QCD Beyond Diagrams

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Photo: Anti Niemi

QCD

$$L = \frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu}) + \overline{\psi}(\not{D} + M)\psi$$

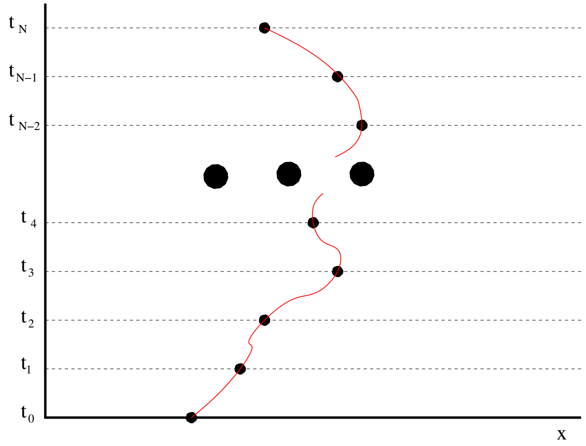
- renormalizable quantum field theory
- describes protons, neutrons, pions, etc.
- only parameters m_q/Λ_{qcd}
  - α_s not a parameter, adjust to get m_p right

QCD: peculiarities invisible to Feynman Diagrams

- confinement: free quarks don't exist
- mass generation: $m_p \propto \Lambda_{qcd}$
- chiral symmetry breaking: $m_\pi << m_\rho$
- Theta: different theories identical perturbatively

Crucial Tool: Path Integrals

Feynman: *Rev. Mod. Phys.*, 20, 367, 1948



$$\int_0^\beta (dx(t))\ e^{-S(x(t))} \qquad \propto \qquad \mathrm{Tr}\ e^{-\beta H}$$

- $S = \int dt\ \dot{x}^2/2 + V(x)$
- $H = \hat{p}^2/2 + V(\hat{x}) \qquad\qquad [\hat{p}, \hat{x}] = i$

D dimensional QM $\equiv D+1$ dimensional stat mech

- the heart of lattice gauge Monte Carlo
- in field theory "paths" become "configurations"

Path integral defined as a limit $a \to 0$

- Typical paths are not differentiable
  - $\langle \dot{x}^2 \rangle = \left\langle \left(\frac{x_{i+1}-x_i}{a} \right)^2 \right\rangle \propto \frac{1}{a} \to \infty$

Perturbation Theory: expand $S = S_0 + gS_I$

- S_0 describes free particle propagation
- S_I couples the fields together

Feynman Diagrams: expand path integral in g

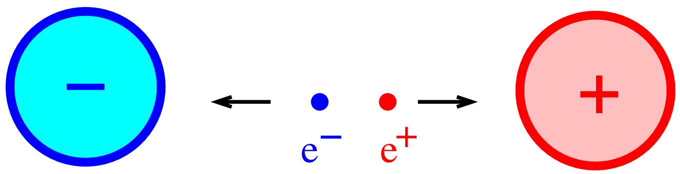
Path integral more general!

Electroweak $\alpha = 1/137 << 1$

- perturbation theory works for all practical purposes

Dyson: *Phys. Rev*, 85, 631 (1952)

- perturbation theory cannot converge
- $e \to ie$ makes like charges attract



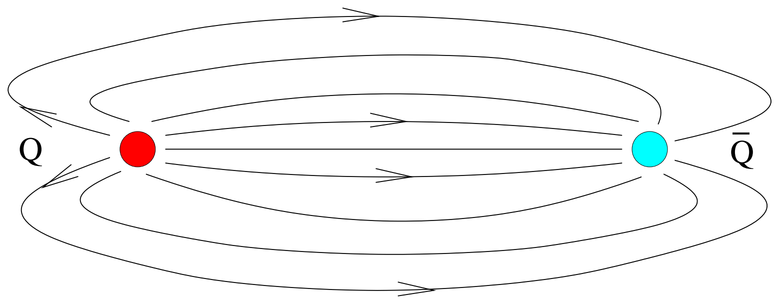
- vacuum unstable

QCD: Quark Confining Dynamics

- $g = 0$ free quarks and gluons
- $g \neq 0$ protons and pions

Spectrum qualitatively different

- must go beyond perturbation theory



Divergences and renormalization

Quantum field theory has infinities

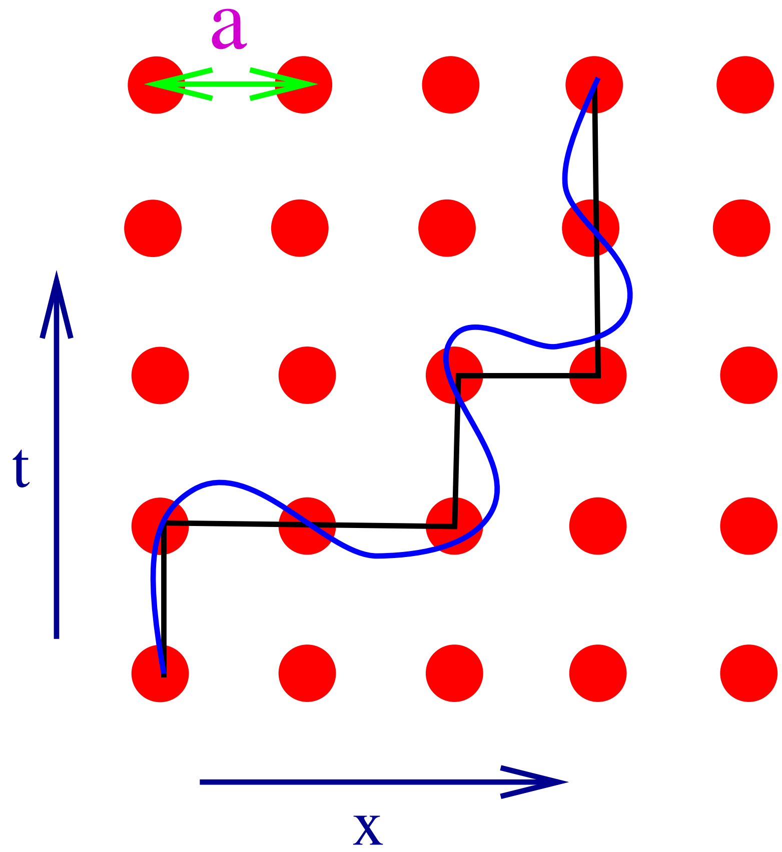
- requires introducing a cutoff
- remove cutoff by a limiting procedure
- adjust couplings holding physical quantities fixed

Most regulators based on perturbation theory

- Pauli-Villars, dimensional, ...
- find a divergent diagram, cut it off

QCD requires a non-perturbative regulator

- lattice gauge theory
- use lattice spacing a for a cutoff


a
t
x

Use the proton mass as a physical observable

- $m_p(g(a), a)$
- $a\frac{d}{da}m_p = 0 = \left(\frac{\partial}{\partial g}m_p\right)\left(a\frac{dg}{da}\right) + a\frac{\partial}{\partial a}m_p$
  - dimensions imply $a\frac{\partial}{\partial a}m_p = -m_p$

$$a\frac{dg}{da} \equiv \beta(g) = \frac{m_p}{\frac{\partial}{\partial g}m_p}$$

How g varies with a for physical limit

$\beta(g)$ has a perturbative expansion!

- $\beta(g) = \beta_0 g^3 + \beta_1 g^5 + O(g^7)$
- $\beta_0 = \frac{1}{16\pi^2}(11 - 2N_f/3)$ N_f fermion flavors
  - Gross, Wilczek, Politzer (1973)
- $\beta_1 = \left(\frac{1}{16\pi^2}\right)^2 (102 - 22N_f/3)$
  - Caswell, Jones (1974)

β_0 and β_1 are universal

- independent of cutoff scheme
- applies to the lattice as well

Solving the differential equation $a\frac{dg}{da} = \beta(g)$

- $a = \frac{1}{\Lambda}\ g^{-\beta_1/\beta_0^2}\ \exp\left(\frac{-1}{2\beta_0 g^2}\right)\ (1 + O(g^2))$
- Λ is an integration constant
- $a \to 0\ \Leftrightarrow\ g \to 0$ "asymptotic freedom"

Particle masses proportional to Λ

$$m_p \propto \Lambda \propto \frac{1}{a}\ g^{-\beta_1/\beta_0^2}\ \exp\left(\frac{-1}{2\beta_0 g^2}\right)$$

Non-perturbative behavior!!

"Dimensional transmutation" Coleman and Weinberg

- renormalization eliminates dimensionless g
- turns it into an overall scale Λ

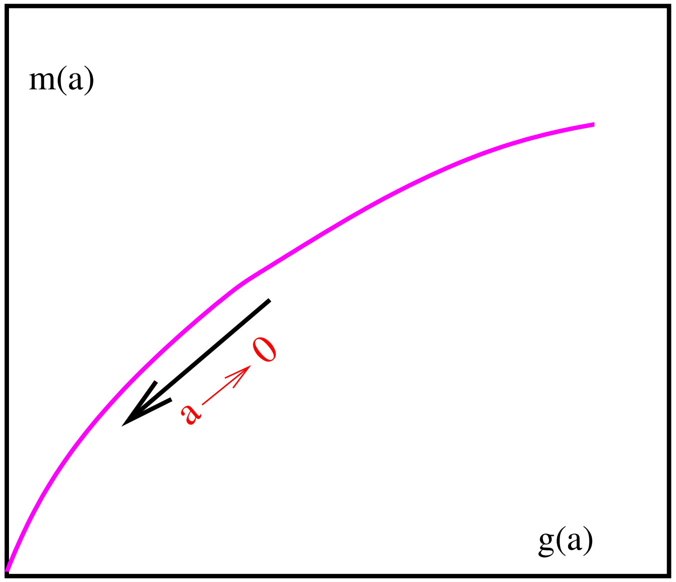
Quark masses also require renormalization

- hold more things fixed
- $m = M\ g^{\gamma_0/\beta_0}\ (1 + O(g^2))$ $\qquad \gamma_0 = \frac{8}{(4\pi)^2}$

M another integration constant: "renormalized mass"

- one M_i for each quark species i
- "bare" mass $m \to 0$ for continuum limit

Continuum limit takes both g and m to zero together



Dimensional “integration constants” M_i and Λ

- can mix, scheme dependent

Electroweak also fits on the lattice, but

- $U(1)$ not asymptotically free?
  - unification? $SO(10)$?
- Higgs not asymptotically free?
  - composite Higgs?
  - interplay with top and gravity?

Classical gauge theories also non-perturbative

Tied to topology

- winding $\frac{g^2}{16\pi^2}\int d^4x \mathrm{Tr} F\tilde{F} = \nu$
- ν is an integer for smooth fields

Topology and zero modes

When $\nu \neq 0$, zero modes of the Dirac equation

- $D\psi(x) = \gamma_\mu(\partial_\mu + igA_\mu)\psi(x) = 0$
- ψ is chiral $\qquad \gamma_5\psi(x) = \pm\psi(x)$

Index theorem

- $\nu = n_+ - n_-$
- other eigenvalues of D in chiral pairs

Fujikawa:

- configurations with $n_+ \neq n_-$ exist
- on these $\mathrm{Tr}\ \gamma_5 \ \equiv \ \sum_i \langle \psi_i \gamma_5 \psi_i \rangle = \nu \neq 0$
  - other modes paired or "above the cutoff"

$\psi \to e^{i\gamma_5\theta}\psi$ not a symmetry!

- changes fermion measure in path integral
  - $d\psi \to |e^{i\gamma_5\Theta}|\ d\psi = e^{i\nu\Theta}\ d\psi$

Topology inserts $e^{i\nu\Theta}$ into the path integral

- a new theory
- explains why η' not a pseudo-Goldstone boson

QCD has a hidden non-perturbative parameter Θ

For each value of Θ

- the perturbative expansion is identical!

$\Theta = \pi$ takes $m \to -m$

- three flavor QCD with negative masses
  - different theory, $\Theta = \pi$

Perturbatively: sign of a fermion mass is a convention

Strong CP puzzle: $\Theta \neq 0$ violates CP

- experimentally Θ small, why?

Confinement and masses

Quarks are confined: what does their mass mean?

Physical particles propagate over long distances

- $E = mc^2 + \frac{1}{2}mv^2 + O(mv^4/c^2)$

Quarks don't propagate alone

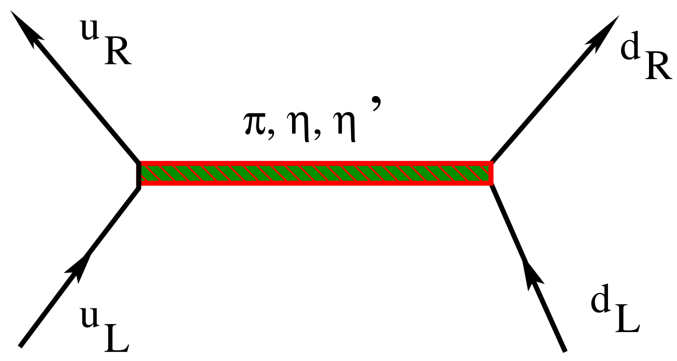
Neutral pseudoscalars in the three flavor theory:

$$\pi_0 = \frac{1}{2}(\overline{u}_L u_R - \overline{u}_R u_L - \overline{d}_L d_R + \overline{d}_R d_L)$$

$$\eta = \frac{1}{2\sqrt{3}}(2\overline{s}_L s_R - 2\overline{s}_R s_L - \overline{u}_L u_R + \overline{u}_R u_L - \overline{d}_L d_R + \overline{d}_R d_L)$$

$$\eta' = \frac{1}{\sqrt{6}}(\overline{s}_L s_R - \overline{s}_R s_L + \overline{u}_L u_R - \overline{u}_R u_L + \overline{d}_L d_R - \overline{d}_R d_l)$$

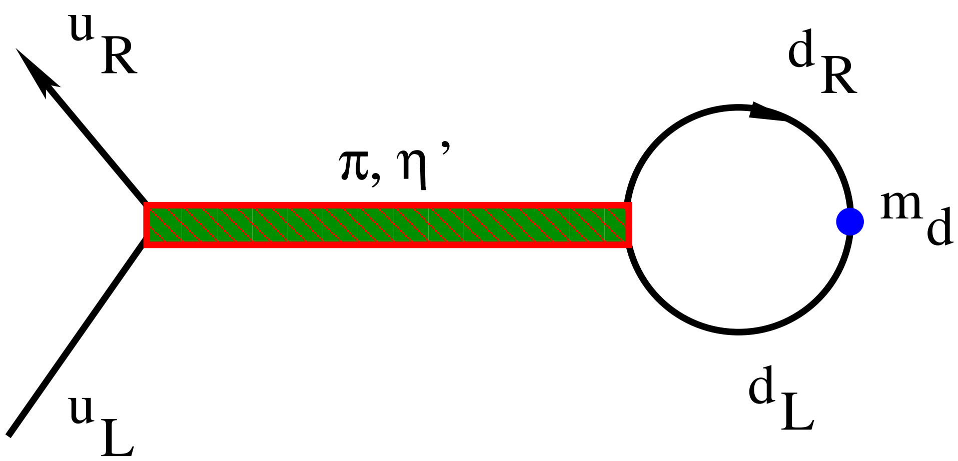
$\overline{u}_L u_R$ or $\overline{d}_L d_R$ can create any of these mesons



- does not vanish for massless quarks
- spin flip process via chiral anomaly
  - "'t Hooft vertex"

Now turn on a small d quark mass

- closing d loop induces $u_L\ u_R$ mixing



Non-zero d quark mass shifts u quark mass

- all quark masses and Λ become entangled

Effect automatically included in lattice simulations

Old point

- ’t Hooft, 1976
- Georgi, McArthur, 1981 (unpublished)
- Choi, Kim, Sze, 1988 (PRL)
- Banks, Nir, Seiberg, 1994 (conference proceedings)
- MC, 2004 (PRL)

Chiral symmetry

- degenerate light quarks
- $M_\pi^2 \propto m_q \Lambda$

Massless quarks imply massless pions

- for degenerate quarks $m_q = 0$ is well defined

What if we make isospin breaking large?

- $M_\pi^2 \propto \frac{m_u+m_d}{2}\Lambda$
- mass gap persists at $m_u = 0$ if $m_d > 0$

Sensible physics for small negative m_u!

- perturbation theory: sign of mass is a convention

Negative quark mass equivalent to $\Theta = \pi$

With degenerate quarks

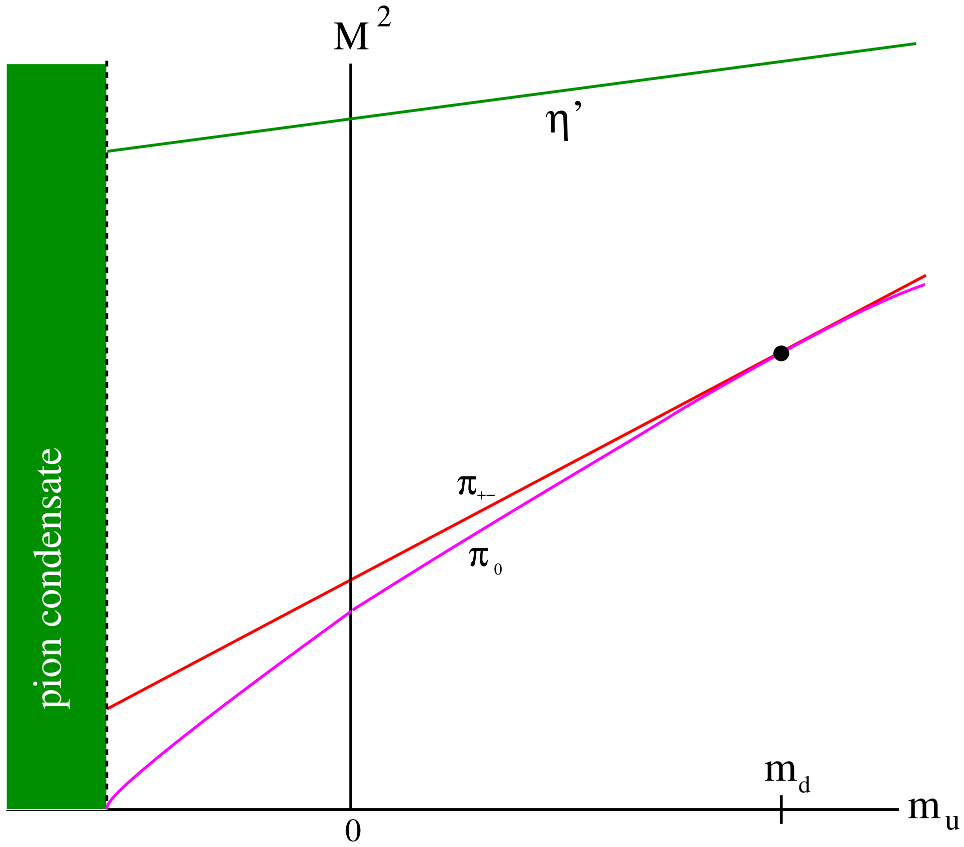
- $\Theta = \pi$ must have spontaneous parity breaking
- many demonstrations of this
  - MC, Annals of Physics 324 (2009), 1573

As the up quark mass varies from $+m_d$ to $-m_d$

  - something interesting must happen!

The Dashen phenomenon

Isospin breaking reduces π_0 mass



- $M^2_{\pi_0}$ vanishes before $m_u = -m_d$

At $M_{\pi_0}^2 < 0$ a pion condensate will form

- $\langle \pi_0 \rangle \neq 0$
- CP broken

Formally at $\Theta = \pi$ $\qquad$ $\prod_q m_q < 0$

Dashen 1971: allowed by current algebra

- Before qcd!

Explicit in sigma models from $(\pi_0,\ \eta)$ mixing

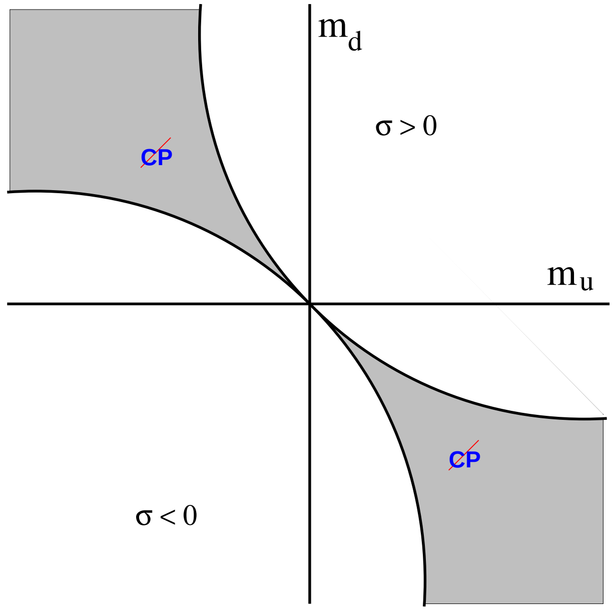
- $$m_{\pi_0}^2 \propto \frac{2}{3}\Bigg(m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\Bigg)$$

- $$m_{\eta}^2 \propto \frac{2}{3}\Bigg(m_u + m_d + m_s + \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\Bigg)$$

π_0 becomes massless at $m_u = \frac{-m_d m_s}{m_d + m_s} > -m_d$

Ising-like transition at $m_u < 0$

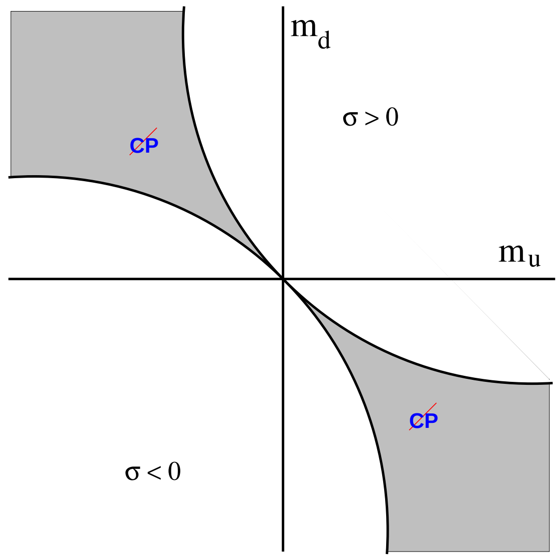
- order parameter $\langle \pi_0 \rangle \neq 0$
- breaks CP spontaneously



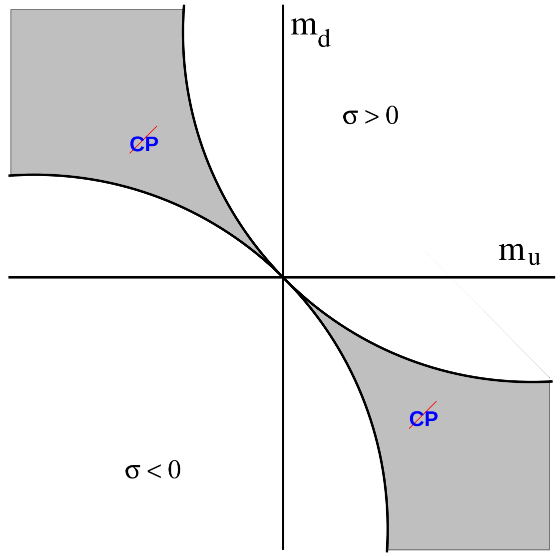
Also seen in 2 flavor Schwinger model

- Funke, Jansen, Kühn (2023)
- Coleman: "half asymptotic" phase

Symmetries of the (m_u, m_d) phase diagram



- $m_u \leftrightarrow m_d$ — Isospin
- $(m_u, m_d) \rightarrow (-m_u, -m_d)$ — Flavored chiral symmetry
  - $\psi \rightarrow e^{i\pi\tau_3\gamma_5}\psi$



NO symmetry under $(m_u, m_d) \to (-m_u, m_d)$

- a strictly non-perturbative effect
- a non-degenerate massless quark
  - not protected from renormalization

Symmetries imply multiplicative mass renormalization

- quark mass difference $m_d - m_u$
- quark mass average $m_d + m_u$

Renormalization factors are not in general equal!!

- m_u can acquire an additive shift ('t Hooft, 1976)
    - depends on scheme

Wilson fermions

- additive renormalization of Kappa critical
- depends on coefficient of Wilson therm

Overlap fermions

- depend on kernel
- Wilson parameter and Kappa

Staggered fermions

- inherent taste degeneracy cancels effect

Should we care if quark masses fuzzy?

- not directly measured in scattering
- related to fuzziness in gauge field topology
    - non-differentiable gauge fields

Only particle masses and scatterings are physical

Summary

Non-perturbative physics is crucial for QCD

Asymptotic freedom and the lattice define the theory

- unlike QED, Higgs field

Different theories with identical perturbative expansions

- $\Theta \neq 0$

Nondegenerate quark masses scheme dependent

- $m_u = 0$ ill defined if $m_d \neq 0$