Non-factorizable corrections to Higgs production in Vector Boson Fusion

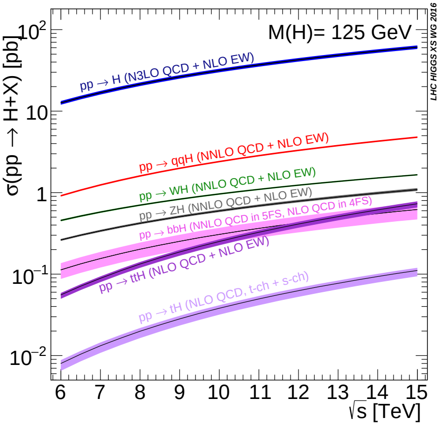
The 20th Workshop of the LHC Higgs Working Group, CERN, 13 - 15 November 2023

Ming-Ming Long | In collaboration with Christian Brønnum-Hansen, Kirill Melnikov and Jérémie Juvin-Quarroz

Outline

Higgs production in VBF

- large cross section



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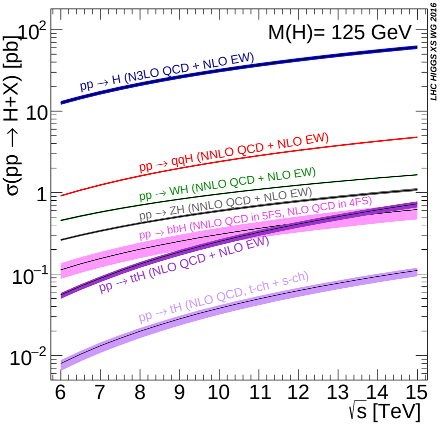

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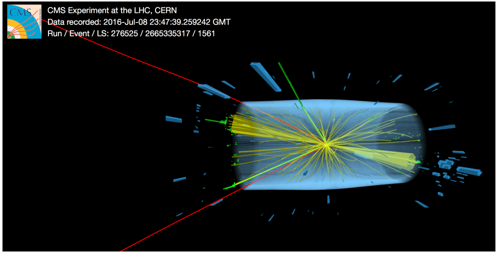

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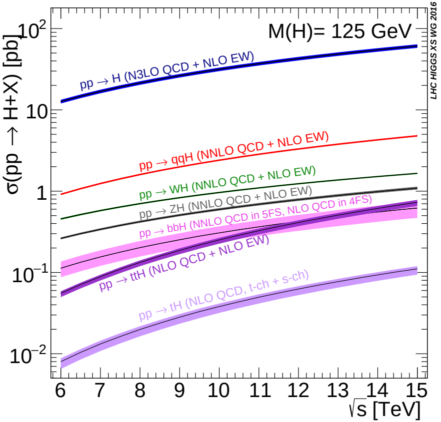

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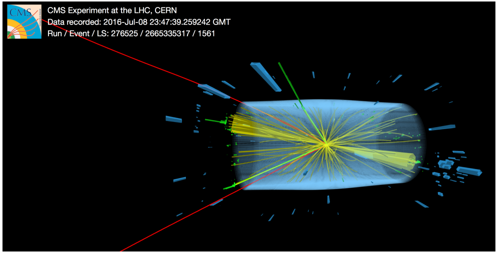

- HVV (anomalous) couplings; CP properties of Higgs; Higgs decays

High-order corrections to VBF

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- Figy, Oleari, Zeppenfeld 2003
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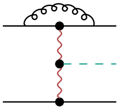
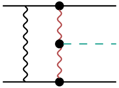
•NLO QCD & EW





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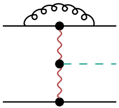
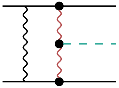
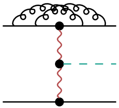

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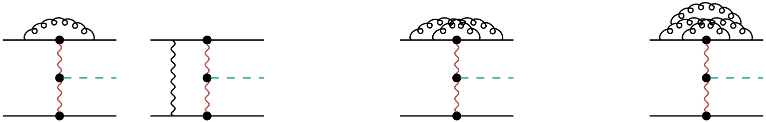

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N3LO

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NF

- Liu, Melnikov, Penin 2019
- Dreyer, Karlberg, Tancredi 2020
- Gates 2023
- Brønnum-Hansen, **MML**, Melnikov, Juvin-Quarroz 2023

Factorizable VS Non-factorizable

- Factorizable corrections are at $\mathcal{O}(\%)$ [Dreyer, Karlberg 2016]

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	$\sigma^{(13\ \mathrm{TeV})}$ [pb]	$\sigma^{(14\ \mathrm{TeV})}$ [pb]	$\sigma^{(100\ \mathrm{TeV})}$ [pb]
LO	$4.099^{+0.051}_{-0.067}$	$4.647^{+0.037}_{-0.058}$	$77.17^{+6.45}_{-7.29}$
NLO	$3.970^{+0.025}_{-0.023}$	$4.497^{+0.032}_{-0.027}$	$73.90^{+1.73}_{-1.94}$
NNLO	$3.932^{+0.015}_{-0.010}$	$4.452^{+0.018}_{-0.012}$	$72.44^{+0.53}_{-0.40}$
N3LO	$3.928^{+0.005}_{-0.001}$	$4.448^{+0.006}_{-0.001}$	$72.34^{+0.11}_{-0.02}$

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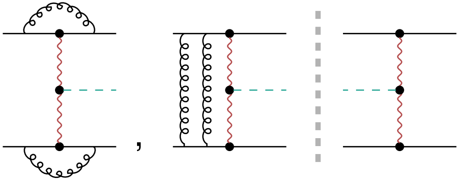

$$\Longrightarrow \frac{(N_c^2-1)^2}{4}, \quad \frac{(N_c^2-1)}{4}$$

Factorizable VS Non-factorizable

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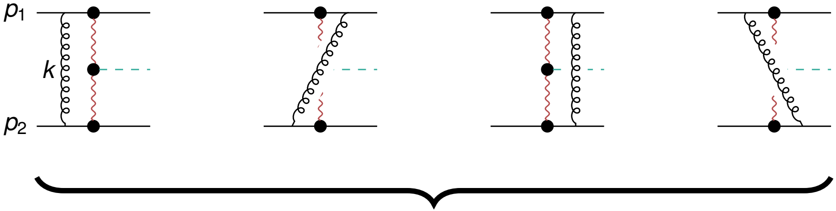
- Non-factorizable corrections are color-suppressed
- π^2 enhancement in non-factorizable contributions [Liu, Melnikov, Penin 2019]



$$\left[\frac{1}{2p_1k+i0}+\frac{1}{-2p_1k+i0}\right]\left[\frac{1}{-2p_2k+i0}+\frac{1}{2p_2k+i0}\right]\Longrightarrow \pi^2$$

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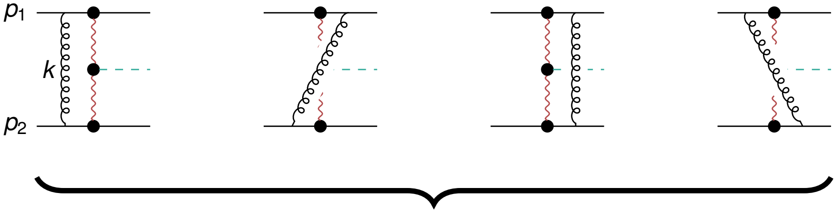

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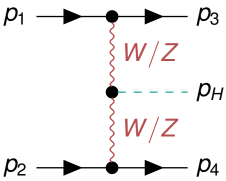
- How to go beyond eikonal approximation?

Forward kinematics

$$q(p_1) + q'(p_2) \to Q(p_3) + Q'(p_4) + H(p_H)$$



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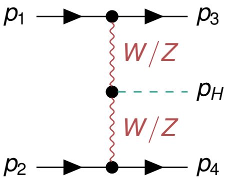

Sudakov decomposition

$$p_i = \alpha_i p_1 + \beta_i p_2 + p_{i,\perp}, \quad i = 3, 4$$

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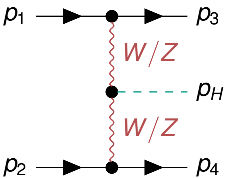

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using $\quad p_{3,4}^2 = 0 \Longrightarrow \beta_3 = \dfrac{\mathbf{p}_{3,\perp}^2}{s\alpha_3}, \alpha_4 = \dfrac{\mathbf{p}_{4,\perp}^2}{s\beta_4}$

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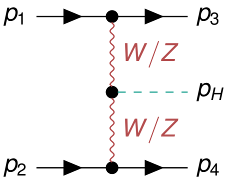

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one has

$$\delta_3\delta_4 \approx \frac{m_H^2 + \mathbf{p}_{H,\perp}^2}{s}, \quad \begin{cases} \delta_3 & = 1 - \alpha_3 \\ \delta_4 & = 1 - \beta_4 \end{cases}$$

Forward kinematics

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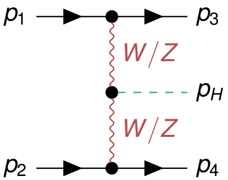

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Forward limit

Forward kinematics

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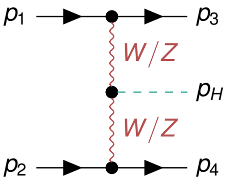

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$$\delta_3\delta_4 \sim \frac{m_H^2}{s} \sim \frac{m_V^2}{s} \sim \frac{\mathbf{p}_{3,\perp}^2}{s} \sim \frac{\mathbf{p}_{4,\perp}^2}{s} \sim \lambda << 1$$

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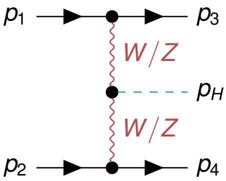

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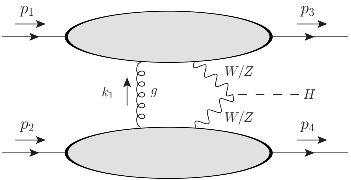
$$\delta_3 \sim \delta_4 \sim \sqrt{\lambda}$$

One-loop amplitudes



$$\mathcal{M}_1 = g_s^2 g_w^2 g_{VVH} T^a_{i_3 i_1} T^a_{i_4 i_2} \mathcal{A}_1$$

the color-stripped amplitude $\mathcal{A}_1$ reads

$$\mathcal{A}_1 = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d} \frac{1}{d_1 d_3 d_4} J^{\mu\nu} \tilde{J}_{\mu\nu}$$

One-loop amplitudes

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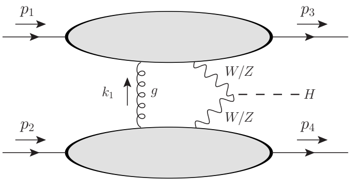

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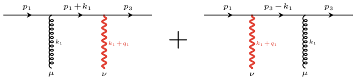
$J^{\mu\nu} =$ 

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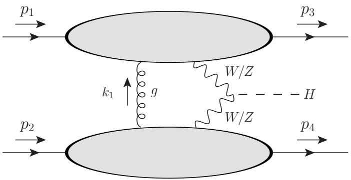

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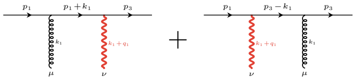
$J^{\mu\nu} =$ 

Expansion by regions

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}, \quad \frac{\mathrm{d}^d k_1}{(2\pi)^d} = -\frac{s}{2} \frac{\mathrm{d}\alpha_1}{2\pi i} \frac{\mathrm{d}\beta_1}{2\pi i} \frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{d-2}}$$

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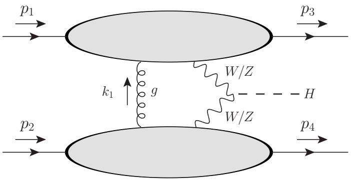

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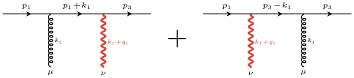

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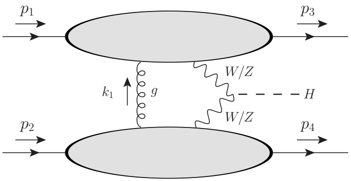
	α_1	β_1	$\mathbf{k}_{1,\perp}$	$\mathcal{M}_1$
G	λ	λ	$\sqrt{\lambda}$	-2
G-S	λ	$\sqrt{\lambda}$	$\sqrt{\lambda}$	-2
S	$\sqrt{\lambda}$	$\sqrt{\lambda}$	$\sqrt{\lambda}$	-2
C	1	λ	$\sqrt{\lambda}$	-3/2
H	1	1	1	0

One-loop amplitudes



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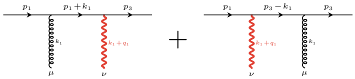
$J^{\mu\nu} =$ 

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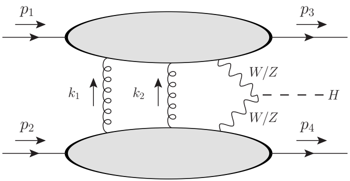
H	1	1	1	0

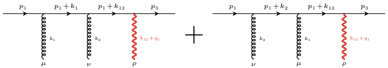
Two-loop amplitudes



$$\mathcal{M}_2 = -ig_s^4 g_w^2 g_{VVH} \left(\frac{1}{2}\{T^a, T^b\}\right)_{i_3 i_1} \left(\frac{1}{2}\{T^a, T^b\}\right)_{i_4 i_2} \mathcal{A}_2$$

$$\mathcal{A}_2 = \frac{1}{2!} \int \frac{\mathrm{d}^d k_1}{(2\pi)^d} \frac{\mathrm{d}^d k_2}{(2\pi)^d} \frac{1}{d_1 d_2 d_3 d_4} J_{\mu\nu\rho} \tilde{J}^{\mu\nu\rho} \,.$$

with the current $J^{\mu\nu\rho}$

$J^{\mu\nu\rho} =$  $+$(permu.)

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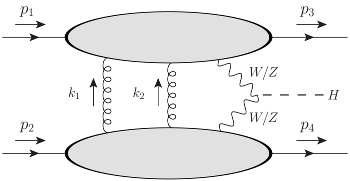
α_i	β_i	$k_{i,\perp}$
λ	λ	$\sqrt{\lambda}$
$\sqrt{\lambda}$	$\sqrt{\lambda}$	1
1	1	

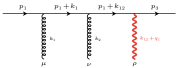
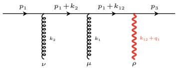
Two-loop amplitudes



$$\mathcal{M}_2 = -ig_s^4 g_w^2 g_{VVH} \left(\frac{1}{2}\{T^a, T^b\}\right)_{i_3 i_1} \left(\frac{1}{2}\{T^a, T^b\}\right)_{i_4 i_2} \mathcal{A}_2$$

$$\mathcal{A}_2 = \frac{1}{2!} \int \frac{\mathrm{d}^d k_1}{(2\pi)^d} \frac{\mathrm{d}^d k_2}{(2\pi)^d} \frac{1}{d_1 d_2 d_3 d_4} J_{\mu\nu\rho} \tilde{J}^{\mu\nu\rho} \, .$$

with the current $J^{\mu\nu\rho}$

$J^{\mu\nu\rho} =$  $+$  $+$(permu.)

$$k_i = \alpha_i p_1 + \beta_i p_2 + k_{i,\perp},$$

α_i	β_i	$k_{i,\perp}$
λ	λ	$\sqrt{\lambda}$
$\sqrt{\lambda}$	$\sqrt{\lambda}$	1
1	1	

Only the Glauber and mixed regions contribute!

Factorization of amplitudes

$$\mathcal{M}_1 = i\frac{g_s^2}{4\pi}\, T^a_{i_3 i_1}\, T^a_{i_4 i_2}\, \mathcal{M}_0\, \mathcal{C}_1, \quad \mathcal{M}_2 = -\frac{1}{2}\frac{g_s^4}{(4\pi)^2}\left(\frac{1}{2}\{T^a, T^b\}\right)_{i_3 i_1}\left(\frac{1}{2}\{T^a, T^b\}\right)_{i_4 i_2} \mathcal{M}_0\, \mathcal{C}_2$$

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The functions $\mathcal{C}_i$ read

$$\mathcal{C}_1 = 2\int \frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{1-2\epsilon}} \frac{(\mathbf{p}_{3,\perp}^2 + m_V^2)(\mathbf{p}_{4,\perp}^2 + m_V^2)}{\Delta_1 \Delta_{3,1} \Delta_{4,1}} \times \Omega_1$$

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with

$$\Omega_i = 1 - \delta_3\left(\frac{m_V^2}{\mathbf{p}_{3,\perp}^2 + m_V^2} + \frac{m_V^2}{\Delta_{3,i}}\right) - \delta_4\left(\frac{m_V^2}{\mathbf{p}_{4,\perp}^2 + m_V^2} + \frac{m_V^2}{\Delta_{4,i}}\right)$$

Numerical results

$$\mathrm{d}\hat{\sigma}_{\mathrm{nf}}^{\mathrm{NNLO}} = \frac{N_c^2 - 1}{4N_c^2}\,\alpha_s^2\,\mathcal{C}_{\mathrm{nf}}\,\mathrm{d}\hat{\sigma}^{\mathrm{LO}}, \qquad \mathcal{C}_{\mathrm{nf}} = \mathcal{C}_1^2 - \mathcal{C}_2\,,$$

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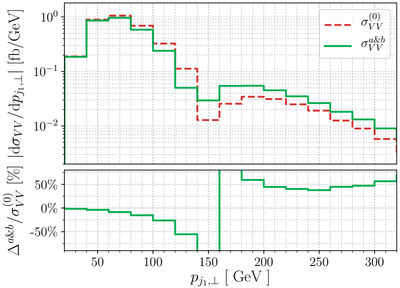

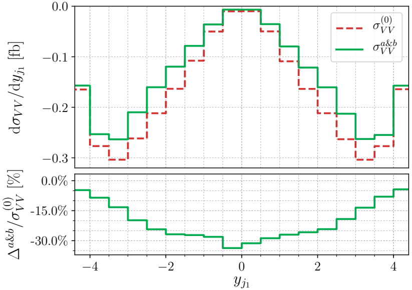

Scale dependence

- Strong μ_R dependence

$$\mu_F = \mu_R = \frac{m_H}{2}\left[1 + \frac{4\mathbf{p}_{H,\perp}^2}{m_H^2}\right]^{1/4}$$

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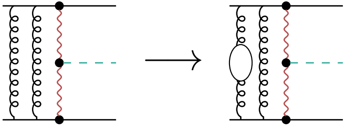
- How to reduce the renormalization scale uncertainties?

- Fermion bubble corrections!



$$\mathcal{C}_{\mathrm{nf}} \to \mathcal{C}_{\mathrm{nf}}(\mu_R)$$

- It will compensate for the μ_R dependence of α_s.

Fermion bubble

We only consider the leading eikonal approximation. To include the effects of running α_s, replace $\Delta_{1,2}$ in $\mathcal{C}_{1,2}$ [Brodsky, Lepage, Mackenzie 1983]

$$\tilde{\Delta}_i = \Delta_i \left(1 + \frac{\beta_0 \alpha_s}{2\pi} \ln \frac{\mathbf{k}_{i,\perp}^2}{\mu_R^2 e^{5/3}}\right)$$

$$\mathcal{C}_{\rm nf} = 4 \int \frac{\mathrm{d}^2\mathbf{k}_{1,\perp}}{(2\pi)} \frac{\mathrm{d}^2\mathbf{k}_{2,\perp}}{(2\pi)} \frac{\Delta_3\Delta_4}{\tilde{\Delta}_1\tilde{\Delta}_2} \left(\frac{\Delta_3\Delta_4}{\Delta_{3,1}\Delta_{4,1}\Delta_{3,2}\Delta_{4,2}} - \frac{1}{\Delta_{3,12}\Delta_{4,12}}\right)$$

we obtain

$$\mathcal{C}_{\rm nf} = \mathcal{C}_{\rm nf}^{(0)} + \frac{\alpha_s\beta_0}{\pi}\left(\mathcal{C}_{\rm nf}^{(0)} \ln\left(\frac{\mu_R^2 e^{5/3}}{m_V^2}\right) + \mathcal{C}_{\rm nf}^{(1)}\right) + \mathcal{O}(\alpha_s^2\beta_0^2)$$

where

$$C_{\rm nf}^{(0)} = \left(C_1^{(0)}\right)^2 - 2C_1^{(1)}, \qquad C_{\rm nf}^{(1)} = C_1^{(0)}C_1^{(1)} - 3C_1^{(2)} + 2\zeta_3$$

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the auxiliary function

$$C_1(\nu) = -2 \int \frac{\mathrm{d}^2\mathbf{k}_{1,\perp}}{2\pi} \frac{\Delta_3\ \Delta_4\ m_V^{2\nu}}{\Delta_1^{1+\nu} \Delta_{3,1} \Delta_{4,1}}$$

One-dimensional integral representation of $C_1^{(0,1,2)}$

$$C_1^{(0)} = \int_0^1 \mathrm{d}t\ \frac{\Delta_x \Delta_y}{r_{12}^2} \left[\ln r_2 - 2 \ln r_{12} + \frac{r_2 - r_1}{r_2} \right]$$

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For 13 TeV at LHC

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$\mathcal{O}(20\%)$ → $\mathcal{O}(5\%)$

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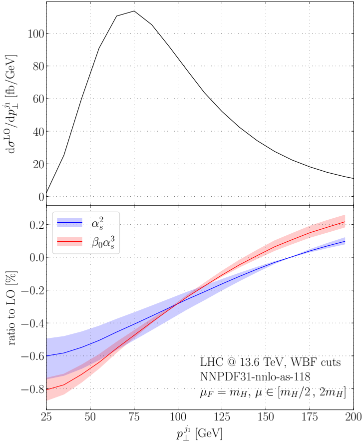

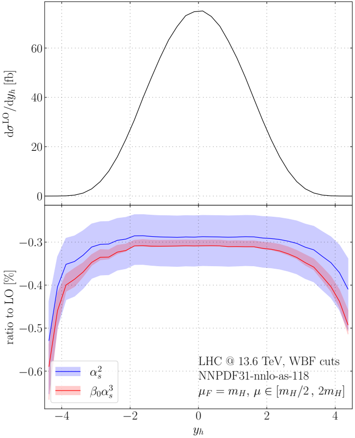

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Summary

- Studies on the Higgs production in VBF are very advanced, thanks to the impressive calculations of factorizable corrections up to N3LO QCD.
- The expansion of the complicated five-point amplitudes around the forward limit is highly non-trivial. But the first power correction is surprisingly compact and relatively simple. That deeply profit from the special kinematics of VBF.
- The new sub-eikonal contribution changes the current estimate of **NNLO non-factorizable** corrections to VBF cross section by about 20%.
- Non-factorizable corrections are color-suppressed but π^2 enhanced. They might be equally important as the N3LO factorizable corrections.

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Karlsruhe Institute of Technology

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- The expansion of the complicated five-point amplitudes around the forward limit is highly non-trivial. But the first power correction is surprisingly compact and relatively simple. That deeply profit from the special kinematics of VBF.
- The new sub-eikonal contribution changes the current estimate of **NNLO non-factorizable** corrections to VBF cross section by about 20%.
- Non-factorizable corrections are color-suppressed but π^2 enhanced. They might be equally important as the N3LO factorizable corrections.
- The strong dependence of renormalization scale of non-factorizable contribution are reduced by computing the three-loop $\mathcal{O}(\beta_0 \alpha_s^3)$ corrections.
- They account for the effects of running coupling constant, reducing the dependence on renormalization scale from $\mathcal{O}(20\%)$ to $\mathcal{O}(5\%)$, and thus stabilizing the theoretical predictions.

Summary

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We have a much better understanding of the NNLO non-factorizable corrections to VBF.

Thank you for your attention!

Setup in Monte Carlo

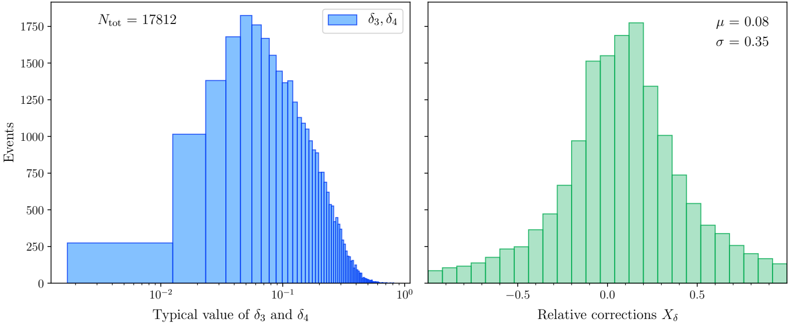
- PDFs: `NNPDF31-nnlo-as-118`
- VBF cuts

anti-k_t	2 jets, $R = 0.4$
jet transverse momentum	$p_{j,\perp} > 25\ \mathrm{GeV}$
jet rapidity	$\lvert y_j \rvert < 4.5$
jet separation	$\lvert y_{j_1} - y_{j_2} \rvert > 4.5$
invariant mass of jets	$M_{jj} > 600\ \mathrm{GeV}$
separate hemispheres	$y_{j_1} y_{j_2} < 0$

Some checks

- reproduce the leading eikonal approximation [Liu, Melnikov, Penin 2019]
- $\delta_{3,4}$ distribution and one-loop check

$$X_\delta = \frac{\mathcal{A}_1 - \mathcal{A}_1^{\text{G\&G-S}}}{\mathcal{A}_1^{\text{G\&G-S}} - \mathcal{A}_1^{\text{G\&G-S}}\big|_{\delta_{3,4}\to 0}}$$



Backup slides

Glauber and mixed regions at one loop

Factorization of integrations:

$$\mathcal{A}_1^{\text{G\&G-S}} = -\langle 3|\gamma^\mu|1]\langle 4|\gamma_\mu|2] \int \frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{d-2}} \frac{1}{\Delta_1 \Delta_{3,1} \Delta_{4,1}} \times \Phi \times \tilde{\Phi}$$

with

$$\Phi = \int \frac{\mathrm{d}\beta_1}{2\pi i} \frac{\Delta_{3,1}}{s\delta_3(\beta_1 - \beta_3) + \Delta_{3,1} + i0} \left[\frac{1}{\beta_1 + \frac{\Delta_1}{s} + i0} + \frac{1}{-\beta_1 + \frac{\Theta_{3,1}}{s\alpha_3} + i0} \right],$$

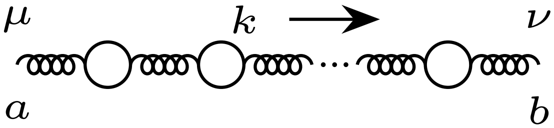
$$\tilde{\Phi} = \int \frac{\mathrm{d}\alpha_1}{2\pi i} \frac{\Delta_{4,1}}{-s\delta_4(\alpha_1 + \alpha_4) + \Delta_{4,1} + i0} \left[\frac{1}{-\alpha_1 + \frac{\Delta_1}{s} + i0} + \frac{1}{\alpha_1 + \frac{\Theta_{4,1}}{s\beta_4} + i0} \right]$$

and

$$\Delta_i = -\mathbf{k}_{i,\perp}^2 \,, \quad \Delta_{3,i} = -(\mathbf{k}_{i,\perp} - \mathbf{p}_{3,\perp})^2 - m_V^2 \,, \quad \Delta_{4,i} = -(\mathbf{k}_{i,\perp} + \mathbf{p}_{4,\perp})^2 - m_V^2 \,,$$

$$\Theta_{3,i} = -\left(\mathbf{k}_{i,\perp}^2 - 2\mathbf{k}_{i,\perp} \cdot \mathbf{p}_{3,\perp}\right) \,, \quad \Theta_{4,i} = -\left(\mathbf{k}_{i,\perp}^2 + 2\mathbf{k}_{i,\perp} \cdot \mathbf{p}_{4,\perp}\right)$$

Dressed gluon propagator



$$G^{\mu\nu}_{ab} = \frac{-ig^{\mu\alpha_1}\delta_{aa_1}}{k^2} Q^{a_1 b_1}_{1,\alpha_1\beta_1} \frac{-ig^{\beta_1\alpha_2}\delta_{b_1 a_2}}{k^2} Q^{a_2 b_2}_{2,\alpha_2\beta_2} \frac{-ig^{\beta_2\alpha_3}\delta_{b_2 a_3}}{k^2} \cdots \frac{-ig^{\beta_{n-1}\alpha_n}\delta_{b_{n-1} a_n}}{k^2} Q^{a_n b_n}_{n,\alpha_n\beta_n} \frac{-ig^{\beta_n\nu}\delta_{b_n b}}{k^2}$$

where the j-th bubble reads explicitly

$$Q^{a_j b_j}_{j,\alpha_j\beta_j} = (-1) T_f \delta^{a_j b_j} g_s^2 \mu_R^{2\epsilon} \int \frac{d^d l_j}{(2\pi)^d} \frac{\mathrm{Tr}\left[\gamma_{\alpha_j}(\hat{k}+\hat{l}_j)\gamma_{\beta_j}\hat{l}_j\right]}{(k+l_j)^2 l_j^2}, \quad T_f = n_f T_r, \quad T_r = \frac{1}{2}$$

we can get

$$G^{\mu\nu}_{ab} = \frac{-i}{k^2} g^{\mu\nu}\delta_{ab}(-1)^n \left\{ \frac{T_f \alpha_s(\mu_R^2)}{3\pi} \left[\log\left(\frac{\mu_R^2}{-k^2}\right) + \frac{5}{3} + \mathcal{O}(\epsilon) \right] \right\}^n$$