The role of lattice QCD in precision physics

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50 Years of Quantum Chromodynamics
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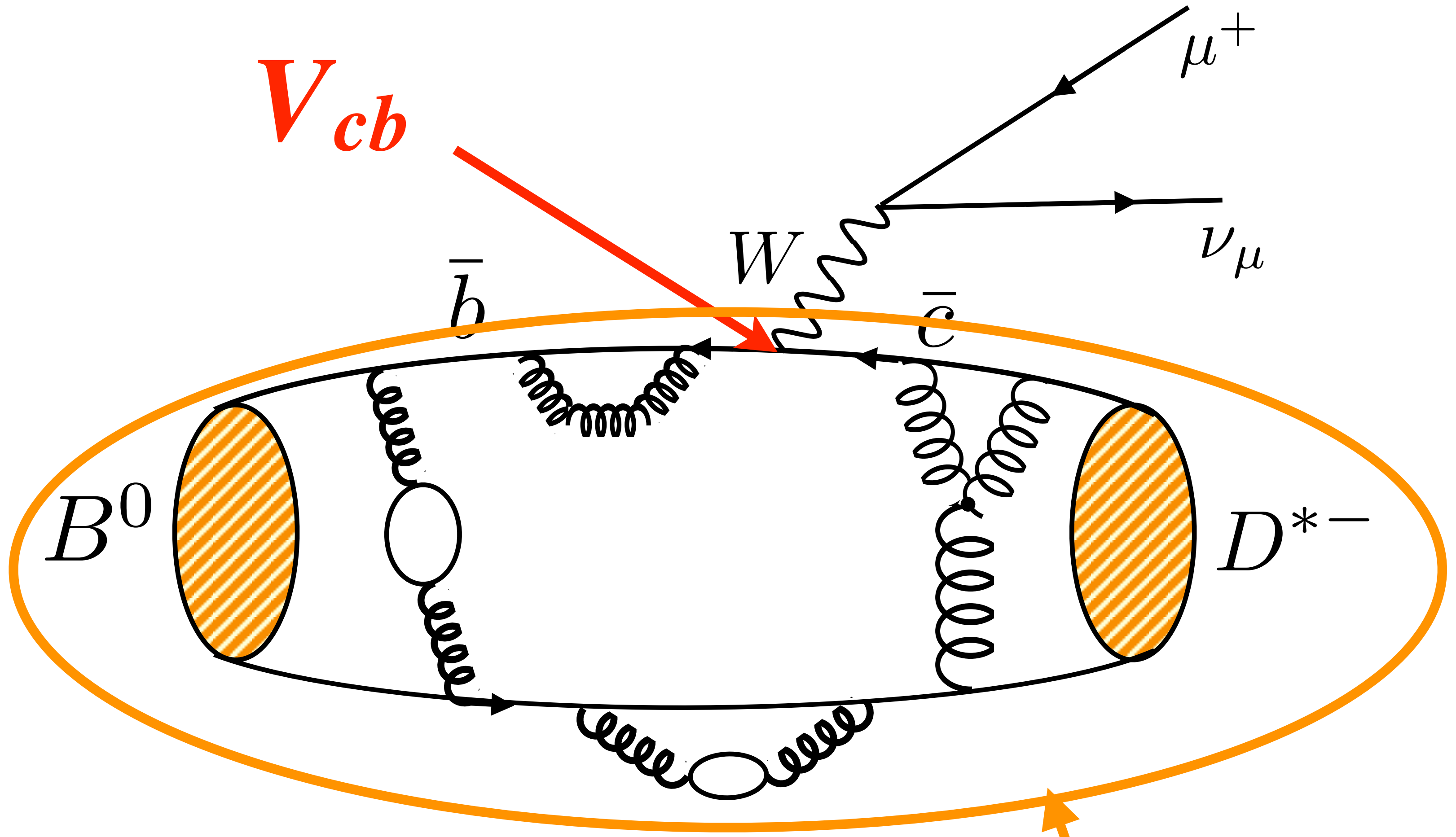
UCLA Mani L. Bhaumik Institute for Theoretical Physics

Outline

- The role of (lattice) QCD in precision physics
- Introduction to lattice QCD
- Success stories: two examples
  - m_q, α_s: inputs for Higgs decay rates
  - $B_{s,d} \to \mu\mu$
- Puzzles: one example
  - hadronic corrections to muon g-2
- Summary and Outlook

The role of lattice QCD in precision physics

example: $B^0 \to D^{*-}\mu^+\nu_\mu$



Experiment vs. SM theory:

(experiment) = (known) x (**CKM factors**) x (had. matrix element)

$\Gamma\left(K^+ \to \ell^+\nu_\ell(\gamma)\right)$

$d\Gamma(B^0 \to D^{*-}\mu^+\nu_\mu), \ldots$

$B(B_s \to \mu\mu), \ldots$

$\Delta m_{d(s)} \quad \cdots$

Two main purposes:

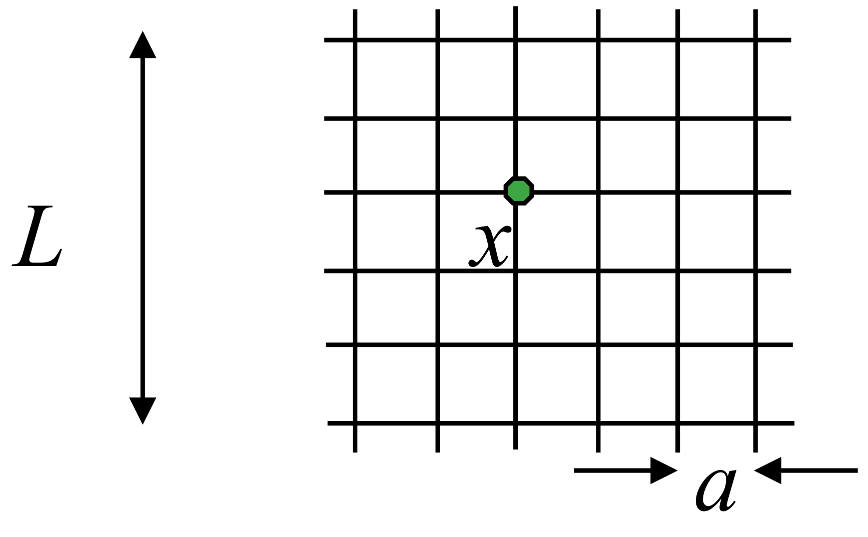
- ✦ combine experimental measurements with LQCD results to determine CKM parameters.
- ✦ confront experimental measurements with SM theory using LQCD inputs.

Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

Lattice QCD Introduction

$$\mathcal{L}_{\rm QCD} = \sum_f \bar\psi_f (\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr} F_{\mu\nu}F^{\mu\nu}$$

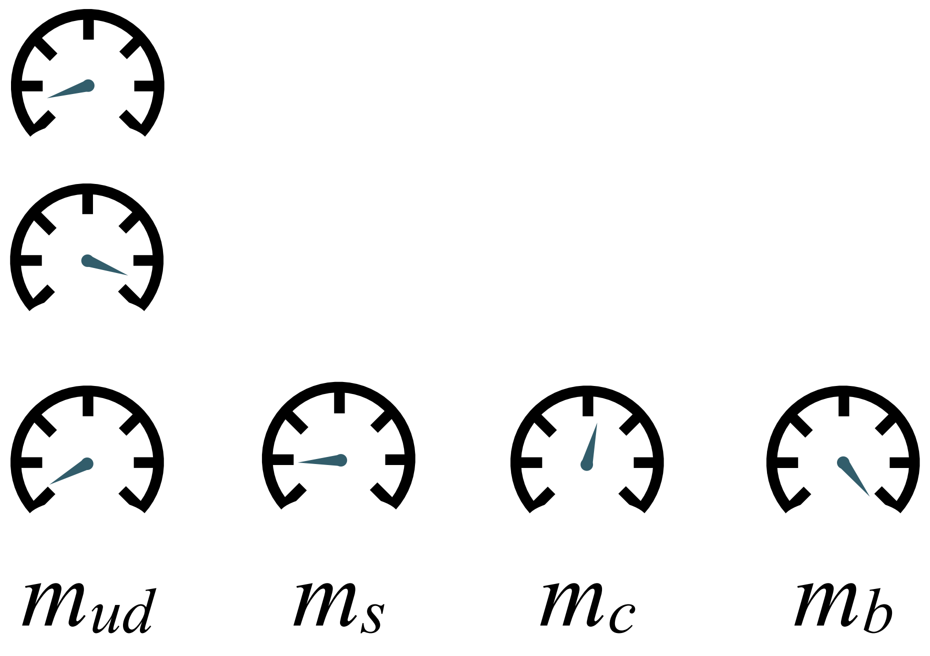


- discrete Euclidean space-time (spacing a)
  derivatives → difference operators, etc…
- finite spatial volume (L)
- finite time extent (T)

Integrals are evaluated numerically using monte carlo methods.

adjustable parameters

- lattice spacing: $a \to 0$
- finite volume, time: $L \to \infty$, $T > L$
- quark masses (m_f): $M_{H,{\rm lat}} = M_{H,{\rm exp}}$
  tune using hadron masses $m_f \to m_{f,{\rm phys}}$
  extrapolations/interpolations



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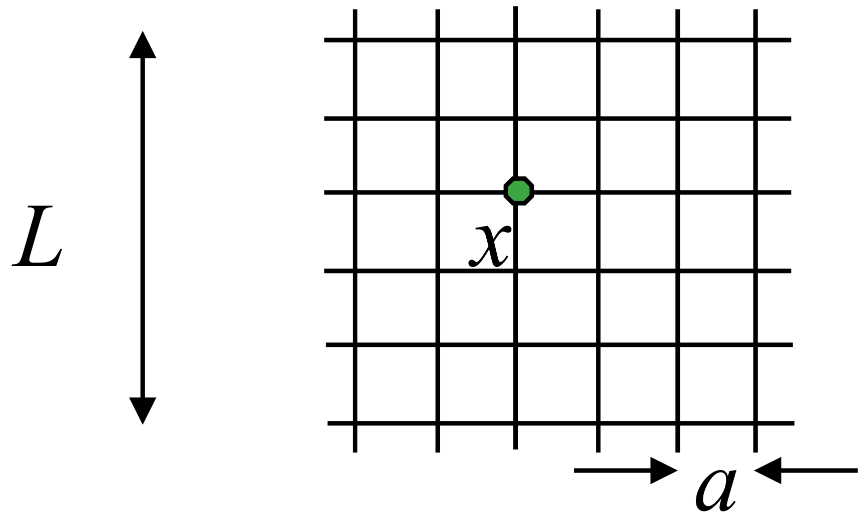

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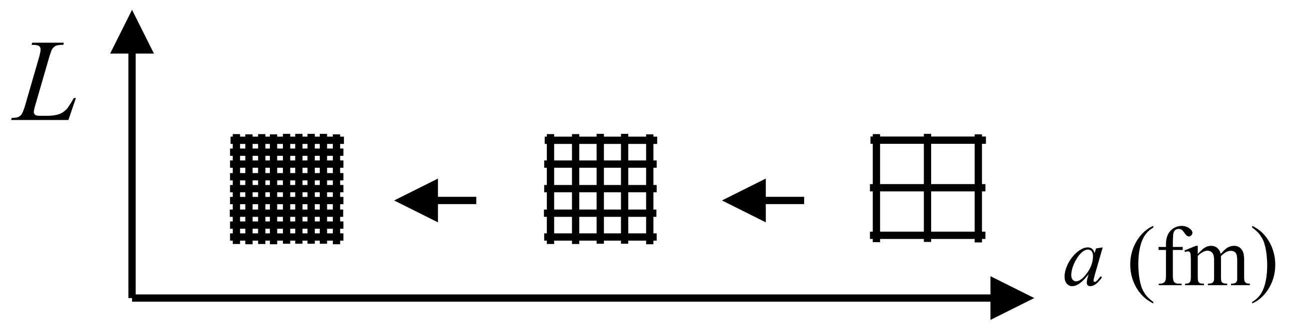
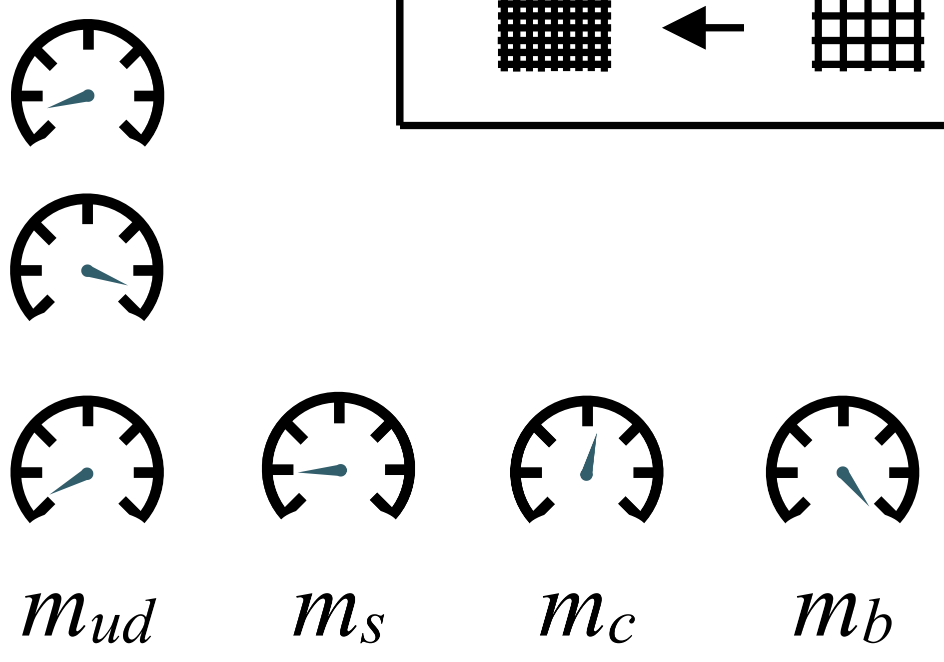
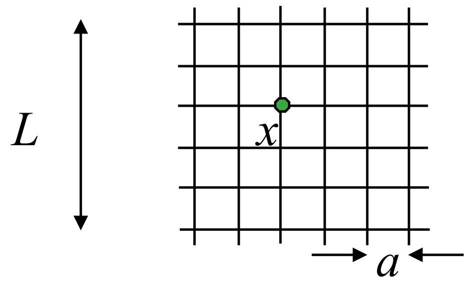


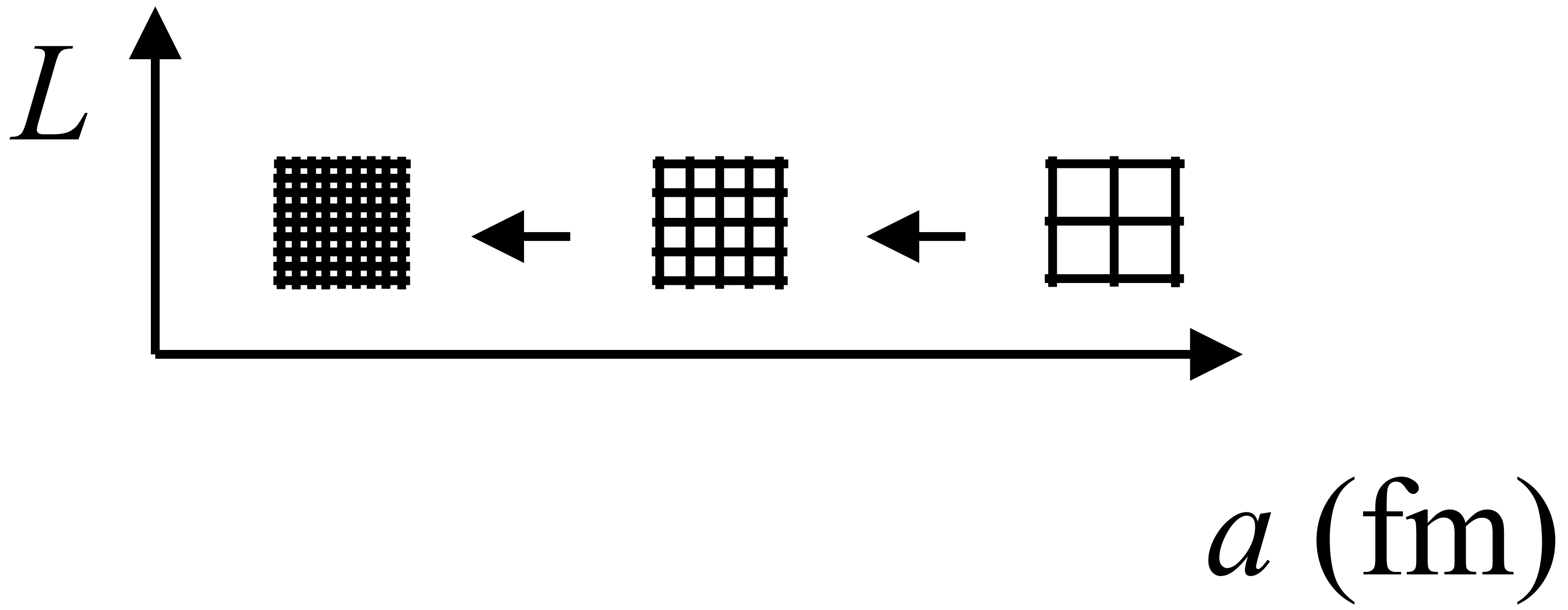


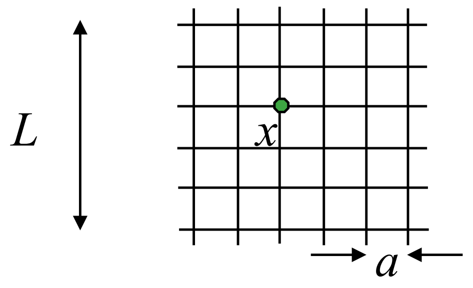


Lattice QCD Introduction

discretization effects — continuum extrapolation

- typical momentum scale of quarks gluons inside hadrons: $\sim\Lambda_{\mathrm{QCD}}$
- make a small to separate the scales: $\Lambda_{\mathrm{QCD}} \ll 1/a$
- Symanzik EFT: $\langle \mathcal{O} \rangle^{\mathrm{lat}} = \langle \mathcal{O} \rangle^{\mathrm{cont}} + O(a\Lambda)^n$, $n \geq 2$
  - provides functional form for extrapolation (depends on the details of the lattice action)
  - can be used to build improved lattice actions
  - can be used to anticipate the size of discretization effects



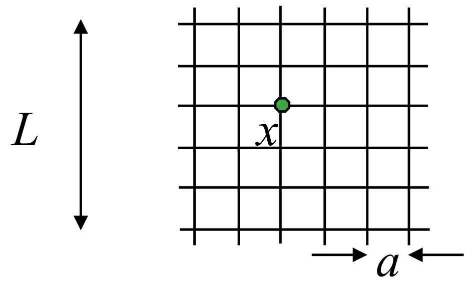


Lattice QCD: quark discretizations

Fermion doubling problem ⇔ chiral symmetry

- Staggered quarks (a.k.a Kogut-Susskind)
  reduce the number of doublers (staggering) but keep some (a.k.a tastes)
  dominant discretization effects due to taste-breaking effects (can be corrected analytically) ~$O(a^2)$
  various improved versions to reduce taste-breaking effects (HISQ,..)
  computationally inexpensive
- (improved) Wilson quarks
  no doublers, but chiral symmetry broken explicitly
  requires improvement to remove $O(a)$ effects (NP improved, twisted mass, …)
  moderate computational cost
- Domain wall quarks (live in 5 dimensions)
  no doublers, chiral symmetry exponentially suppressed
  small $O(a^2)$ discretization effects
  high computational cost

- new ideas:
  workshop on novel fermion actions
  https://indico.mitp.uni-mainz.de/event/314/

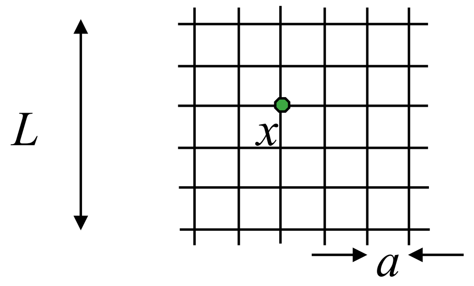


Lattice QCD Introduction

systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on Effective Field Theory (EFT) descriptions of QCD ➙ ab initio

- finite a: Symanzik EFT
- light quark masses: ChPT
- heavy quark effects: HQET
- finite L: finite volume EFT



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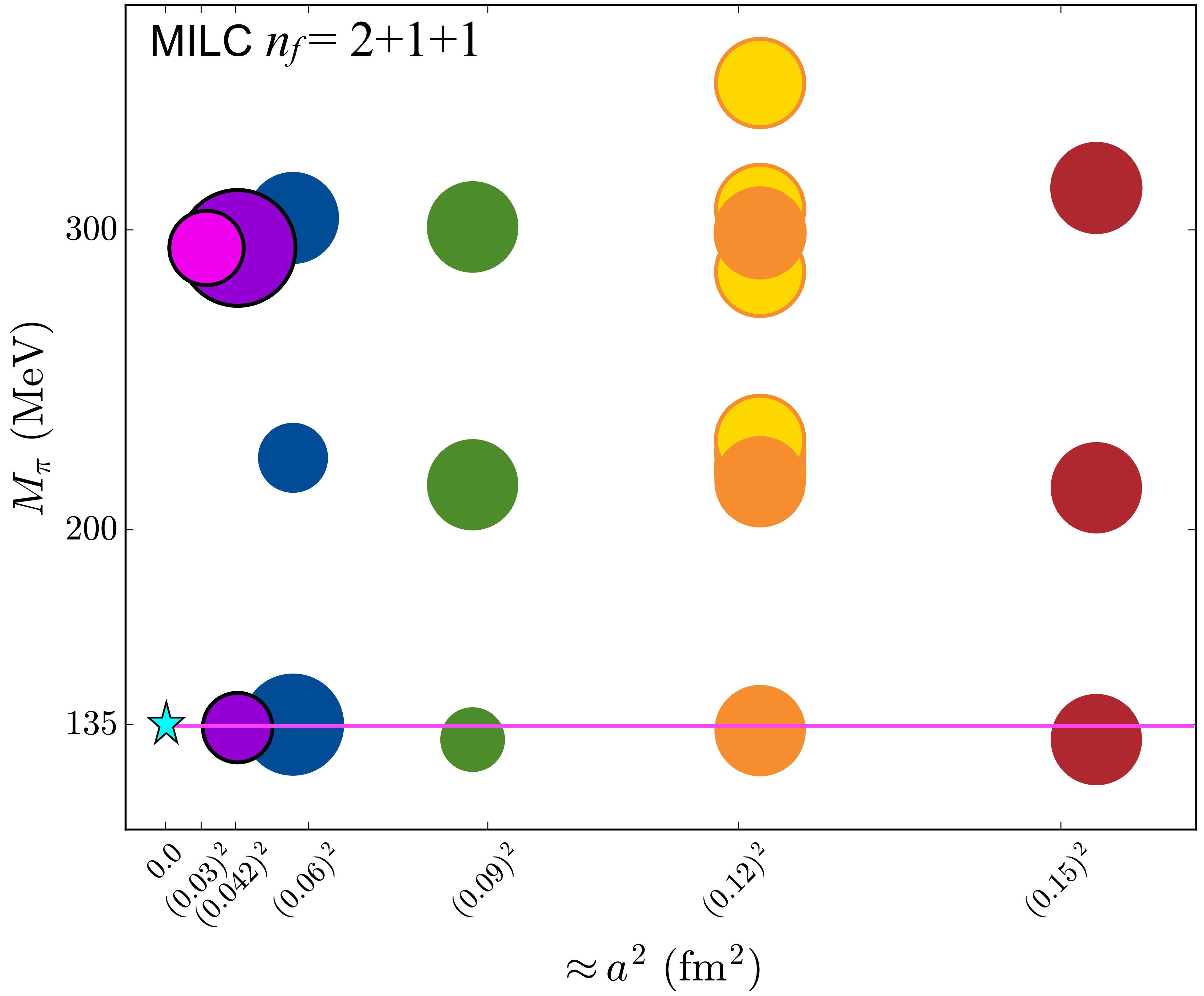
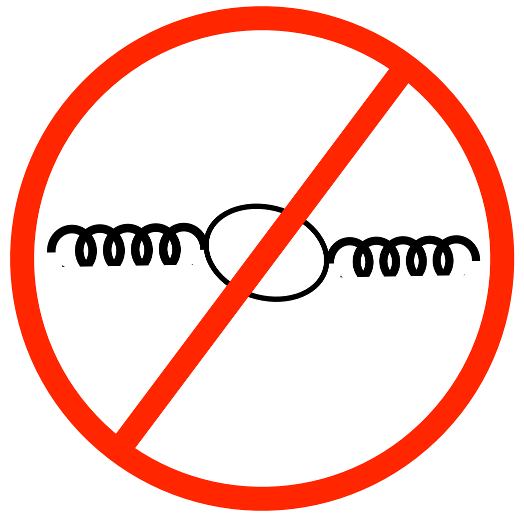
In practice:
stability and control over systematic errors depends on the underlying simulation parameters, available computational resources, analysis choices, …

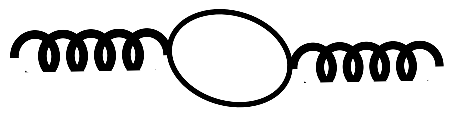


a selective view of (L)QCD history

- 1971 and 1974 — Discovery of charm (J/ψ)
- 1973 — Gross, Politzer, Wilczek *Asymptotic Freedom* — Kobayashi & Maskawa
- 1974 — Wilson: *Confinement of quarks:* gauge theory on space-time lattice
- 1977 — Discovery of beauty (Υ)
- 1979 — Creutz: *Monte Carlo study of quantized SU(2) gauge theory*
- 1981 — Hamber & Parisi, Weingarten: first quenched LQCD calculations of hadron masses



- 1984-1985 — **Bernard et al (UCLA group)**; Cabbibo, Martinelli, Petronzio; Brower et al:
  Weak Matrix Elements (ϵ_K, $\Delta I = 1/2$ rule, ϵ'/ϵ,…)
- 1989 — Sharpe review at Lattice 1989 conference on Weak Matrix Elements
- 2003 — First lattice QCD simulations that include realistic sea quark effects



⋮

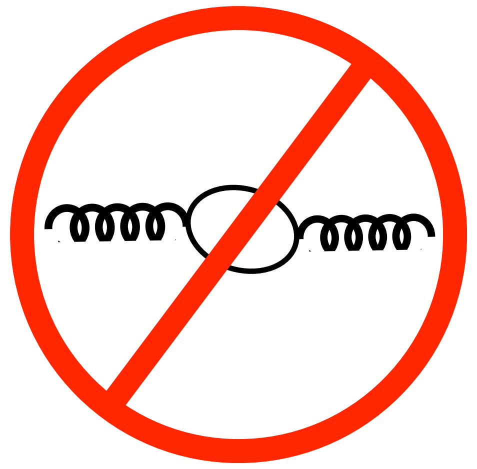
Status 1989

Sharpe @ Lattice 1989 [Nuc. Phys. B (Proc. Suppl.) 17 (1990)]

What?	Why?	Who? [6]	Level
Nucleon matrix elements			
f_π/m_N, f_K/f_π	check	MANY	2
Axial vector matrix elements: g_A ...	check	Sömmer [7]	2
EM form factors: $G_M(q^2)$, ...	check	Wilcox, Draper/Liu [8]	2
Structure functions	check	Rossi [9]	1
Neutron Electric Dipole Moment	measure θ_{QCD}	Goksch [10]	1
Heavy-light mesons			
f_D, f_B, B_D, B_B	$\overline{D}D$ and $\overline{B}B$ mixing	Eichten, Martinelli [11]	1-2
$D \to Ke\nu$, $(B \to \pi e\nu)$, ...	measure V_{cs}, V_{ub}	El Khadra, [12] Sachrajda [13]	1-2
$D \to K\pi$	check	Sachrajda, Simone	1
K decay and mixing amplitudes			
B_K	extract δ from ϵ	Bernard, Kilcup, [14] Martinelli	3
$K \to \pi\pi$ ($\Delta I = 1/2$ rule)	check	Bernard, Kilcup, Martinelli	2
ϵ'	over-determine δ	Kilcup, Bernard	2

Table 1: Work done on weak matrix elements in the year preceding September 1989

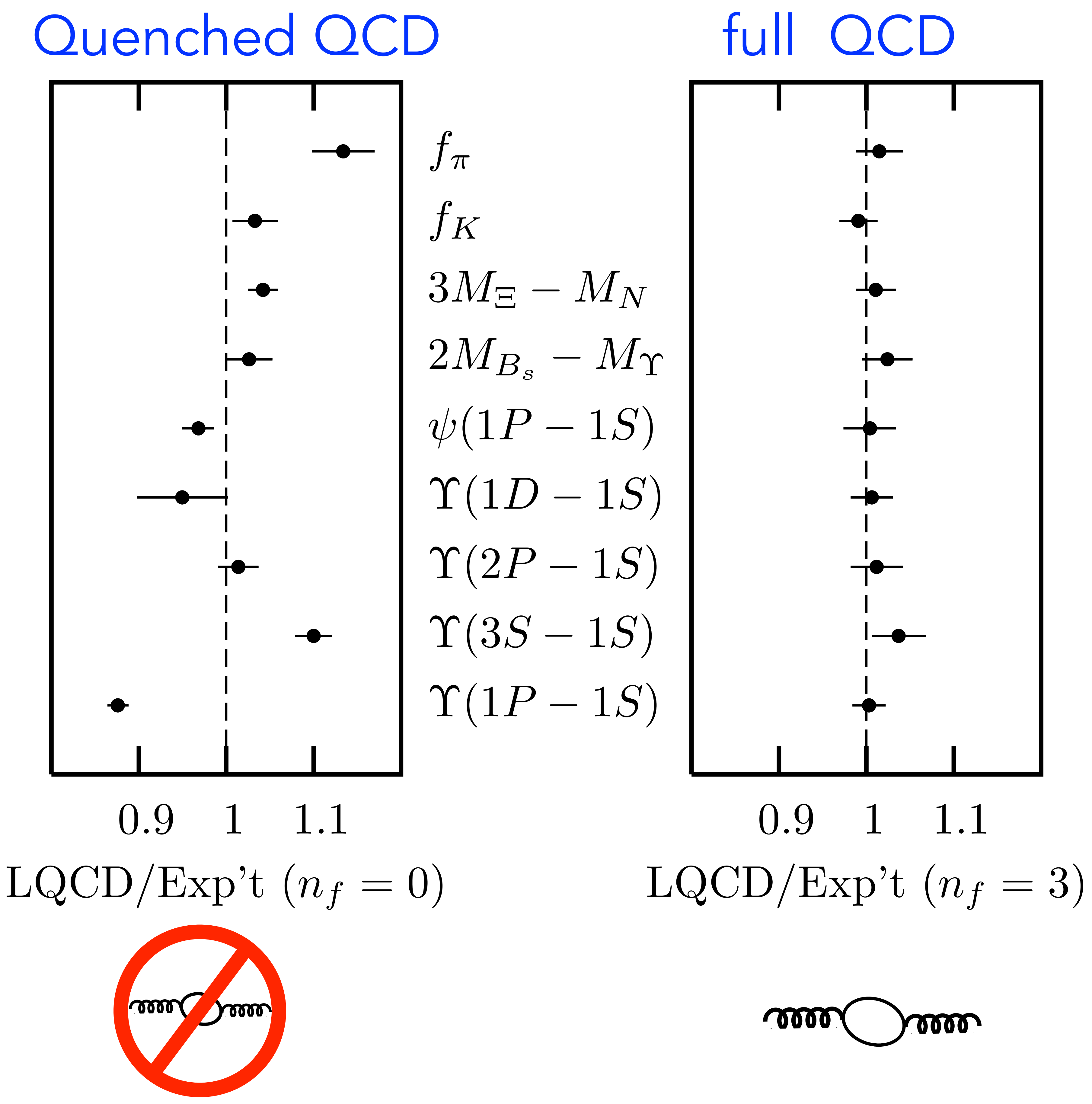
All lattice QCD simulations use the quenched approximation: $n_f = 0$



2003-2005: first "realistic" lattice QCD results

based on simulations with three flavors of sea quarks ($n_f = 2 + 1$):

C. Davies et al [HPQCD, MILC, Fermilab Lattice, hep-lat/0304004, 2004 PRL]



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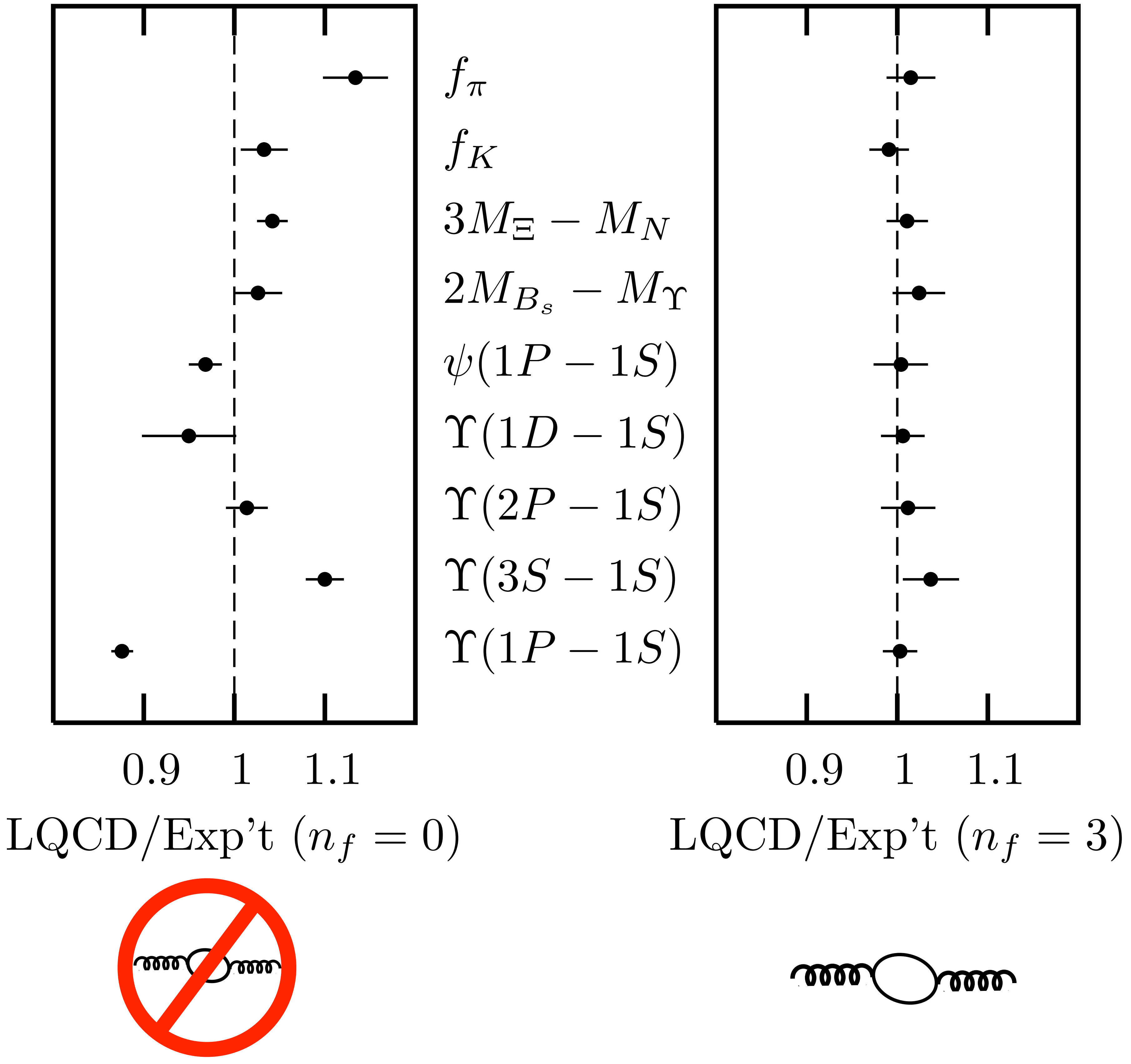
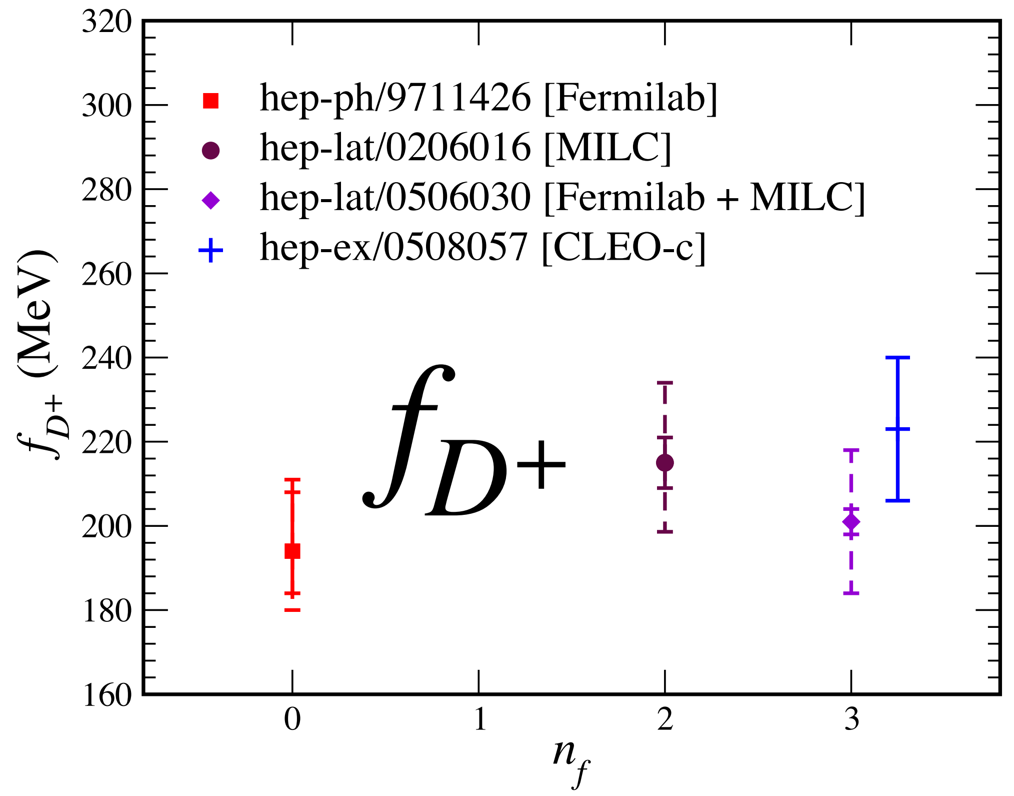
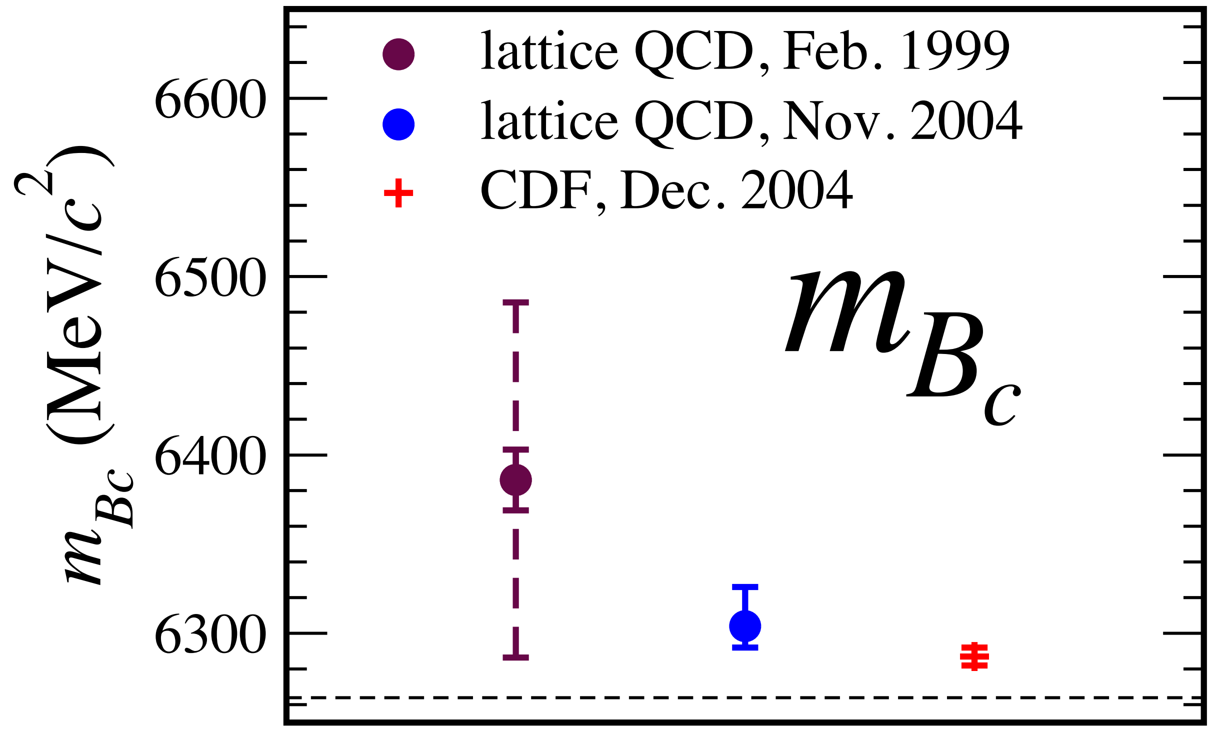
A. Kronfeld et al [Fermilab Lattice, MILC, HPQCD, hep-lat/0509169,Int.J.Mod.Phys 2006]

Quenched QCD   full QCD

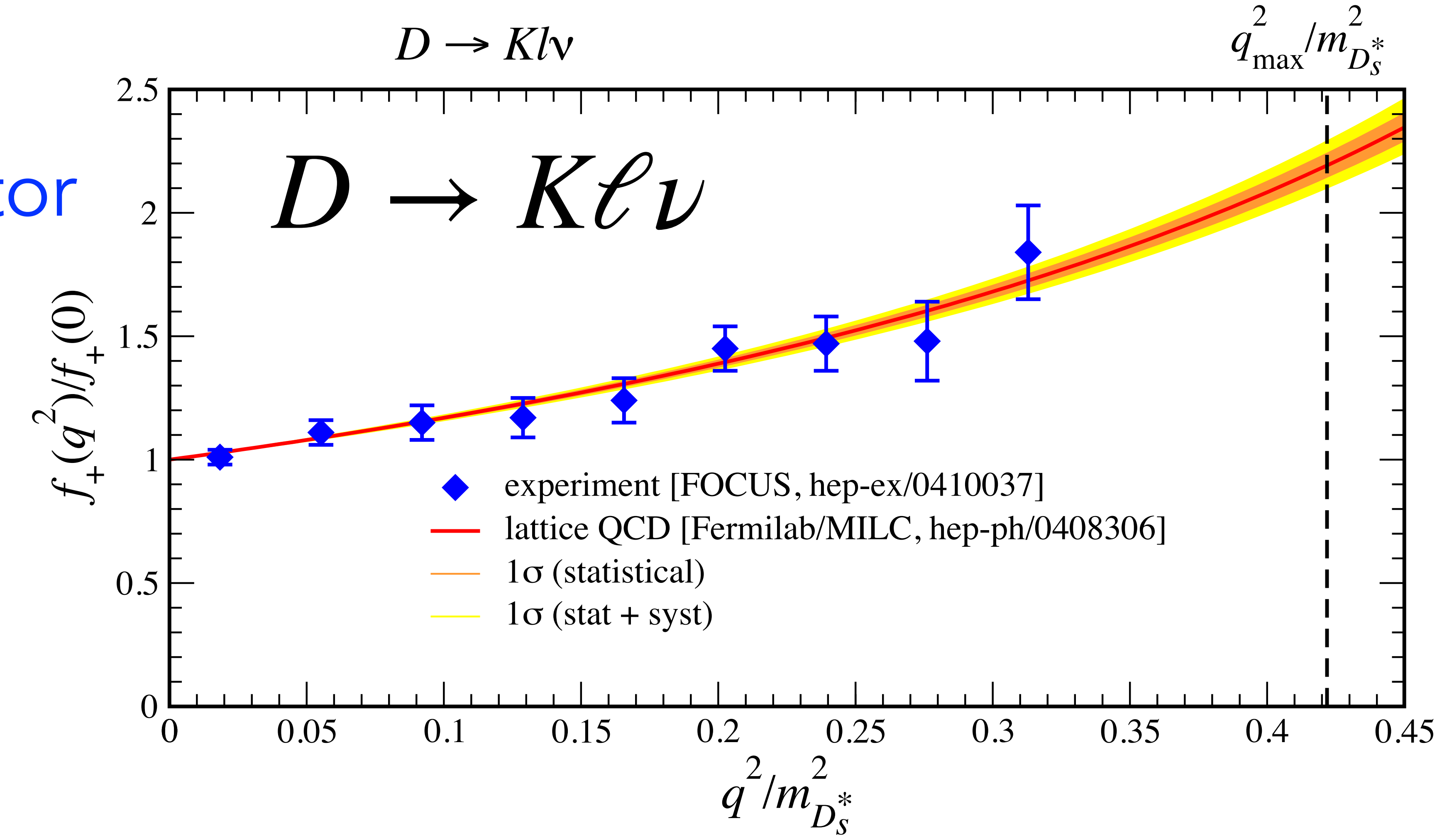


First lattice QCD *predictions,* confirmed by experiment:





shape of form factor



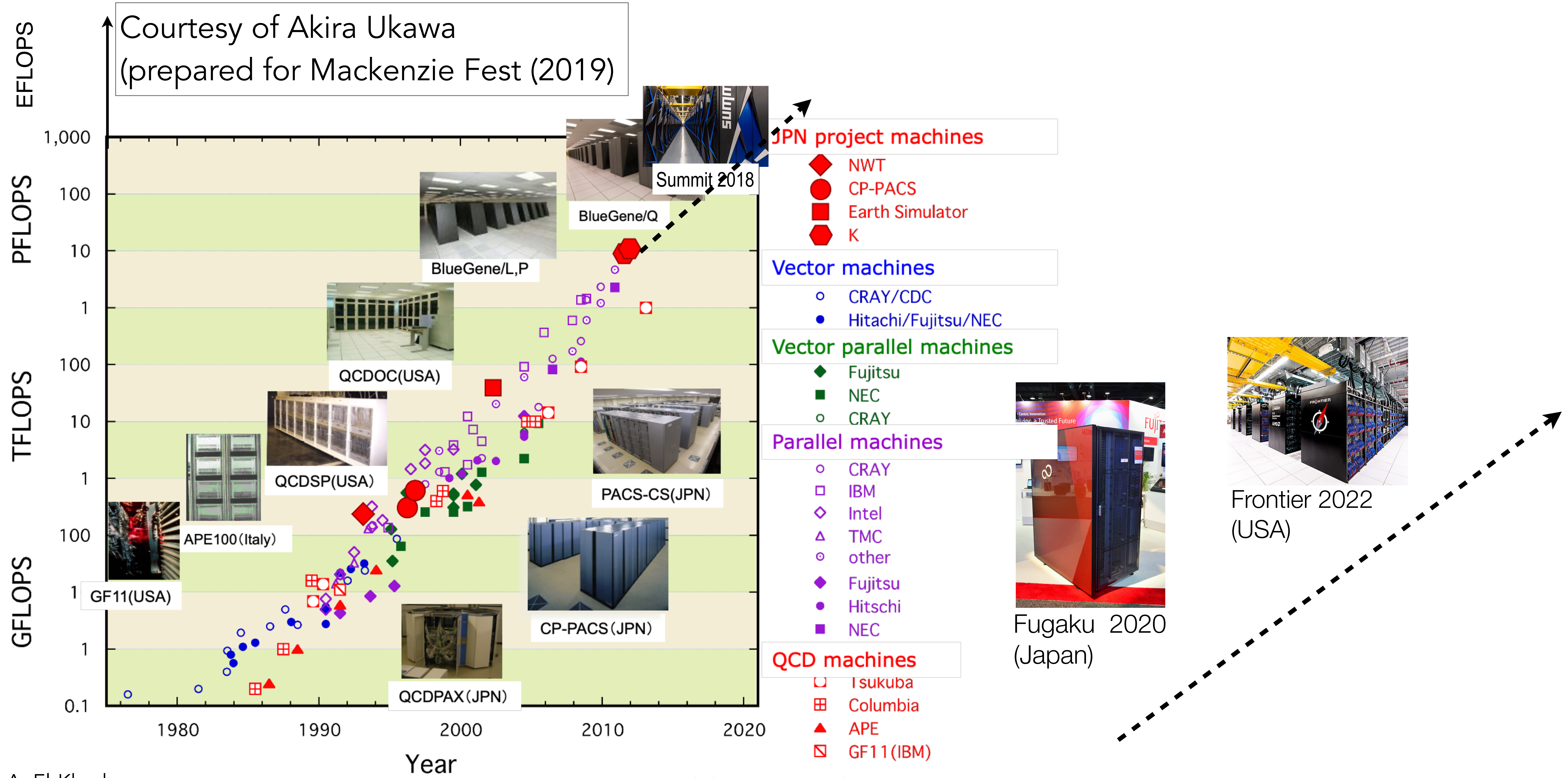
Snowmass 2013 ➠ present

https://www.usqcd.org/documents/13flavor.pdf and [J. Butler et al, arXiv:1311.1076]

Quantity	CKM element	2013 expt. error	2007 forecast lattice error	2013 lattice error	2018 forecast lattice error	2021 FLAG Average
f_K/f_π	$\|V_{us}\|$	0.2%	0.5%	0.4%	0.15%	0.18 %
$f_+^{K\pi}(0)$	$\|V_{us}\|$	0.2%	–	0.4%	0.2%	0.18 %
f_D	$\|V_{cd}\|$	4.3%	5%	2%	$< 1\%$	0.3 %
f_{D_s}	$\|V_{cs}\|$	2.1%	5%	2%	$< 1\%$	0.2 %
$D \to \pi\ell\nu$	$\|V_{cd}\|$	2.6%	–	4.4%	2%	0.7 % [from 2212.12648]
$D \to K\ell\nu$	$\|V_{cs}\|$	1.1%	–	2.5%	1%	0.6 %
$B \to D^*\ell\nu$	$\|V_{cb}\|$	1.3%	–	1.8%	$< 1\%$	~1.5 % [from 2105.14019, 2304.03137, 2306.05657]
$B \to \pi\ell\nu$	$\|V_{ub}\|$	4.1%	–	8.7%	2%	~3 %
f_B	$\|V_{ub}\|$	9%	–	2.5%	$< 1\%$	0.7 % (0.6 % for f_{B_s})
ξ	$\|V_{ts}/V_{td}\|$	0.4%	2–4%	4%	$< 1\%$	1.3 %
Δm_s	$\|V_{ts}V_{tb}\|^2$	0.24%	7–12%	11%	5%	4.5 %
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	$< 1\%$	1.3 %

QED corrections dominant source of theory error

Timeline of computational resources



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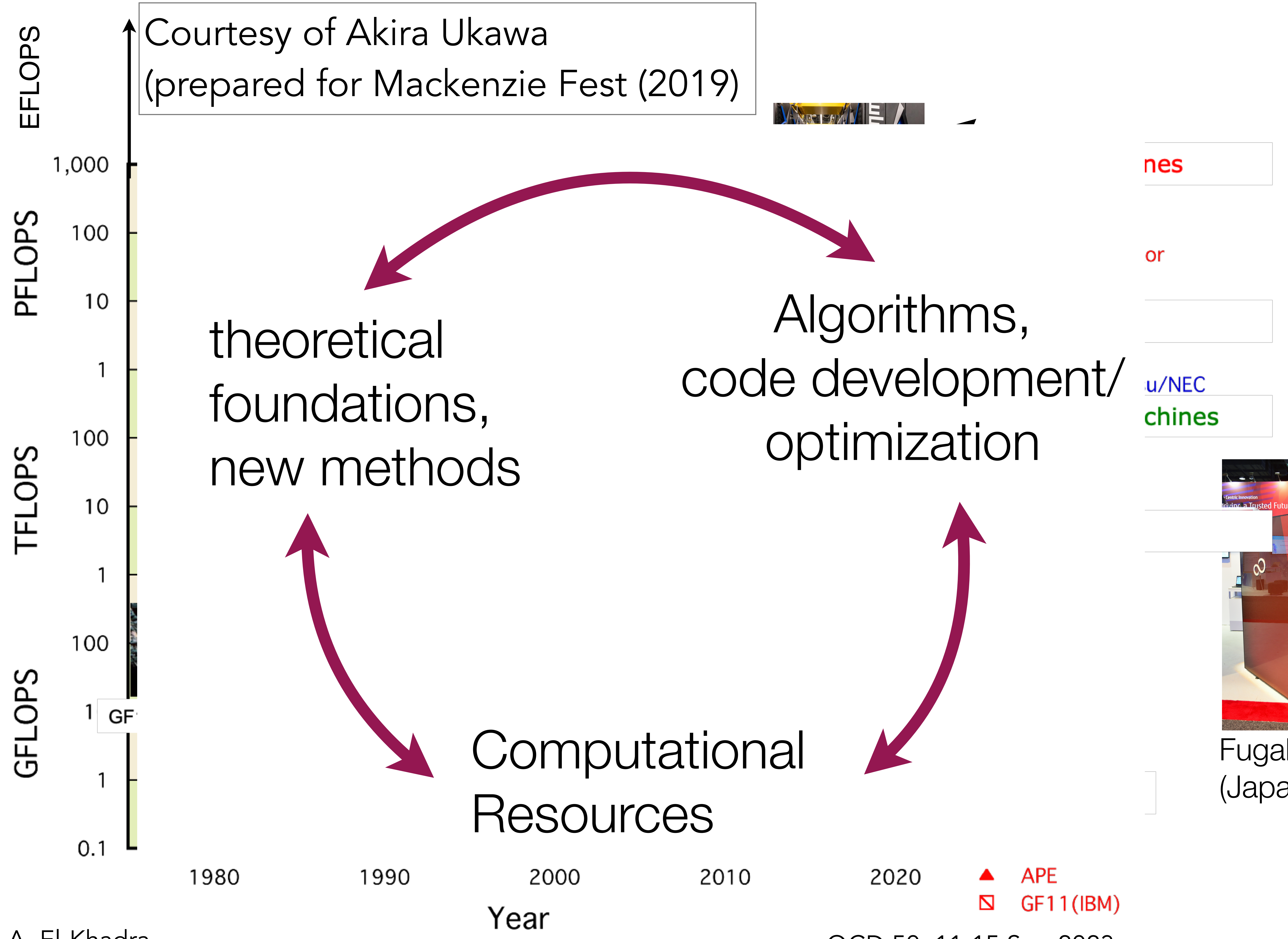
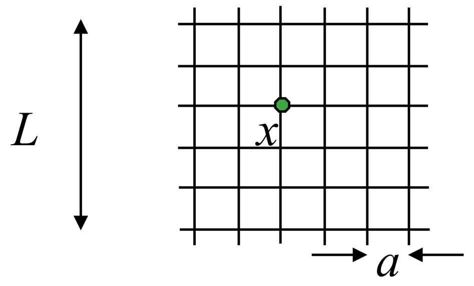


Fugaku 2020 (Japan)

Frontier 2022 (USA)



Lattice QCD Introduction

The State of the Art

Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,…) in some cases with

- sub percent precision.
- total errors that are commensurate (or smaller) than corresponding experimental uncertainties.

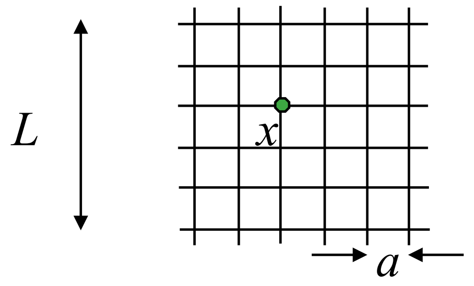
Progress due to a virtuous cycle of theoretical developments, improved algorithms/methods and increases in computational resources (``Moore's law")

FLAG 2021 Flavour Lattice Averaging Group

Flavor Lattice Averaging Group: S. Aoki et al [FLAG 2021 review, arXiv:2111.09849, EPJC 2022]

- quality criteria for inclusion in averages
- consider sys. and stat. error correlations

- reviews over 60 quantities
- ~ biannual schedule + web update



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Scope of LQCD calculations is increasing due to continual development of new methods:

- nucleon matrix elements
- nonleptonic kaon decays ($K \to \pi\pi$, ϵ',…)
- resonances, scattering ($\pi\pi \to \rho$,…)
- long-distance effects (ΔM_K, …)
- QED corrections
- radiative decay rates
- structure: PDFs, GPDs, TMDs, …
- inclusive decay rates ($B \to X_c \ell \nu$,…)
- …

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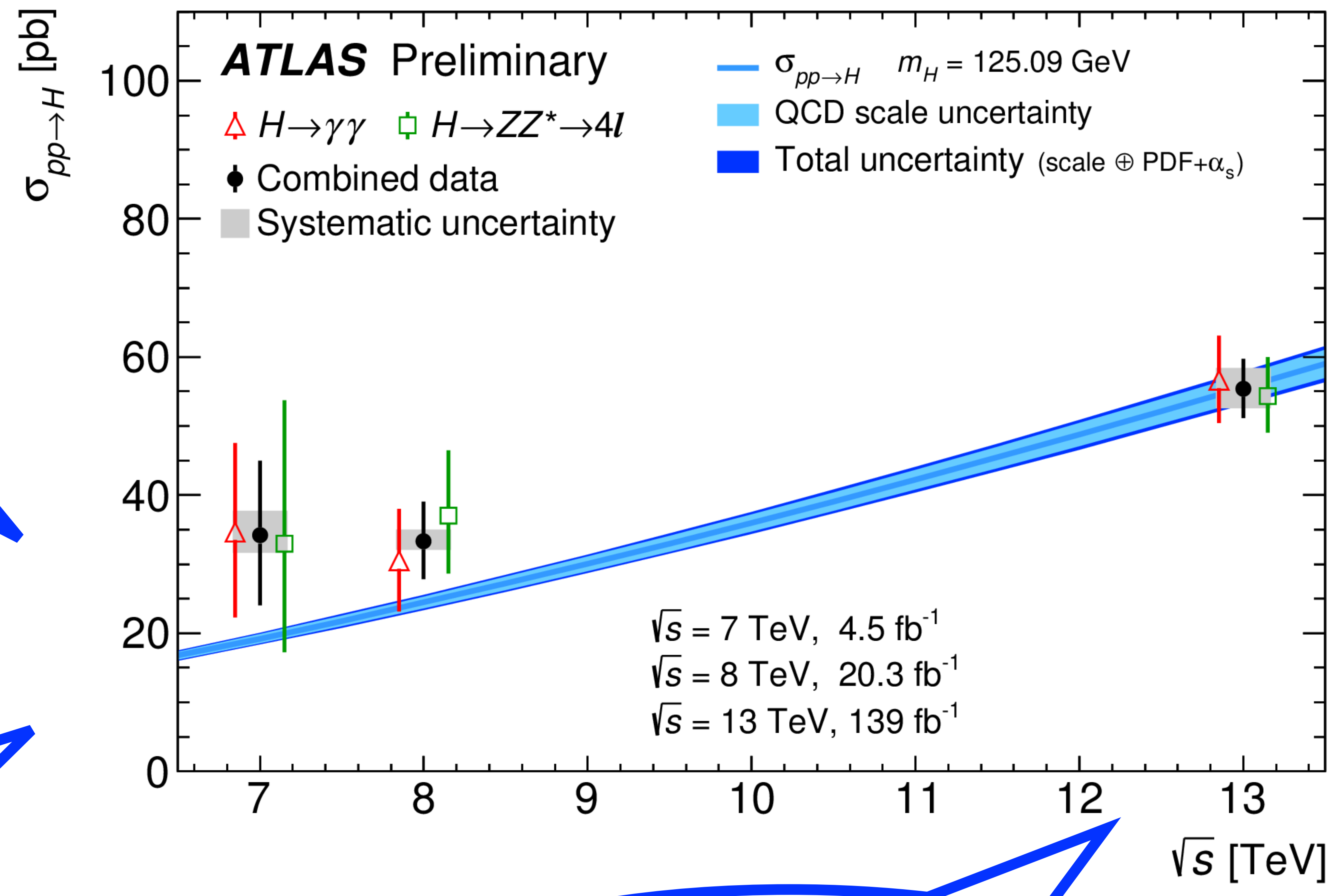
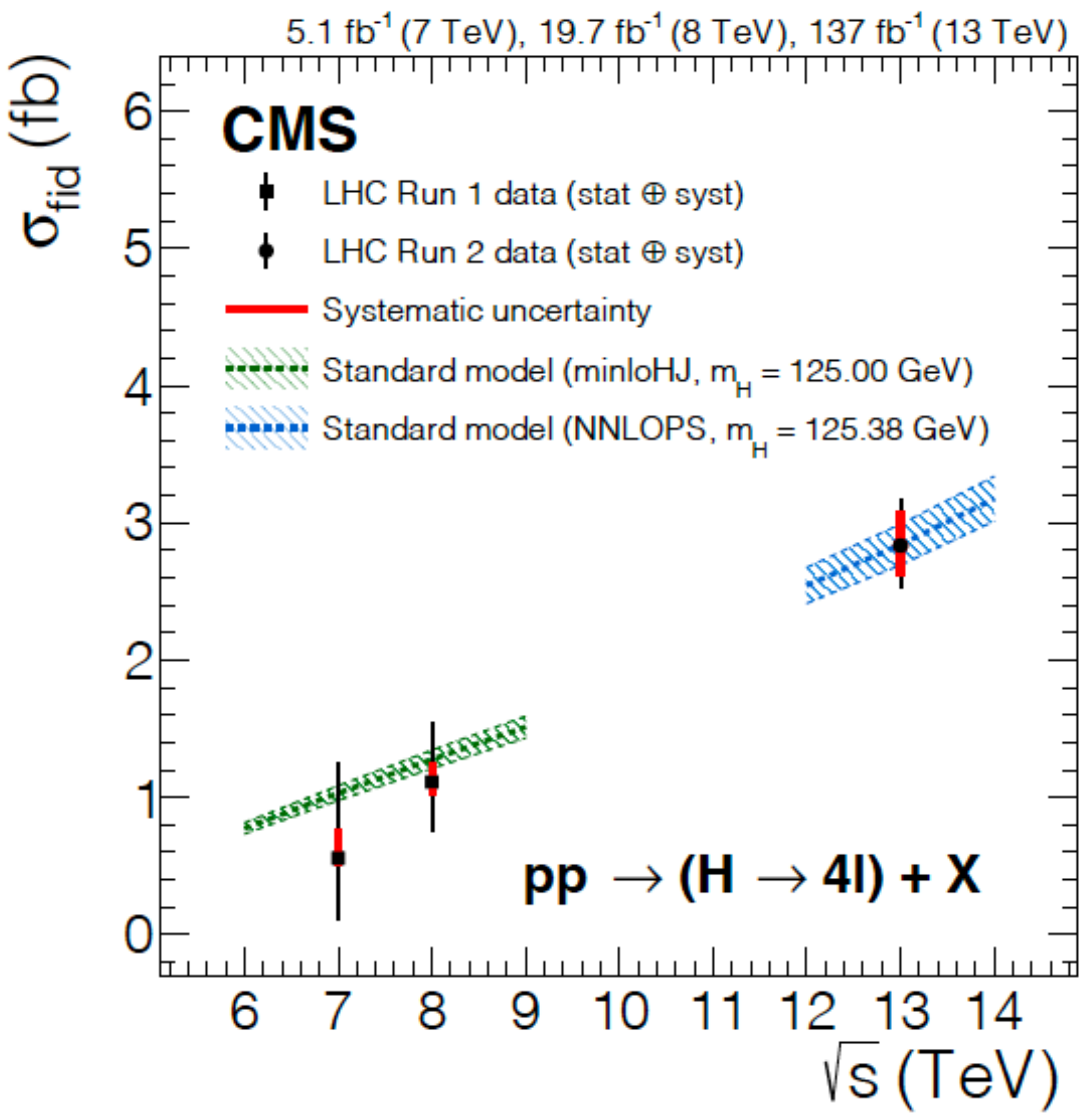
Higgs production and decay

Radja Boughezal @ P5 SLAC town hall

The computation of the Higgs cross sections and decay modes is an excellent example that highlights all of the theoretical advances needed to maximize the potential of the LHC program.

Electroweak corrections at 2 loops





QCD@N3LO

PDFs@NNLO

Precision determination of quark masses

Precision extraction of α_S

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[Radja Boughezal @ P5 SLAC town hall]

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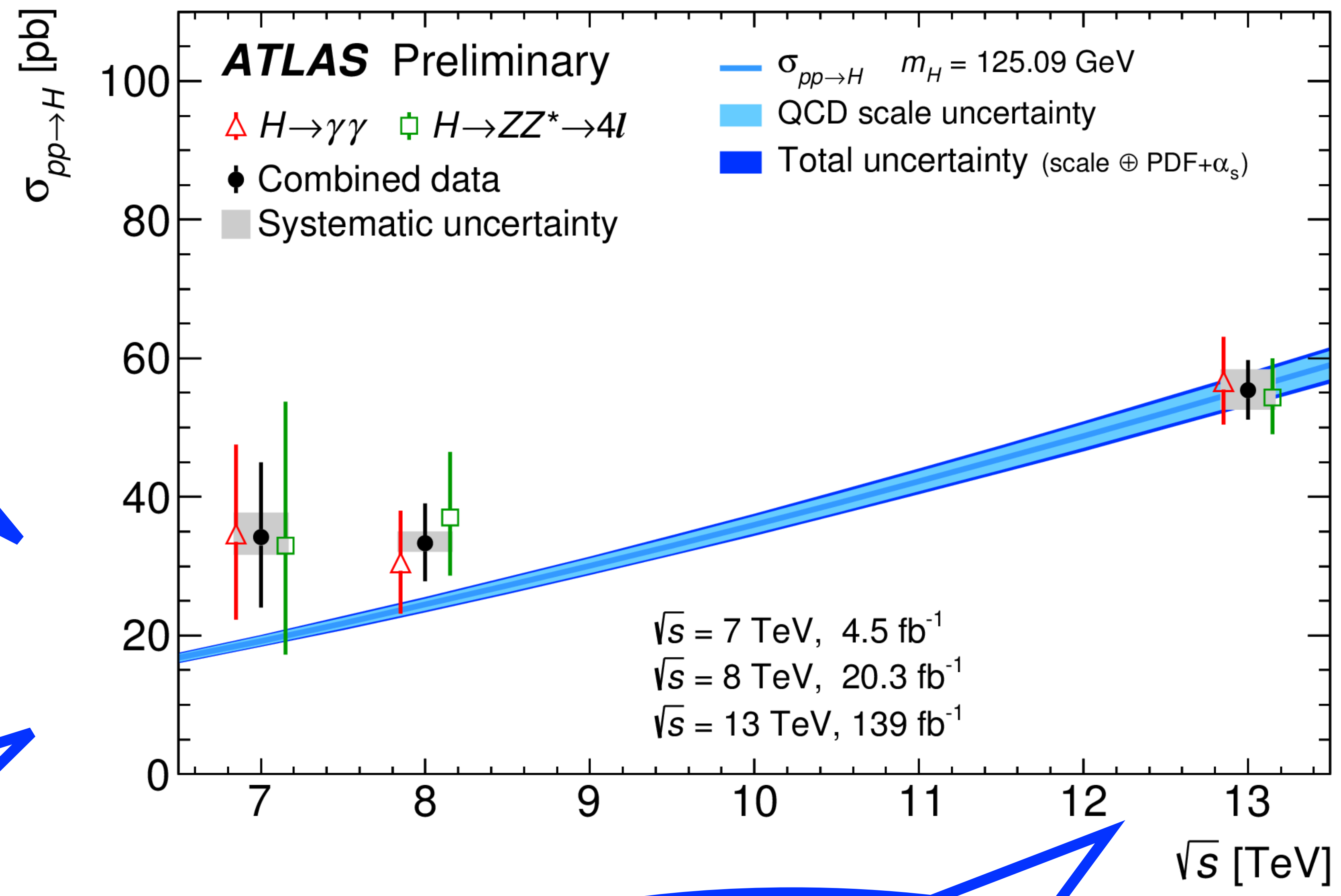

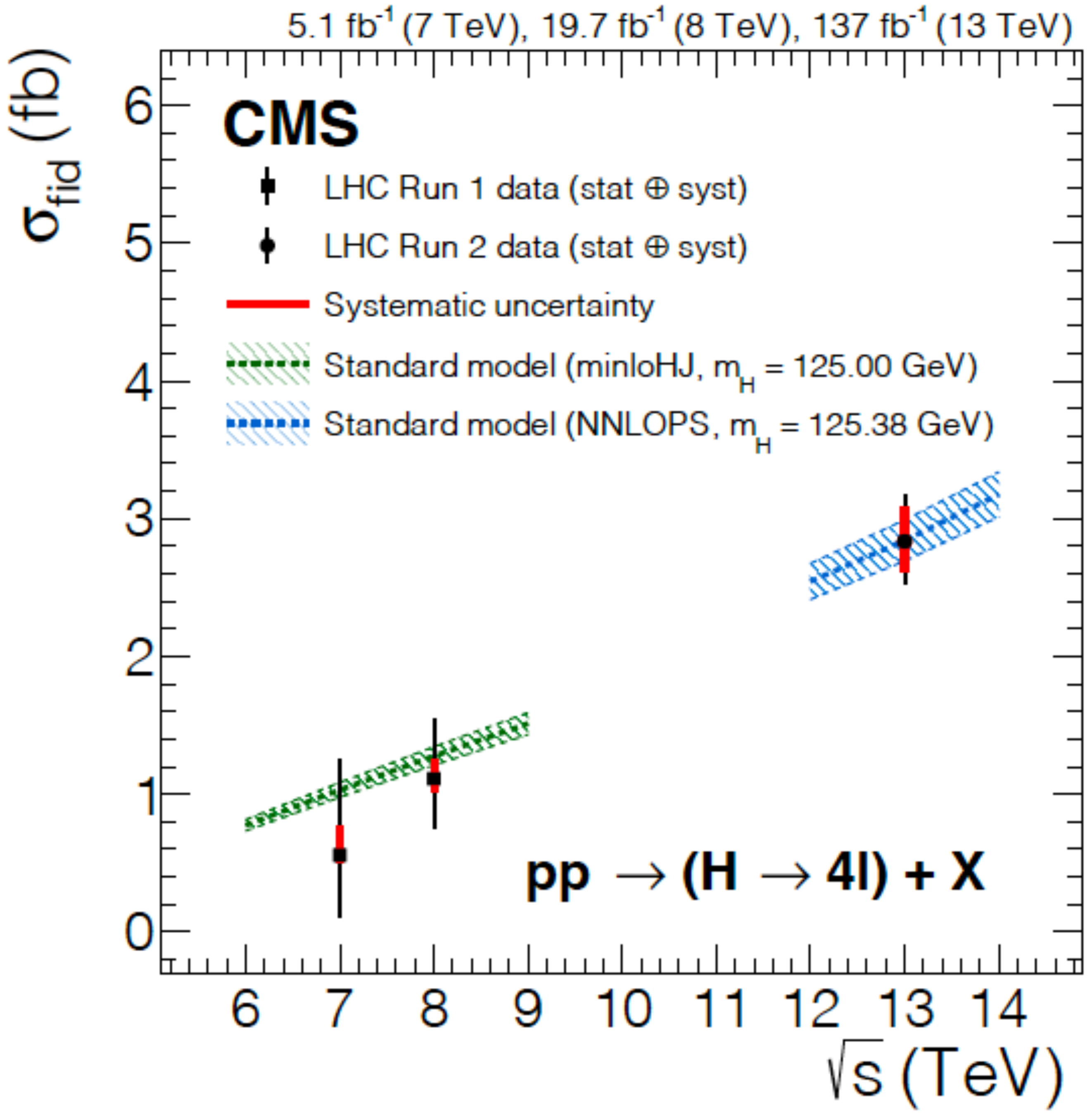

QCD@N3LO

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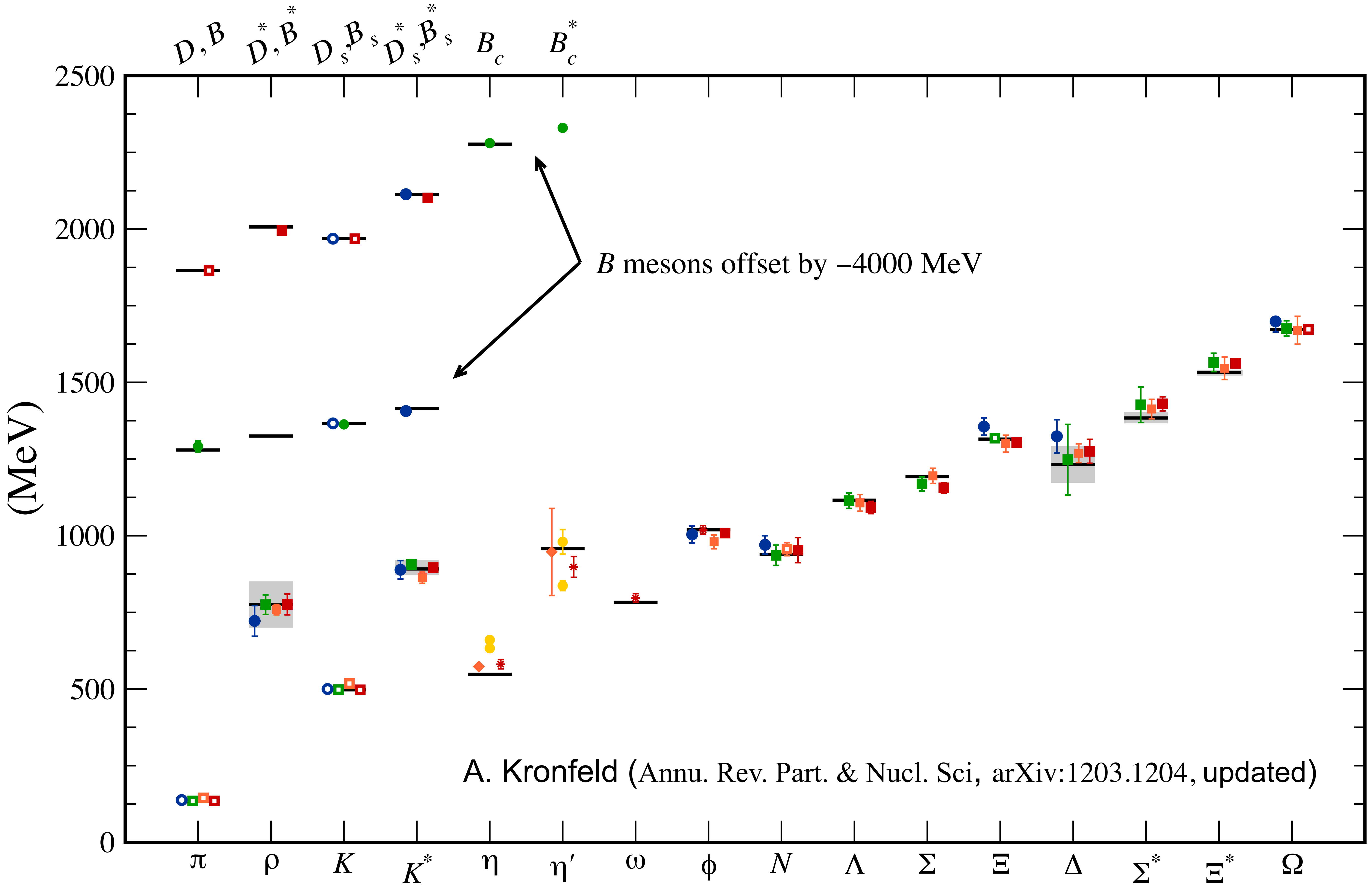
Future lattice QCD inputs

Lattice QCD inputs

quark masses and α_s

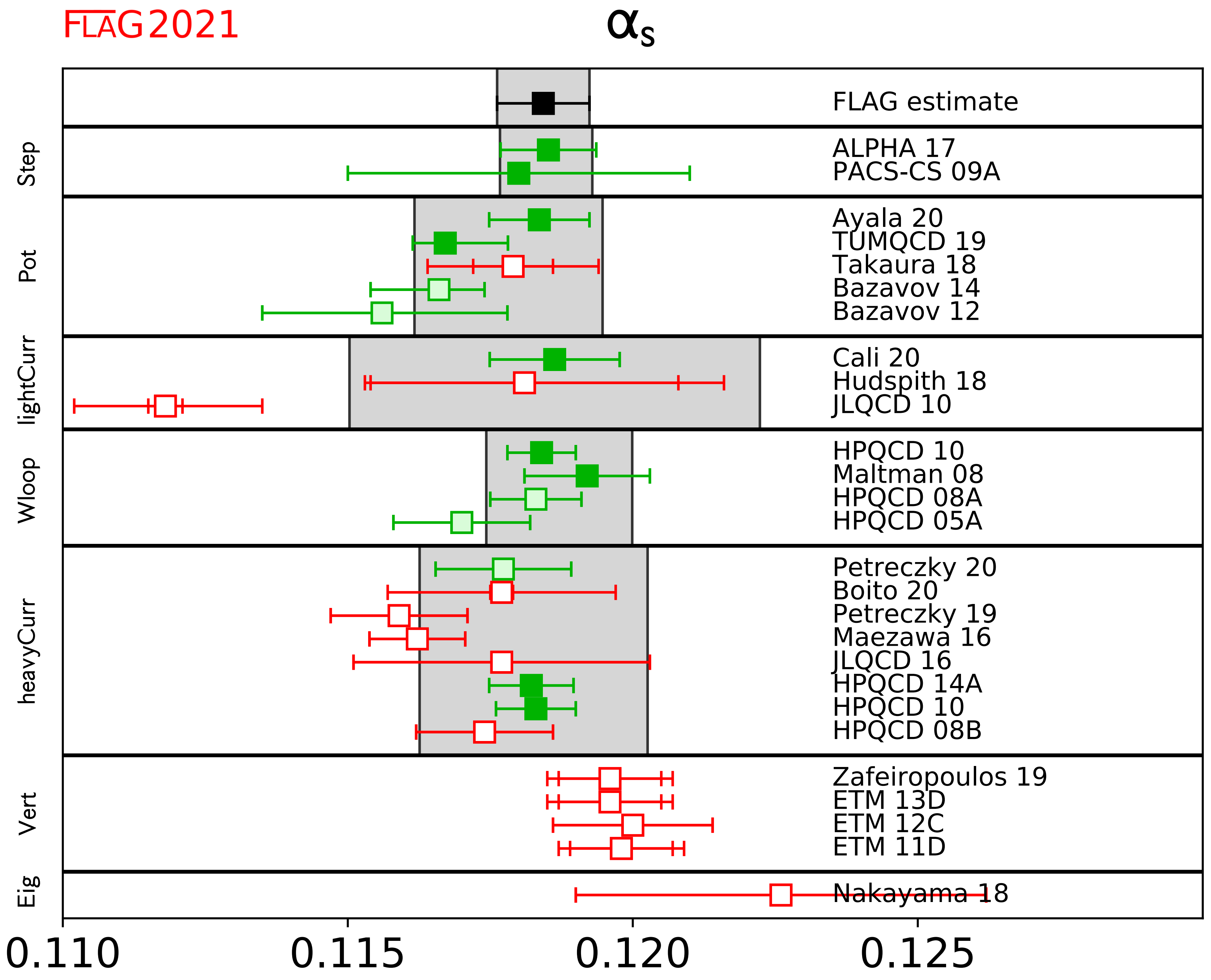
- Inputs to the (lattice) QCD lagrangian
- **bare** quark masses, m_{ud}, m_s, m_c, m_b: fixed with exp. measured hadron masses, e.g., $M_\pi, M_K, M_{D_s}, M_{B_s}$
- lattice spacing in physical units (scale setting): f_π (or M_Ω or …) ➠ α_s

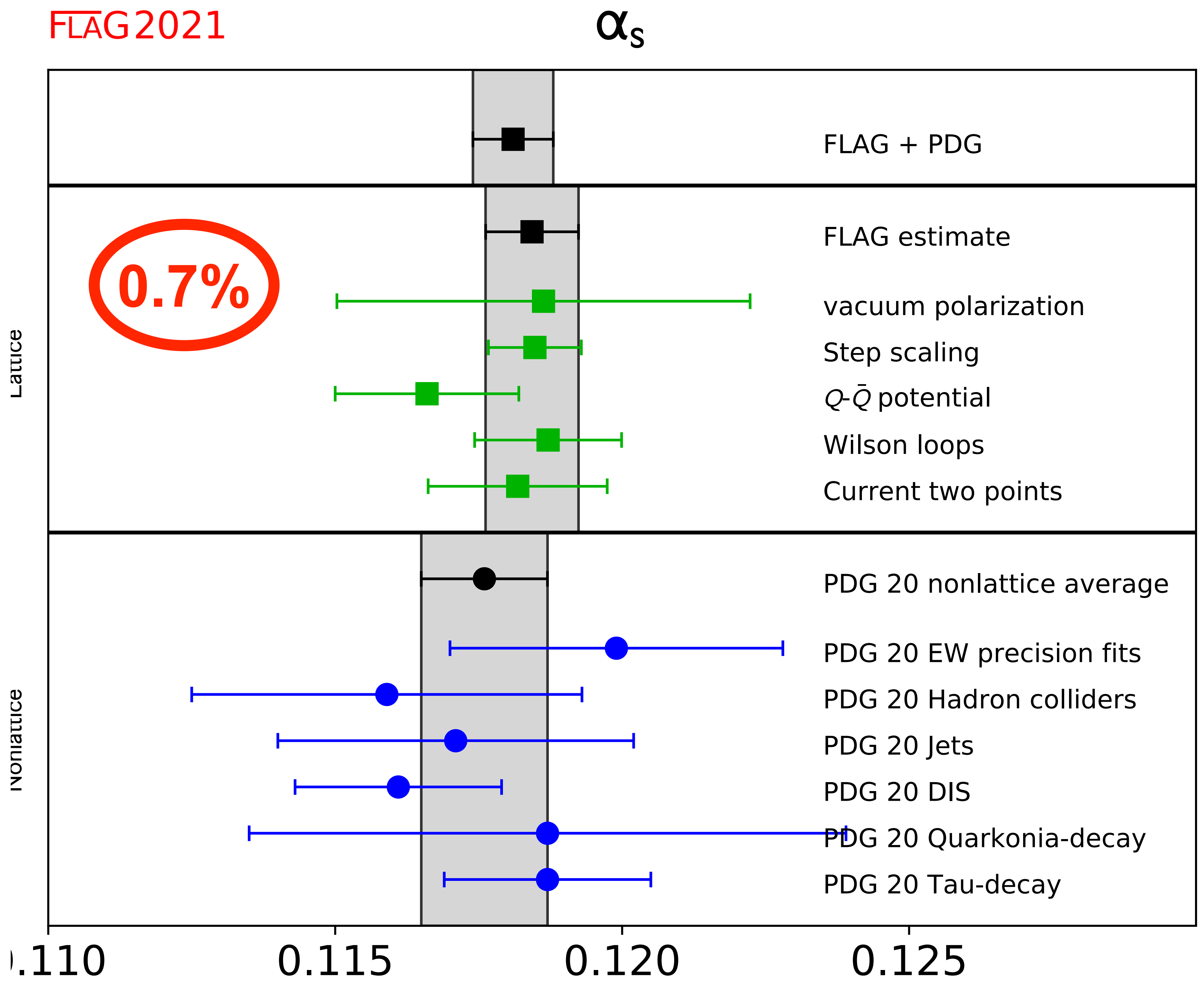


- all other quantities are pre/post dictions that can be compared to experiment.
- determinations of **renormalized** α_s from many different observables/methods: Wilson loops, current correlators, HQ potential, step scaling,…
- m_q:
  different intermediate renormalization schemes (nonperturbative or perturbative) before matching to $\overline{MS}$

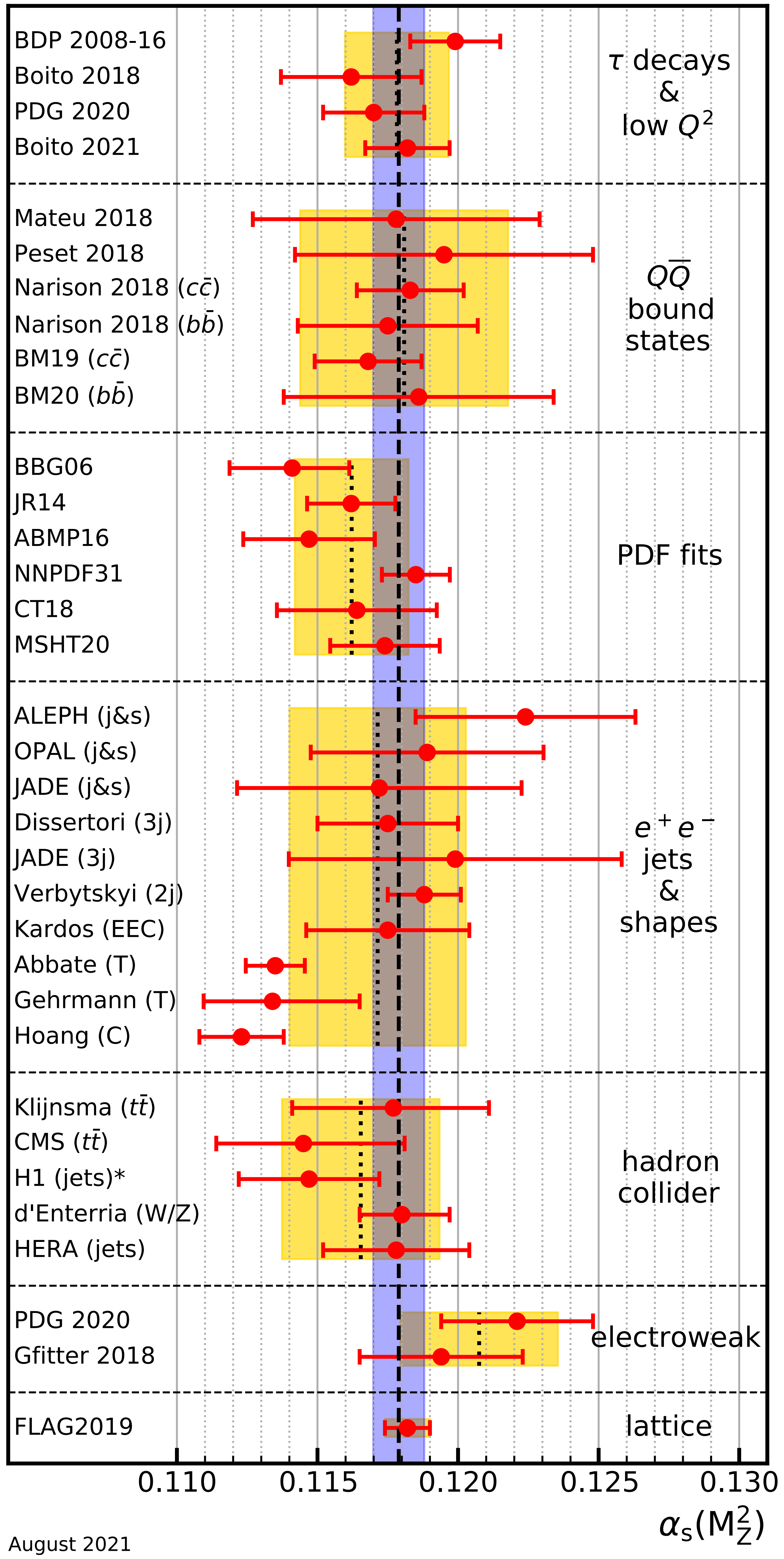
α_s

S. Aoki et al [FLAG 2021 review, arXiv:2111.09849, EPJC 2022]

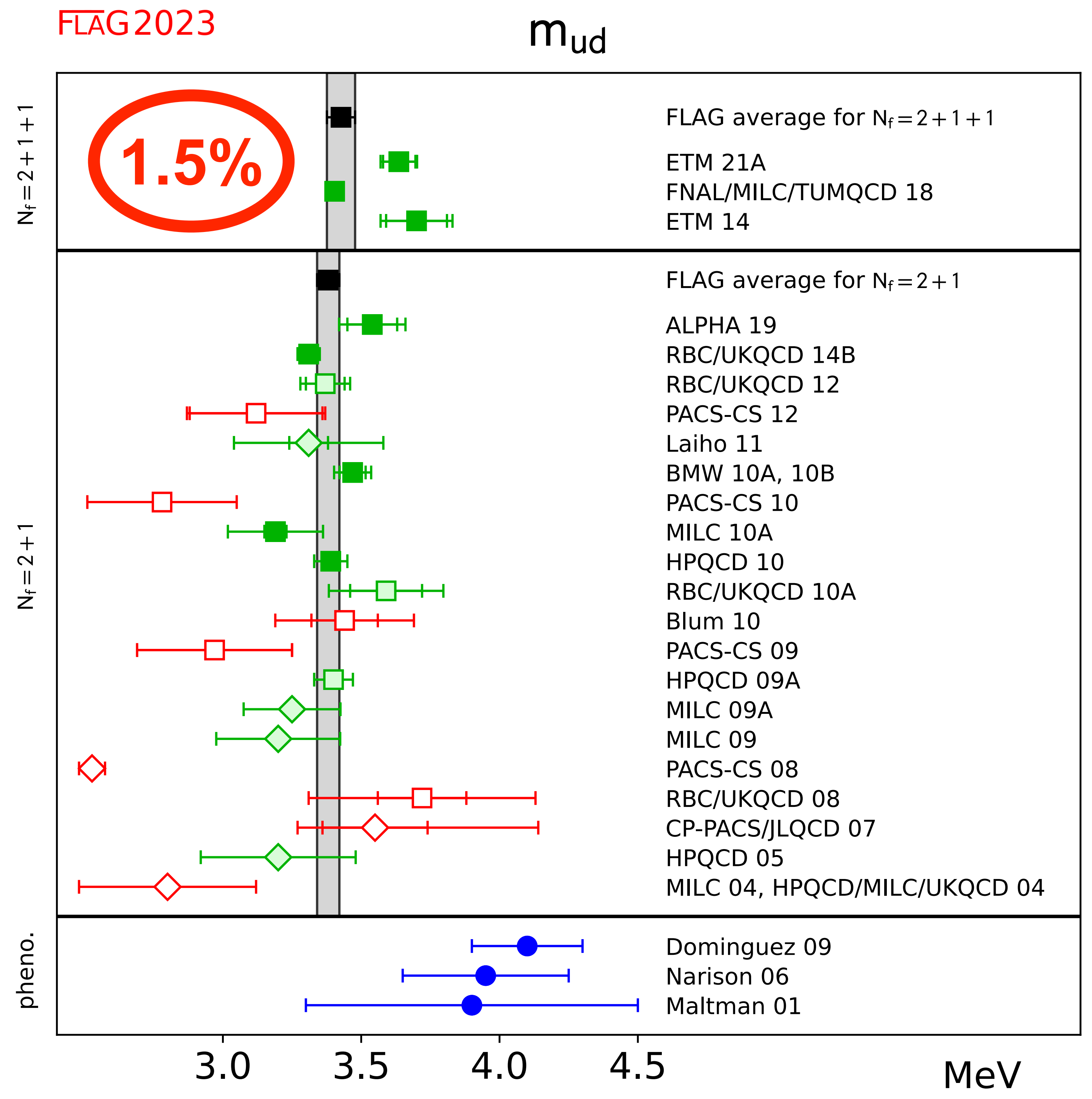


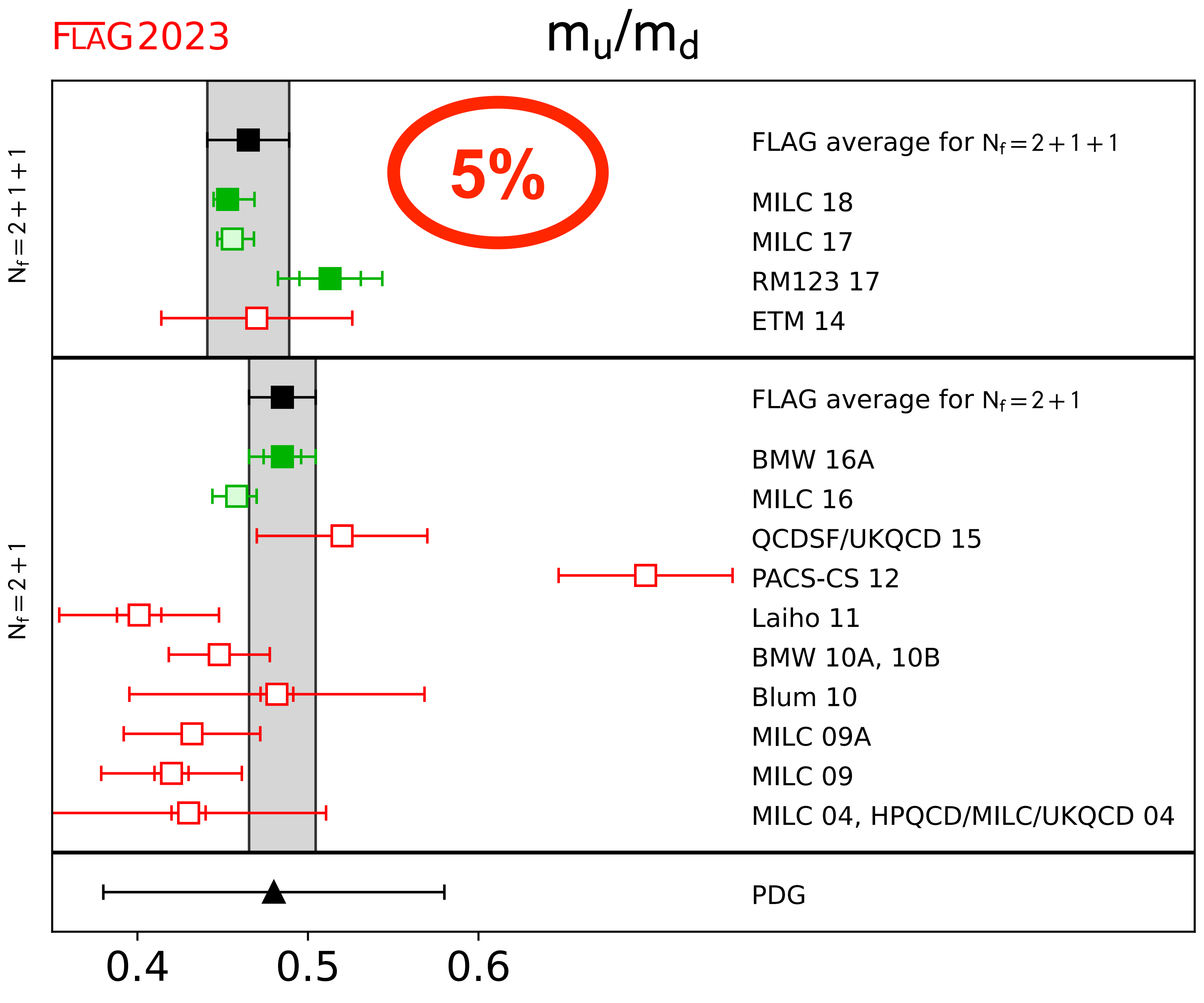


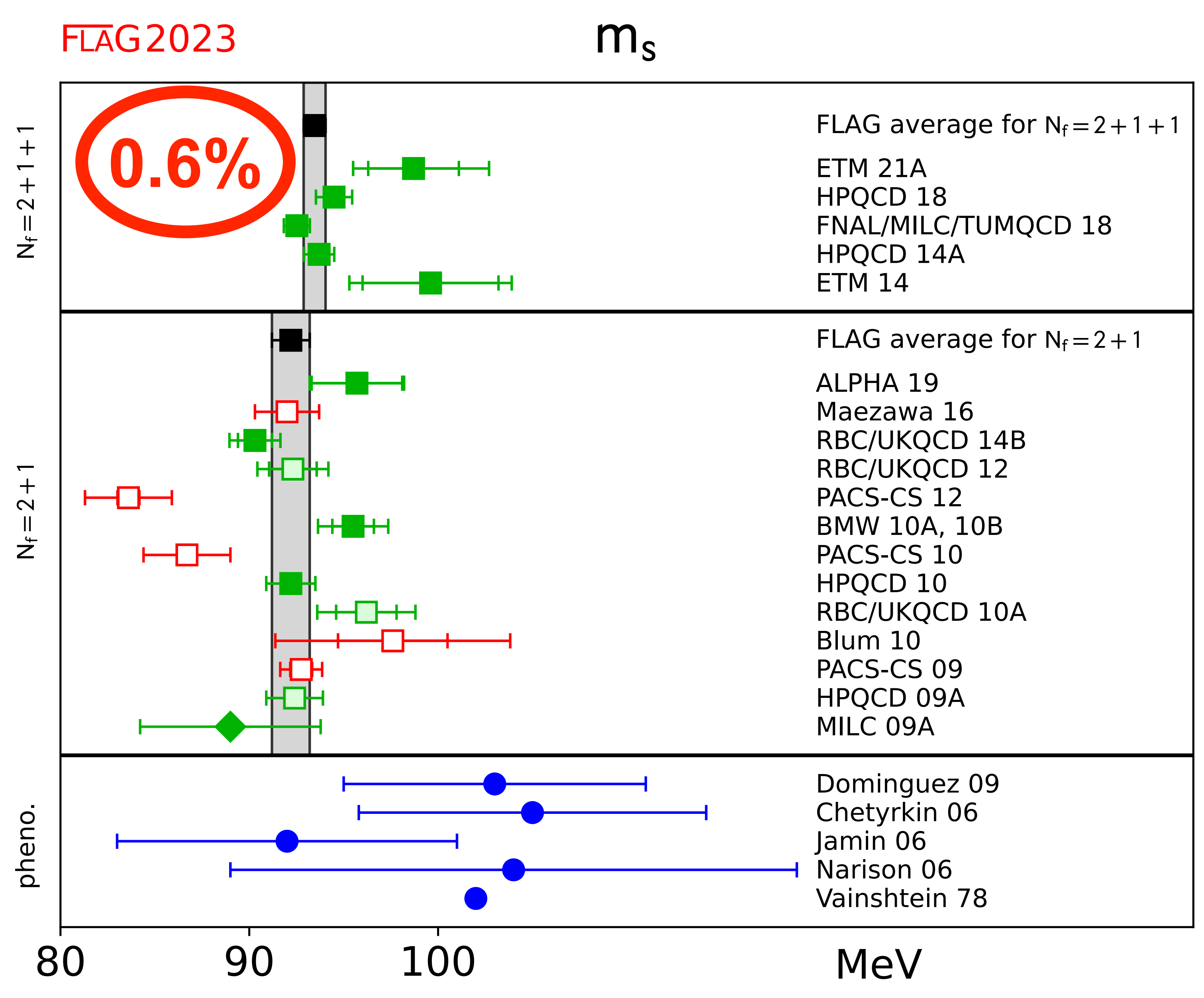
J. Huston, K. Rabbertz, G. Zanderighi [PDG QCD review]



quark masses

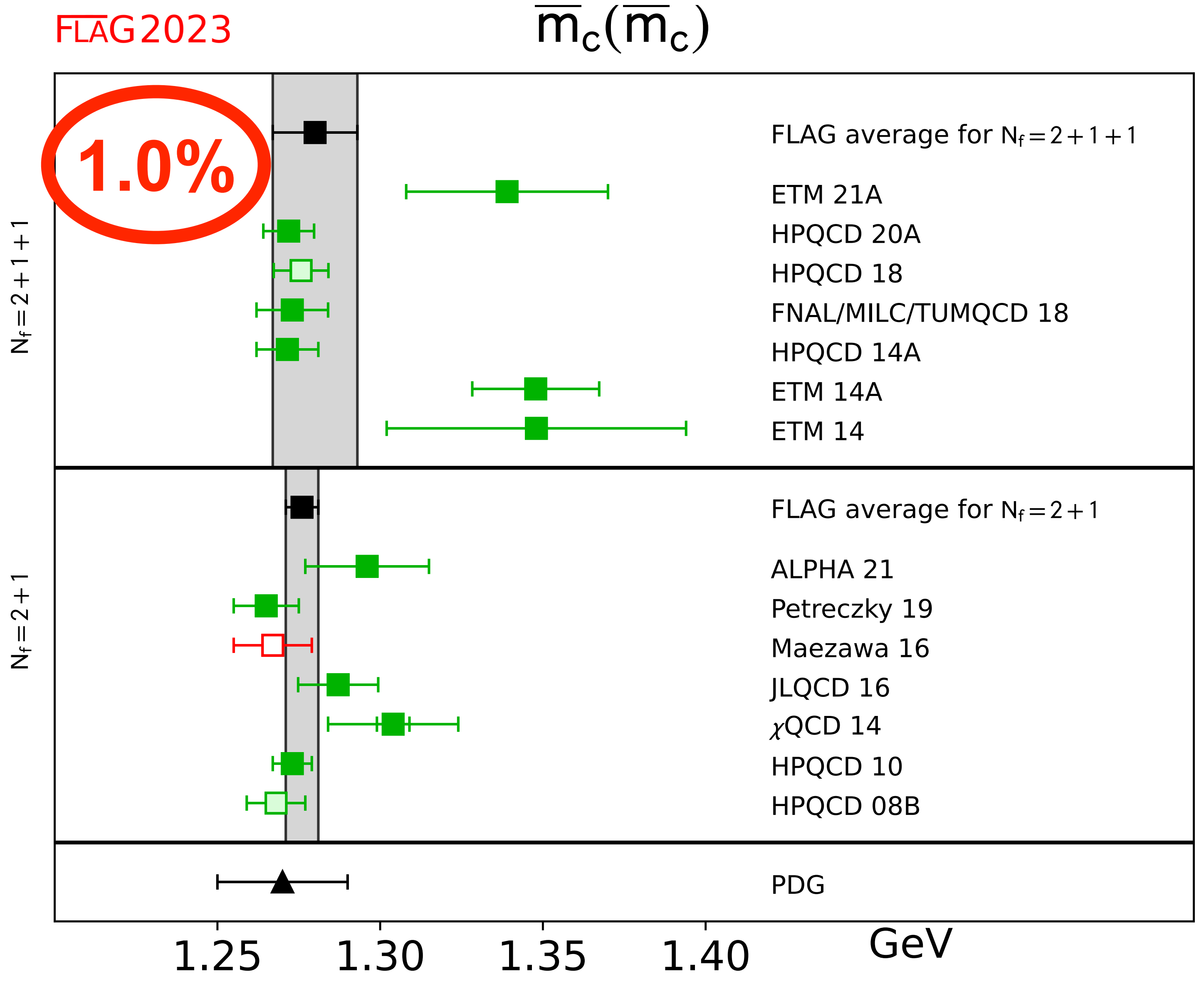


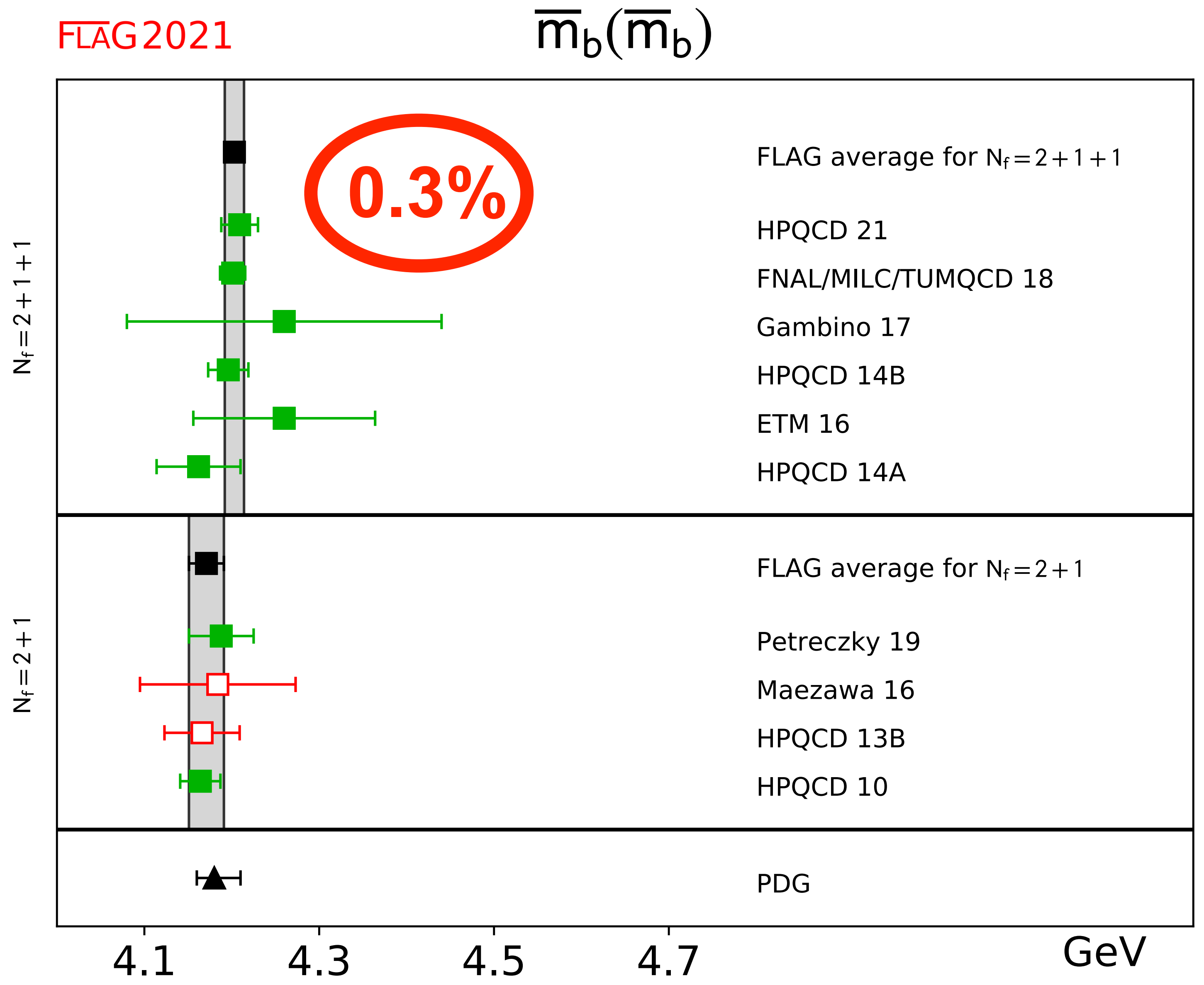




S. Aoki et al [FLAG 2021 review, arXiv:2111.09849, EPJC 2022]

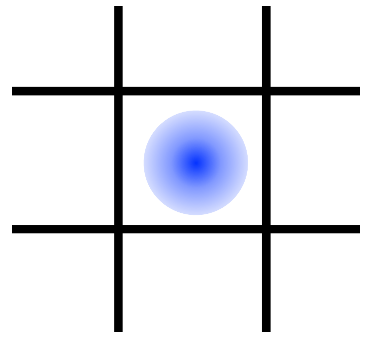
PDG quark mass listings are (still) outdated.





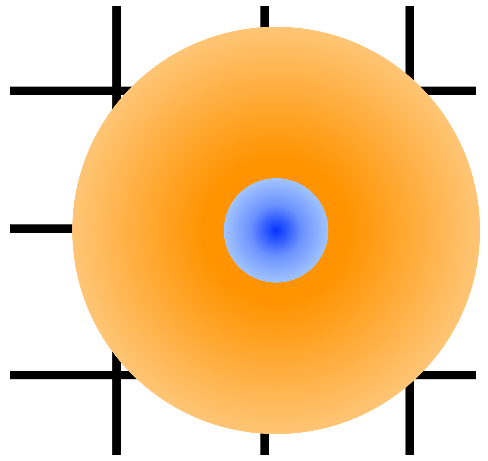
Finding Beauty

b quark



$m_b \gtrsim a^{-1} \gg \Lambda$ ➟ leading discretization errors $\sim (am_b)^2$
(using same action as for light quarks)

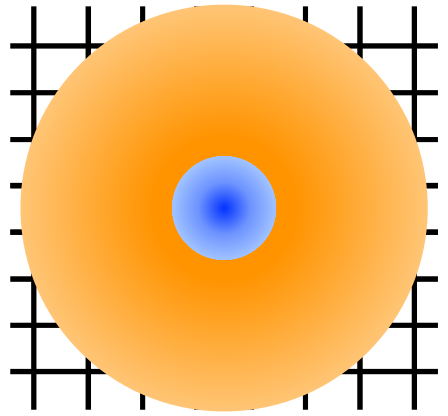
B meson



use EFT (HQET, NRQCD) ➟ Λ/m_b expansion

EFTs co-developed continuum/lattice

- lattice HQET, NRQCD: use EFT to construct lattice action
  complicated continuum limit
  nontrivial matching and renormalization ➟ (few-5)% errors
- relativistic heavy quark approach: Fermilab (1996), also Tsukuba (2003), RHQ (2006)
  matching relativistic lattice action via HQET to continuum
  nontrivial matching and renormalization ➟ (1-3)% errors

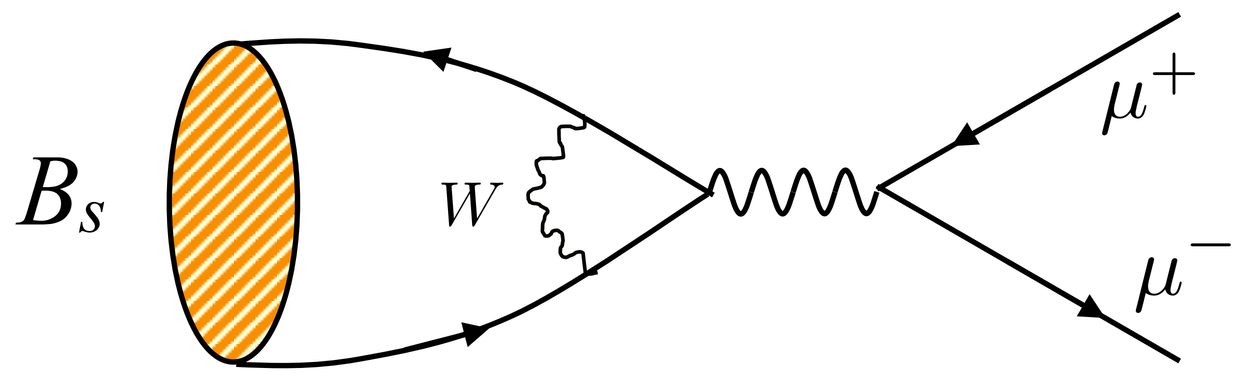


$a^{-1} > m_b \gg \Lambda$ + highly improved light quark action
➟ same action for all quarks
➟ simple renormalization (Ward identities) ➟ < 1% errors

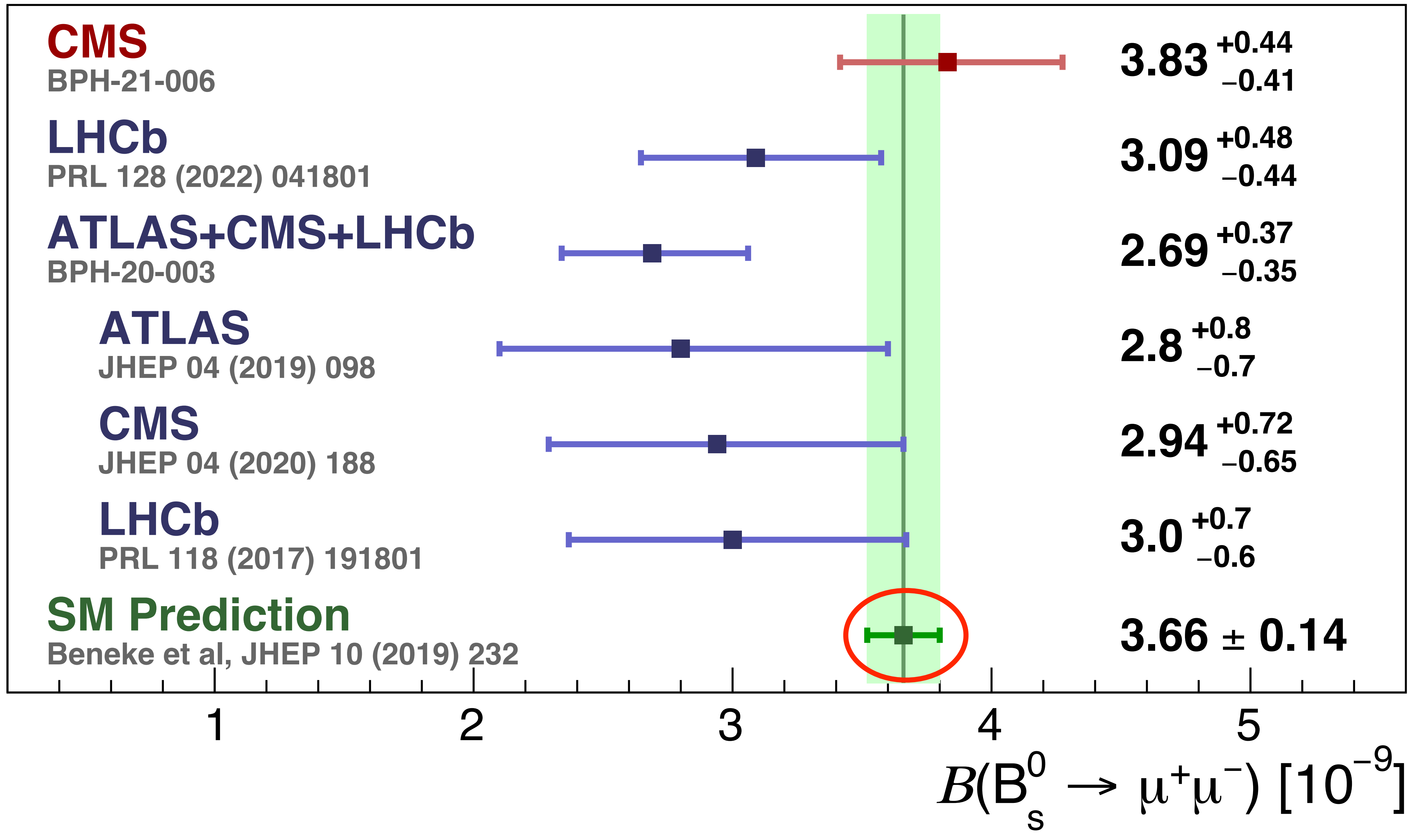
Rare leptonic decay $B_s \to \mu\mu$

SM prediction for rare leptonic decay rate

[Beneke et al, arXiv:1908.07011, JHEP 2019]



Silvano Tosi @ LHCP2023



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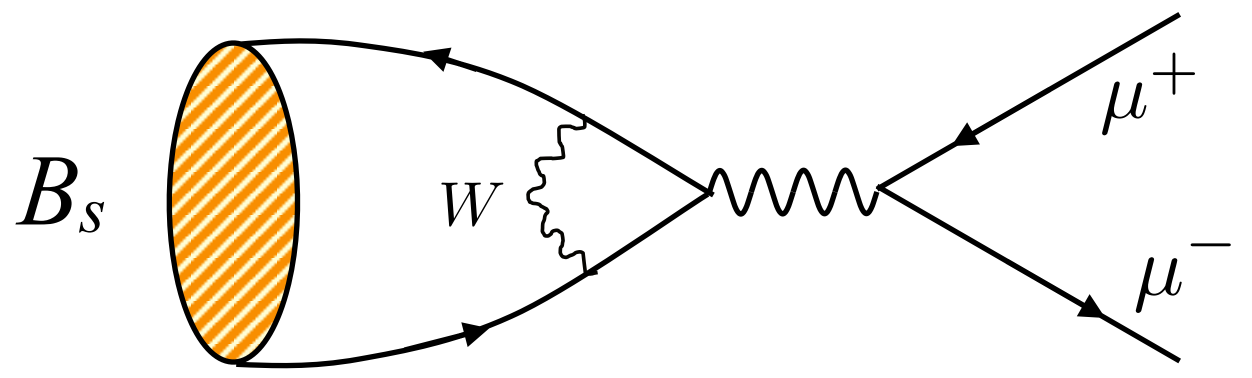
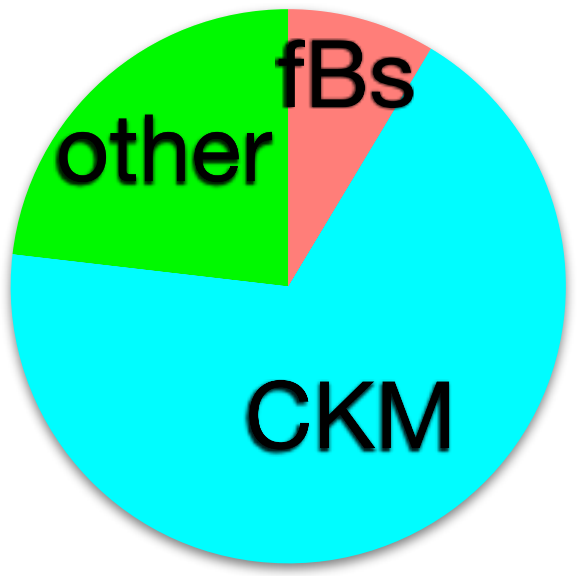
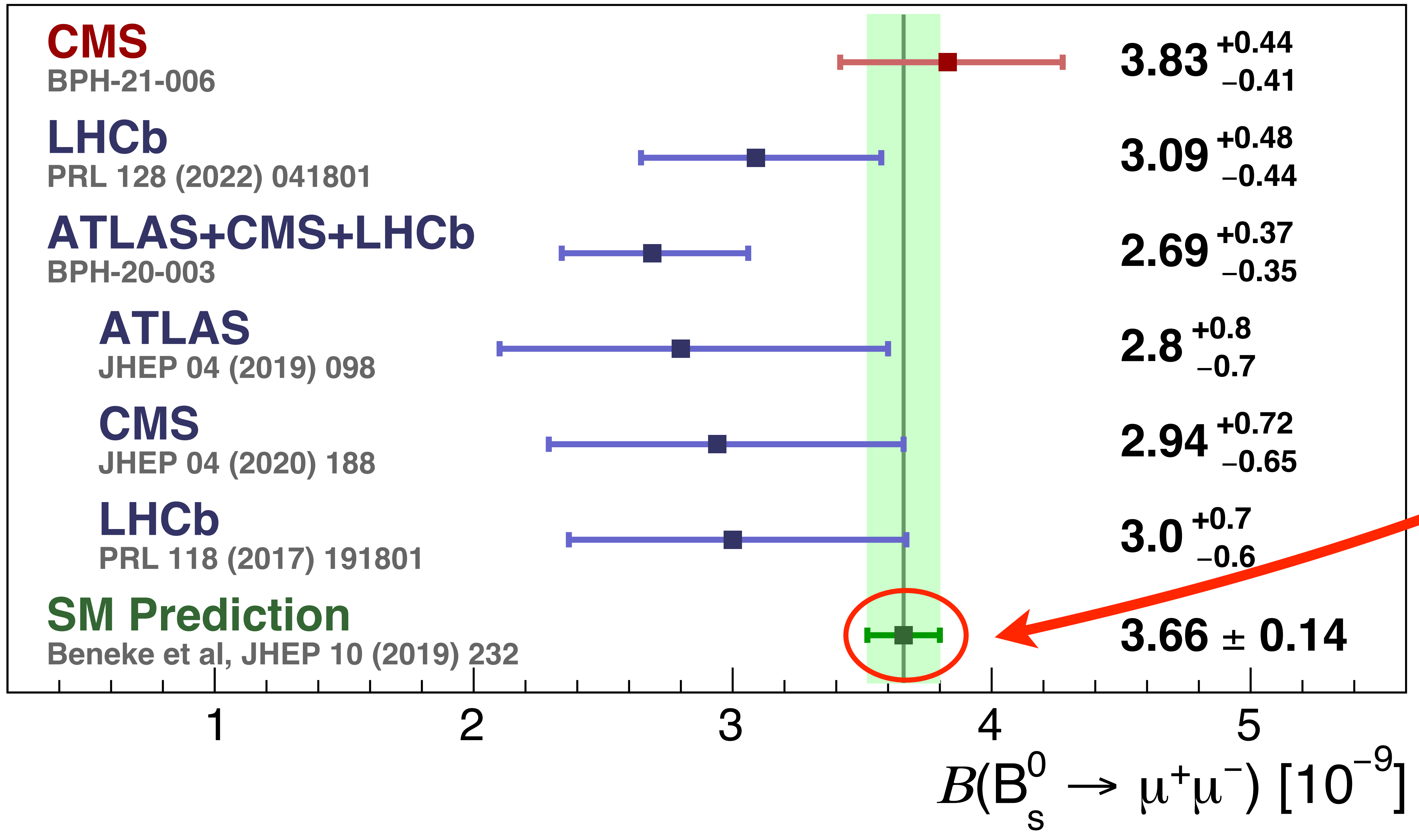


error^2

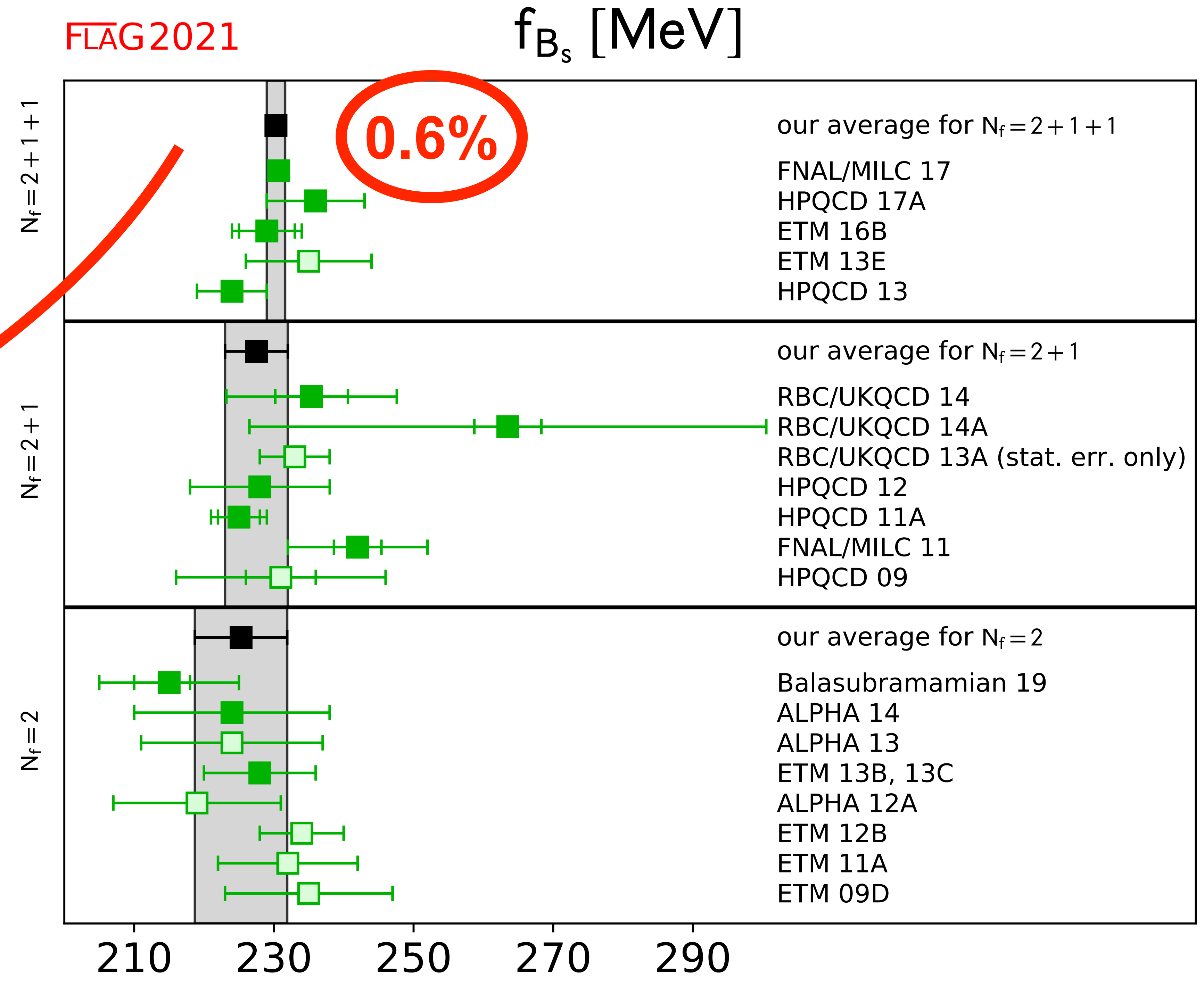


- includes structure-dependent QED corrections
- dominant uncertainty due to $|V_{cb}|$
- LQCD decay constant sub dominant source of uncertainty

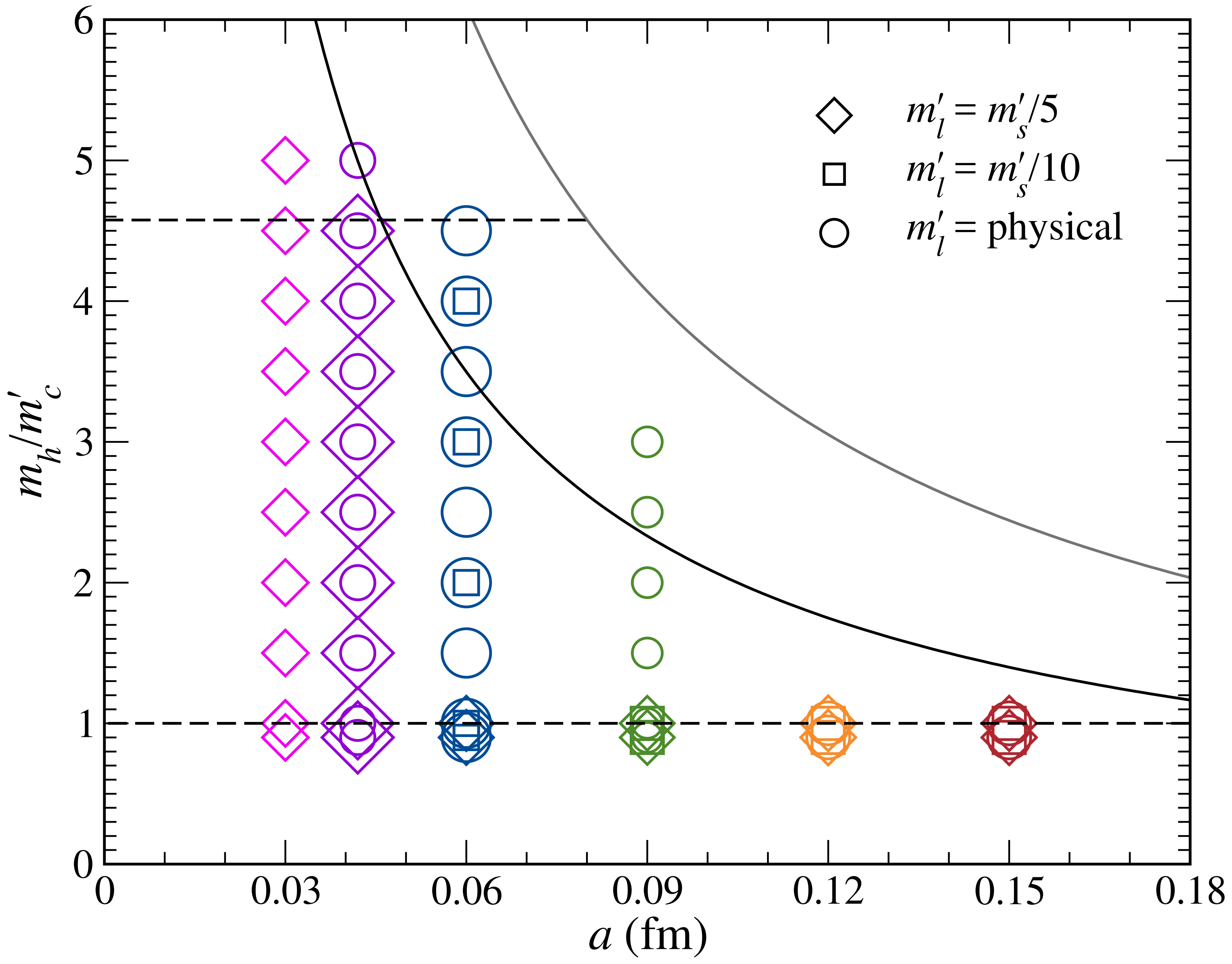
Silvano Tosi @ LHCP2023

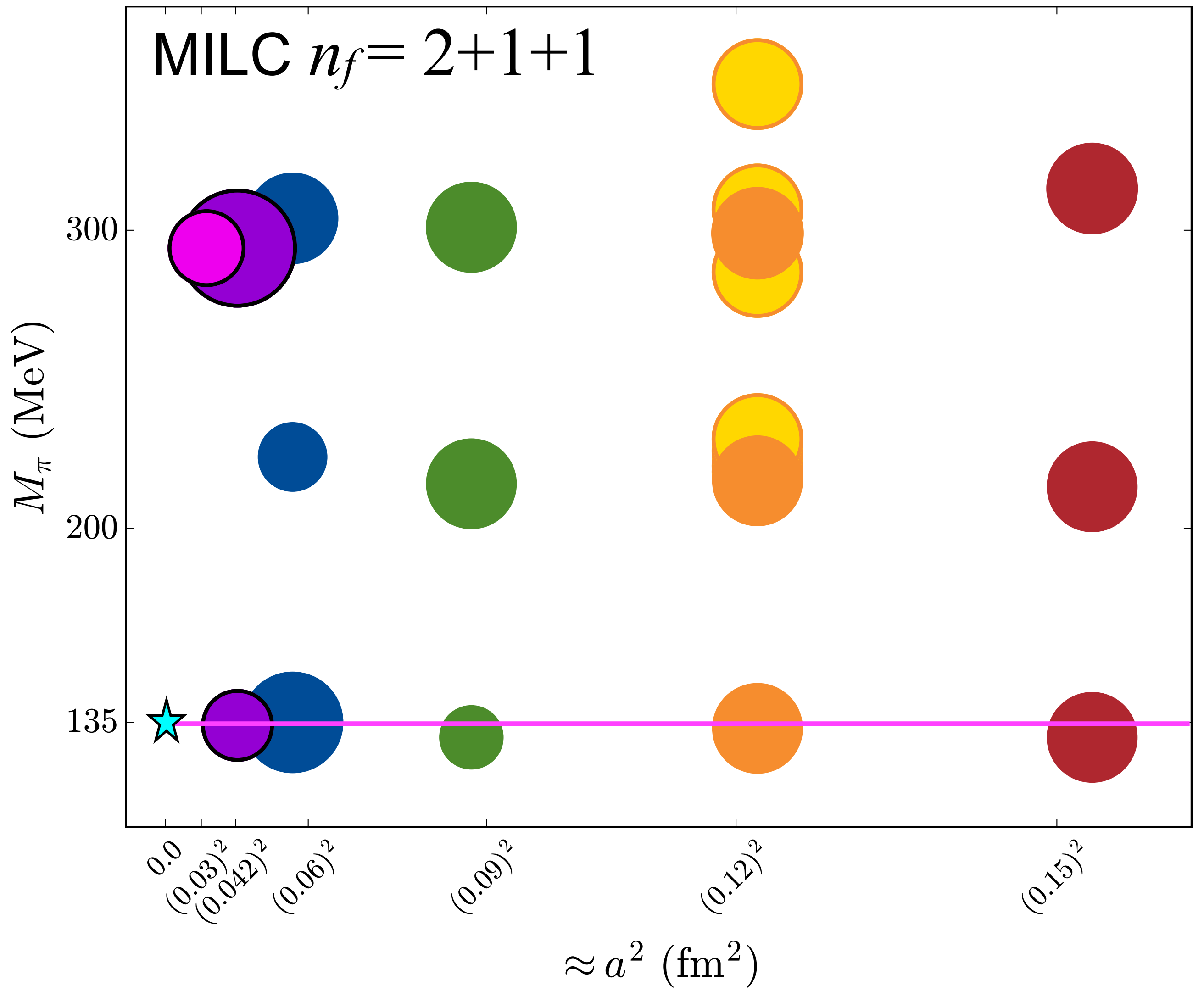


S. Aoki et al [FLAG 2021 review, arXiv:2111.09849, EPJC 2022]

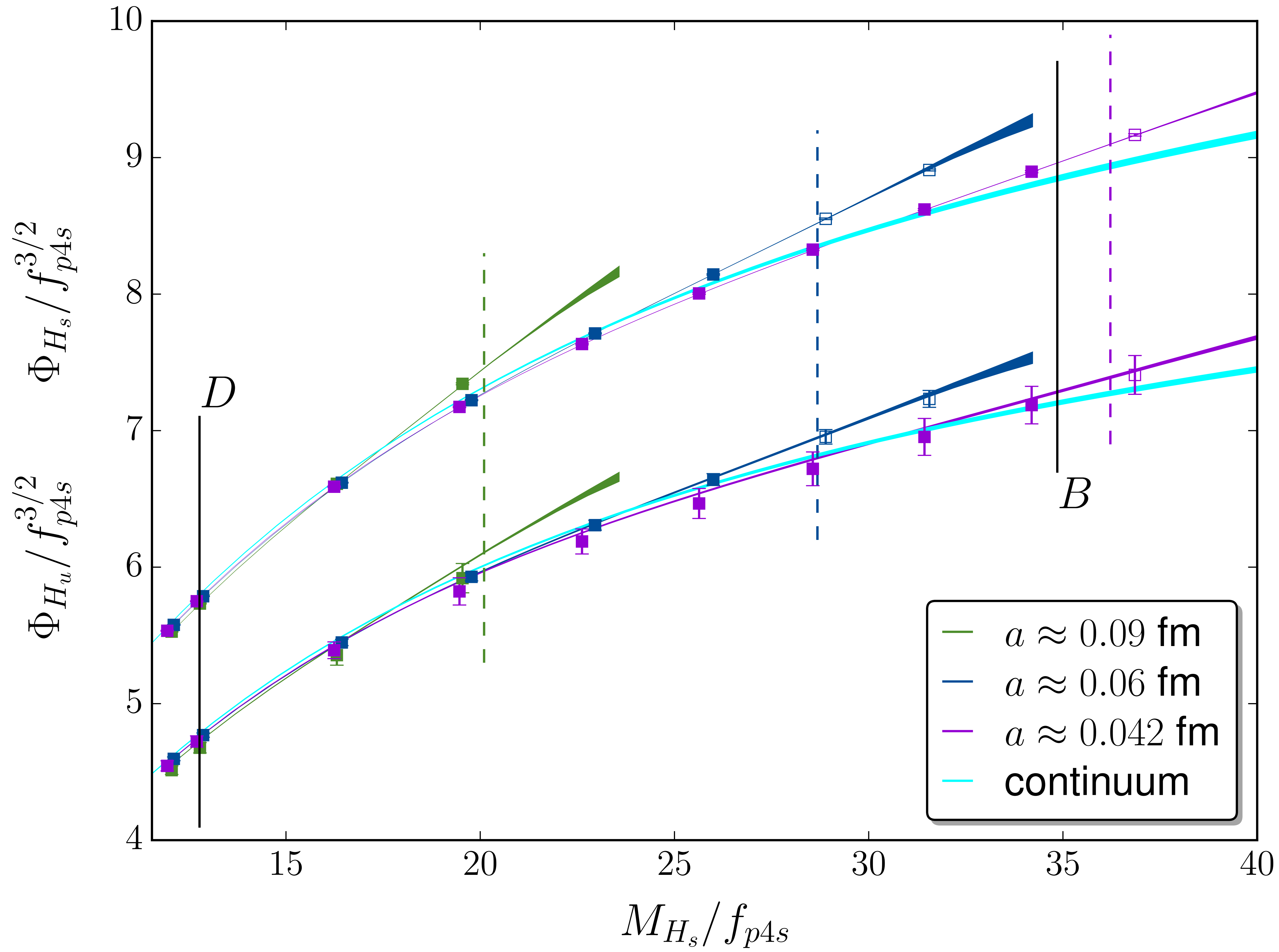


B, D meson decay constant results





A. Bazavov et al [FNAL/MILC, arXiv:1712.09262, 2018 PRD]



small errors due to **physical light quark masses**
improved quark action with small discretization errors even for heavy quarks
no renormalization (Ward identity)

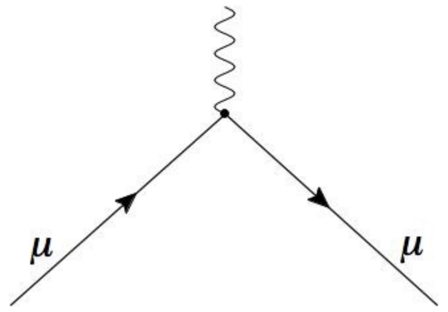
Outline

- The role of (lattice) QCD in precision physics
- Introduction to lattice QCD
- Success stories: two examples
  - m_q, α_s: inputs for Higgs decay rates
  - $B_{s,d} \to \mu\mu$
- Puzzles: one example
  - hadronic corrections to muon g-2
- Summary and Outlook

Anomalous magnetic moment

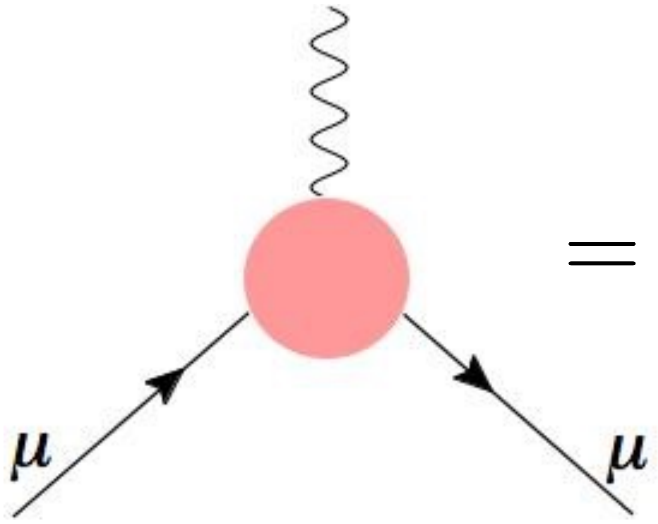
The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g \frac{e}{2m} \vec{S}$

Dirac (leading order): $g = 2$



$$= (-ie)\,\bar{u}(p')\gamma^\mu u(p)$$

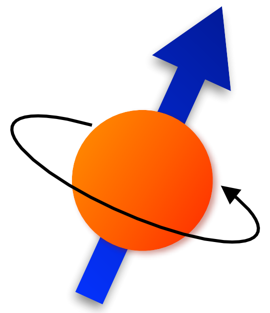
Quantum effects (loops):



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m} F_2(q^2)\right] u(p)$$

Note: $F_1(0) = 1$ and $g = 2 + 2\,F_2(0)$

Anomalous magnetic moment: $a \equiv \frac{g-2}{2} = F_2(0)$



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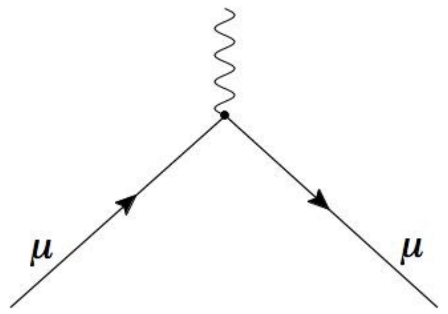

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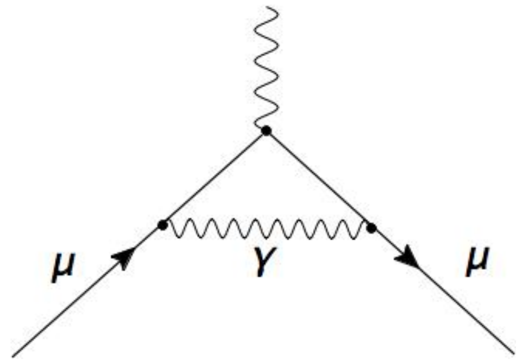
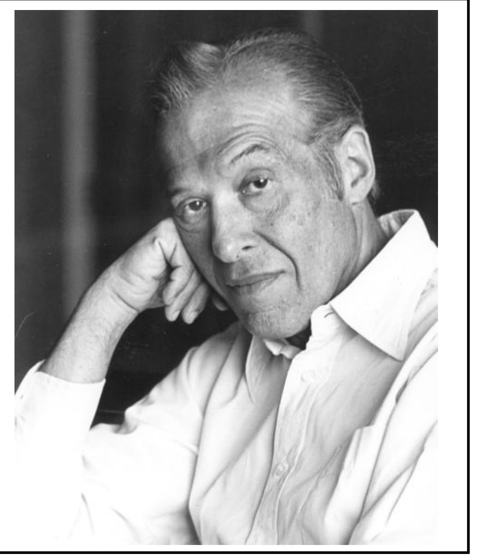
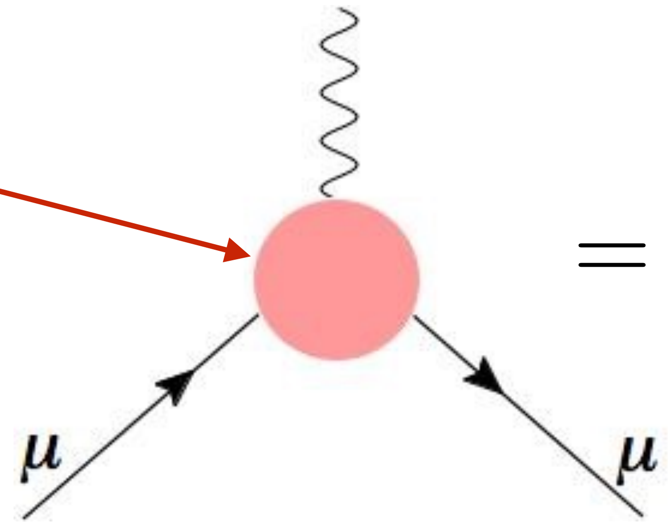


$$\Rightarrow \quad g = 2\left(1 + \frac{\alpha}{2\pi}\right)$$



All SM particles contribute



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

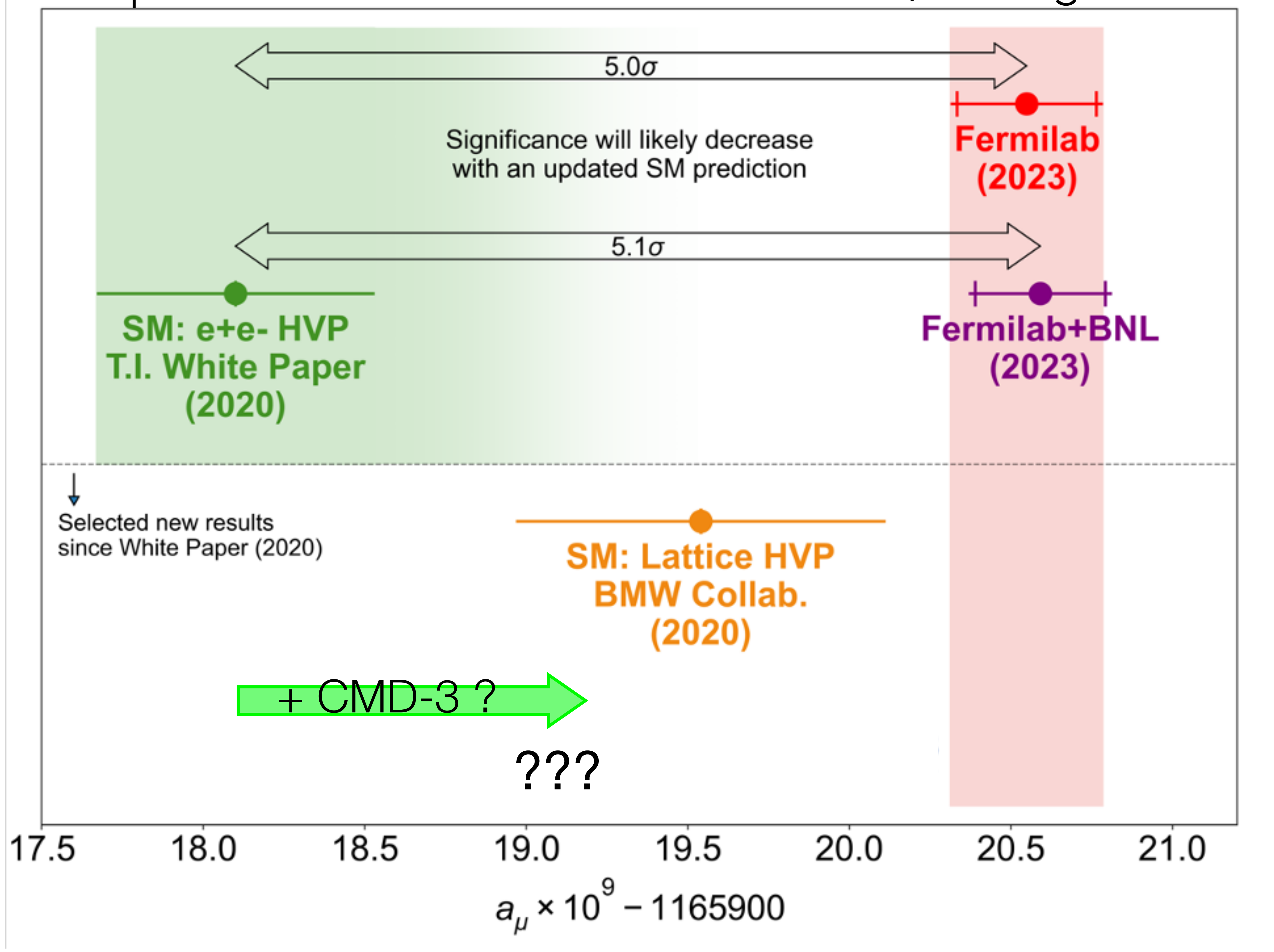
Note: $F_1(0) = 1$ and $g = 2 + 2\,F_2(0)$

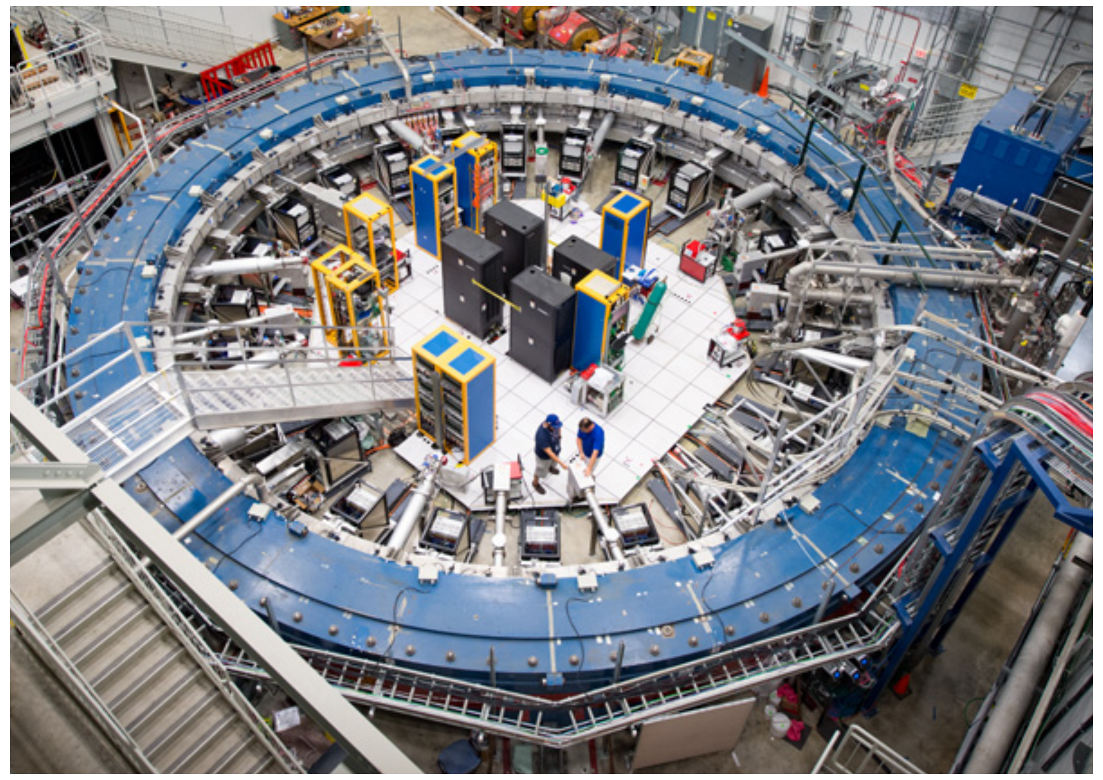
Anomalous magnetic moment: $a \equiv \frac{g-2}{2} = F_2(0)$ $= \frac{\alpha}{2\pi} + O(\alpha^2) + \ldots = 0.00116\ldots$

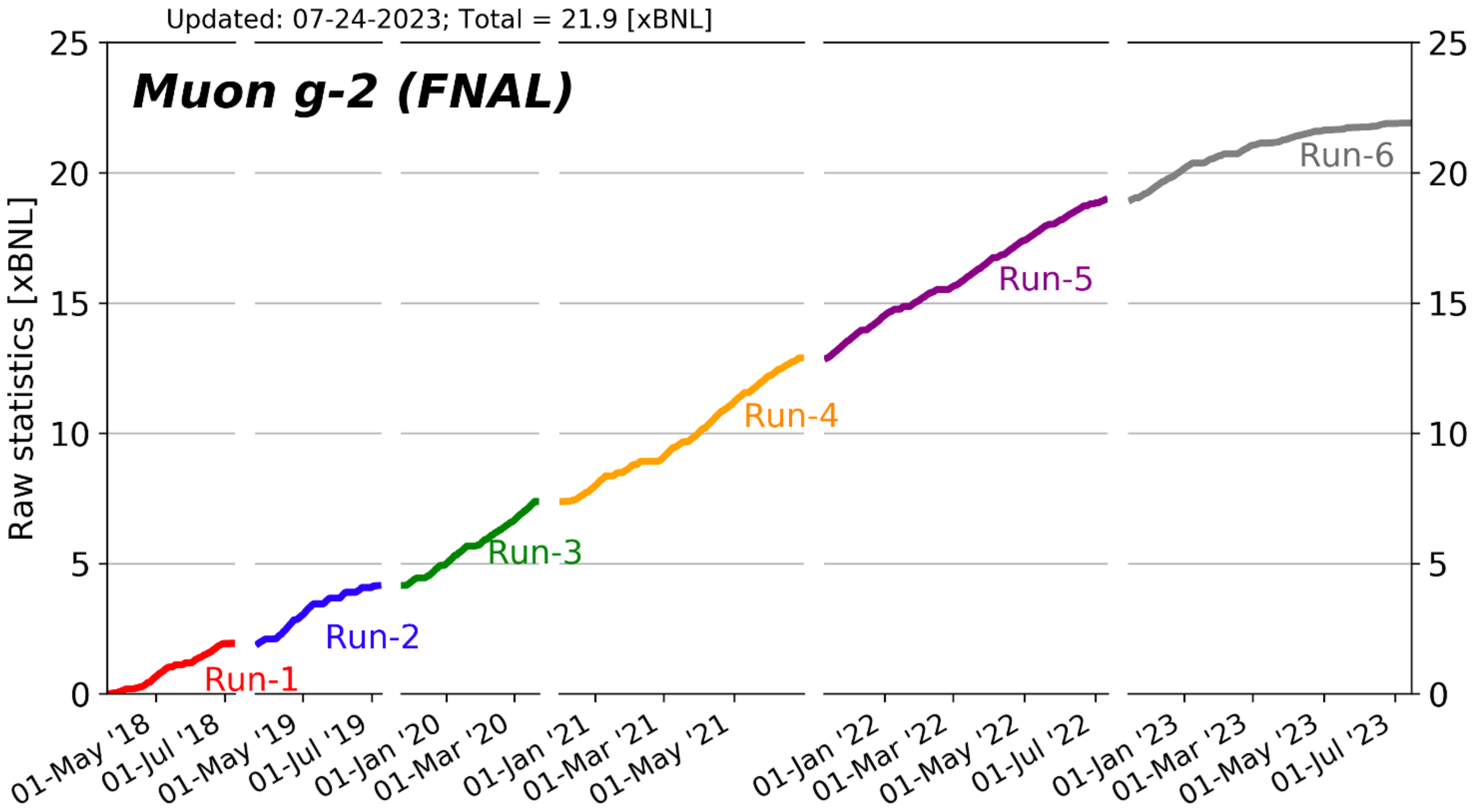
Fermilab muon g-2 experiment

- The Fermilab experiment released the measurement result from their run 2&3 data on 10 Aug 2023. [D. Aguillard et al, 2308.06230]
- Run 6 completed summer 2023.

adapted from J. Mott @ Scientific Seminar, 10 Aug 2023





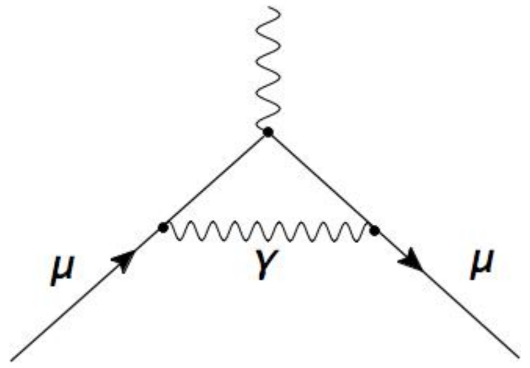


Muon g-2: SM contributions

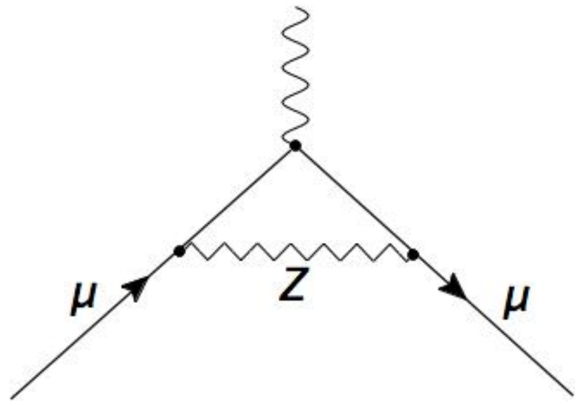
$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

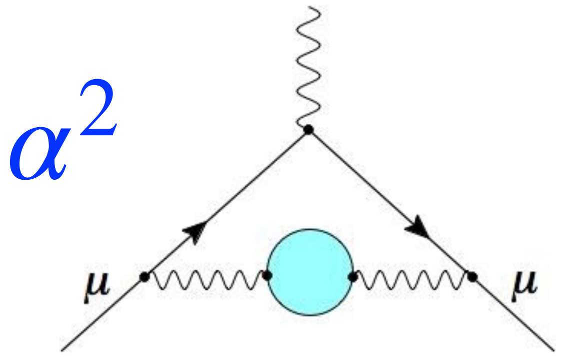
QED 

+… (5 loops) $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

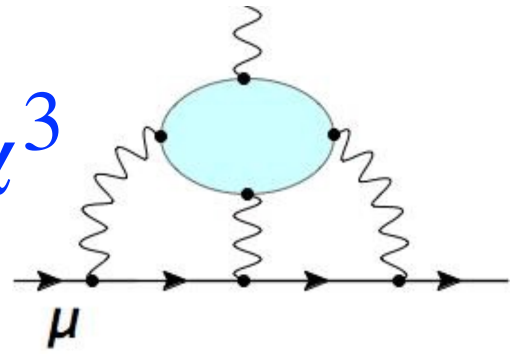
EW 

+… (2 loops) $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic corrections

HVP α^2 

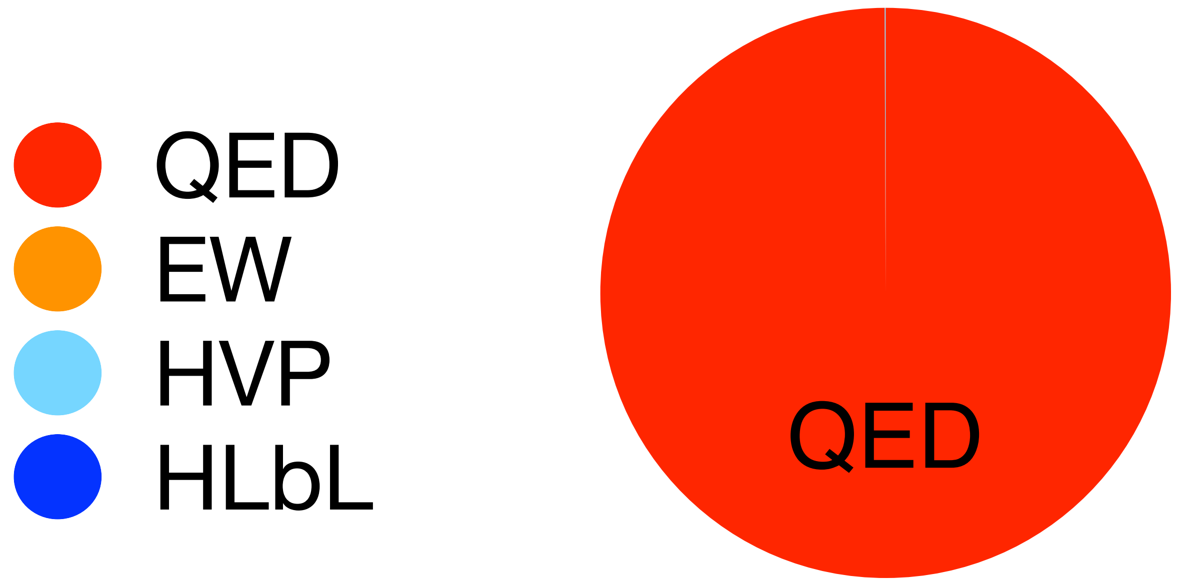
+… (NNLO) $6845\,(40) \times 10^{-11}$ [0.6%] 0.34 ppm

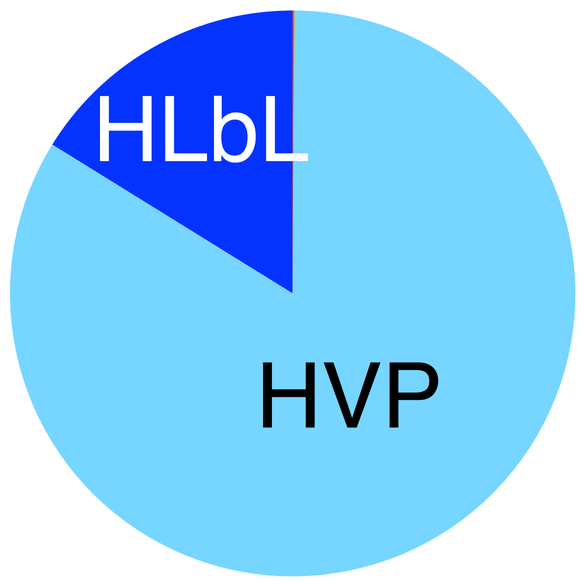
HLbL α^3 

+… (NLO) $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

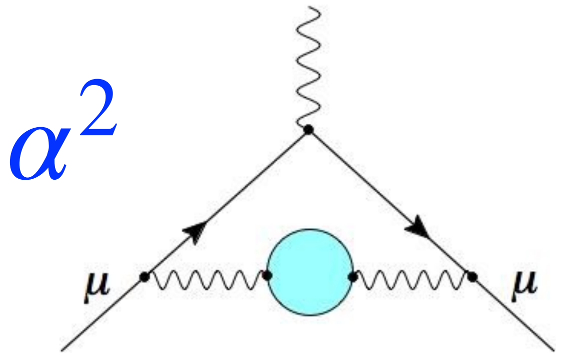
contribution 

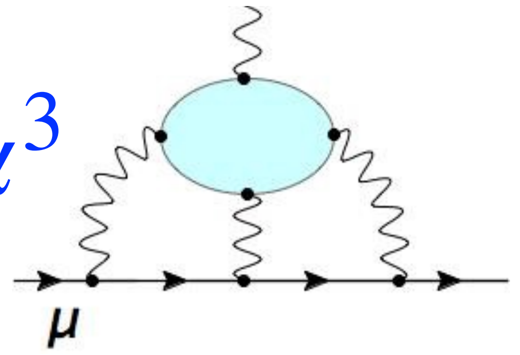
error² 

$116\,584\,718.9\,(1) \times 10^{-11}$ — 0.001 ppm

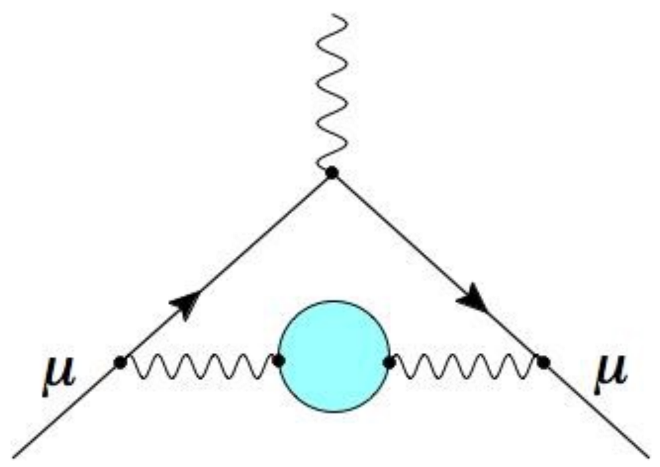
$153.6\,(1.0) \times 10^{-11}$ — 0.01 ppm

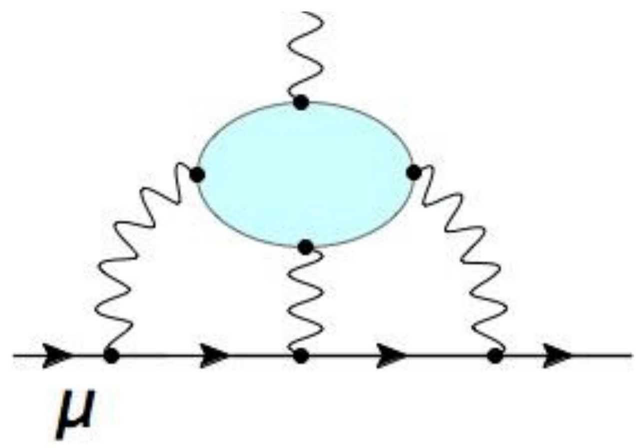
Hadronic corrections

HVP α^2  +… (NNLO) — $6845\,(40) \times 10^{-11}$ [0.6%] — 0.34 ppm

HLbL α^3  +… (NLO) — $92\,(18) \times 10^{-11}$ [20%] — 0.15 ppm

Muon g-2: hadronic corrections



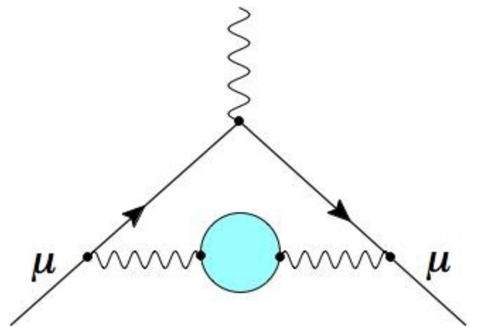


- Hadronic contributions are obtained by integrating over all possible virtual photon momenta, integral is weighted towards low q^2.
- Cannot use perturbation theory to reliably compute the hadronic bubbles
- Two-point & four-point functions:

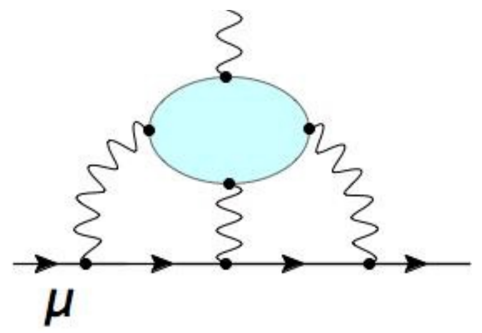
  HVP: $\langle 0 | T\{j_\mu j_\nu\} | 0 \rangle$ HLbL: $\langle 0 | T\{j_\mu j_\nu j_\rho j_\sigma\} | 0 \rangle$

Two independent approaches

1. Dispersive, data-driven
2. Lattice QCD



Hadronic Corrections

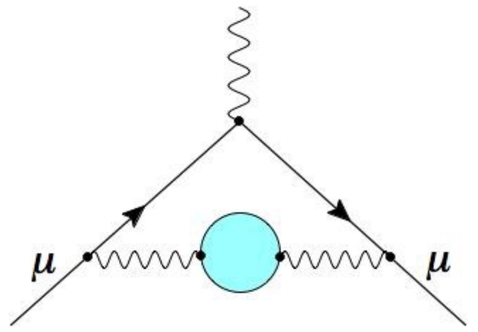


Two independent approaches:

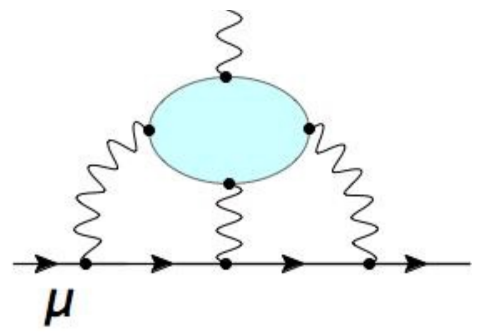
- **For HVP:** use dispersion relations to rewrite integral in terms of hadronic cross section:

Many experiments (over 20+ years) have measured the e^+e^-cross sections for the different channels over the needed energy range with increasing precision.

For HLbL: new dispersive formulation



Hadronic Corrections



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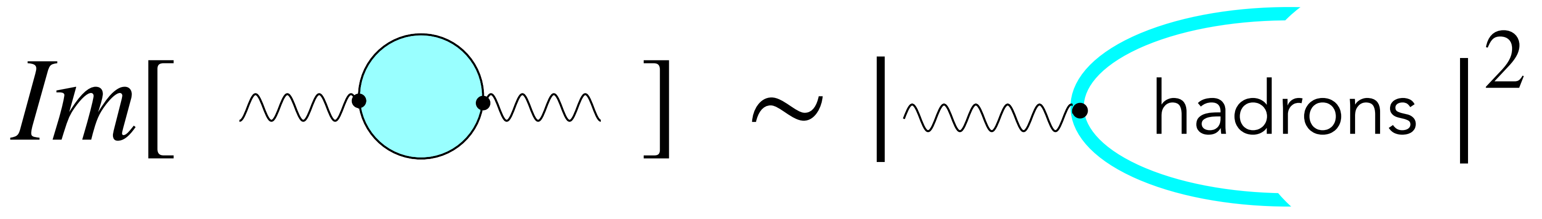

$$\Rightarrow \quad a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\,\hat{\Pi}(q^2) \;=\; \frac{m_\mu^2}{12\pi^3}\int ds \frac{\hat{K}(s)}{s}\,\sigma_{\exp}(s)$$

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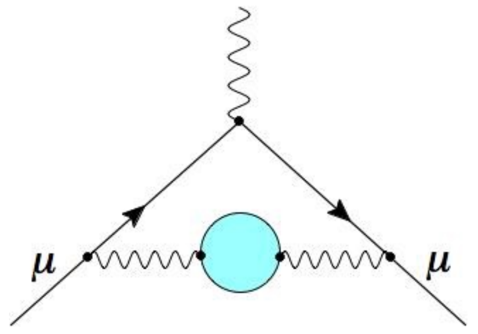
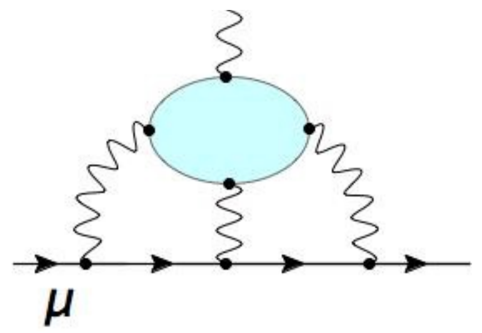


Hadronic Corrections



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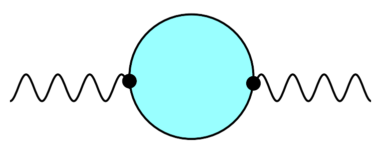
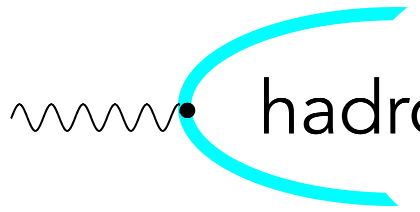
$Im[$  $] \sim |$ 

$|^2$ ➠

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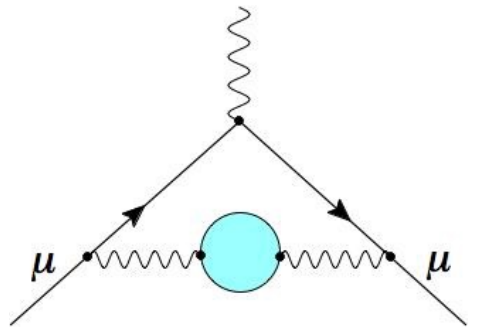
For HLbL: new dispersive formulation

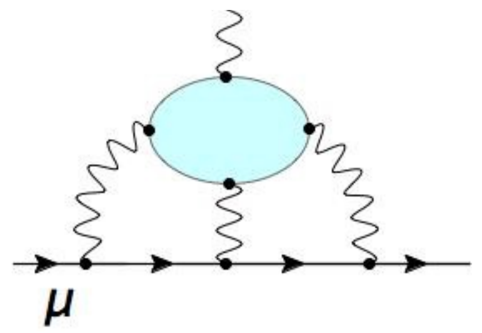
- Direct calculation using Euclidean Lattice QCD

➠ $$a_\mu^{\rm HVP,LO} = 4\,\alpha^2 \int_0^\infty dt\, C(t)\, \tilde{w}(t)$$

  - *ab-initio* method to quantify QCD effects
  - already used for simple hadronic quantities with high precision
  - requires large-scale computational resources
  - allows for entirely SM theory based evaluations



Hadronic Corrections



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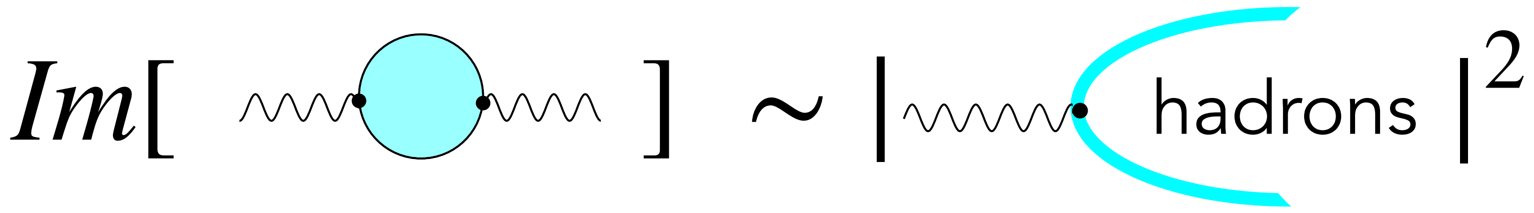

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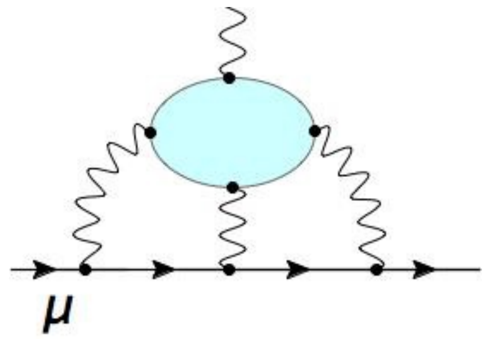
Muon g-2 Theory Initiative

Steering Committee

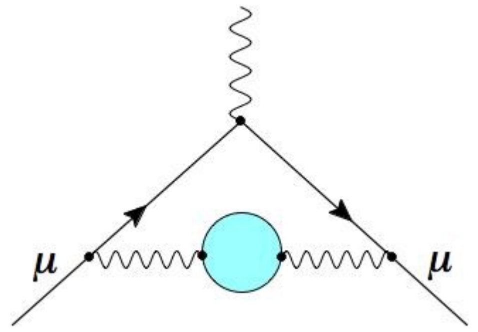
- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK)
  J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston)
  Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

https://muon-gm2-theory.illinois.edu

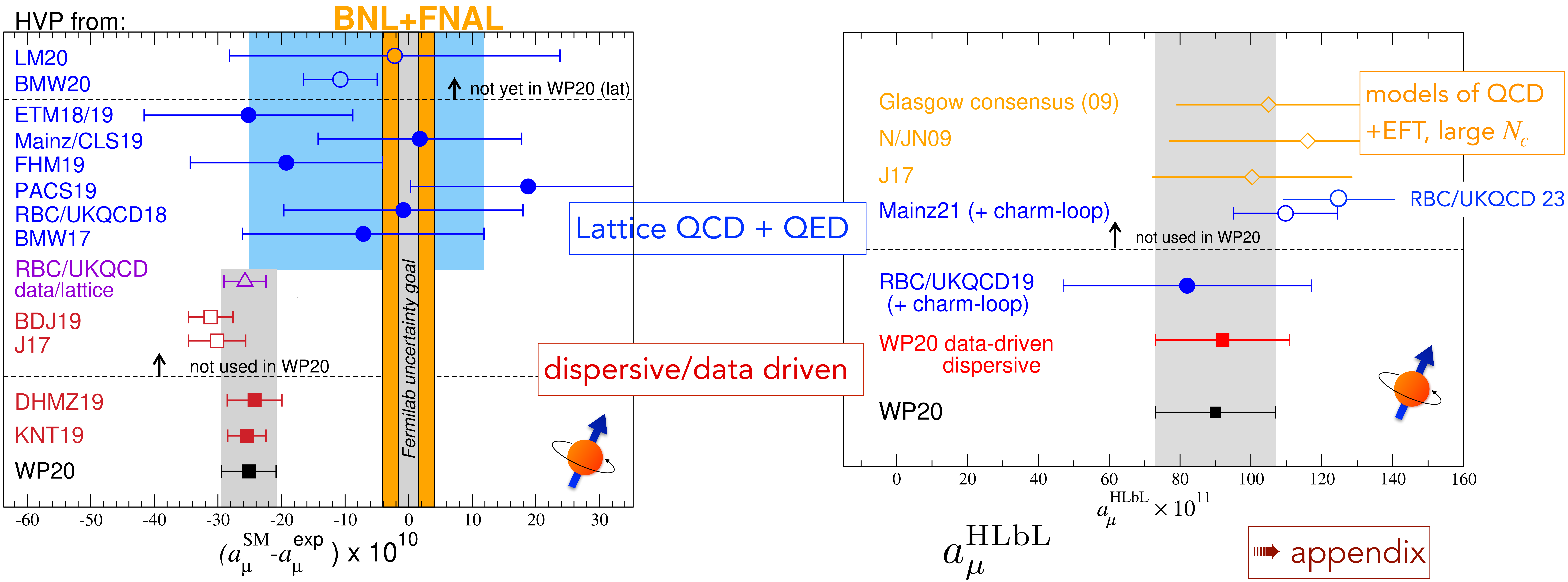
- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Lattice HVP at high precision workshop (virtual): 16-20 November 2020
  Fourth plenary workshop @ KEK (virtual): 28 June - 02 July 2021
  Fifth plenary workshop @ Higgs Centre (Edinburgh): 5-9 September 2022
  Sixth plenary workshop @ University of Bern: 4-8 September 2023
  Seventh plenary workshop @ KEK or KMI (Japan): 9-13 September 2024
  Eight plenary workshop: 2025 — seeking proposals



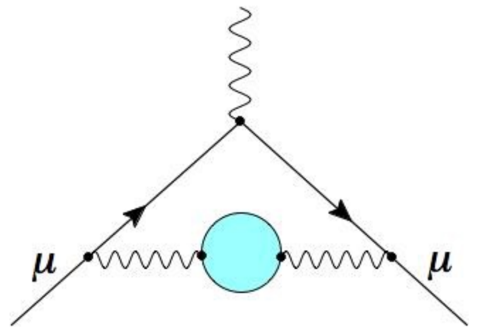
Hadronic Corrections: Comparisons



Before February 2023

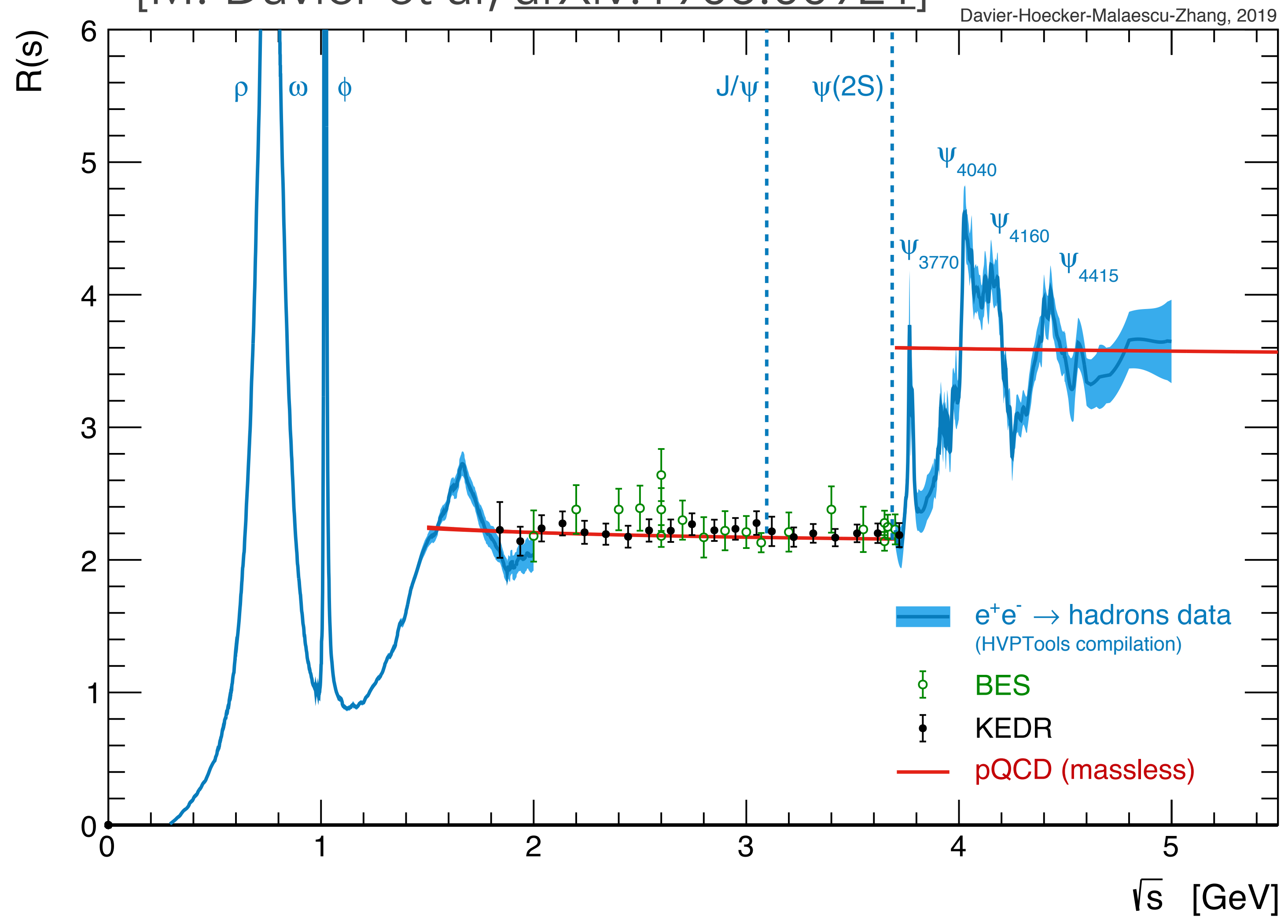


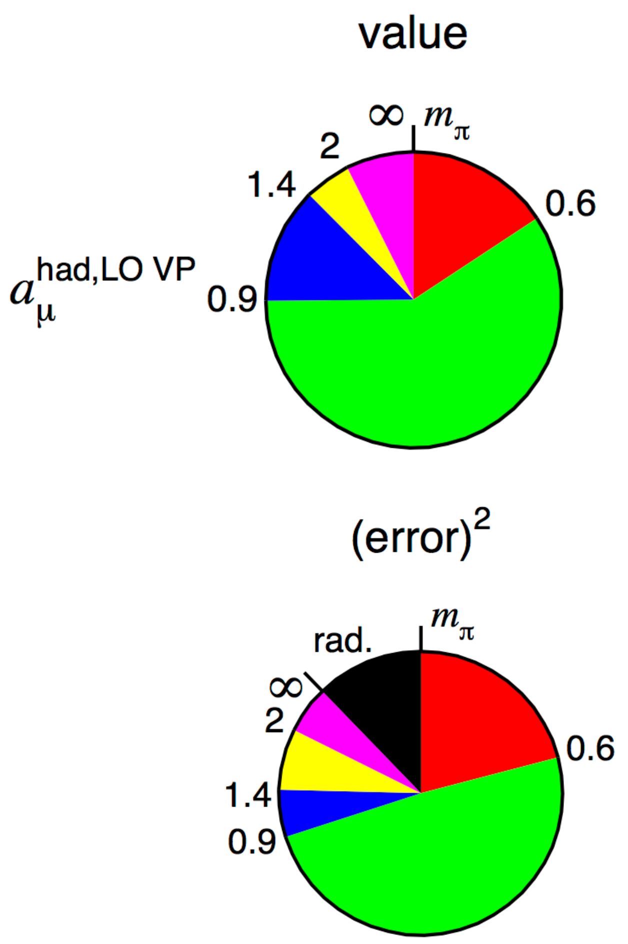
$$a_\mu^{\rm SM} = a_\mu^{\rm HVP} + \left[a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HLbL}\right]$$



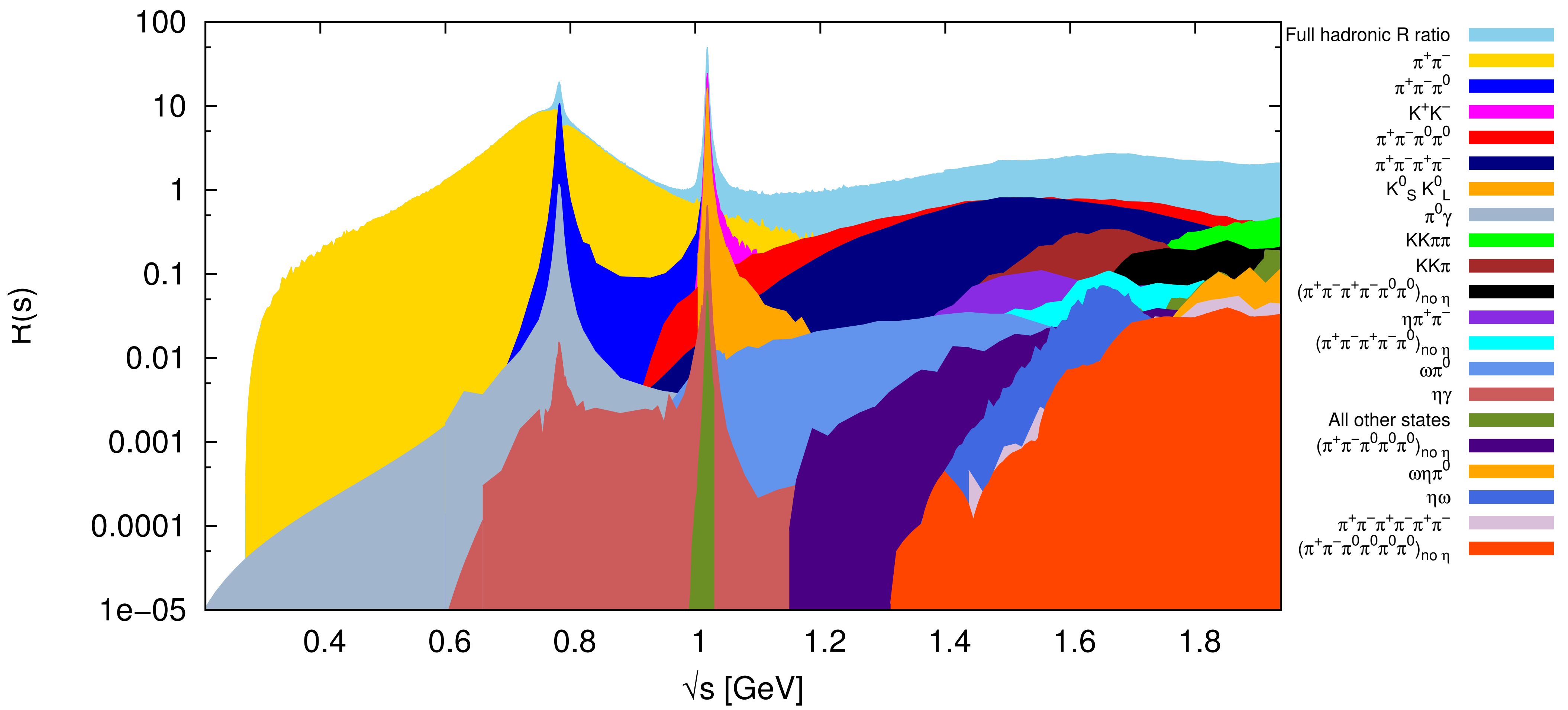
HVP: data-driven

[M. Davier et al, arXiv:1908.00921]





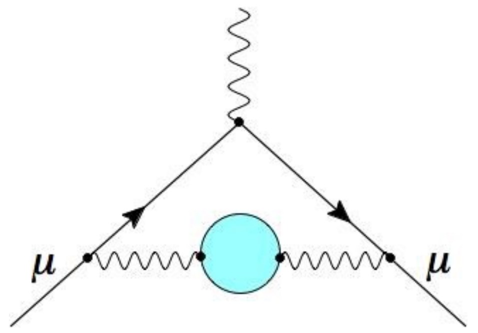
[A. Keshavarzi et al, arXiv:1802.02995]



- $\sigma_{\rm had}(s)$ defined to include real & virtual photons
- direct integration method: no modelling of $\sigma_{\rm had}(s)$, summing up contributions from all hadronic channels
- total hadronic cross section $\sigma_{\rm had}(s)$ from > 100 data sets in more than 35 channels summed up to $\sqrt{s} \sim 2\,{\rm GeV}$
- $\sqrt{s} > 2$ GeV: inclusive data + pQCD + narrow resonances
- two independent compilations (DHMZ, KNT) using the direct integration method

Tensions between BaBar and KLOE data sets:

- Cross checks using analyticity and unitarity relating pion form factor to $\pi\pi$ scattering
- Combinations of data sets affected by tensions
  ➟ conservative merging procedure



HVP: data-driven

see appendix

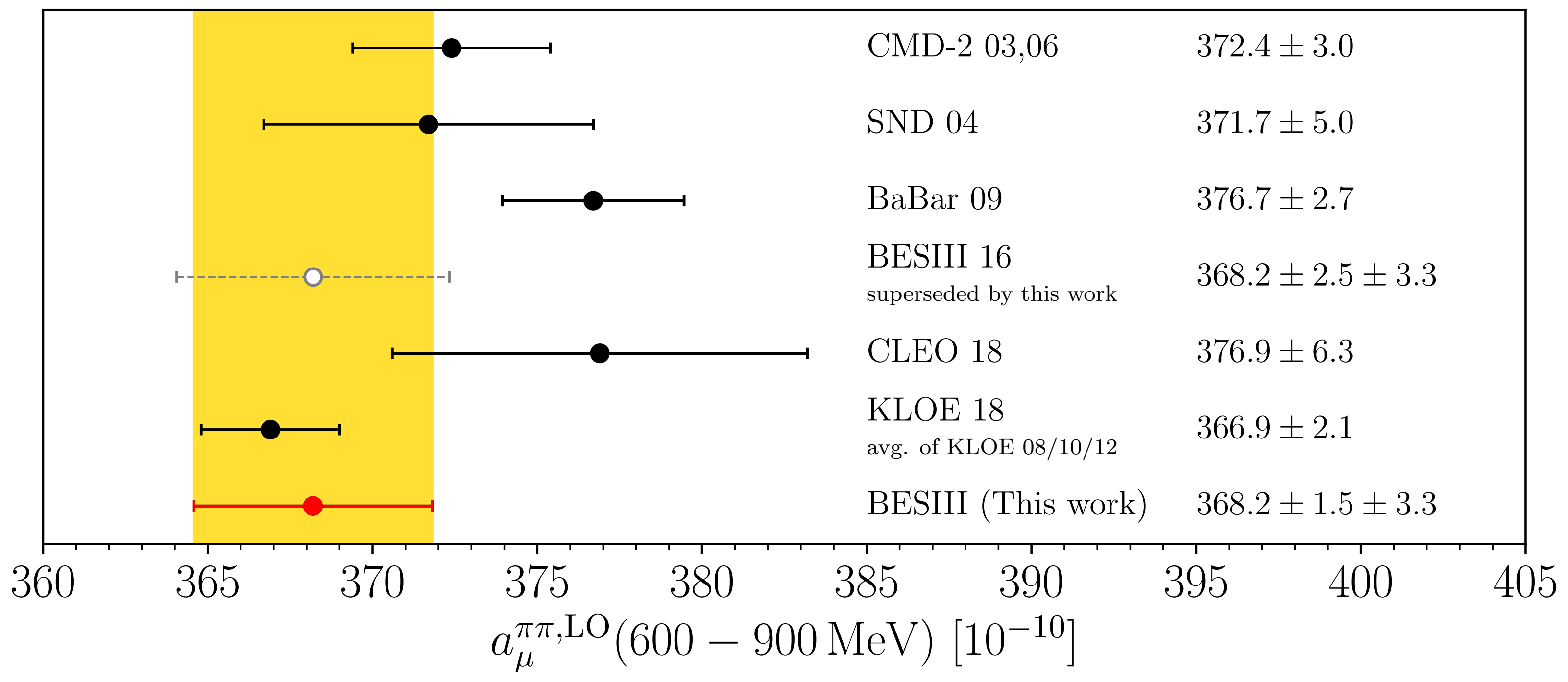
In 2020 WP:

Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

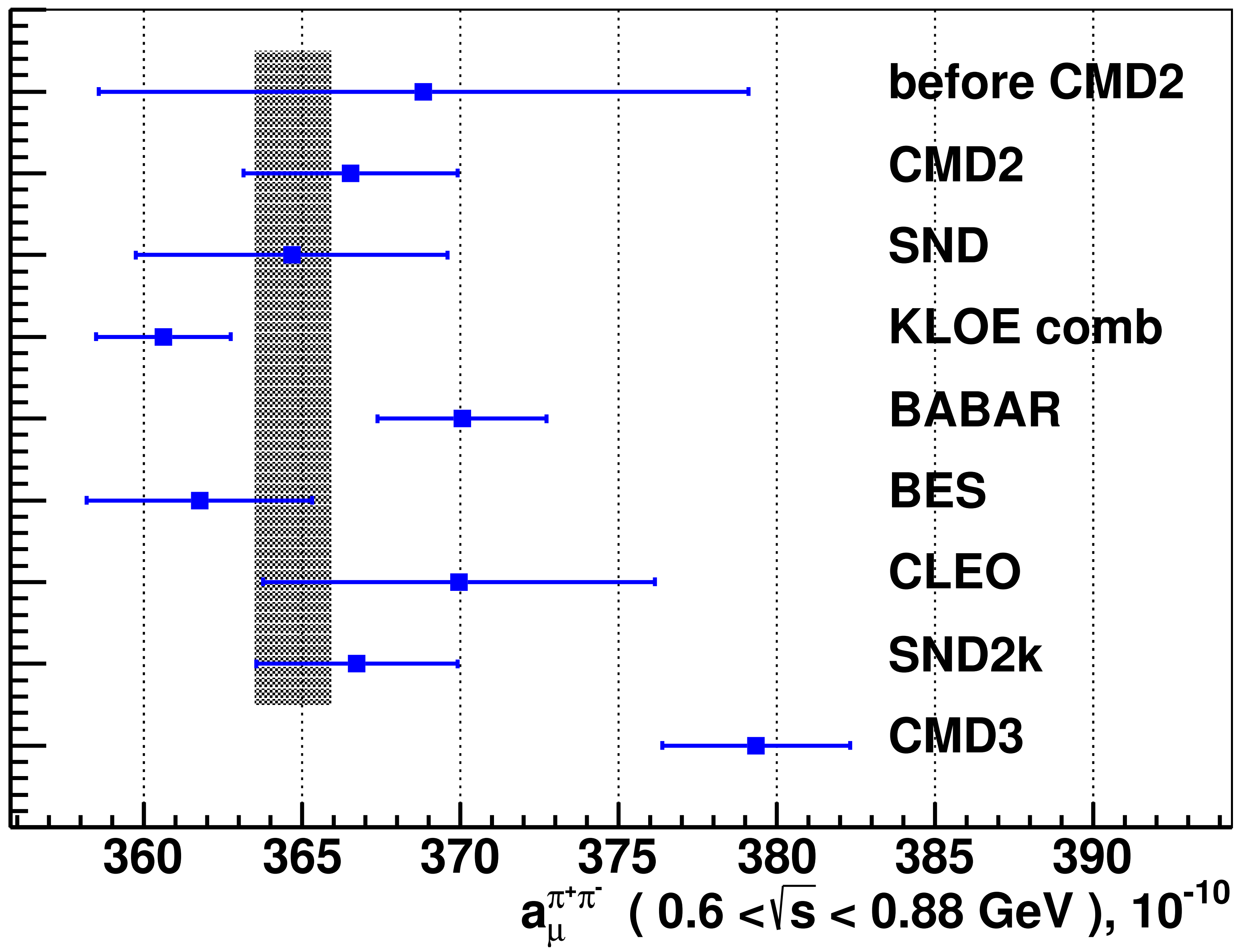
- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs
- include correlations between systematic errors
- cross checks from unitarity & analyticity constraints [Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections [Campanario et al, 2019]

$$a_\mu^{\rm HVP,LO} = 693.1\,(2.8)_{\rm exp}\,(0.7)_{\rm DV+pQCD}\,(2.8)_{\rm BaBar-KLOE} \times 10^{-10}$$
$$= 693.1\,(4.0) \times 10^{-10}$$

[M. Ablikim et al (BES III), arXiv:2009.05011]

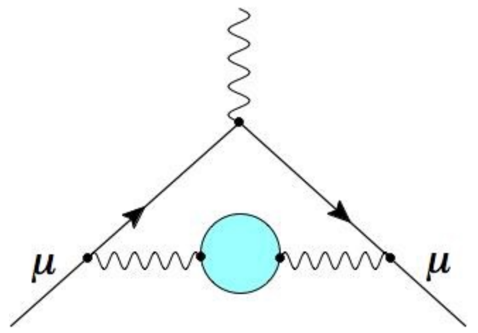


New: from CMD-3 [F. Ignatov et al, arXiv:2302.08834]



A new puzzle!

- discrepancies between experiments now $\gtrsim (3-5)\,\sigma$
  this needs to be understood/resolved
- (virtual) scientific seminar + discussion panel on CMD-3 measurement
  March 27 (8:00 –11:00 am US CDT)
  2nd CMD-3 discussion meeting (
- 6th Muon g-2 Theory Initiative workshop (4-8 Sep 2023, Bern)



HVP: data-driven

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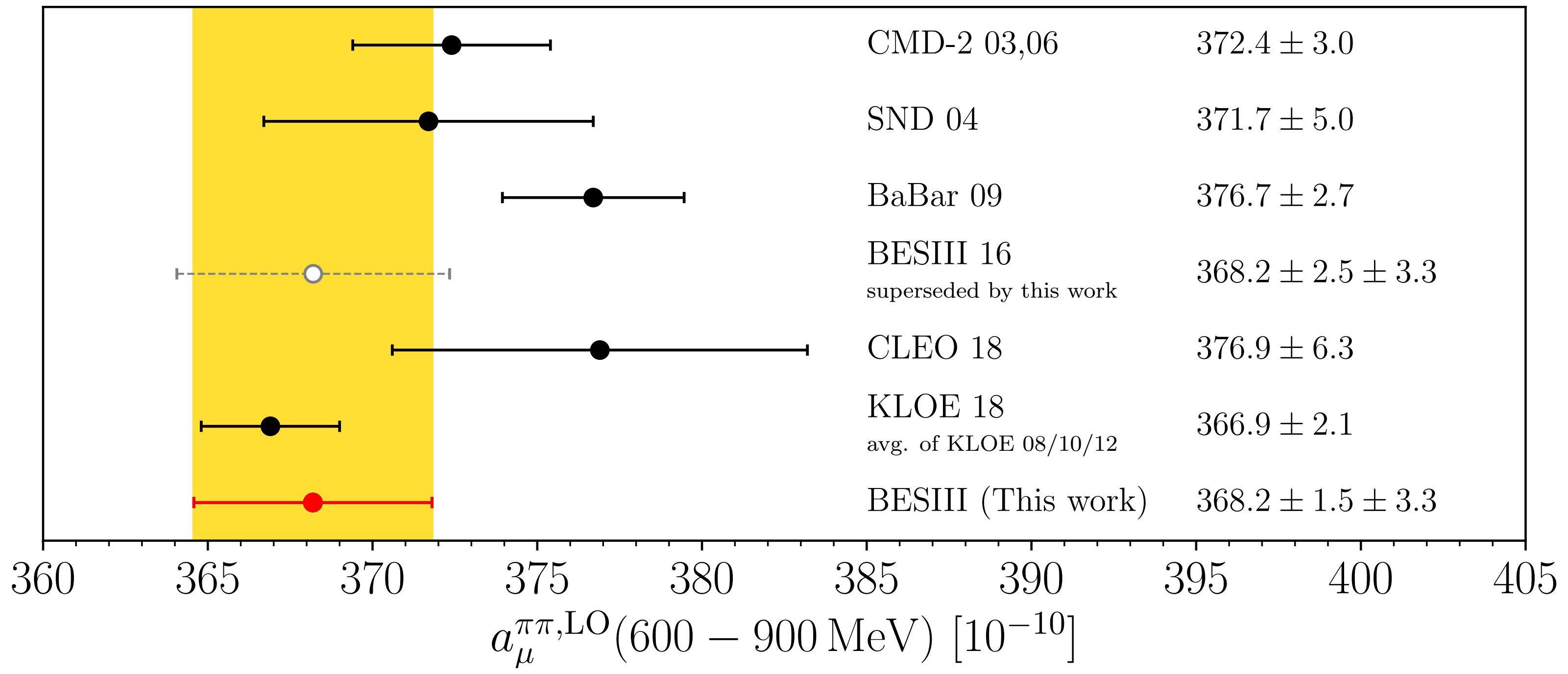
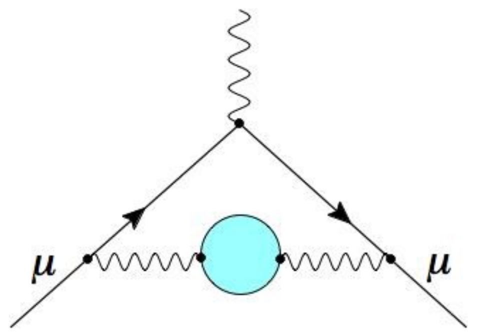


Ongoing work on experimental inputs:

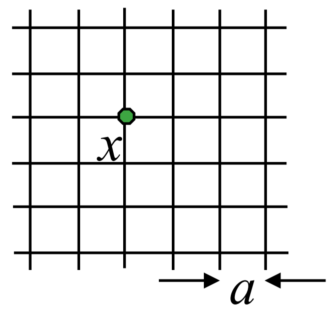
- BaBar: new analysis of large data set in $\pi\pi$ channel
- KLOE: new analysis of large data in $\pi\pi$ channel
- SND: new results for $\pi\pi$ channel, other channels in progress
- BESIII: new results in 2021 for $\pi\pi$ channel, continued analysis for $\pi\pi\pi$, …
- Belle II: arXiv:2207.06307 (Snowmass WP)
  Better ultimate statistics than BaBar or KLOE; similar or better systematics for low-energy cross sections

Ongoing work on theoretical aspects:

- better treatment of structure dependent radiative corrections (NLO) in $\pi\pi$ and $\pi\pi\pi$ channels
  so far: FsQED (scalar QED + pion form factor)
  tests of radiative corrections using exp. measurement of charge asymmetry [Ignatov + Lee, arXiv:2204.12235]
  new dispersive treatment [Colangelo at al, arXiv:2207.03495]
- Developing NNLO Monte Carlo generators (STRONG 2020 workshop https://agenda.infn.it/event/28089/)
- including τ decay data: requires nonperturbative evaluation of IB correction [M. Bruno et al, arXiv:1811.00508]

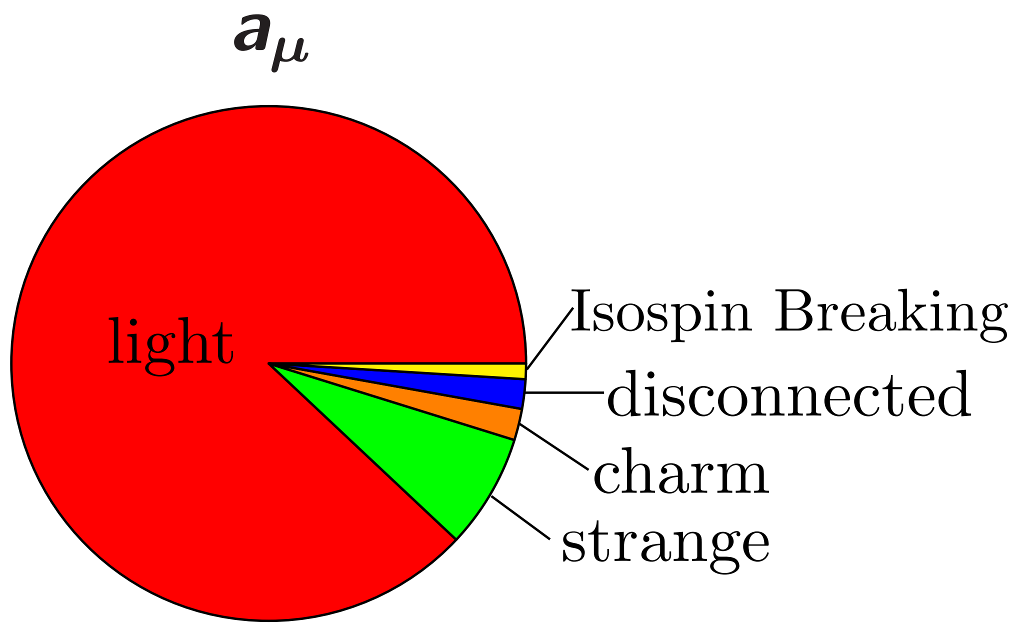


Lattice HVP: Introduction



Calculate $a_\mu^{\rm HVP}$ in Lattice QCD:

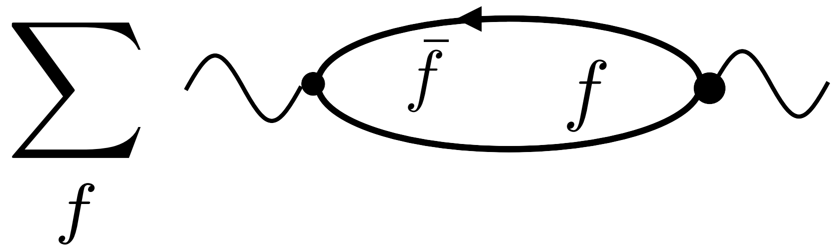
$$a_\mu^{\rm HVP,LO} = \sum_f a_{\mu,f}^{\rm HVP,LO} + a_{\mu,\rm disc}^{\rm HVP,LO}$$



- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

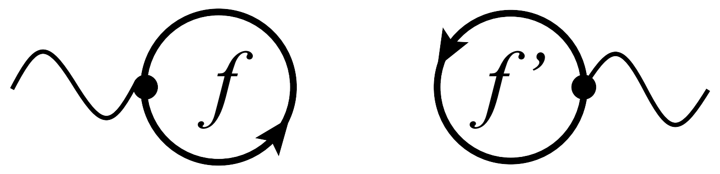
  Note: almost always $m_u = m_d$

f = ud, s, c, b



- light-quark connected contribution: $a_\mu^{\rm HVP,LO}(ud)$ ~90% of total
- *s,c,b*-quark contributions $a_\mu^{\rm HVP,LO}(s,c,b)$ ~8%, 2%, 0.05% of total

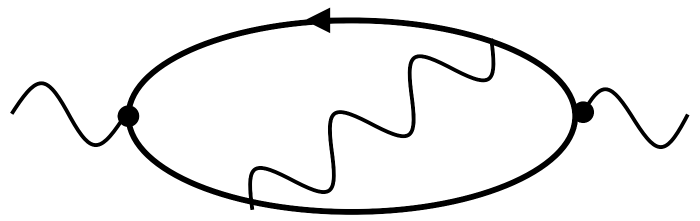
+



- disconnected contribution: $a_{\mu,\rm disc}^{\rm HVP,LO}$ ~2% of total

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:

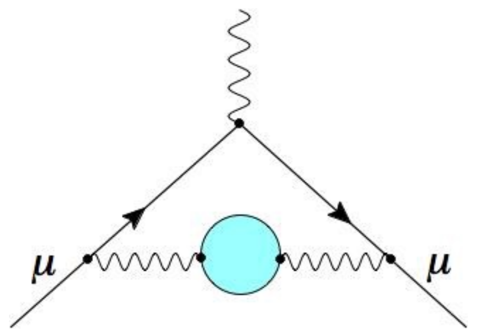
+



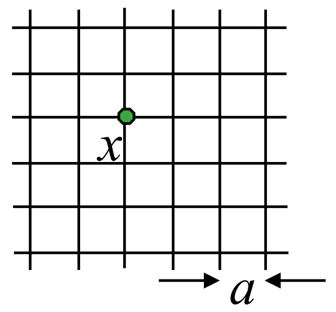
+ …

- Isospinbreaking (QED + $m_u \neq m_d$) corrections: $\delta a_\mu^{\rm HVP,LO}$ ~1% of total

$$a_\mu^{\rm HVP,LO} = a_\mu^{\rm HVP,LO}(ud) + a_\mu^{\rm HVP,LO}(s) + a_\mu^{\rm HVP,LO}(c) + a_{\mu,\rm disc}^{\rm HVP,LO} + \delta a_\mu^{\rm HVP,LO}$$

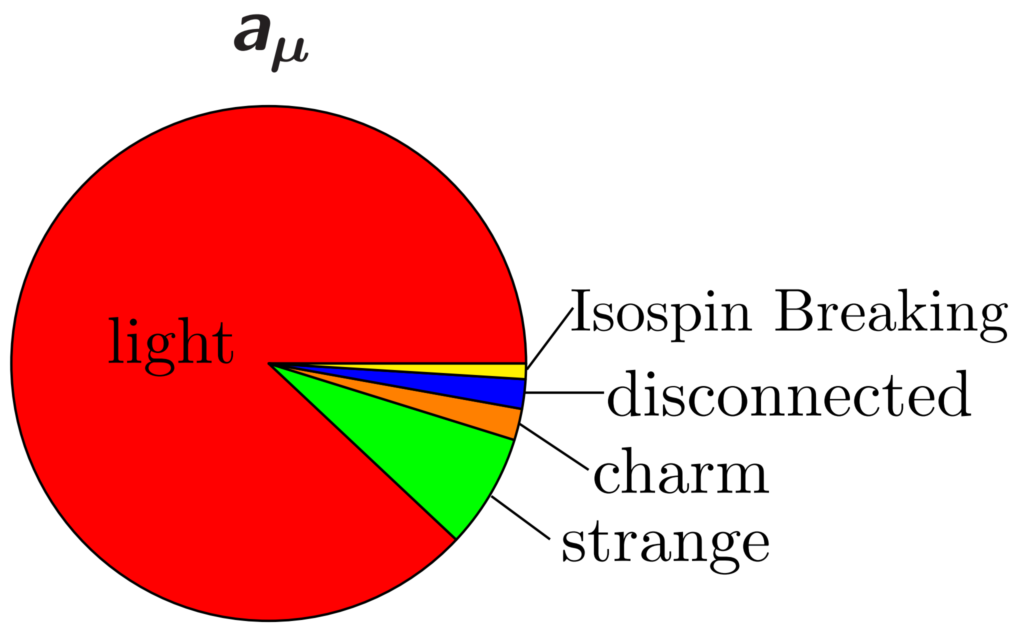


Lattice HVP: challenges



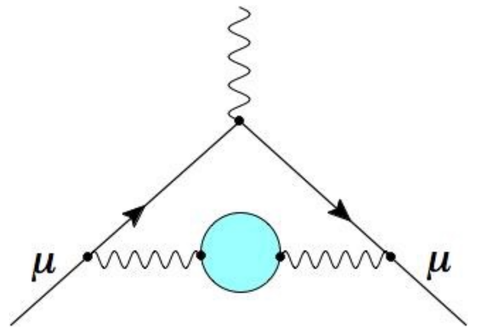
Calculate $a_\mu^{\rm HVP}$ in Lattice QCD: $$a_\mu^{\rm HVP,LO} = \sum_f a_{\mu,f}^{\rm HVP,LO} + a_{\mu,{\rm disc}}^{\rm HVP,LO}$$

$$a_\mu^{\rm HVP,LO} = a_\mu^{\rm HVP,LO}(ud) + a_\mu^{\rm HVP,LO}(s) + a_\mu^{\rm HVP,LO}(c) + a_{\mu,{\rm disc}}^{\rm HVP,LO} + \delta a_\mu^{\rm HVP,LO}$$

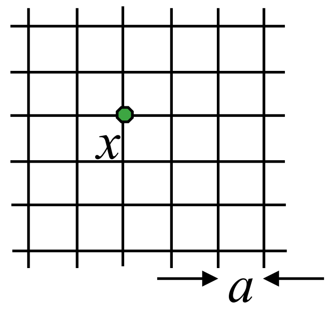


- $a_\mu^{\rm HVP,LO}$ needed with $< 0.5\,\%$ precision
- subpercent statistical precision:
  exponentially growing noise-to-signal in $C(t)$ as $t \to \infty$
  affects light-quark contributions
- sizable finite volume effects
- sensitivity to scale setting uncertainty
- control discretization effects
- quark-disconnected diagrams: control noise
- include isospin-breaking effects
  Separation of $a_\mu^{\rm HVP,LO}$ into $a_\mu^{\rm HVP,LO}(ud)$ and $\delta a_\mu^{\rm HVP,LO}$ is scheme dependent.

- light-quark connected contribution:
  $a_\mu^{\rm HVP,LO}(ud)$ ~90% of total
- *s*,*c*,*b*-quark contributions
  $a_\mu^{\rm HVP,LO}(s,c,b)$ ~8%, 2%, 0.05% of total
- disconnected contribution:
  $a_{\mu,{\rm disc}}^{\rm HVP,LO}$ ~2% of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  $\delta a_\mu^{\rm HVP,LO}$ ~1% of total



Windows in Euclidean time



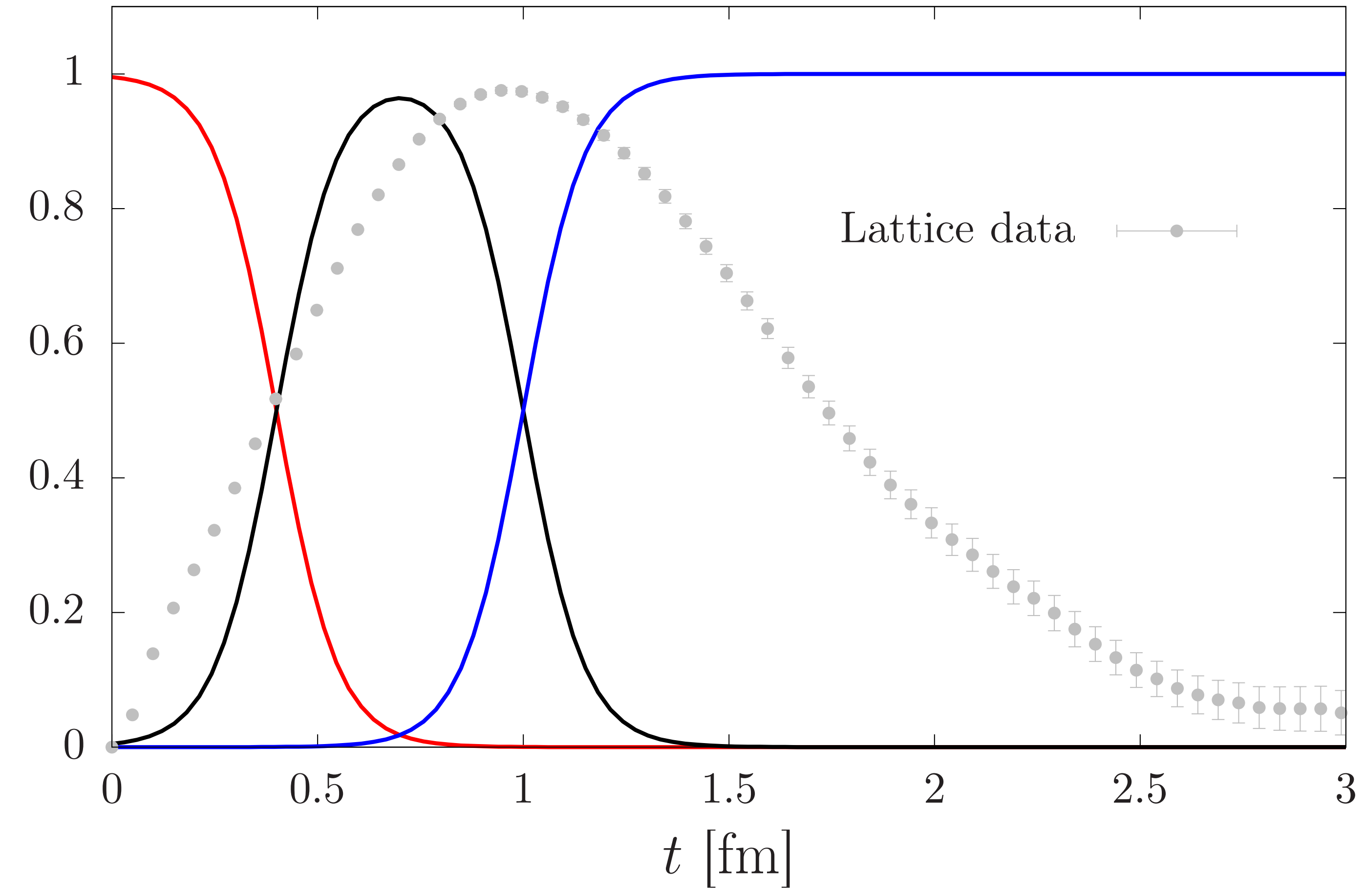
$$a_\mu^{\mathrm{HVP,LO}} = 4\,\alpha^2 \int_0^\infty dt\, C(t)\, \tilde{w}(t)$$

- Use windows in Euclidean time to consider the different time regions separately
  [T. Blum et al, arXiv:1801.07224, 2018 PRL]

Short Distance (SD) $t: 0 \to t_0$
Intermediate (W) $t: t_0 \to t_1$
Long Distance (LD) $t: t_1 \to \infty$

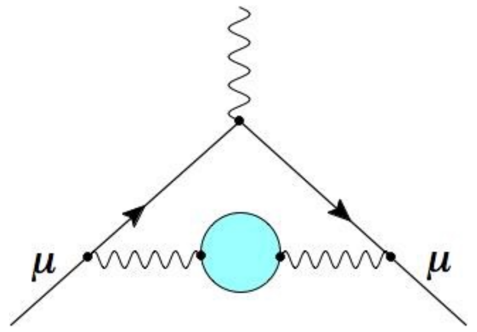
$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}$

H. Wittig @ Lattice 2021

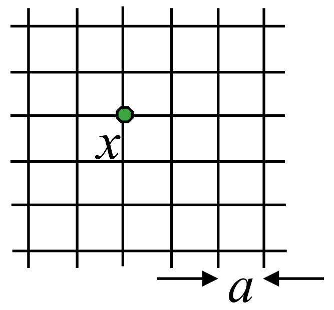


- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD; compare to disperse approach
- Internal cross check: compute each window separately (in continuum, infinite volume limits,…) and combine:

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$



Lattice HVP: results



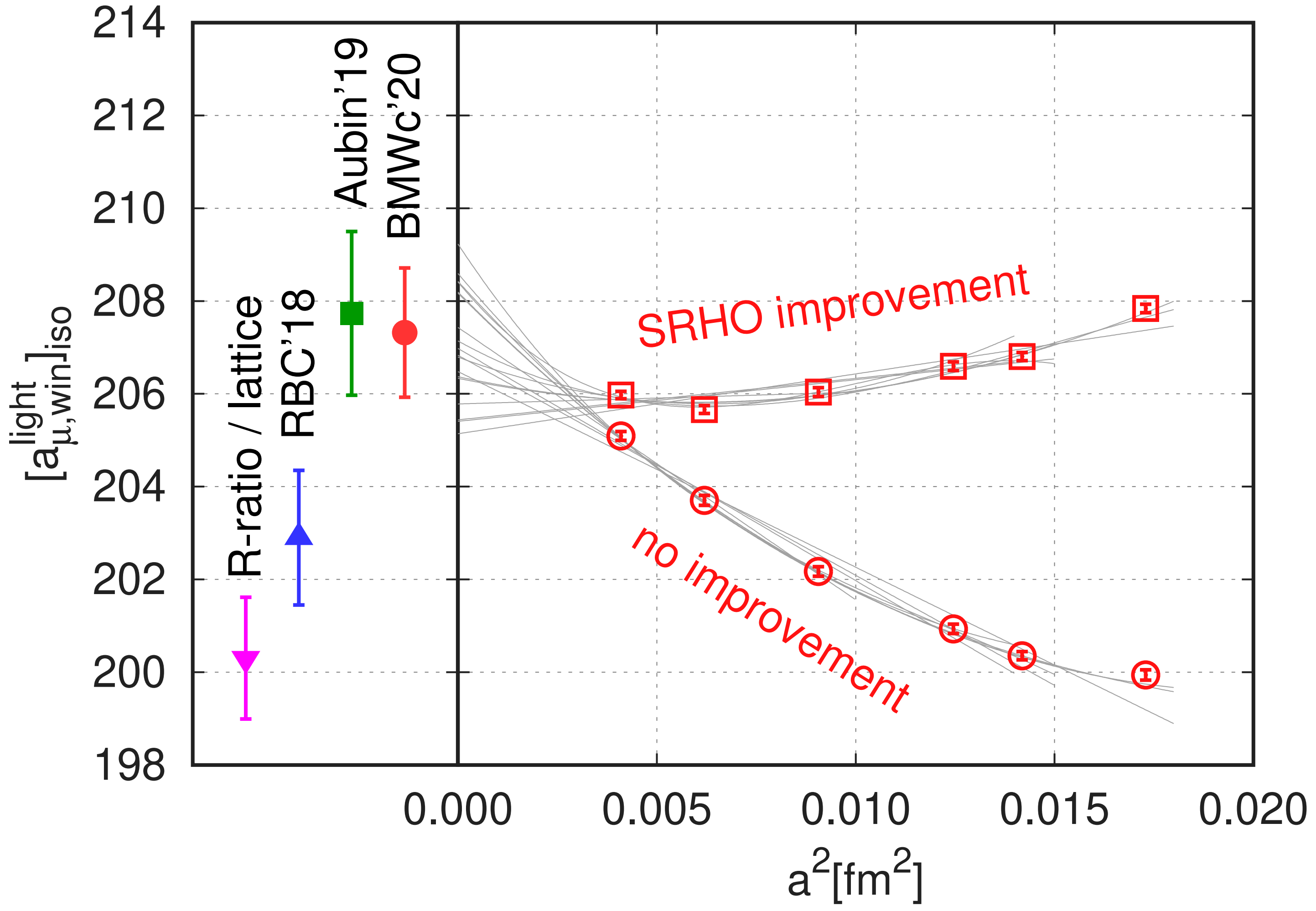
$$a_\mu^{\mathrm{HVP,LO}} = 4\,\alpha^2 \int_0^\infty dt\, C(t)\, \tilde{w}(t)$$

In 2020 WP:

- Lattice HVP average at $2.6\,\%$ total uncertainty: $a_\mu^{\mathrm{HVP,LO}} = 711.6\,(18.4) \times 10^{10}$
- BMW 20 [Sz. Borsanyi et al, arXiv:2002.12347, 2021 Nature] first LQCD calculation with sub-percent ($0.8\,\%$) error in tension with data-driven HVP (2.1σ)
- Further tensions for intermediate window:

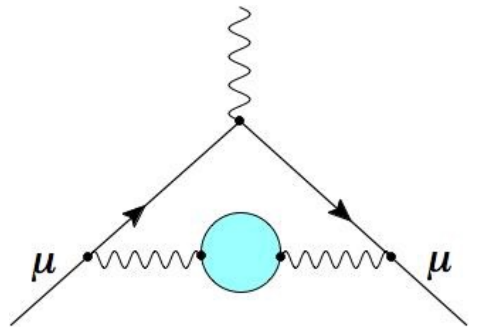
  -3.7σ tension with data-driven evaluation

  -2.2σ tension with RBC/UKQCD18



Staggered fermions:

- taste-breaking effects (which yield taste splittings) are significant (sometimes dominant) source of discretization errors
- possible to use EFT schemes (ChPT, Chiral Model, MLLGS) to correct for taste-splitting effects before taking continuum extrapolation: continuum limit should not be affected



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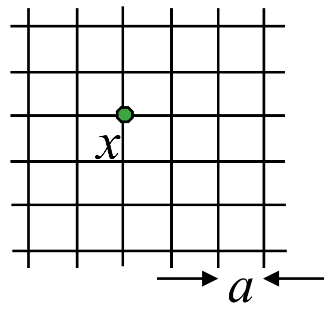
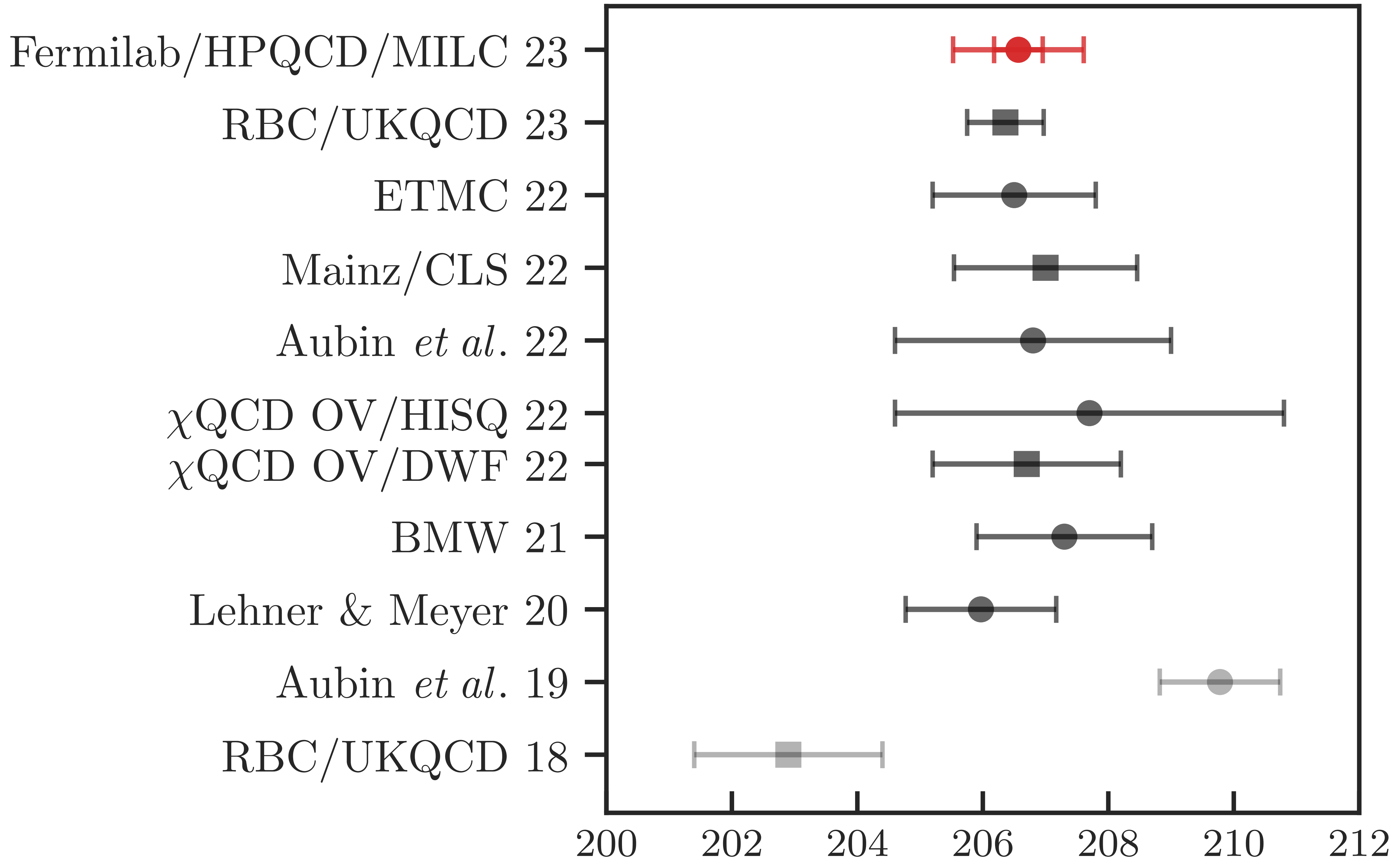
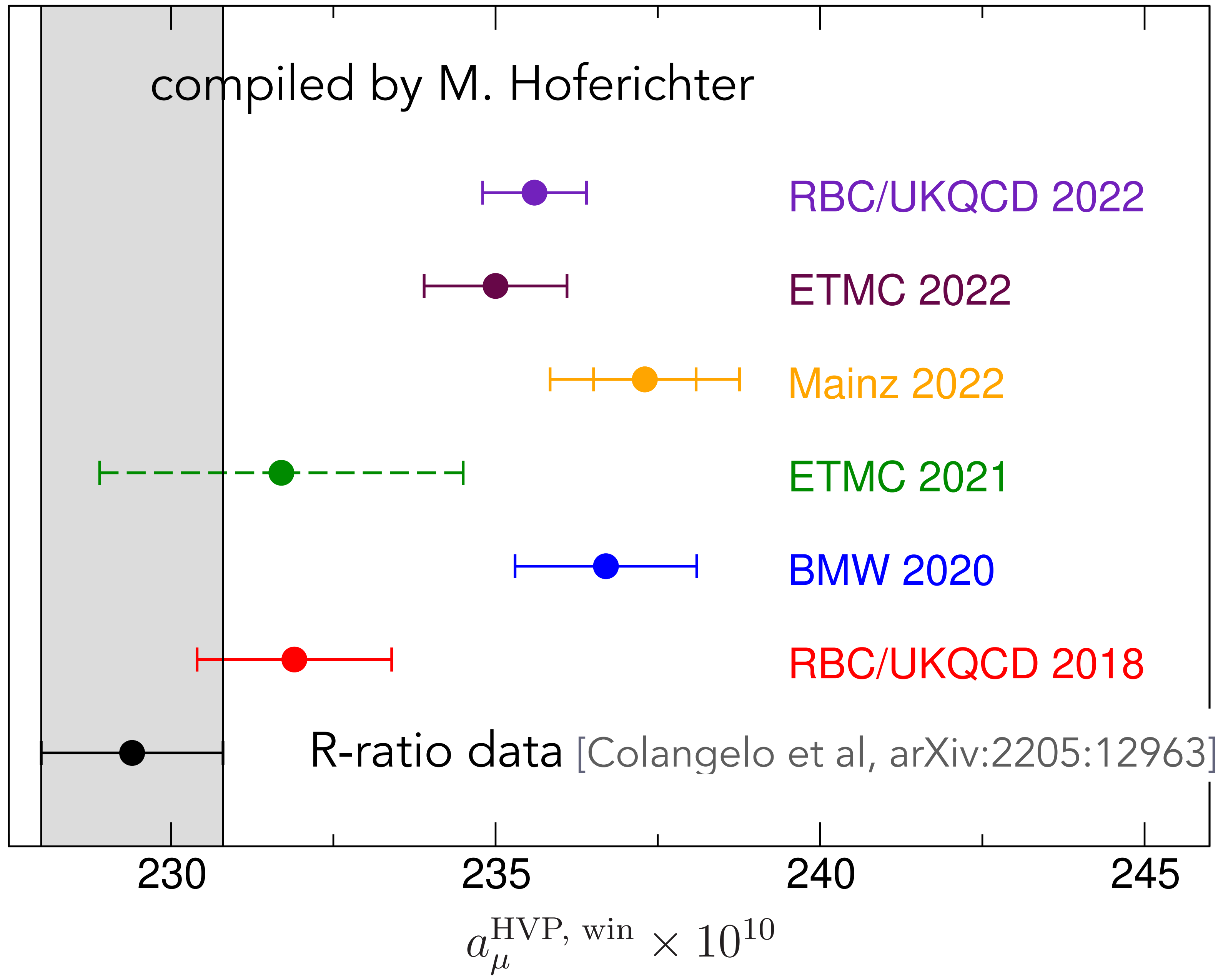


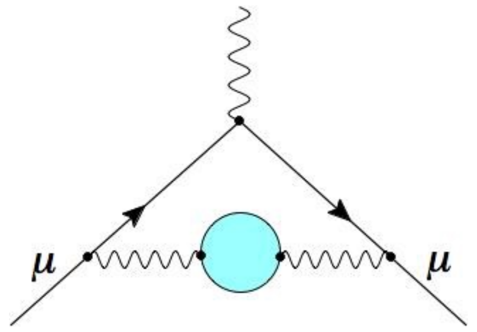
- new results in 2022/2023 for intermediate window, a_μ^W from six different lattice groups.
- blind analyses: Fermilab/HPQCD/MILC + RBC/UKQCD
- lattice-only comparison of light-quark connected contribution to intermediate window:



- LQCD results including all contributions



$\sim (3.5 - 4)\sigma$ tensions between LQCD and (pre-2023) data-driven evaluations



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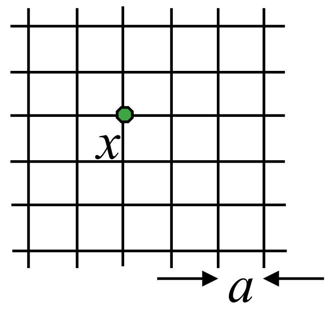

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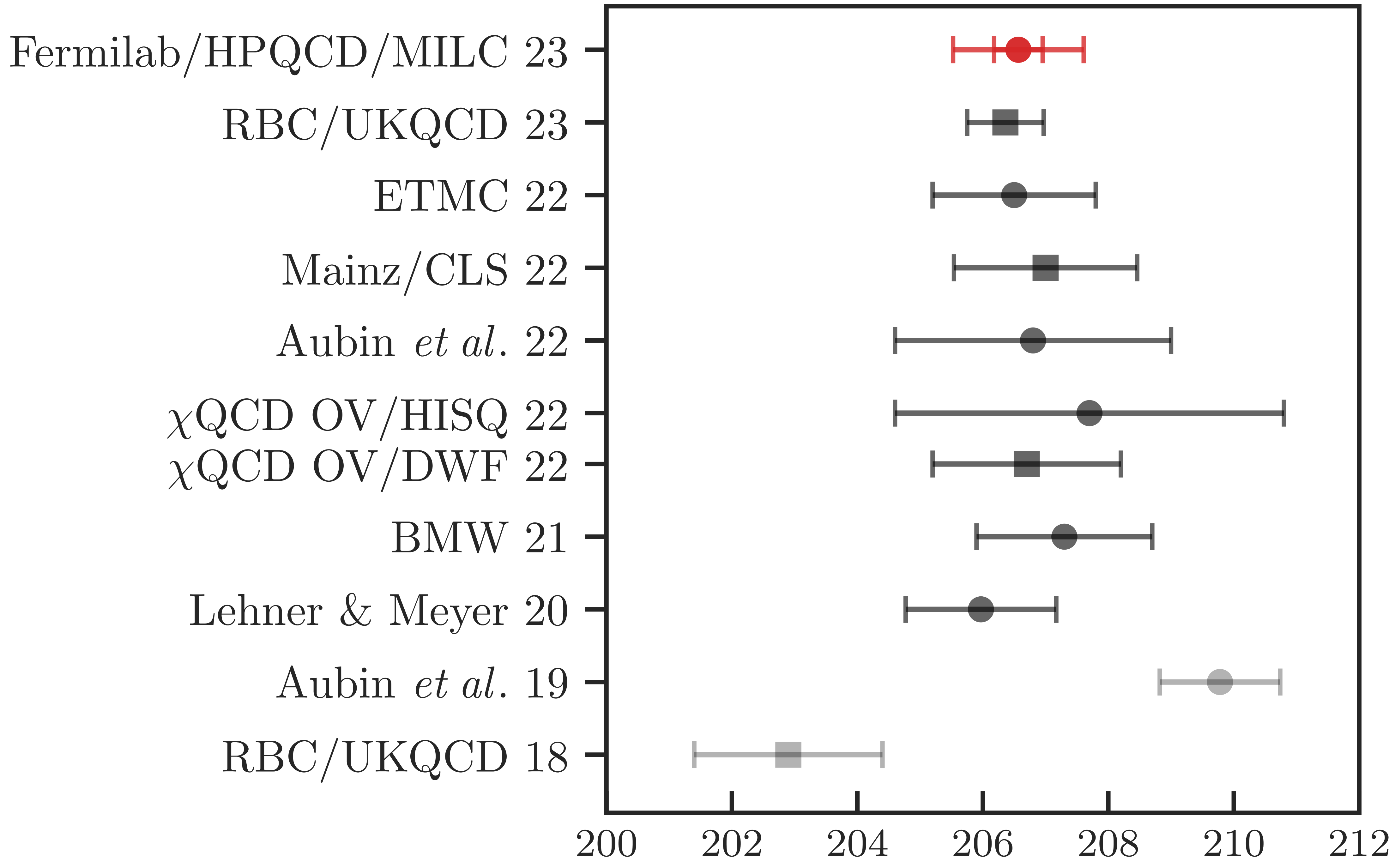
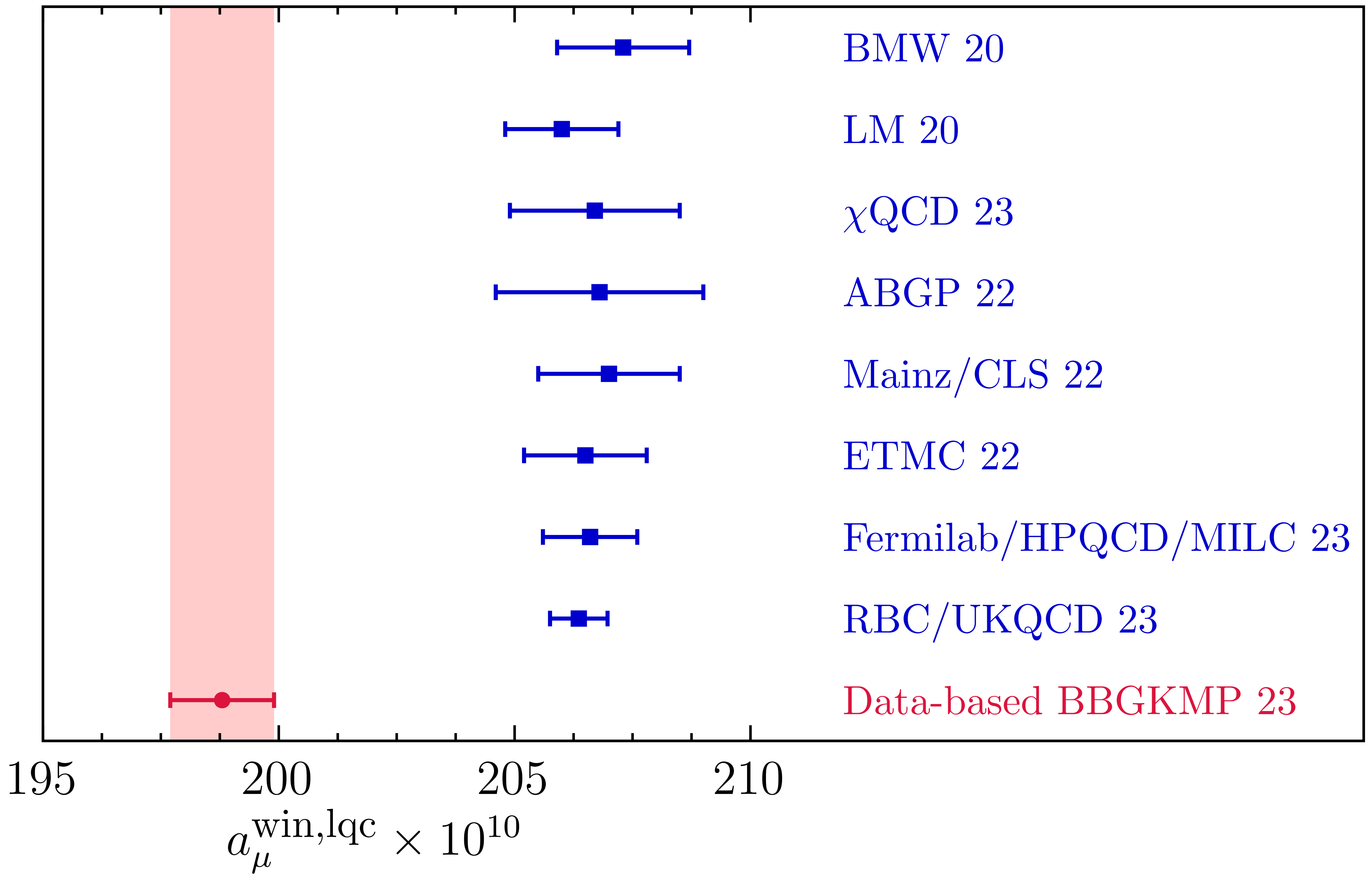
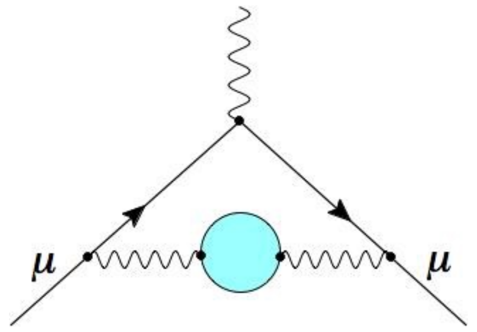


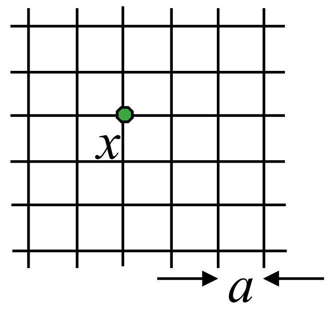
- dispersive evaluation of light-quark connected contribution [G. Benton, et al, arXiv:2306.16808]



➠ primary source of tension



Lattice HVP: outlook



Ongoing work:

Evaluations of short-distance windows [ETMC, RBC/UKQCD]

Proposals for computing more windows:

- Use linear combinations of finer windows to locate the tension (if it persists) in $\sqrt{s}$ [Colangelo et al, arXiv:12963]
- Use larger windows, excluding the long-distance region $t \gtrsim 2\,\mathrm{fm}$ to maximize the significance of any tension [Davies at at, arXiv:2207.04765]

For total HVP:

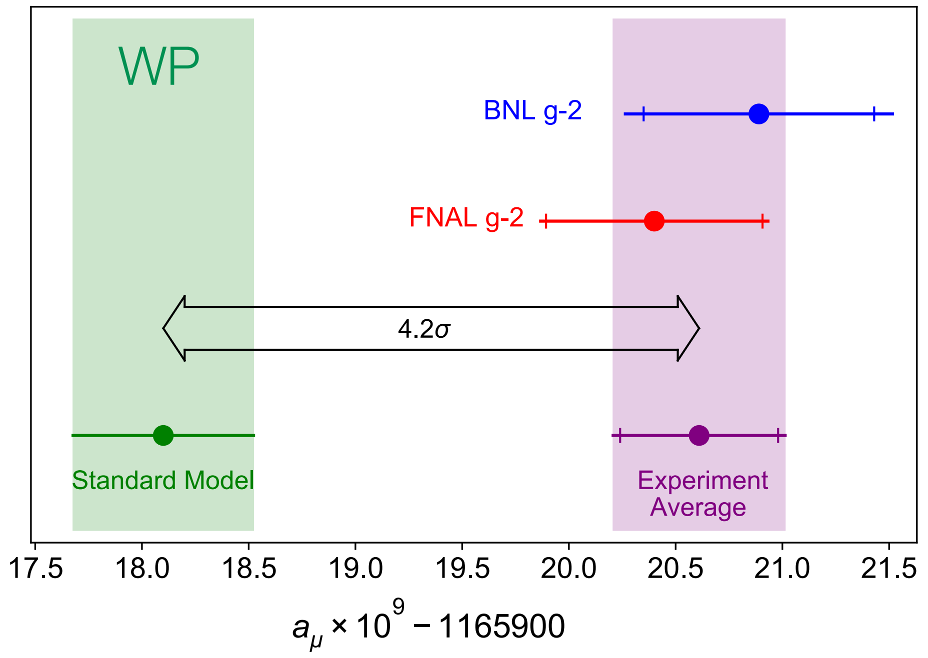
- independent lattice results at sub-percent precision: coming soon!
- Including $\pi\pi$ states for refined long-distance computation (Mainz, RBC/UKQCD, FNAL/MILC)
- include smaller lattice spacings to test continuum extrapolations

➠ if no tensions between independent lattice results, $\sim 0.5\,\%$ feasible

Near-term Timeline


Physics Reports

The anomalous magnetic moment of the muon in the Standard Model



Theory Initiative:

- CMD-3 seminar (virtual): 27 March 2023 at 8:00am US CDT
- 2nd CMD-3 discussion meeting
- 8/9/2023: Status of Muon g-2 Theory in SM

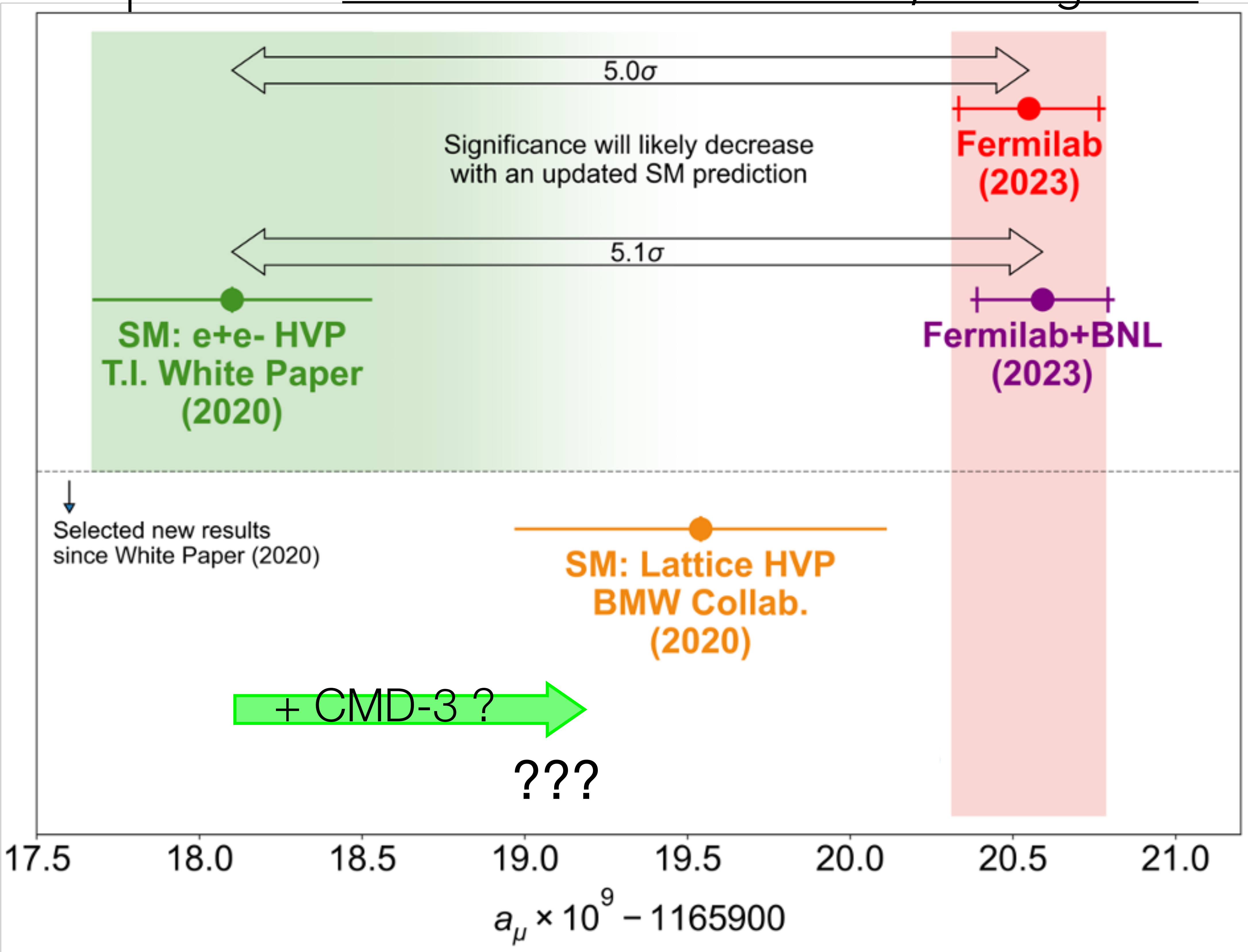
★ TI workshops:

Jun 2021 @ KEK (virtual) Sep 2022 @ Higgscentre
Sep 2023 @ Bern Sep 2024 @ KEK or KMI

muon g-2: SM theory vs experiment

- The Fermilab experiment released the measurement result from their run 2&3 data on 10 Aug 2023. [D. Aguillard et al, 2308.06230]
- Run 6 completed summer 2023, final measurement result expected in 2025

adapted from J. Mott @ Scientific Seminar, 10 Aug 2023



← consolidated LQCD prediction (coming soon)

← new measurements by BaBar, KLOE, SND,….

Summary & Outlook

$m_{ud},\ m_u/m_d,\ m_s$ ⬅ $a_\mu^{\text{HVP LO}}$ a_μ^{HLbL} d_n MEs for light nuclei

$m_c,\ m_b$

$\langle \bar{B}^0_q | \mathcal{O}_i^{\Delta B=2} | B^0_q \rangle$ PDFs, GPDs, TMDs,.. $\langle NN | O_i | NN \rangle$

$\alpha_{\overline{MS}}(m_z)$

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$ $\Lambda_b \to (p, \Lambda_c, \Lambda)\, \ell\nu$ $D_s \to \ell\nu\gamma$ $B \to \pi\pi\, \ell\nu\ \ldots$

$\hat{B}_K, \ldots$ nucleon form factors, .. $K^+ \to \ell^+\nu\,(\gamma)$ $B \to X_c \ell \nu$, other inclusive decay rates, $\ldots$

$f_{B_{(s)}}\ , \ldots$ $g_A,\ g_T,\ g_S$

$\pi\pi \to \rho$ $\pi \to \ell^+\ell^-,\ K \to \gamma\gamma$

$f^{K\to\pi}(q^2)$ $f^{B\to D}_{+,0,T}(q^2), \ldots$ $\langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ $K^+ \to \pi^+\ell^+\ell^-$ $\ldots$

$f_{K^\pm}$ $f^{B\to\pi}_{+,0,T}(q^2), \ldots$ $\langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ ϵ'/ϵ $\Delta M_K,\ \epsilon_K$ $K^+ \to \pi^+\nu\bar{\nu}$

Complexity →

- ✓ LQCD flagship results
- First complete LQCD results, large(ish) errors
- First results, physical params, incomplete systematics
- new methods, pilot projects, unphysical kinematics
- new ideas, first studies

Summary & Outlook

$m_{ud},\, m_u/m_d,\, m_s$ $a_\mu^{\rm HVP\ LO}$ $a_\mu^{\rm HLbL}$ d_n MEs for light nuclei

$m_c,\, m_b$ $\langle \bar B_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$ PDFs, GPDs, TMDs,..

$\alpha_{\overline{MS}}(m_z)$ $\langle NN | O_i | NN \rangle$

$\langle \bar D^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$ $\Lambda_b \to (p, \Lambda_c, \Lambda)\, \ell\nu$ $D_s \to \ell\nu\gamma$ $B \to \pi\pi\, \ell\nu$ ...

$\hat B_K, \ldots$ nucleon form factors, .. $K^+ \to \ell^+\nu\,(\gamma)$ $B \to X_c \ell\nu$, other inclusive decay rates, ...

$f_{B_{(s)}}, \ldots$ $g_A,\, g_T,\, g_S$

$\pi\pi \to \rho$ $\pi \to \ell^+\ell^-,\, K \to \gamma\gamma$

$f^{K\to\pi}(q^2)$ $f_{+,0,T}^{B\to D}(q^2), \ldots$ $\langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ $K^+ \to \pi^+ \ell^+ \ell^-$ ...

$f_{K^\pm}$ $f_{+,0,T}^{B\to\pi}(q^2), \ldots$ $\langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ ϵ'/ϵ $\Delta M_K,\, \epsilon_K$ $K^+ \to \pi^+ \nu\bar\nu$

Complexity →

✓ LQCD flagship results

First complete LQCD results, large(ish) errors

First results, physical params, incomplete systematics

new methods, pilot projects, unphysical kinematics

new ideas, first studies

Outlook

- Experimental program beyond 2025:
  - J-PARC: Muon g-2/EDM
  - Fermilab: future muon campus experiments?
  - Belle II, BESIII, Novosibirsk,…
  - Chiral Belle (?)
- Data-driven/dispersive program beyond 2025:
  - development of NNLO MC generators
  - for HLbL, improved experimental/lattice inputs together with further development of dispersive approach
- MUonE will provide a space-like measurement of HVP
- Lattice QCD beyond 2025:
  - access to future computational resources (coming Exascale) will enable improvements of all errors (statistical and systematic)
  - concurrent development of better methods and algorithms (gauge-field sampling, noise reduction) will accelerate progress
  - beyond g-2: a rich program relevant for all areas of HEP

Topics not covered (incomplete list)

- PDFs: huge progress and much new theoretical work since 2013
  [X. Ji arXiv:1305.1535, PRL 2013]
- hot QCD
- hadron spectroscopy, exotics, scattering phase shifts
- inclusive decay rates (appendix)
- Semileptonic B-meson decay form factors + baryons ffs
- B mixing
- First and second row CKM unitarity
- QED corrections and radiative decay rates
- kaon mixing, $\Delta M_K, \epsilon'$
- nucleon matrix elements and charges
- two and few nucleon systems
- …

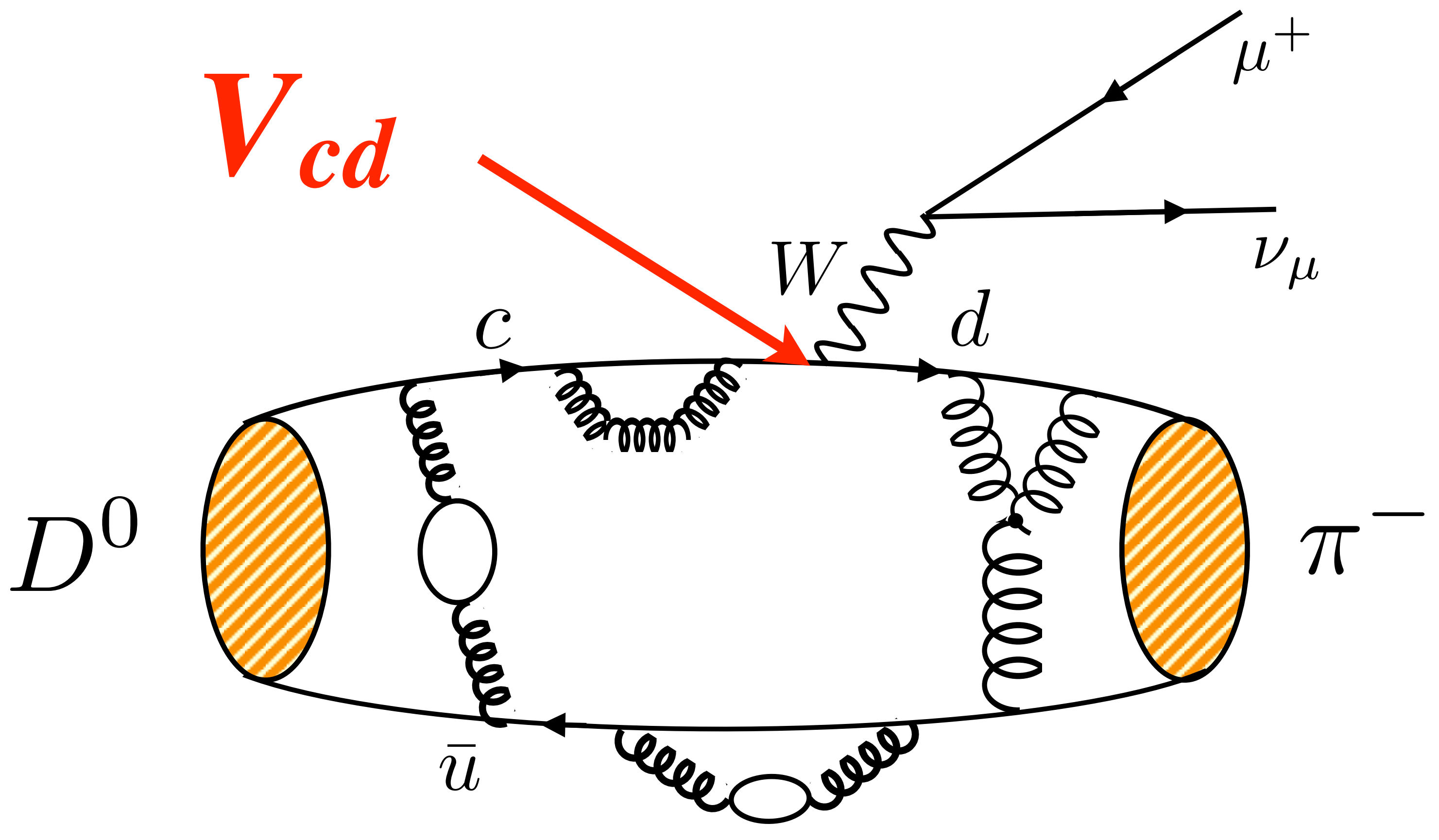
Thank you!

Farah Willenbrock

Appendix

Semileptonic D, D_s meson decay

example: $D^0 \to \pi^- \mu^+ \nu_\mu$



$$\frac{d\Gamma(D^0 \to \pi^- \mu^+ \nu_\mu (\gamma))}{dq^2} = (\text{known}) \times S_{\text{EW}} (1 + \delta_{\text{EM}}) \times |V_{cd}|^2 \times f_+(q^2)^2$$

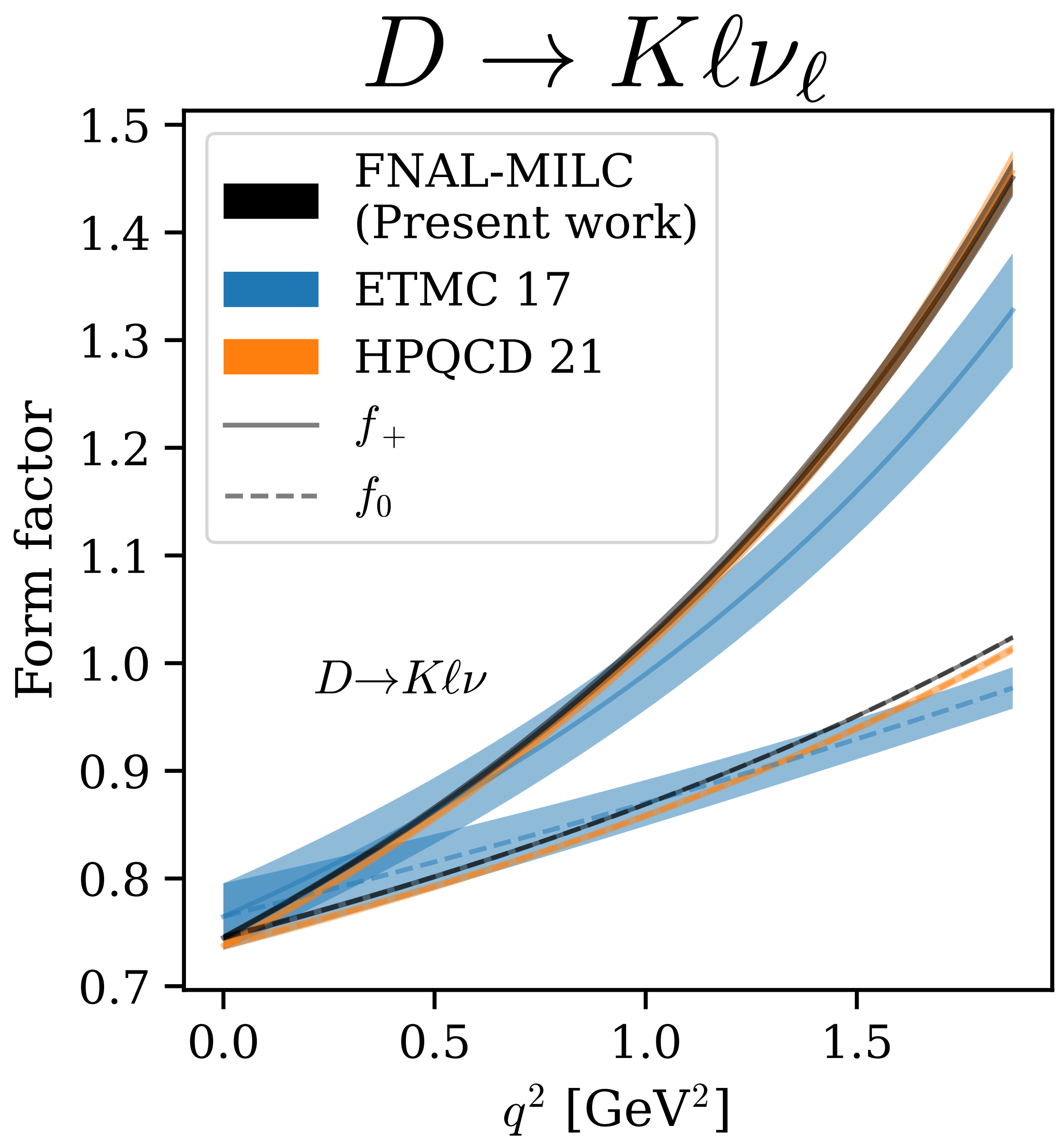
- calculate the form factors over entire q^2 range + model-independent parametrization of shape (z-expansion).
- account for EW+EM corrections in experimental rate
  - EW: [Sirlin, Nuc. Phys. 1982] ~ 1.8%
  - EM: Structure dependent: has not been calculated!
    - ➠ use guidance from $K_{\ell 3}$ ~ 1% take correction as uncertainty, inflated by $\times$ 2
  - Long distance: [Kinoshita, PRL 1959] ~ 2.4% ➠ removed with PHOTOS

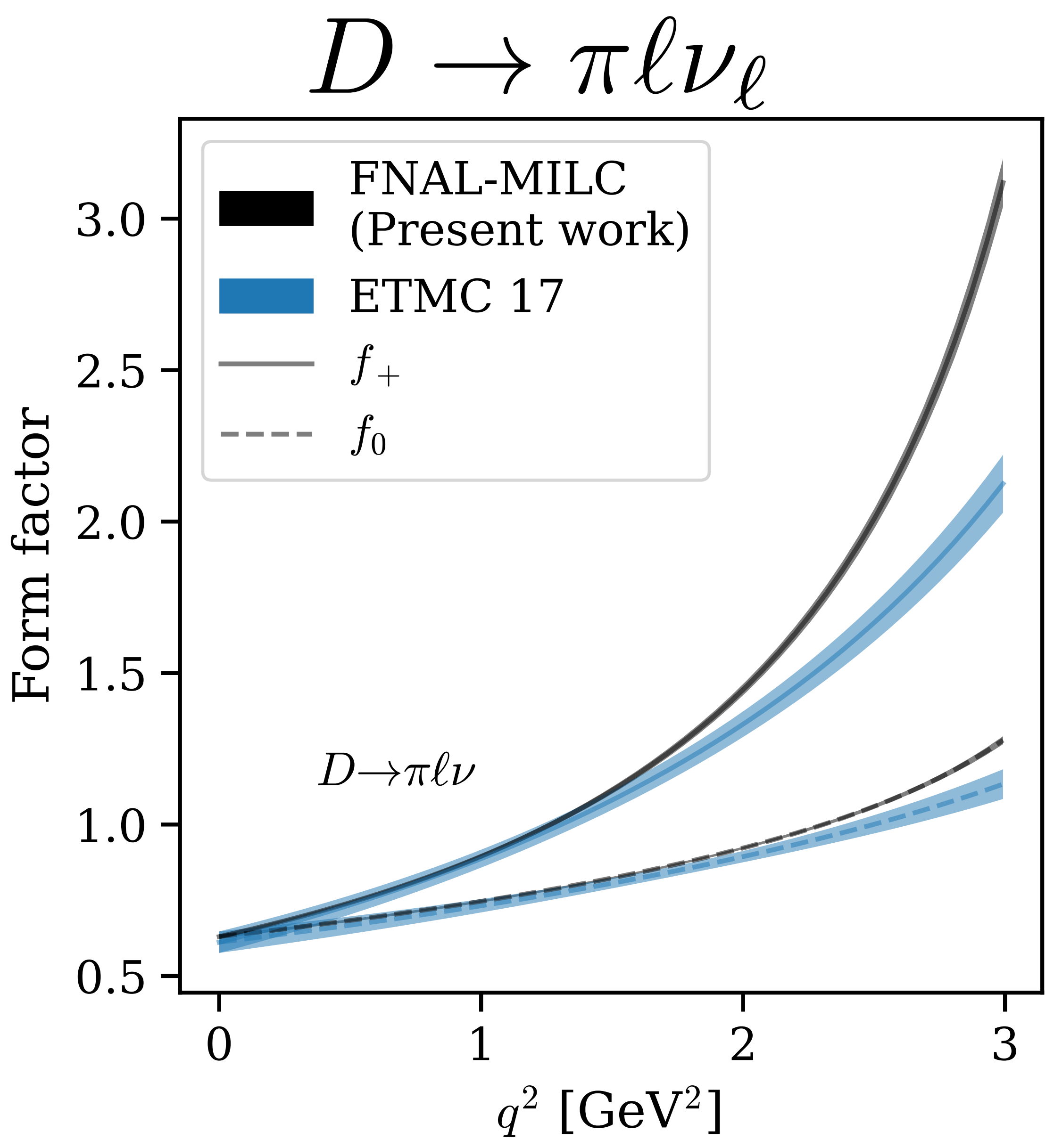
Semileptonic D meson decay form factors

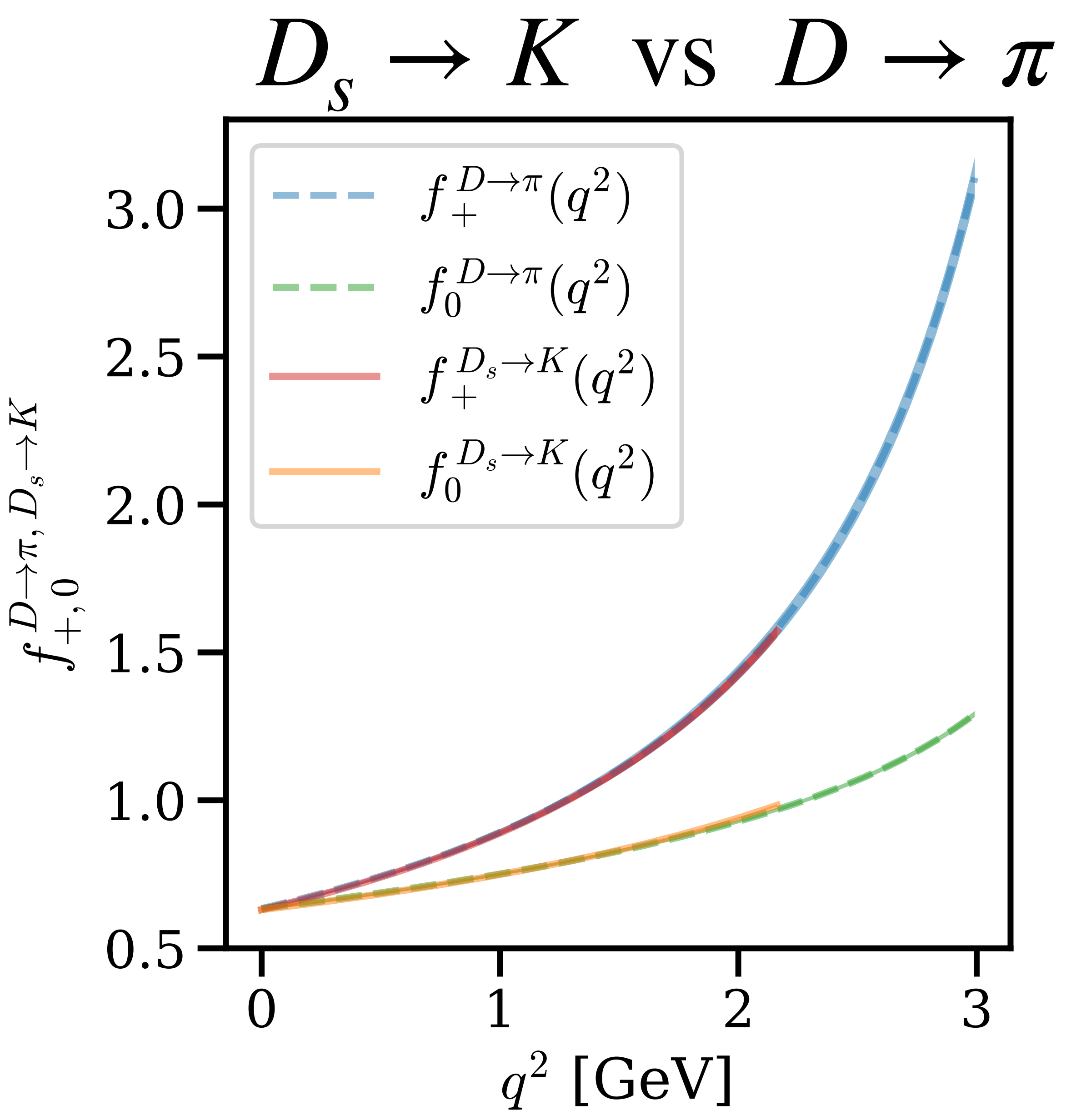
ETM [arXiv:1706.03017, PRD 2017; arXiv:1706.03657, EPJC 2017]
HPQCD [arXiv:2104.09883,2207.12468]

FNAL/MILC [arXiv:2212.12648]

Will Jay (MIT)





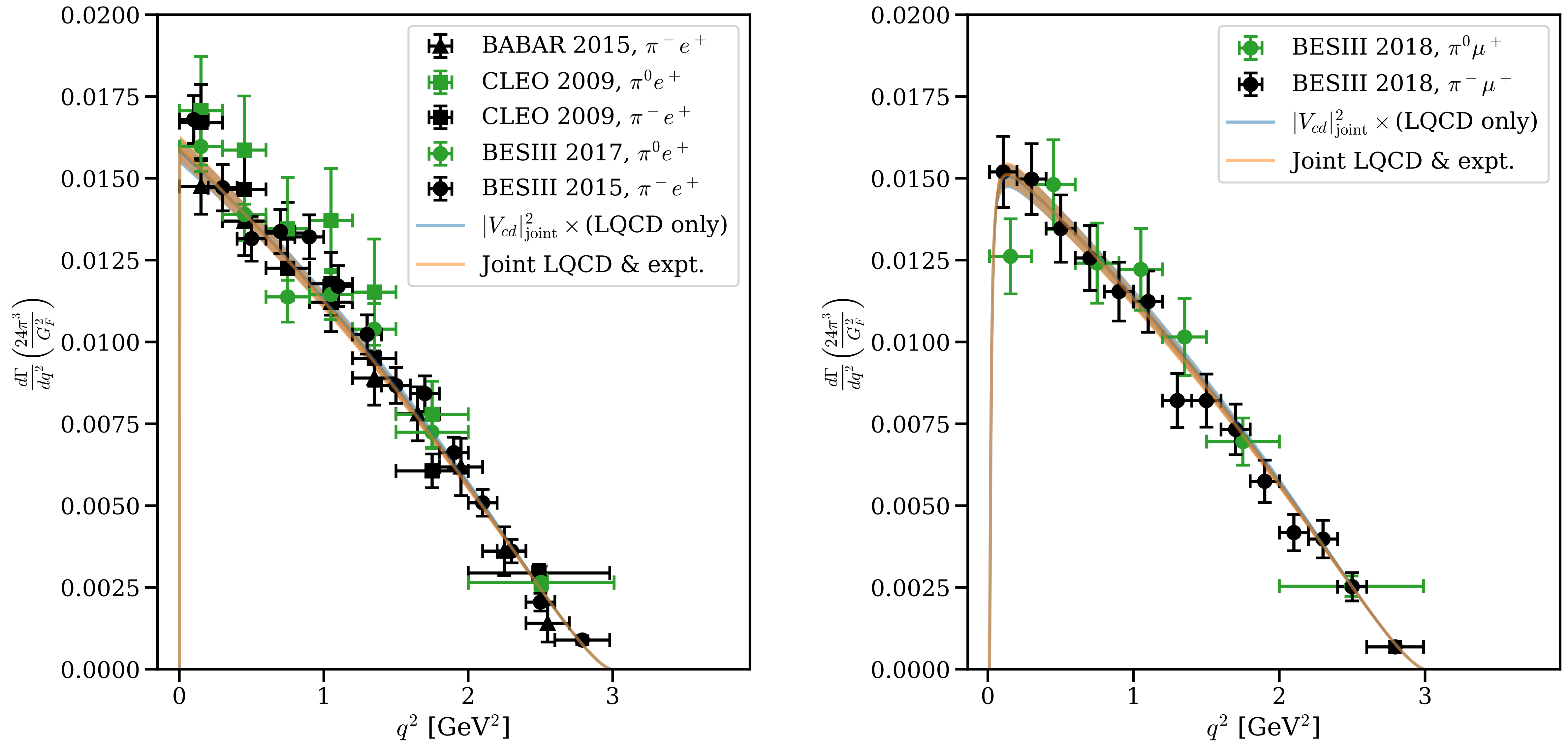


⭐ Compare shape of LQCD form factor with experiment and fit LQCD form factors + experimental diff. rates to determine $|V_{cd}|$ or $|V_{cs}|$.

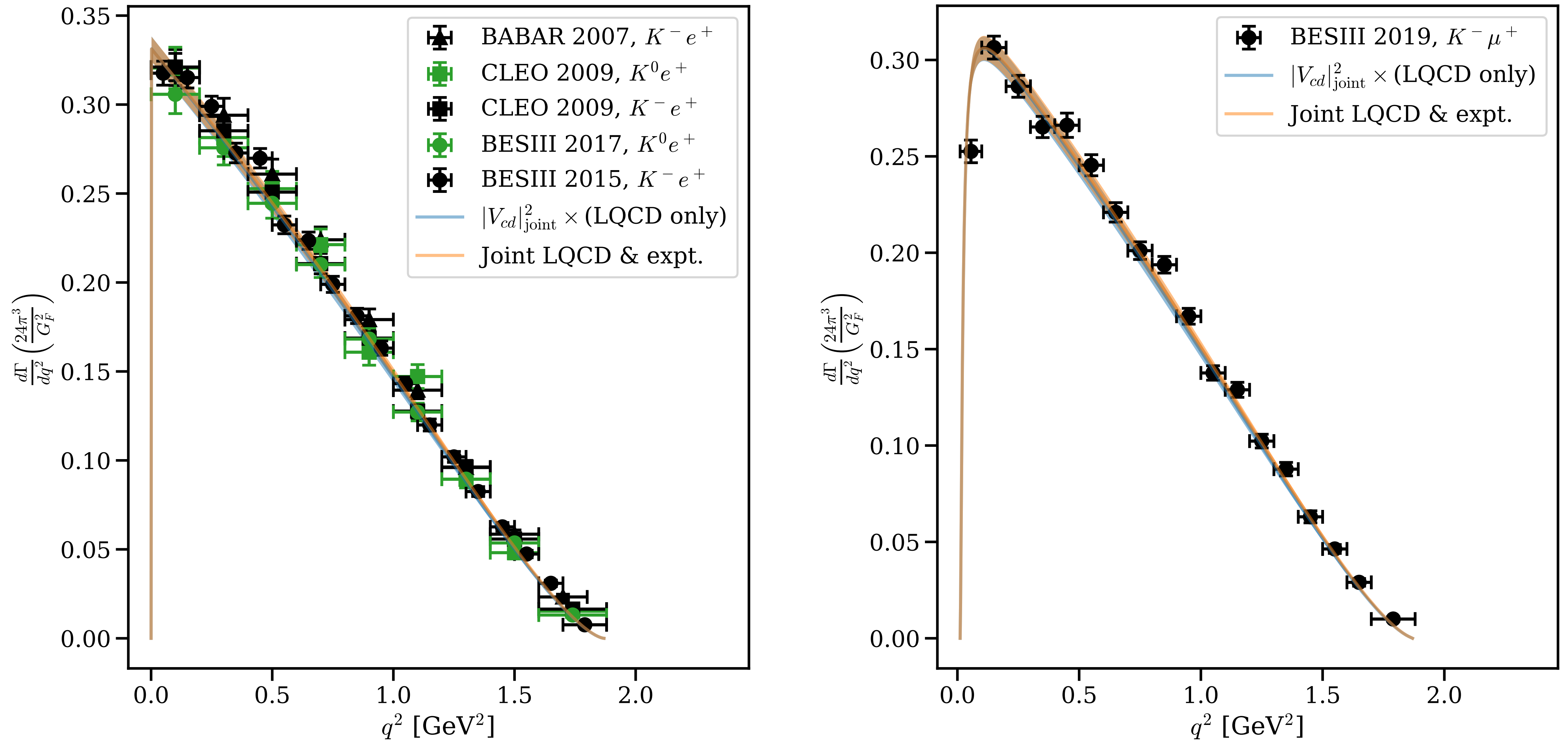
joint fits to exp. $d\Gamma/dq^2$ + LQCD form factors

FNAL/MILC [arXiv:2212.12648]

$D \to \pi \ell \nu_\ell$



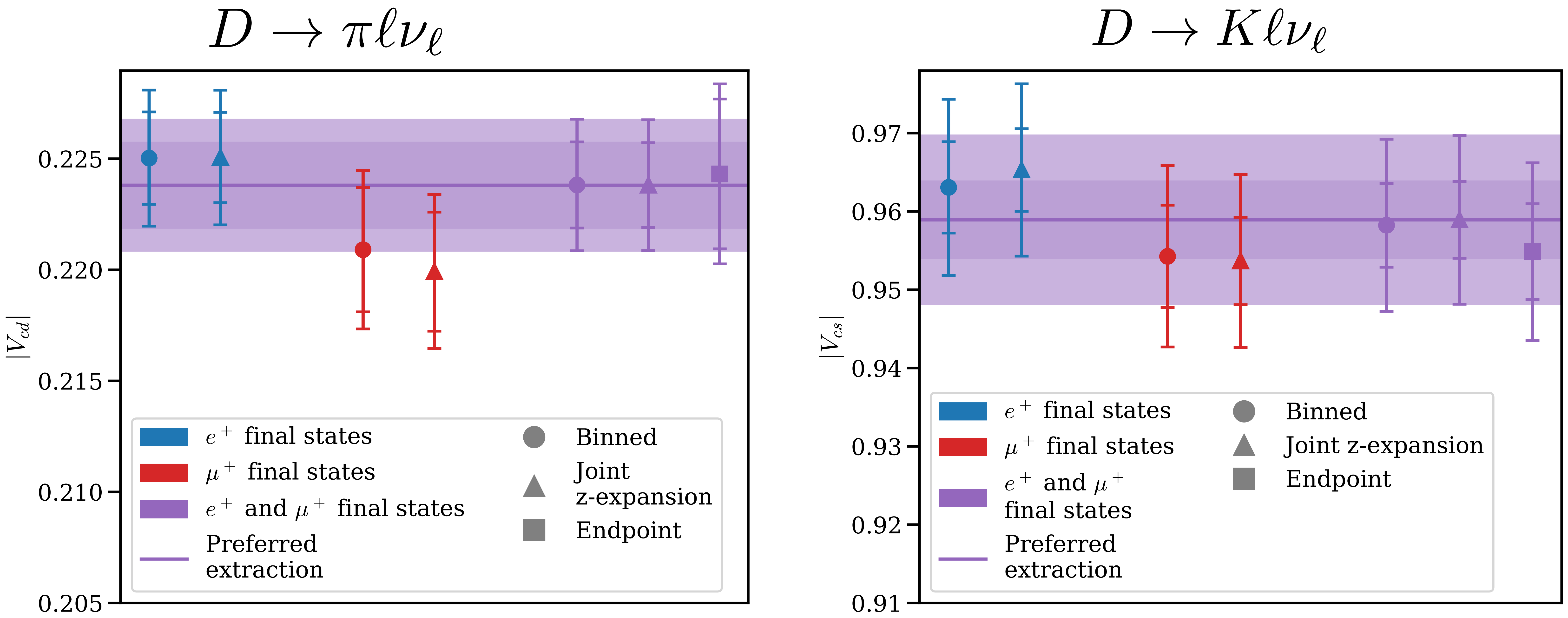
$D \to K \ell \nu_\ell$



- Compare shape of LQCD form factor with experiment and fit LQCD form factors + experimental diff. rates to determine $|V_{cd}|$ or $|V_{cs}|$.
- can also extract CKM elements from exp. average of $|V_{cq}| f_+(0)$
- similar analysis with Λ_c decay form factors [Meinel, arXiv:1611.09696, 2017 PRL].
- also: D-meson tensor form factors [ETM, arXiv:1803.04807, 2018 PRD]

$|V_{cd}|$ and $|V_{cs}|$ determinations

FNAL/MILC [arXiv:2212.12648]



- ☆ Compare shape of LQCD form factor with experiment and fit LQCD form factors + experimental diff. rates to determine $|V_{cd}|$ or $|V_{cs}|$ or perform binned analysis.
- ☆ can also extract CKM elements from exp. average of $|V_{cq}| f_+(0)$
- ☆ similar analysis with Λ_c decay form factors [Meinel, arXiv:1611.09696, 2017 PRL].
- ☆ also: D-meson tensor form factors [ETM, arXiv:1803.04807, 2018 PRD]

$|V_{cd}|$ and $|V_{cs}|$ determinations

For illustration: experimental averages [HFLAV 2019, arXiv:1909.12524, EPJC2021]:

$$[S_{\rm EW}(1+\delta_{\rm EM})]^{1/2}|V_{cs}|f_+^{DK}(0) = 0.7180\,(33)_{\rm exp} \quad [S_{\rm EW}(1+\delta_{\rm EM})]^{1/2}|V_{cd}|f_+^{D\pi}(0) = 0.1426\,(18)_{\rm exp}$$

From joint exp + LQCD fits:

HPQCD [arXiv:2104.09883]

$$|V_{cs}| = 0.9663\,(39)_{\rm exp}(53)_{\rm LQCD}(19)_{\rm EW}(40)_{\rm EM}$$

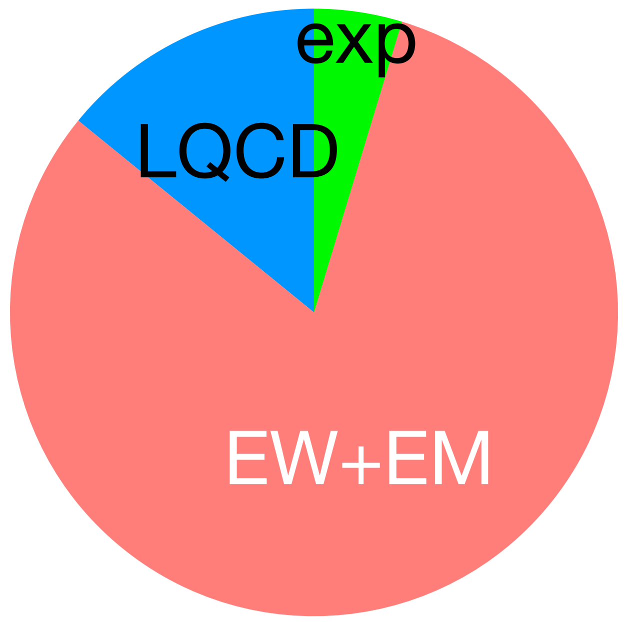
ETM [arXiv:1706.03657, EPJC 2017] $|V_{cd}| = 0.2341(74)_{\rm exp+LQCD}$

FNAL/MILC [arXiv:2212.12648]

$$|V_{cs}| = 0.9589(23)_{\rm exp}(40)_{\rm LQCD}(15)_{\rm EW}(05)_{\rm SIB}[95]_{\rm QED}$$

$$|V_{cd}| = 0.2238(11)_{\rm exp}(15)_{\rm LQCD}(04)_{\rm EW}(02)_{\rm SIB}[22]_{\rm QED}$$

1.1%



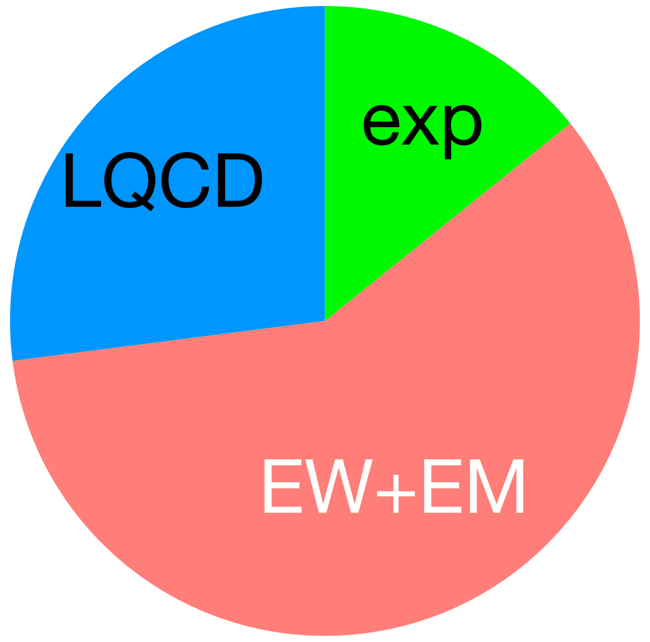
error^2

exp
EW+EM
LQCD

2nd row CKM unitarity test:

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = -0.029(22)$$

1.3%

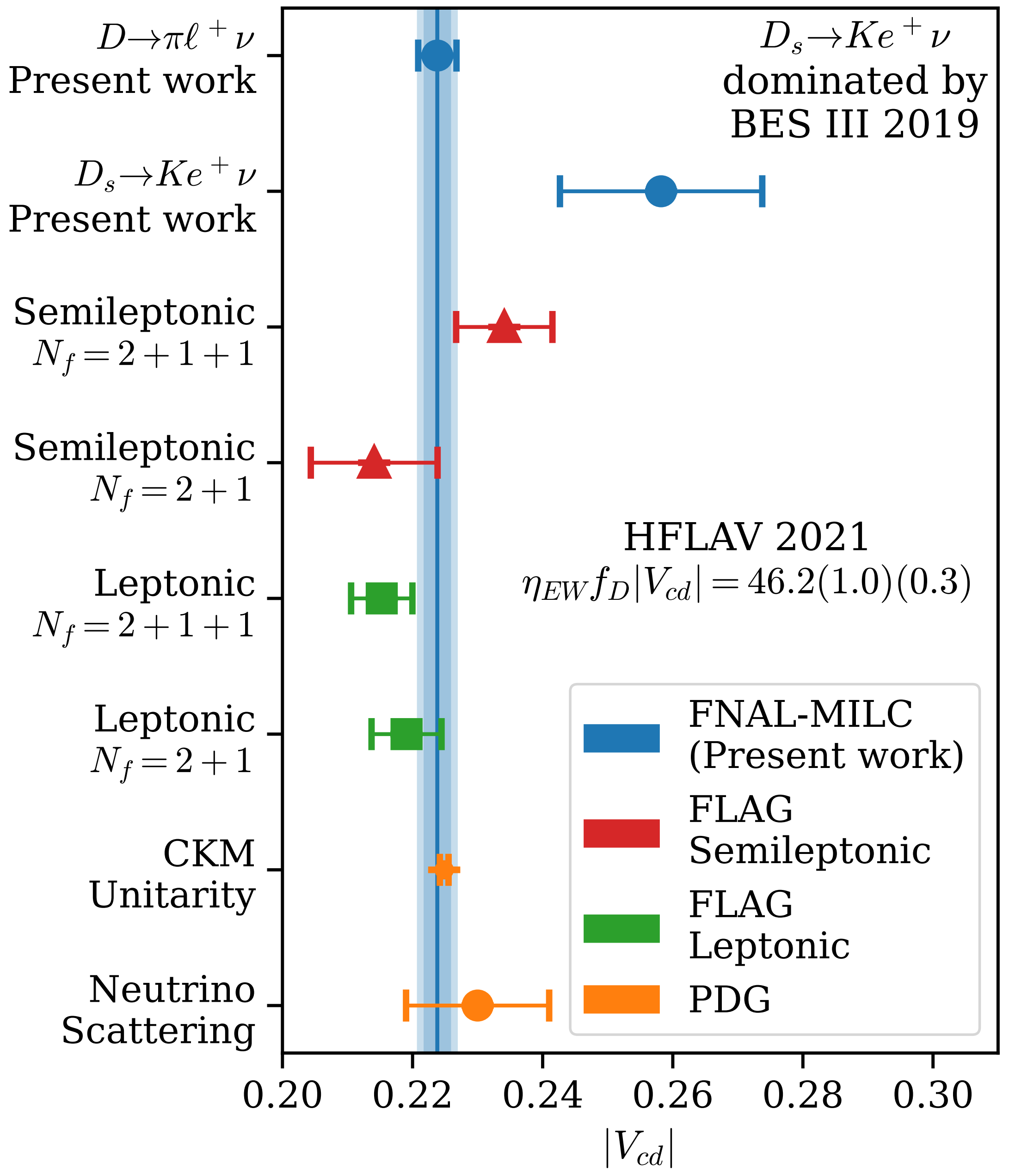


error^2

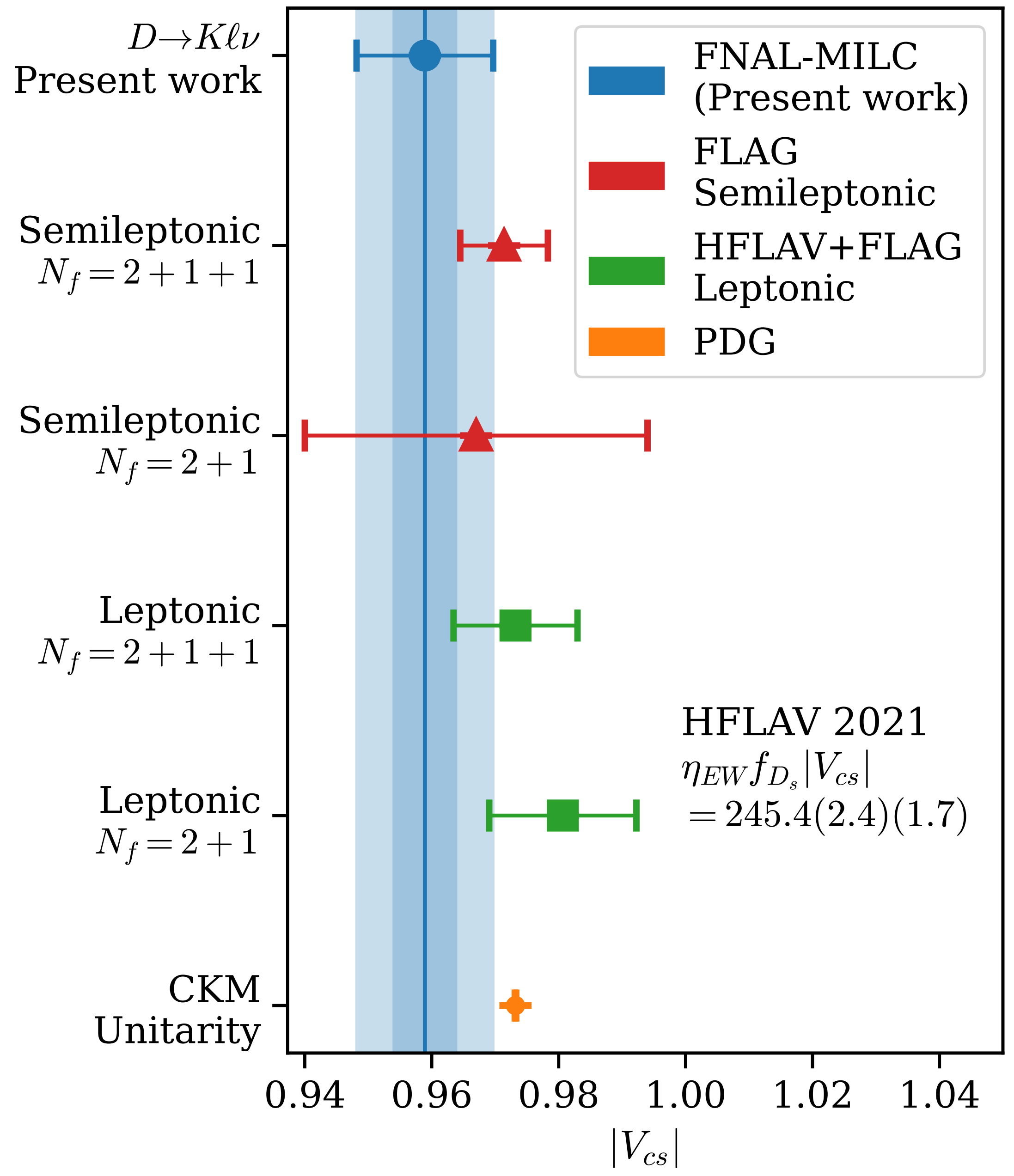
Comparisons

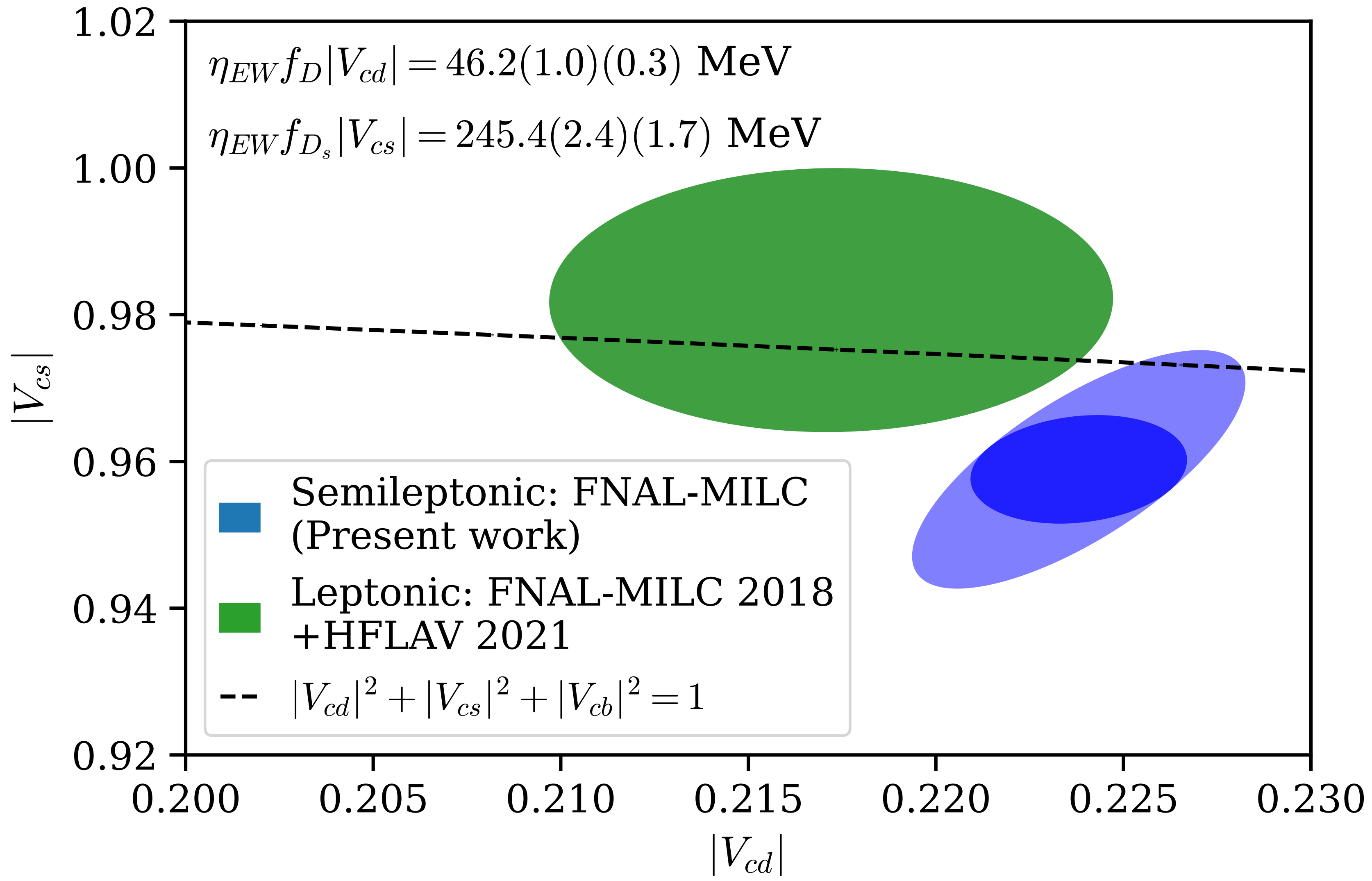
FNAL/MILC [arXiv:2212.12648]

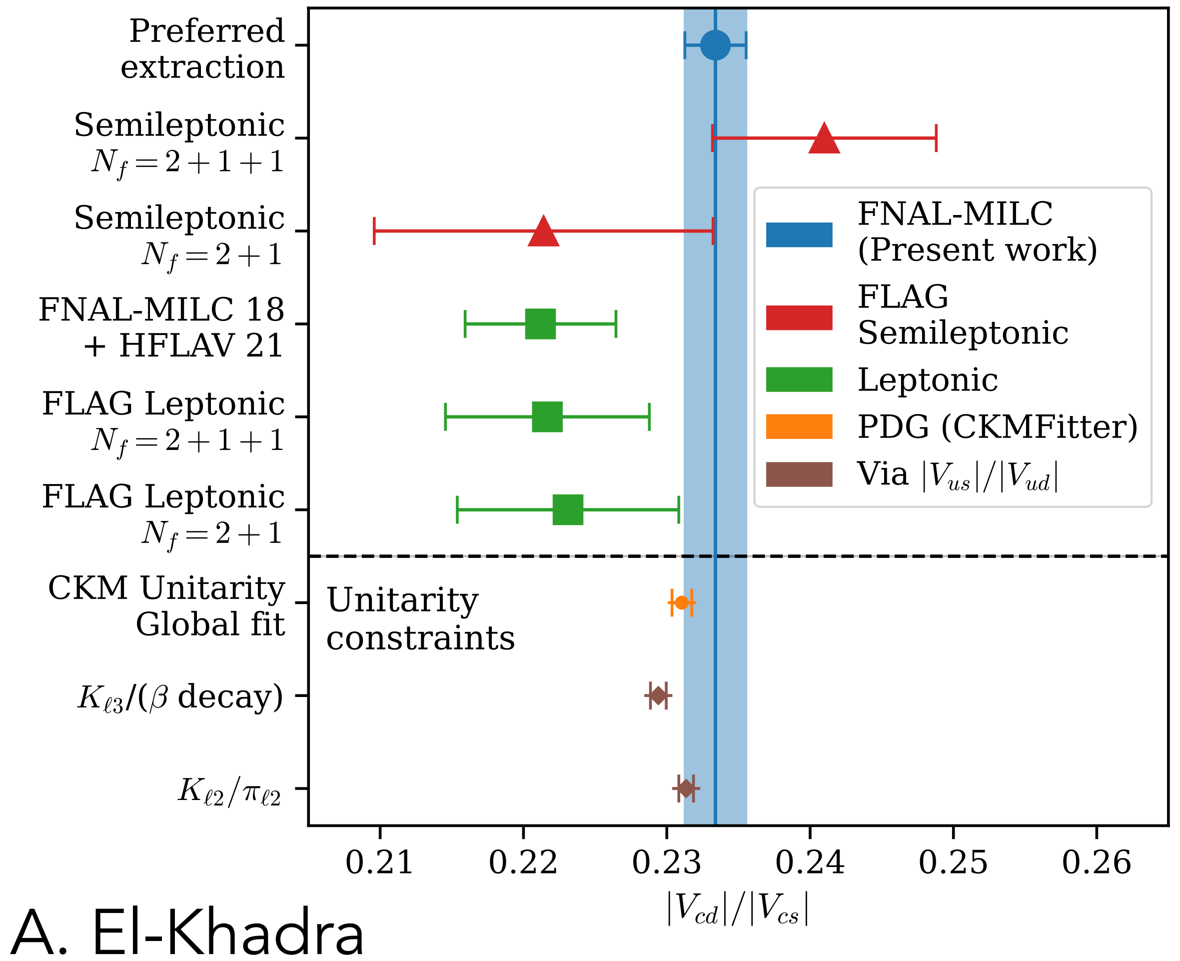
$|V_{cd}|$



$|V_{cs}|$

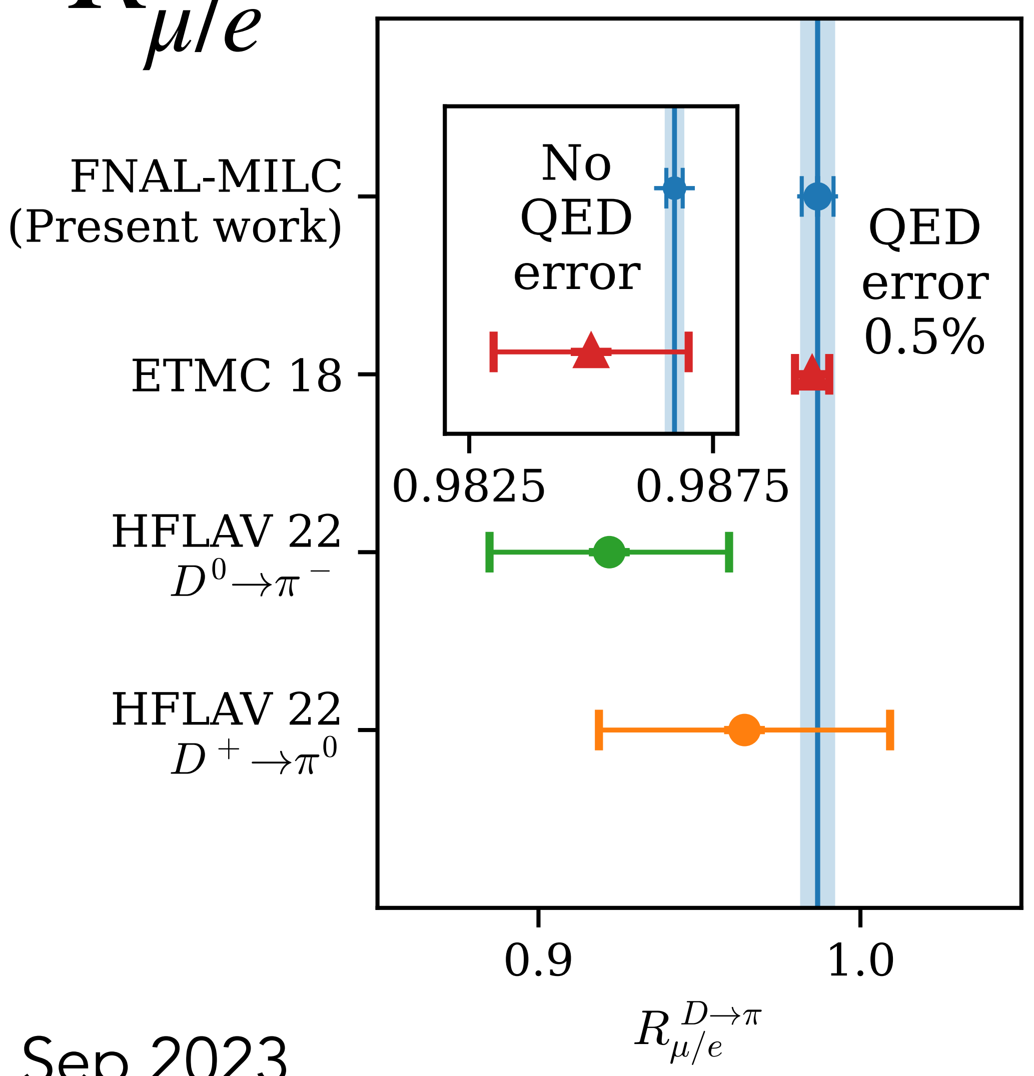




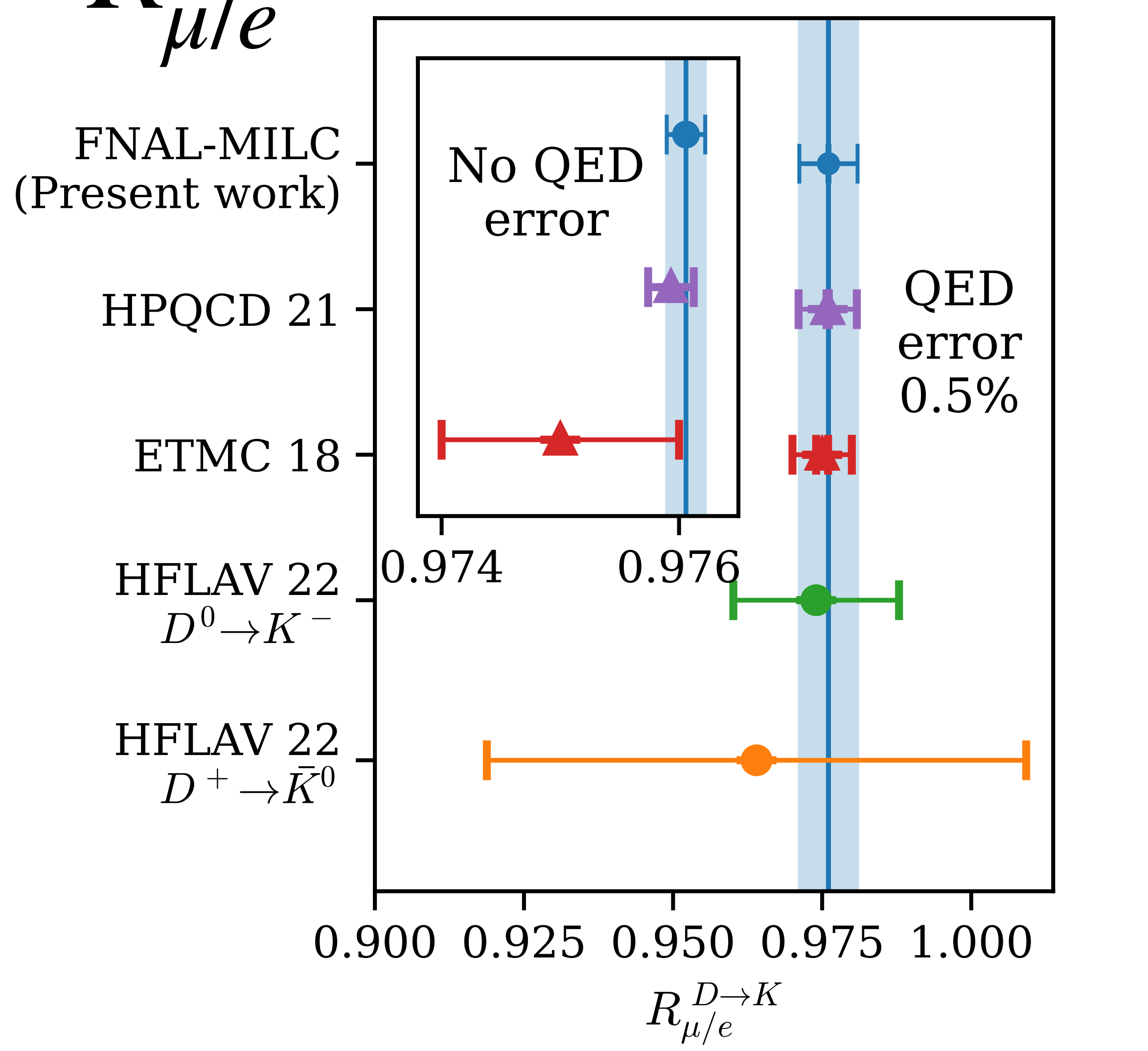


$|V_{cd}/V_{cs}|$

$R_{\mu/e}^{D\to\pi}$



$R_{\mu/e}^{D\to K}$

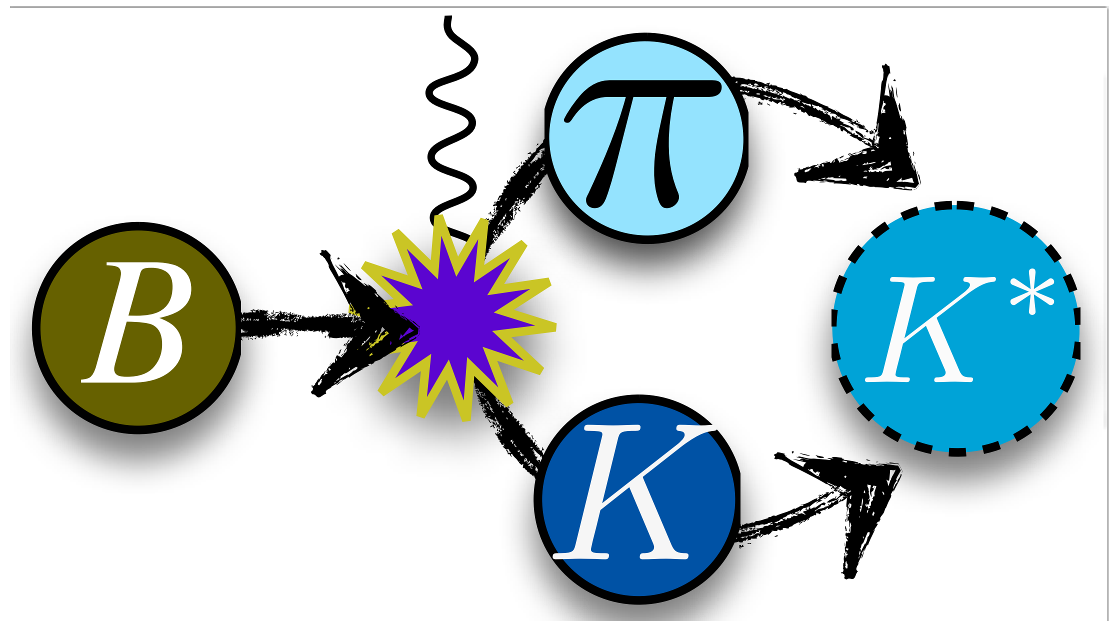


Semileptonic B decays to vector mesons: $B \to K^* \ell\ell$

existing LQCD results for $B \to K^*, B_s \to \phi$ form factors assume stable K^*, ϕ (narrow width approximation)
[R. Horgan et al, arXiv:1310.3887, 1310.3722, 1501.00367]

Formalism for multi-channel 1→ 2 transition amplitudes:
[Briceno, Hansen, Walker-Loud, arXiv:1406.5965, PRD 2015;1502.04314, PRD 2015,…]

weak current



[Figure by R. Briceno]

pilot study [Agadjanov et al, arXiv:1605.03386, NPB 2016]

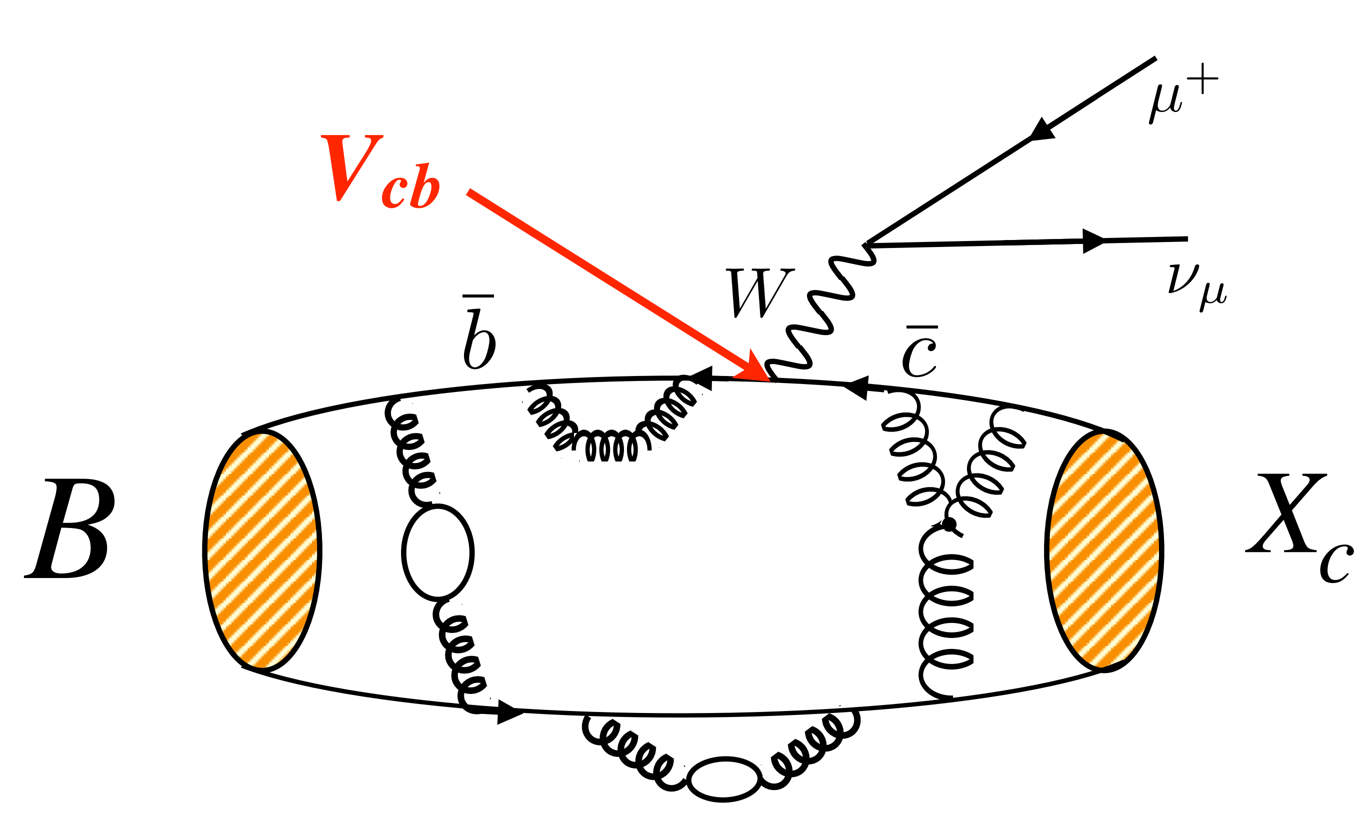
Limitations:
- q^2 reach: small recoil
- invariant mass of two-hadron system: $< 3\ m_H$
- recent work to extend formalism to 3 hadrons
  [M. Hansen et al, arXiv:2101.10246]

studies of $K\pi$ scattering
[G. Rendon et al, arXiv:1811.10750;
D. Wilson et al, arXiv:1904.03188]

preliminary results for $B \to \pi\pi\ell\nu$ form factor with $m_\pi \simeq 320\,\mathrm{MeV}$
[L. Leskovec et al, arXiv:2212.08833]

Inclusive decay rates with lattice QCD

For example: $B \to X_c \ell \nu_\ell$



Sum over final states:

$$X_c = D, D^*, D\pi, D\pi\pi, D^{**}, \ldots$$

Use OPE + pert. QCD to write $d\Gamma$ as a double expansion:

$$d\Gamma \sim \sum_n c_n \frac{\langle O_n \rangle}{m_b^n}$$

- c_n are calculated in perturbation theory
- $\langle O_n \rangle$ are matrix elements of local operators

Target: $d\Gamma \sim |V_{cb}|^2 L^{\mu\nu} W_{\mu\nu}$

$$W_{\mu\nu} = \frac{1}{2M_B} \int d^4x e^{-iqx} \langle B | J_\mu^\dagger(x) J_\nu(0) | B \rangle$$

Start with Euclidean four-point function:

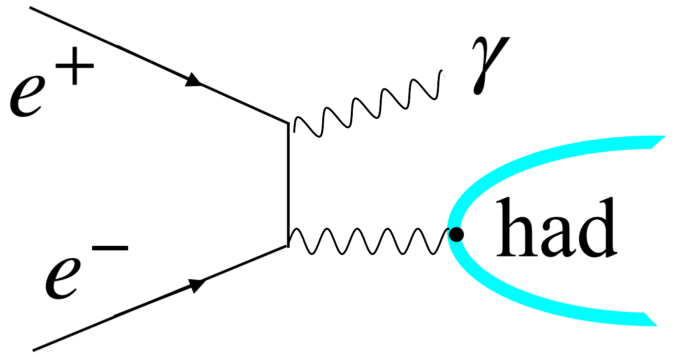
$$C_4(q,\tau) = \sum_x e^{iqx} \frac{1}{2M_B} \langle B | J_\mu^\dagger(x) J_\nu(0) | B \rangle$$

- new methods to perform inverse Laplace transform
  [Liu & Dong (PRL 1994);Liu (PRD 200);Jian et al (1710.11145); Hansen, Meyer, Robaina (1703.01881, PRD 2017); M. Hansen et al, arXiv:1903.06476; P. Gambino & S. Hashimoto, arXiv:2005.13730; J. Bulava et al, arXiv:2111.12774]

- first application to $B \to X_c \ell \nu$
  good agreement with OPE
  [P. Gambino et al, arXiv:2203.11762]

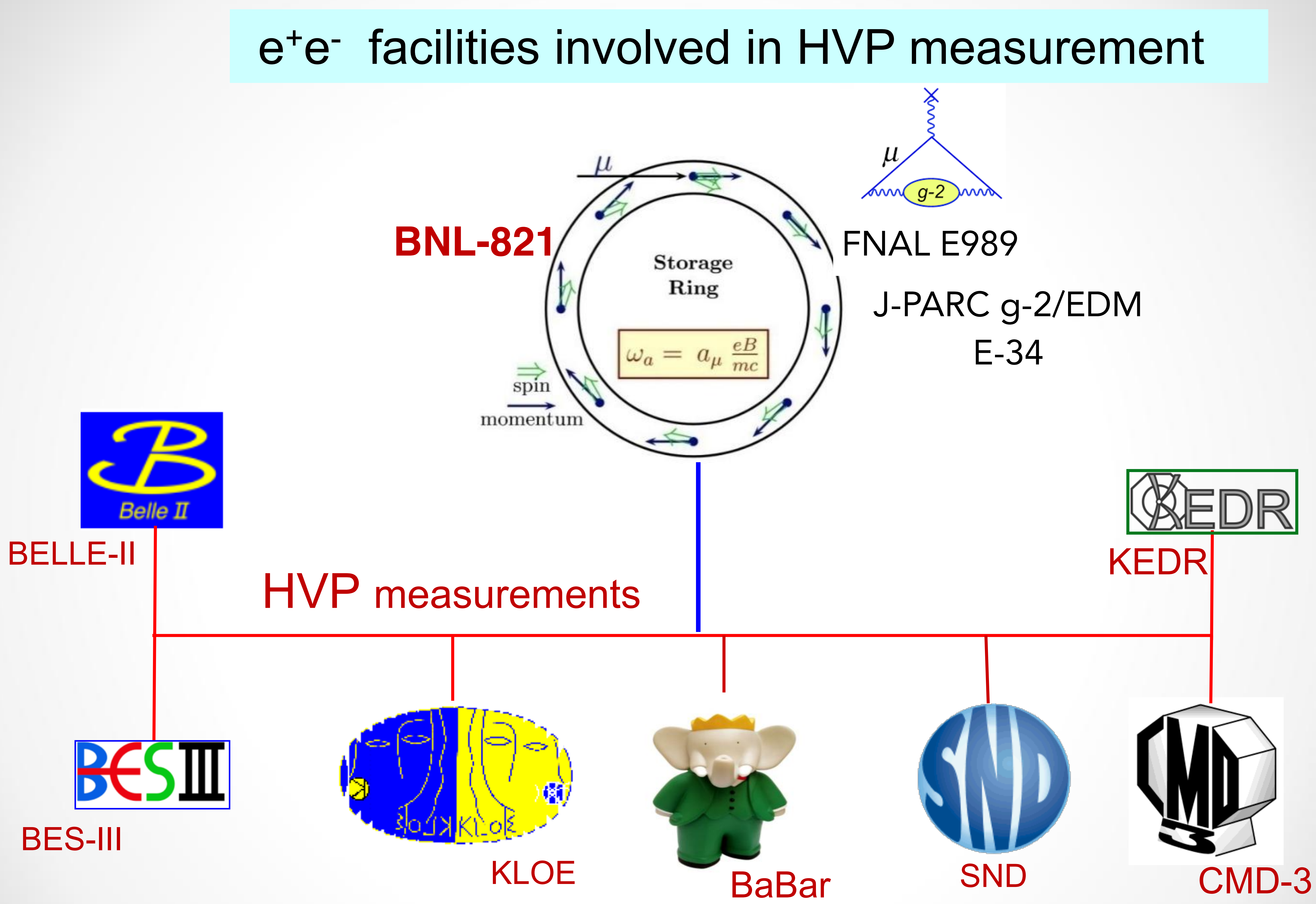
Experimental Inputs to HVP

- two exp. approaches
  - ``Direct scan": change CM energy of e^+e^- beams
  - ``Radiative Return":
    with fixed e^+e^- CM energy, select events with initial state radiation (ISR)



- complemented by:
  - MC generators for $\sigma_{\text{had}}(s)$ (e.g. PHOKARA)
  - detailed studies of radiative corrections (now known through NLO)

S. Serednyakov (for SND) @ HVP KEK workshop

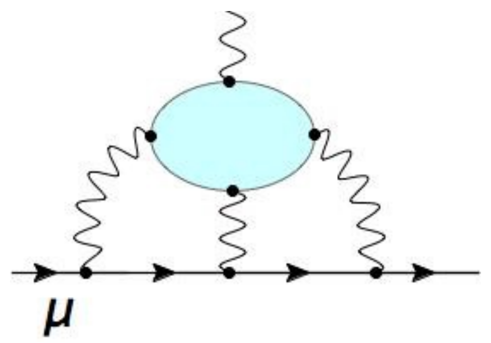


HVP: data-driven

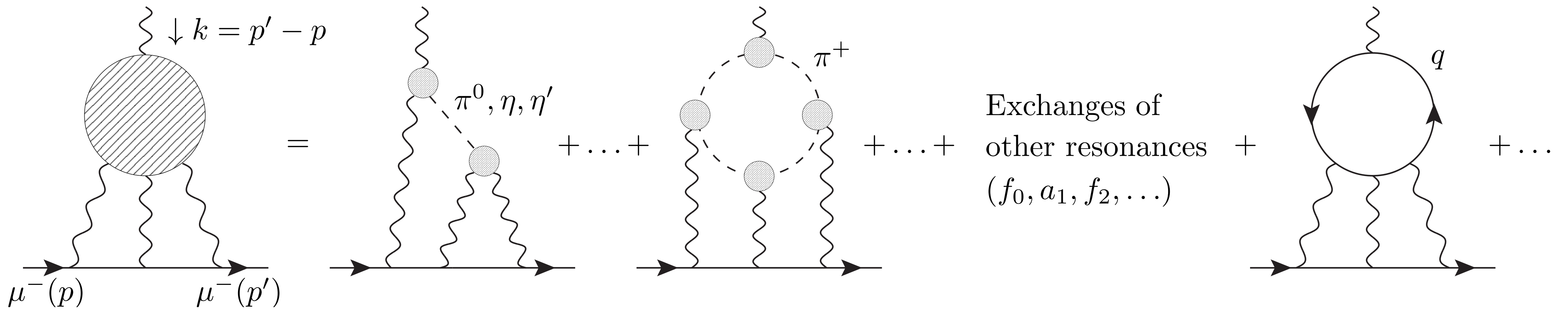
T. Teubner @ Zurich workshop

New results for $\sigma_{\mathrm{had}}(s)$:

- **pi+pi-pi0**, BESIII (2019), arXiv:1912.11208
- **pi+pi-** [covariance matrix erratum], BESIII (2020), Phys.Lett.B 812 (2021)
- **K+K-pi0**, SND (2020), Eur.Phys.J.C 80 (2020) 12, 1139
- **etapi0gamma** (res. only), SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- **pi+pi-**, SND (2020), JHEP 01 (2021) 113
- **etaomega** → pi0gamma, SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- **pi+pi-pi0**, SND (2020), Eur.Phys.J.C 80 (2020) 10, 993
- **pi+pi-pi0**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112003
- **pi+pi-2pi0omega**, BaBar (2021), Phys. Rev. D 103, 092001
- **etaetagamma**, SND (2021), Eur.Phys.J.C 82 (2022) 2, 168
- **etaomega**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **pi+pi-pi0eta**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **omegaetapi0**, BaBar (2021), Phys. Rev. D 103, 092001
- **pi+pi-4pi0**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **pi+pi-pi0pi0eta**, BaBar (2021), Phys.Rev.D 103 (2021) 9, 092001
- **pi+pi-3pi0eta**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **2pi+2pi-3pi0**, BaBar (2021), Phys. Rev. D 103, 092001
- **omega3pi0**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **pi+pi-pi+pi-eta**, BaBar (2021), Phys. Rev. D 103, 092001
- **inclusive**, BESIII (2021), Phys.Rev.Lett. 128 (2022) 6, 062004
- …



Hadronic Light-by-light

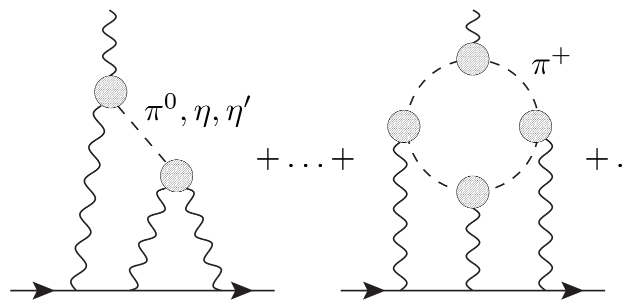


Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

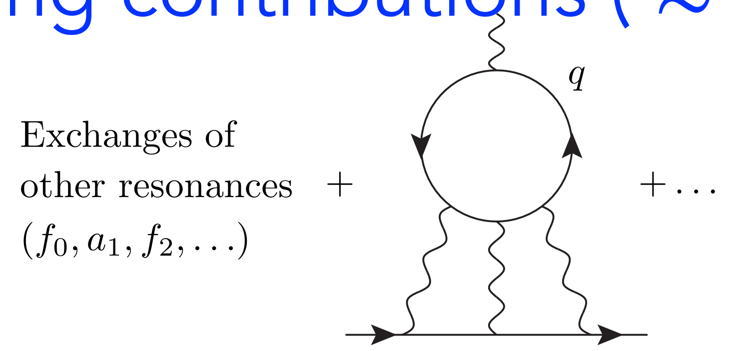
- ✦ model independent
- ✦ significantly more complicated than for HVP
- ✦ provides a framework for data-driven evaluations
- ✦ can also use lattice results as inputs

Dominant contributions ($\approx 75\,\%$ of total):



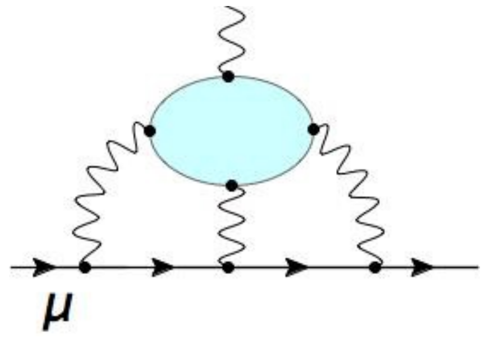
- ✦ Well quantified with $\approx 6\,\%$ uncertainty
- ✦ η, η' pole contributions: Canterbury approximants only
- ✦ Ongoing work: consolidation of η, η' pole contributions using disp. relations and LQCD

Subleading contributions ($\approx 25\,\%$ of total):

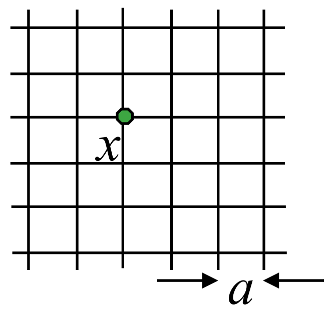


- ✦ Not yet well known
  - ➠ dominant contribution to total uncertainty
- ✦ Ongoing work:
  - Implementation of short-distance constraints (now at 2-loop)
  - DR implementation for axial vector contributions
  - new dispersive formalism for higher spin intermediate states [Luedtke, Procura, Stoffer, 2023, in progress]
  - Mainz and BESIII ramping up $\gamma^{(*)}\gamma^*$ programs [A. Denig and C. Redmer @ Higgscentre workshop]

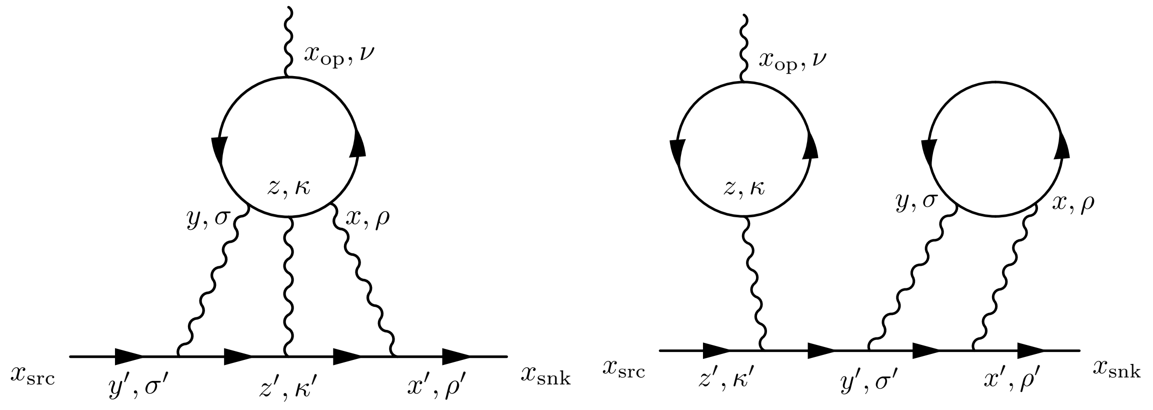
Dispersive, data-driven evaluation of HLbL with $\leq 10\,\%$ total uncertainty feasible by ~2025.

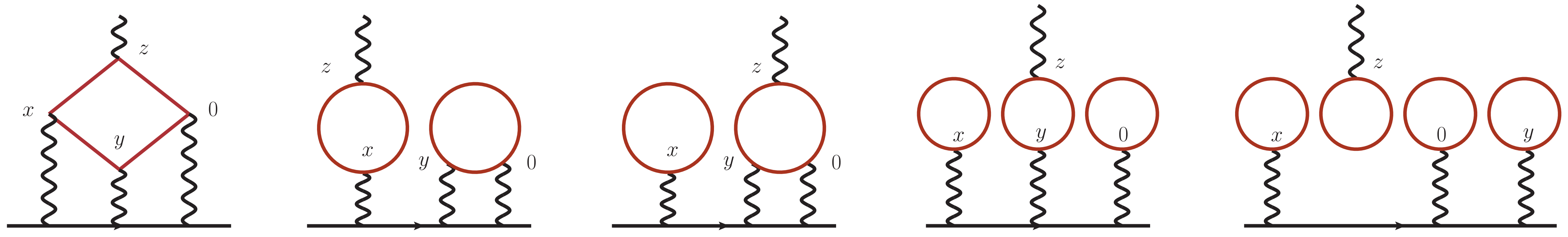


Hadronic Light-by-light: lattice



Lattice QCD+QED: Two independent and complete direct calculations of a_μ^{HLbL}





- ✦ RBC/UKQCD
  [T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123, 2020 PRL]
- ✦ QCD + $\mathrm{QED_L}$ (finite volume) stochastic

  DWF ensembles at/near phys mass,
  $a \approx 0.08 - 0.2\,\mathrm{fm},\ L \sim 4.5 - 9.3\,\mathrm{fm}$

- ✦ Mainz group
  [E. Chao et al, arXiv:2104.02632]
- ✦ QCD + QED (infinite volume & continuum) analytic

  CLS (2+1 Wilson-clover) ensembles
  $m_\pi \sim 200 - 430\,\mathrm{MeV},\ a \approx 0.05 - 0.1\,\mathrm{fm},\ m_\pi L > 4$

- ✦ Cross checks between RBC/UKQCD & Mainz approaches in White Paper at unphysical pion mass
- ✦ Both groups are continuing to improve their calculations, adding more statistics, lattice spacings, physical mass ensemble (Mainz)
- ✦ new result from RBC/UKQCD [T. Blum et al, arXiv:2304.04423] using QCD + QED (inf.): $a_\mu^{\mathrm{HLbL}} = 124.7\,(11.5)\,(9.9) \times 10^{-11}$ consistent with previous calculations
- ✦ ongoing LQCD calculations of π, η, η' transition form factors to determine pseudo scalar pole contributions [Mainz, ETMC, BMW]

Lattice HLbL results with $10\,\%$ total uncertainty feasible by ~2025

muon g-2 Summary

- consistent results from independent, precise LQCD calculations for light-quark connected contribution to **intermediate window** a_μ^{W} (~ 1/3 of $a_\mu^{\mathrm{HVP,LO}}$) ➠ $3-4\,\sigma$ tension with data-driven results?
- still need independent LQCD results for long-distance contribution, total HVP: coming soon
  ➠ develop method average for lattice HVP results, **assess tensions (if any)** with data-driven average
- Programs and plans in place to improve by 2025:
  - data-driven HVP: **if differences are resolved/understood,** $\sim 0.3\,\%$
    new measurements from BaBar, KLOE, SND, Belle II,…. will shed light on current discrepancies (blind analyses are paramount!)
    improved treatment of structure dependent radiative corrections (NLO) in $\pi\pi$ and $\pi\pi\pi$ channels
  - lattice HVP: **if no tensions** between independent lattice results, $\sim 0.5\,\%$
  - dispersive HLbL and lattice HLbL: no puzzles, steady progress, $\sim 10\,\%$
- **IF tensions/differences between data-driven HVP and lattice HVP are resolved,** SM prediction will likely match precision goal of the Fermilab experiment.
- IF NOT, will need detailed comparisons, explore connections between HVP, $\sigma(e^+e^-)$, $\Delta\alpha$, global EW fits.

- BSM implications ➠ appendix

➠ **continued coordination by Theory Initiative: workshops, WPs, …**

Connections

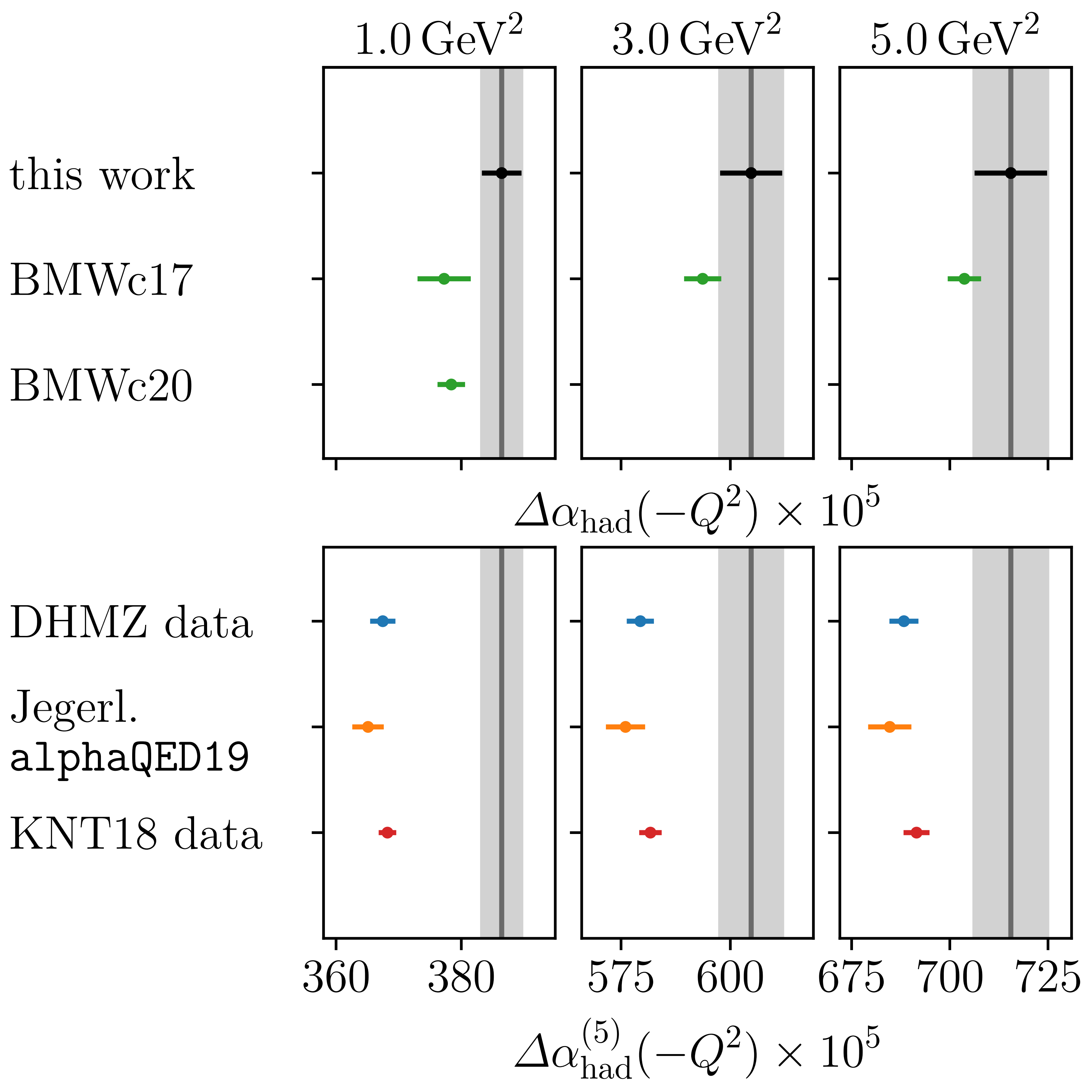
$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.
- a shift in a_μ^{HVP} also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits
  [Passera, et al, 2008, Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]
  If the shift in a_μ^{HVP} is in the low-energy region ($\lesssim 1\,\text{GeV}$), the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \Leftrightarrow a_\mu^{\text{HVP}} \Leftrightarrow \Delta\alpha_{\text{had}}(M_Z^2)$$

H. Wittig @ Higgscentre workshop



Dispersion integral: $\Delta\alpha_{\text{had}}^{(5)}(q^2) = -\frac{\alpha\, q^2}{3\pi} \oint_{m_{\pi^0}^2}^{\infty} ds\, \frac{R(s)}{s(s-q^2)}$

Lattice QCD:

$$\Delta\alpha_{\text{had}}(-Q^2) = \frac{\alpha}{\pi}\frac{1}{Q^2}\int_0^\infty dt\, G(t)\left[Q^2t^2 - 4\sin^2\left(\tfrac{1}{2}Q^2t^2\right)\right]$$

- Direct lattice calculation of $\Delta\alpha(-Q^2)$ on the same gauge ensembles used in Mainz/CLS 22
  [Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]
- Tension of $\sim 3\sigma$ observed with data-driven evaluation of $\Delta\alpha_{\text{had}}(-Q^2)$ for $Q^2 \gtrsim 3\,\text{GeV}^2$
  → consistent with tension for window observable

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \Leftrightarrow a_\mu^{\text{HVP}} \Leftrightarrow \Delta\alpha_{\text{had}}(M_Z^2)$$

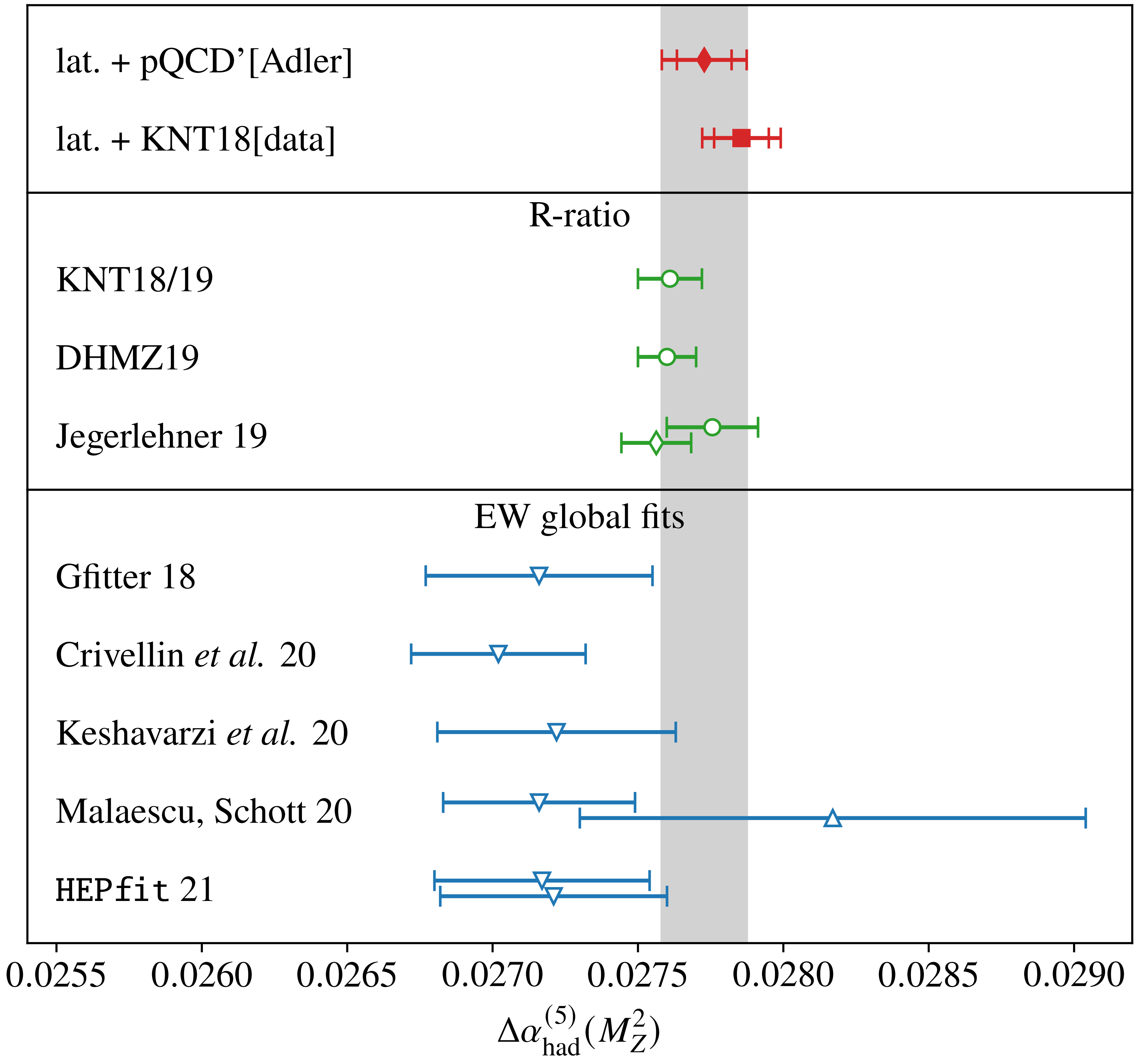
H. Wittig @ Higgscentre workshop

Adler function approach, aka. "Euclidean split technique"

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \Delta\alpha_{\text{had}}^{(5)}(-Q_0^2)$$
$$+[\Delta\alpha_{\text{had}}^{(5)}(-M_Z^2) - \Delta\alpha_{\text{had}}^{(5)}(-Q_0^2)]$$
$$+[\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) - \Delta\alpha_{\text{had}}^{(5)}(-M_Z^2)]$$

$$\Rightarrow \quad \Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = 0.027\,73(9)_{\text{lat}}(2)_{\text{btm}}(12)_{\text{pQCD}}$$

[Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]



- Agreement between lattice QCD and evaluations based on the R-ratio

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

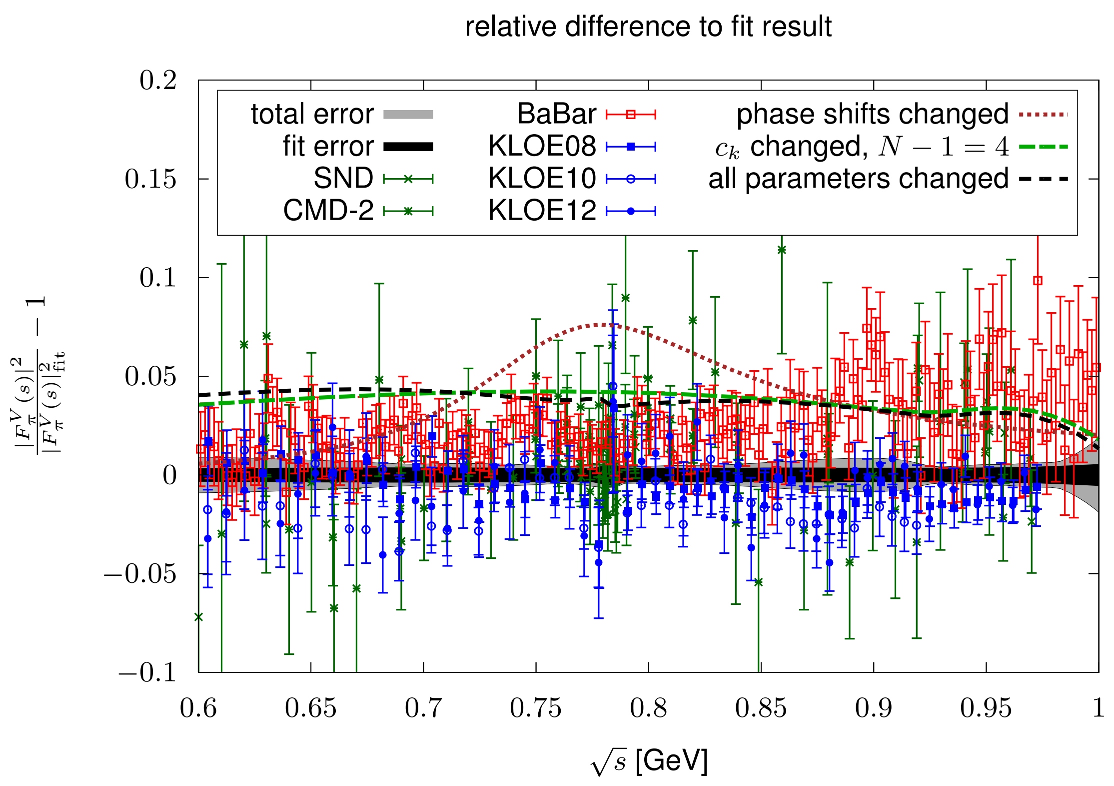
- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.
- a shift in a_μ^{HVP} also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits [Passera, et al, 2008, Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]
  If the shift in a_μ^{HVP} is in the low-energy region ($\lesssim 1\,\text{GeV}$), the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.
- A shift in a_μ^{HVP} from low ($\lesssim 2\,\text{GeV}$) energies
  ➠ $\sigma(e^+e^- \to \pi\pi)$
  must satisfy unitarity & analyticity constraints ➠ $F_\pi^V(s)$
  can be tested with lattice calculations
  [Colangelo, Hoferichter, Stoffer, arXiv:2010.07943]

Peter Stoffer @ Lattice HVP workshop

Constraints on the two-pion contribution to HVP arXiv:2010.07943 [hep-ph]

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

- "low-energy" scenario: local changes in cross section of $\sim 8\%$ **around ρ**
- "high-energy" scenario: impact on **pion charge radius** and space-like VFF $\Rightarrow$ chance for **independent lattice-QCD checks**
- requires **factor ~ 3 improvement** over χQCD result:
  $\langle r_\pi^2 \rangle = 0.433(9)(13)\,\text{fm}^2$
  → arXiv:2006.05431 [hep-ph]

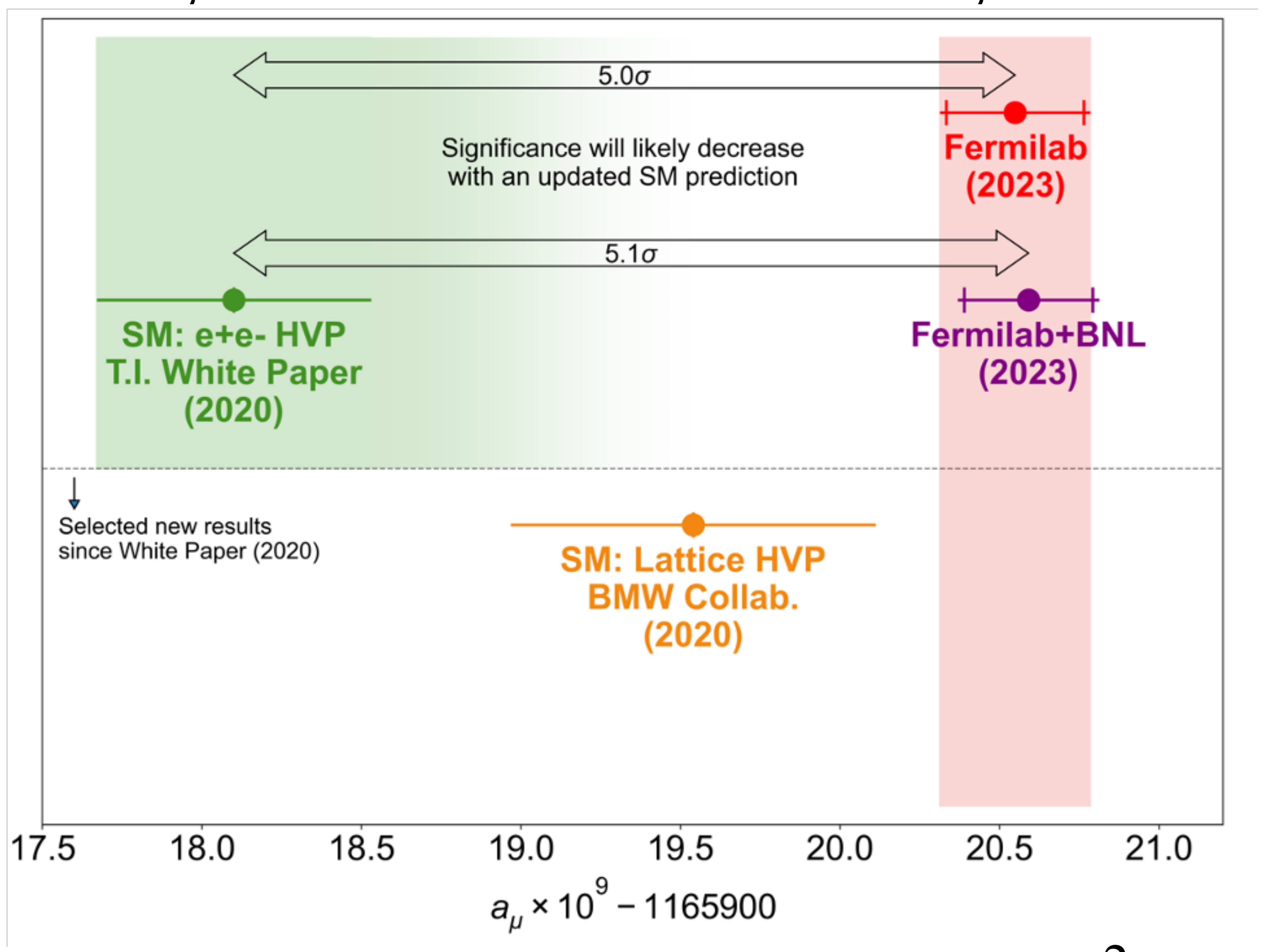


Beyond the SM possibilities

a_μ is loop-induced, conserves CP & flavor, flips chirality.

The difference between Exp-WP2020 is large:

$$\Delta a_\mu = 249\,(48) \times 10^{-11} \;>\; a_\mu(\mathrm{EW})$$



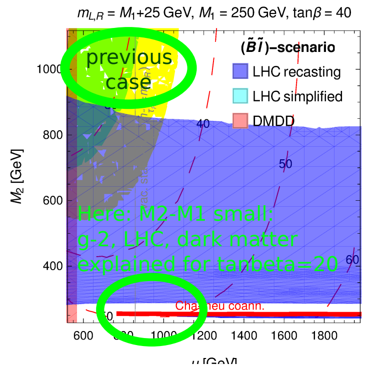
Generically expect: $a_\mu^{\mathrm{NP}} \sim a_\mu^{\mathrm{EW}} \times \dfrac{M_W^2}{\Lambda^2} \times \text{couplings}$

Can be accommodated by many BSM theories (800+ papers)

D. Stöckinger @ g-2 Days (http://pheno.csic.es/g-2Days21/)

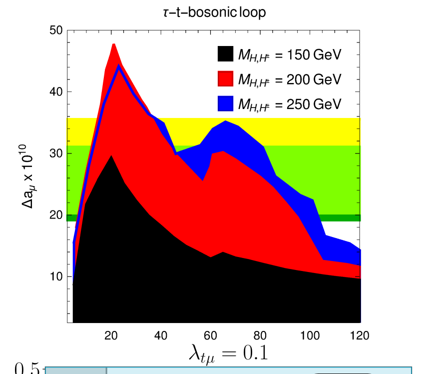
SUSY: MSSM, MRSSM

- MSugra. . . many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns



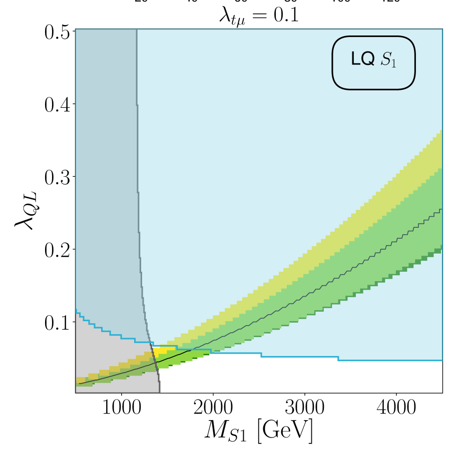
Two-Higgs doublet model

- Type I, II, Y, Type X(lepton-specific), flavour-aligned



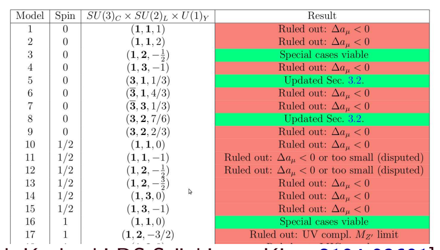
Lepto-quarks, vector-like leptons

- scenarios with muon-specific couplings to μ_L and μ_R



Simple models (one or two new fields)

- Mostly excluded
- light N.P. (ALPs, Dark Photon, Light $L_\mu - L_\tau$)



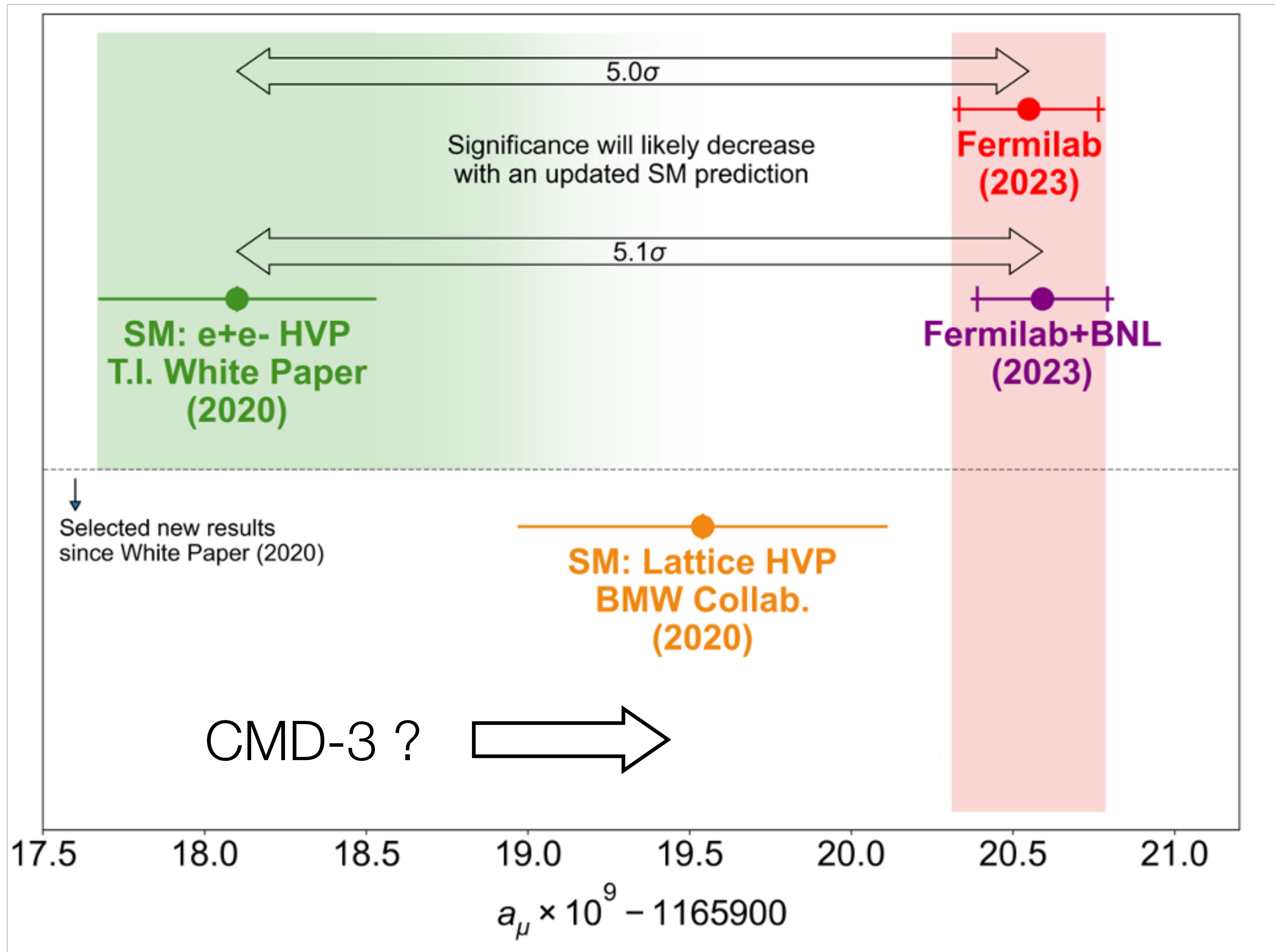
[Athron,Balazs,Jacob,Kotlarski,DS,Stöckinger-Kim, 2104.03691]

Beyond the SM possibilities

a_μ is loop-induced, conserves CP & flavor, flips chirality.

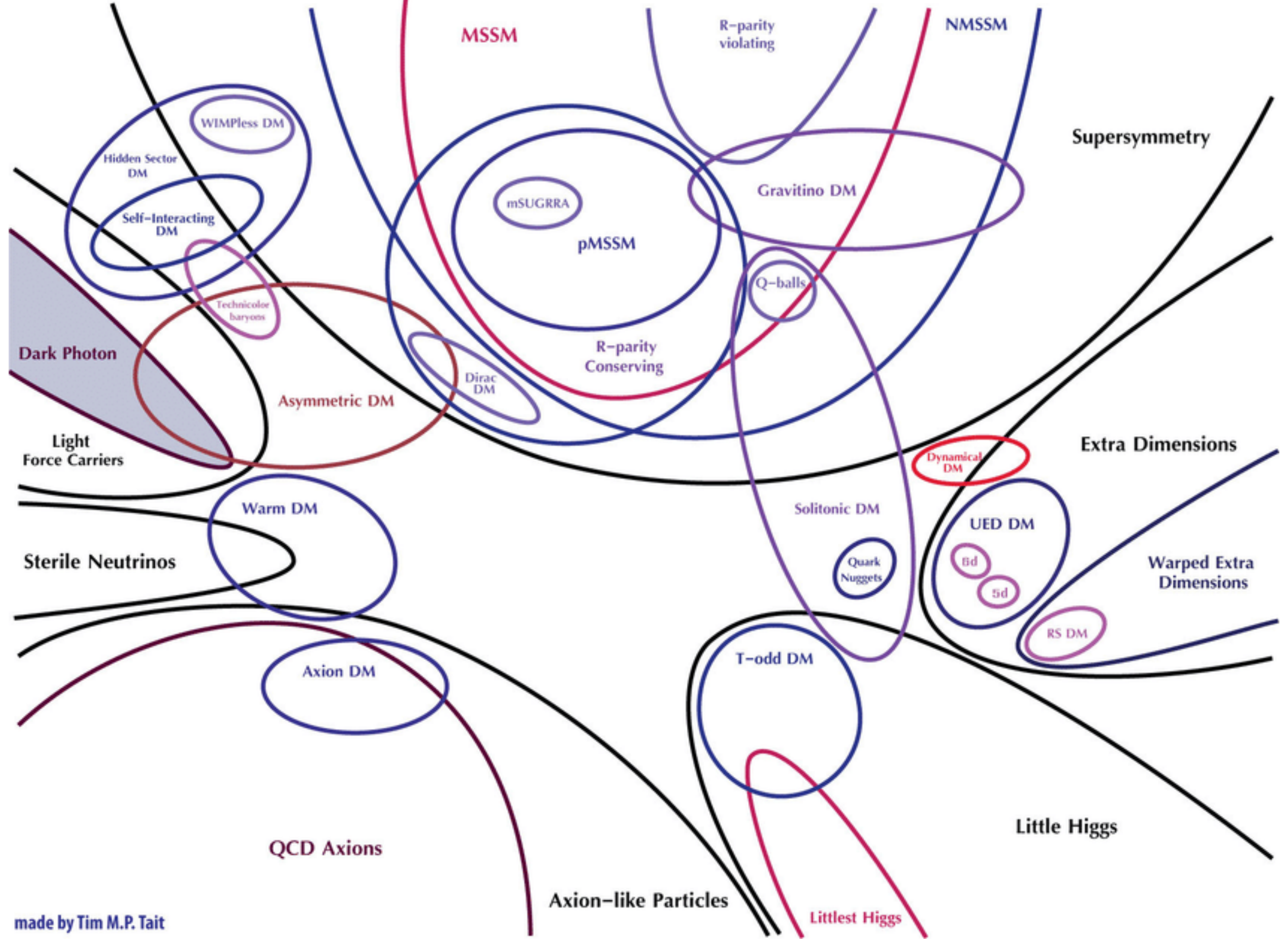
The difference between Exp-WP2020 is large:

$$\Delta a_\mu = 249\,(48) \times 10^{-11} \; > \; a_\mu(\mathrm{EW})$$



Generically expect: $a_\mu^{\mathrm{NP}} \sim a_\mu^{\mathrm{EW}} \times \frac{M_W^2}{\Lambda^2} \times$

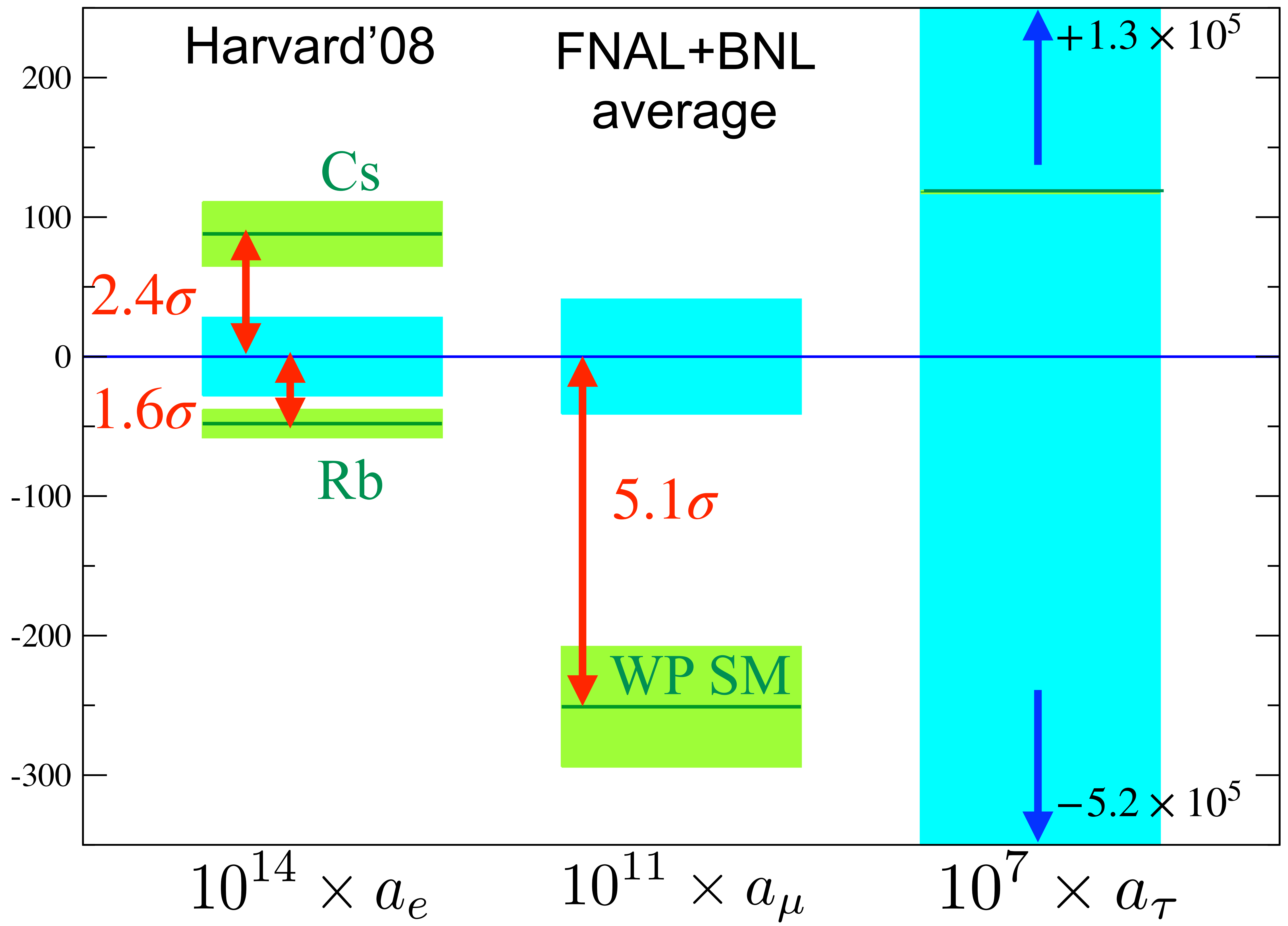
Can be accommodated by many BSM theories (1k+ papers)



- Can new physics hide in the low-energy $\sigma(e^+e^- \to \pi\pi)$ cross section?
  ➠ No [Luzio, et al, arXiv:2112.08312]
- New boson at ~ 1GeV decays into $\mu^+\mu^-, e^+e^-$, affects $\sigma(e^+e^- \to \pi\pi)$ indirectly [L. Darmé et al, arXiv:2112.09139]
- Neutral, long-lived hadrons, heretofore undetected? [Farrar, arXiv:2206.13460]
- Z′ at < 1 GeV, coupling to 1st gen matter particles [Coyle, Wagner, arXiv:2305.02354]

Lepton moments summary

$$a_\ell^{\rm SM} - a_\ell^{\rm Exp}$$



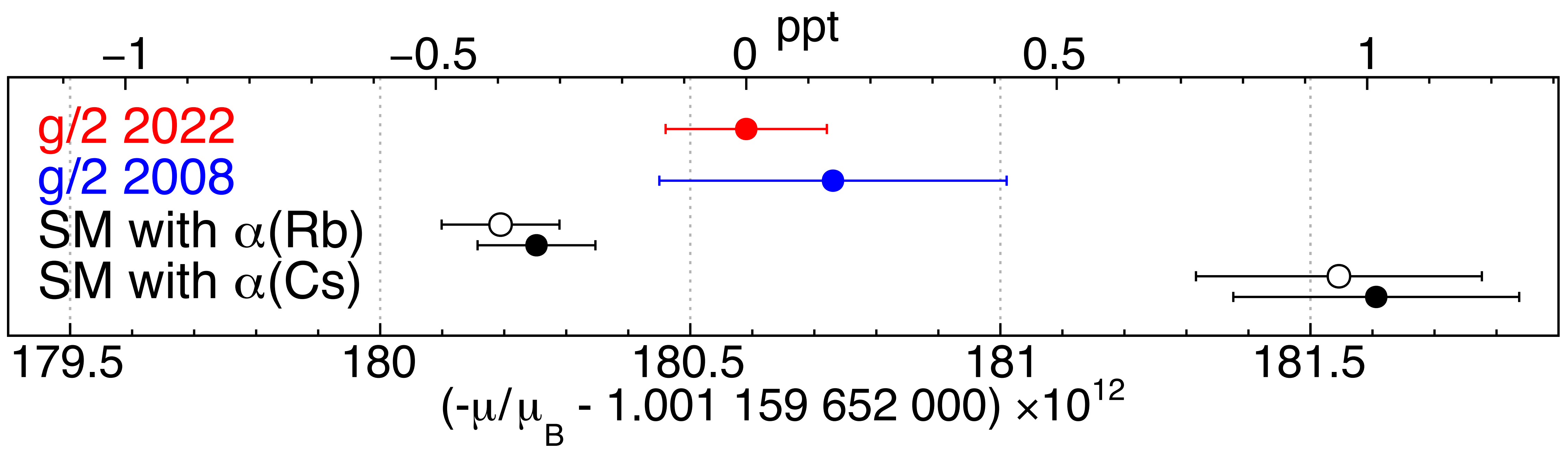
Cs: α from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: α from Paris group [Morel et al, Nature 588, 61–65(2020)]

Sensitivity to heavy new physics: $a_\ell^{\rm NP} \sim \frac{m_\ell^2}{\Lambda^2}$

$(m_\mu/m_e)^2 \sim 4 \times 10^4$

[X. Fan et al (Gabrielse group), arXiv:2209.13084]



Prospects for tau moment measurement:

Chiral Belle arXiv:2205.12847

- ☆ use polarized e^- beam
- ☆ with $40ab^{-1}$ measurement of a_τ at 10^{-5} feasible
- ☆ with more statistics measurement at 10^{-6} possible