

WG2+ WG3 extended scalars: overview

Tania Robens

Rudjer Boskovic Institute

on behalf of **WG2 conveners:**

S. Heim, G. Ortona, K. Mimasu, D. Barducci

and **WG3 Extended Higgs Sector conveners:**

M. d'Alfonso, S. Laurila, TR, N. Rompotis, R. Santos, L. Zivkovic

**The 20th Workshop of the LHC Higgs Working Group
CERN**

15. November '23

Factsheet

Conveners

- **WG2:** S. Heim (ATLAS); G. Ortona (CMS); K. Mimasu, D. Barducci (TH)
- **WG3, extended scalars:** L. Zivkovic, N. Rompotis (ATLAS); M. d'Alfonso, S. Laurila (CMS); T. Robens, R. Santos (TH)

Meetings

- 23.6.22, <https://indico.cern.ch/event/1173518/>
- 11.1.23, <https://indico.cern.ch/event/1230456/>
- 26.9.23, <https://indico.cern.ch/event/1327545/>

e-groups

Ihc-higgs-properties, Ihc-higgs-neutral-extended-scalars



Joint activities with WG2: CP violation and Higgs Sector

[slide stolen from K. Mimasu, Summary of WG2 CPV activity, General assembly '22]

Joint WG2/WG3 activity

- CPV in Higgs interactions often means extended scalar sector
- Many interesting signatures of spontaneous/explicit CPV in extended Higgs sectors

Discovery of BSM Higgs in multiple decay channels \Rightarrow CPV

Classes	C_1	C_2	C_3	C_4	C_5
Decays	$h_3 \rightarrow h_2 Z$	$h_2 \rightarrow h_1 Z$	$h_3 \rightarrow h_1 Z$	$h_3 \rightarrow h_2 Z$	$h_3 \rightarrow ZZ$
	$h_2 \rightarrow h_1 Z$	$h_1 \rightarrow ZZ$	$h_1 \rightarrow ZZ$	$h_2 \rightarrow ZZ$	$h_2 \rightarrow ZZ$
	$h_3 \rightarrow h_1 Z$	$h_2 \rightarrow ZZ$	$h_3 \rightarrow ZZ$	$h_3 \rightarrow ZZ$	$h_1 \rightarrow ZZ$

h_{125} -style CP properties study for BSM scalars \Rightarrow CPV

- Undoubtedly complementarity with h_{125} CP properties
- Establish some benchmark models & identify regions of parameter space where one or the other can provide complementary sensitivity

Joint activities with WG2: CP violation and Higgs Sector

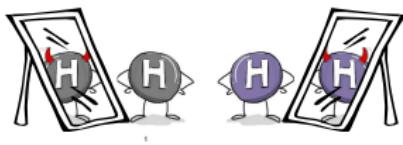
three meetings over the last 2 years, ~ 20 talks

CPV in Higgs interactions: WG2/WG3 (extended Higgs) joint meeting

WG3: Mariarosaria d'Alfonso, Santeri Laurila, Tania Robens, Nikos Rompotis, Rui Santos, Shufang Su & Lidija Zivkovic

WG2: Nicolas Berger, Mauro Donega, Ken Mimasu & Daniele Barducci

23rd June 2022



Joint WG2/WG3 activity

Todays meeting!

- Received several kick-off meeting contributions that overlapped with WG3 (extended Higgs sector) interests
- Many interesting signatures of spontaneous/explicit CPV in extended Higgs sectors
- From mixing of would-be CP-even/odd eigenstates

Discovery of BSM Higgs
in multiple decay channels
⇒ CPV

WG3 Proposal for CP violating benchmarks in the C2HDM ~ 2015

[Fontes et al.; PRD 92 (2015) 055014]

h_{123} -style CP properties
study for BSM scalars
⇒ CPV

Decay angular distributions etc.

6

[Slides from K. Mimasu, <https://indico.cern.ch/event/1173518/>]

goal:

study CPV in models with extended Higgs sectors
will result in whitepaper/ report/ ...



Topics covered this year

- very open call \Rightarrow CP violation in SM EFTs and BSM
- \Rightarrow large variety of topics

Examples

- Studies of specific CP violating couplings and prospects at LHC and beyond:

Sarmah, Bhardwaj, Barrue, Menen, Barman, Sahoo

- general discussion and parameter ranges in specific models:

Osland, de Giorgi

time is limited

\Rightarrow will concentrate on a few examples in the following

List of all talks this year [slide by R. Santos]

WG2+WG3 meetings

Thursday 23 Jun 2022

- ⌚ Electroweak Baryogenesis and Dark Matter with an Inert Doublet, [Sven Fabian](#).
- ⌚ BSM Higgs Flavoured Correlations, [Arturo de Giorgi](#).
- ⌚ P-even, CP-violating Signals in Scalar Mediated Processes, [Venus Keus](#).
- ⌚ Direct and indirect probes of Higgs CP violation, [Stefania Gori](#).
- ⌚ CP-violation in ttPhi: asymmetries and interferences, [Duarte Azevedo](#).
- ⌚ Electroweak phase transition in a dark sector with CP-violation, [Lisa Biermann](#).
- ⌚ Di-Higgs-Production and Baryogenesis in the C2HDM, [Milada Muhleitner](#).

Wednesday 11 Jan 2023

- ⌚ Study of anomalous gauge-Higgs couplings using Z boson polarization at LHC, [Priyanka Sarmah](#).
- ⌚ Constraining Higgs-Higgs-Z couplings in the 3HDM, [Per Osland](#).
- ⌚ Machine-enhanced CP-asymmetries in the Higgs sector, [Akanksha Bhardwaj](#).
- ⌚ Simulation-based inference in the search for CP violation in leptonic WH production Higgs, [Ricardo Barrué](#).

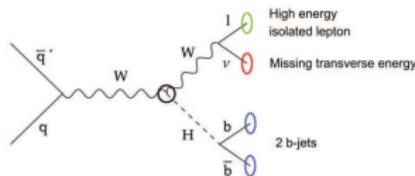
Tuesday 26 Sept 2023

- ⌚ Flavour and Higgs physics in Z2-symmetric 2HD models near the decoupling limit, [Arturo de Giorgi](#).
- ⌚ Classifying the CP properties of the ggH coupling in H+2j production, [Marco Menen](#).
- ⌚ Non-linear top-Higgs CP violation, [Akanksha Bhardwaj](#).
- ⌚ Analysis of interference effects in the di-top final state for CP-mixed scalars in extended Higgs sectors, [Romal Kumar](#).
- ⌚ Returning CP-observables to the frames they belong, [Rahool Kumar Barman](#).
- ⌚ Probing CP violation in H -> tau+ tau- gamma, [Dibyakrupa Sahoo](#).
- ⌚ Search for an invisible scalar in tt final states at the LHC, [Rodrigo Capucha](#).

Example: R. Barre, *Simulation-based inference in the search for CP violation in leptonic WH production Higgs*

CP violation in HWW interaction

Goal: optimize search for CP violation in the HWW interaction via WH production



SMEFT, Warsaw basis, **1** dimension-6 CP-odd operator

$$O_{\tilde{H}\tilde{W}} = \frac{c_{\tilde{H}\tilde{W}}}{\Lambda^2} H^\dagger H \epsilon_{\mu\nu\rho\sigma} W^{I\mu\nu} W^{I\rho\sigma}$$

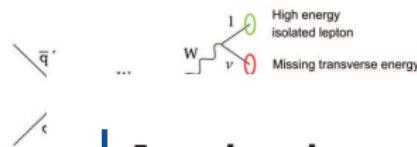
WG2+WG3 meeting - CPV in Higgs sector
11/1/23

3

Example: R. Barrue, *Simulation-based inference in the search for CP violation in leptonic WH production Higgs*

CP violation in HWW interaction

Goal: optimize search for CP violation in the HWW interaction via WH production

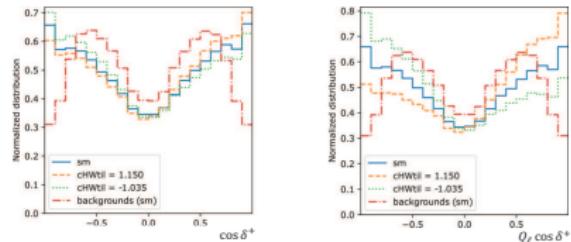


SMEFT, Warsaw basis, 1 dimens

O_{HW}

WG2+A

Angular observables



- Symmetric for SM signal and backgrounds, asymmetric for $c_{\bar{H}W} \neq 0$
- Can extract sign of $c_{\bar{H}W}$, weighting by lepton charge increases asymmetry

[3]: R. Godbole et al., 'Jet substructure and probes of CP violation in VH production', arXiv:1409.5449

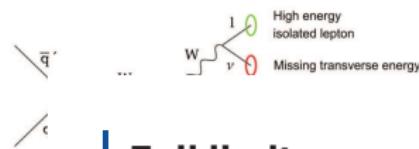
WG2+WG3 meeting - CPV in Higgs sector
11/1/23

9

Example: R. Barrue, *Simulation-based inference in the search for CP violation in leptonic WH production Higgs*

CP violation in HWW interaction

Goal: optimize search for CP violation in the HWW interaction via WH production



Full limits

SMEFT, Warsaw basis, 1 dimens

O_{HW}

WG2+A

Determined expected limits w/ full likelihood ratio (**shape-only**)

- Properly takes into account the effect of terms $\propto c_{HW}^2$ in the likelihood ratio

Observable	c_{HW} S+B 95% CL ($L = 300 \text{ fb}^{-1}$)
1D: p_{TW}	[-0.192, 0.216]
2D: $p_{TW} \times m_{T_{\ell\nu b\bar{b}}}$	[-0.36, 0.384]
1D: $Q_\ell \cos \delta^+$	[-0.264, 0.216]
2D: $p_{TW} \times Q_\ell \cos \delta^+$	[-0.096, 0.072]
MVA: SALLY, 48 input variables	[-0.144, 0.12]
MVA: SALLY, 48 input variables + $p_{z_\ell} Q_\ell \cos \delta^+, Q_\ell \cos \delta^-, \cos \theta^+$	[-0.168, 0.096]

2D combination of p_{TW} and $Q_\ell \cos \delta^+$ yields the best limits

- SALLY no longer optimal when quadratic effects included

WG2+WG3 meeting - CPV in Higgs sector
11/1/23

13

Example: M. Menen, *Classifying the CP properties of the ggH coupling in H+2j production* [arXiv:2309.03146]

BSM framework

Free parameters:

- Higgs characterisation model: Higgs H assumed to be mixed CP state
- Effective Higgs-gluon coupling:

[Artoisenet et al. '13](#)

$$\mathcal{L}_{ggH} = -\frac{1}{4v} \left(-\frac{\alpha_s}{3\pi} c_g G_{\mu\nu}^a G^{\mu\nu,a} + \frac{\alpha_s}{2\pi} \tilde{c}_g G_{\mu\nu}^a \tilde{G}^{\mu\nu,a} \right) H$$

- Effective CP-even (c_g) and CP-odd (\tilde{c}_g) coupling modifiers
- SM obtained for $c_g = 1$, $\tilde{c}_g = 0$
- Higgs-gluon coupling corresponds to top-Yukawa in the heavy top limit and if there are no low-mass BSM particles in the ggF loop $\Rightarrow c_g = c_t$, $\tilde{c}_g = \tilde{c}_t$
- We impose a cut $p_T^H < 200\text{GeV}$ to remain in the heavy top limit

Example: M. Menen, *Classifying the CP properties of the ggH coupling in H+2j production* [arXiv:2309.03146]

BSM framework

Free parameters:

- Higgs characterisation model: ...
- Effective Higgs-gluon coupling

$$\mathcal{L}_{ggH} = -\frac{1}{4v} \left(-\frac{\alpha_s}{3\pi} \right)$$

- Effective CP-even (c_g) and CP-odd (\tilde{c}_g)
- SM obtained for $c_g = 1$, $\tilde{c}_g = 0$
- Higgs-gluon coupling corresponds if there are no low-mass BSM |
- We impose a cut $p_T^H < 200\text{GeV}$

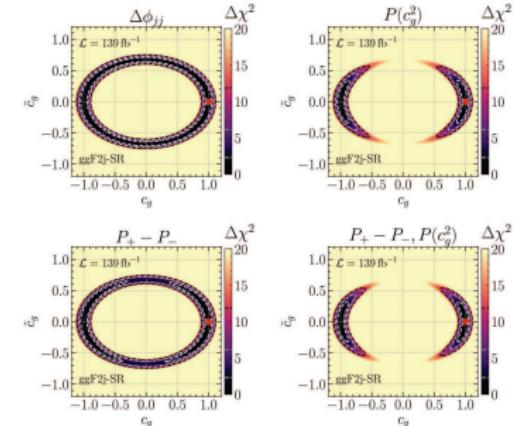
➤ Ellipse from total rate

➤ $\Delta\phi_{jj}$ alone is not able to resolve the ellipse

➤ 2D-limits dominated by the $P(c_g^2)$ classifier (low interference contribution)

➤ $|\tilde{c}_g| \leq 0.32 @ 1\sigma$

ggF2j signal region



Example: R. Capucha, *Search for an invisible scalar in tt final states at the LHC* [arXiv:2308.00819]

DM Lagrangian and CP-observables

- Analysis performed within the context of **simplified models of DM production** at the LHC. The **DMsimp** model was used.

$$\mathcal{L}_{SM}^{Y_0} = \frac{y_{33}^f}{\sqrt{2}} (g_{u33}^S + i g_{u33}^P \gamma^5) t Y_0 \quad \text{Backovic et al. - 1508.05327}$$

$$\mathcal{L}_{X_D}^{Y_0} = \bar{X}_D (g_{X_D}^S + i g_{X_D}^P \gamma^5) X_D Y_0$$

➢ **CP-even:** $g_{u33}^S = 1$, $g_{u33}^P = 0$. **CP-odd:** $g_{u33}^S = 0$, $g_{u33}^P = 1$. **CP-mixed:** $g_{u33}^{S/P} \neq 0$ (**CP-violating interaction**).

➢ $t(\bar{t}) \rightarrow W^+ b$ ($W^- \bar{b}$) and $W^+ (W^-) \rightarrow l^+ \nu_l (\bar{l}^- \bar{\nu}_l)$: **dileptonic final state**, with $l = e, \mu$.

➢ BR ($Y_0 \rightarrow X_D \bar{X}_D$) ≈ 1 . We focus only on the tops and mediator interaction.

- Several **observables** have been proposed to **probe the CP-nature of the Higgs** in the Higgs-top couplings. To illustrate our findings, we considered the **azimuthal angle difference of the charged leptons** from the tops decay, $\Delta\phi_{l+l-}$, and the **b_4 variable** in the laboratory frame (LAB)

$$b_4 = (p_t^x p_{\bar{t}}^z) / (|p_t^x| |p_{\bar{t}}^z|) \quad \text{Gunion, He - hep-ph/9602226, Buckley, Gonçalves - 1511.06451}$$

- In order to evaluate this variable, the kinematic reconstruction of the $t\bar{t}$ system needs to be accomplished.

4

Example: R. Capucha, *Search for an invisible scalar in tt final states at the LHC* [arXiv:2308.00819]

DM Lagrangian and CP-observables

- Analysis performed within the context of **simplified models of DM production** at the LHC. The **DMsimp** model was used.

$$\mathcal{L}_S^Y$$

$$\mathcal{L}_A^Y$$

Results – heavier masses

- CP-even:** $g_{u33}^S = 1$, $g_{u33}^P = 0$. **CP-odd:** $g_{u33}^S = 0$, $g_{u33}^P \neq 0$
- $t(\bar{t}) \rightarrow W^+ b$ ($W^- \bar{b}$) and $W^+ (W^-) \rightarrow l^+ \nu_l (l^-)$
- $\text{BR}(Y_0 \rightarrow X_D \bar{X}_D) \approx 1$. We focus only on the $t\bar{t}$

- Several observables** have been proposed to **probe** findings, we considered the **azimuthal angle** ϕ variable in the laboratory frame (LAB)

- In order to evaluate this variable, the kinematic

- Results extended to a **massive DM mediator**, with $m_{Y_0} = 1, 10, 125$ GeV. As expected, **exclusion limits worsen as masses increase** in both scenarios, since the $t\bar{t}Y_0$ production cross section decreases for heavier Y_0 masses.
- The observable choice can have some impact on the exclusion limits, even in scenario 1, for heavier masses, because of the cross section decrease.

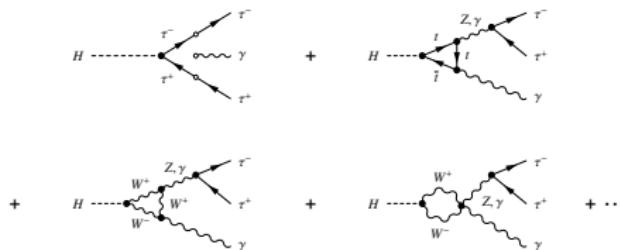
Scenario 1

Exclusion Limits from $\Delta\phi_{l^+l^-}$	$L = 200 \text{ fb}^{-1}$		$L = 3000 \text{ fb}^{-1}$	
	(68% CL)	(95% CL)	(68% CL)	(95% CL)
$m_{Y_0} = 1 \text{ GeV}$	$g_{u33}^S \in [-0.073, +0.073]$	$[-0.142, +0.142]$	$[-0.038, +0.038]$	$[-0.068, +0.068]$
	$g_{u33}^P \in [-0.89, +0.89]$	$[-1.65, +1.65]$	$[-0.43, +0.43]$	$[-0.83, +0.83]$
$m_{Y_0} = 10 \text{ GeV}$	$g_{u33}^S \in [-0.198, +0.198]$	$[-0.368, +0.372]$	$[-0.098, +0.098]$	$[-0.188, +0.188]$
	$g_{u33}^P \in [-0.87, +0.87]$	$[-1.65, +1.65]$	$[-0.44, +0.44]$	$[-0.83, +0.83]$
$m_{Y_0} = 125 \text{ GeV}$	$g_{u33}^S \in [-0.328, +0.322]$	$[-0.608, +0.612]$	$[-0.162, +0.162]$	$[-0.308, +0.308]$
	$g_{u33}^P \in [-1.48, +1.49]$	$[-2.77, +2.78]$	$[-0.75, +0.75]$	$[-1.41, +1.41]$

Example: D. Sahoo, *Probing CP violation in $H \rightarrow \tau^+ \tau^- \gamma$*

The 3-body decay $H \rightarrow \tau^+ \tau^- \gamma$ offers an alternative methodology.

Decay proceeds via both tree and loop diagrams



$\text{Br}(H \rightarrow \tau^+ \tau^- \gamma)_{\text{SM}} \sim 3.24 \times 10^{-3}$ with $E_\gamma > 5 \text{ GeV}$ and angular separation $> 5^\circ$ in rest frame of H

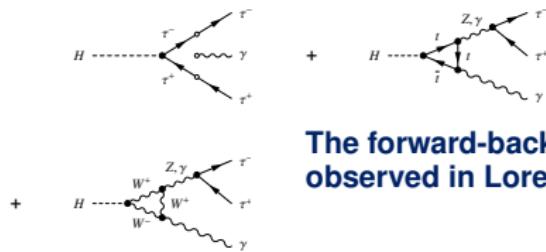
[See for example Phys. Rev. D 55, 5647-5656 (1997); Phys. Rev. D 90, no.11, 113006 (2014); Eur. Phys. J. C 74, no.11, 3141 (2014); JHEP 12, 111 (2016).]

6/13

Example: D. Sahoo, *Probing CP violation in $H \rightarrow \tau^+ \tau^- \gamma$*

The 3-body decay $H \rightarrow \tau^+ \tau^- \gamma$ offers an alternative methodology.

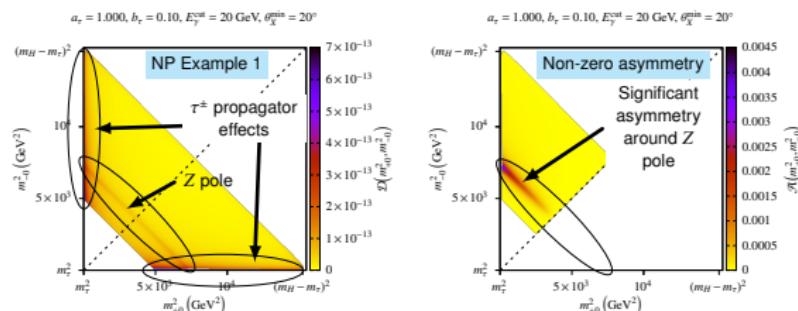
Decay proceeds via both tree and loop diagrams



$$\text{Br}(H \rightarrow \tau^+ \tau^- \gamma)_{\text{SM}} \sim 3.24 \times 10^{-3} \text{ with } E$$

[See for example Phys. Rev. D 55, 5647-5656 (1997); JHEP 12, 111 (2016).]

The forward-backward asymmetry can be easily observed in Lorentz invariant Dalitz plot distribution.



Example: A. de Giorgi, *Flavour and Higgs physics in Z2-symmetric 2HD models near the decoupling limit* [Nucl.Phys.B 994 (2023) 116323]

Higgs-Fermion Couplings

$$-\mathcal{L}_Y^{\text{eff}} \supset M_f \bar{f} f + \frac{M_f}{v} h (\kappa_f \bar{f} f + \tilde{\kappa}_f \bar{f} i \gamma_5 f) + \dots,$$

$$\kappa_u = \kappa_d = \kappa_e = 1 - \zeta_f |\tilde{\lambda}_6| \cos(\rho) \frac{v^2}{\tilde{m}_2^2},$$

Stronger than flavour symmetries!
(see last year presentation or
[2109.07490](#))

$$\tilde{\kappa}_u = \tilde{\kappa}_d = \tilde{\kappa}_e = -\zeta_f |\tilde{\lambda}_6| \sin(\rho) \frac{v^2}{\tilde{m}_2^2},$$

$$\rho \equiv \arg \left[\tilde{\lambda}_6^* e^{-ik/2} \right]$$

Universal deviation for each fermion-type
+
Correlations among different ones

	Type I	Type II	Type III (X)	Type IV (Y)
ζ_u	$\cot \beta$	$\cot \beta$	$\cot \beta$	$\cot \beta$
ζ_d	$\cot \beta$	$-\tan \beta$	$\cot \beta$	$-\tan \beta$
ζ_e	$\cot \beta$	$-\tan \beta$	$-\tan \beta$	$\cot \beta$

Example: A. de Giorgi, *Flavour and Higgs physics in Z₂-symmetric 2HD models near the decoupling limit* [Nucl.Phys.B 994 (2023) 116323]

Higgs-Fermion Couplings

$$-\mathcal{L}_Y^{\text{eff}} \supset M_f \bar{f} f + \frac{M_f}{\pi} h (\kappa_f \bar{f} f + \tilde{\kappa}_f \bar{f} i \gamma_5 f) + \dots,$$

$$\kappa_u = \kappa_d = \kappa_e = 1 - \zeta_f \left| \tilde{\lambda}_6 \right| \cos(\rho) \frac{v^2}{\tilde{m}_2^2}$$

Universal deviation for

**Stronger than flavour
symmetries!**
(see last year presentation or
2109.07490)

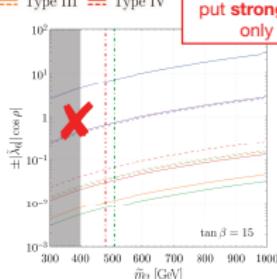
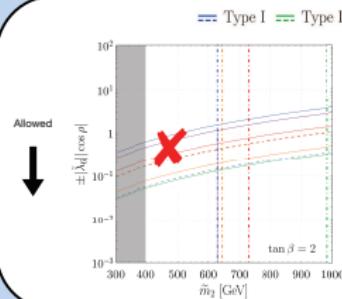
	Type I
ζ_u	$\cot \beta$
ζ_d	$\cot \beta$
ζ_c	$\cot \beta$

Arthur de Groot, ar00c 2304 10580

Bounds on the parameters

$$g_{hVV} = \frac{2m_V^2}{v^2} \left[1 - \frac{1}{2} \left| \tilde{\lambda}_6 \right|^2 \left(\frac{v^2}{\tilde{m}_2^2} \right)^2 \right] \equiv \frac{2m_V^2}{v^2} \kappa_V \quad \left| \tilde{\lambda}_6 \right| \left(\frac{v^2}{\tilde{m}_2^2} \right) \leq 0.17.$$

Bounds on Vector-Bosons put stronger constraints only on Type-I

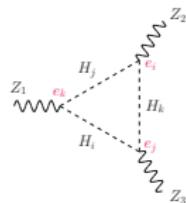


Arturo de Giorgi, ar29c-2304-10560

Example: P. Osland, *Constraining Higgs-Higgs-Z couplings in the 3HDM*

CP violation and alignment

In a CP-violating 2HDM, all pairs of neutral scalars couple to the Z , allowing the triangle diagram



The existence of these couplings induces a CP-violating amplitude,

Example: P. Osland, *Constraining Higgs-Higgs-Z couplings in the 3HDM*

CP violation and alignment

Also other diagrams, but importantly

f_4^Z is proportional to the invariant $\text{Im } J_2 \propto e_1 e_2 e_3$

CP-violating

In the alignment limit, two of the e_i vanish, the ZZZ amplitude vanishes

$$e_i \rightarrow v \quad \xrightarrow{\hspace{1cm}} \quad e_j, e_k \rightarrow 0 \quad \text{Im } J_2 \rightarrow 0$$

Example: P. Osland, *Constraining Higgs-Higgs-Z couplings in the 3HDM*

CP violation and alignment

Also other diagrams, but importantly

f_4^Z is proportional to the invariant $\text{Im } J_2 \propto e_1 e_2 e_3$

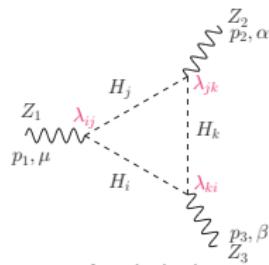
CP-violating

In the alignment limit, two of the e_i vanish, the ZZZ amplitude vanishes

$$\begin{array}{c} \downarrow \\ e_i \rightarrow v \end{array} \quad \xrightarrow{\hspace{1cm}} \quad e_j, e_k \rightarrow 0 \quad \text{Im } J_2 \rightarrow 0$$

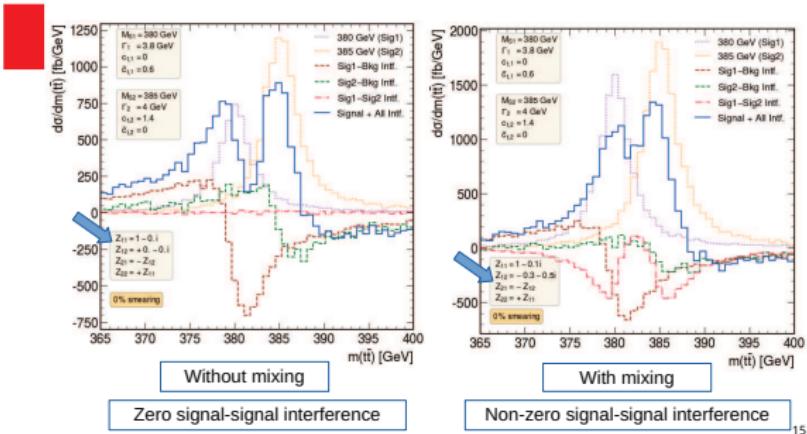
CP violation and alignment

In a 3HDM

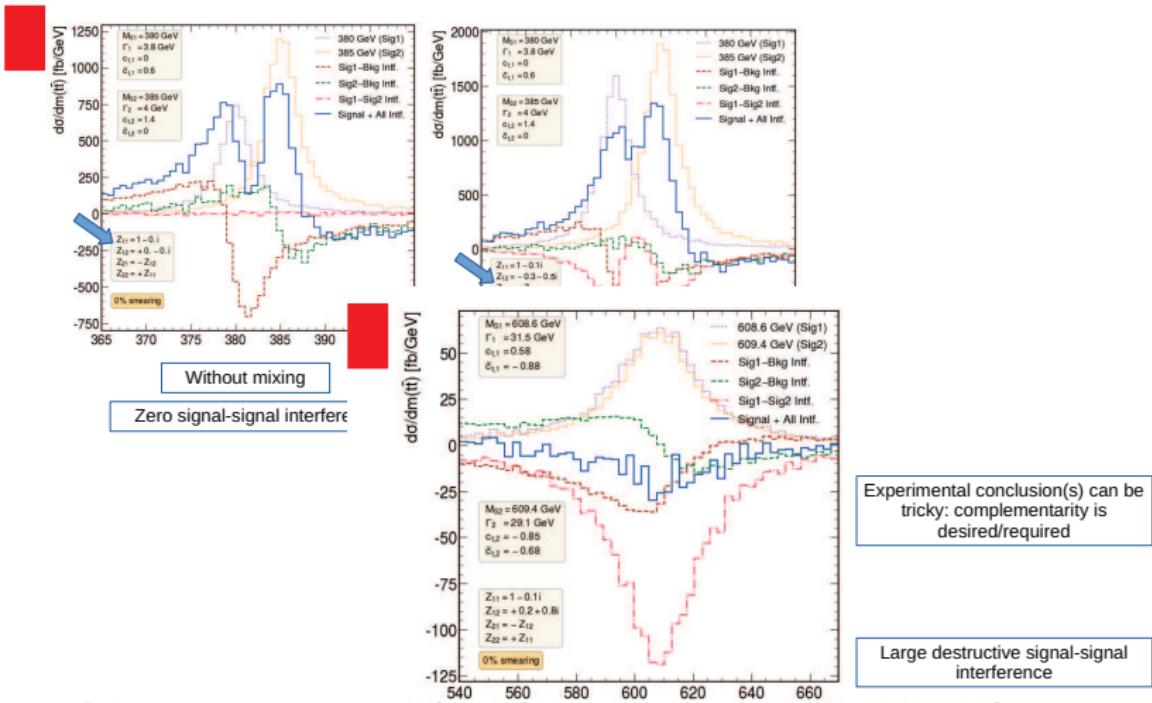


contributions proportional to $\lambda_{ij}\lambda_{jk}\lambda_{ki}$
This does not vanish in the alignment limit!

Example: R. Kumar, *Analysis of interference effects in the di-top final state for CP-mixed scalars in extended Higgs sectors*



Example: R. Kumar, *Analysis of interference effects in the di-top final state for CP-mixed scalars in extended Higgs sectors*



Further plans

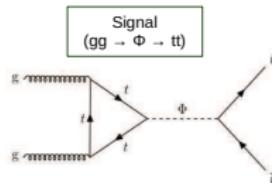
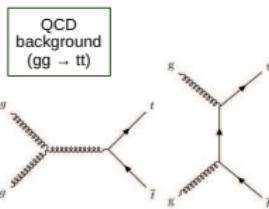
- very open calls ⇒ **large variety of topics**
- **we will continue these open calls, hopefully several/ year**
- iff there is a YREP5, we will contribute w **summary of state of the art and maybe benchmarks**
- currently no other plans

Comments ? Suggestions ?

Example: R. Kumar, *Analysis of interference effects in the di-top final state for CP-mixed scalars in extended Higgs sectors*

Di-top final state

- Total amplitude:
 $\mathcal{A} = \mathcal{A}(gg \rightarrow t\bar{t}) + \mathcal{A}(gg \rightarrow \Phi \rightarrow t\bar{t})$
- Signal-background interference
 $\propto \text{Re}[\mathcal{A}(gg \rightarrow \Phi \rightarrow t\bar{t})\mathcal{A}^*(gg \rightarrow t\bar{t})]$
large destructive contribution
- Invariant mass distribution of the top quarks ($m_{t\bar{t}}$) significantly distorted → peak-dip structure

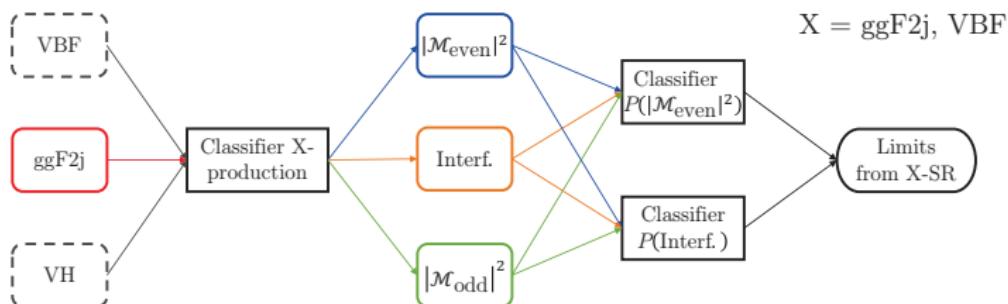


4

Example: M. Menen, *Classifying the CP properties of the ggH coupling in H+2j production* [arXiv:2309.03146]

Analysis strategy

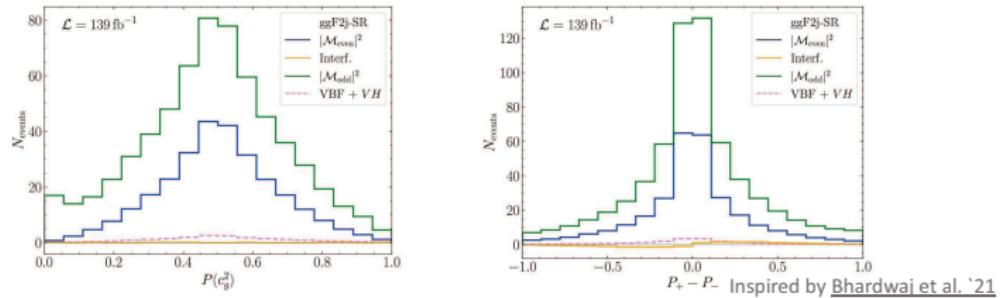
$$|\mathcal{M}_{\text{ggF2j}}|^2 = c_g^2 |\mathcal{M}_{\text{even}}|^2 + \underbrace{2c_g \tilde{c}_g \text{Re}[\mathcal{M}_{\text{even}} \mathcal{M}_{\text{odd}}^*]}_{\text{Interference}} + \tilde{c}_g^2 |\mathcal{M}_{\text{odd}}|^2$$



- Train a CP-even and a CP-odd classifier in a ggF2j-SR and a VBF-SR

Example: M. Menen, *Classifying the CP properties of the ggH coupling in H+2j production* [arXiv:2309.03146]

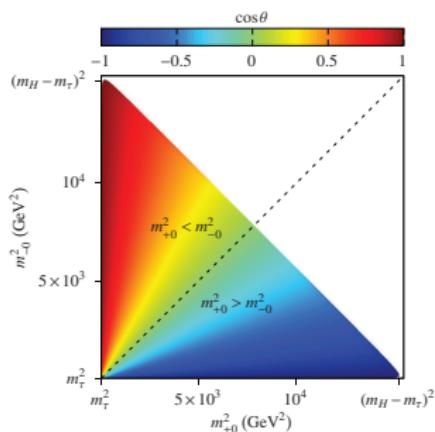
ggF2j signal region



- $P(c_g^2)$ differentiates between $c_g^2 |M_{\text{even}}|^2$ and $\tilde{c}_g^2 |M_{\text{odd}}|^2$
- Kinematically very similar, but some separation in outer bins
- Interference term cancels out
- $P_+ - P_-$ differentiates between positive and negative interference
- Interference barely visible due to low cross section & looks more VBF-like
- CP-even terms are symmetric

Example: D. Sahoo, *Probing CP violation in $H \rightarrow \tau^+ \tau^- \gamma$*

The amplitude square can be expressed using Lorentz invariant mass-squares.



- Only 3 Lorentz invariant mass-squares:

$$m_{\pm}^2 \equiv (p_H - p_0)^2 = (p_+ + p_-)^2,$$

$$m_{+0}^2 \equiv (p_H - p_-)^2 = (p_+ + p_0)^2,$$

$$m_{-0}^2 \equiv (p_H - p_+)^2 = (p_- + p_0)^2,$$

$$m_{+-}^2 + m_{+0}^2 + m_{-0}^2 = m_H^2 + 2 m_\tau^2 \; .$$

∴ Only 2 *independent* mass-squares.

- In the GJ frame,

$$m_{\pi^0}^2 = M^2 - M'^2 \cos \theta,$$

$$m_{-0}^2 = M^2 + M'^2 \cos \theta,$$

where $M^2 = \frac{1}{2} (m_H^2 + 2 m_\tau^2 - m_{+-}^2)$,

$$M'^2 = \frac{1}{2} \left(m_H^2 - m_{+-}^2 \right) \left(1 - \frac{4m_\tau^2}{m_{+-}^2} \right)^{\frac{1}{2}}.$$