NLO electroweak corrections for di-Higgs production

20th Workshop of the LHC Higgs Working Group, 2023

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CERN, November 14, 2023

University of Zürich

Based on: [JHEP 08 (2022) 259, JHEP 06 (2023) 063, JHEP 10 (2023) 033]

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Outline

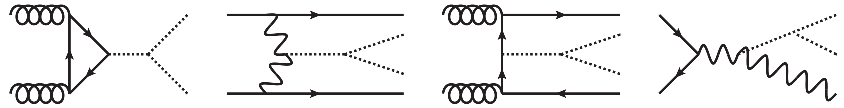
Introduction

Higgs Self Coupling

- Standard Model Higgs potential:

$$\boxed{V(H) = \frac{1}{2}m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4}, \quad \text{where } \lambda = m_H^2/(2v^2) \approx 0.13.$$

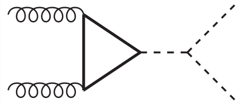
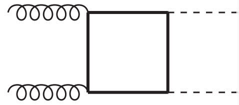
- Want to measure λ, to determine if $V(H)$ is consistent with nature.
  - Challenging! Cross-section $\approx 10^{-3} \times H$ prod.
  - $-1.24 < \lambda/\lambda_{SM} < 6.49$ [CMS '22] ; $-0.6 < \lambda/\lambda_{SM} < 6.6$ [Atlas '22]
- λ appears in various production channels:



- Gluon fusion – dominant, 10x
- VBF
- $t\bar{t}$ associated production
- H-strahlung

Gluon Fusion

- Leading order (1 loop) partonic amplitude:

$$\mathcal{M}^{\mu\nu} \sim \mathcal{A}_1^{\mu\nu}(\mathcal{F}_{tri} + \mathcal{F}_{box1}) + \mathcal{A}_2^{\mu\nu}(\mathcal{F}_{box2})$$

  - $\mathcal{F}_{tri}$ contains the dependence on λ at LO
- Form factors:
  - LO: known exactly [Glover, van der Bij '88]
  - Beyond LO... no fully-exact (analytic) results to date
    - QCD: numerical evaluation, expansion in various kinematic limits
    - EW: first steps: this work (heavy top expansion, high-energy expansion) [Davies, Mishima, Schönwald, Steinhauser, Zhang '22]
    - see also Yuakwa corrections in (partial) HTL [Mühlleitner,Schlenk,Spira '22]

$gg \to HH$ Beyond LO

NLO QCD:

- large-m_t [Dawson, Dittmaier, Spira '98] [Grigo, Hoff, Melnikov, Steinhauser '13]
- numeric [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke '16] [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher '19]
- large-m_t + threshold exp. Padé [Gröber, Maier, Rauh '17]
- high-energy expansion [Davies, Mishima, Steinhauser, Wellmann '18,'19]
- small-p_T expansion [Bonciani, Degrassi, Giardino, Gröber '18]
  + high-energy expansion [Bagnaschi, Degrassi, Gröber '23]

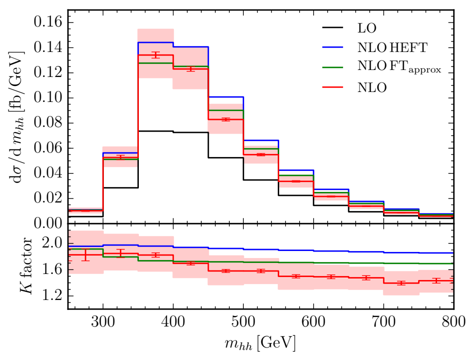
NNLO QCD:

- large-m_t virtuals [de Florian, Mazzitelli '13] [Grigo, Hoff, Steinhauser '15, Davies; Steinhauser '19]
- HTL+numeric real ("FTapprox") [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18]
- large-m_t reals [Davies, Herren, Mishima, Steinhauser '19 '21]
- light fermion corrections at $p_T = 0$ [Davies, Schönwald, Steinhauser '23]

N3LO QCD:

- Wilson coefficient C_{HH} [Spira '16; Gerlach, Herren, Steinhauser '18]
- HTL [Chen, Li, Shao, Wang '19]

$gg \to HH$ Beyond LO



[Borowka, Greiner, Heinrich, Jones, Kerner '16]

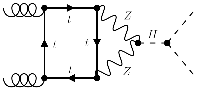
Total cross section (14TeV):

	σ_{LO}	σ_{NLO}	σ_{NNLO}
B-i HTL	–	$38.32^{+18.1\%}_{-14.9\%}$	$39.58^{+1.4\%}_{-4.7\%}$
FTapprox	–	$34.25^{+14.7\%}_{-13.2\%}$	$36.69^{+2.1\%}_{-4.9\%}$
Full	$19.85^{+27.6\%}_{-20.5\%}$	$32.88^{+13.5\%}_{-12.5\%}$	–

Electroweak Corrections to Di-Higgs Production

Full Electroweak Corrections in the Large-m_t Expansion

- Sample Feynman diagrams involving:
  - SM fields: $\{t, b, H, \gamma, Z, W^\pm, \chi, \phi^\pm\}$
  - ghosts: $\{u^\gamma, u^Z, u^\pm\}$



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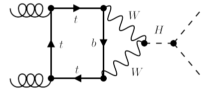

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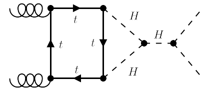

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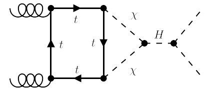

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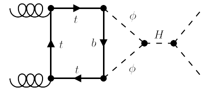

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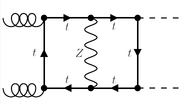

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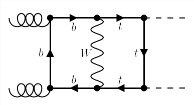

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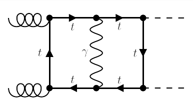

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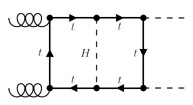

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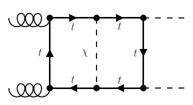

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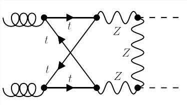

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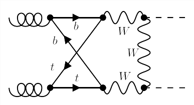

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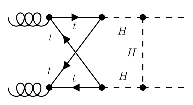

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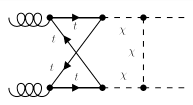

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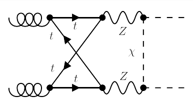

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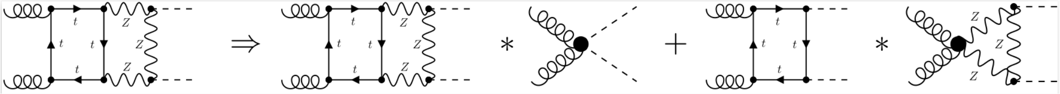
Goal: obtain analytic expressions in the large-m_t expansion

Large-m_t Expansion and Renormalization

- Expand and calculate in general R_ξ gauge with `qgraf` [Nogueira '93] , `tapir` [Gerlach, Herren, Lang '23] , `q2e&exp` [Harlander, Seidensticker, Steinhauser '97-'99] , `form` [Ruijl, Ueda, Vermaseren '17] , `LiteRed` [Lee '12] and `MATAD` [Steinhauser '01] .
- Expansion hierarchy: $m_t^2 \gg \xi_W m_W^2, \xi_Z m_Z^2 \gg s, t, m_H^2, m_W^2, m_Z^2$

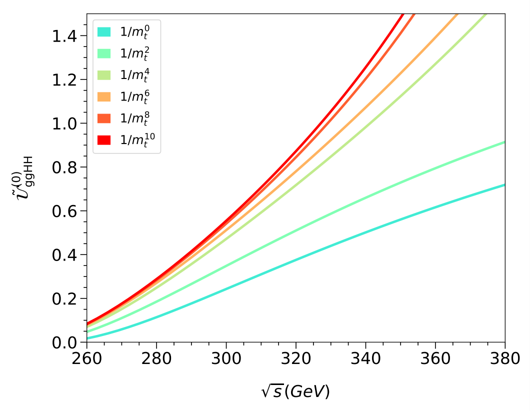


- We renormalize the input parameters $\{e,\ m_W,\ m_Z,\ m_t,\ m_H\}$ and the Higgs wave function on-shell and transform to the G_μ scheme.
  - ξ_W, ξ_Z, μ^2 cancel analytically

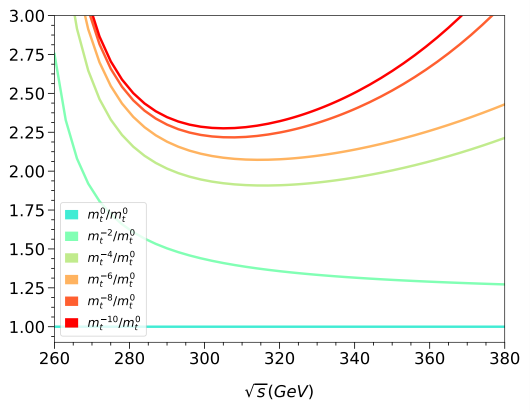
LO Matrix Elements for $gg \to HH$

$$\mathcal{M} = \frac{1}{8^2 2^2} \sum_{\text{col}} \sum_{\text{pol}} |\mathcal{A}|^2 = \frac{1}{16} \left(X_0^{ggHH} \right)^2 \tilde{U}_{ggHH}$$

$$\tilde{U}_{ggHH} = \tilde{U}^{(0)}_{ggHH} + \frac{\alpha}{\pi} \tilde{U}^{(0,1)}_{ggHH}$$



$\tilde{U}_{ggHH}$ up to different expansion orders in $1/m_t$.



Different expansion orders normalized to m_t^0.

We see a nice convergence up to roughly $\sqrt{s} = 2m_t \approx 350$,GeV.

NLO Electroweak Matrix Elements for $gg \to HH$

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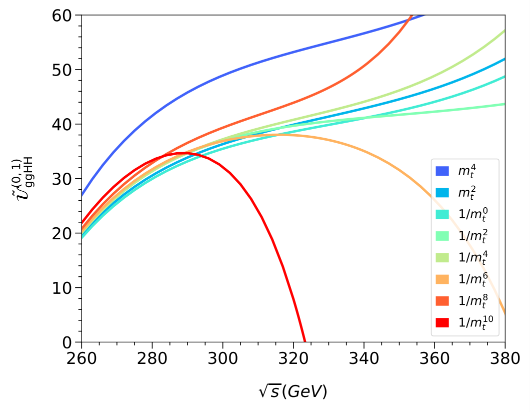
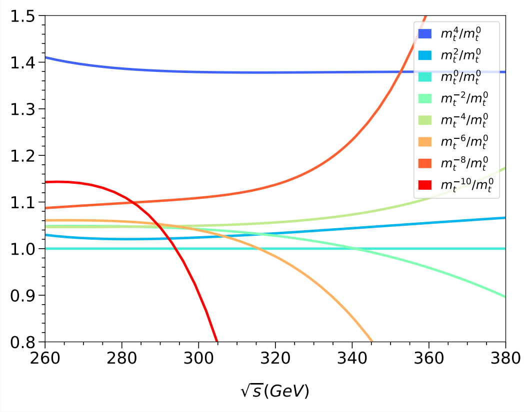


$\tilde{U}_{ggHH}$ up to different expansion orders in $1/m_t$.

We do not see such a nice convergence at NLO.



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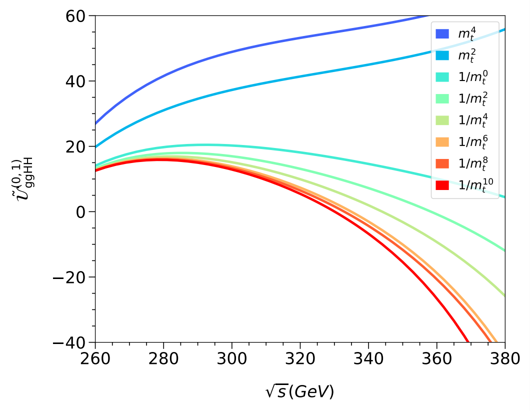
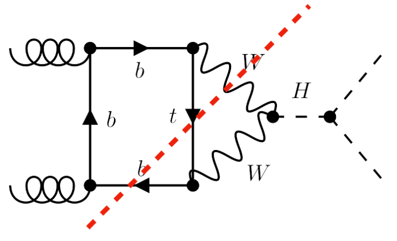


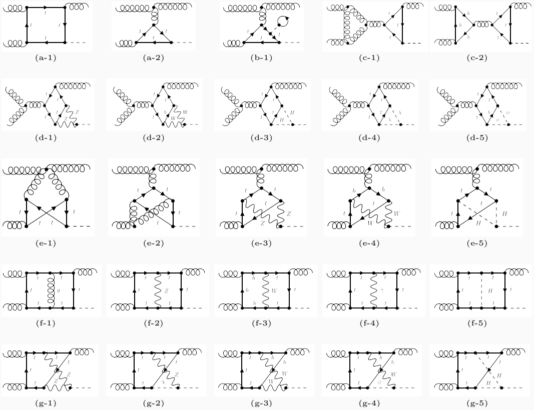


Cut through W-t-b affects convergence of the large-m_t expansion:

$m_t + m_b + m_W \approx 250\,\text{GeV}$

We can restore convergence by excluding diagrams with W-t-b cuts.

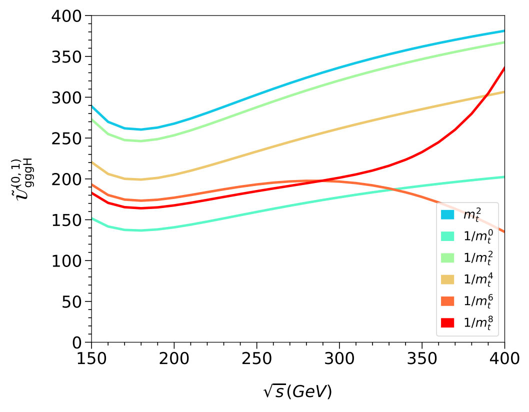
$$\mathcal{M} = \frac{1}{8^2 2^2} \sum_{\text{col}} \sum_{\text{pol}} |\mathcal{A}|^2 = \frac{3}{32} \left(X_0^{gggH} \right)^2 s \, \tilde{U}_{gggH}$$



Graphs contributing to $gg \to Hg$.

We observe a nice convergence at NLO.

$$\tilde{U}_{gggH} = \tilde{U}^{(0)}_{gggH} + \frac{\alpha}{\pi} \tilde{U}^{(0,1)}_{gggH}$$

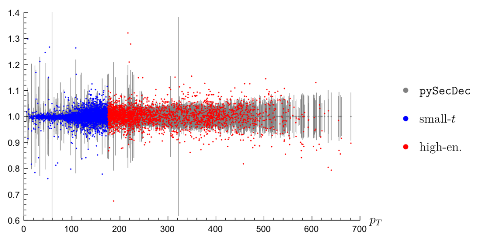


Different expansion orders in $1/m_t$.

Beyond the Large-m_t Expansion

- QCD corrections can be effectively covered by combining the expansions in:
  - the large-m_t limit
  - the high-energy limit [Davies, Mishima, Steinhauser, Wellmann '18]
  - forward kinematics ($p_T \to 0$ or $t \to 0$) [Bonciani, Degrassi, Giardino, Gröber '18; Davies, Mishima, Schönwald, Steinhauser '23]

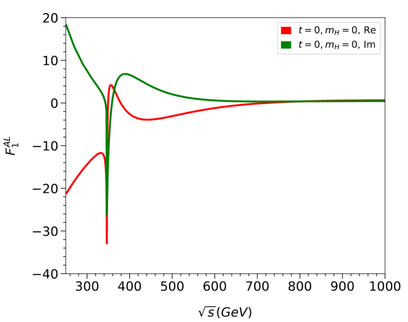
NLO $\mathcal{V}_{fin}$



[Davies, Mishima, Schönwald, Steinhauser, JHEP 06 (2023)]

based on 56 expansion terms in the high-energy and 5 terms in the small-t expansion

First steps to NNLO



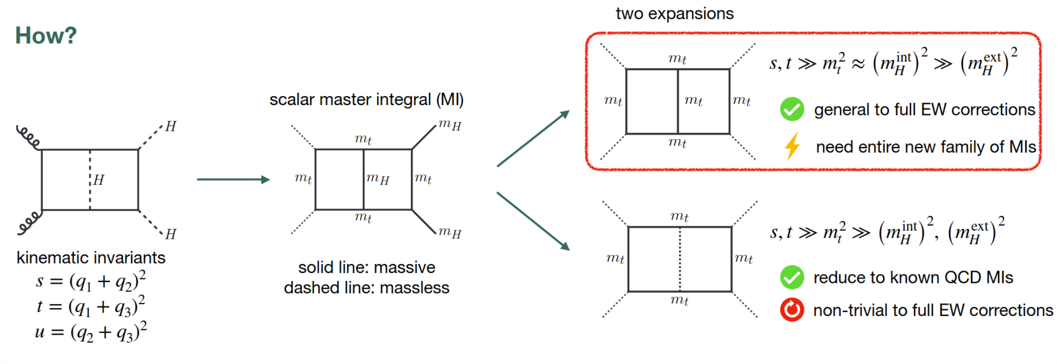
[Davies, Schönwald, Steinhauser, Phys.Lett.B 845 (2023)]

Light fermion contributions to $gg \to HH$ at NNLO for $t = 0$.

NNLO corrections are needed to address top mass scheme uncertainty.

Beyond the Large-m_t Expansion – High Energy Expansion

- Start with diagrams with internally propagating Higgs:
  - expansion parameter not small $\alpha_t = \alpha m_t^2/(2 s_W^2 m_W^2) \sim \alpha_s/2$
  - only planar integrals contribute in this subset



Beyond the Large-m_t Expansion – High Energy Expansion

Analytic high-energy expansion:

- Expansion hierarchy: $s, t \gg m_t^2 \approx (m_H^2)^{int} \gg (m_H^2)^{ext}$
- We get a system of differential equations for 140 master integrals

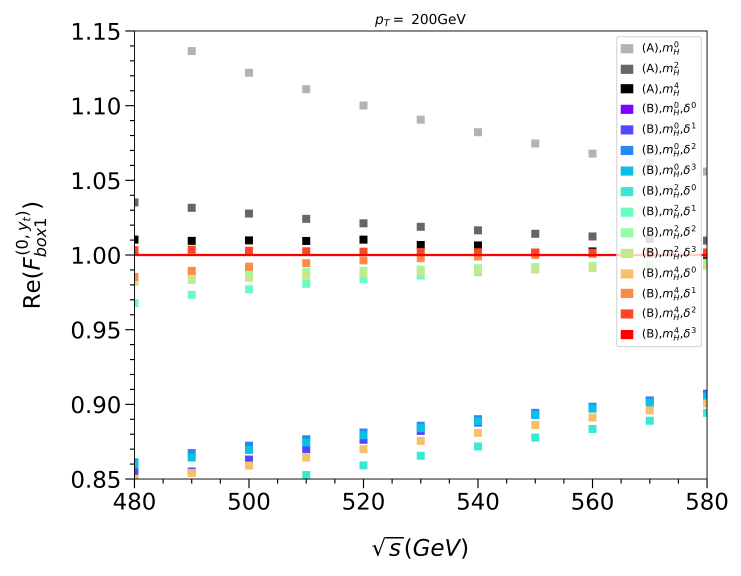
$$\boxed{\frac{\partial}{\partial m_t^2}\vec{I} = M(s,t,m_t^2,\epsilon)\cdot\vec{I}}, \quad \text{with } \vec{I} = (I_1,\ldots,I_{140})$$

- Plug in power-log ansatz for each master integral

$$\boxed{I_n = \sum_{i=-2}^{0}\sum_{j=-1}^{60}\sum_{k=0}^{i+4} c_n^{ijk}(s,t)\epsilon^i (m_t^2)^j \ln^k(m_t^2)}$$

- Solve the system of linear equations for a small set of boundary constants with `Kira` and `FireFly` [Klappert, Lange, Maierhöfer, Usovitsch '21] .
- Solve boundary master integrals in the asymptotic limit $m_t \to 0$ with Mellin-Barnes methods and symbolic summation using `Asy` [Pak, Smirnov '11] , `MB.m` [Czakon '05] , `HarmonicSums` [Ablinger '10] and `Sigma` [Schneider '07] .

Beyond the Large-m_t Expansion – High Energy Expansion



$$\mathcal{A}^{\mu\nu} = T_1^{\mu\nu} F_{box1} + T_2^{\mu\nu} F_{box2}$$

- We benchmark against the expansion to $O(m_H^4, \delta^3, m_t^{116})$, with $\delta = 1 - m_H/m_t$.
- Convergence of different expansion orders at fixed $p_T = 200\,\text{GeV}$.

Conclusions and Outlook

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Conslusions:

- We have calculated full NLO electroweak corrections to $gg \to HH$ in the large-m_t expansion.
  - The convergence of these expansions is hindered by W-t-b cuts.
  - We estimate the the EW corrections can reach a few tens of percent corrections in this region.
- We have calculated parts of the leading-Yukawa corrections in the high-energy region and see a good convergence of our approach.

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Outlook:

- Calculate the full EW corrections in the
  1. high-energy expansion.
  2. small-t expansion.
- Provide a numerical program, which can be incorporated into Monte-Carlo studies.

Backup

Padé-Improved High-Energy Expansion

The master integrals for both methods are computed as an expansion in $m_t \ll s, |t|$.

The expansions diverge for $\sqrt{s} \sim 750\text{GeV}$ ("A"), $\sqrt{s} \sim 1000\text{GeV}$ ("B").

The situation can be improved using Padé Approximants:

- Approximate a function using a rational polynomial

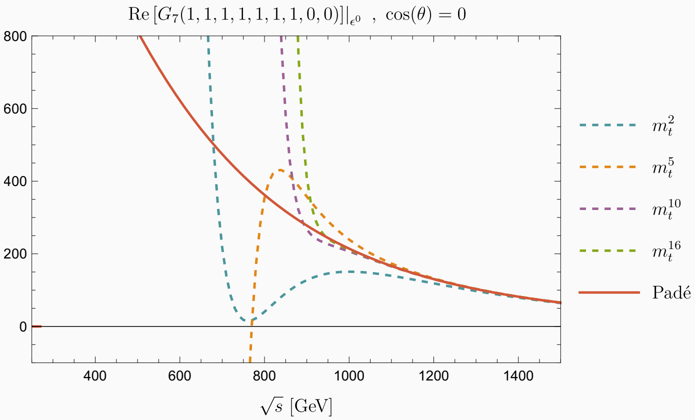
$$\boxed{f(x) \approx \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n}{1 + b_1 x + b_2 x^2 + \cdots + b_m x^m}},$$

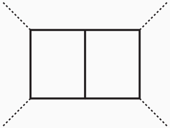
where a_i, b_j coefficients are fixed by the series coefficients of $f(x)$.

We compute a set of various Padé Approximants:

- combine to give a central value and error estimates
- a deeper input expansion $\Rightarrow$ larger $n + m$ $\Rightarrow$ smaller error
- here, m_t^{120} expansion allows for very high-order Padé Approximants

Master Integrals Results

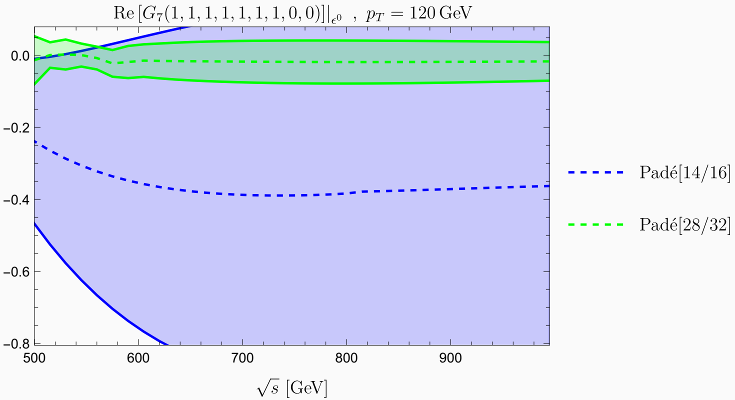




$$\cos(\theta) = \frac{s + 2t - 2m_h^2}{s\sqrt{1 - 4m_h^2/s}}$$

- Fixed order m_t expansions diverge at $\sqrt{s} \sim 1000\,\text{GeV}$.
- The Padé approximation extends the range of validity.

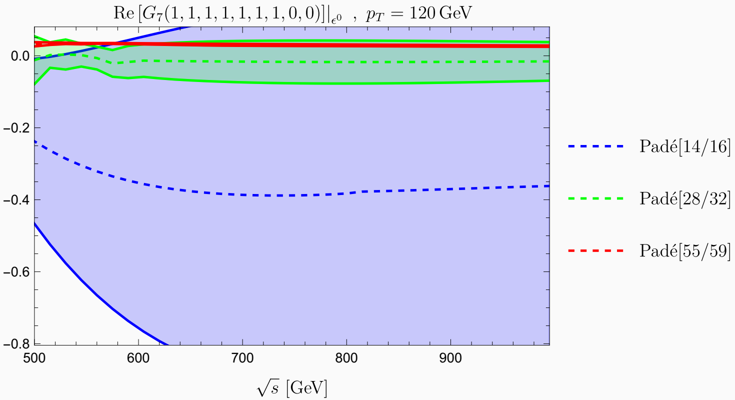
Padé Improvement



$$p_T^2 = \frac{tu - m_h^4}{s}$$

- Lower order Padé approximantions cannot reach low values of p_T.
- For QCD corrections expansions up to m_t^{32} were available: $p_T \gtrsim 150\,\mathrm{GeV}$
- With expansions up to m_t^{120} we reach: $p_T \gtrsim 120\,\mathrm{GeV}$.
- Error estimate from Padé approximations is reliable.

Padé Improvement



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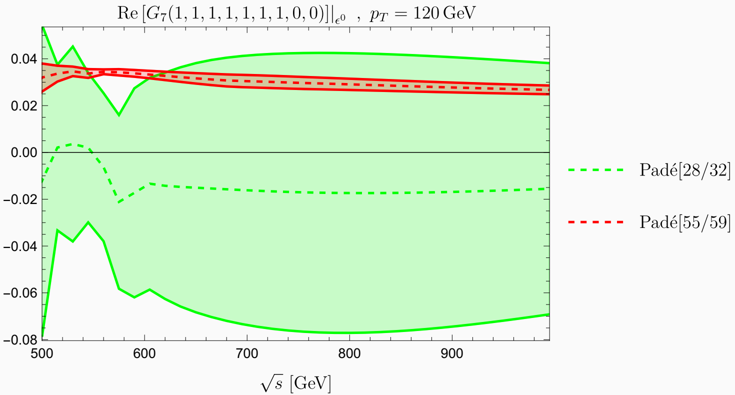

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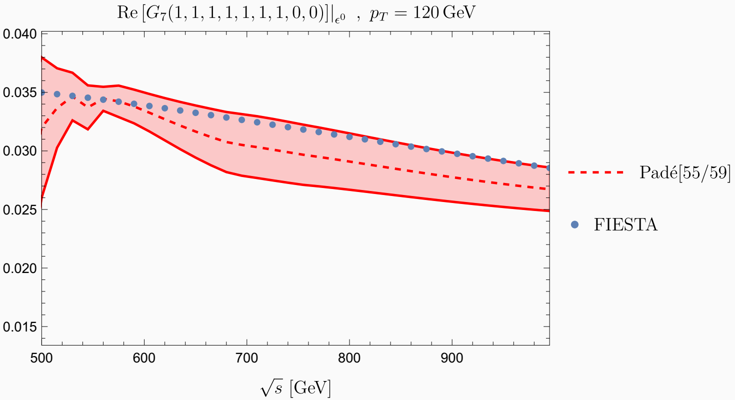

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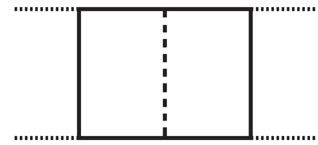
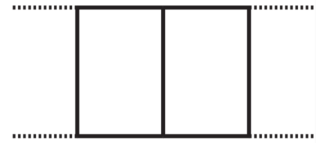
- Error estimate from Padé approximations is reliable.

Comparison to the $m_H \to 0$ Expansion



Approach A:

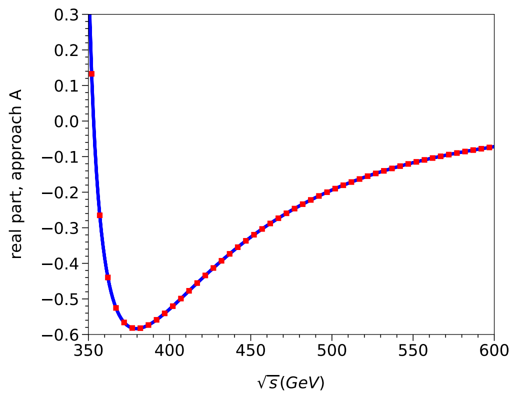
- middle line massless $m_H^{\text{int}} \approx 0$
- calculated in the context of QCD corrections [Davies, Mishima, Steinhauser, Wellmann '18, '19]

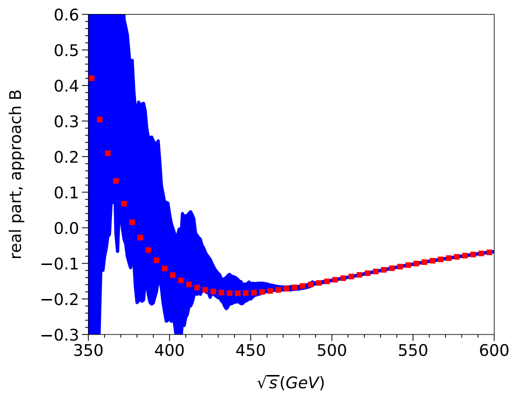


Approach B:

- middle line massive $m_H^{\text{int}} \approx m_t$

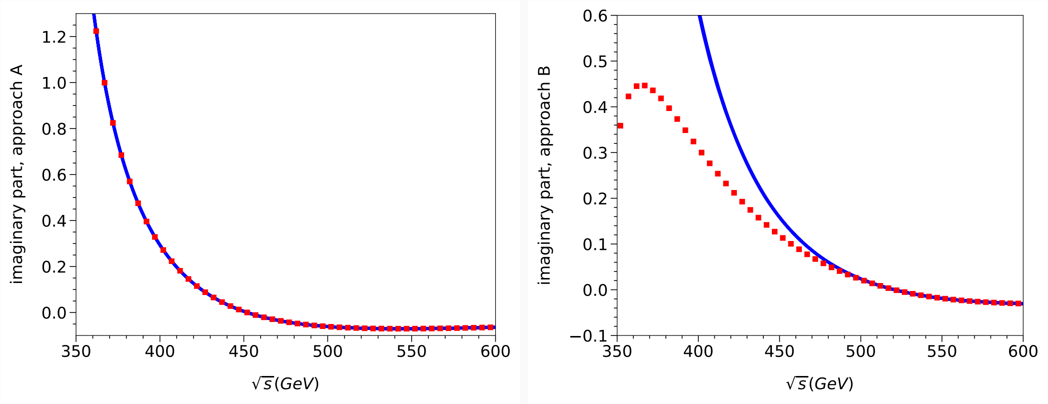
Comparison with Approach A





Approach A: threshold at $\sqrt{s} = 2m_t = 346\,\text{GeV}$ Approach B: threshold at $\sqrt{s} = 3m_t = 519\,\text{GeV}$

Comparison with Approach A



Approach A: threshold at $\sqrt{s} = 2m_t = 346\,\text{GeV}$ Approach B: threshold at $\sqrt{s} = 3m_t = 519\,\text{GeV}$