

# II. Physikalisches Institut – Computerarithmetik

Niklas Grün (n.gruen@stud.uni-goettingen.de)

## Darstellung von Zahlen

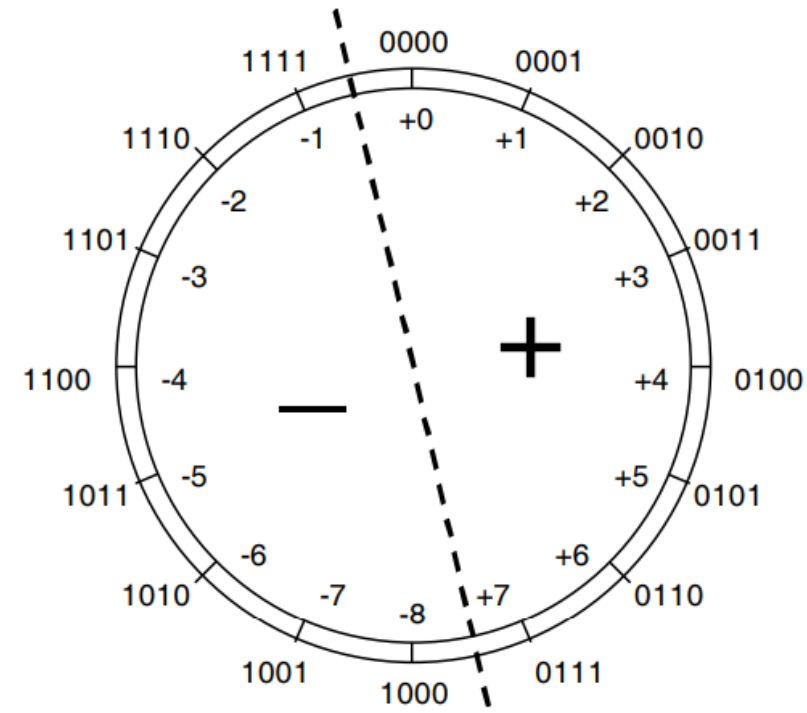
Darstellung einer positiven ganzen Zahl in Radix r

$$x = (x_{k-1}x_{k-2}\dots x_1x_0)_r := \sum_{i=0}^{k-1} x_i \cdot r^i \quad \text{mit } 0 \leq x_i \leq r-1$$

Negative Zahlen

Signed magnitude: ein Bit für das Vorzeichen

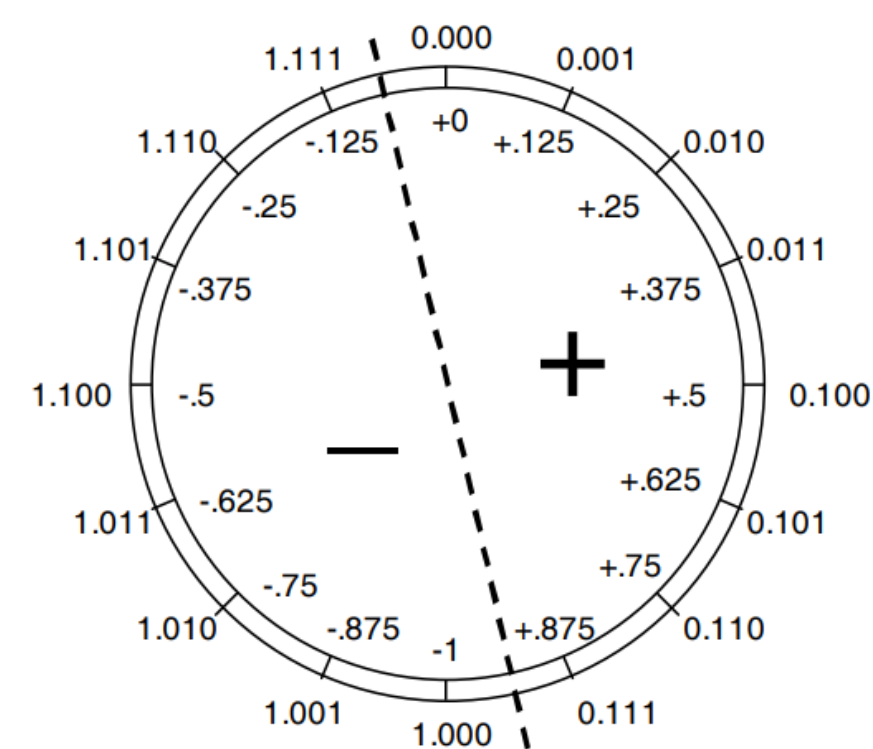
2's-complement (= bitweiser Flip +1):  
MSB negatives Vorzeichen



Fixed-Point

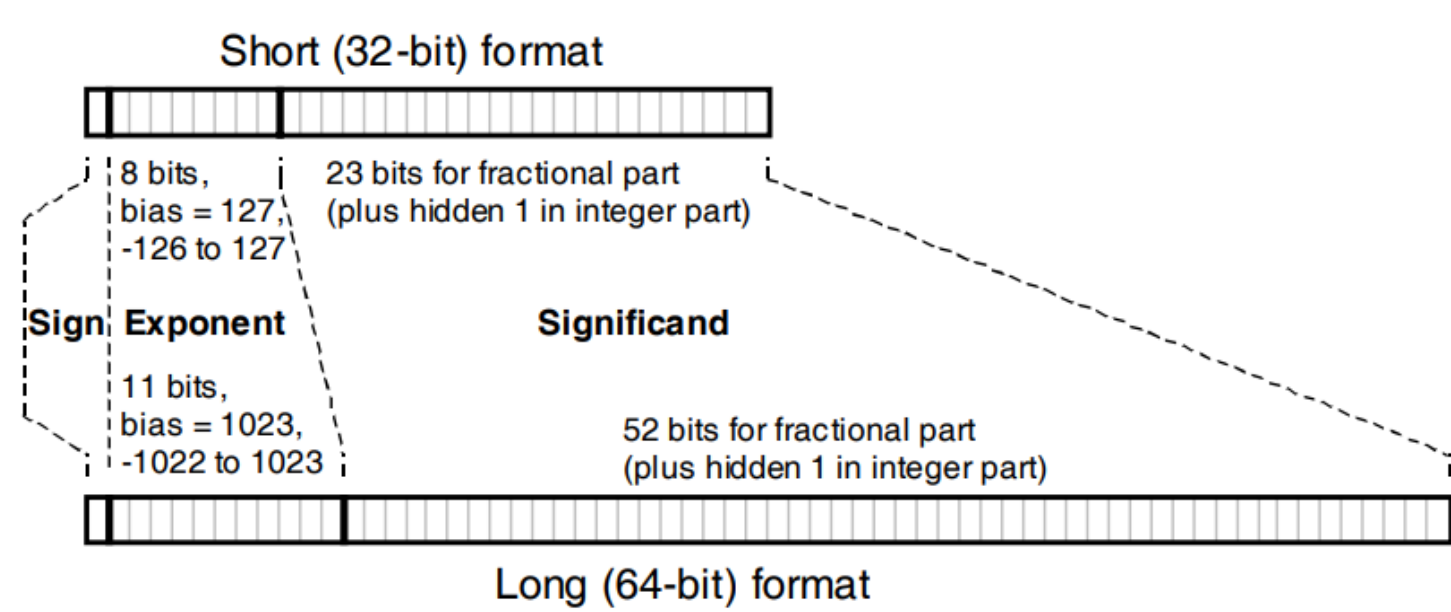
$$x = (\underbrace{x_{k-1}x_{k-2}\dots x_1x_0}_{\text{whole/integral}} \cdot \underbrace{x_{-1}x_{-2}\dots x_{-\ell}}_{\text{fractional}})_r := \sum_{i=-\ell}^{k-1} x_i r^i$$

$$0.175 \approx (00101100)_2 = 0.171875$$



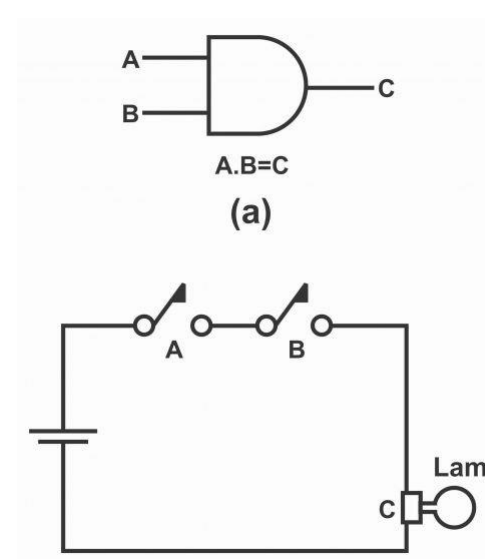
Floating-Point Arithmetic

$1,2 \cdot 10^{-38}$  bis  $3,4 \cdot 10^{38}$   
bzw.  $2,2 \cdot 10^{-308}$  bis  $1,8 \cdot 10^{308}$



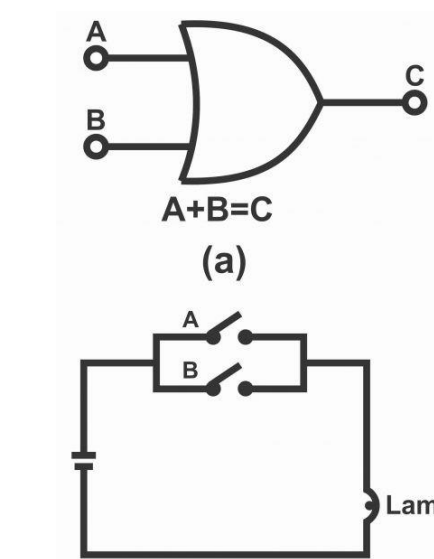
## Von Logic Gates zum Adder

AND



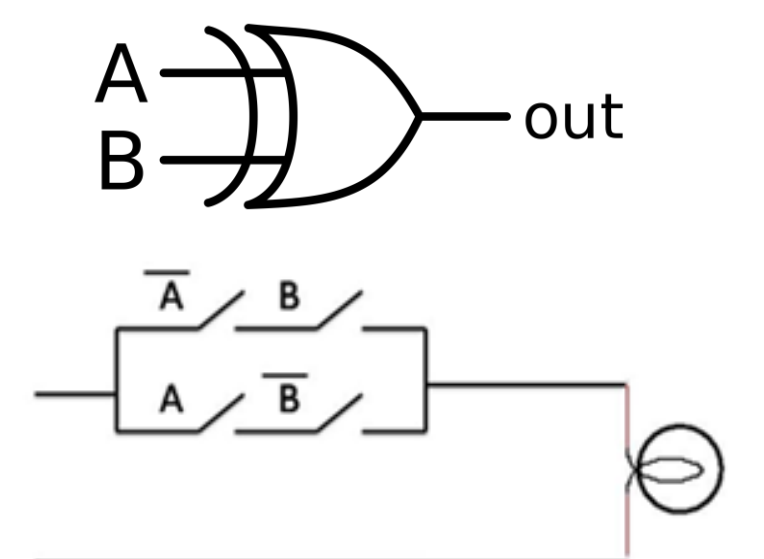
A	B	C
1	1	1
1	0	0
0	1	0
0	0	0

OR



A	B	C
1	1	1
1	0	1
0	1	1
0	0	0

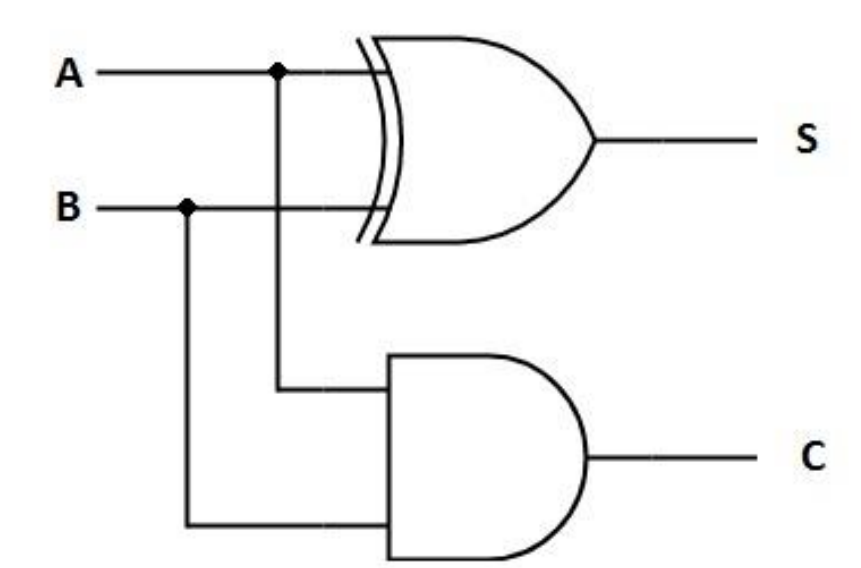
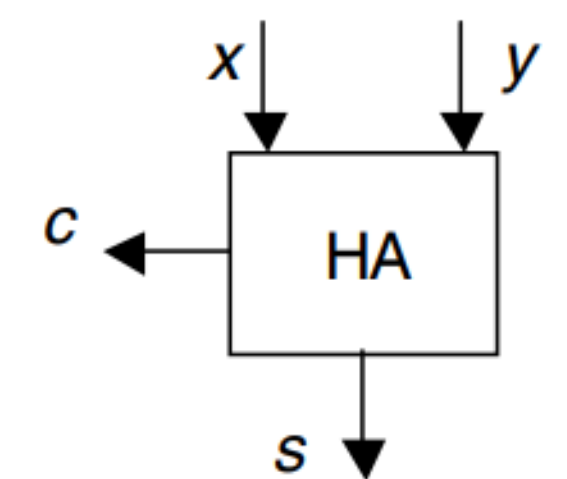
XOR



A	B	C
1	1	0
1	0	1
0	1	1
0	0	0

Half-Adder (HA)

Inputs		Outputs	
x	y	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

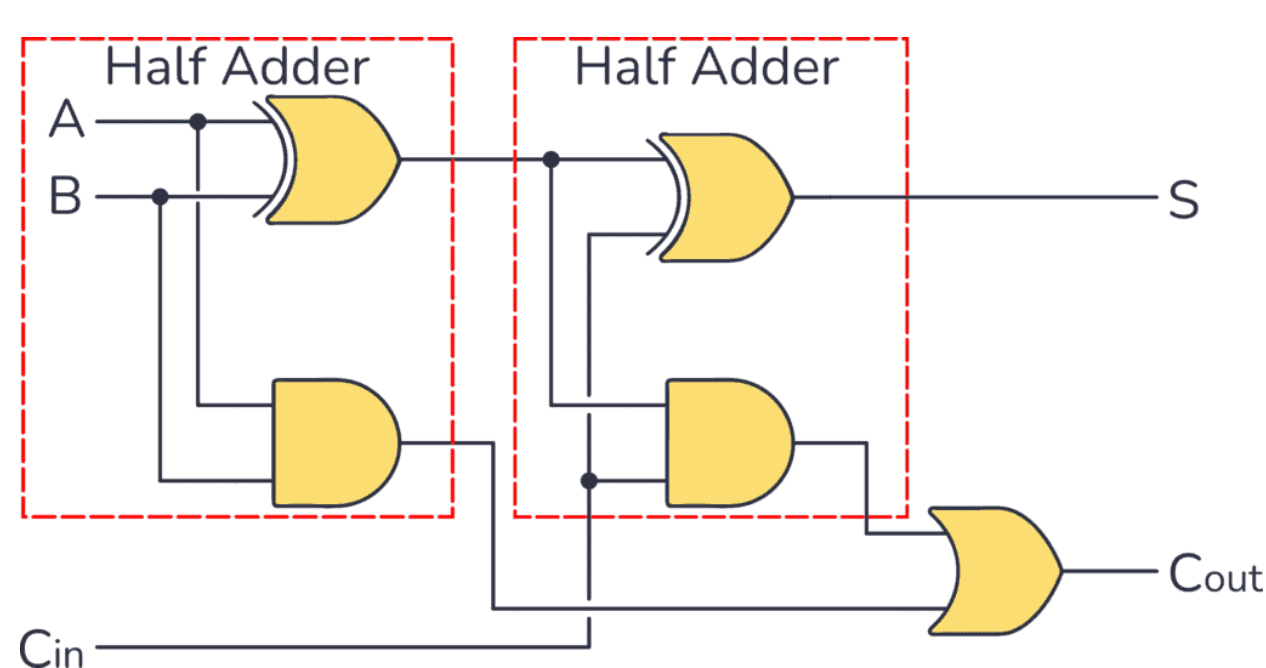
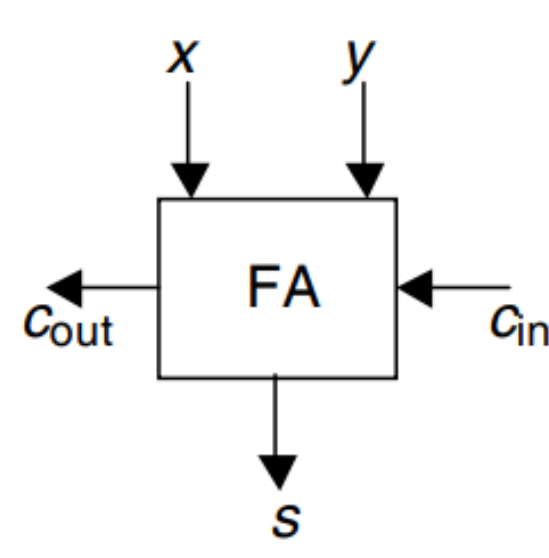


Sum Bit = XOR  
Carry Bit = AND

# Computerarithmetik

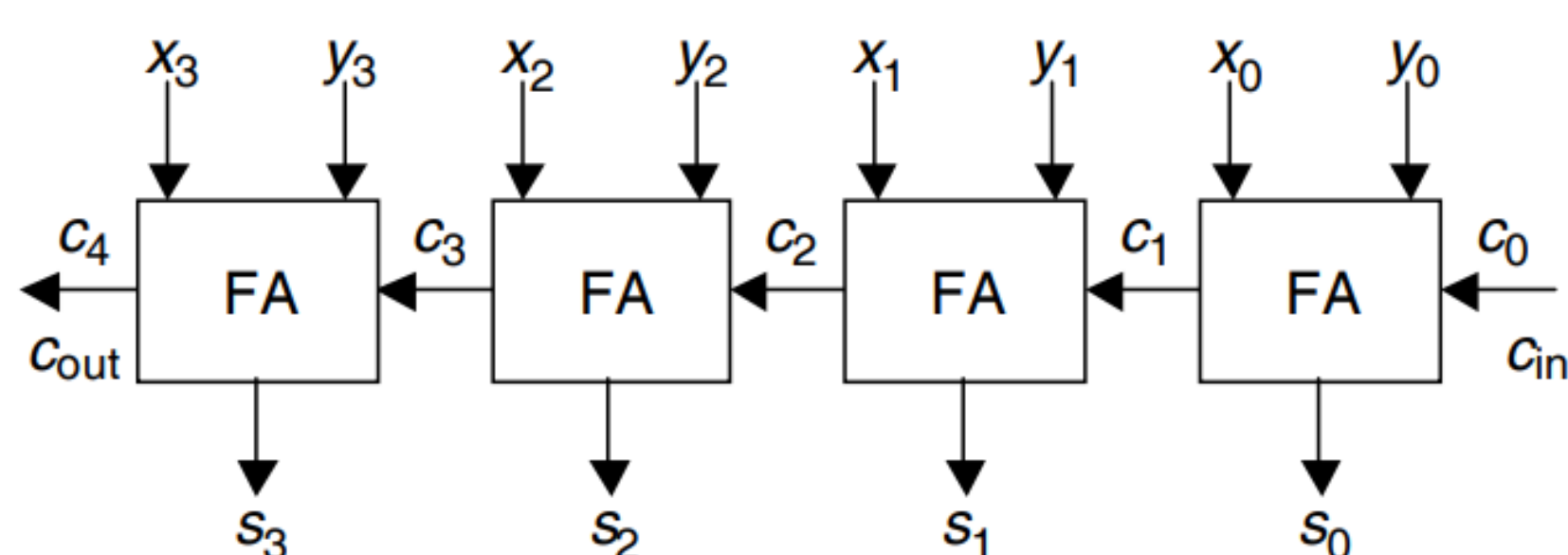
Full-Adder

Inputs		Outputs		
x	y	c <sub>in</sub>	c <sub>out</sub>	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



Ripple-Carry:

n FA hintereinander, addiere zwei n-Bit binäre Zahlen

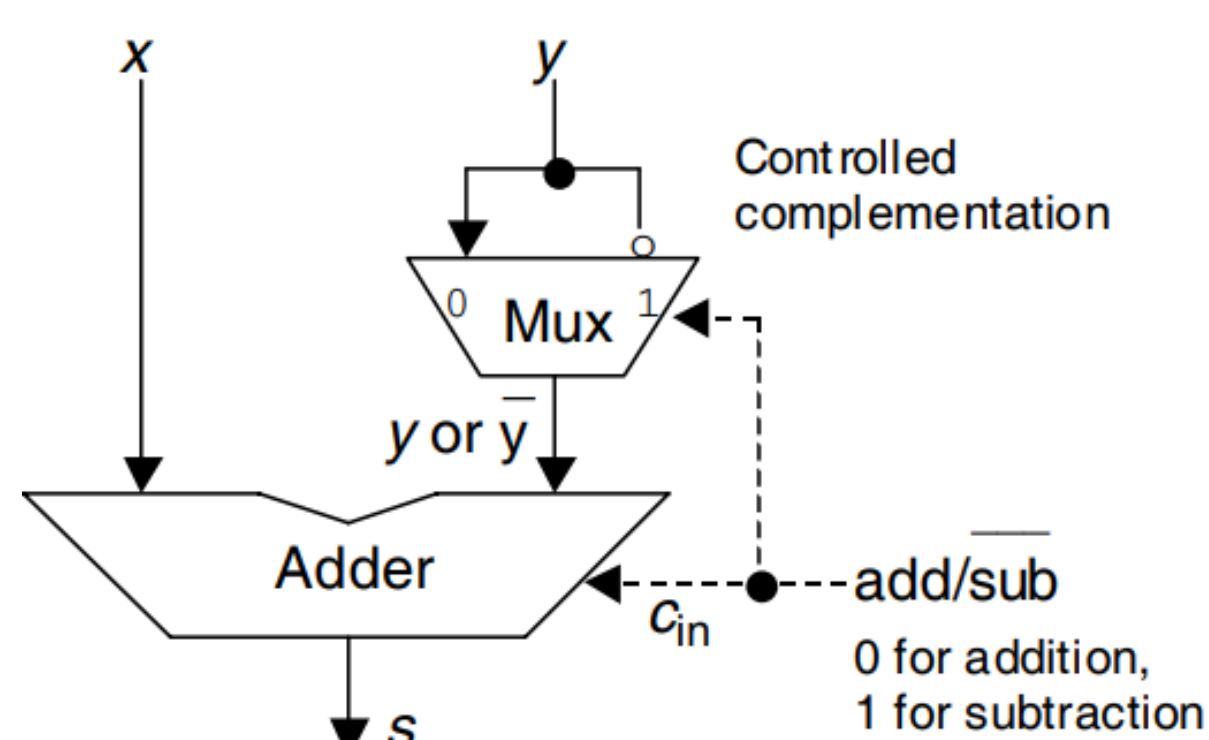


Subtraktion:

Spezialfall der Addition:

$$a - b = a + (-b)$$

mit  $(-b)$  in 2's-complement



$(-2) \cdot 3$

1110 · 0011

00000110  
-1110

001000110  
000100011  
000010001  
+1110

111010001  
111101000  
111110100

11111010  
8 bit 2's-complement

Multipliziere zwei n-Bit binäre Zahlen in 2's-complement

- Bilde neue Variable der Länge 2n+1-Bit
- 10 heißt subtrahieren und verschieben
- 11 oder 00 heißt nur verschieben
- 01 heißt addieren und verschieben
- Vorzeichen beibehalten

## Full Adder, Ripple-Carry & Subtraktion

## Booth Algorithm