

# $gg \rightarrow ZH$ : Theory Status and Prospects

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The problem with  $gg \rightarrow ZH$

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# VH Production at the LHC

$pp \rightarrow VH$  is the most sensitive process to  $H \rightarrow b\bar{b}$  [ATLAS-2007.02873, CMS-1808.08242]

- Work in progress on  $H \rightarrow c\bar{c}$  [ATLAS-2201.11428, CMS-2205.0555]
- Probe of VVH coupling
- Larger scale uncertainties in  $ZH$

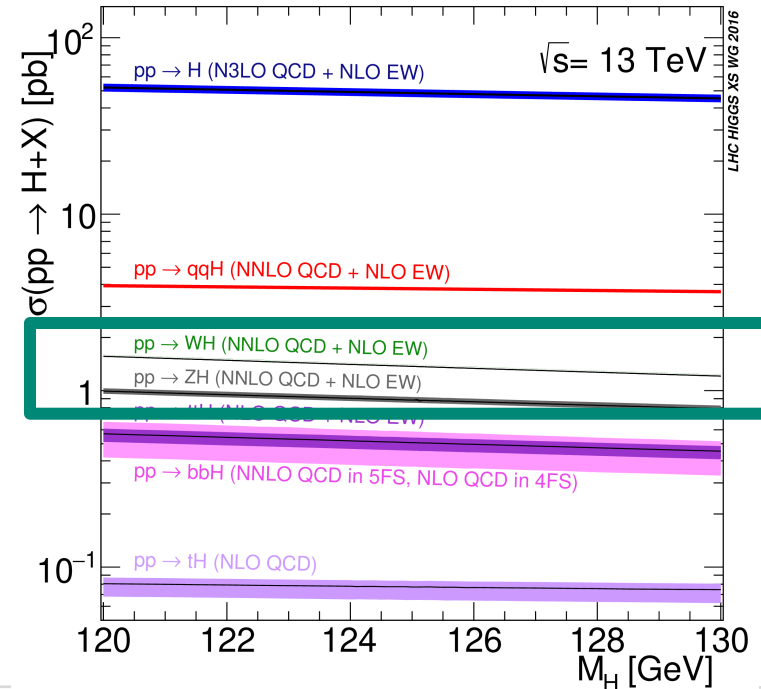
$pp \rightarrow WH$

$\sqrt{s}$ [TeV]	$\sigma_{\text{NNLO QCD} \otimes \text{NLO EW}}$ [pb]	$\Delta_{\text{scale}}$ [%]	$\Delta_{\text{PDF} \oplus \alpha_s}$ [%]
13	1.358	+0.51 -0.51	1.35
14	1.498	+0.51 -0.51	1.35
27	3.397	+0.29 -0.72	1.37

$pp \rightarrow ZH$

$\sqrt{s}$ [TeV]	$\sigma_{\text{NNLO QCD} \otimes \text{NLO EW}}$ [pb]	$\Delta_{\text{scale}}$ [%]	$\Delta_{\text{PDF} \oplus \alpha_s}$ [%]
13	0.880	+3.50 -2.68	1.65
14	0.981	+3.61 -2.94	1.90
27	2.463	+5.42 -4.00	2.24

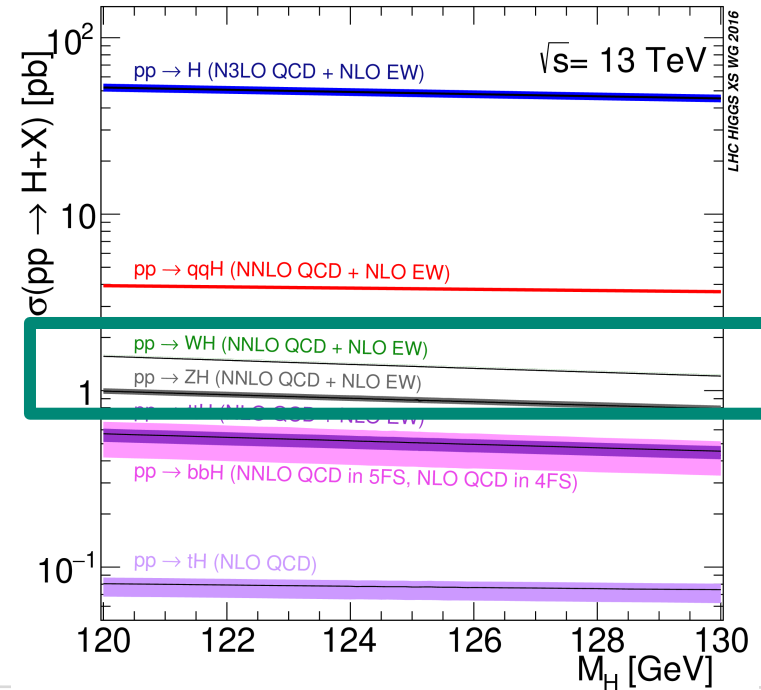
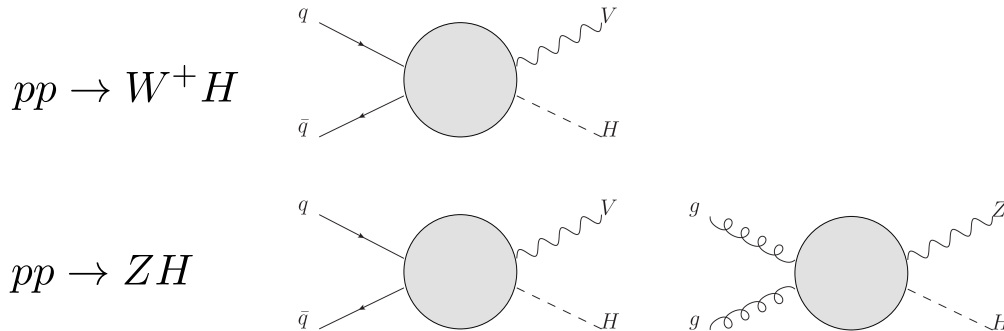
[Cepeda et al. - 1902.00134]



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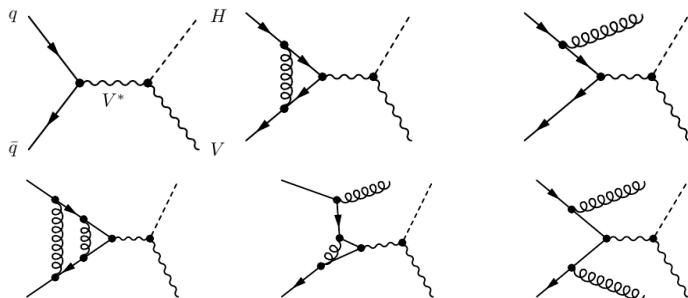


# Theoretical Predictions for $pp \rightarrow ZH$

**LO:** quark-initiated tree-level contribution

**QCD effects:** mainly due to Drell-Yan (DY) production followed by  $Z^* \rightarrow ZH$  decay

## ■ Drell-Yan



■ Known through N3LO (+30% wrt LO)

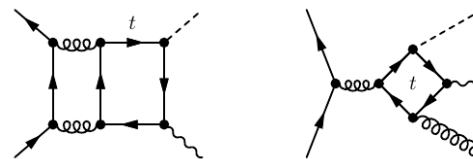
[Han, Willenbrock ('91) ; Hamberg, van Neerven, Matsuura ('92) ; Brein, Djouadi, Harlander – 0307206; Baglio, Duhr, Mistlberger, Szafron - 2209.06138]

**EW corrections:** known through NLO (-(5-10%) wrt LO)

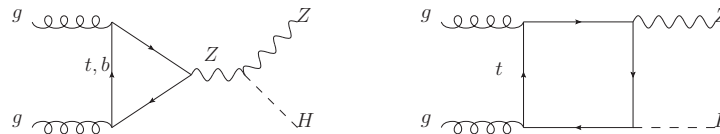
[Dittmaier et al. - 1211.5015]

■ Non Drell-Yan - quark-initiated  $O(1\%)$  wrt LO

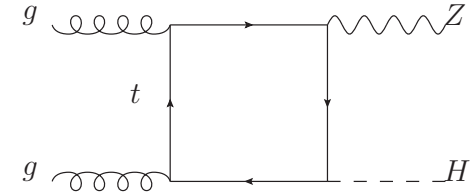
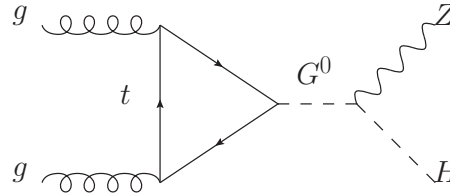
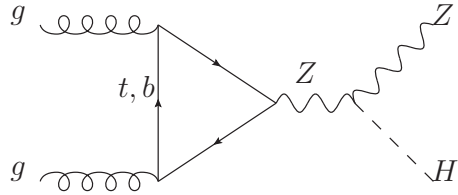
[Brein, Harlander, Wiesemann, Zirke - 1111.0761]



■ Non Drell-Yan - gluon-initiated



# $gg \rightarrow ZH$ @ LO

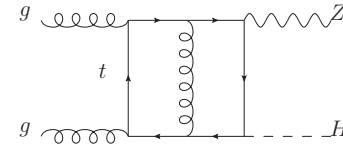
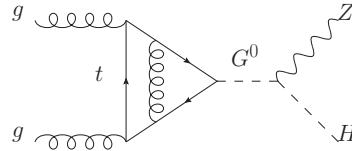
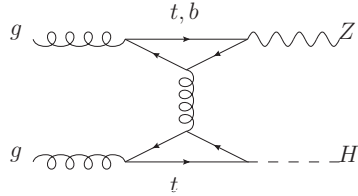


- Top-quark loops give dominant contribution [Kniehl ('90) - Dicus, Kao ('88)]
- $O(\alpha_s^2)$  correction to  $\sigma(pp \rightarrow ZH)$
- NNLO suppression wrt to  $q\bar{q} \rightarrow ZH$  compensated by larger gluon luminosity
- Contributes to  $\sim 6\%$  of  $\sigma(pp \rightarrow ZH)$  for  $\sqrt{s} = 14$  TeV
- Only LO included in MC  $\rightarrow$  scale variation leads to **25%** relative uncertainties

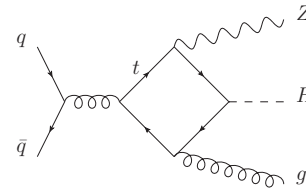
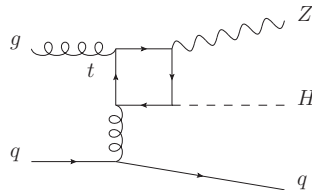
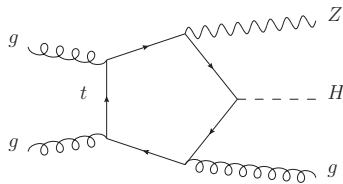
$\sqrt{s}$ [TeV]	$\sigma_{\text{NNLO QCD} \otimes \text{NLO EW}}$ [pb]	$\Delta_{\text{scale}}$ [%]	$\Delta_{\text{PDF} \oplus \alpha_s}$ [%]
13	0.123	+24.9 -18.8	4.37
14	0.145	+24.3 -19.6	7.47
27	0.526	+25.3 -18.5	5.85

# $gg \rightarrow ZH$ @ NLO in QCD - Ingredients

## Virtual corrections ( $2 \rightarrow 2$ , two loops) - interference with LO

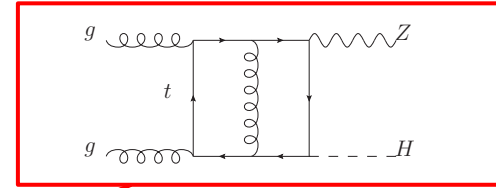
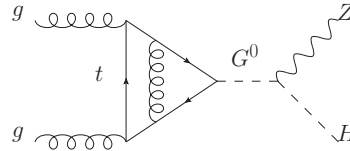
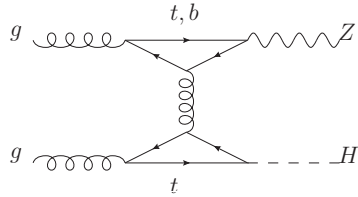


## Real emission ( $2 \rightarrow 3$ , one loop) - squared amplitudes



# $gg \rightarrow ZH$ @ NLO in QCD - Ingredients

## Virtual corrections ( $2 \rightarrow 2$ , two loops) - interference with LO



**Main problem in the NLO calculation**  
Multi-scale  $(m_Z, m_H, m_t, s, t)$  two-loop box integrals  
No full analytic results



# Two-loop Massive Boxes for $gg \rightarrow ZH$

**Numerical Evaluation** [Chen, Heinrich, Jones, Kerner, Klappert, Schlenk - 2011.12325]

- Exact results
- Demanding in terms of computing resources and time
- Issues with flexibility

**Analytic Approximations:** exploit hierarchies of masses/kinematic invariants

- Reduce the number of scales in Feynman integrals
- Proliferation of integrals
- Restricted to specific phase-space regions

■ Limit  $m_t \rightarrow \infty$

[Altenkamp, Dittmaier, Harlander, Rzehak, Zirke - 1211.50]

■ Large mass expansion

[Hasselhuhn, Luthe, Steinhauser - 1611.05881]

■ High-energy expansion:  $m_Z^2, m_H^2 \ll m_t^2 \ll \hat{s}, \hat{t}$

[Davies, Mishima, Steinhauser - 2011.12314]

■ Small-mass expansion:  $m_Z, m_H \rightarrow 0$

[Wang, Xu, Xu, Yang - 2107.08206]

■ pT expansion:  $m_Z^2, m_H^2, p_T^2 \ll m_t^2, \hat{s}$

[Alasfar, Degrossi, Giardino Groeber, MV - 2103.06225]

# Latest Complete NLO Predictions

## ■ Small-mass expansion [Wang, Xu, Xu, Yang - 2107.08206]

### Virtual corrections

- I. Expansion in  $m_Z, m_H \rightarrow 0$  limit
- II. Elliptic integrals evaluated numerically

### Real emission

- ◆ Automated evaluation with GoSam  
[Cullen et al. - 1404.7096]

## ■ Sector decomposition $\oplus$ High-Energy expansion

[Chen, Davies, Heinrich, Jones, Kerner, Mishima, Schlenk, Steinhauser - 2204.05225]

### Virtual corrections

- ◆ pySecDec for  $p_T < 200$  GeV
- ◆ HE exp for  $p_T > 200$  GeV

### Real emission

- ◆ GoSam & in-house C++ code

## ■ pT expansion $\oplus$ High-Energy expansion [Degrassi, Gröber, MV, Zhao - 2205.02769]

### Virtual corrections

- ◆ pT exp for  $|\hat{t}| < 4m_t^2$
- ◆ HE exp for  $|\hat{t}| > 4m_t^2$

### Real emission

- ◆ RECOLA2 [Denner, Lang, Uccirati - 1711.07388]
- ◆ MadGraph5 [Alwall et al. - 1405.0301]

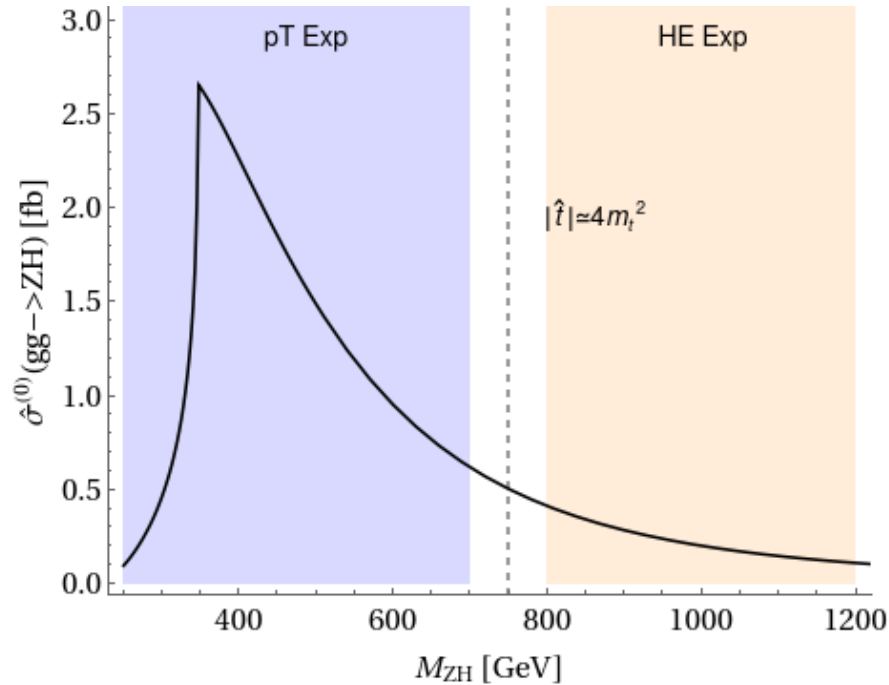
# Comparing Validity Ranges

■  $p_T$  exp: valid for

$$p_T^2 \lesssim 4m_t^2$$

or

$$|\hat{t}| \lesssim 4m_t^2$$



■ High-Energy exp:

$$|\hat{t}| \gtrsim 4m_t^2$$

[Davies, Mishima, Steinhauser - 2011.12314]

[Alasfar, Degrassi, Giardino  
Groeber, MV - 2103.06225]

**The two expansions can be combined**

(Needed refinement using Padé approximants)

[Bellafronte, Degrassi, Giardino, Groeber, MV -2103.06225]

■ Accuracy at % level or below

⇒ OK for phenomenology

■ Evaluation time for a phase-space point  
below 0.1 s ⇒ suitable for Monte Carlo

# Results

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# $gg \rightarrow ZH$ @ NLO QCD – Inclusive Cross Section

[Wang, Xu, Xu, Yang - 2107.08206]

$\sqrt{s} = 13\text{TeV}$

$\mu_r = \mu_f$	$\sigma_{\text{LO}}^{gg}$	$\sigma_{\text{NLO}}^{gg}$	$\sigma_{pp \rightarrow ZH}^{\text{no } gg}$	$\sigma_{pp \rightarrow ZH}$	$\sigma_{\text{NLO}}^{gg, m_t \rightarrow \infty}$	$\sigma_{pp \rightarrow ZH}^{m_t \rightarrow \infty}$
$M_{ZH}/3$	73.56(7)	129.4(3)	784.0(7)	913.4(7)	133.6(6)	917.6(9)
$M_{ZH}$	51.03(5)	101.7(2)	781.1(7)	882.9(7)	106.0(4)	887.2(8)
$3M_{ZH}$	36.62(4)	80.4(2)	780.7(8)	861.1(8)	84.0(3)	864.8(9)

[Chen et al. - 2204.05225]

$\mu_r = \mu_f = M_{ZH}$

$\sqrt{s}$	LO [fb]	NLO [fb]
13 TeV	52.42 <sup>+25.5%</sup> <sub>-19.3%</sub>	103.8(3) <sup>+16.4%</sup> <sub>-13.9%</sub>
13.6 TeV	58.06 <sup>+25.1%</sup> <sub>-19.0%</sub>	114.7(3) <sup>+16.2%</sup> <sub>-13.7%</sub>
14 TeV	61.96 <sup>+24.9%</sup> <sub>-18.9%</sub>	122.2(3) <sup>+16.1%</sup> <sub>-13.6%</sub>

■ General agreement between the three results when same inputs are adopted

[Degrassi, Gröber, MV, Zhao - 2205.02769]

$\sqrt{s} = 13\text{TeV}$   
 $\mu_r = \mu_f = M_{ZH}/2$

Top-mass scheme	LO [fb]	$\sigma_{\text{LO}}/\sigma_{\text{LO}}^{\text{OS}}$	NLO [fb]	$\sigma_{\text{NLO}}/\sigma_{\text{NLO}}^{\text{OS}}$	$K = \sigma_{\text{NLO}}/\sigma_{\text{LO}}$
On-Shell	64.01 <sup>+27.2%</sup> <sub>-20.3%</sub>	—	118.6 <sup>+16.7%</sup> <sub>-14.1%</sub>	—	1.85
$\overline{\text{MS}}, \mu_t = M_{ZH}/4$	59.40 <sup>+27.1%</sup> <sub>-20.2%</sub>	0.928	113.3 <sup>+17.4%</sup> <sub>-14.5%</sub>	0.955	1.91
$\overline{\text{MS}}, \mu_t = m_t^{\overline{\text{MS}}}(m_t^{\overline{\text{MS}}})$	57.95 <sup>+26.9%</sup> <sub>-20.1%</sub>	0.905	111.7 <sup>+17.7%</sup> <sub>-14.6%</sub>	0.942	1.93
$\overline{\text{MS}}, \mu_t = M_{ZH}/2$	54.22 <sup>+26.8%</sup> <sub>-20.0%</sub>	0.847	107.9 <sup>+18.4%</sup> <sub>-15.0%</sub>	0.910	1.99
$\overline{\text{MS}}, \mu_t = M_{ZH}$	49.23 <sup>+26.6%</sup> <sub>-19.9%</sub>	0.769	103.3 <sup>+19.6%</sup> <sub>-15.6%</sub>	0.871	2.10

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■ NLO corrections are the same size as LO

[Degrandi, Gröber, MV, Zhao - 2205.02769]

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■ Scale uncertainties reduced by ~30% wrt LO

[Degrassi, Gröber, MV, Zhao - 2205.02769]

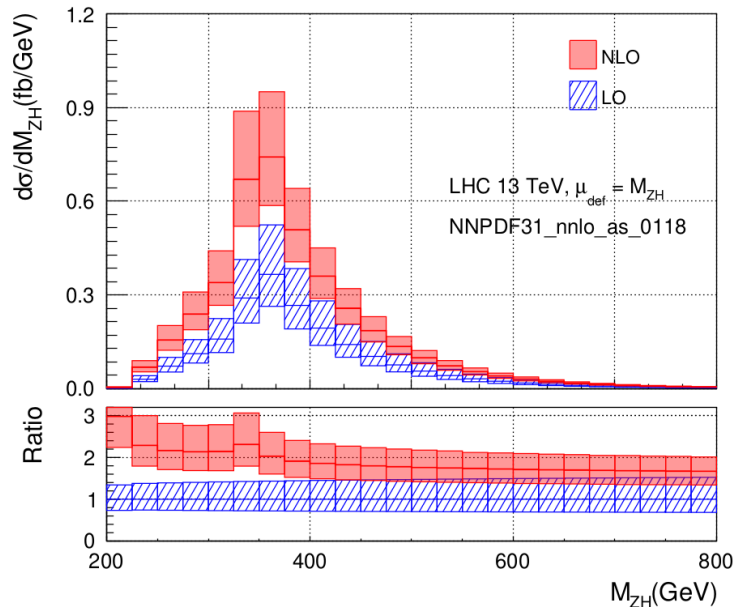
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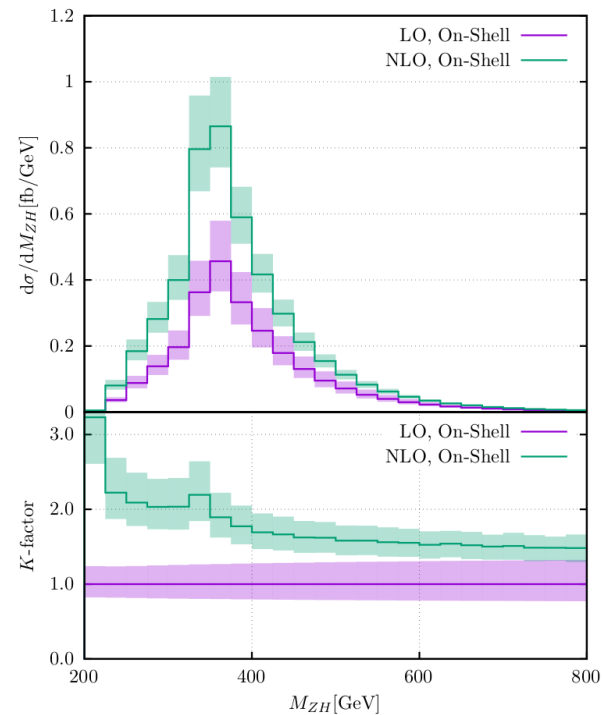
$\mu_r = \mu_f = M_{ZH}/2$

# Invariant-Mass Distribution

- K-factor not flat in the low-energy region



[Wang, Xu, Xu, Yang - 2107.08206]



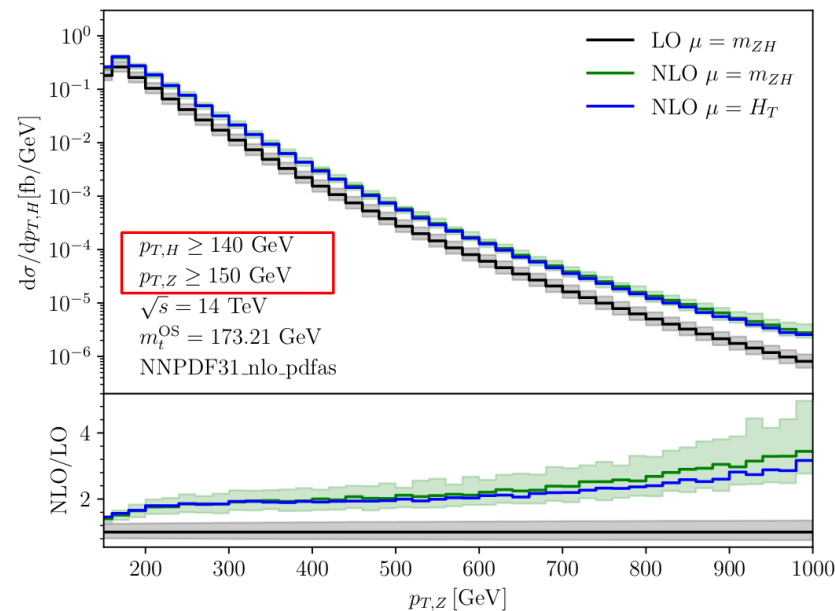
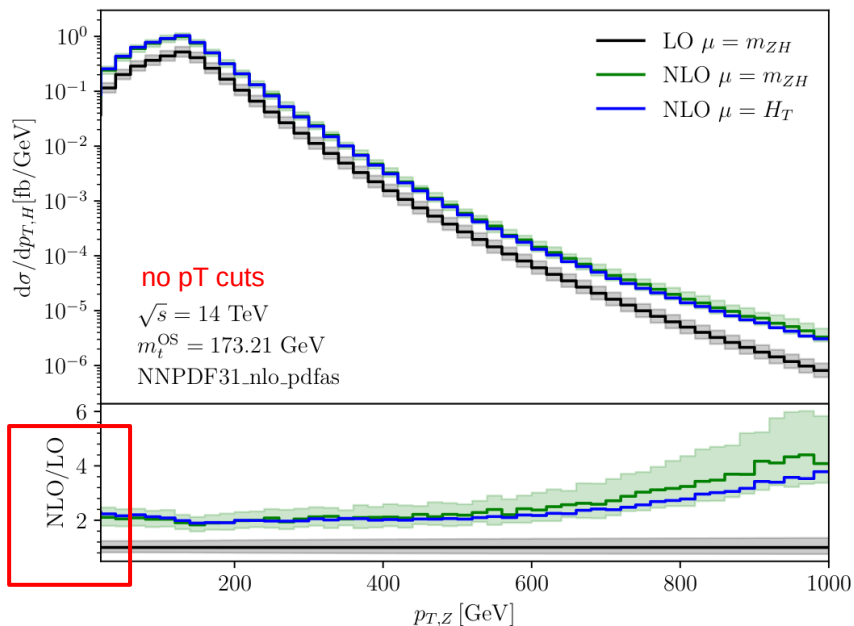
[Degrassi, Gröber, MV, Zhao - 2205.02769]



# High-Energy Tails I – pT Distributions

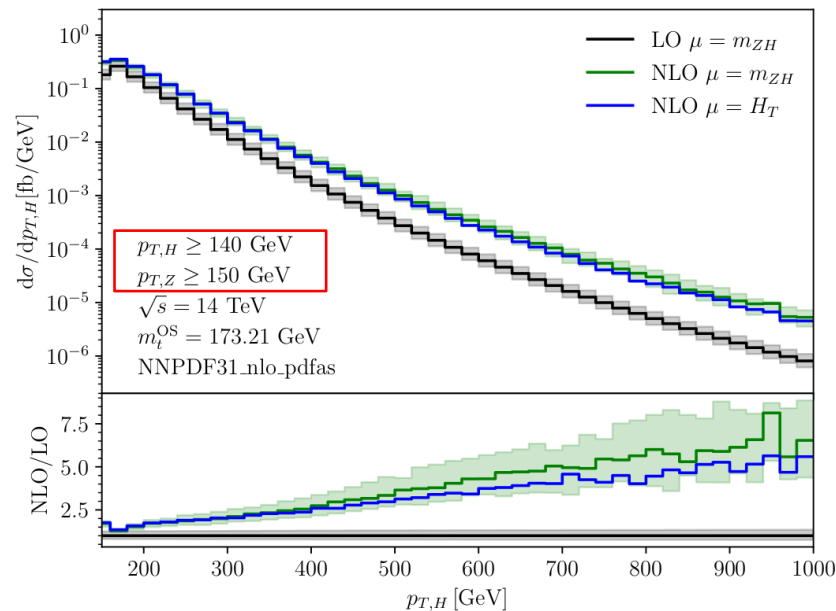
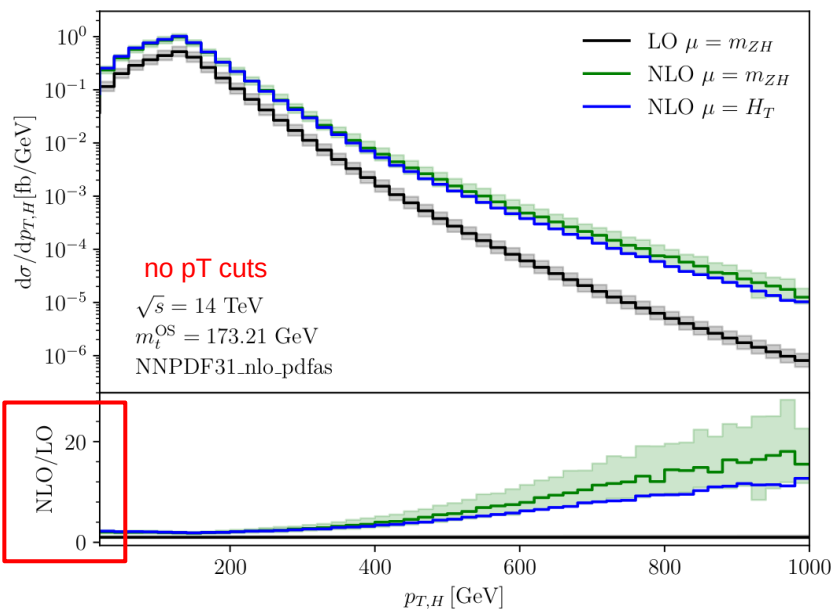
■ K-factor increasing for  $p_{T,Z} > 600$  GeV

■ Not very sensitive to pT cuts



# High-Energy Tails I – p<sub>T</sub> Distributions

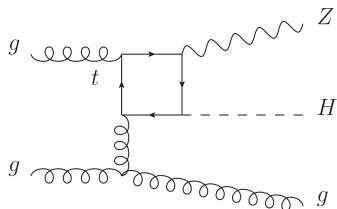
- Very large NLO corrections for  $p_{T,H} > 400$  GeV
- Still K-factor of  $\sim 5$  after p<sub>T</sub> cuts



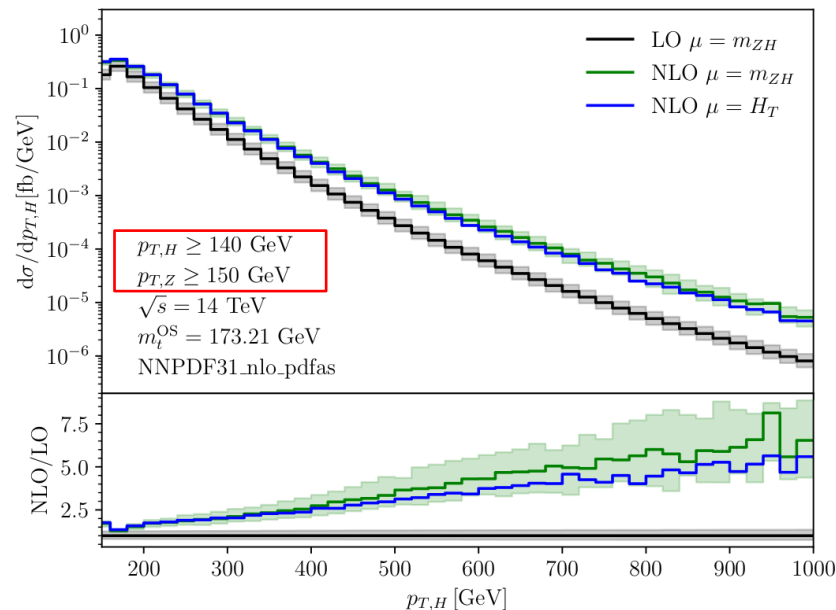
# High-Energy Tails I – pT Distributions

- Very large NLO corrections for  $p_{T,H} > 400$  GeV
- Still K-factor of  $\sim 5$  after pT cuts

- The pT cuts remove 2  $\rightarrow$  3 configurations with a hard jet and a soft Z
- These are very likely in the high-energy region

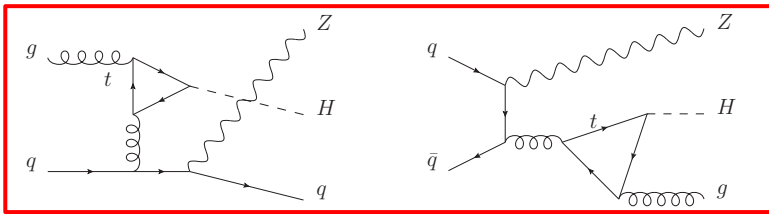


Observed in  
[Hespel, Maltoni, Vryonidou  
-1503.01656]



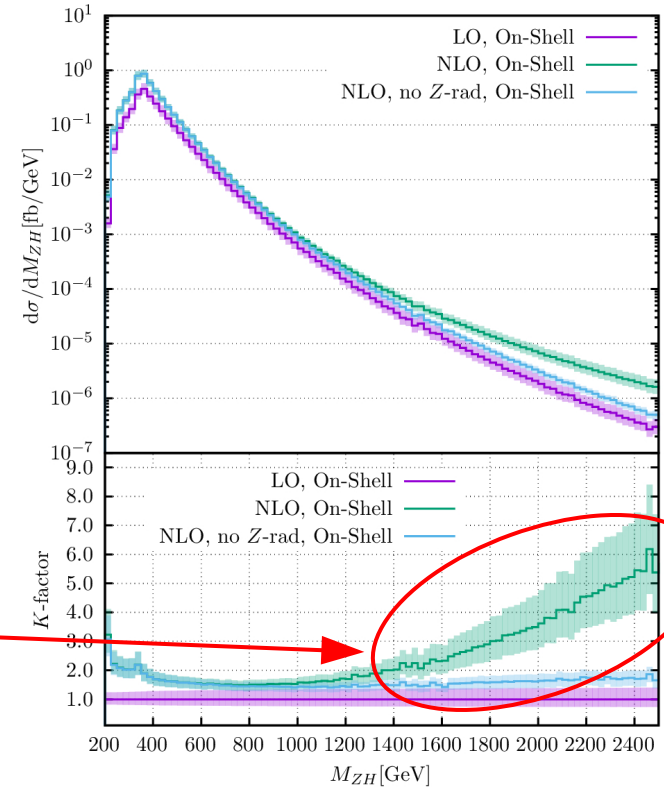
# High-Energy Tails II – Z Radiation

- K-factor rapidly increasing for  $M_{ZH} > 1 \text{ TeV}$
- Effect due to real-emission diagrams where the Z is radiated from an open fermion line
- Not included in [Wang, Xu, Xu, Yang - 2107.08206] [Chen et al. - 2204.05225] (can be attributed to DY process)



Dominant

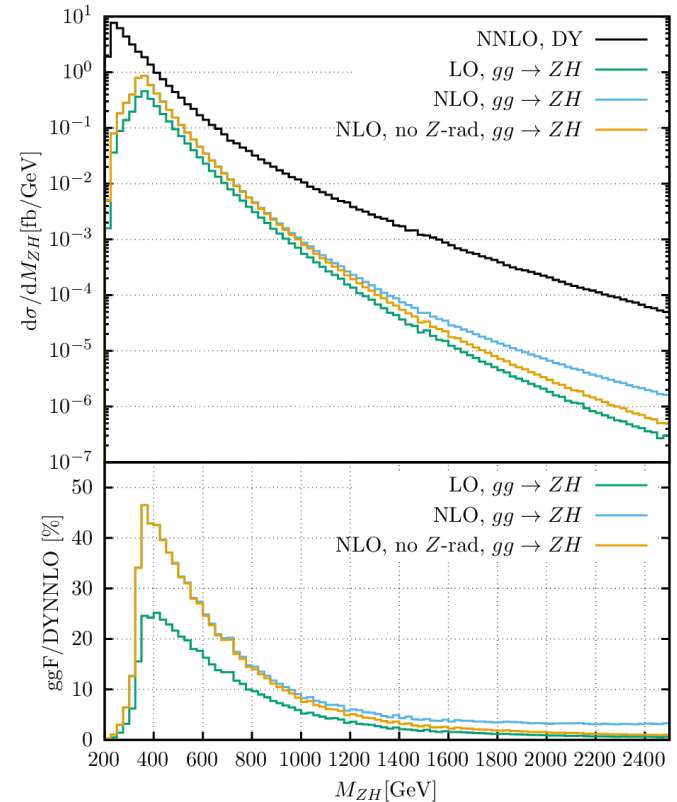
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[Degrassi, Gröber, MV, Zhao - 2205.02769]

# $gg \rightarrow ZH$ @ NLO vs Drell-Yan contribution

- $gg \rightarrow ZH$  is almost 50% of DY near  $M_{ZH} \sim 2 m_t$
- Because of Z-radiated diagrams the  $gg$  contribution falls off as rapidly as the DY one (ratio constant at  $\sim 2\%$ )
- DY obtained using **vh@nnlo**  
[Harlander et al - 1802.04817]



# Top Mass Scheme Uncertainty

- Envelope of deviations of  $\overline{M_S}$  schemes wrt OS result  
Same method already used for HH production  
[Baglio et al. - 1811.05692, 2003.03227]

- Uncertainty sensitive to the binning of top-pair threshold peak

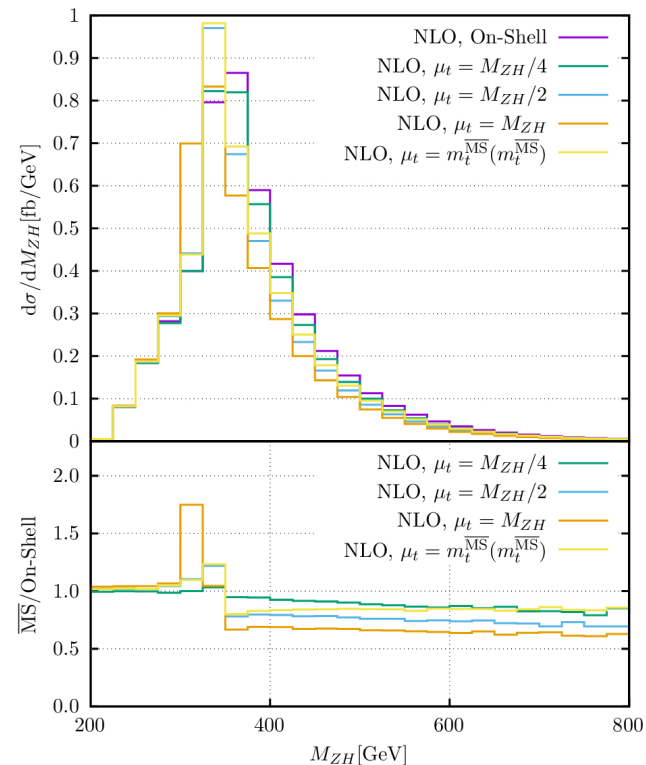
Bin Width [GeV]	LO	NLO
1	64.01 <sup>+15.6%</sup> <sub>-35.9%</sub>	118.6 <sup>+17.2%</sup> <sub>-27.0%</sub>
5	64.01 <sup>+15.3%</sup> <sub>-35.6%</sub>	118.6 <sup>+14.7%</sup> <sub>-24.9%</sub>
25	64.01 <sup>+14.0%</sup> <sub>-33.1%</sub>	118.6 <sup>+10.9%</sup> <sub>-20.8%</sub>
100	64.01 <sup>+2.0%</sup> <sub>-25.3%</sub>	118.6 <sup>+0.6%</sup> <sub>-13.7%</sub>
$\infty$	64.01 <sup>+0%</sup> <sub>-23.1%</sub>	118.6 <sup>+0%</sup> <sub>-12.9%</sub>

Avoid overestimate of uncertainty



- Top-mass uncertainty  $\sim$  scale uncertainty

- Agreement with [Chen et al. - 2204.05225] for  $M_{ZH} > 400$  GeV



# Conclusions

- Three (almost) independent predictions are in good agreement
- $gg \rightarrow ZH$  receives large NLO QCD corrections
- Enhanced importance wrt  $qq \rightarrow ZH$  in differential distributions
- Combinations of different approaches (numerical, expansions...) crucial for virtual corrections

# Outlook

- Agree on a unified prediction - work in progress from [\[Chen et al. - 2204.05225\]](#)
  - Improve understanding of high-energy tails of distributions [\[Degrassi, Gröber, MV, Zhao - 2205.02769\]](#)
  - Understand better top-mass scheme uncertainties
  - Computation of NNLO QCD corrections may be desirable for complete control over scale uncertainties
-

Thank you for your attention

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# Backup

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# pT Expansion - Calculation Overview

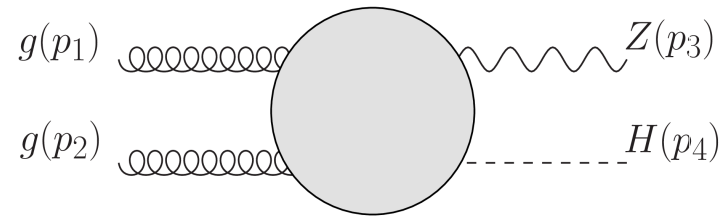
1. Generation of Feynman diagrams -  $O(100 \text{ diags})$  (FeynArts [Hahn - 0012260])
2. Lorentz decomposition of the amplitude: **projectors** and **scalar form factors** ( FeynCalc [Mertig et al. ('91) ; Shtabovenko et al. - 1601.01167] ): contractions, Dirac traces...

$$\mathcal{A}_{\mu\nu\rho} = \sum_{i=1}^6 \mathcal{P}_{\mu\nu\rho}^{(i)} F^{(i)} \qquad F^{(i)} = \sum_{i=1}^n C^{(i)} I^{(i)}(\hat{s}, \hat{t}, m_Z^2, m_H^2, m_t^2)$$

3. Expansion of the form factors in the limit of small pT
4. Decomposition of scalar integrals using integration-by-parts (IBP) identities ( LiteRed [Lee - 1310.1145] )
5. Evaluation of master integrals

Steps implemented in **Mathematica** code on a **desktop machine**

# pT Expansion - Details



- We assume the limit of a **forward kinematics**

$$(p_1 + p_3)^2 \rightarrow 0 \Leftrightarrow \hat{t} \rightarrow 0 \Rightarrow p_T \rightarrow 0$$

- Then Taylor-expand the form factors in the ratios

$$\frac{m_H^2}{\hat{s}}, \frac{m_Z^2}{\hat{s}}, \frac{p_T^2}{\hat{s}} \ll 1$$

$$\frac{p_T^2}{4m_t^2} \ll 1$$

Expansion at  
integrand level

- Now scalar loop integrals depend on fewer scales

$$I(\hat{s}, \hat{t}, m_Z^2, m_H^2, m_t^2) \rightarrow I'(\hat{s}, \hat{t}, m_t^2)$$

- The new scalar integrals are decomposed in MIs using IBP relations

- The MIs depend on the ratio  $\hat{s}/m_t^2 \Rightarrow$  **only one scale**

$$I(\hat{s}, \hat{t}, m_Z^2, m_H^2, m_t^2) \rightarrow I'(\hat{s}, \hat{t}, m_t^2) \rightarrow \text{MI}(\hat{s}/m_t^2)$$

- 52 MIs already known in the literature

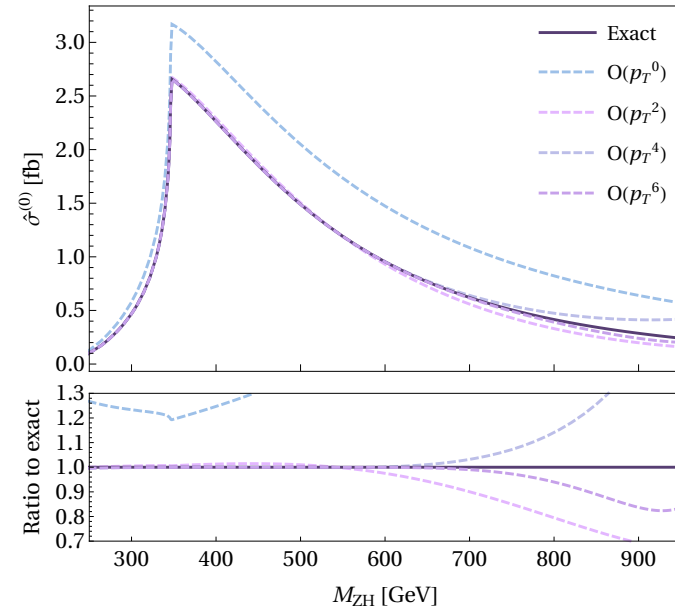
**SAME MIs FOR**  $gg \rightarrow HH$ ,  $gg \rightarrow ZH$ ,  $gg \rightarrow ZZ$

# pT Expansion - LO Validation

- Three orders sufficient for very good accuracy
- Reliable results for  $M_{ZH} \lesssim 700$  GeV
- For  $M_{ZH} \gtrsim 700$  GeV the assumption

$$p_T^2 \ll 4m_t^2$$

can be violated  $\Rightarrow$  the  $p_T$  expansion **diverges**



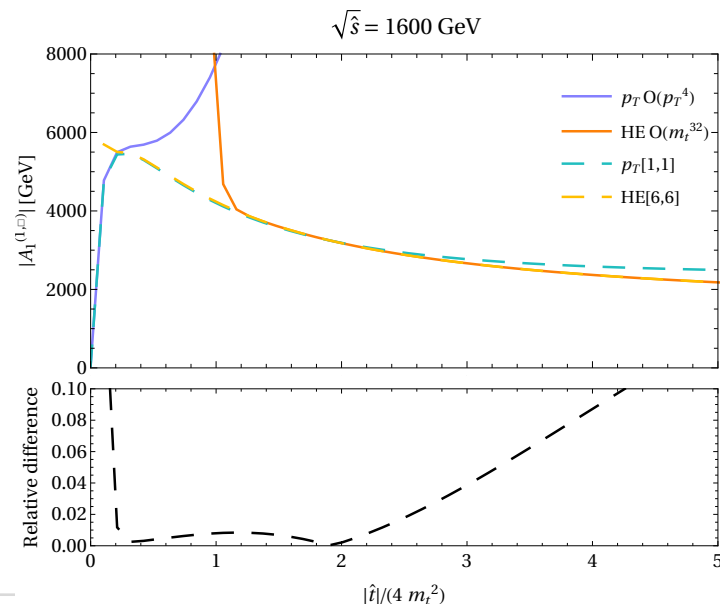
# Merging pT and HE Expansions at NLO

Improve the convergence of a series expansion by matching the coefficients of the **Pade approximant** [m/n] [e.g. Fleisher, Tarasov ('94)]

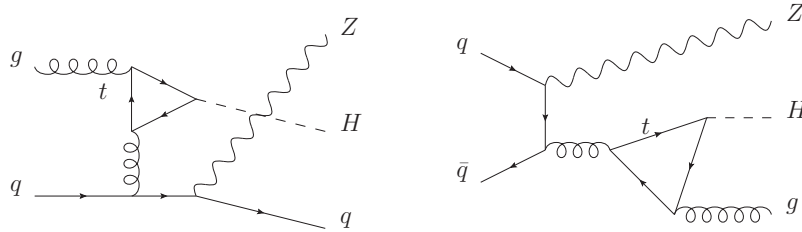
$$f(x) \stackrel{x \rightarrow 0}{\simeq} c_0 + c_1 x + \dots + c_q x^q \quad f(x) \simeq [m/n](x) = \frac{a_0 + a_1 x + \dots + a_m x^m}{1 + b_1 x + \dots + b_n x^n} \quad (q = m + n)$$

[Bellafronte, Degrassi, Giardino, Gröber, MV -2103.06225]

- For each FF we merged the following results
  - pT exp improved by [1/1] Padé
  - HE exp improved by [6/6] Padé
- Padé results are stable and comparable in the region  $|\hat{t}| \sim 4 m_t^2 \rightarrow$  can switch without loss of accuracy (% level or below)
- Evaluation time for a phase-space point below 0.1 s  $\Rightarrow$  suitable for Monte Carlo



# High-Energy Tails II – Z Radiation



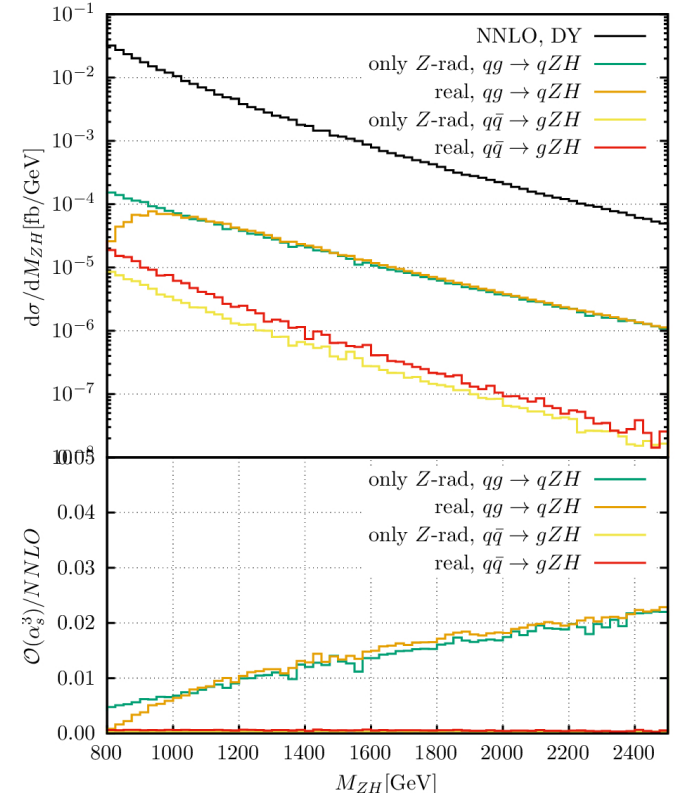
In the high-energy tail ( $M_{ZH} > 1$  TeV)

## ■ $qg \rightarrow ZHq$ channel

- Z-radiated diagrams dominate
- Non-negligible contribution (up to 2% wrt DY)

## ■ $q\bar{q} \rightarrow ZHg$ channel

- Z-radiated diagrams dominate
- Negligible (PDF suppression)



# Master Integrals – pT Expansion

## ■ 50 MIs expressed in terms of Generalized Polylogarithms (GPLs)

[Bonciani, Mastrolia, Remiddi ('03) - Aglietti et al. ('06) - Anastasiou et al. ('06) - Caron-Huot, Henn ('14) - Becchetti, Bonciani ('17) - Bonciani, Degrassi, Vicini ('10)]

## ■ Two elliptic integrals [von Manteuffel, Tancredi ('17)]

Semi-analytical evaluation implemented in FORTRAN routine

[Bonciani, Degrassi, Giardino, Gröber ('18)]

