Computing Feynman Parton Distributions on Lattice through Large-momentum effective theory

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50 years of QCD symposium, UCLA

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Outline

- Power of partons
- A brief history of partons from light-cone correlator
- Partons in Feynman's way: Euclidean formulation
- Large momentum expansion (or EFT)
- Applications
  - Gluon polarization
  - PDFs
  - GPDs
  - TMDPDFs
  - LFWFs
- Outlook

Power of partons

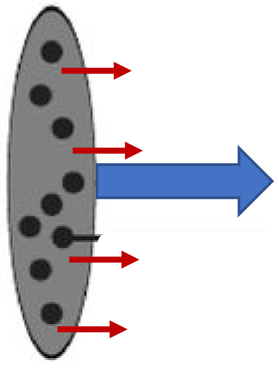
Feynman's parton model

- In high-energy scattering, proton travels at v~c, one can assume the proton travels exactly at v=c, or the proton momentum is

$$p=E=\infty$$

(Infinite momentum frame, IMF)

- Proton may be considered as a collection of interaction-free particles: partons



Parton distribution functions (PDF)

- Every parton has k=∞, however,

  x=k/p = finite, $\in$ [0,1]

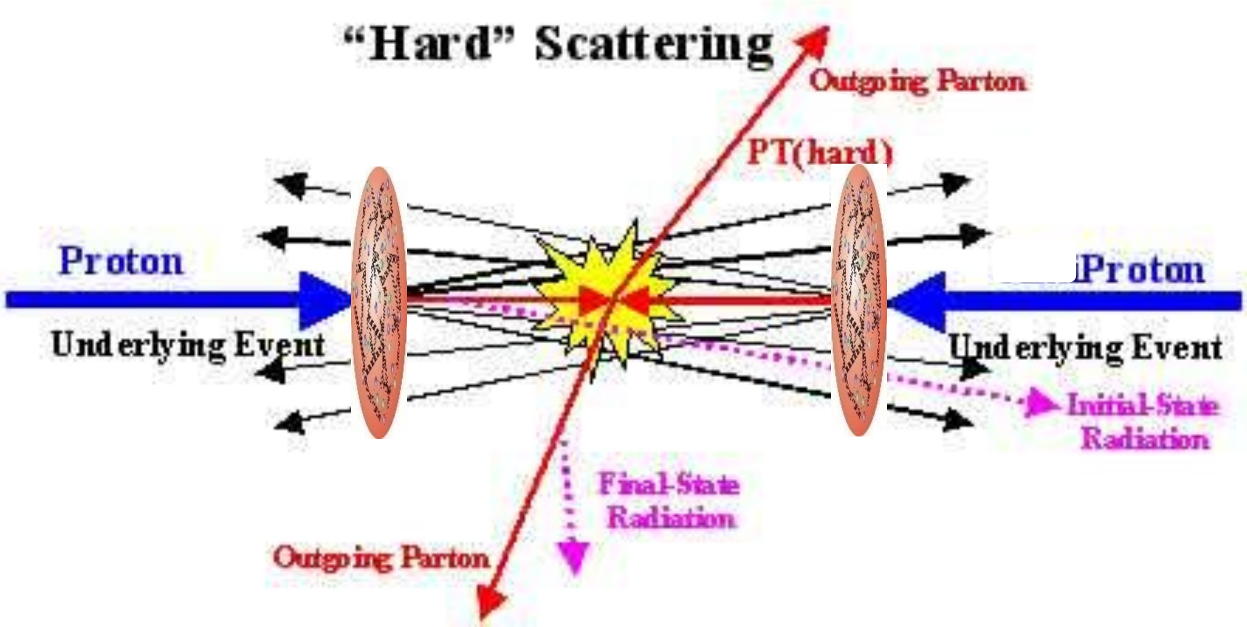
- Parton distribution function

  $f(x)$

is the probability of finding parton in a proton, carrying x fraction of the momentum of the parent.

- PDF is a bound state property of the proton, essential to explain the results of high-energy collisions.

Hard scattering & factorization

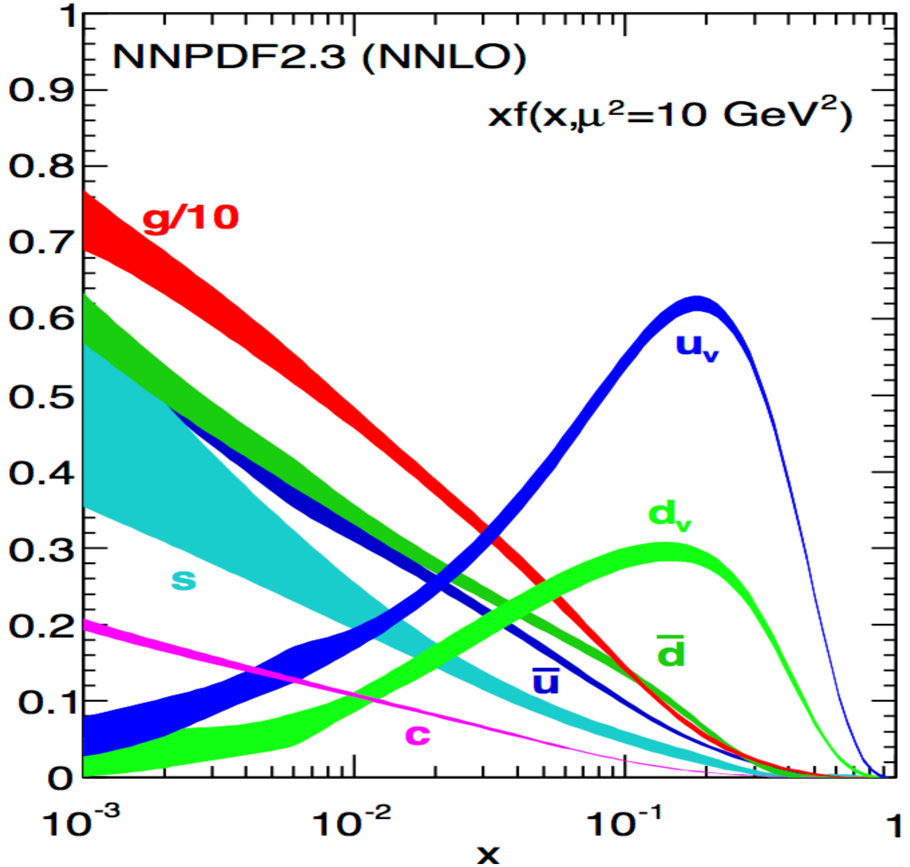


- **Factorization:** The scattering cross sections are factorized in terms of PDFs and parton x-section.

$$\sigma = \int dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}$$

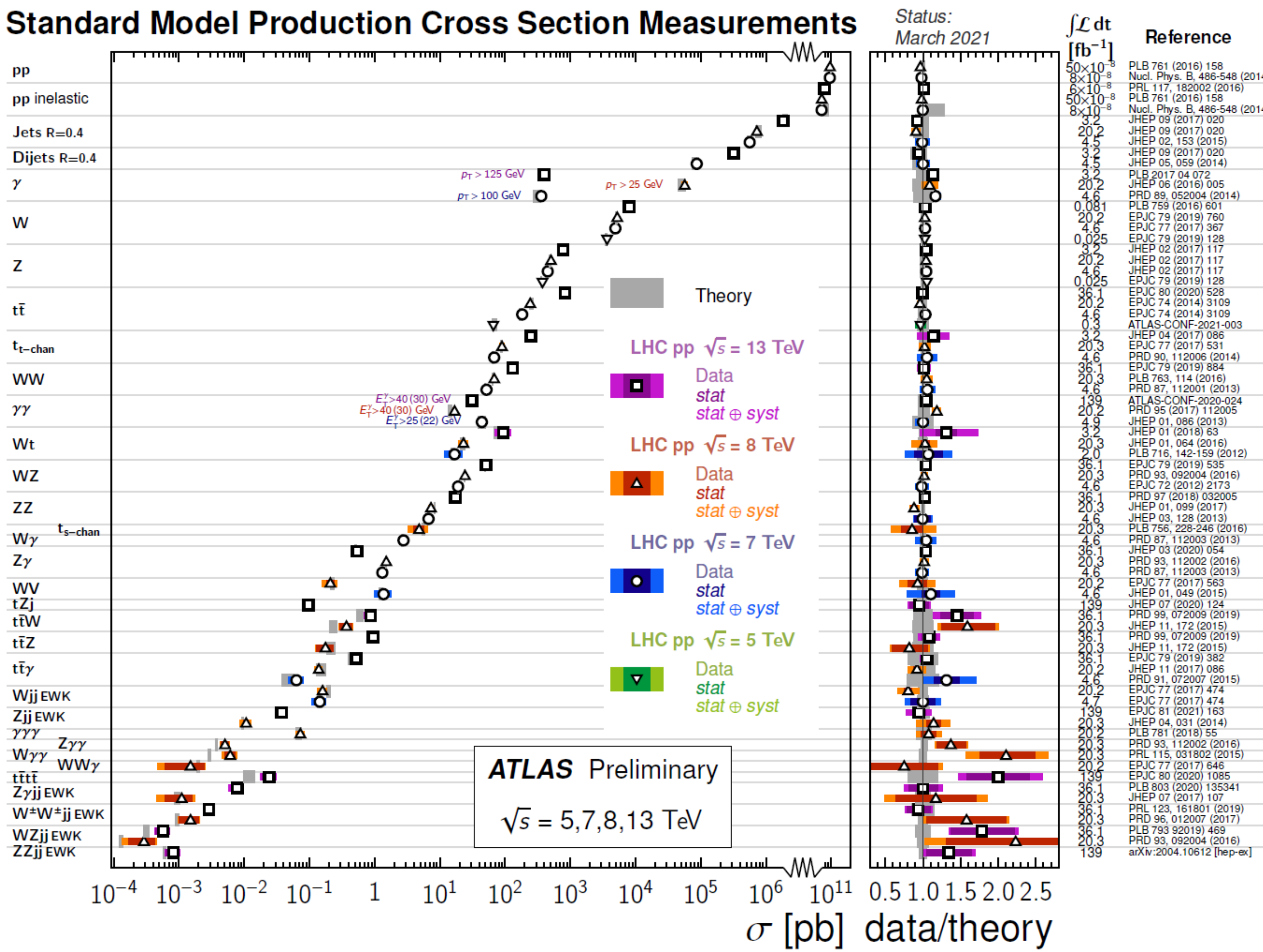
Phenomenological PDFs

- Use experimental data (50+ yrs) to extract PDFs



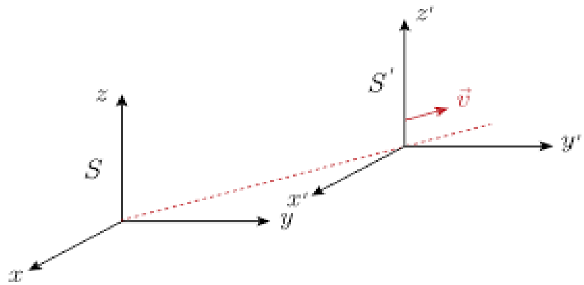
J. Gao, et al, Phys. Rept. 742 (2018) 1-121

Exp. data vs. Standard Model theory



A brief history of partons from light-cone correlator

Weinberg's rules



- What does an object look like when travelling at infinite momentum or speed of light?
- S. Weinberg (scalar QFT)

Dynamics at infinite momentum

Phys. Rev. 150 (1966) 1313-1318

- All kinematic infinities can be removed from the calculations, resulting a set of rules for perturbation theory ("old-fashioned p.t.")
- The result is similar to a "non-relativistic" theory.

More Weinberg's rules...

- L. Susskind, K. Bardakci, and M. B.Halpern,...
- S. J. Chang and S. K. Ma (1969)

Feynman rules and quantum electrodynamics at infinite momentum,

Phys. Rev. 180 (1969) 1506-1513

- J. Kogut and D. Soper

Quantum Electrodynamics in the Infinite Momentum Frame, Phys.Rev.D1 (1970) 2901-2913

Chang and Ma's discovery

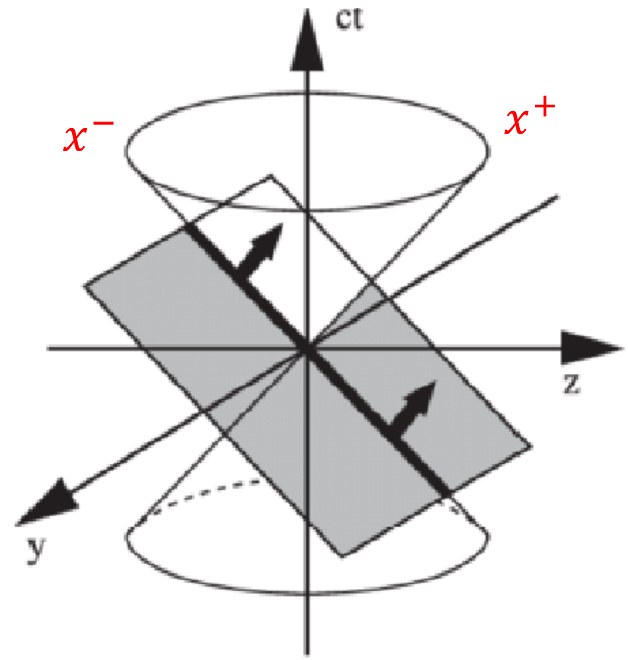
- All Weinberg's rules in the P=∞ limit can be obtained by quantizing the theory with "new coordinates"

$$x^+ = \frac{x^0 + x^3}{\sqrt{2}}, \quad x^- = \frac{x^0 - x^3}{\sqrt{2}}$$

by treating

x^+ as the new "time"

x^- as the new "space"



Dirac's form of dynamics

- The Weinberg's rules exactly correspond to what Dirac proposed in 1949.
- Paul A.M. Dirac,

Forms of Relativistic Dynamics,

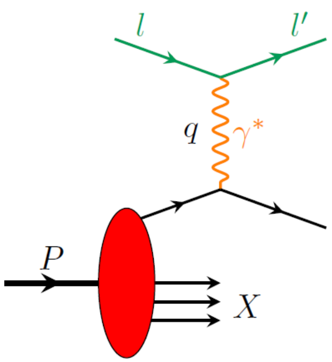
Rev. Mod. Phys. 21 (1949) 392-399.

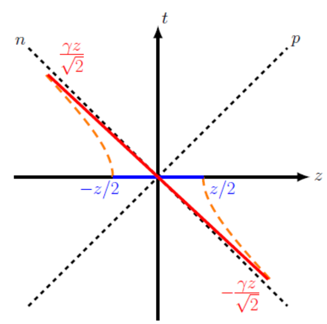
"Front form"

or Light-front quantization (LFQ)

Light-cone dominance

- In DIS, it was realized the process is dominated by light-cone correlations
- In Feynman's parton model, the worldline of the final state parton goes along the lightcone





- The light-cone correlation is captured by light-front quantization

Feynman's parton & LF wave functions

- Feynman's parton are Fock components in the light-front wave function

$$|P\rangle = \sum_{m,n}^{\infty} \int \Pi_{i=1}^{n} [dx_i d^2\vec{k}_{i\perp}] \Pi_{i=1}^{m} [d^2\vec{k}_{i\perp}]$$
$$\times \delta(x_1 + ... + x_n - 1)\delta^2(\vec{k}_{1\perp} + ... + \vec{k}_{n+m\perp})$$
$$\phi(x_1, \vec{k}_{1\perp}, ..., 0, \vec{k}_{n+m\perp}) a^\dagger_{x_1,\vec{k}_{1\perp}} ... a^\dagger_{x_n,\vec{k}_{n\perp}} |\mathbf{0}_{nm}\rangle$$

- And parton density is just the particle number distribution

$$G_{f/h}(x;Q) = \sum_n \int d[\mu_n] \left|\Psi^{(Q)}_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)\right|^2 \sum_i \delta(x - x_i)\, \delta_{i,f}$$

It is hard to solve LF wave functions

- LF QCD is still a strong-coupling problem! there is no demonstration that the weak coupling expansion actually works for QCD.

  K. Wilson et. al. Phys. Rev. D49 (1994)

- LF breaks various symmetries that make renormalization difficult (infinite number of renormalization constant)
- Difficulty with vacuum zero-modes.

Partons and critical phenomena

- Fourier trans. of PDFs gives a small-x behavior,

  f(x) → $x^{-\alpha}$

- When FT back to position space, one has

  C$(\lambda) \sim \lambda^{\alpha-1}$

This corresponds to "infinite correlation" length

C$(\lambda) \sim \exp(-\frac{\xi}{\lambda})$ with $\xi \rightarrow \infty$

No condensed matter theorists directly solve critical phenomena at T=Tc!

Factorization & collinear modes

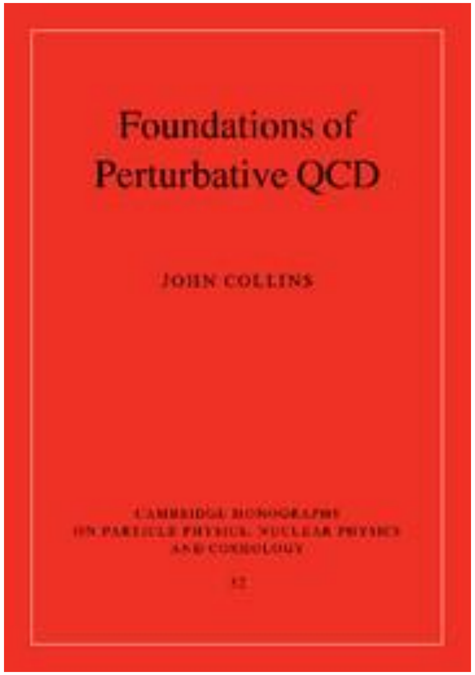
- In lagrangian formulation of parton physics, the partons are represented by collinear modes in QCD

$$\psi(\lambda n)\,,\ n^2 = 0$$

λ is the distance along the LF

- Parton physics is related to correlations of these fields along n with distance λ.

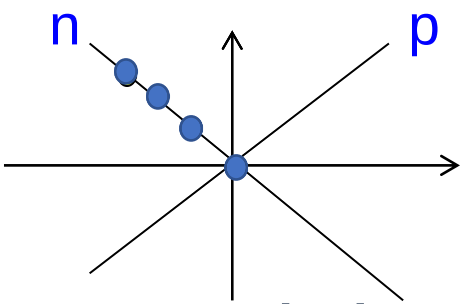
e.g. Soft-collinear Effective Theory (SCET)



Partons as LF correlations

- Probes (operators) are **light-cone correlations**

$$\hat{O} = \phi_1(\lambda_1 \mathrm{n})\phi_2(\lambda_2 n)\ldots\phi_k(\lambda_k n)$$



- The matrix elements are independent of hadron momentum, and they can be calculated in the **states in the rest frame**.

("Heisenberg picture")

Real-time Monte Carlo in path integrals?

- Monte Carlo simulations have not been very successful with quantum real-time dynamics.

$$\exp(-iHt)$$

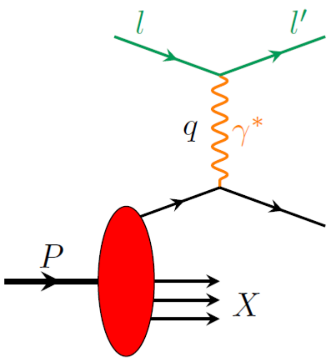
an oscillating phase factor!

- "Sign problem": Hubbard model for high Tc.
- Signals are exponentially small!
- Quantum computer?

Partons in Feynman's way:
a Euclidean formulation

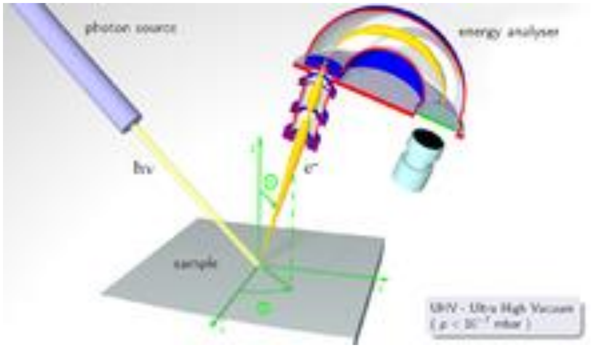
Origin of Parton Model

- Electron-proton deep-inelastic scattering (DIS)



- Knock out scattering in NR systems
    - e-scattering on atoms
    - ARPES in CM systems
    - Neutron scattering on liquid He

…



Momentum distribution in NR systems

- Knock-out reactions in NR systems probes momentum distribution

$$n(\vec{k}) = \left|\psi(\vec{k})\right|^2$$

$$\sim \int \psi^*(\vec{r})\psi(0)e^{i\vec{k}\vec{r}}d^3r$$

$$\sim \int \langle\Omega\left|\hat{\psi}^+(\vec{r})\hat{\psi}(0)\right|\Omega\rangle e^{i\vec{k}\vec{r}}d^3r$$

- Mom.dis. are related to Euclidean correlations, generally amenable for Monte Carlo simulations.

Difference between relativistic and NR systems

- NR cases, the energy transfer is small.

$$q^0 \sim \frac{1}{M} \sim 0$$

- Relativistic systems:

  In DIS, if we choose a frame in which the virtual photon energy is zero

$$q^\mu = (0, 0, 0, -Q)\,,$$
$$P^\mu = \left(\frac{Q}{2x_B} + \frac{M^2 x_B}{Q}, 0, 0, \frac{Q}{2x_B}\right)\,,$$

In the Bjorken limit, $P^z \sim Q \rightarrow \infty$

Feynman's partons

- Momentum distribution in a hadron state depends on the center-of-mass momentum P^z:

$$n(\vec{k}, P^z)$$

  because of relativity.

- Consider the dependence on the z-component

$$\mathrm{n}(k^z, P^z) = \int d^2k_\perp \, n(k^z, k_\perp, P^z)$$

PDF is a result of the P^z → ∞ limit,

$$\mathrm{n}(k^z, P^z) \rightarrow_{P^z \rightarrow \infty} f(x) \quad with\, x = \frac{k^z}{P^z},$$

Partons in Euclidean formulation

- Calculate the Euclidean correlation

$$C(\lambda) = \langle P^z = \infty | \overline{\psi}(z)\Gamma\psi(0) | P^z = \infty \rangle$$

$$\lambda = \lim_{P^z \to \infty, z \to 0} (zP^z).$$

- Parton distribution

$$f(x) = \frac{1}{2P^+} \int \frac{d\lambda}{2\pi} e^{ix\lambda} C(\lambda) \ .$$

Relation between two parton formalisms

- Partons as light-front field correlations
  - Use LF collinear field operators
  - Parton physics as LF correlations
  - Independent of external state momentum
- Partons as mom. distribution in infinite-P hadrons
  - Use infinite-momentum states to select parton modes
  - Euclidean correlations
  - Can select with different operators: universality class

Large-momentum expansion

$P^z = \infty$ as a limit

- Directly computing at $P^z = \infty$ is very difficult . It must be studied in the context of a QFT (only a field theory can support ∞ momentum modes).
- However, one can study the momentum density as a $P^z \to \infty$ limit. If the limit exists (analytic), the limiting process is controlled by expansion

$$n(k^z, P^z) = f(x) + b(x)\left(\frac{M}{P^z}\right)^2 + \cdots$$

where $x = \frac{k^z}{P^z}$, M is a bound-state scale,

P^z is a large-momentum scale

Convergence of the $P^z = \infty$ expansion

- Naively, $\epsilon = \left(\frac{M}{P^z}\right)^2$ is an expansion parameter

  $M = 1\text{ GeV}, P^z = 2\text{ GeV}$

  $\epsilon = 1/4$

  the expansion may already work.

Similar to other expansions in QCD

- Lattice QCD: approximate a continuum

  theory by a discrete one.

  cut-off $\Lambda \to \infty$, on lattice $\Lambda = \pi / a$

  expansion in a ~ 0.1 fm ~ 2 GeV

- HQET: $\epsilon = \Lambda_{QCD} / m_Q$

  using $m_Q = \infty$ to approximate

  expansion at m_c ~ 1.5 GeV!

QFT subtleties

- When there is a UV cut-off Λ_{UV}, $\mathrm{n}(k^z, P^z)$ is not analytic at $P^z = \infty$!
- There are two possible $P^z \to \infty$ limits:
  1. $P^z \ll \Lambda_{UV} \to \infty$, IMF limit (lattice QCD)
  2. $P^z \gg \Lambda_{UV} \to \infty$ LFQ limit (HEP PDF)

mom.dis. is calculated with the limit 1.

and PDF is defined in limit 2.

- Solution: matching in EFT

The difference is perturbative!

Large momentum expansion

- Thus for finite momentum, one can have a large momentum expansion for large P_z (Ji, 2013)

$$\tilde{f}(y, P^z) = \int Z(y/x, xP^z/\mu) f(x, \mu) dx + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}^2}{y^2(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(1-y)^2(P^z)^2}\Big),$$

- Z contains all perturbative contributions with logarithms in Pz. Large pert effects implied higher twist-contributions and break down of the LaMET
- Large momentum effective theory (LaMET)

Momentum renormalization group equation

- Mom.dis. n(k^z, P^z) has a non-trivial dependence on P^z (H is frame-dependent).
- At large P^z, this dependence shall be calculable in perturbation theory

$$\frac{\partial O(P^z)}{\partial \ln P^z} = \gamma_P(\alpha_s) O(P^z) \, ,$$

$$P^z \frac{\partial}{\partial P^z} \tilde{q}(y, P^z, \mu) = \int_0^1 \frac{dt}{|t|} P_{qq}(t) \times \tilde{q}\left(\frac{y}{t}, tP^z, \mu\right) - 2\gamma_F \tilde{q}(y, P^z, \mu) \ .$$

Momentum RGE

- DGLAP evolution is related to the change of mom.dis. with different CoM motion.

Universality

- One can practically choose ANY composite operator, so long as Pz large enough, they give the same collinear or soft physics.
- For different operators, flowing into the fixed point of large momentum will have different rates (which is faster?), but the limit is the same.

Applications

App1: Gluon total helicity ΔG

- In QCD factorization, one can show that the gluon polarization is a matrix element of non-local light-cone correlation.

A. Manohar,Phys. Rev. Lett. 66 (1991) 2684

$$\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-) \times \mathcal{L}^{ab}(\xi^-, 0)\tilde{F}^+_{\alpha,b}(0)|PS\rangle,$$

- No one knows how to calculate this for a long time.

LaMET calculations

- In LaMET theory, one can start with the local
- operator $\vec{E} \times \vec{A}$, any physical gauge

(gauge choices shall allow transverse polarized gluons):

  - Coulomb gauge $\nabla \cdot \vec{E} = 0$
  - Axial guage A_z=0
  - Temporal gauge A_0 =0

- Their matrix elements in the large momentum limit all go to ΔG (Weizsacker & Williams)

Ji, Zhang, Zhao, Phys. Rev. Lett.,111,112002(2013)

Y. Hatta, Ji, Zhao, Phys.Rev. D89 (2014).

First calculation (Y. Yang et al, PRL(2017))

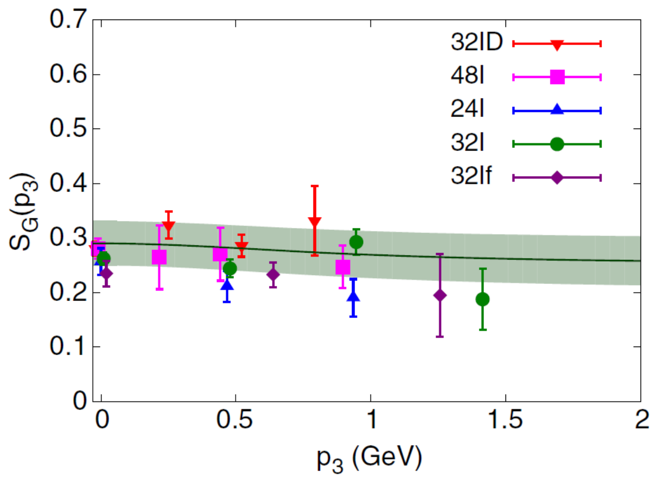
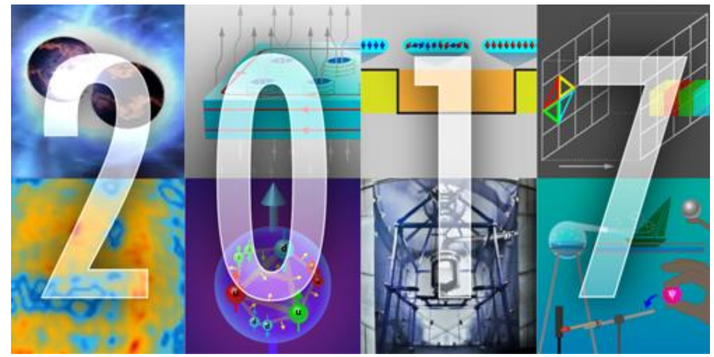


FIG. 4. The results extrapolated to the physical pion mass as a function of the absolute value of $\vec{p} = (0, 0, p_3)$, on all the five ensembles. All the results have been converted to $\overline{\text{MS}}$ at $\mu^2 = 10\ \text{GeV}^2$. The data on several ensembles are shifted horizontally to enhance the legibility. The green band shows the frame dependence of the global fit [with the empirical form in Eq. (11)] of the results.

Need more controlled calculations



Gluons Provide Half of the Proton's Spin

The gluons that bind quarks together in nucleons provide a considerable chunk of the proton's total spin. That was the conclusion reached by Yi-Bo Yang from the University of Kentucky, Lexington, and colleagues (see Viewpoint: **Spinning Gluons in the Proton**). By running state-of-the-art computer simulations of quark-gluon dynamics on a so-called spacetime lattice, the researchers found that 50% of the proton's spin comes from

App2: Feynman PDF

- PDF can be obtained from large momentum limit of a correlation

  $$\left\langle P \middle| \bar{\psi}(z) \Gamma W(z,0) \psi(0) \middle| P \right\rangle$$

  - Γ can be γ^0 or γ^3 or any combination.
  - W is a straight-line Wilson link

This is the starting point of quarsi-PDF,

X. Ji (2013)

Factorization was conjectured. The full-proved given by
Ma and Qiu, PRD98 (2018) 074021

Pion PDF (BNL/ANL, X Gao et al, PRL, 2022)

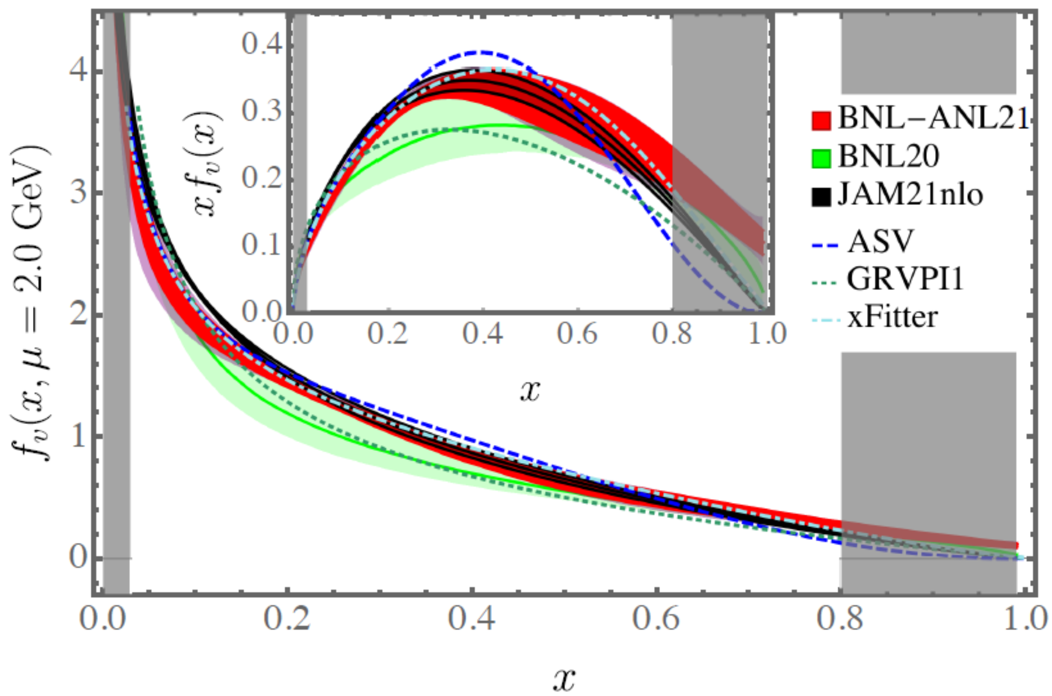
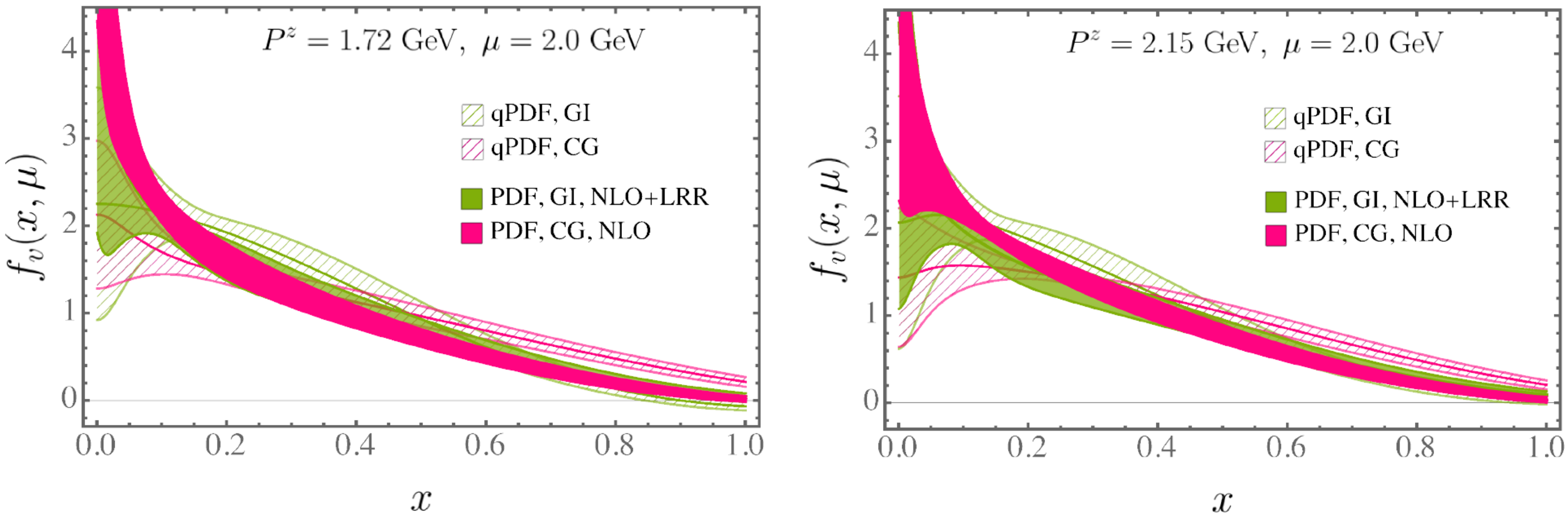


FIG. 4. Comparison of our prediction of $f_v(x)$, BNL-ANL21, to global fits and BNL20. The shaded regions $x < 0.03$ and $x > 0.8$ are excluded by requiring that estimates of $\mathcal{O}(\alpha_s^3)$ and power corrections be smaller than 5% and 10%, respectively.

LaMET is only approach to predict x dependence

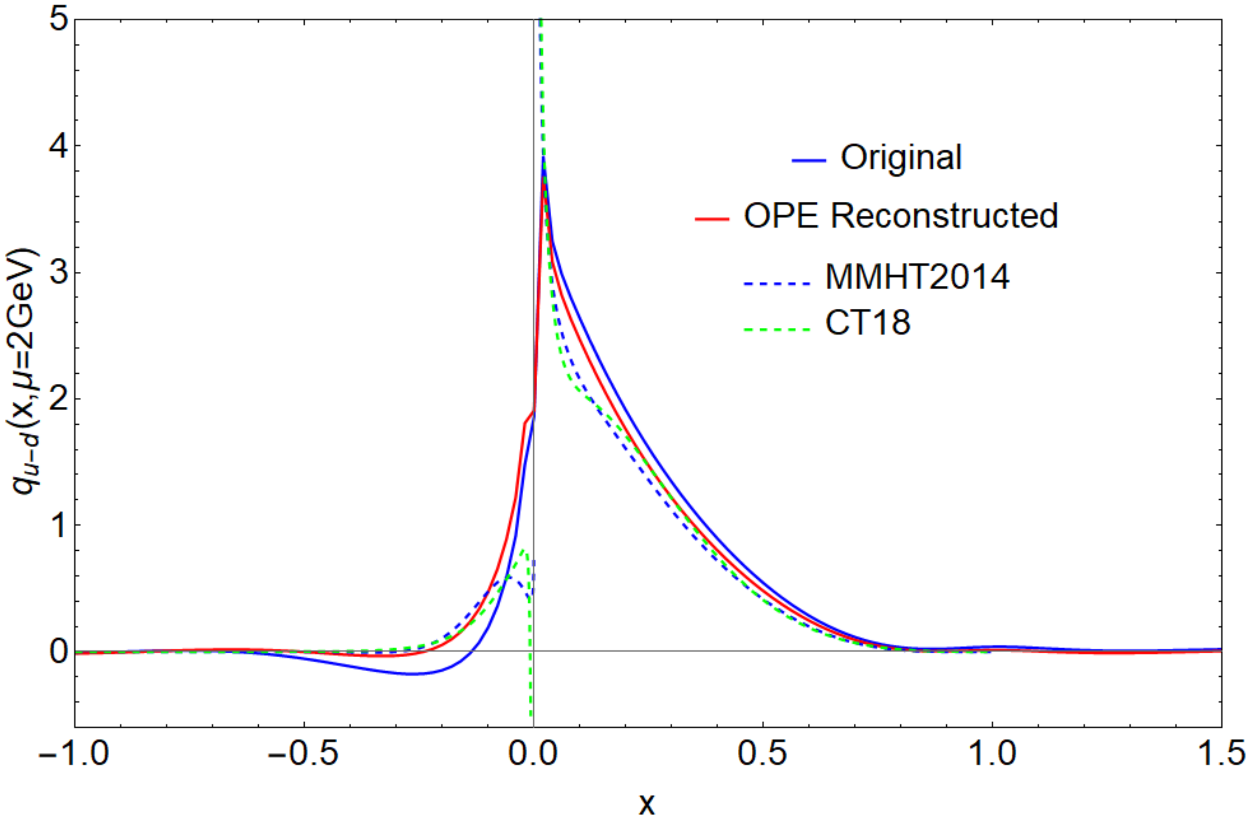
Universality

Comparison of the GI and CG quasi-PDF methods:



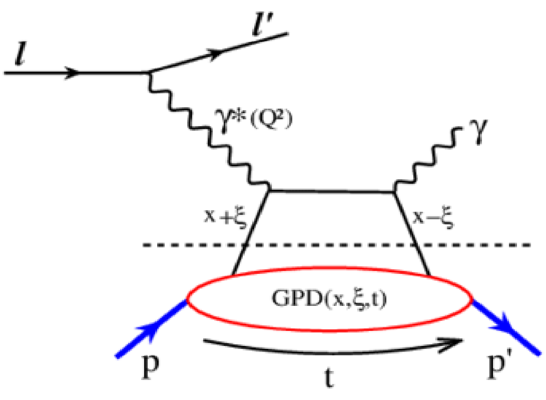
While the quasi-PDFs are different by at least 1σ, the matched results are consistent for $x \gtrsim 0.2$, demonstrating the universality in LaMET !

Proton u(x)-d(x) PDF (LPC, unpublished)



App3: Generalized parton distributions (GPD)

- GPD are form factors of parton distributions, providing coordinate space distribution and angular momentum of partons.
- An experimental process that was proposed to measure GPD: Deeply virtual Compton scattering (Ji, 1996). Other experiments including meson productions have also been studied (talk by J. W. Qiu)



Kinematic variables: x, ξ, t

GPD on lattice

- Present no additional difficulty compared with PDF
- One considers the off-forward matrix elements

$$\langle P' | \bar{\psi}(z) \Gamma W(z,0) \psi(0) | P \rangle$$

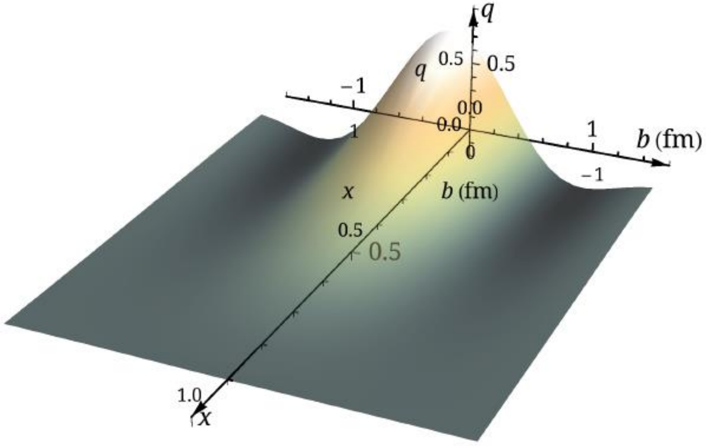
- Two more variables results
  - Momentum transfer, t
  - Skewness, ξ, need momentum transfer in z- direction.
- It has been very difficult to model these dependences in the literature. Great oppo to get guidance from lattice.
- Matching: Liu et al., PRD100 (2019) 034006

Quark transverse-plan distributions in the proton from lattice (HWLin, PRL2021)

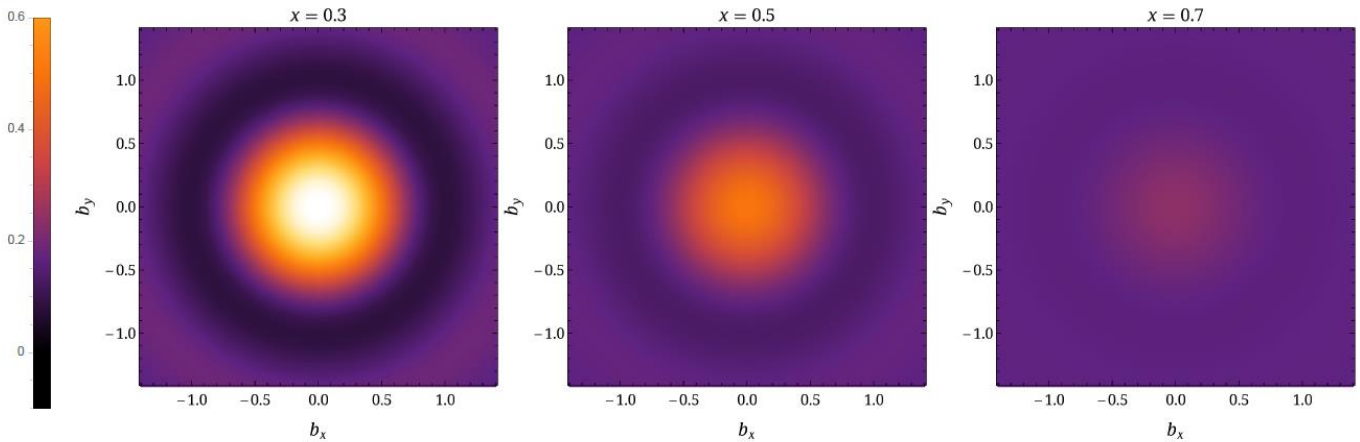
- MILC configurations

  $a = 0.09$ fm, $m_\pi = 140$ MeV

  $P_z = 2.2$ GeV



$$\mathrm{q}(x,b) = \int \frac{d\mathbf{q}}{(2\pi)^2} H(x,\xi=0,t=-\mathbf{q}^2) e^{i\mathbf{q}\cdot\mathbf{b}},$$



App4: Transverse-momentum-dependent PDFs

- Important nucleon observable, many phenomenology related to spin physics (Sivers effect etc).

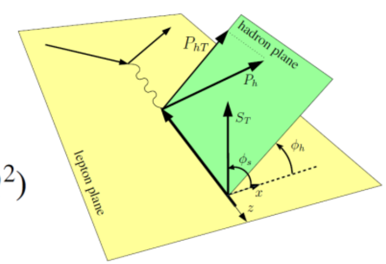
TMD HANDBOOK, R. Boussarie et al, e-Print: 2304.03302

TMDs from global analyses

e.g., semi-inclusive deep inelastic scattering: $l + p \longrightarrow l + h(P_h) + X$

$$\frac{d\sigma^W}{dx dy dz_h d^2\mathbf{P}_{hT}} \sim \int d^2\mathbf{b}_T \, e^{i\mathbf{b}_T\cdot\mathbf{P}_{hT}/z}$$

$$\times f_{i/p}(x, \mathbf{b}_T, Q, Q^2)\; D_{h/i}(z_h, \mathbf{b}_T, Q, Q^2)$$



Kang, Prokudin, Sun and Yuan, PRD 93 (2016)

$$f_{i/p}(x, \mathbf{b}_T, \mu, \zeta) = f^{\rm pert}_{i/p}(x, b^*(b_T), \mu, \zeta)$$

$$\times \left(\frac{\zeta}{Q_0^2}\right)^{g_K(b_T)/2} f^{\rm NP}_{i/p}(x, b_T)$$

$g_K(b_T)/2$ ⟶ Collins-Soper kernel (NP part)

$f^{\rm NP}_{i/p}(x, b_T)$ ⟶ Intrinsic TMD

$Q_0 \sim 1$ GeV

Non-perturbative when $b_T \sim 1/\Lambda_{\rm QCD}$!

Factorization formula for the quasi-TMDs

$$\frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \ \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right]$$
$$\times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\rm QCD}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and YZ, PRD99 (2019);
- Ebert, Stewart, YZ, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Vladimirov and Schäfer, PRD 101 (2020);
- Ebert, Schindler, Stewart and YZ, JHEP 04, 178 (2022).

Matching coefficient:

- Independent of spin;
  - Vladimirov and Schäfer, PRD 101 (2020);
  - Ebert, Schindler, Stewart and YZ, JHEP 09 (2020);
  - Ji, Liu, Schäfer and Yuan, PRD 103 (2021).
- No quark-gluon or flavor mixing, which makes gluon calculation much easier.

One-loop matching for gluon TMDs:

Schindler, Stewart and YZ, JHEP **08** (2022);

Zhu, Ji, Zhang and Zhao, JHEP **02** (2023).

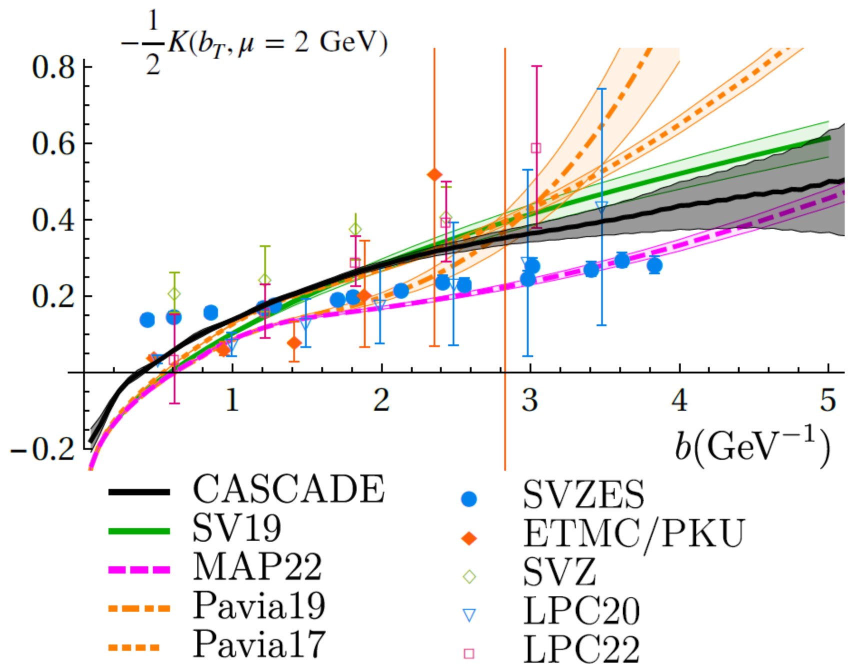
Factorization formula for the quasi-TMDs

$$\frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right]$$
$$\times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\text{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

- ✳ Collins-Soper kernel; $\gamma_\zeta(\mu, b_T) = \frac{d}{d\ln\tilde{P}^z}\ln\frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{C\left(\mu, x\tilde{P}^z\right)}$
- ✳ Flavor separation; $\frac{f_{i/p}^{[s]}(x, \mathbf{b}_T)}{f_{j/p}^{[s']}(x, \mathbf{b}_T)} = \frac{\tilde{f}_{i/p}^{\text{naive}[s]}(x, \mathbf{b}_T)}{\tilde{f}_{j/p}^{\text{naive}[s']}(x, \mathbf{b}_T)}$
- ✳ Spin-dependence, e.g., Sivers function (single-spin asymmetry);
- ✳ Full TMD kinematic dependence.
- ✳ Twist-3 PDFs from small b_T expansion of TMDs. Ji, Liu, Schäfer and Yuan, PRD 103 (2021).
- ✳ Higher-twist TMDs. Rodini and Vladimirov, JHEP 08 (2022).

Collins Soper kernel from Lattice QCD

Comparison between lattice results and global fits

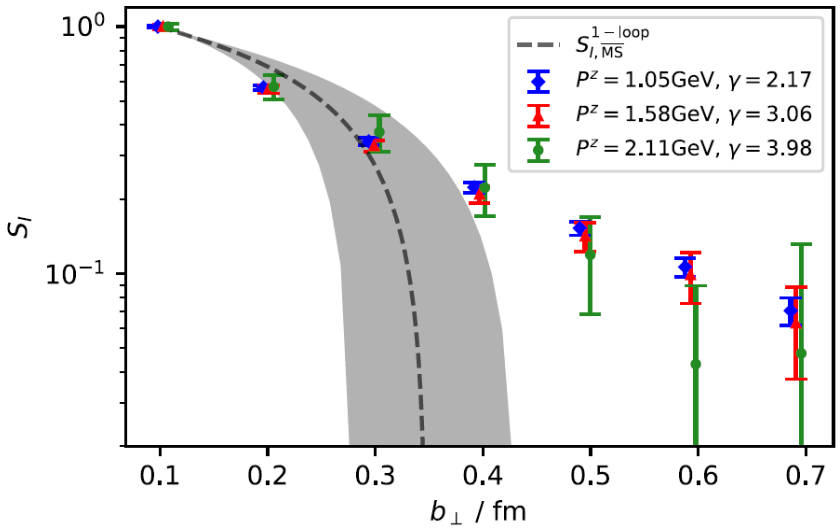


MAP22: Bacchetta, Bertone, Bissolotti, et al., JHEP **10** (2022)
SV19: I. Scimemi and A. Vladimirov, JHEP **06** (2020)
Pavia19: A. Bacchetta et al., JHEP **07** (2020)
Pavia 17: A. Bacchetta et al., JHEP **06** (2017)
CASCADE: Martinez and Vladimirov, PRD **106** (2022)

Approach	Collaboration
Quasi beam functions	P. Shanahan, M. Wagman and YZ (SWZ21), PRD **104** (2021)
Quasi TMD wavefunctions	Q.-A. Zhang, et al. (LPC20), PRL **125** (2020).
	Y. Li et al. (ETMC/PKU 21), PRL **128** (2022).
	M.-H. Chu et al. (LPC22), PRD 106 (2022)
Moments of quasi TMDs	Schäfer, Vladmirov et al. (SVZES21), JHEP **08** (2021), 2302.06502

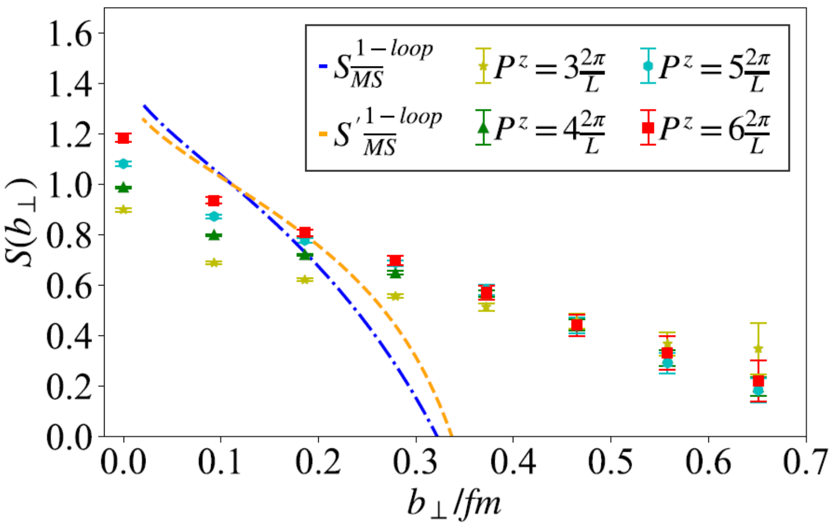
Lattice result of the reduced soft factor

$a = 0.10$ fm,
$m_\pi = 547$ MeV,
$P^z_{\rm max} = 2.11$ GeV



Q.-A. Zhang, et al. (LPC), PRL 125 (2020).

$a = 0.09$ fm,
$m_\pi = 827$ MeV,
$P^z_{\rm max} = 3.3$ GeV



Y. Li et al., PRL 128 (2022).

Tree-level approximation:

$$H(x, x', \mu) = 1 + \mathcal{O}(\alpha_s) \quad \Rightarrow S^r_q(b_T) = \frac{F(b_T, P^z)}{[\tilde{\Phi}(b^z = 0, b_T, P^z)]^2}$$

TMDPDF

- LPC (J.C.He et al, 2211.02340)

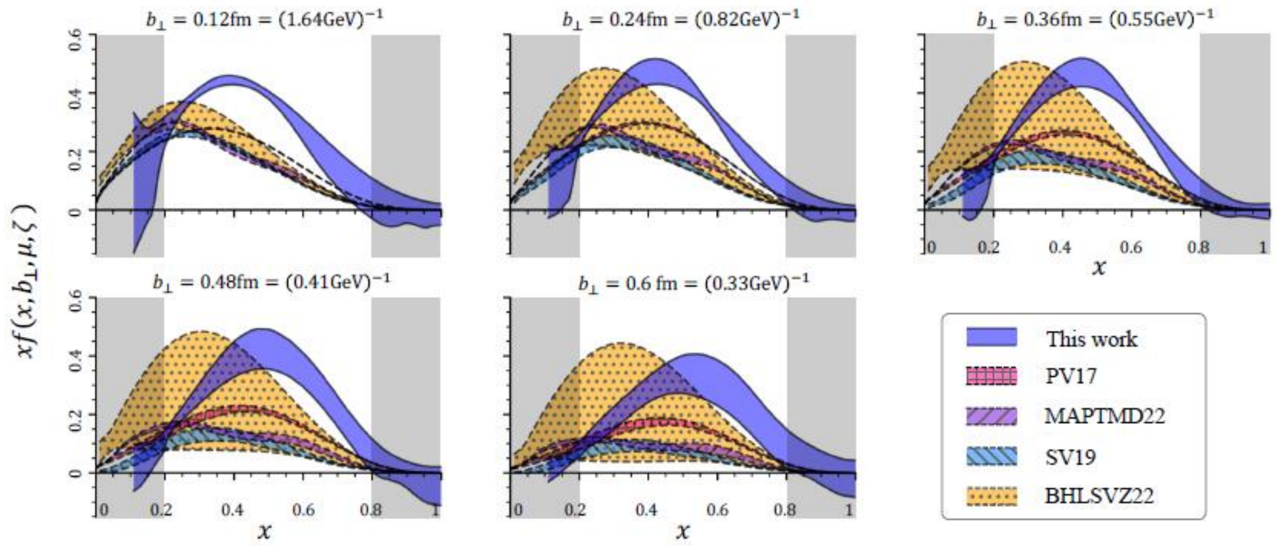


FIG. 5. Our final results for isovector unpolarized TMDPDFs $xf(x, b_\perp, \mu, \zeta)$ at renormalization scale $\mu = 2$ GeV and rapidity scale $\sqrt{\zeta} = 2$ GeV, extrapolated to physical pion mass 135 MeV and infinite momentum limit $P^z \to \infty$, compared with PV17 [6], MAPTMD22 [9], SV19 [7] and BHLSVZ22 [8] global fits (slashed bands). The colored bands denote our results with both statistical and systematic uncertainties, the shaded grey regions imply the endpoint regions where LaMET predictions are not reliable.

App 5: Light-Front Wave Function

- Light-front quantized theory is formal (undefined!) and cannot be solved without regularizing light-cone singularities.
- If the regularization breaks Lorentz symmetry (almost all regulators in the LFQ literature do), theory ends up non-renormalizable.
- LFQ can be defined through large-momentum effective theories, including wave functions.

(X. Ji & Y. Liu, 2022)

Transverse Momentum Dependent Wave Functions from Lattice QCD

(Lattice Parton Collaboration (LPC))

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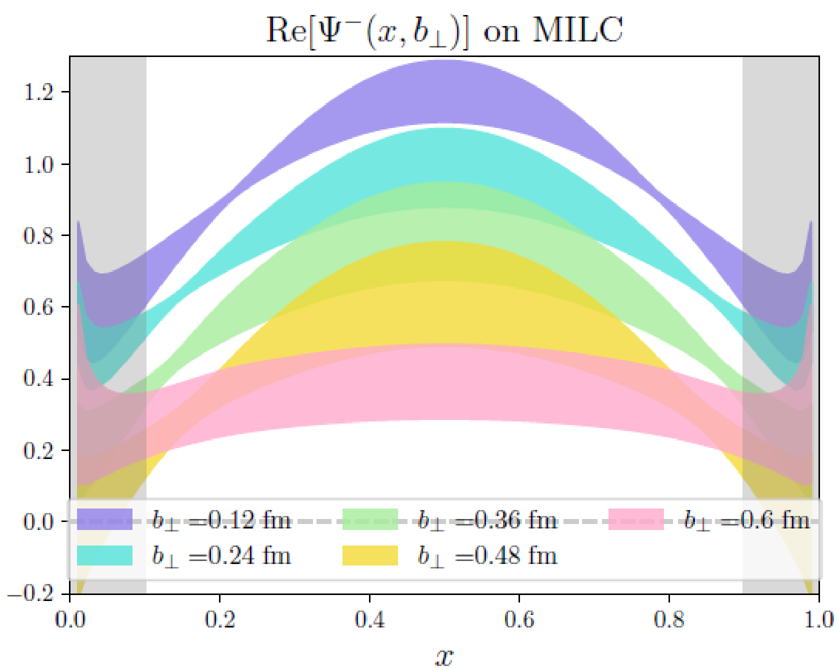
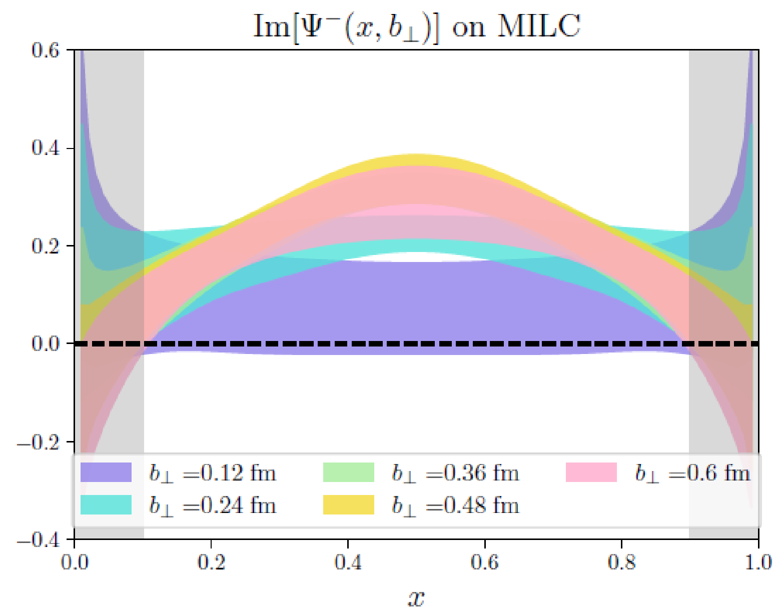
We present a first lattice QCD calculation of the transverse momentum dependent wave functions (TMDWFs) in large-momentum effective theory. Numerical simulations are based on 2+1+1 flavors of highly improved staggered quarks action with lattice spacing a=0.121 fm from MILC Collaboration, and another 2 +1 flavor clover fermions and tree-level Symanzik gauge action configuration generated by CLS Collaboration with a=0.098 fm. We present the result for soft function that incorporates the one-loop perturbative contributions and a coherent normalization. Based on the obtained soft function, we simulate the equal-time quasi-TMDWFs on the lattice, and extract the physical TMDWFs. A comparison with the phenomenological parameterization is made and consistent behaviors between the two lattice ensembles and phenomenological model are found. Our studies provide crucial *ab initio* theory inputs for making precise predictions for exclusive processes under QCD factorization.

Lowest Fock state LFWF





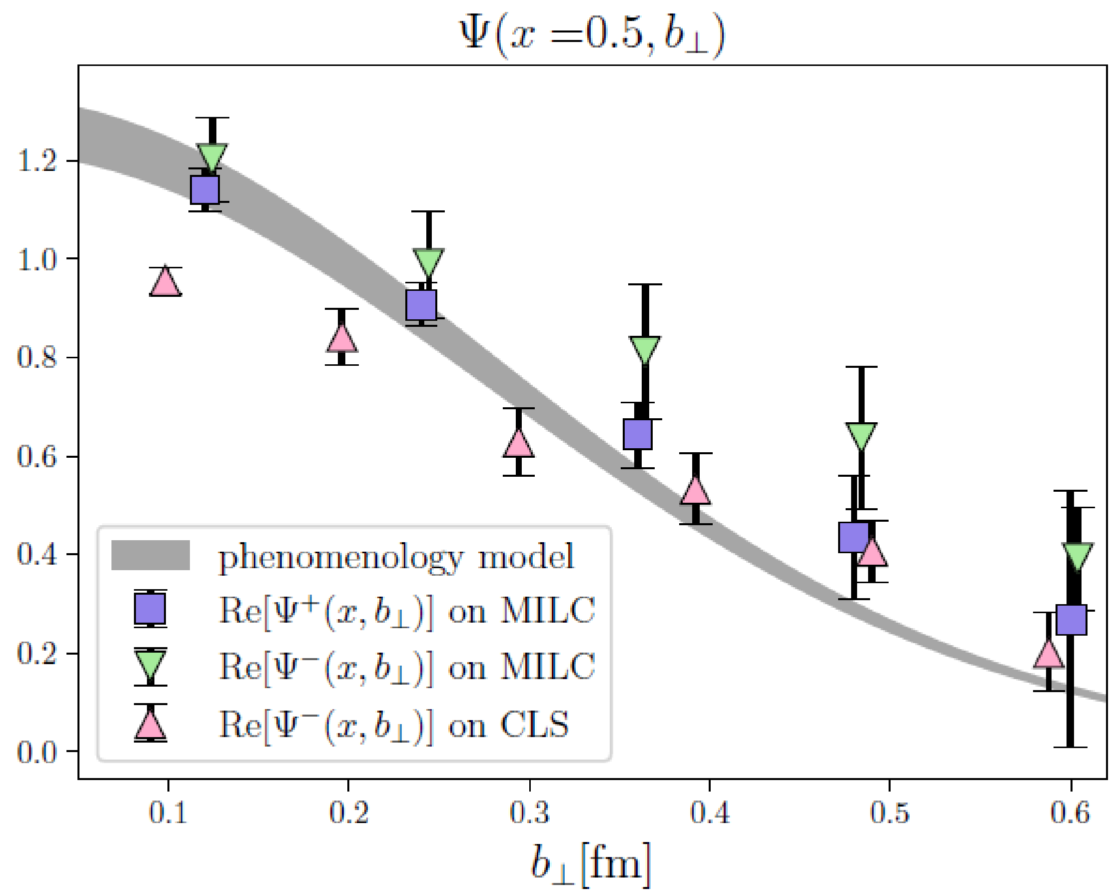


FIG. 5. Comparison of the transverse momentum distribution in our results with $\{\zeta, \mu\} = \{(6\ \mathrm{GeV})^2, 2\ \mathrm{GeV}\}$ and phenomenological model at $x = 0.5$ point.

Outlook

- Partons can be calculated on lattice using a large-momentum hadron state

$$\frac{\Lambda_{QCD}}{P} \ll 1$$

- LaMET3.0 (~ 5% error)

  Improved non-pert renormalization

  two-loop matching, small-x and threshold resummation, twist-3 renormalon corrections

  P=3 GeV

- ~1% accuracy in 10-20 years?