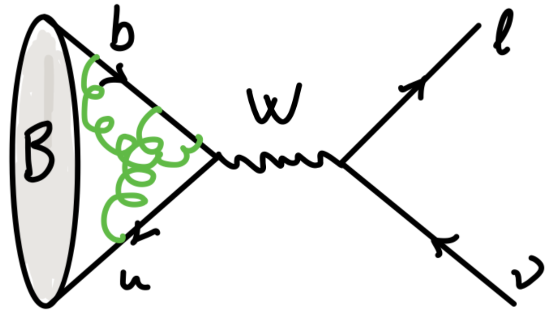
On the importance of QED corrections @FCC-ee: the $B \to l\nu$ case

Claudia Cornella (JGU Mainz)

based on 2212.14430 and ongoing work with M. Neubert and M. König

Why $B \to \ell\nu$?



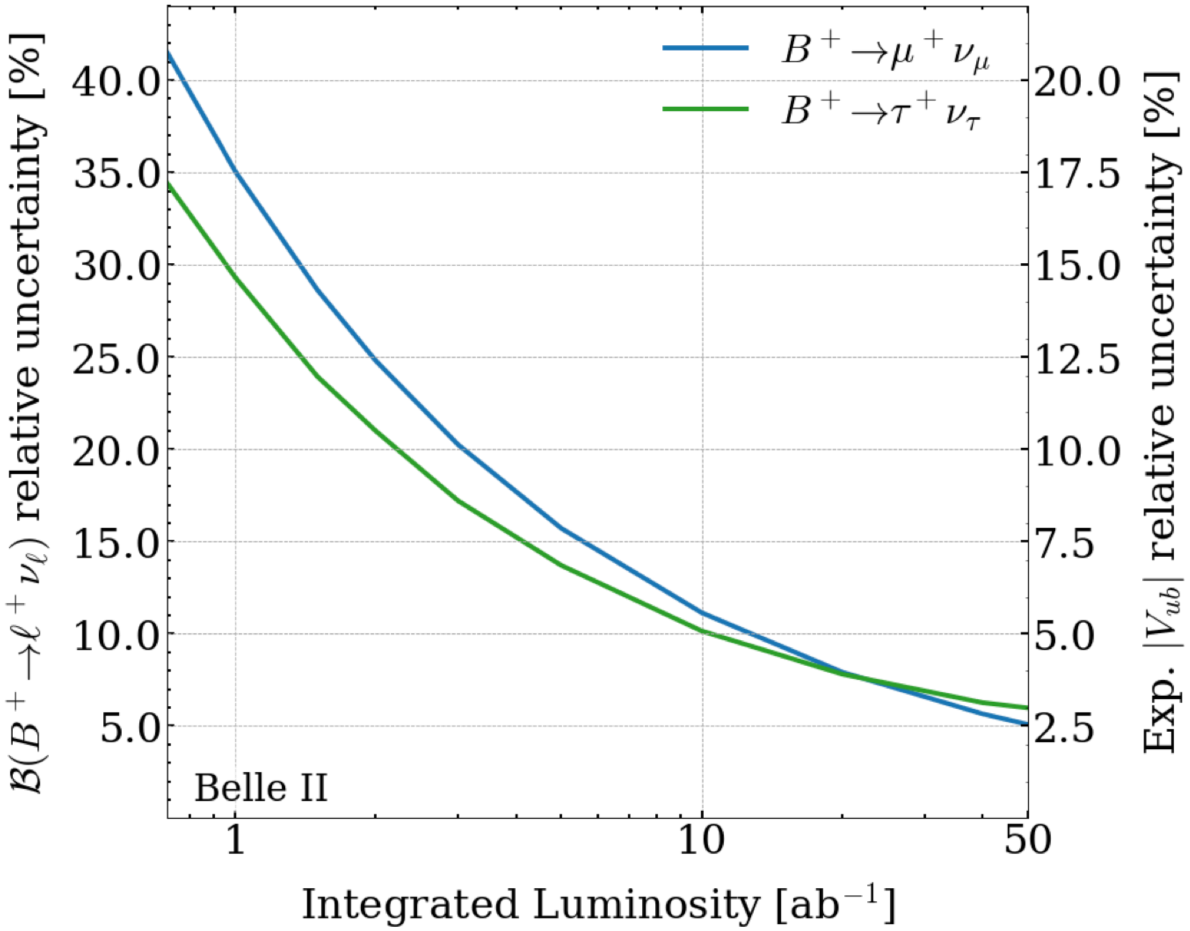
$$\Gamma \sim m_\ell^2 f_{B_u}^2 |V_{ub}|^2$$

- direct determination of $|V_{ub}|$, largely unaffected by hadronic uncertainties
- helicity suppressed → powerful probe of (pseudo)scalar NP.
- testing Lepton Flavor Universality in charged currents (τ vs μ)

Experimental prospects

- at the moment only **Belle II** can do precision studies of this mode:
  - will measure τ, μ channels with ~5% uncertainty @50ab^{-1}

    [Belle II Physics Book, Snowmass 2207.06307]
  - projections assume a cut $E_\gamma^{\max}$~100 MeV on final state radiation



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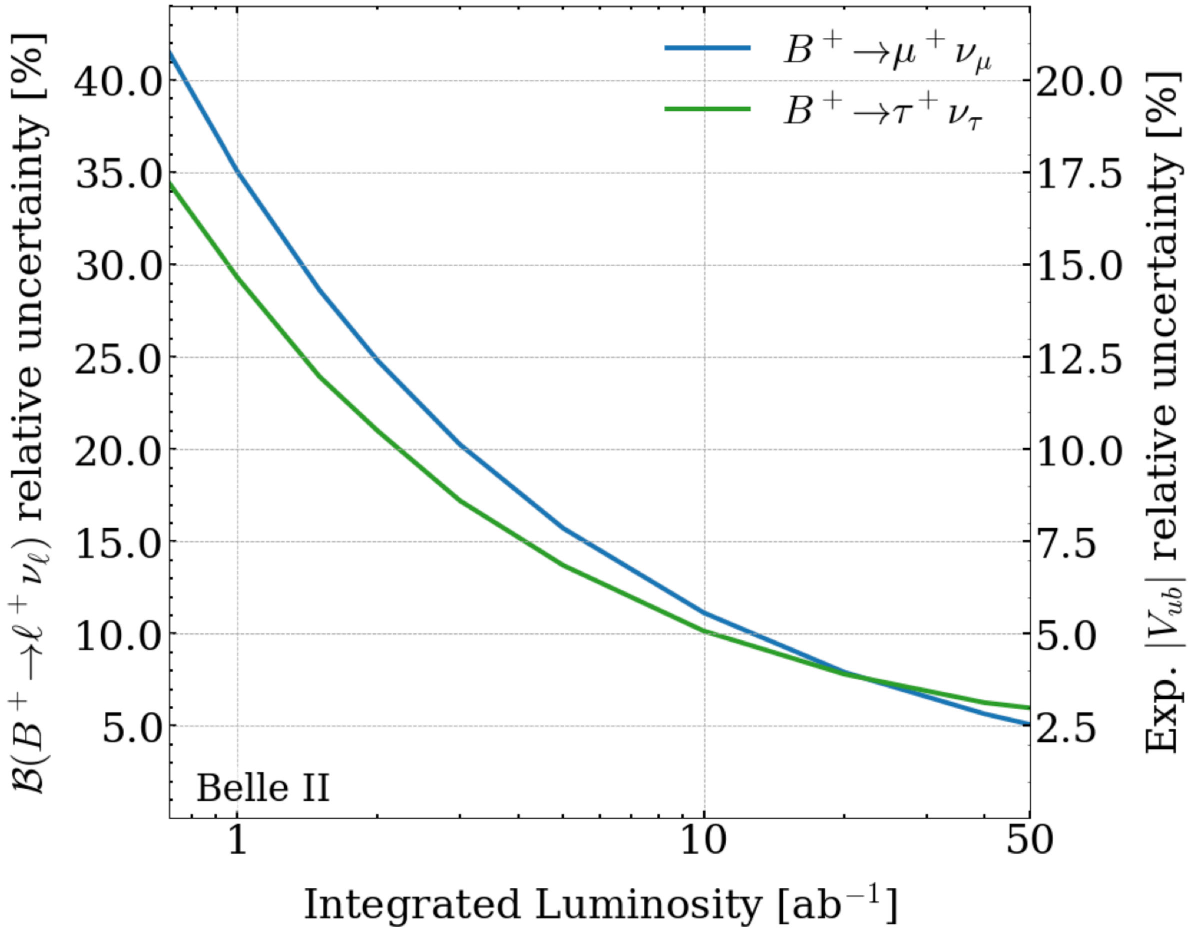

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  - 15 times the $B^{\pm}$ of Belle II (→ factor of 4 reduction in uncertainty)
  - precise measurement also for much smaller $E_\gamma^{\max}$

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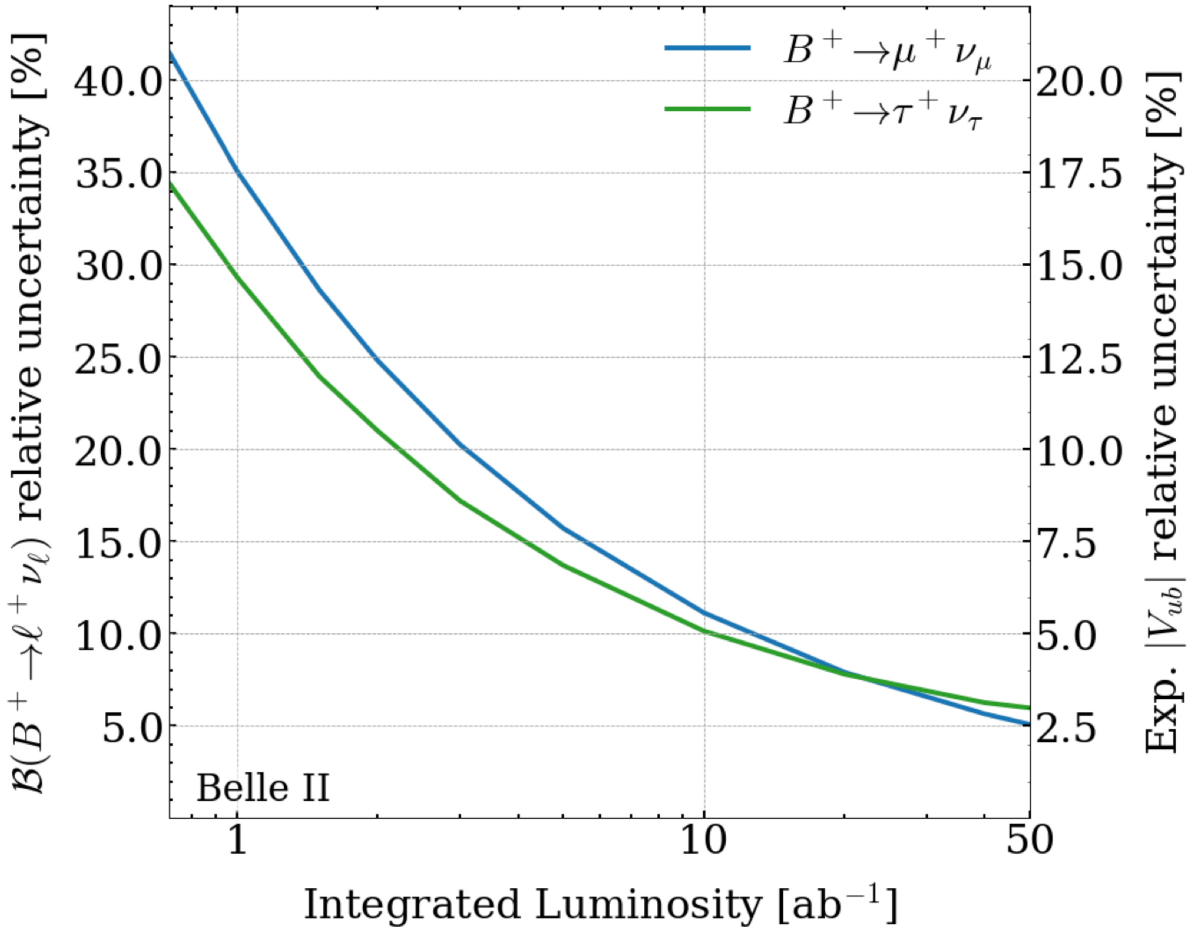

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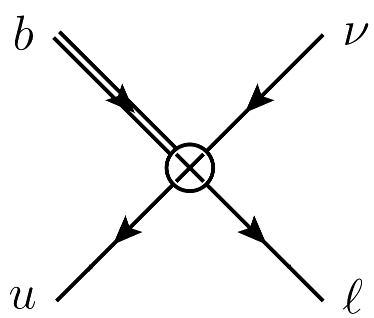
To profit from this precision, need to carefully assess all other corrections, including QED effects!

QED corrections to exclusive B decays

- QED effects are well under control for $\mu > m_b$ as well as for $\mu \ll \Lambda_{\rm QCD}$:
  - all short distance ($\mu > m_b$) QED effects can be included in the **weak effective Lagrangian**



$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} K_{\rm EW}(\mu) V_{ub} (\bar{u}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell)$$

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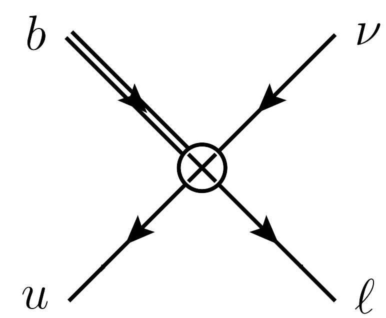

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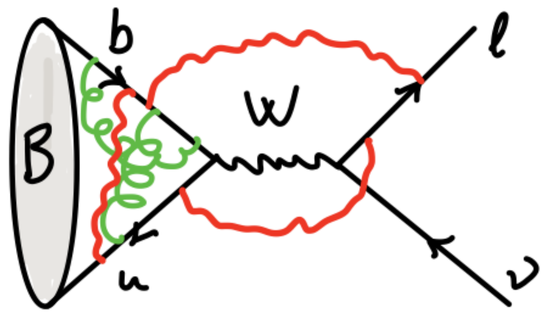
- Things are more complicated for $\Lambda_{\rm QCD} < \mu < m_b$: very active research topic.

  QED factorization theorems available only for a few processes:

  - $B_s \to \mu^+\mu^-$ [Beneke, Bobeth, Szafron, 1708.09152,1908.07011]
  - $B \to \pi K, B \to D\pi$ [Beneke, Böer et al 2008.10615,2107.03819]
  - $B_s \to \mu^+\mu^-\gamma$ [Beneke, Bobeth, Wang 2008.12494]

QED corrections to exclusive B decays

Main **challenges** in formulating a factorization theorem:



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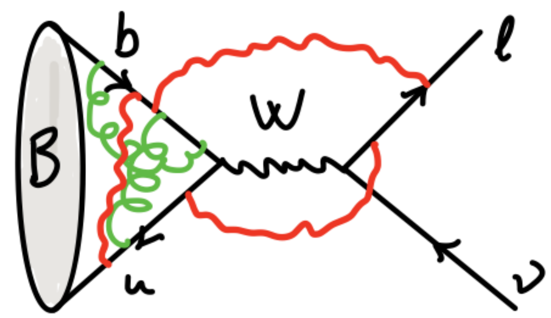

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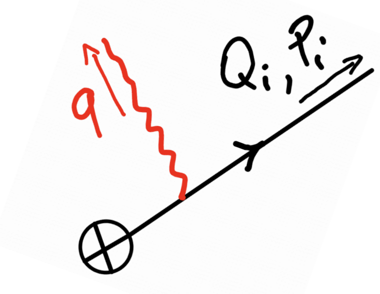
- beyond leading power convolutions have **endpoint divergences**.
  - cannot be dealt with using "standard" renormalization, require appropriate subtractions
  - relevant here: the chiral suppression makes $B \to \mu\bar{\nu}$ a genuine next-to-leading power process!

Beyond a point-like B

Pointlike approximation captures all **structure-independent** infrared effects.

soft radiative amplitude:
("eikonal" approximation)

$$A^{i\to f+\gamma} \sim A^{i\to f} \sum_i eQ_i \frac{p_i \cdot \epsilon_i}{\eta_i\, p_i \cdot q}$$



$$\Rightarrow \qquad \Gamma^{i\to f+\gamma} = \Gamma^{i\to f}_{\text{tree}} \left(\frac{2E_\gamma^{\max}}{m_B}\right)^{-\frac{\alpha}{\pi}\sum_{l,m} Q_l Q_m f(\beta_{lm})} \approx \Gamma^{i\to f}_{\text{tree}} \left[1 + \frac{\alpha}{\pi}\log^2(\ldots) + \ldots\right]$$

UV cutoff in the point-like approximation

"Coulomb" logs

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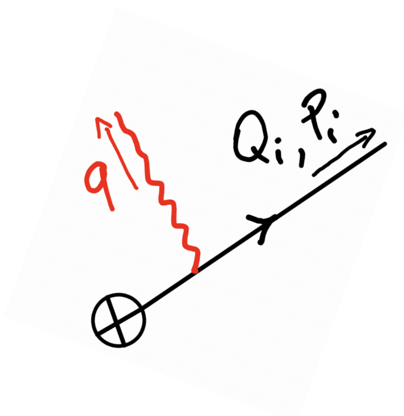

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Generally not a limiting factor when aiming for a few % precision at the rate level, or for ratios of observables where structure-dependence cancels in the ratio…

Beyond a point-like B

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For example in $B_s \to \mu\mu$: [Beneke at al. 2017]

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i\mathcal{A} = {} & m_\ell f_{B_q} \mathcal{N}\, C_{10}\, [\bar{\ell}\gamma_5 \ell] + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q\, m_\ell\, m_{B_q} f_{B_q} \mathcal{N}\, [\bar{\ell}(1+\gamma_5)\ell] \\
& \times \left\{ \int_0^1 du\, (1-u)\, C_9^{\rm eff}(u m_b^2) \int_0^\infty \frac{d\omega}{\omega}\, \phi_+(\omega) \left[\ln \frac{m_b \omega}{m_\ell^2} + \ln \frac{u}{1-u} \right] \right. \\
& \left. \quad - Q_\ell C_7^{\rm eff} \int_0^\infty \frac{d\omega}{\omega}\, \phi_+(\omega) \left[\boxed{\ln^2 \frac{m_b \omega}{m_\ell^2}} - 2 \ln \frac{m_b \omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \right\} + \ldots
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structure-dependent double log!

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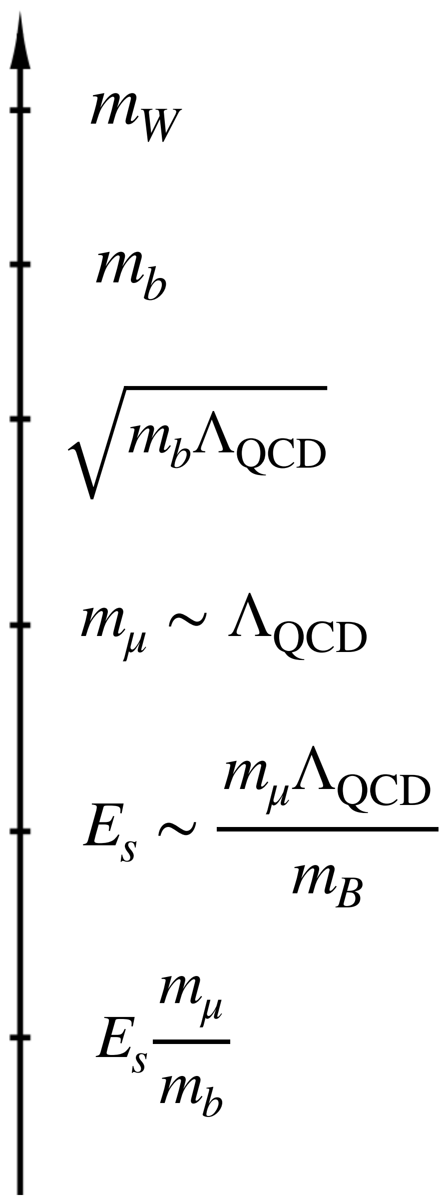
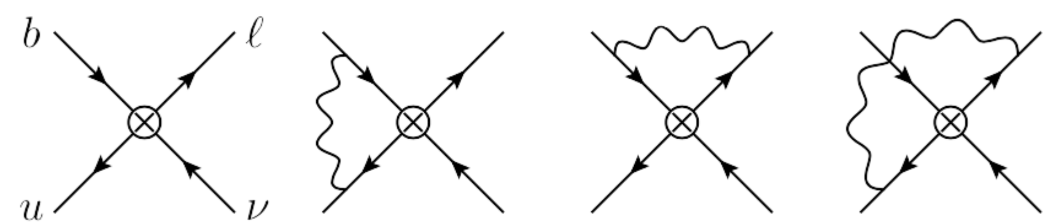
- In most cases, like for $B \to l\nu$, all double logs are accounted for in the point-like treatment.

  However, structure-dependent **single** logs are present and need to be included!

Scales

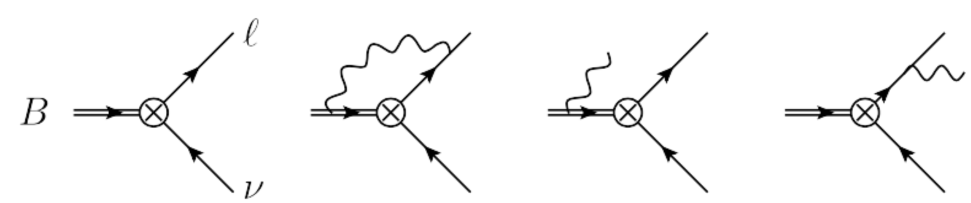
In the presence of QED corrections, $B \to \mu\bar{\nu}$ is sensitive to many scales:





description of meson by $|b\bar{q}\rangle + \ldots$ [Beneke et al (*2019*), JHEP10 232]

description of meson by point-like scalar



[Dai et al (*2022*), Phys.Rev.D 105 3]

Strategy

In general the rate is a complicated function of all these scales.

We want to **factorize** it, i.e. to write is as “product” of single-scale objects.

$\Rightarrow$ need an appropriate **effective description** in the vicinity of each scale!

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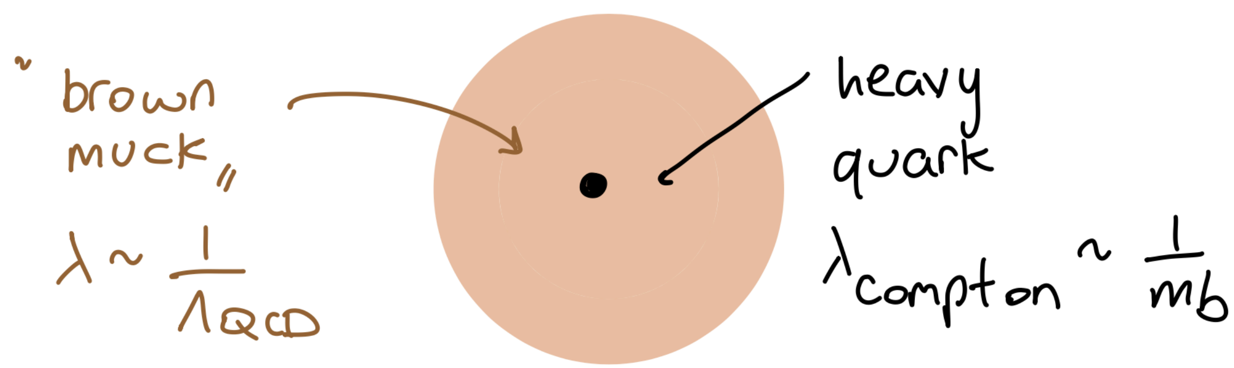
Next:

- describe the **EFT construction** across all scales
- discuss the **factorization & refactorization** of the **“virtual” amplitude**
- sketch the **low-energy theory** describing real emissions

Heavy Quark Effective Theory

- Below m_b, radiation is too soft to affect the heavy quark momentum.



$$b(x) \to e^{-im_b v\cdot x} h_v(x)$$

- **heavy quark** behaves as an almost **static source** with fixed v and $\lambda \sim 1/m_b$

- the **dynamical** dofs are the "**wiggles**" of the heavy quark around its mass shell due to the kicks of the surrounding light partons, with $\lambda \sim 1/\Lambda_{\text{QCD}}$

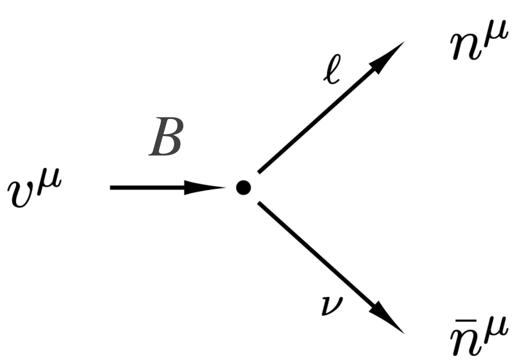
- $\Lambda_{\text{QCD}} \gg m_b \Rightarrow$ integrate out long-distance modes.
  The resulting EFT is **Heavy Quark Effective Theory**.

Soft Collinear Effective Theory

- Remaining fields can have large E, but small p^2
  $\Rightarrow$ the right EFT is **Soft-Collinear Effective Theory**

  - *Idea*: integrate out fields with **large invariant mass** - either heavy **fields**, or light fields with **large virtualities**.

  - **effective fields** have $p^2 \sim 0$, but **individual components** can be large*(here $\sim m_B$):



$$p_\mu^\mu = \frac{m_B}{2}\left(1 + \frac{m_\ell^2}{m_B^2}, 0, 0, +1 - \frac{m_\ell^2}{m_B^2}\right) \approx \frac{m_B}{2}(1, 0, 0, +1) = \frac{m_B}{2} n^\mu \qquad \text{“collinear”}$$

$$p_\nu^\mu = \frac{m_B}{2}\left(1 - \frac{m_\ell^2}{m_B^2}, 0, 0, -1 + \frac{m_\ell^2}{m_B^2}\right) \approx \frac{m_B}{2}(1, 0, 0, -1) = \frac{m_B}{2} \bar{n}^\mu \qquad \text{“anti-collinear”}$$

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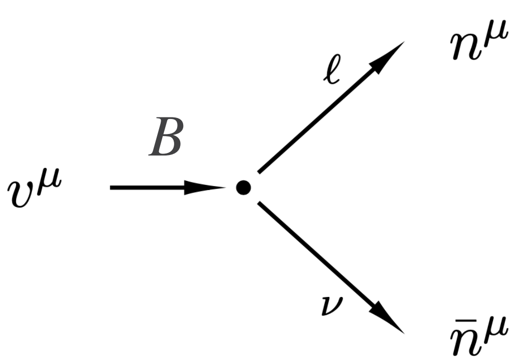

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- Lots of **field theory implications**: **non-local operators**, Wilson lines, power counting $\neq$ mass dimension, multiple fields per particle…. technical, but powerful!

Non-local operators

$$\left\langle 0 \middle| \left(\frac{1}{n\cdot\partial} q_s\right) \ldots h_v \middle| B \right\rangle \sim \frac{1}{\lambda_B} \sim \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^{-1}\right)$$

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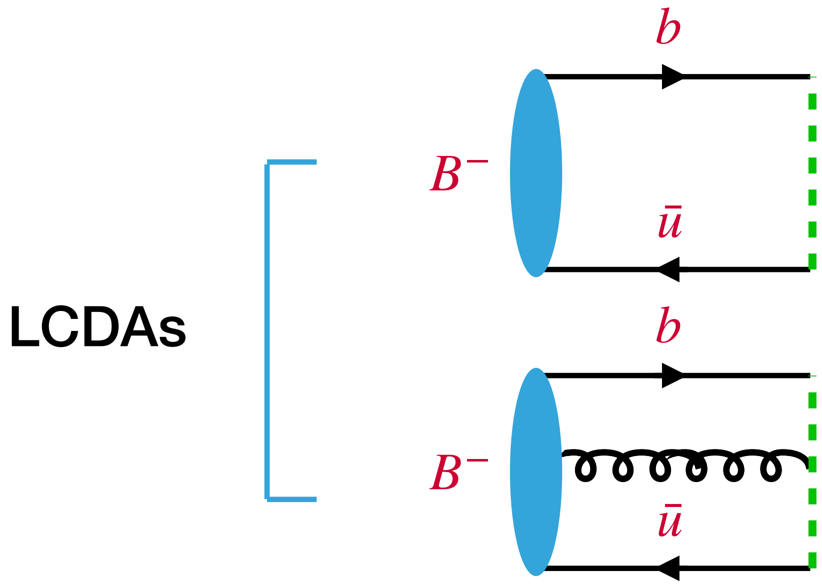
  The “momentum-sharing” among partons is described by non-perturbative objects, the **Light-Cone Distribution Amplitudes** of the B.



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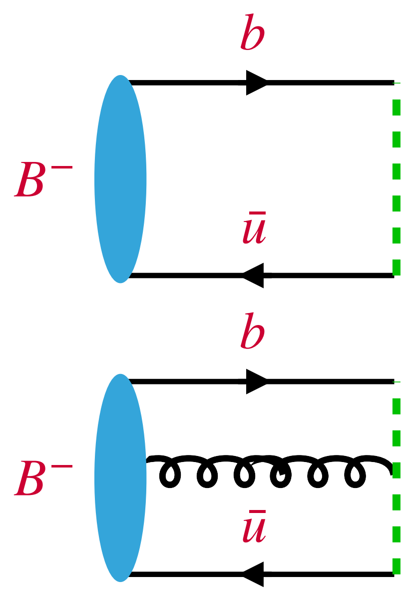
LCDAs



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- **Single logarithmic** corrections to $B \to \mu\bar{\nu}$ at $\mathcal{O}(\alpha)$ are sensitive to **2- and 3-particle LCDAs**. This introduces a hadronic **source of uncertainty** in QED corrections.

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- …but **not for** $B \to \mu\bar{\nu}$

For left-handed currents,"power enhanced" contributions come with **evanescent** Dirac structures:

$$\left(\bar{v}\frac{\not{n}}{2}\gamma_\perp^\mu\gamma_\perp^\nu P_L u\right)_h \left(\bar{u}\gamma_\mu^\perp\gamma_\nu^\perp\left[\frac{v-a\gamma_5}{2}\right]v\right)_\ell = 2(v-a)\left(\bar{v}\frac{\not{n}}{2}P_L u\right)_h \left(\bar{u}P_R v\right)_\ell + \mathcal{O}(\epsilon)$$

$\Rightarrow$ structure-dependent contributions to $B \to \mu\bar{\nu}$ carry the **same suppression** as the tree level result!

Factorization formula (virtual corrections)

$$\mathscr{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = \sum_j \underbrace{H_j\, S_j\, K_j}_{} + \sum_i \underbrace{H_i \otimes J_j \otimes S_i \otimes K_i}_{},$$

SCET-1 operators with soft spectator (A-type)

SCET-1 operators with hc spectator (B-type)

- hard function: matching corrections at $\mu \sim m_b$
- hard-collinear function: matching corrections at $\mu \sim (m_b \Lambda_{\text{QCD}})^{1/2}$
- collinear function: leptonic matrix elements, $\mu \sim m_\mu$
- soft (& soft-collinear*) function: HQET B meson matrix elements

Endpoint divergences

- Neglecting $\mathcal{O}(\alpha\alpha_s)$ corrections, two main contributions

$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell)\, \bar{u}(p_\ell) P_L v(p_\nu)$$

$$\left[H_A(m_b) S_A + \int d\omega \int_0^1 dx\, H_B(m_b, x)\, J_B(m_b\omega, x) S_B(\omega) \right] \qquad \omega = n \cdot p_u$$

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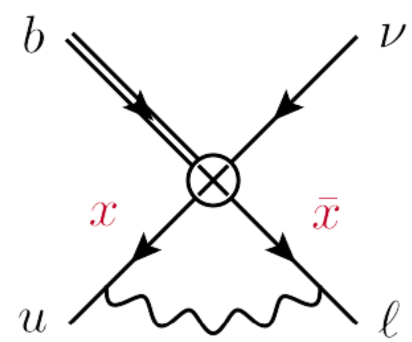
- Hard and jet function share a variable x = collinear momentum fraction carried by the spectator
- $H_B \sim x^{-\epsilon}, J_B \sim x^{-1-\epsilon}$

$\Rightarrow H_B \otimes J_B \sim \int_0^1 dx\, x^{-1}$ has an **endpoint divergence** in $x = 0$!



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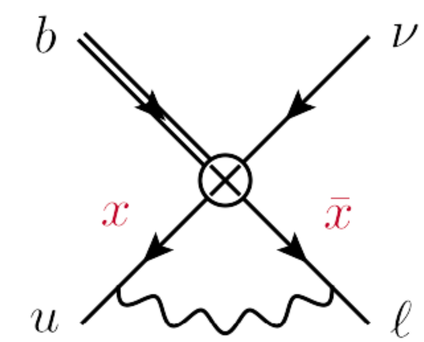

- This cannot be removed with RG techniques, but is systematically treatable with **refactorization-based subtraction (RBS) scheme**

[Liu, Neubert 2019; Liu, Mecaj, Neubert, Wang 2020; Beneke et al. 2022; Liu, Neubert, Schnubel, Wang 2022]

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$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell)\, \bar{u}(p_\ell) P_L v(p_\nu)$$

$$\left[H_A(m_b) S_A + \int d\omega \int_0^1 dx\, H_B(m_b, x)\, J_B(m_b\omega, x)\, S_B(\omega) \right]$$

$\omega = n \cdot p_u$

$$\int_0^1 dx\, \left[H_B(m_b, x) J_B(m_b\omega, x) - \theta(\lambda - x) [[H_B(m_b, x)]] [[J_B(m_b\omega, x)]] \right]$$

$0 < \lambda < 1$ $\quad$ $[[f]] =$ singular part of f for $x \to 0$

- Remove the divergence from $H_B \otimes J_B$ with a plus subtraction

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$$\Lambda = \lambda\, m_b \qquad H_A(m_b) S_A^{(\Lambda)}$$

$$\int_0^1 dx\, \left[H_B(m_b, x) J_B(m_b\omega, x) - \theta(\lambda - x)[[H_B(m_b, x)]][[J_B(m_b\omega, x)]] \right]$$

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- Remove the divergence from $H_B \otimes J_B$ with a plus subtraction

- Add the subtraction term back, combining it with the other terms in the factorization formula

Decay constant

$$S_A^{(\Lambda)} \equiv \langle 0|O_A^{(\Lambda)}|B^-\rangle = \langle 0|\bar{u}_s \not{\bar{n}} P_L h_{v_B} Y_n^{(\ell)\dagger}|B^-\rangle + Q_\ell Q_u \frac{\alpha}{2\pi}\frac{e^{\epsilon\gamma_E}\Gamma(\epsilon)}{\epsilon(1-\epsilon)}\int d\omega\, \phi_-(\omega)\left(\frac{\mu^2}{\omega\Lambda}\right)^\epsilon$$
$$= \langle 0|\bar{u}_s \not{\bar{n}} P_L h_{v_B}\theta(i\bar{n}\cdot D_s - \Lambda) Y_n^{(\ell)\dagger}|B^-\rangle$$

From the soft function $S_A^{(\Lambda)}$, **prescription** to define a "new" decay constant F:

$$S_A^{(\Lambda)} = -\frac{i\sqrt{m_B}}{2} F(\mu,\Lambda)\,\langle 0|\,Y_v^{(B)} Y_n^{(\ell)\dagger}\,|0\rangle$$

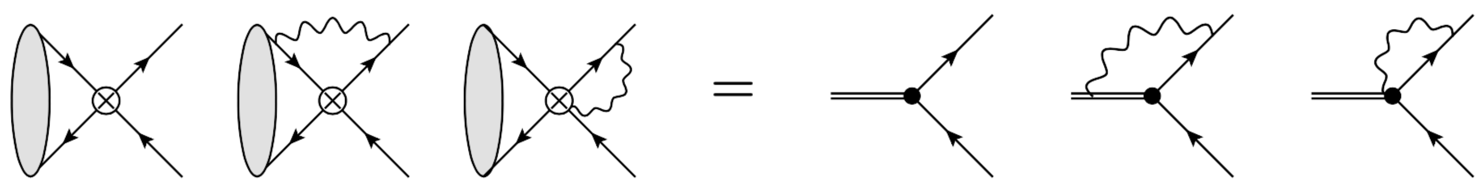
Evolution in μ and Λ well-defined and insensitive to IR regulators:

$$\frac{d\ln F}{d\ln\mu} = C_F\frac{3\alpha_s}{4\pi} - \frac{3\alpha}{4\pi}\left(Q_\ell^2 - Q_b^2 + \frac{2}{3}Q_\ell Q_u \ln\frac{\Lambda^2}{\mu^2}\right)$$

$$\frac{d\ln F}{d\ln\Lambda} = Q_\ell Q_u\frac{\alpha}{2\pi}\left[\int d\omega\,\phi_-(\omega)\ln\frac{\omega\Lambda}{\mu^2} - 1 + \ldots\right]$$

$\mu < \Lambda \sim m_\mu$: understanding the low-energy theory

- Below $\mu \sim \Lambda_{\rm QCD}$ quarks hadronize: move to effective description with a **Yukawa theory**, meson treated as a **heavy scalar**



- Since $\Lambda_{\rm QCD} \sim m_\mu$, integrate out the **muon** in the same step and describe it as a **boosted heavy lepton** field: $\ell(x) = e^{-i m_\ell v_\ell \cdot x} \chi_{v_\ell}(x)$

- all interactions of charged fields with photons can be moved into Wilson lines, and decoupled via field redefinitions

$$Y_v^{(\rm s)}(x) = \mathcal{P} \exp\left\{ ie \int_{-\infty}^{0} ds\, v \cdot A_s(x+sv) \right\}$$
$$Y_v^{(\rm sc)}(x) = \mathcal{P} \exp\left\{ ie \int_{-\infty}^{0} ds\, v \cdot A_{\rm sc}(x+sv) \right\}$$

$\mu < \Lambda \sim m_\mu$: understanding the low-energy theory

- Real corrections are matrix elements of these Wilson lines, e.g.

$$W_s(\omega_s,\mu) = \left[\sum_{n_s=0}^{\infty}\prod_{i=1}^{n_s}\int d\Pi_i(q_i)\right]\left|\langle n_s\gamma_s(q_i)|\, Y_v^{(\mathrm{s})}Y_n^{(\mathrm{s})\dagger}\,|0\rangle\right|^2 \delta\left(\omega_s - q_0^{(\mathrm{s})}\right) ,$$

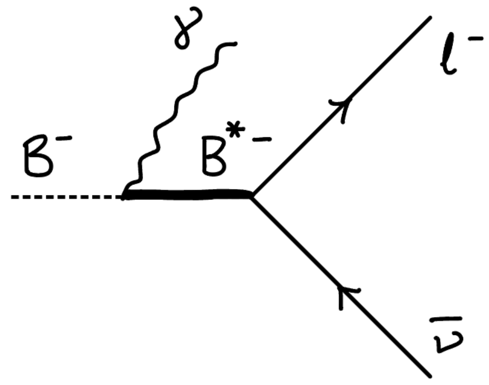
  - Convolute them with the measurement function to get the complete radiative function:

$$S(E_s,\mu) = \int_0^\infty d\omega_s \int_0^\infty d\omega_{sc}\, \theta\left(\frac{E_s}{2} - \omega_s - \omega_{sc}\right) W_s(\omega_s,\mu)W_{sc}(\omega_{sc},\mu)$$

  - Integration and renormalisation of the bare functions can be carried out in Laplace space ⇒ resummation of ultra-soft and ultrasoft-collinear logs.

Importance of the cut on final state radiation

- If $E_\gamma^{\text{cut}} \ll \Lambda_{\text{QCD}}$, this description is sufficient.
  For $E_\gamma^{\text{cut}} \sim \Lambda_{\text{QCD}}$, structure effects (B*) potentially relevant!



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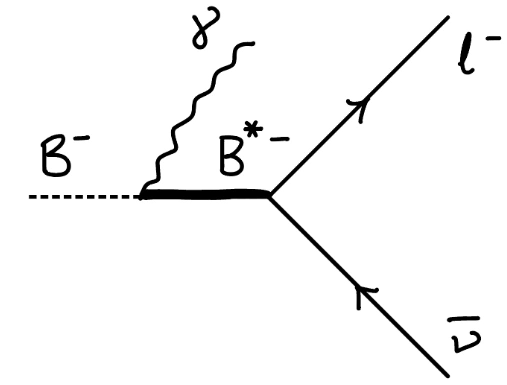
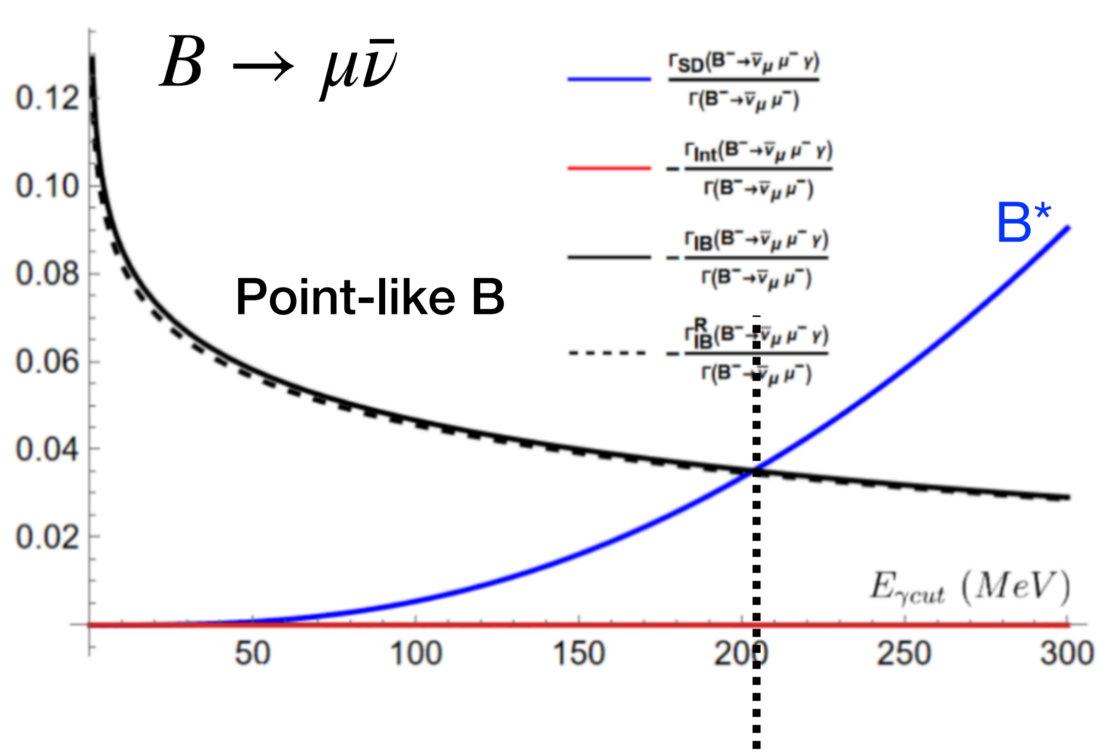
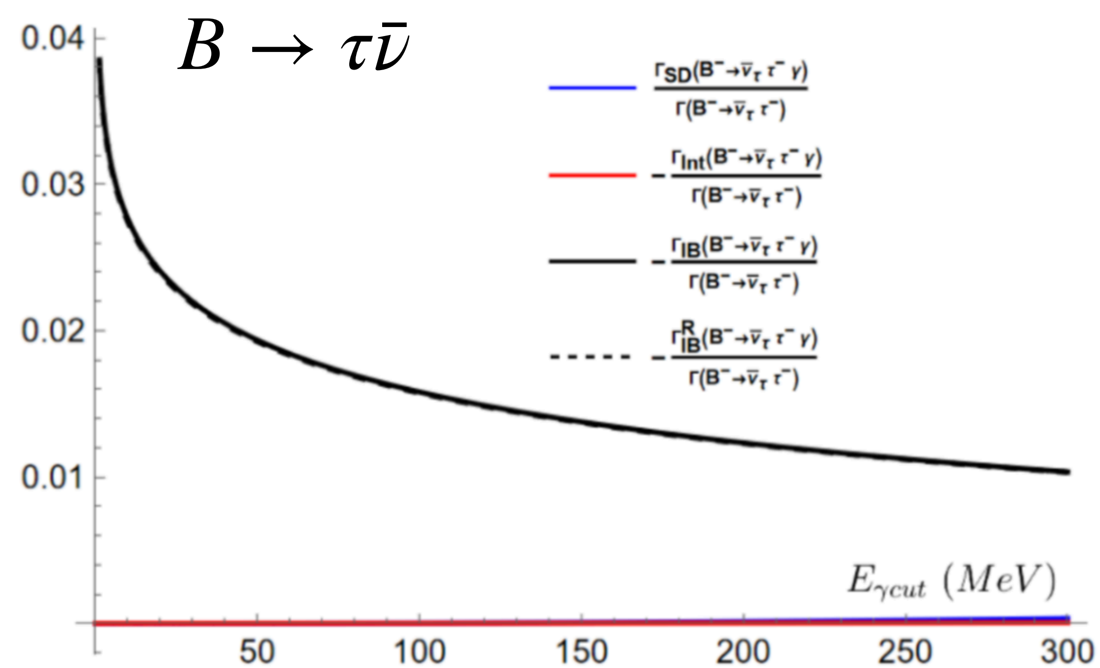


Indeed, relevant for the μ channel:





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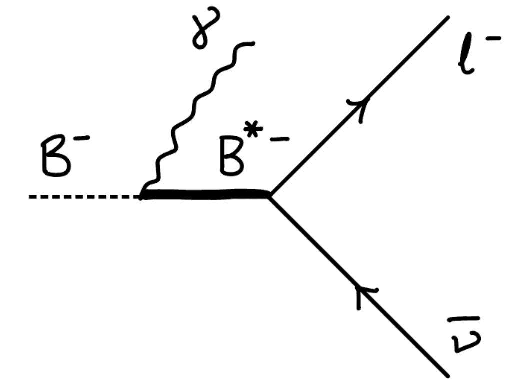

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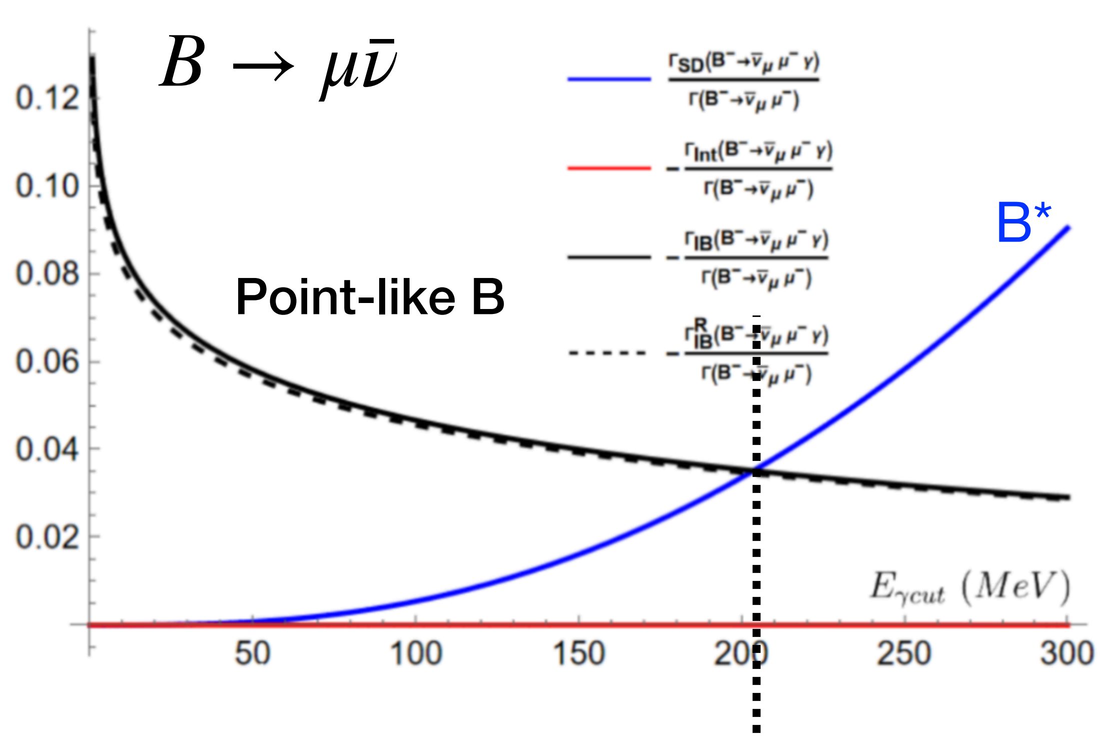

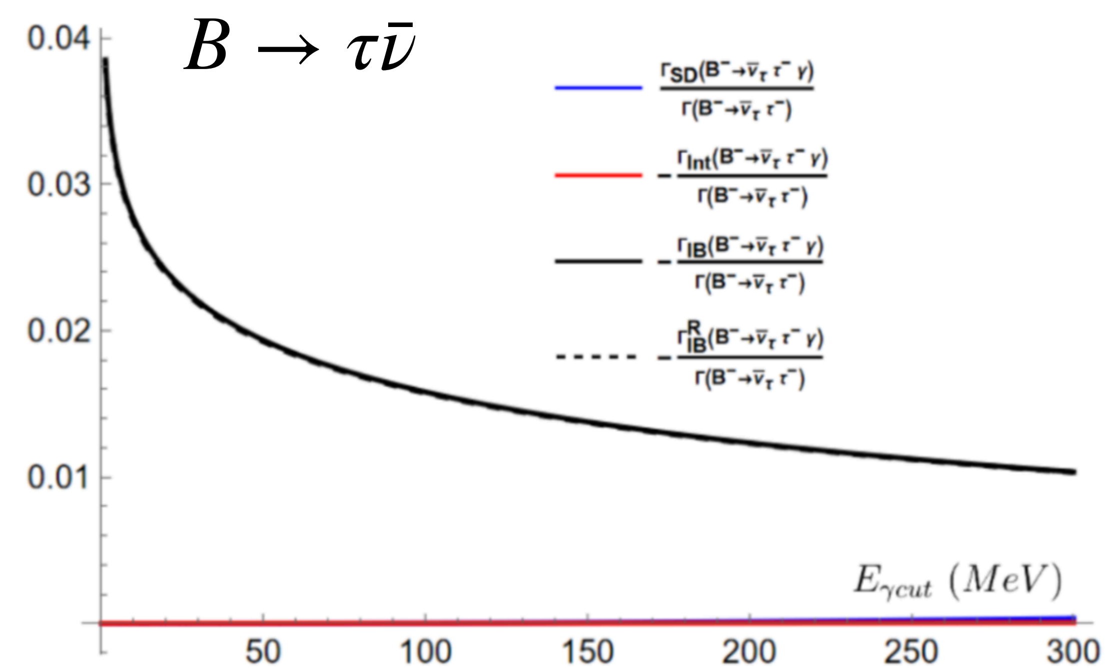

- source of **uncertainty**:
  - reliance on assumptions (pole dominance…) and model-dependent estimates
  - can be **controlled** by choosing a **tighter cut**! Achievable with **FCC-ee**.

Conclusions

- FCC-ee is an exciting opportunity for precision B physics.
  - increased statistics & potential for more exclusive measurements
- To profit from it, a precise estimate of QED effects beyond pointlike is needed
  - significant on-going effort to understand structure-dependent effects
- Here I discussed a specific example, $B \to \mu\bar{\nu}$, but there's a lot to do
  - This program has to be carried out for many other channels
  - Possible help from lattice?

Back-up

Virtual corrections to $B \to \mu\bar{\nu}$

Amplitude including virtual QED corrections at $\mathcal{O}(\alpha)$:

$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = i\sqrt{2}G_F\,K_{\text{EW}}(\mu)\,V_{ub}\,\frac{m_\ell}{m_b}\,\sqrt{m_B}\,F(\mu,m_b)\,\bar{u}(p_\ell)P_L\,v(p_\nu)\Big[\mathcal{M}_{2p}(\mu)+\mathcal{M}_{3p}(\mu)\Big]$$

IR divergence cancels against real soft photon emission

$$\mathcal{M}_{2p}(\mu) = 1 + \frac{C_F\,\alpha_s}{4\pi}\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2}-2\right]$$
$$+\frac{\alpha}{4\pi}\left\{Q_b^2\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2}-2\right]-Q_\ell Q_b\left[\frac{1}{2}\ln^2\frac{m_b^2}{\mu^2}+2\ln\frac{m_b^2}{\mu^2}-3\ln\frac{m_\ell^2}{\mu^2}+1+\frac{5\pi^2}{12}\right]\right.$$
$$\left.+2Q_\ell Q_u\int_0^\infty d\omega\,\phi_-(\omega)\ln\frac{m_b\omega}{\mu^2}+Q_\ell^2\left[\frac{1}{\epsilon_{\text{IR}}}\left(\ln\frac{m_B^2}{m_\ell^2}-2\right)+\frac{1}{2}\ln^2\frac{m_\ell^2}{\mu^2}-\frac{1}{2}\ln\frac{m_\ell^2}{\mu^2}+2+\frac{5\pi^2}{12}\right]\right\}$$

$$\mathcal{M}_{3p}(\mu) = \frac{\alpha}{\pi}\,Q_\ell Q_u\int_0^\infty d\omega\int_0^\infty d\omega_g\,\phi_{3g}(\omega,\omega_g)\left[\frac{1}{\omega_g}\ln\left(1+\frac{\omega_g}{\omega}\right)-\frac{1}{\omega+\omega_g}\right]$$

significant hadronic uncertainties

[CC, König, Neubert 2022]