How to study n-Higgs doublet models

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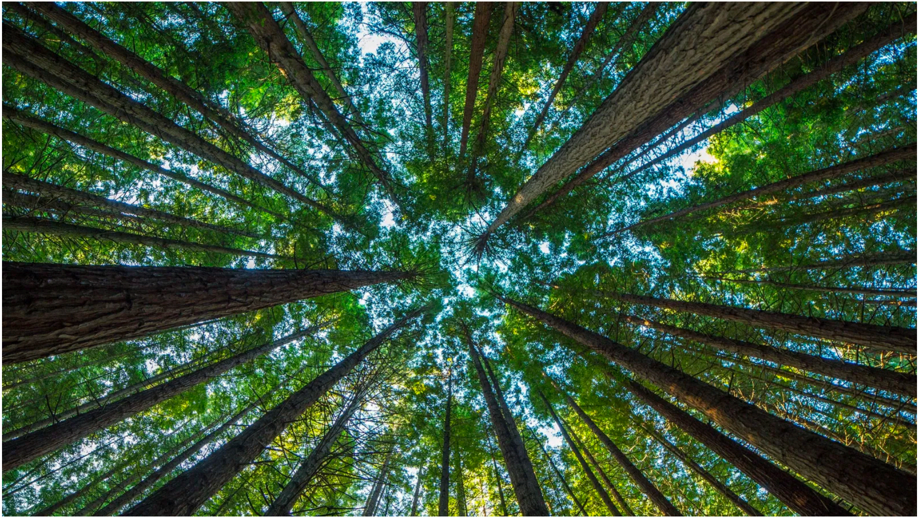
Sometimes we can't see the forest for the trees



Electroweak symmetry breaking

- Gauge symmetry hidden after electroweak symmetry breaking.
- Principal ideal: Keep gauge symmetry manifest.
- Let us revisit the Standard Model.

- SM: One Higgs-boson doublet

$$\varphi(x) = \begin{pmatrix} \varphi^+(x) \\ \varphi^0(x) \end{pmatrix}$$

- Domain: $\varphi^+, \varphi^0 \in \mathbb{C}$
- SM potential

$$V_{SM} = -\mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2$$

- Stable, partially electroweak symmetry breaking minimum for
  $\mu^2 > 0, \qquad \lambda > 0, \qquad v = \sqrt{\frac{\mu^2}{\lambda}}.$
- Conventionally, gauge rotation and expansion around minimum

$$\varphi(x) = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\left(v + h(x)\right) \end{pmatrix}$$

$$V_{SM} = \frac{\lambda}{4} v^2 - \frac{\mu^2}{2} v^2 + \left(\lambda v^3 - \mu^2 v\right) h + \left(\frac{3}{2}\lambda v^2 - \frac{\mu^2}{2}\right) h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4$$

- Gauge symmetry hidden!

- However, we can keep all expressions gauge invariant.

- Define a real gauge-invariant bilinear

$$K = \varphi^\dagger \varphi$$

- Domain: $K \geqslant 0$

$$V_{SM} = -\mu^2 K + \lambda K^2$$

- Minimum: $\langle K \rangle = \frac{\mu^2}{2\lambda}$, electroweak symmetry unbroken for $\langle K \rangle = 0$, partially broken for $\langle K \rangle > 0$.

- What do we have achieved?

  - Simple, real, potential - polynomial of degree two.

  - Gauge invariance manifest:
    Symmetries not hidden by unphysical gauge-degrees of freedom.

Why study NHDM's?

- T. D. Lee introduced 2HDM getting new source for CP violation.
- Dark matter models realized by NHDM's.
- Neutrino mixing models realized by NHDM's.
- Supersymmetry requieres at least two doublets.
- Pragmatically: Number of doublets not restricted.

Bilinears in the NHDM

- Higgs doublets $\varphi_i = \begin{pmatrix} \varphi_i^+ \\ \varphi_i^0 \end{pmatrix}$, $i = 1, \ldots, n$.
- Arrange all $SU(2)_L \times U(1)_Y$ invariants into hermitian $n \times n$ matrix

$$\underline{K} = \begin{pmatrix} \varphi_1^\dagger \varphi_1 & \cdots & \varphi_n^\dagger \varphi_1 \\ \vdots & \ddots & \vdots \\ \varphi_1^\dagger \varphi_n & \cdots & \varphi_n^\dagger \varphi_n \end{pmatrix}$$

- Basis for $\underline{K}$ are Gell-Mann matrices λ_α, $\lambda_0 = \sqrt{\frac{2}{n}} \mathbb{1}_n$,

$$\underline{K} = \frac{1}{2} K_\alpha \lambda_\alpha, \qquad \alpha = 0, 1, \ldots, n^2 - 1$$

- One-to-one correspondance between Higgs-boson doublets and Hermitean matrix $\underline{K}$ with rank $\leqslant 2$.

Example: two doublets

- $K_\alpha = \mathrm{tr}\,(\underline{K}\sigma_\alpha)$, $(\mu = 0, \ldots, 3)$ translates to

$$\begin{aligned} K_0 &= \varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 \\ K_1 &= \varphi_1^\dagger \varphi_2 + \varphi_2^\dagger \varphi_1 \\ K_2 &= i\left(\varphi_2^\dagger \varphi_1 - \varphi_2^\dagger \varphi_2\right) \\ K_3 &= \varphi_1^\dagger \varphi_1 - \varphi_2^\dagger \varphi_2 \end{aligned}$$

- Inversion gives scalar products

$$\begin{aligned} \varphi_1^\dagger \varphi_1 &= \frac{1}{2}(K_0 + K_3) \\ \varphi_2^\dagger \varphi_1 &= \frac{1}{2}(K_1 - iK_2) \\ \varphi_1^\dagger \varphi_2 &= \frac{1}{2}(K_1 + iK_2) \\ \varphi_2^\dagger \varphi_2 &= \frac{1}{2}(K_0 - K_3) \end{aligned}$$

- We write the bilinears as $K_0, \; \boldsymbol{K} = \begin{pmatrix} K_1 \\ \vdots \\ K_{n^2-1} \end{pmatrix}$.
- Example 2HDM Bilinears:

$$K_0, \qquad \boldsymbol{K} = \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix}$$

- General NHDM potential

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0 \, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K}$$

- Real parameters

$$\xi_0, \; \eta_{00}, \; \boldsymbol{\xi}, \; \boldsymbol{\eta}, \; E = E^{\mathrm{T}}$$

- As stated,

  Gauge-symmetry manifest
  Real parameters
  Reduced power of potential

C. Nishi **PRD 74** (2006)
MM, A. Manteuffel, O. Nachtmann, F. Nagel **EPJC 48** (2006)

Change of basis

- Consider the following unitary mixing of the doublets

$$\begin{pmatrix} \varphi_1'(x)^{\mathrm{T}} \\ \vdots \\ \varphi_n'(x)^{\mathrm{T}} \end{pmatrix} = U \begin{pmatrix} \varphi_1(x)^{\mathrm{T}} \\ \vdots \\ \varphi_n(x)^{\mathrm{T}} \end{pmatrix}$$

- Bilinears transform as

$$K_0' = K_0, \qquad \boldsymbol{K}' = R(U)\boldsymbol{K},$$

with $U^\dagger \lambda_a U = R_{ab}(U)\lambda_b$, $R \in SO(n^2-1)$, proper rotations in $\boldsymbol{K}$-space.

- Under change of basis $K_0' = K_0$, $\boldsymbol{K}' = R(U)\boldsymbol{K}$ potential remains invariant if

$$\xi_0' = \xi_0, \quad \eta_{00}' = \eta_{00},$$
$$\boldsymbol{\xi}' = R\,\boldsymbol{\xi}, \quad \boldsymbol{\eta}' = R\,\boldsymbol{\eta}, \quad E' = R\,E\,R^{\mathrm{T}}.$$

Symmetries

- Symmetry desirable to restrict NHDM.
- Symmetries easily formulated in terms of bilinears.

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} K_0{}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K}$$

- Transformation $K_0 \to K_0$, $\boldsymbol{K} \to \bar{R}\boldsymbol{K}$, $\bar{R} \in O(n^2 - 1)$ is symmetry of potential iff

$$\boldsymbol{\xi} = \bar{R}\, \boldsymbol{\xi}, \qquad \boldsymbol{\eta} = \bar{R}\, \boldsymbol{\eta}, \qquad E = \bar{R}\, E\, \bar{R}^{\mathrm{T}}$$

- $\bar{R} \in O(n^2 - 1)$, keeping kinetic terms invariant.
- Context with unitary mixing of doublets with matrix U: $U^\dagger \lambda_a U = \bar{R}_{ab}(U) \lambda_b$.

I. F. Ginzburg, M. Krawczyk, PRD 72 (2005)
I. P. Ivanov and C. C. Nishi, PRD 82 (2010)
MM, O. Nachtmann, JHEP 11 151 (2011)
V. Keus, S.F. King, S. Moretti, JHEP 1401 (2014)
B. Grzadkowski, MM, J. Wudka, JHEP 1111 (2011)
P. M. Ferreira, H. E. Haber, MM, O. Nachtmann, J. P. Silva, Int.J.Mod.Phys. A26 (2011)

CP symmetry

- CP transformation of the doublet fields

$$\varphi_i(x) \longrightarrow \varphi_i^*(x'), \qquad i = 1, \ldots n, \quad x = (t, \boldsymbol{x})^{\mathrm{T}}, \quad x' = (t, -\boldsymbol{x})^{\mathrm{T}}$$

- In terms of bilinears

$$K_0(x) \longrightarrow K_0(x'), \qquad \boldsymbol{K}(x) \longrightarrow \bar{R}\, \boldsymbol{K}(x')$$

- $\bar{R}$ is defined by the (generalized) Gell-Mann matrices

$$\lambda_a^{\mathrm{T}} = \bar{R}_{ab} \lambda_b, \qquad a, b \in \{1, \ldots, n^2 - 1\}.$$

2HDM: $\bar{R} = \mathrm{diag}(1, -1, 1)$

3HDM: $\bar{R} = \mathrm{diag}(1, -1, 1, 1, -1, 1, -1, 1)$

4HDM: $\bar{R} = \mathrm{diag}(1, -1, 1, 1, -1, 1, -1, 1, 1, -1, 1, -1, 1, -1, 1)$

MM, O. Nachtmann, PRD 100 (2019)

- CP symmetry conditions

$$\xi = \bar{R}\,\xi, \qquad \eta = \bar{R}\,\eta, \qquad E = \bar{R}\,E\,\bar{R}^{\mathrm{T}}.$$

- CP symmetry respected by vacuum if

$$\bar{R}\langle \boldsymbol{K} \rangle = \langle \boldsymbol{K} \rangle$$

- Basis invariant condition derived, generalized CP studied in 2HDM.

C. Nishi **PRD 74** (2006),
MM, A. Manteuffel, O. Nachtmann, EPJC57 (2008)
P. M. Ferreira, H. E. Haber, MM, O. Nachtmann, J. P. Silva, Int.J.Mod.Phys. A26 (2011)

Example: Generalized CP symmetry in the 2HDM

- Definition

$$\varphi_i(x) \xrightarrow{\mathrm{CP_g}} U_{ij}\, \varphi_j^*(x'), \quad i,j = 1,2$$

G.Ecker, W.Grimus, W.Konetschny, **NPB 191** (1981)

- The bilinears transform as

$$K_0(x) \xrightarrow{\mathrm{CP_g}} K_0(x'), \quad \boldsymbol{K}(x) \xrightarrow{\mathrm{CP_g}} \bar{R}\boldsymbol{K}(x')$$

with improper rotation $\bar{R}$.

O. Nachtmann, A. Manteuffel, MM **EPJC 57** (2007), O. Nachtmann, MM **JHEP 0905** (2009),
P. Ferreira, J. Silva, **PR D83** (2011)

- Requiring $\bar{R}^2 = \mathbb{1}_3$ there are two types

(i) $\bar{R} = -\mathbb{1}_3$, point reflection

(ii) $\bar{R} = R^{\mathrm{T}}\, \bar{R}_2\, R$, orthogonal equivalent to $\bar{R}_2$ reflection

Maximally CP invariant model

- 2HDM based on point reflection symmetry.
- Every 2HDM has 5 Higgs bosons: ρ', h', h'', $H^{\pm}$.
- Potential invariant under point reflections

$$\boldsymbol{K}(x) \xrightarrow{\mathrm{CP}_g^{(i)}} -\boldsymbol{K}(x')$$

$$V = \xi_0 K_0 + \cancel{\boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K}} + \eta_{00} {K_0}^2 + \cancel{2 K_0 \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K}} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K},$$

hence

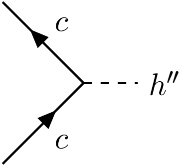
$$\boldsymbol{\xi} = \boldsymbol{\eta} = 0$$

- Note, potential also invariant under reflections on planes.

- Transformation for original Higgs doublets

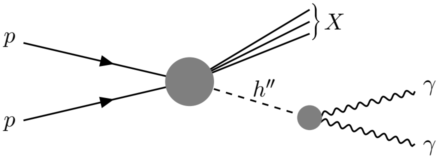
$$\begin{pmatrix} \varphi_1(x)^{\mathrm{T}} \\ \varphi_2(x)^{\mathrm{T}} \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \varphi_1(x')^{*\mathrm{T}} \\ \varphi_2(x')^{*\mathrm{T}} \end{pmatrix}$$

- Yukawa couplings: One family gives vanishing couplings.
- Absence of FCNC fixes couplings.
- Yukawa coupling of 2nd family prop. to 3rd family mass!
- In particular h'' to $c\bar{c}$ coupling



$$= \quad \frac{m_t}{v_0}\gamma_5$$

- Drell–Yan Higgs production

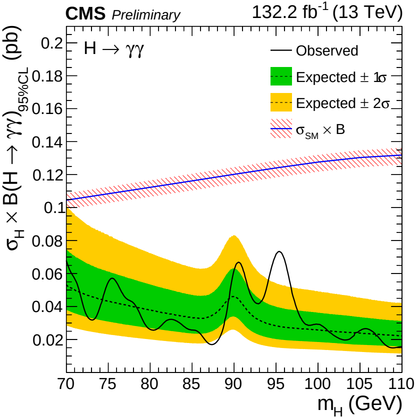


$$p + p \to h'' + X \; .$$
$$\hookrightarrow \gamma + \gamma$$

- For $m_{h''} = 95.4$ GeV, cross section about 0.01 pb.

O. Nachtmann, MM, arXiv:2309.04869

- Possible resonance measured



CMS collaboration, CMS-PAS-HIG-20-002 (2023)

Conclusion

- Main idea: Keep gauge invariance manifest, bilinears are the clue.

- Study of models, 2HDM, 3HDM, NHDM, NMSSM:
  Stability, electroweak symmetry breaking, mass matrices, symmetries, radiative corrections.
- Surprising results, example, CP symmetry mixing the doublets.

- CP reflection model gives argument for family replication.

Thank you for your attention!

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Stability in the NHDM

- Formulation of stability - V bounded from below - in terms of the biliners.
- $K_0 = \sqrt{\frac{2}{n}}(\varphi_1^\dagger \varphi_1 + \ldots + \varphi_n^\dagger \varphi_n)$
- For $K_0 > 0$ we define

$$\boldsymbol{k} = \frac{\boldsymbol{K}}{K_0}$$

- Potential V reads

$$V = K_0 \underbrace{\left(\xi_0 + \boldsymbol{\xi}^\mathrm{T} \boldsymbol{k}\right)}_{J_2(\boldsymbol{k})} + {K_0}^2 \underbrace{\left(\eta_{00} + 2\boldsymbol{\eta}^\mathrm{T} \boldsymbol{k} + \boldsymbol{k}^\mathrm{T} E \boldsymbol{k}\right)}_{J_4(\boldsymbol{k})}$$

- Stability determined by behavior of V in the limit $K_0 \to \infty$.
- Stability conditions derived depending on the functions J_4 and J_2.
  MM, O. Nachtmann JHEP 1502 (2015),
  MM, O. Nachtmann PRD 92 075017 (2015)

Electroweak symmetry breaking

- EW symmetry breaking given by global minimum

$$\langle\phi\rangle = \left\langle \begin{pmatrix} \varphi_1^+ & \varphi_1^0 \\ \vdots & \\ \varphi_n^+ & \varphi_n^0 \end{pmatrix} \right\rangle = \begin{pmatrix} v_1^+ & v_1^0 \\ \vdots & \\ v_n^+ & v_n^0 \end{pmatrix}, \quad \langle \underline{K} \rangle = \langle\phi\rangle\langle\phi\rangle^\dagger$$

- Unbroken $SU(2)_L \times U(1)_Y$ corresponds to $\langle \underline{K} \rangle$ of rank 0,
  $\underline{K} = 0, \qquad \varphi_i = 0$.

- Fully broken EW symmetry corresponds to $\langle \underline{K} \rangle$ of rank 2

- $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ corresponds to $\langle \underline{K} \rangle$ of rank 1

EW symmetry breaking in the 2HDM

- $K_0 = K_1 = K_2 = K_3 = 0$

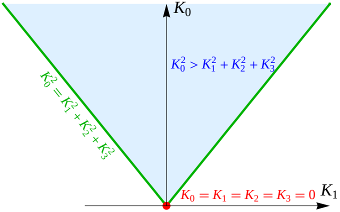
  $\varphi_1 = \varphi_2 = 0$
  $SU(2)_L \times U(1)_Y$ **unbroken**

- $K_0^2 > K_1^2 + K_2^2 + K_3^2$

  φ_1, φ_2 linear independent
  Not possible to arrange $\varphi_1^+ = \varphi_2^+ = 0$
  $SU(2)_L \times U(1)_Y$ **fully broken**

- $K_0^2 = K_1^2 + K_2^2 + K_3^2$

  φ_1, φ_2 linear dependent
  Possible to arrange $\varphi_1^+ = \varphi_2^+ = 0$
  $SU(2)_L \times U(1)_Y$ **partially broken.**



Minkowski space structure in the 2HDM

- Recall matrix $\underline{K}$ in the 2HDM

$$\underline{K} = \phi\phi^\dagger = \begin{pmatrix} \varphi_1^\dagger\varphi_1 & \varphi_2^\dagger\varphi_1 \\ \varphi_1^\dagger\varphi_2 & \varphi_2^\dagger\varphi_2 \end{pmatrix}$$

- We find

$$\begin{aligned} \operatorname{tr}(\underline{K}) &= \varphi_1^\dagger\varphi_1 + \varphi_2^\dagger\varphi_2 = K_0 \geqslant 0 \\ \det(\underline{K}) &= (\varphi_1^\dagger\varphi_1)(\varphi_2^\dagger\varphi_2) - (\varphi_2^\dagger\varphi_1)(\varphi_1^\dagger\varphi_2) = K_0^2 - K_1^2 - K_2^2 - K_3^2 \geqslant 0 \end{aligned}$$

- Minkowski-type four vector out of bilinears

$$(K^\mu) := \begin{pmatrix} K_0 \\ \boldsymbol{K} \end{pmatrix}, \quad \boldsymbol{K} = \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix}$$

with $K_0 > 0$ and $K_0^2 - \boldsymbol{K}^2 \geqslant 0$

Change of basis

- Consider the following unitary mixing of the doublets

$$\begin{pmatrix} \varphi_1(x)^{\mathrm{T}} \\ \vdots \\ \varphi_n(x)^{\mathrm{T}} \end{pmatrix} \to U \begin{pmatrix} \varphi_1(x)^{\mathrm{T}} \\ \vdots \\ \varphi_n(x)^{\mathrm{T}} \end{pmatrix}$$

- Bilinears transform as

$$\phi \to U\phi, \qquad \underline{K} = \phi\phi^\dagger \to U\underline{K}U^\dagger$$

hence,

$$\begin{aligned} K_0 &= \mathrm{tr}\,(\underline{K}\lambda_0) \to \mathrm{tr}\left(U\underline{K}U^\dagger\lambda_0\right) = \mathrm{tr}\left(U\underline{K}\lambda_0 U^\dagger\right) = \mathrm{tr}\,(\underline{K}\lambda_0) = K_0, \\ K_a &= \mathrm{tr}\,(\underline{K}\lambda_a) \to \mathrm{tr}\left(U\underline{K}U^\dagger\lambda_a\right) = \mathrm{tr}\Big(\underbrace{U^\dagger\lambda_a U}_{\equiv R_{ab}\lambda_b}\underline{K}\Big) = R_{ab}K_b \end{aligned}$$

- Change of basis correspond to proper rotations $R = (R_{ab}) \in SO(n^2-1)$.

- Under a change of basis

$$K_0 \to K_0, \quad \boldsymbol{K} \to R\boldsymbol{K}$$

The potential

$$V = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} \boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} E \boldsymbol{K}$$

changes to

$$V' = \xi_0 K_0 + \boldsymbol{\xi}^{\mathrm{T}} R\boldsymbol{K} + \eta_{00} {K_0}^2 + 2 K_0\, \boldsymbol{\eta}^{\mathrm{T}} R\boldsymbol{K} + \boldsymbol{K}^{\mathrm{T}} R^{\mathrm{T}} E R\boldsymbol{K}$$

- Potential invariant, $V = V'$, iff

$$\boldsymbol{\xi} = R\,\boldsymbol{\xi}, \quad \boldsymbol{\eta} = R\,\boldsymbol{\eta}, \quad E = R\,E\,R^{\mathrm{T}}.$$

Yukawa couplings in the MCPM

- At least two families for non-vanishing couplings.
- Absence of FCNC fixes couplings.
- Yukawa couplings

$$\mathscr{L}_{\text{Yuk},l}(x) = -c_{l\,3}\left\{\bar{l}_{3\,R}(x)\,\varphi_1^\dagger(x)\begin{pmatrix}\nu_{3\,L}(x)\\ l_{3\,L}(x)\end{pmatrix} - \bar{l}_{2\,R}(x)\,\varphi_2^\dagger(x)\begin{pmatrix}\nu_{2\,L}(x)\\ l_{2\,L}(x)\end{pmatrix}\right\} + c.c.$$

- Via EWSB $c_{l\,3}$ fixed, $m_{l_3} = c_{l\,3}\frac{\mathsf{v}}{\sqrt{2}}$, $\mathsf{v} \approx 246$ GeV.
- Yukawa coupling of 2nd family prop. to 3rd family mass!