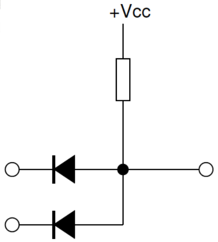
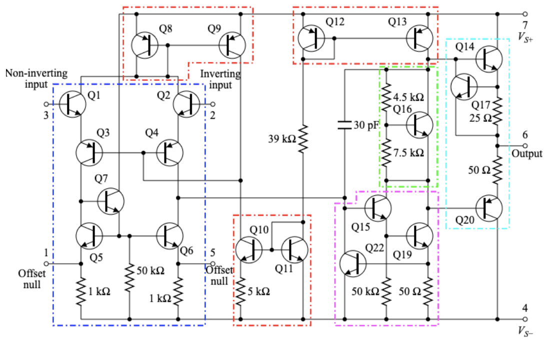
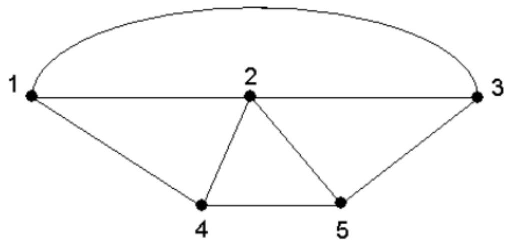
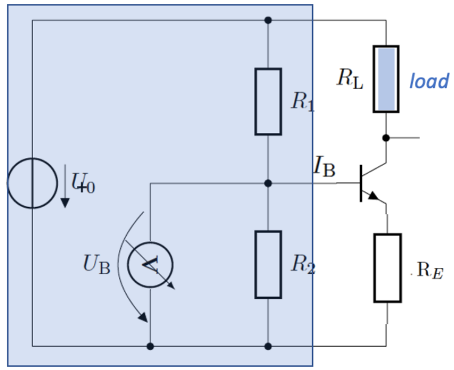
Introduction into Electronics

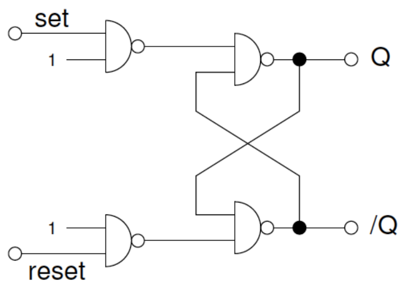
(3) Digital electronics, appendix
(4) Circuit analysis, circuit topologies



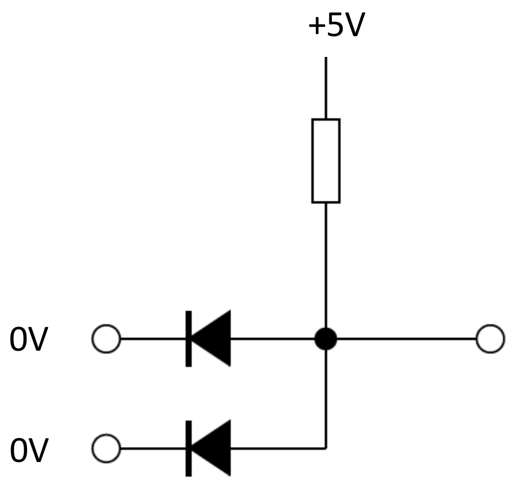








Appendix: Simple diode-based AND gate



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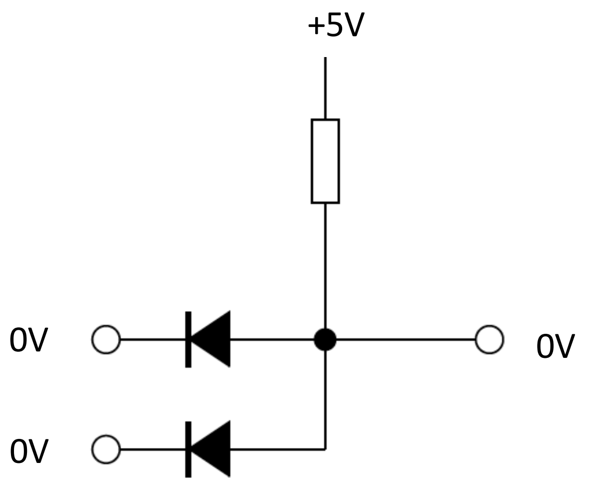

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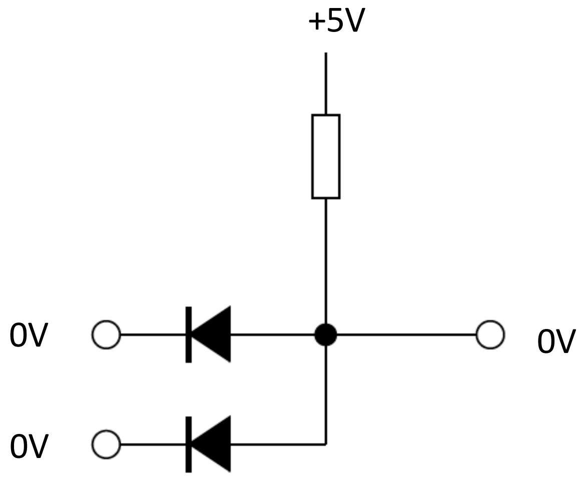

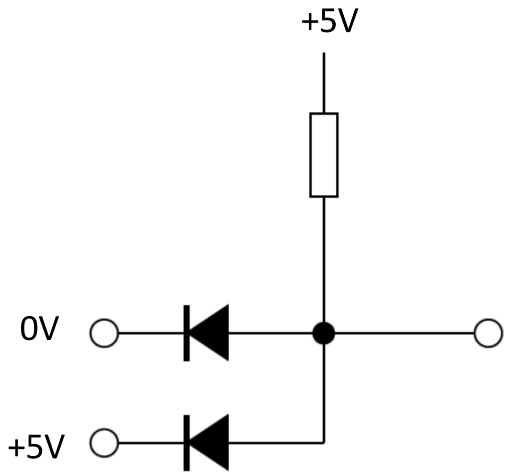

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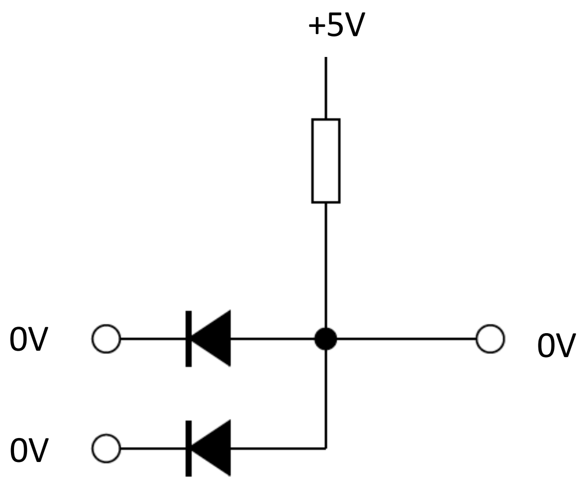

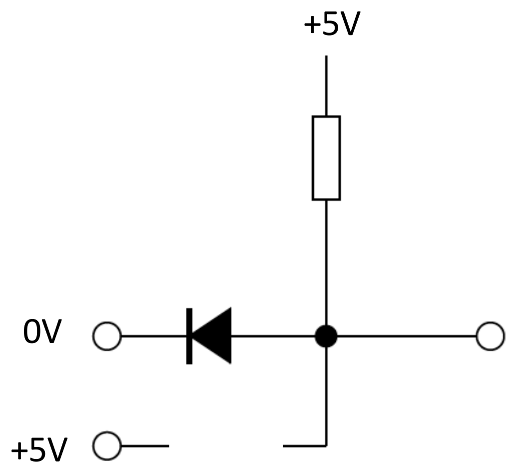

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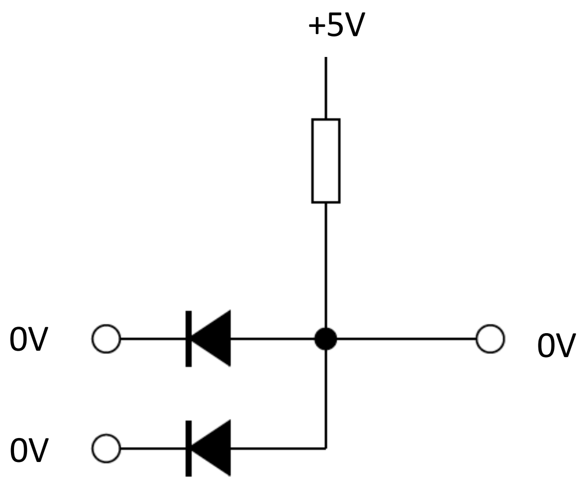

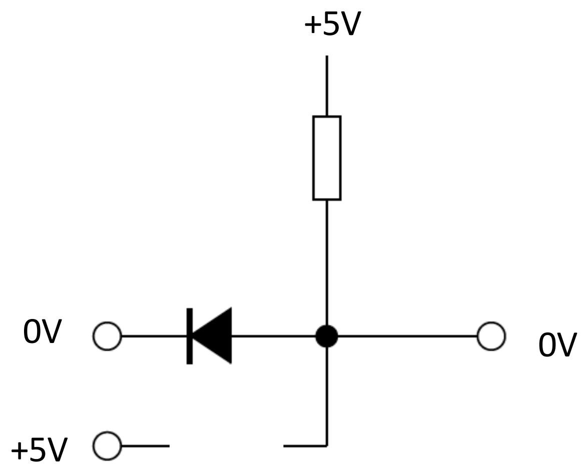

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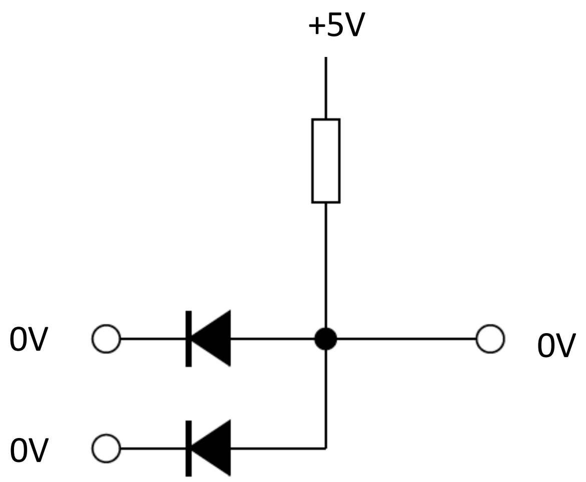

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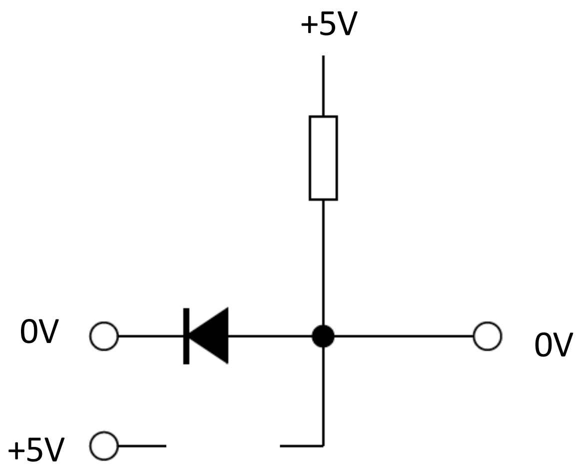

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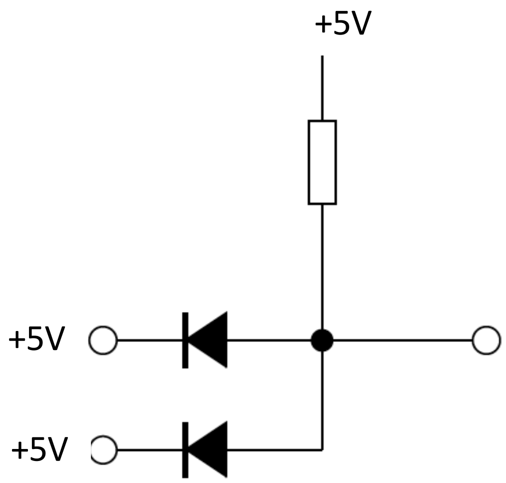

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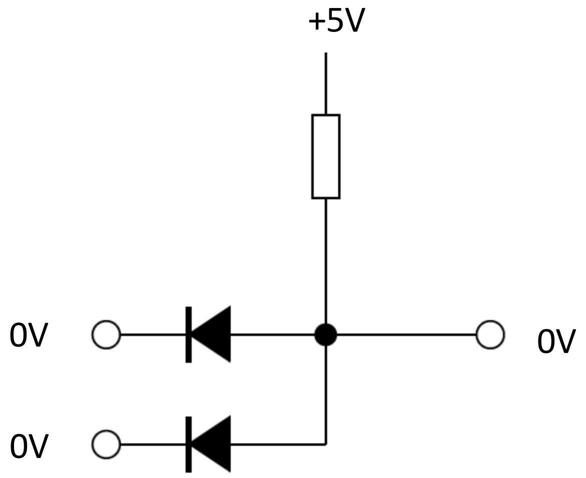

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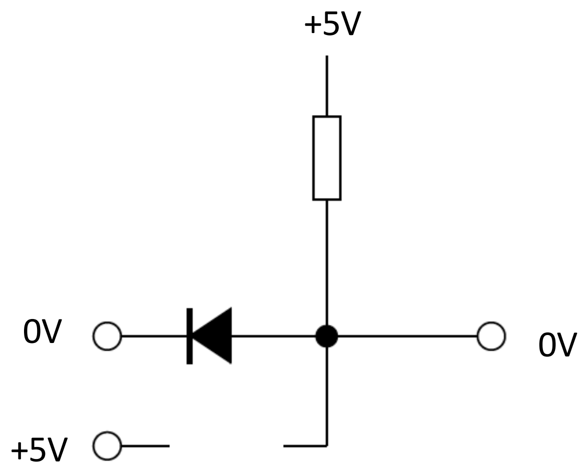

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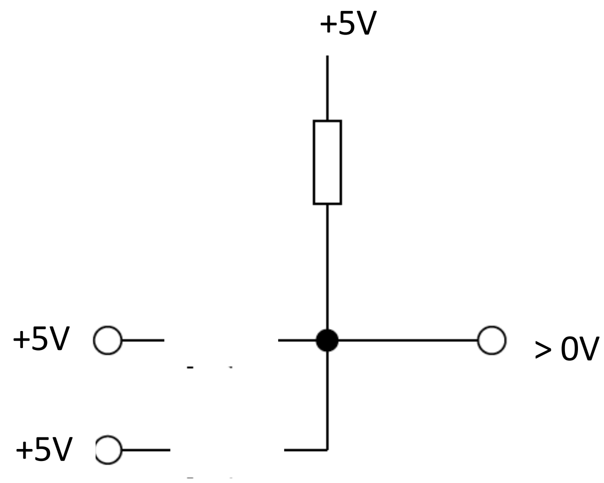

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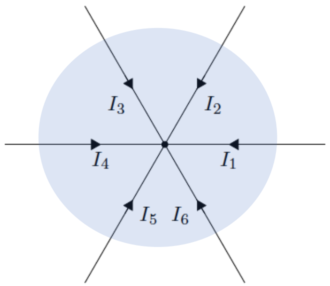
Circuit analysis

Basic circuit analysis can be performed based on the two Kirchhoff laws:

Junction rule or
Kirchhoff's Current Law (KCL):

The currents flowing out of any closed
Region of a circuit sum to 0

$$\sum_{k=1}^{n} I_k = 0$$



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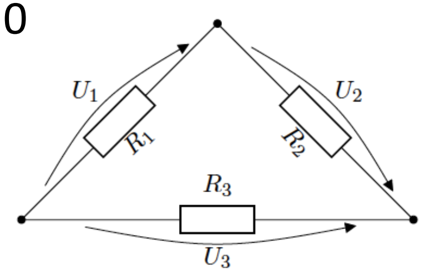

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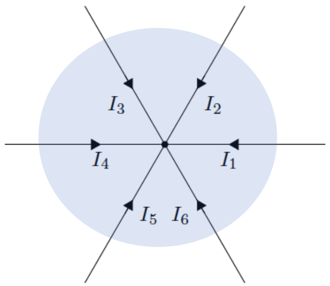

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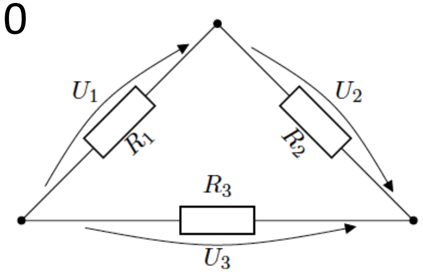
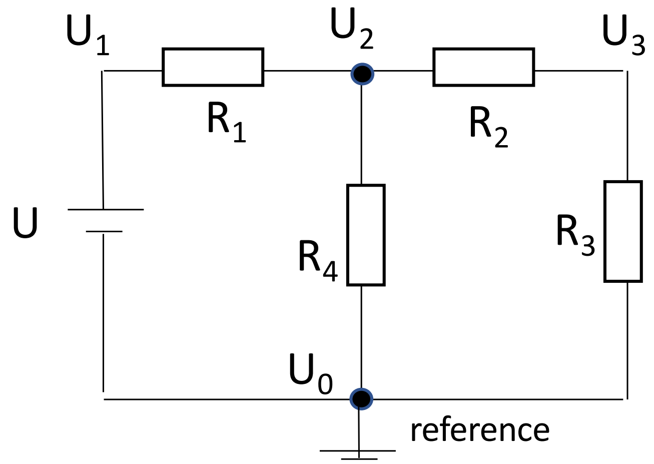


How can we apply these laws to calculate the circuits in an efficient way ?

→ Nodal analysis: identify nodes and applies KCL for each node
→ Mesh current analysis: identify essential meshes, assign mesh current and apply KVL
→ Thevenin equivalent: replace part of the network by source + resistance in series
→ Norton equivalent: replace part of the network by source and resistance in parallel

Nodal Analysis

Node: section of the circuit which connects components.
Aim: determine the voltage at each node relative to a reference node, then use them to derive the other relevant quantities



Steps:
- Identify all nodes and assign voltage variables (treat floating or dependent sources as super nodes with internal equation)
- Choose reference node
- Write a KCL equation at each node
- Solve the system of equations (e.g. via matrix inversion)

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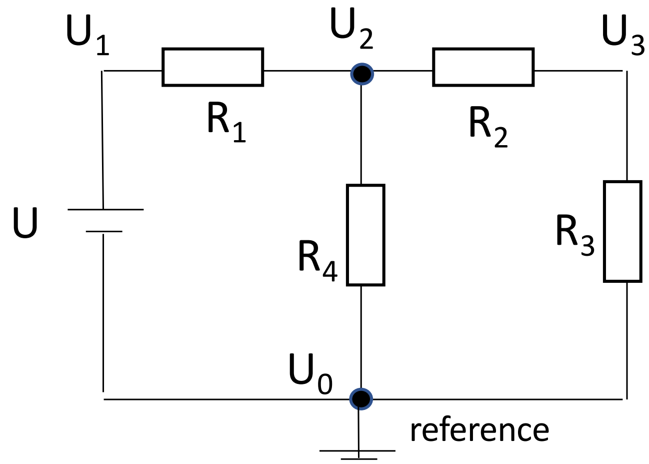

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Example: U_0: $U_0 = 0V$

U_1: $U_1 = U$

U_2 KCL: $\frac{U_1 - U_2}{R_1} = \frac{U_2 - U_0}{R_4} + \frac{U_2 - U_3}{R_2}$

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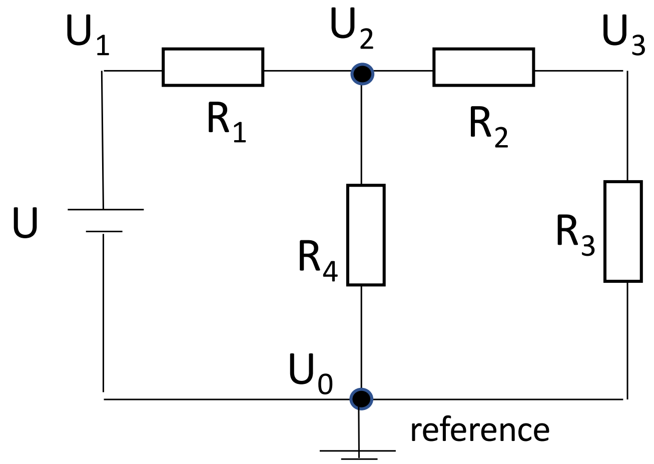

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- conveniently using conductance $G = \frac{1}{R}$ -

$$\rightarrow \begin{bmatrix} G_1 + G_2 + G_4 & -G_2 \\ G_2 & -G_2 - G_3 \end{bmatrix} \begin{pmatrix} U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} G_1 U \\ 0 \end{pmatrix}$$

- invert matrix or solve by substitution -

$$\rightarrow U_3 = \frac{G_1 G_2 U}{G_2(G_1 + G_4) + G_3(G_1 + G_2 + G_4)} \rightarrow U_2$$

Mesh current Analysis

Essential mesh: loop in the circuit that does not contain any other loop.
Aim: determine the current through each mesh
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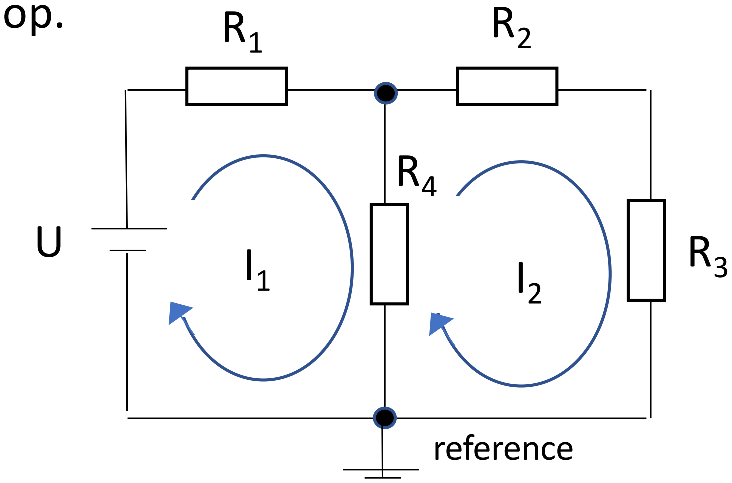

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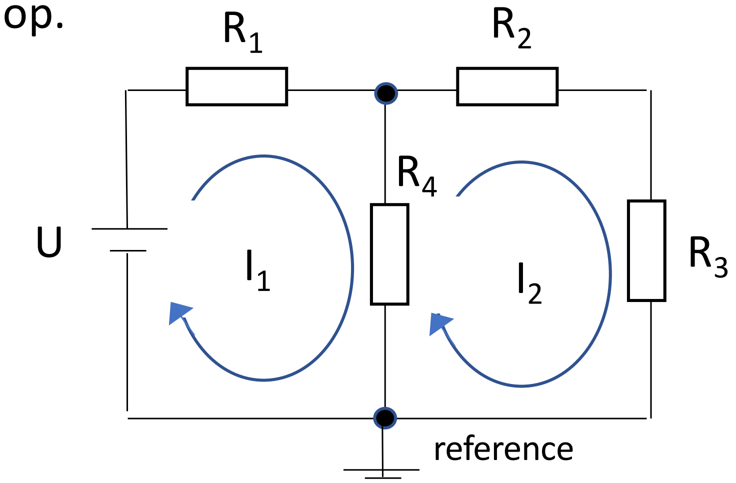

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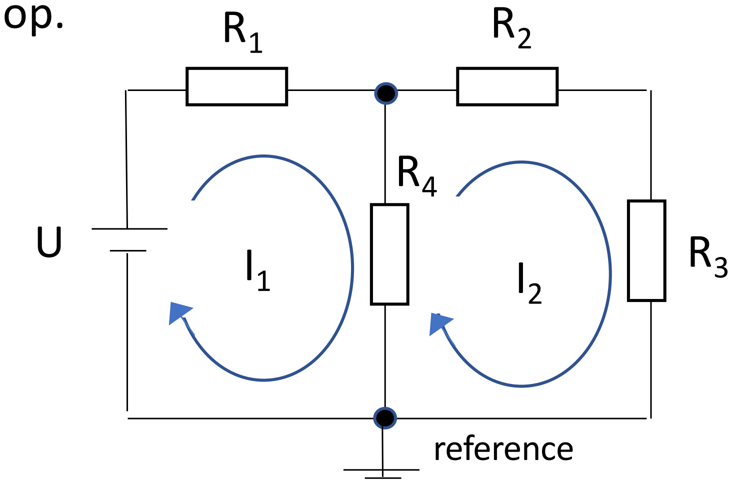

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$$\rightarrow \begin{bmatrix} R_1 + R_4 & -R_4 \\ -R_4 & R_2 + R_3 + R_4 \end{bmatrix} \binom{I_1}{I_2} = \binom{U}{0}$$

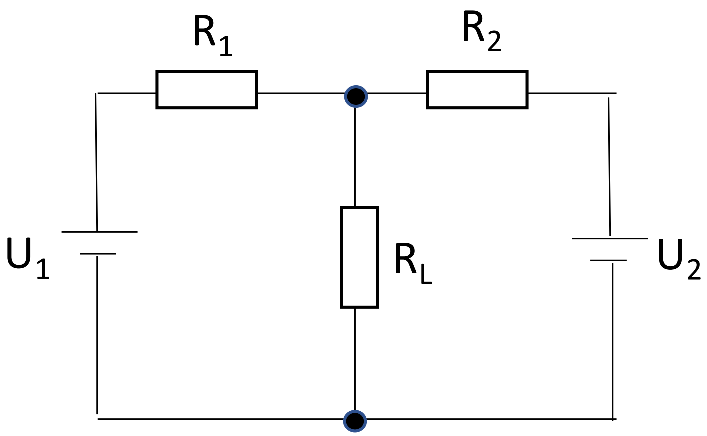
- invert matrix or solve by substitution -

$$\rightarrow I_1 = \frac{(R_2+R_3+R_4)\,U}{(R_1+R_4)(R_2+R_3+R_4)-R_4^{\,2}} \rightarrow I_2$$

Thevenin equivalent circuit

Thevenin's theorem: Any linear circuit containing several voltages and resistances can be replaced by just one single voltage in series with a single resistance connected across the load“

Example circuit:



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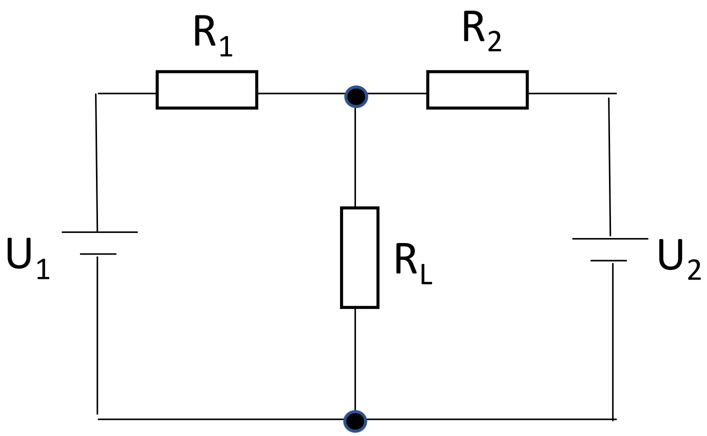
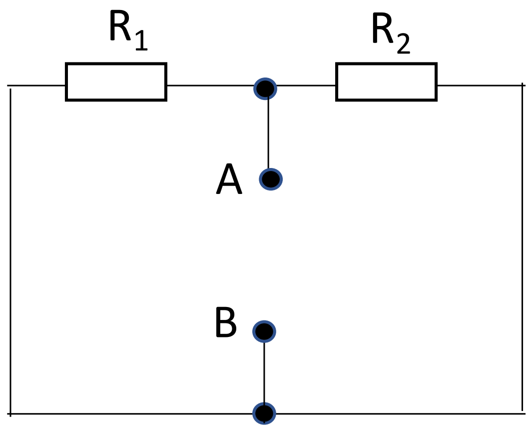


Step1: remove load and shorten sources. Then calculate total Thevenin resistance R_T wrt A/B



$$R_T = R_1||R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

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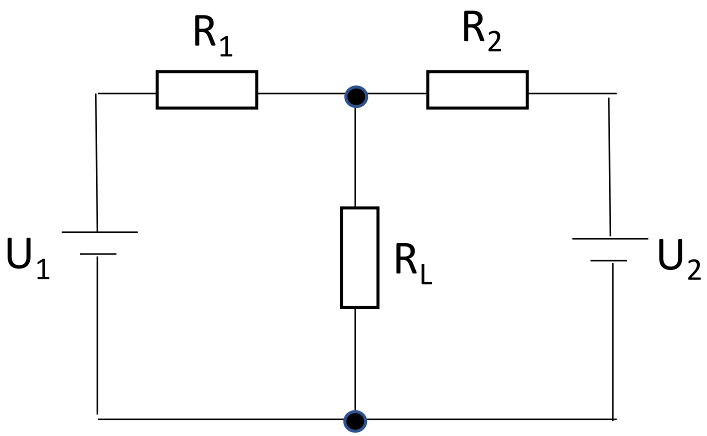

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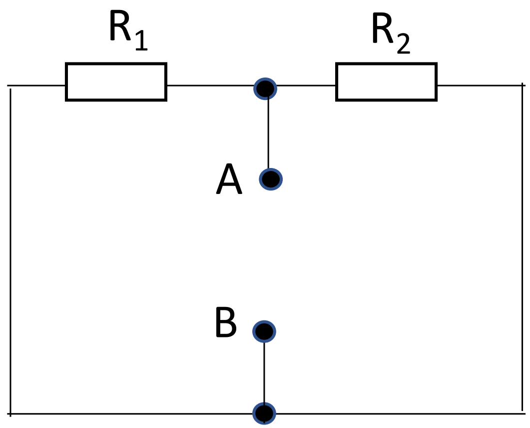

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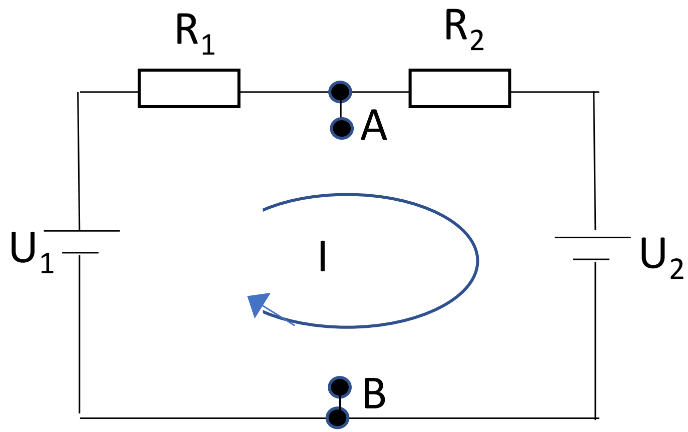
Step2: Reconnect sources. Calculate Thevenin voltage U_T



$$I = \frac{U_1 - U_2}{R_1 + R_2}$$

$$U_T = U_{AB} = U_1 - IR_1 = U_2 + IR_2$$

$$= \frac{U_1(R_1+R_2) - (U_1 - U_2)R_1}{R_1+R_2} = \frac{R_1 U_2 + U_1 R_2}{R_1+R_2}$$

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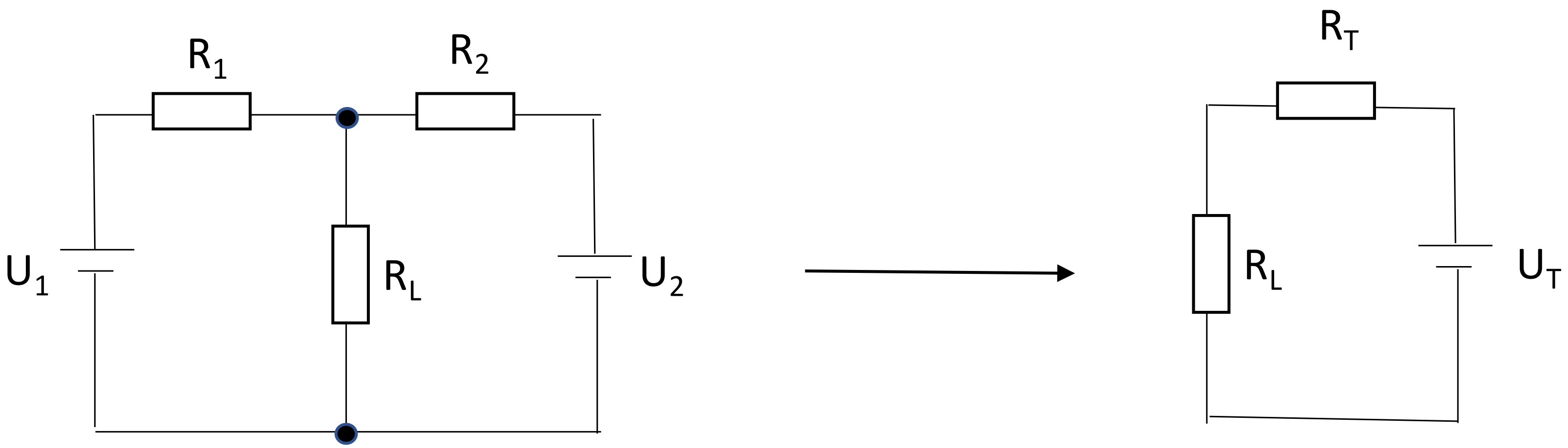
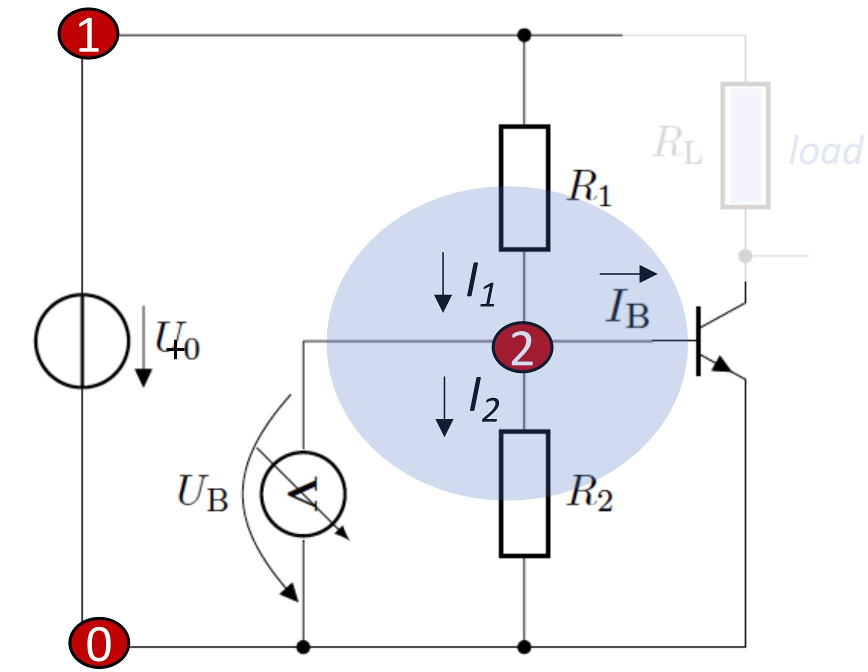


$$R = \frac{R_1 R_2}{R_1 + R_2} \qquad U = \frac{R_1 U_2 + U_1 R_2}{R_1 + R_2}$$

Example: nodal analysis

Revisiting Voltage divider biasing circuit



Reminder: The voltage U_B across R_2 forward-biases the BE junction

Here: can use **nodal analysis**:
Apply Kirchhoff's current law (KCL) at node 2

Reminder the $I_B - U_B$ relation does not follow Ohms law but a diode-like input characteristics (which for this exercise, we pretend not to know)

U1: $U_1 = U_0$ (1)

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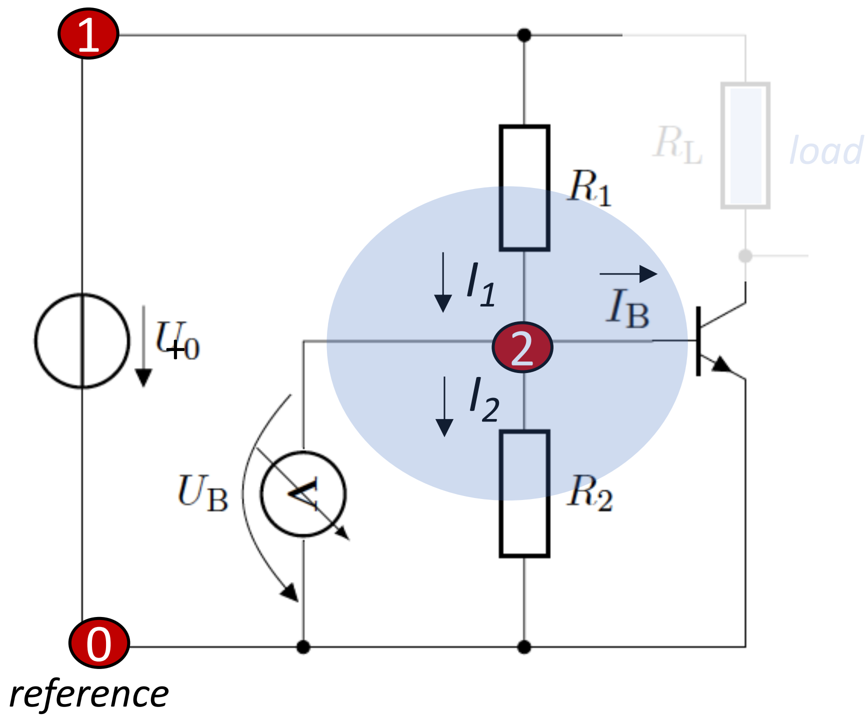

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U_2 KCL: $I_1 = I_2 + I_B$ (2)

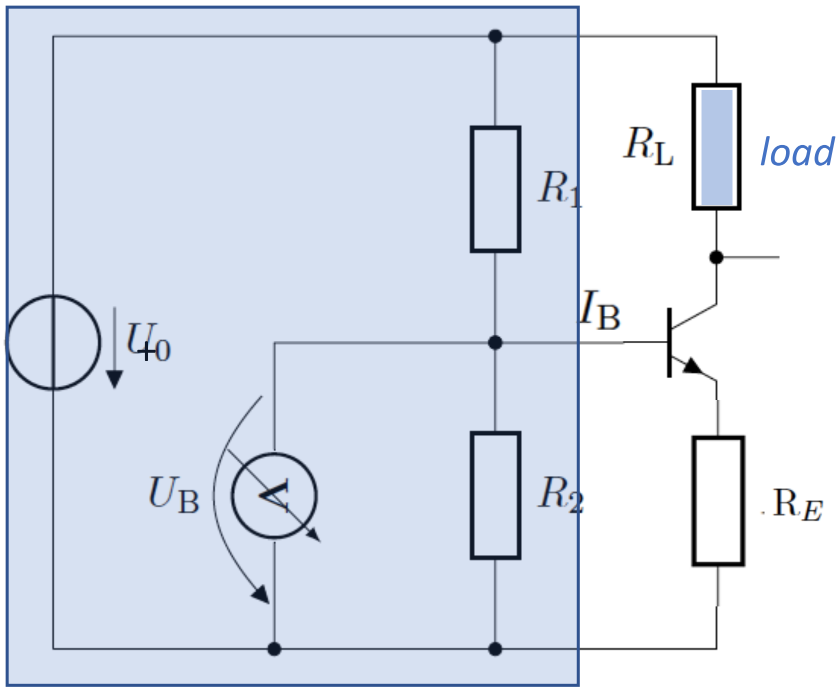
(1) & (2) + Ohm's law $\rightarrow \frac{U_0 - U_B}{R_1} = \frac{U_B}{R_2} + I_B \rightarrow U_0 - U_B = U_B \frac{R_1}{R_2} + I_B R_1 \rightarrow U_B \frac{R_1 + R_2}{R_2} = U_0 - I_B R_1$

$$\rightarrow U_B = U_0 \frac{R_2}{R_1 + R_2} - I_B \frac{R_1 R_2}{R_1 + R_2}$$

Example: Thevenin equivalent

Voltage divider biasing with emitter resistance

Reminder: We can stabilize the working point by adding an emitter resistance



In order to simplify the calculations, we want to replace the voltage divider by the Thevenin equivalent wrt the emitter resistance

- Thevenin voltage: $U_T = \; U_0 \frac{R_2}{R_1+R_2}$

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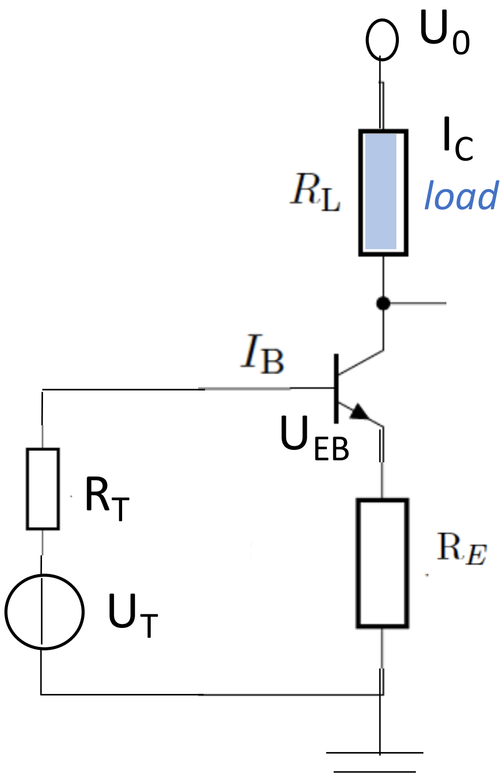

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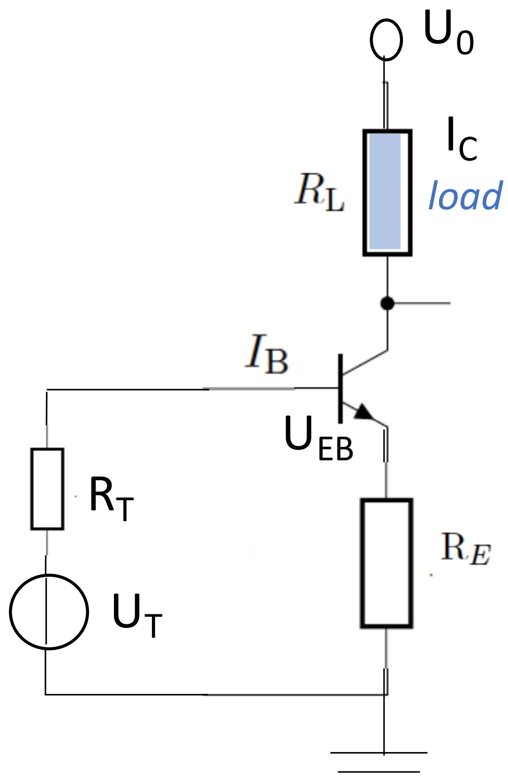

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Now we can analyze the mesh (KVL):

$$U_T = I_B R_T + U_{BE} + R_E(I_B + Ic)$$

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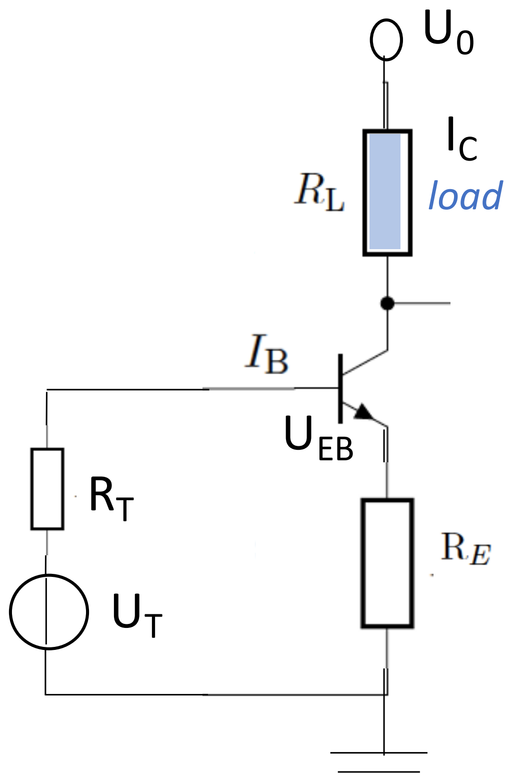

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Now we can analyze the mesh (KVL):

$$U_T = I_BR_T + U_{BE} + R_E(I_B + Ic)$$

with $I_C = \beta\, I_B$ (transfer characteristics):

$$U_T = I_BR_T + U_{BE} + R_E\, I_B\,(1 + \beta)$$

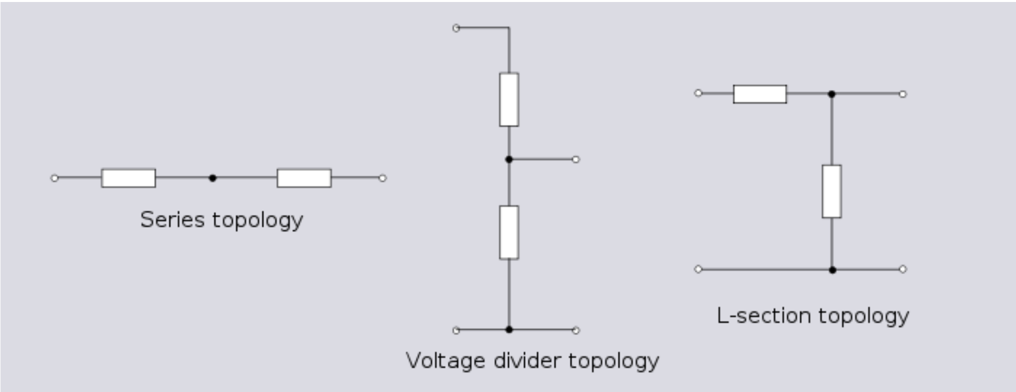
$$\rightarrow I_B = \frac{U_T - U_{BE}}{R_T + R_E(1+\beta)} \rightarrow I_C = \beta I_B \rightarrow ....$$

Circuit topologies

The formal layout of the equations following the application of Kirchhoff's rules applied to a circuit does not depend on the type of device connected in the branches. It only depends on the topology of the circuit.

Circuit Topology describes how components in a network are connected. Circuits with different physical layout can have the same topology.

Example: three circuits with the same topology:



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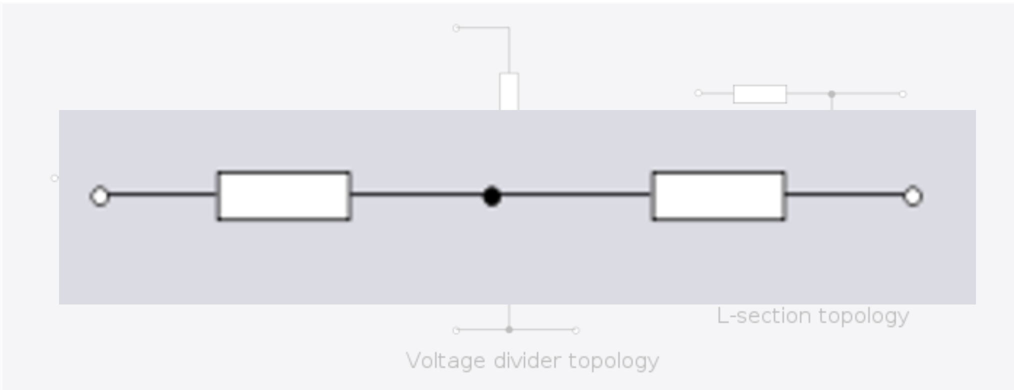

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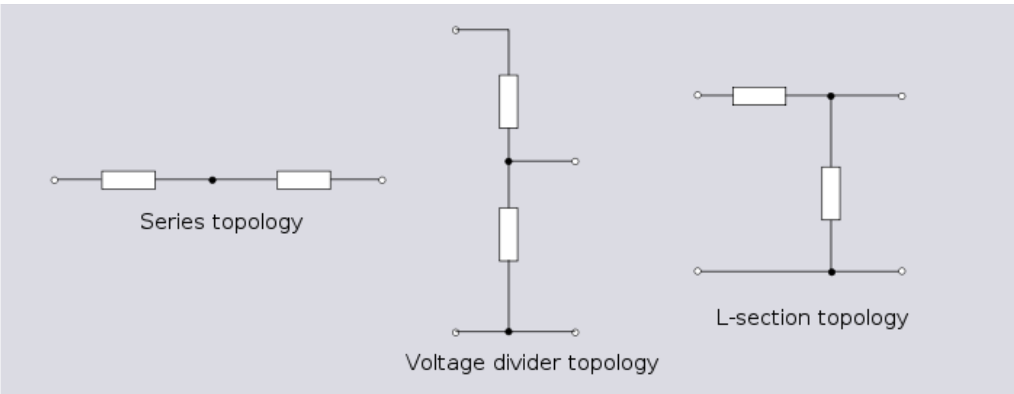
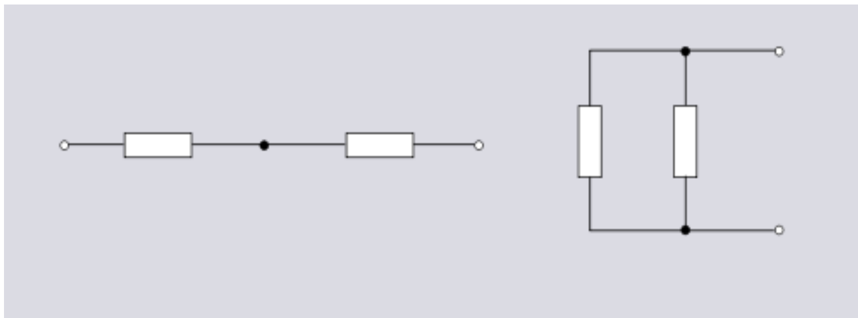


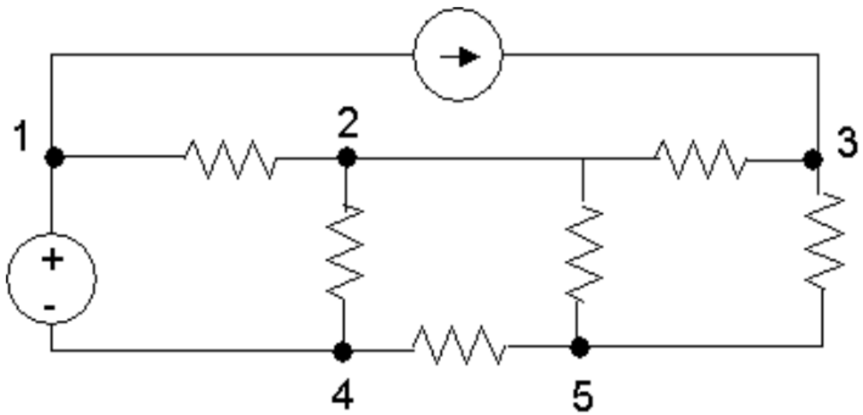
Example: There are only two topologies for a network with 2 branches: series and parallel.



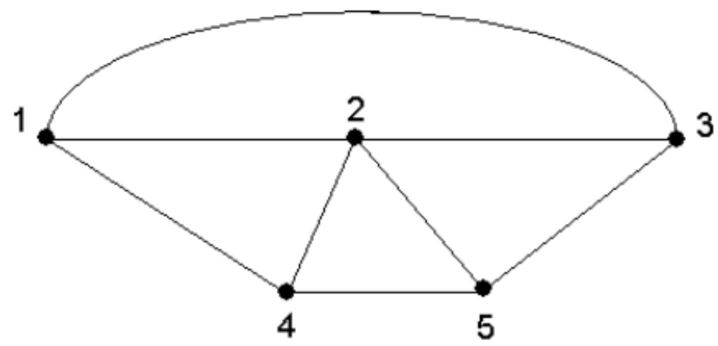
Graph theory

Mathematical ***Graph theory*** is used to analyze the topology:
Graphs represents the aspects of a network connected to its topology. The devices are left out.
→ a line represents a device. A nod represents all points at the same potential.

Example: ***circuit*** ***graph***



→



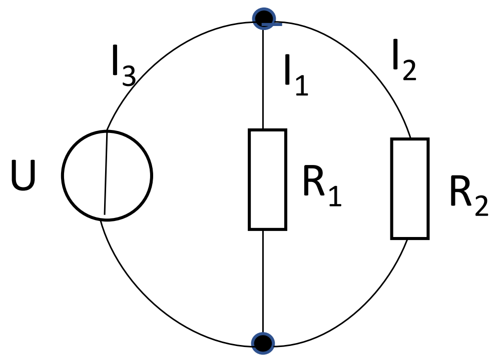
The graph representation makes it easier to analyze complex circuits.
Graphs are ***equivalent*** if they can be transformed into each other by translation, rotation, reflection, stretching or crossing/knotting the branches,

Graph theory: dual graph

*Each graph has a **dual graph** with the following elements exchanged:*

- *R ←→ 1/R*
- *L ←→ C*
- *U ←→ I*
- *U source ←→ I source*
- *mesh ←→ node*

graph



$$I_3 = -I_1 - I_2 = U(\frac{1}{R_1} + \frac{1}{R_2})$$

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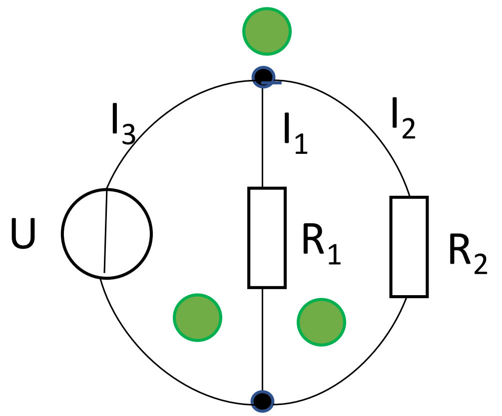

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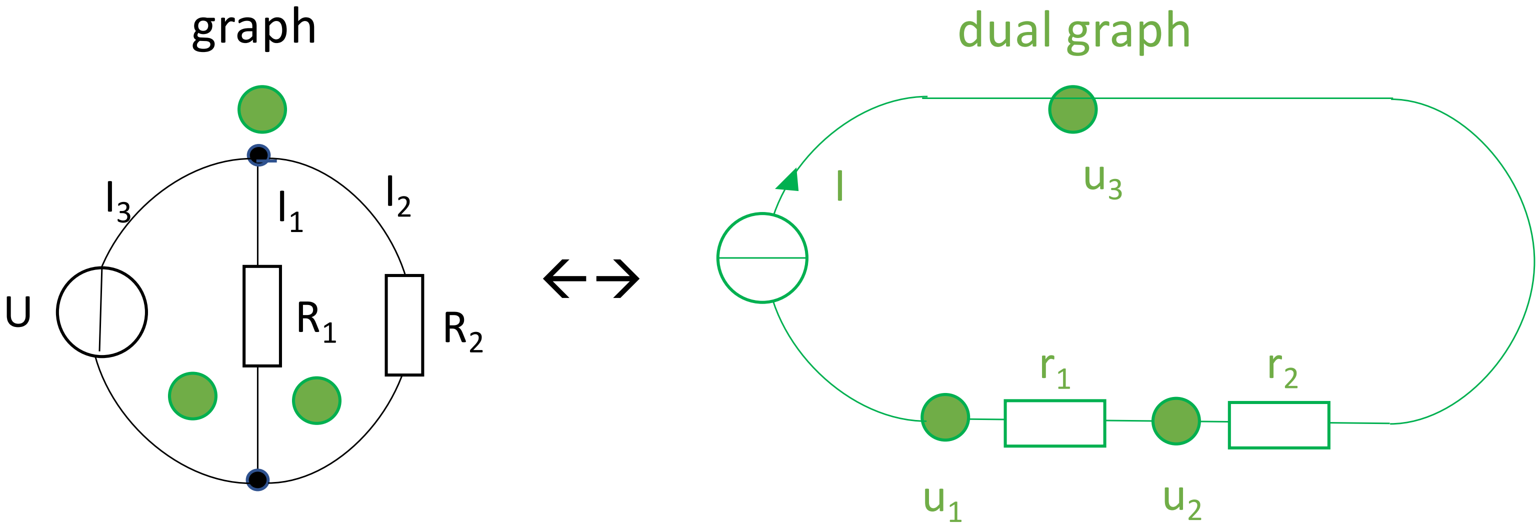
converted parallel circuit into serial circuit



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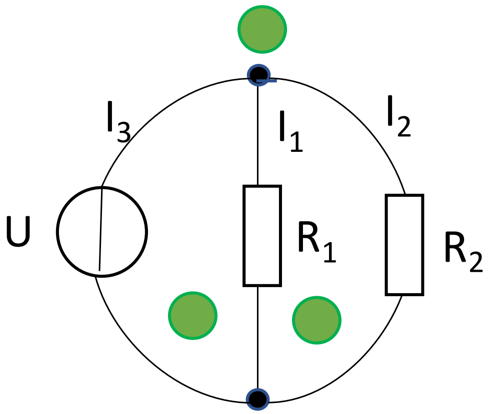
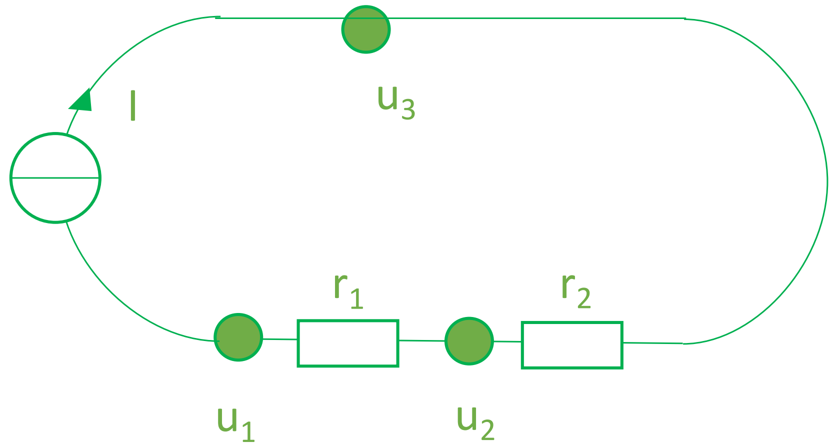


←→

dual graph



converted parallel circuit into serial circuit

*both solutions are equivalent as U ←→ **I***

$$I_3 = -I_1 - I_2 = U(\frac{1}{R_1} + \frac{1}{R_2})$$

$$u_3 - u_1 = I(r_1 + r_2) = I(\frac{1}{R_1} + \frac{1}{R_2})$$

Introduction into Electronics

(1) Electrical circuits
(2) Analog electronics
(3) Digital electronics
(4) Circuit analysis, circuit topologies