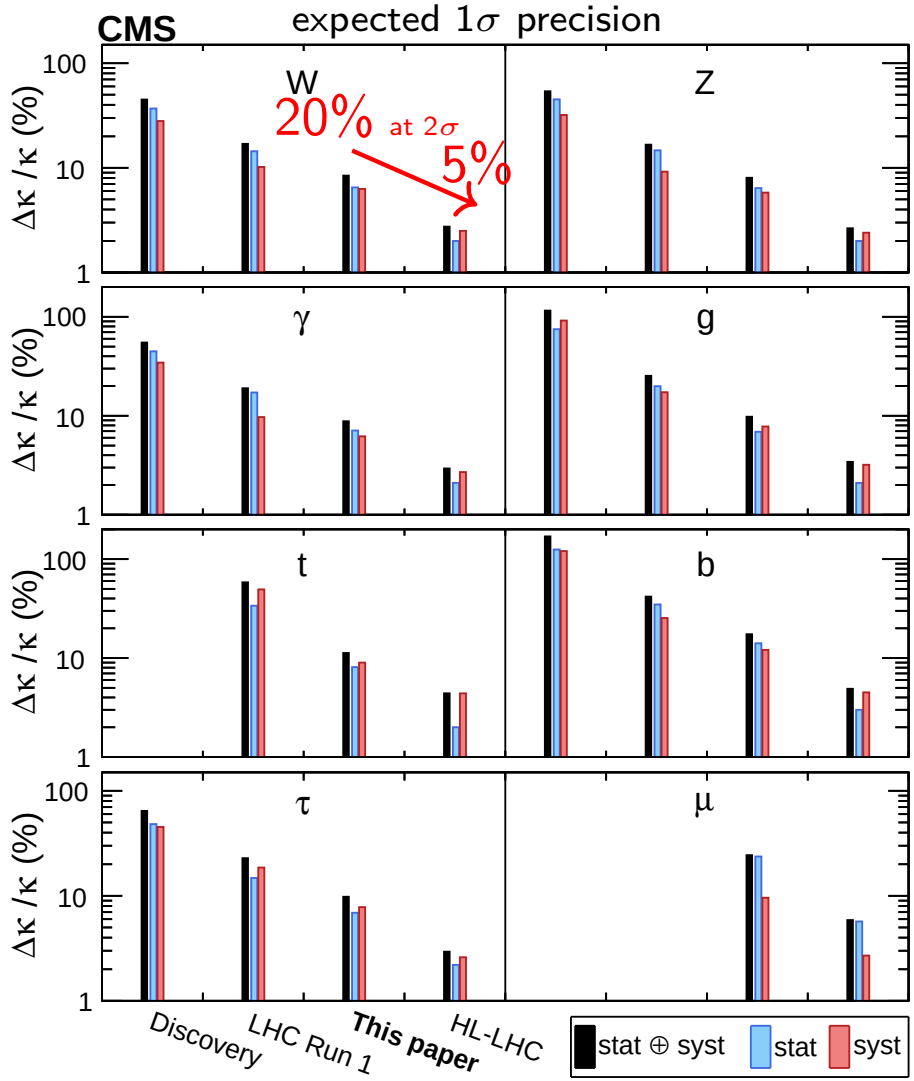
Gauthier Durieux
(UCLouvain)

Gegenbauer Goldstones, JHEP 01 (2022) 076, [2110.06941]
Gegenbauer's Twin, JHEP 05 (2022) 140, [2202.01228]
Charting the Higgs self-coupling boundaries, JHEP 12 (2022) 148, [2209.00666]
with Matthew McCullough and Ennio Salvioni

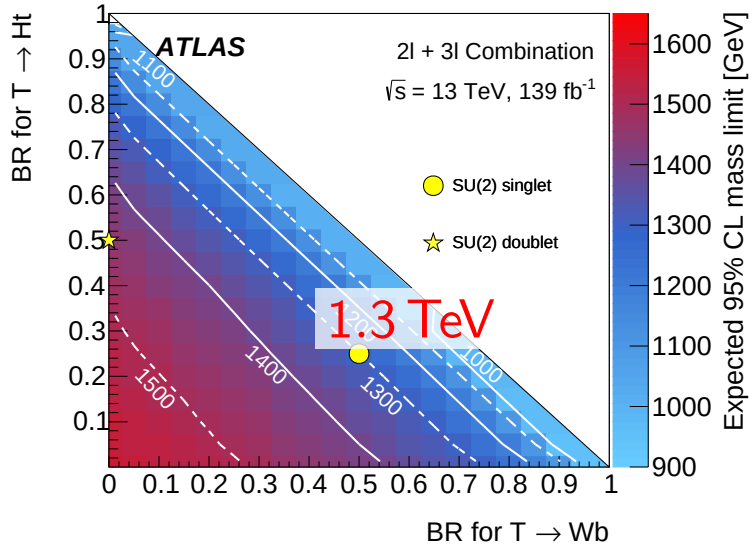
LHC Higgs WG: 20th workshop – 13 Nov 2023

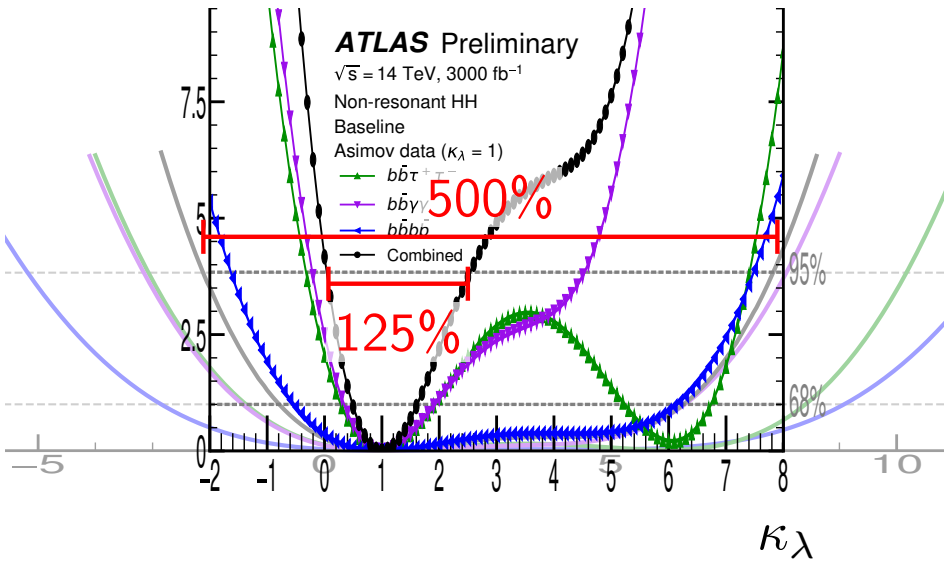
LHC present and future

[EXOT-2018-58]



[CMS-HIG-22-001]





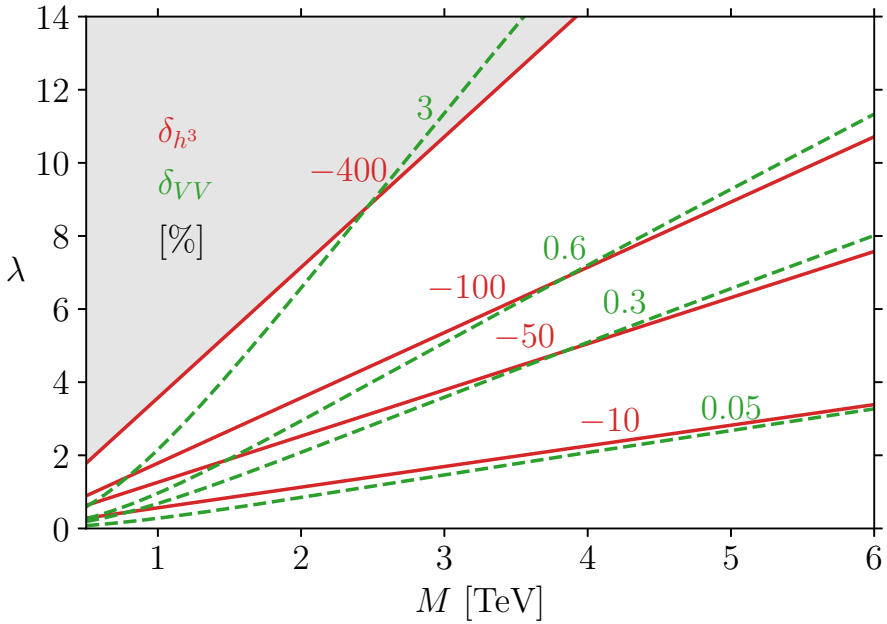
[HDBS-2022-03]
[ATL-PHYS-PUB-2022-053]

Structurally large $\delta\kappa_\lambda/\delta\kappa_V$

see also: [Di Luzio, Gröber, Spannowsky '17]
[Gupta, Rzehak, Wells '13] [Falkowski, Rattazzi '19]
[Logan, Rentala '15] [Chala, Krause, Nardini '18] [etc.]

loop factor (or v^2/M_X^2) allowed dimensionally btw. H^6 and D^2H^4

custodial weak-quadruplet scalar



$$\lambda\, H^* H^* (\epsilon H) \Phi + \lambda \frac{1}{\sqrt{3}} H^* H^* H^* \widetilde{\Phi}$$

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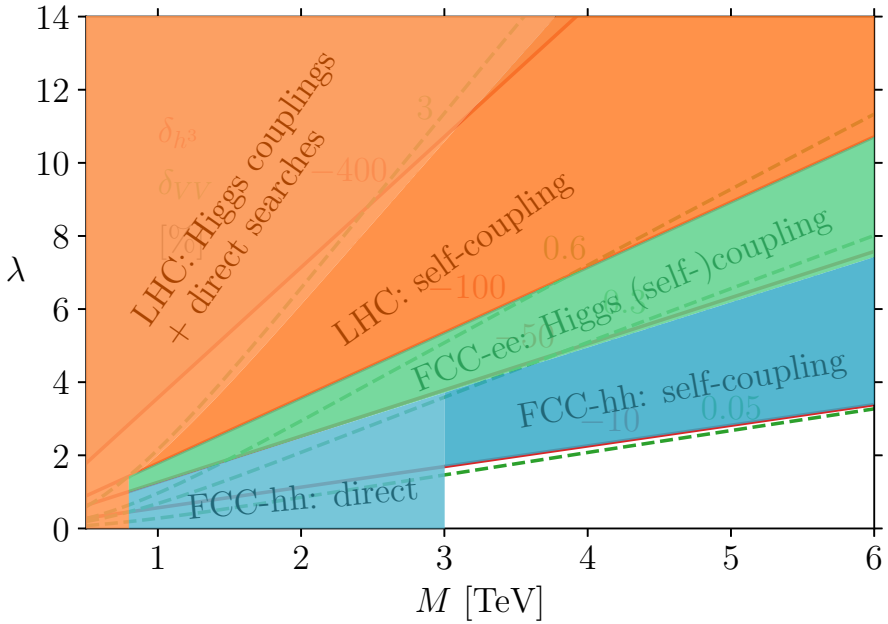
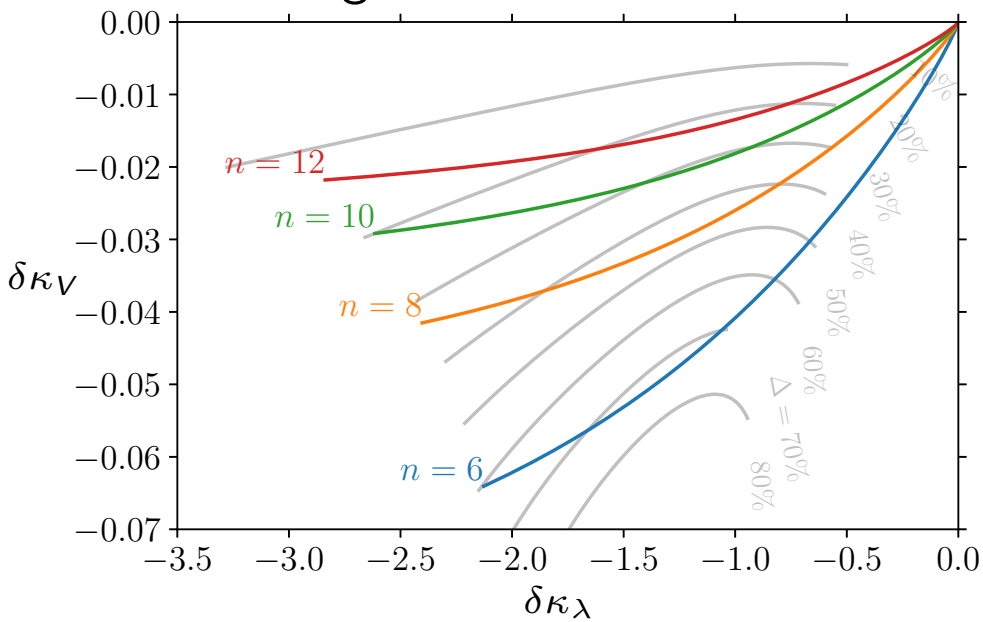


Gegenbauer's Twin



$$\lambda\, H^* H^* (\epsilon H)\Phi + \lambda \frac{1}{\sqrt{3}} H^* H^* H^* \widetilde{\Phi}$$

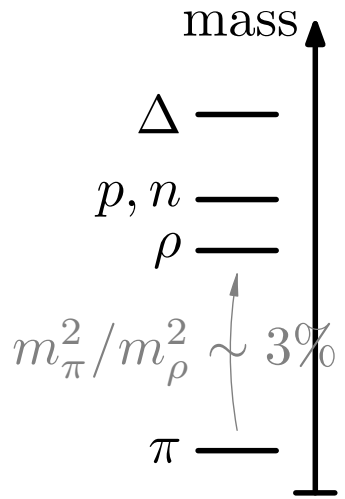
- dim$\gg$6 operators may be very relevant
- vacuum stability limiting the $\delta\kappa_\lambda/\delta\kappa_V$ ratio

Naturalness exacerbated

$$\frac{m_h^2}{M_X^2} \sim \%$$

$$\delta\kappa_V \sim \%$$

Composite Higgs



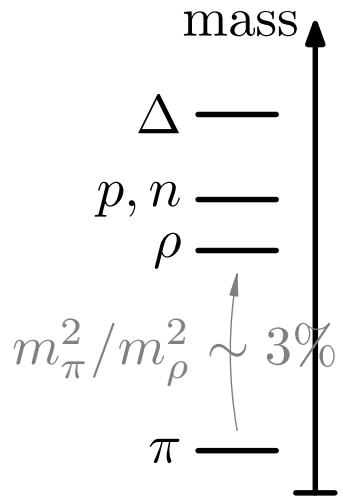
the Higgs as
pseudo-Nambu-Goldstone boson (pNGB)
of a new strong sector

e.g. global SO(5) $\to$ SO(4) spontaneous breaking
at scale f

small mass from
explicit SO(5) breaking
by e.g. the SM

small $\delta\kappa_V$ implies $v^2/f^2 \ll 1$
and requires fine-tuning in minimal models

Composite Higgs



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small(ish) m_h^2/M_X^2!

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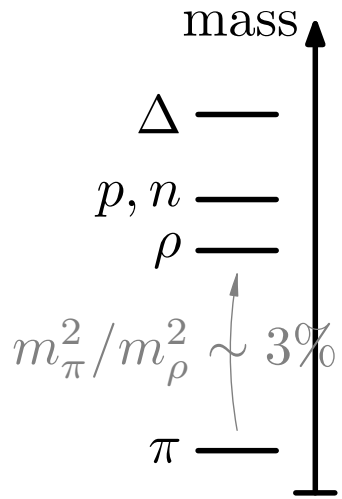


the Higgs as
pseudo-Nambu-Goldstone boson (pNGB)
of a new strong sector

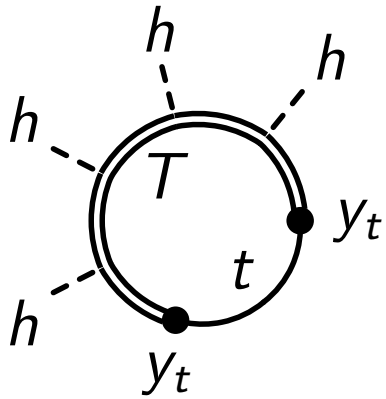
e.g. global SO(5) → SO(4) spontaneous breaking
at scale f

small(ish) m_h^2/M_X^2!

no small $\delta\kappa_V$ or v/f!

and requ models

Minimal composite Higgs

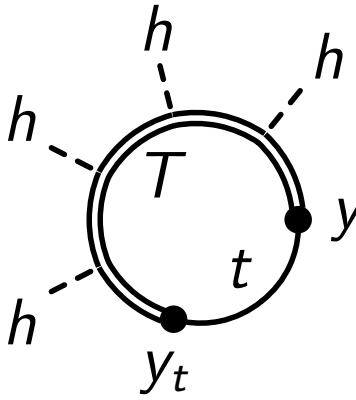
$$V(h) \sim$$ 

$$+ \cdots \sim \boldsymbol{\kappa}\, \frac{y_t^2 N_c}{16\pi^2} f^2 M_T^2 \left(-\sin^2 \frac{h}{f} + \boldsymbol{\delta}\, \sin^4 \frac{h}{f} \right)$$

$$\rightarrow \frac{v^2}{f^2} = \sin^2 \frac{\langle h \rangle}{f} = \frac{1}{2\delta} \qquad \text{vs.} \qquad |\delta\kappa_V| \simeq \frac{v^2}{2f^2} \lesssim 5\%$$

$$\rightarrow \frac{m_h^2}{M_T^2} = \kappa \underbrace{\frac{4y_t^2 N_c}{16\pi^2} \left(1 - \frac{1}{2\delta}\right)}_{\sim 7\%} \qquad \text{vs.} \qquad M_T \gtrsim 1.5\,\text{TeV}$$

Minimal composite Higgs

$$V(h) \sim$$ 

$$y_t + \cdots \sim \boldsymbol{\kappa}\, \frac{y_t^2 N_c}{16\pi^2} f^2 M_T^2 \left(-\sin^2 \frac{h}{f} + \boldsymbol{\delta}\, \sin^4 \frac{h}{f} \right)$$

Gegenbauer fix

$$\rightarrow \frac{v^2}{f^2} = \sin^2 \frac{\langle h \rangle}{f} = \frac{1}{2\delta} \qquad 1/\delta \lesssim 0.20 \qquad |\delta\kappa_V| \simeq \frac{v^2}{2f^2} \lesssim 5\%$$

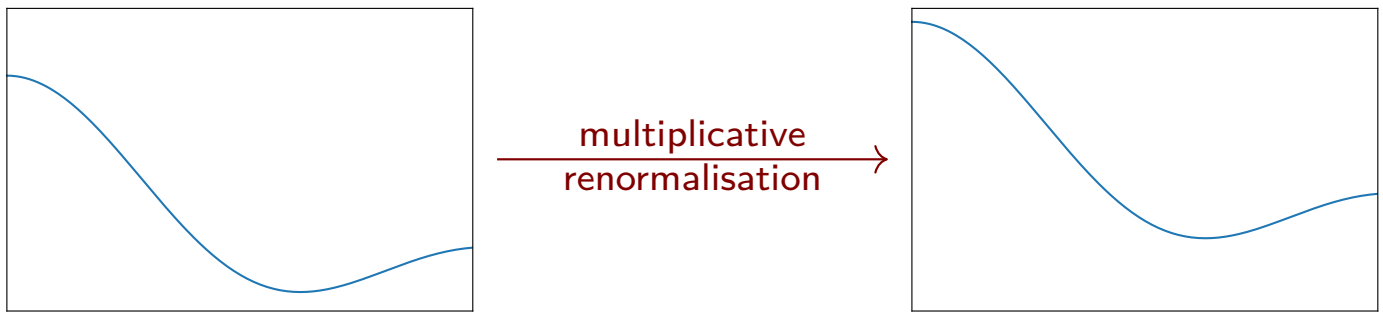
$$\rightarrow \frac{m_h^2}{M_T^2} = \kappa \underbrace{\frac{4y_t^2 N_c}{16\pi^2}\left(1 - \frac{1}{2\delta}\right)}_{\sim 7\%} \qquad \kappa \lesssim 0.10 \qquad M_T \gtrsim 1.5\,\text{TeV}$$

Twin fix

Few percent fine-tuning wrt. $\delta \lesssim 1$, $\kappa \simeq 1$ expectation

Structurally small vev

radiatively stable
low-energy pNGB potential



with deepest minimum close to the origin

Radiatively stable $SO(N+1) \to SO(N)$ potentials

$$\vec{\phi} \equiv (\frac{\vec{h}}{h} \sin\frac{h}{f},\ \cos\frac{h}{f}),\quad h \equiv |\vec{h}|$$

Linear one-loop correction to $V(\frac{h}{f})$:

$$\frac{\Lambda^2}{32\pi^2 f^2}\left(V'' + (N-1)\cot\frac{h}{f}\ V'\right)$$

Radiative stability at one-loop and linear order order if $\propto V$

Differential equation of Gegenbauer polynomials

$$V(\tfrac{h}{f}) \propto G_n^{(N-1)/2}(\cos\tfrac{h}{f})$$

Radiatively stable $SO(N+1) \to SO(N)$ potentials

Explicit $SO(N+1) \to SO(N)$ breaking by an irrep spurion K:

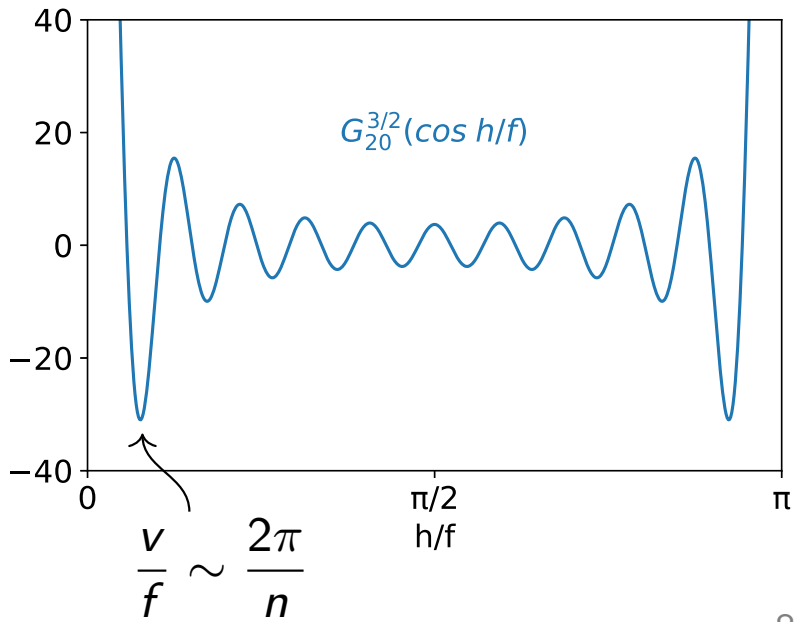
$$K^{i_1 \ldots i_n}\ \phi_{i_1} \cdots \phi_{i_n} \qquad \text{(symmetric traceless)}$$

$$\vec{\phi} \equiv (\tfrac{\vec{h}}{h} \sin \tfrac{h}{f},\ \cos \tfrac{h}{f}), \quad h \equiv |\vec{h}|$$

No other invariant, linear in K, can be constructed,
so all-loop linear renormalisation can only be multiplicative.

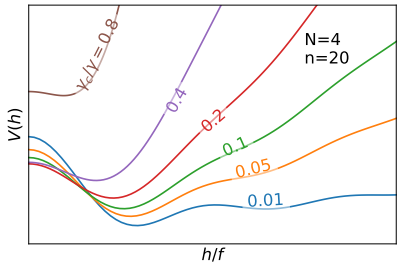
Obtain Gegenbauer polynomials:

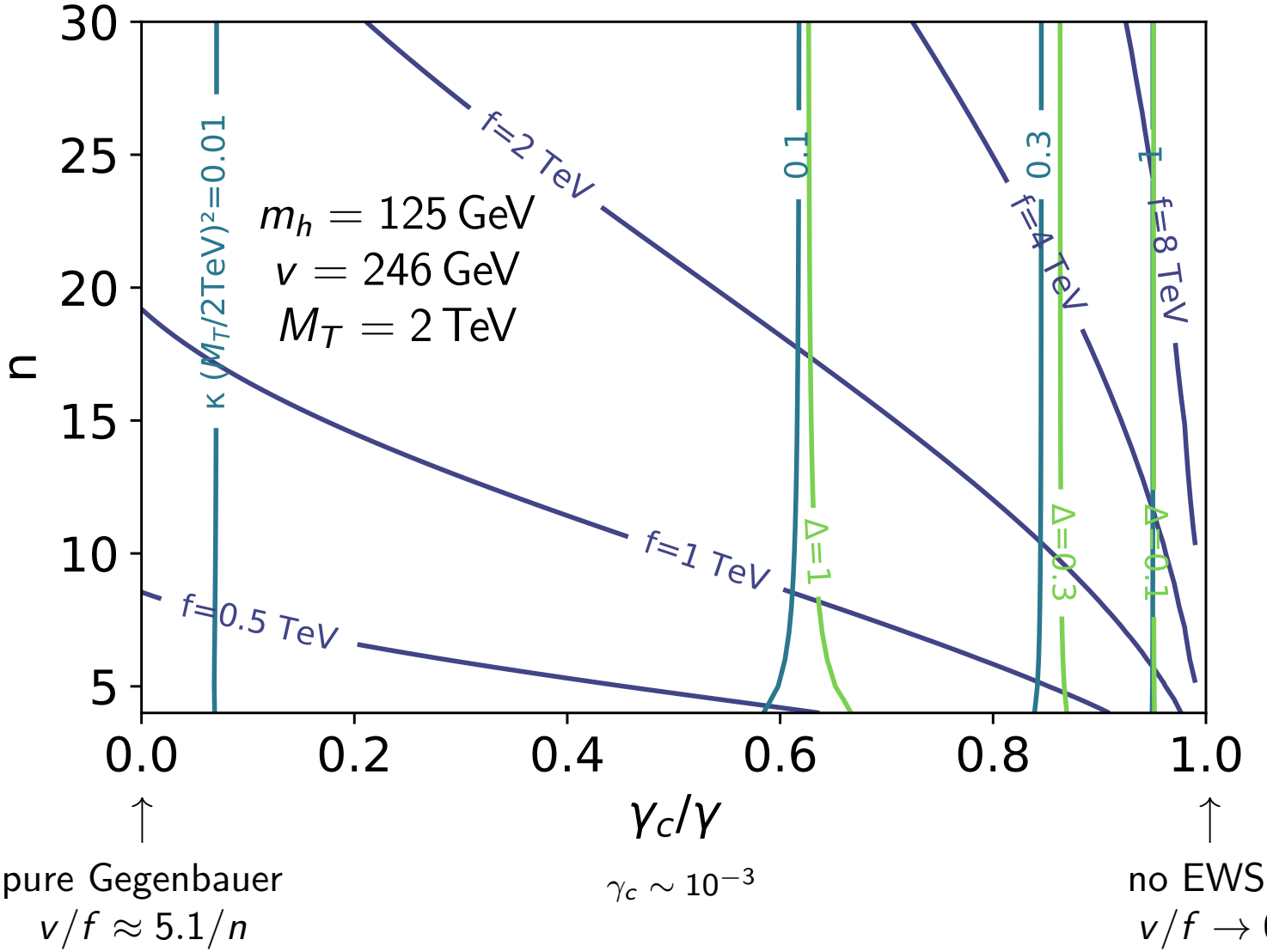
$$K^{i_1 \ldots i_n}\ \phi_{i_1} \cdots \phi_{i_n} \ \propto\ G_n^{(N-1)/2}(\cos \tfrac{h}{f})$$



The Gegenbauer Higgs

$$V(h) = \kappa \frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \left[\sin^2 \frac{h}{f} + \gamma\, G_n^{3/2}\left(\cos \frac{h}{f}\right)\right]$$





pure Gegenbauer
$v/f \approx 5.1/n$

$\gamma_c \sim 10^{-3}$

no EWSB
$v/f \to 0$

The Gegenbauer Higgs

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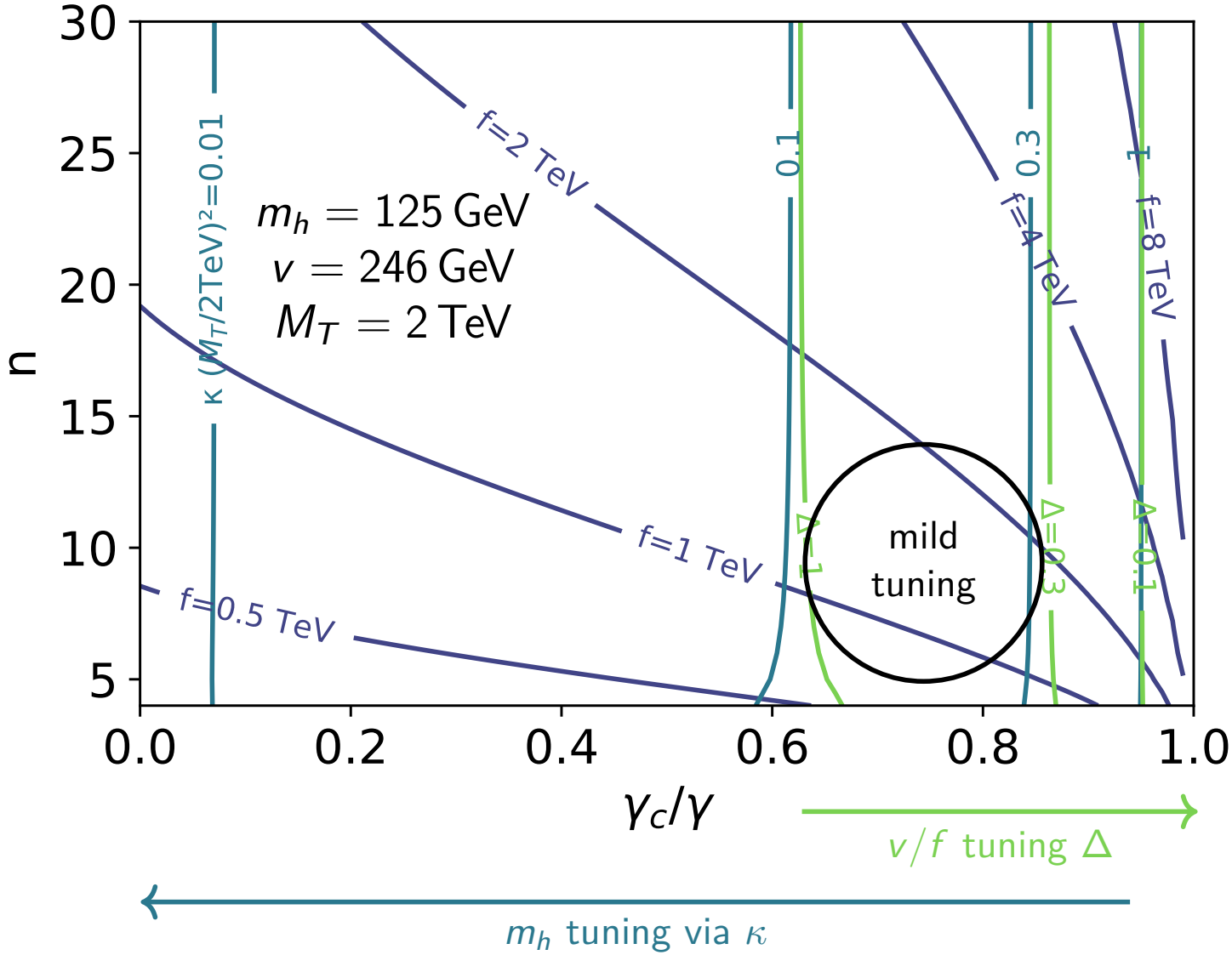

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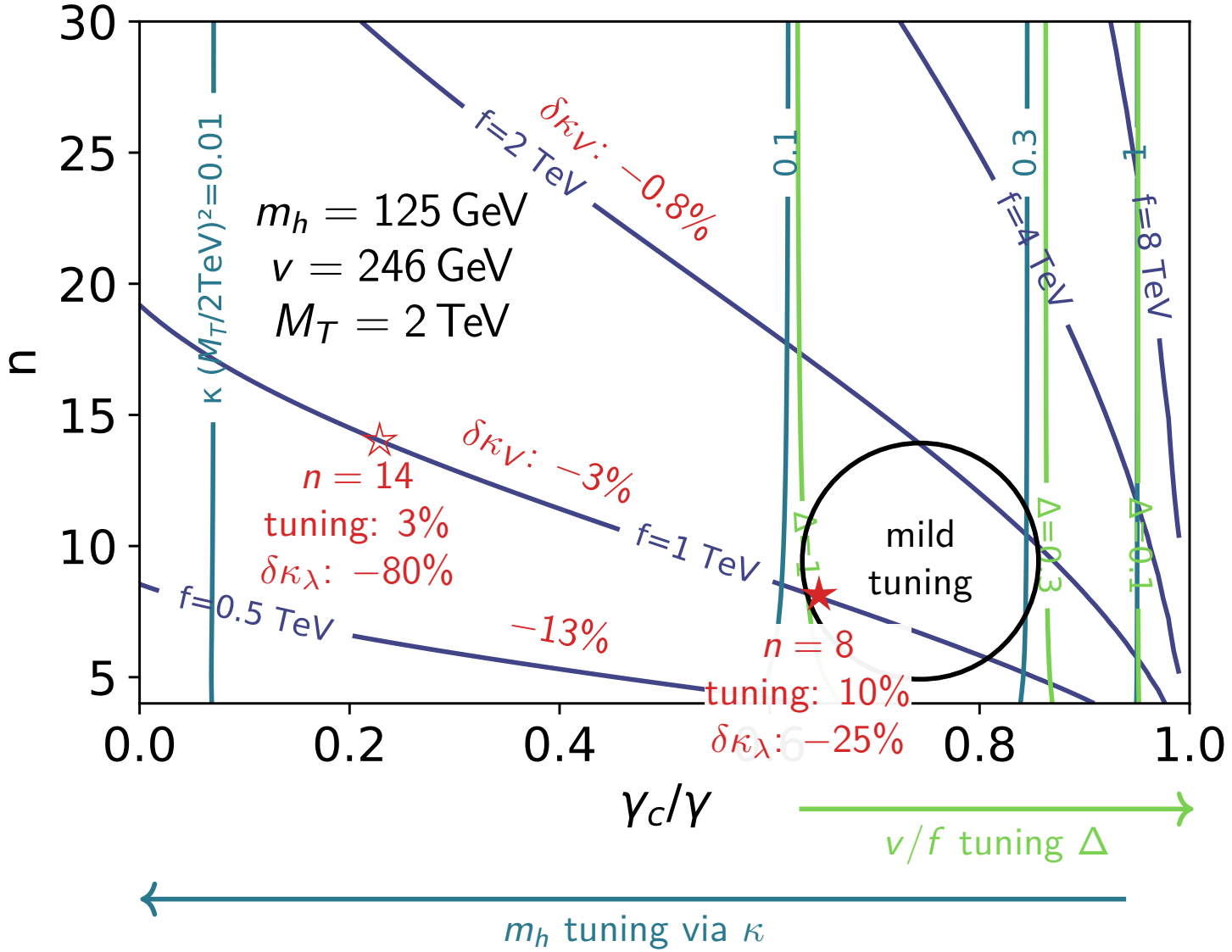
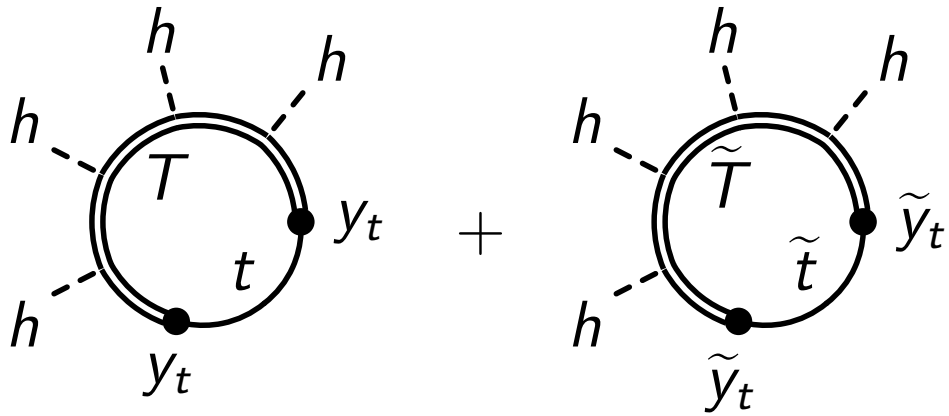


Structurally smaller mass

[Chacko, Goh, Harnik '05]
[Barbieri, Greco, Rattazzi, Wulzer '15]



$$\frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \sin^2\frac{h}{f} + \frac{N_{\widetilde{c}} \widetilde{y}_t^2}{16\pi^2} f^2 M_{\widetilde{T}}^2 \cos^2\frac{h}{f}$$

if twin parity enforces $y_t = \widetilde{y}_t$ and $M_T = M_{\widetilde{T}}$
no M_T^2 sensitivity

Structurally smaller mass

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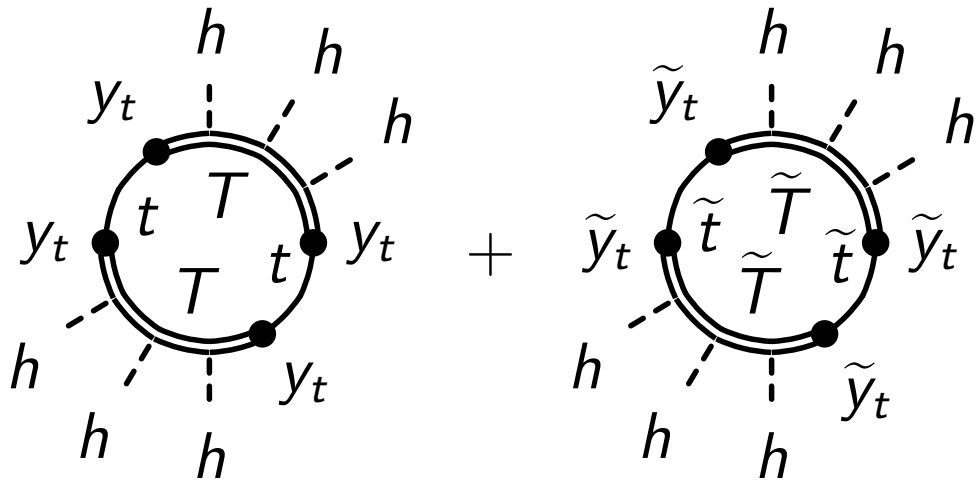
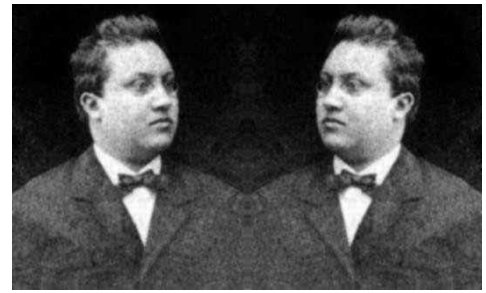


$$\frac{N_c y_t^4}{16\pi^2} f^4 \sin^4\frac{h}{f} \log M_T + \frac{N_c \widetilde{y}_t^4}{16\pi^2} f^4 \cos^4\frac{h}{f} \log M_{\widetilde{T}}$$

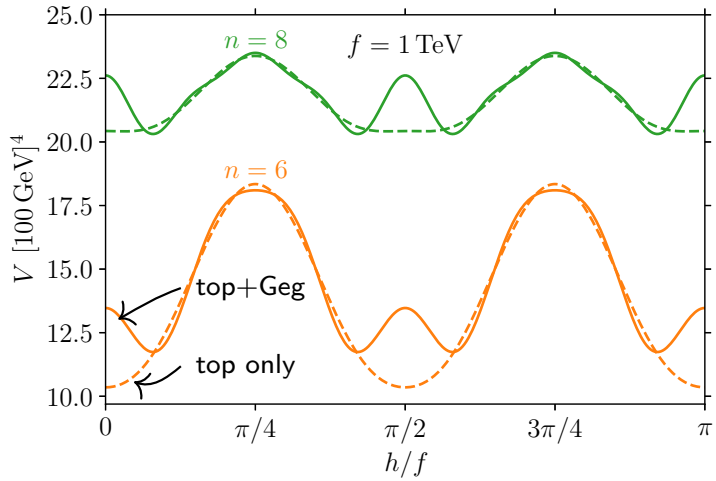
retaining $\log M_T$ sensitivity only

Gegenbauer's Twin



Leopold B. Gegenbauer
1849–1903

- global $\mathrm{SO}(8) \supset \mathrm{SO}(4) \times \widetilde{\mathrm{SO}(4)}$
- spontaneous $\mathrm{SO}(8) \to \mathrm{SO}(7)$
  - 7 NGBs
  - 6 eaten by $W^\pm, Z$ and $\widetilde{W}^\pm, \widetilde{Z}$
  - 1 Higgs: $\vec{\phi} = (\vec{0}_3,\ \sin\frac{h}{f};\ \vec{0}_3,\ \cos\frac{h}{f})^T$ in unitary gauge
- explicit breaking from the top sector is insufficient
- explicit $\mathrm{SO}(8) \to \mathrm{SO}(4) \times \widetilde{\mathrm{SO}(4)}$
  - radiative stability from irrep spurion
  - $G_n^{3/2}(\cos\frac{2h}{f})$ potential



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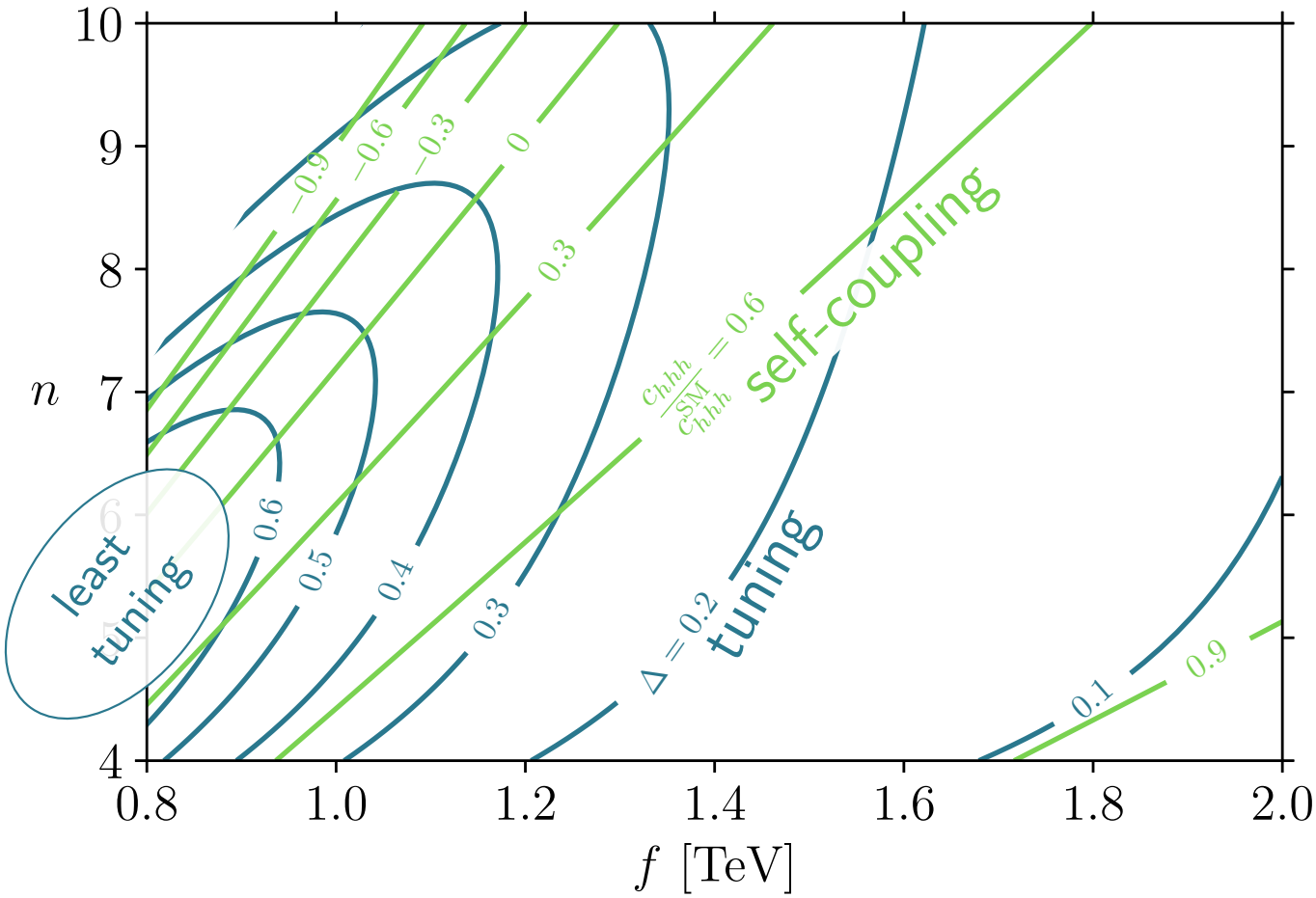
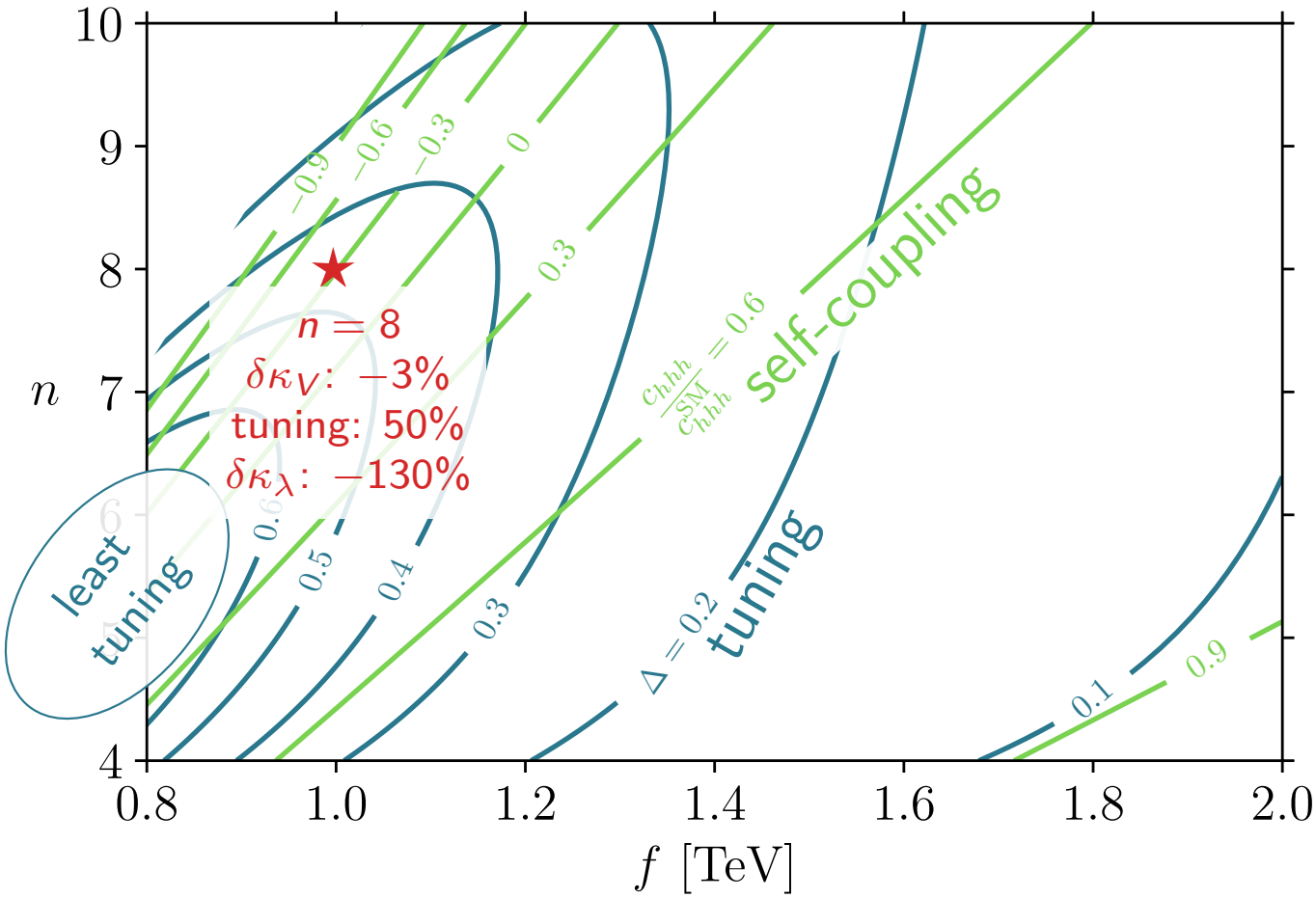


(and possibly large M_T, with unitarity violating H scattering towards 6 TeV)

Gegenbauer's Twin



(and possibly large M_T, with unitarity violating H scattering towards 6 TeV)

The Higgs self-coupling and Gegenbauer naturalness

Classes of model exist with structurally large $\delta\kappa_\lambda/\delta\kappa_V$.

The custodial quadruplet scalar is an example.

Gegenbauer models, motivated by naturalness, have it too.

Key is an explicit breaking of the global pNGB-Higgs symmetry by a large irrep.

$\delta\kappa_\lambda$ could be the first signal of new physics at colliders!