Gravitational waves from cosmological phase transitions

Chiara Caprini
(University of Geneva, CERN, CNRS)

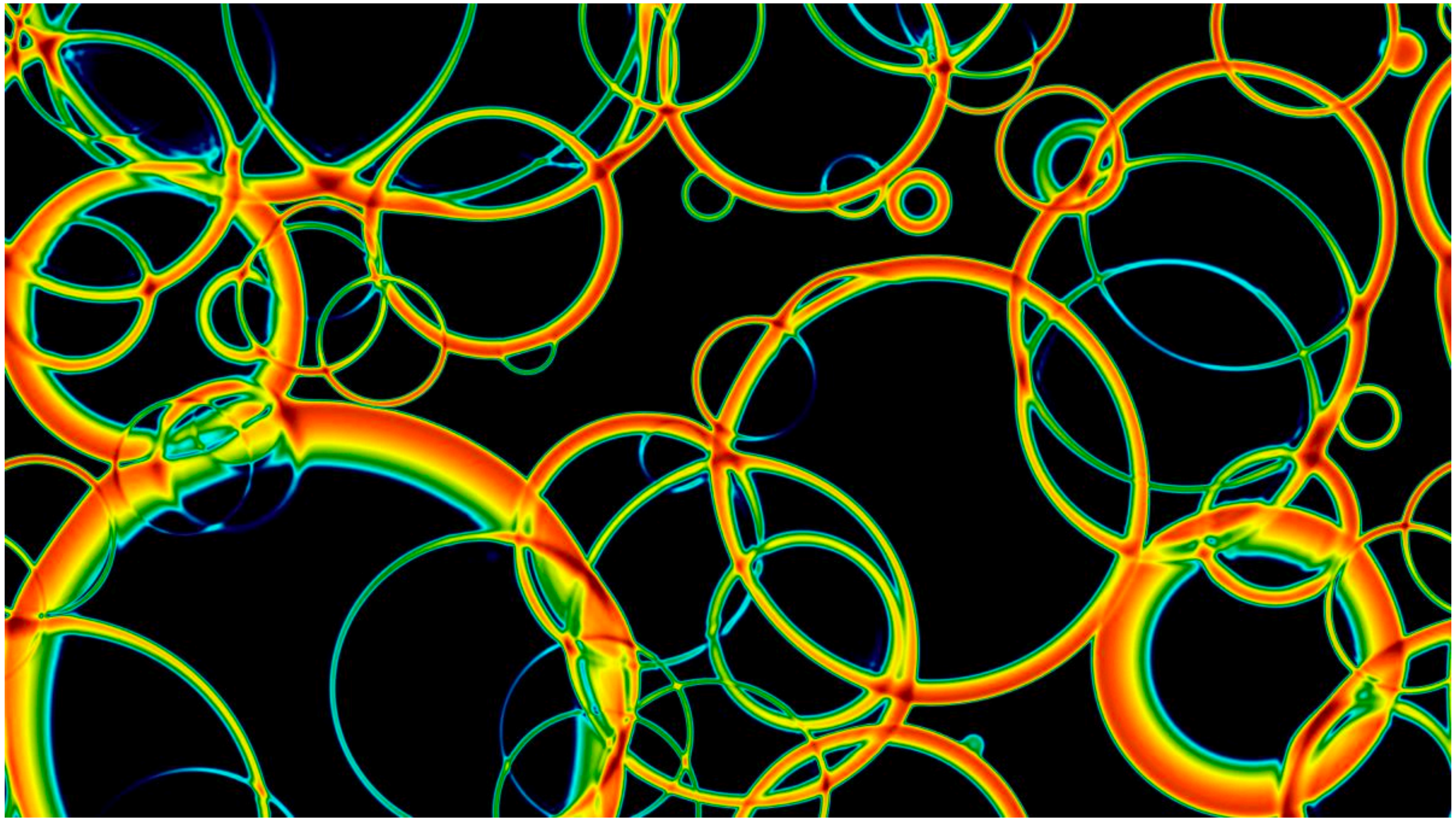


Image credit:
David Weir
(University of
Helsinki)

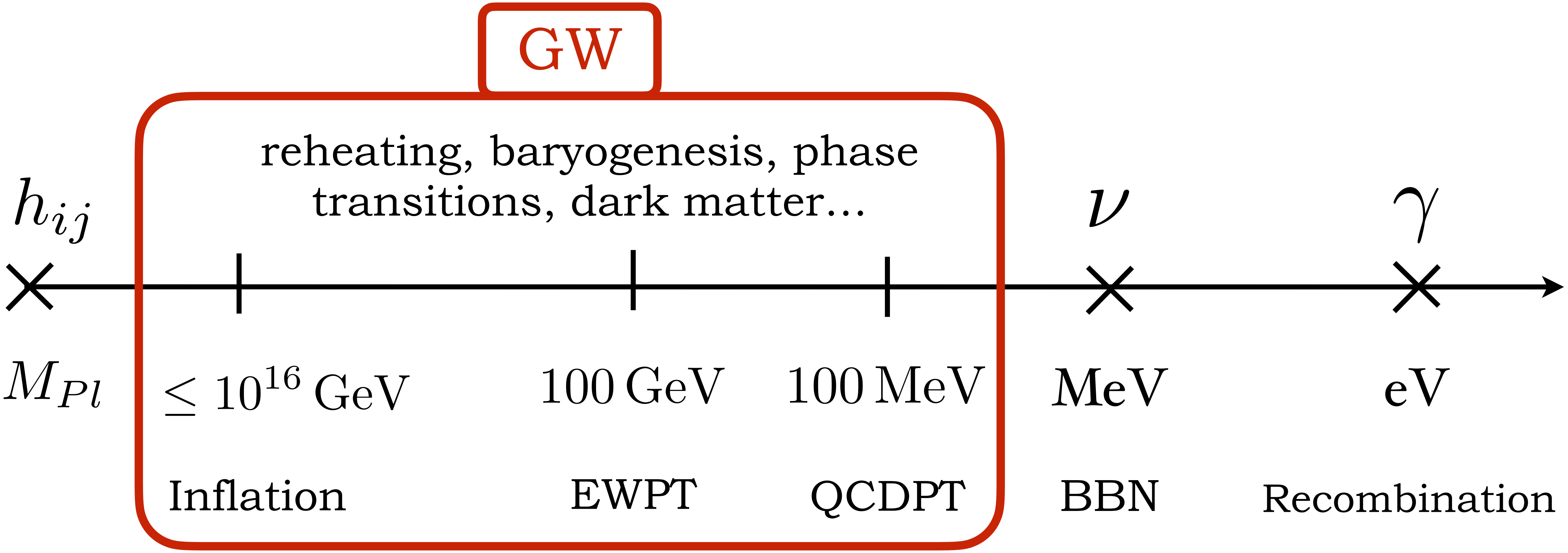
Cosmological gravitational waves

- because of the weakness of the gravitational interaction the universe is transparent to GW

$$\frac{\Gamma(T)}{H(T)} \sim \frac{G^2 T^5}{T^2/M_{Pl}} \sim \left(\frac{T}{M_{Pl}}\right)^3 < 1$$

- GW emission processes in the early universe form a fossil radiation, whose detection would bring *direct information from very early stages of the universe evolution,* to which we have no access through em radiation

- amazing discovery potential, linked to high energy physics



GW

reheating, baryogenesis, phase transitions, dark matter…

h_{ij} ν γ

M_{Pl} $\leq 10^{16}\,\mathrm{GeV}$ $100\,\mathrm{GeV}$ $100\,\mathrm{MeV}$ MeV eV

Inflation EWPT QCDPT BBN Recombination

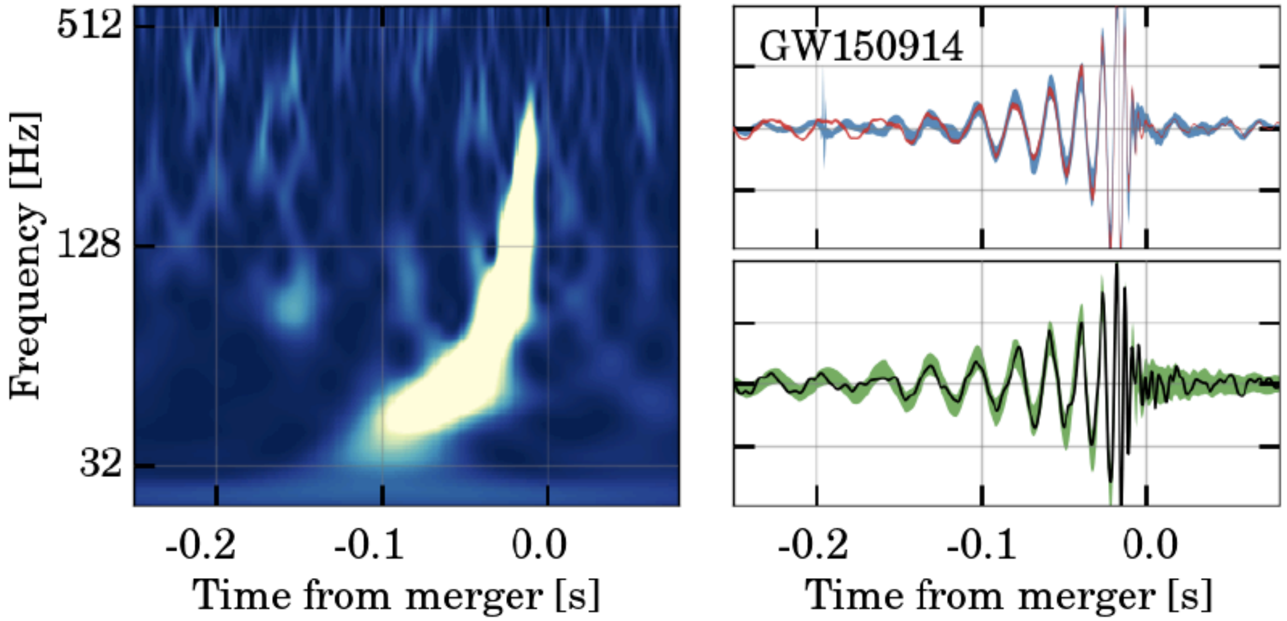
Cosmological gravitational waves

- GWs from astrophysical binaries: frequency of emission set (more or less) by Kepler’s law

$$f(\tau) = \frac{1}{\pi}\left(\frac{G\,M_c}{c^3}\right)^{-5/8}\left(\frac{5}{256\,\tau}\right)^{3/8}$$

Chirp mass

Time to coalescence



LIGO/Virgo arXiv:1811.12907

- GWs from the early universe: frequency of connected to the *Hubble scale*

the characteristic length/time scale of the GW generating process cannot be larger than the causal horizon at the generation time

$$\ell_* \leq H_*^{-1}$$

GW frequency

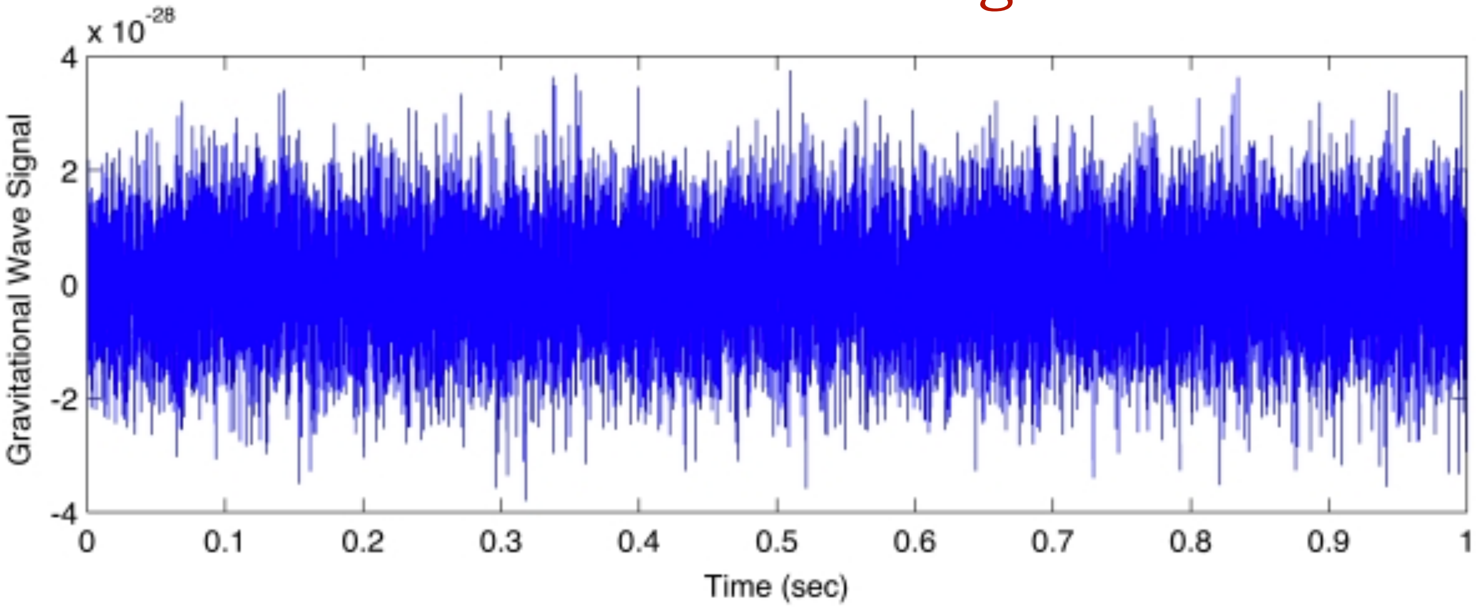
$$f_* \simeq \frac{1}{\ell_*} \geq H_*$$

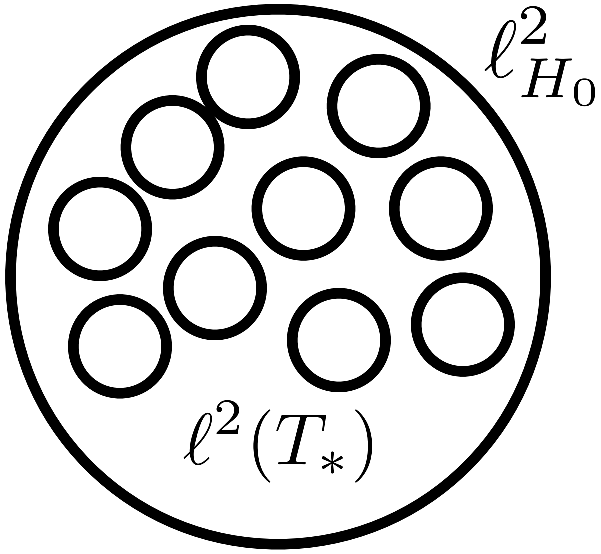
Cosmological gravitational waves

1. GW signals from the primordial universe have *too small correlation scale with respect to the detector resolution* -> only the statistical properties of the signal can be accessed

$$\Theta(T_* = 100\,\mathrm{GeV}) \simeq 10^{-12}\mathrm{deg}$$

Stochastic GW background

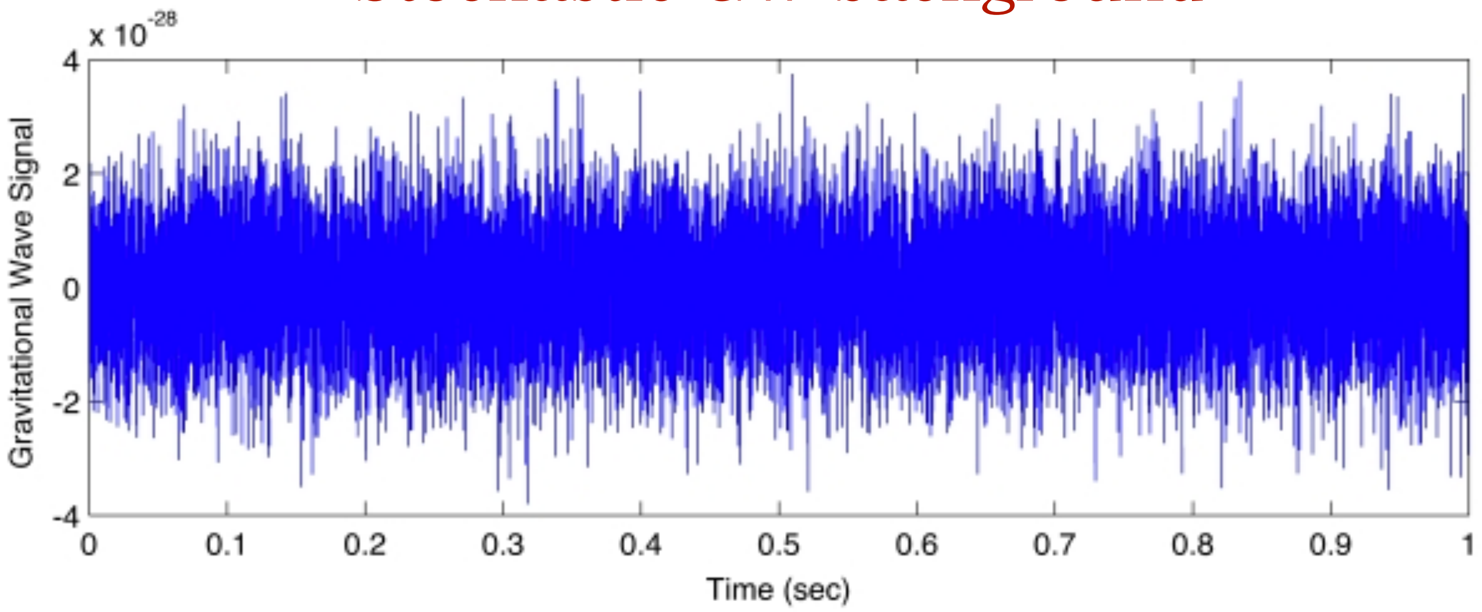


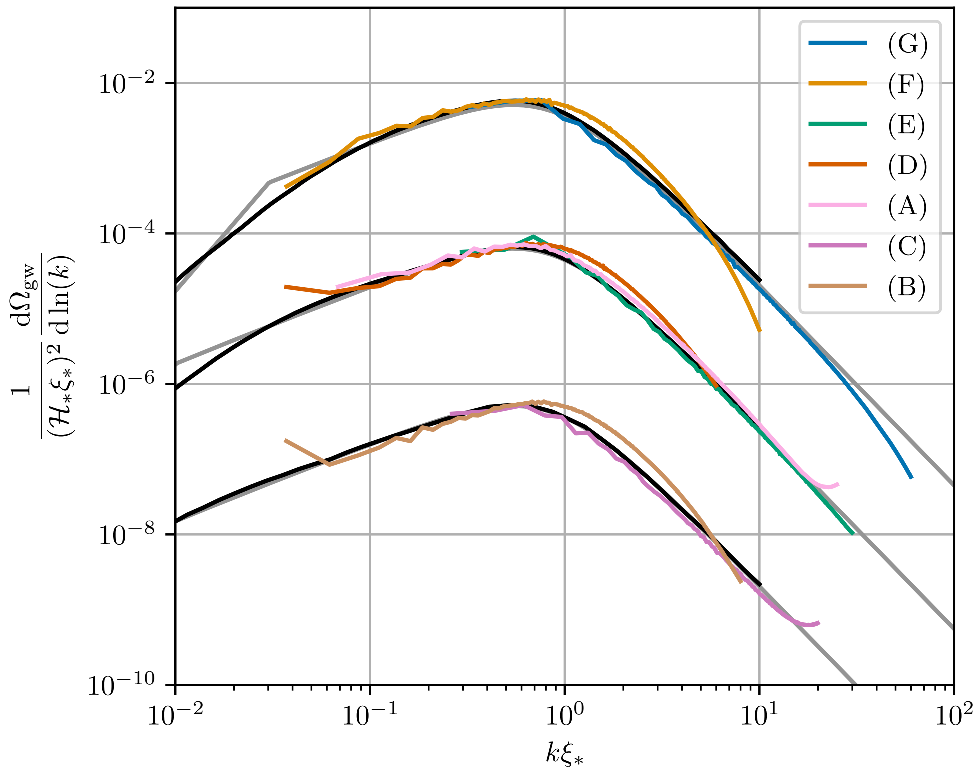


Cosmological gravitational waves

1. GW signals from the primordial universe have *too small correlation scale with respect to the detector resolution* -> only the statistical properties of the signal can be accessed -> frequency power spectrum

Stochastic GW background





2. The *specific frequency range* of a GW detector allows it to probe GW generating processes occurring at *specific energy scales*

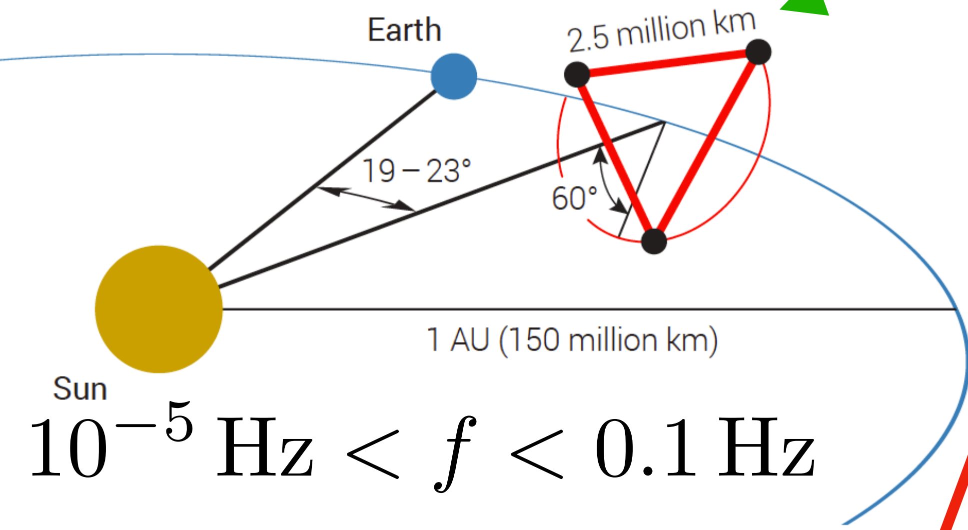
$$f_* \simeq \frac{1}{\ell_*} \geq H_*$$

after redshift:

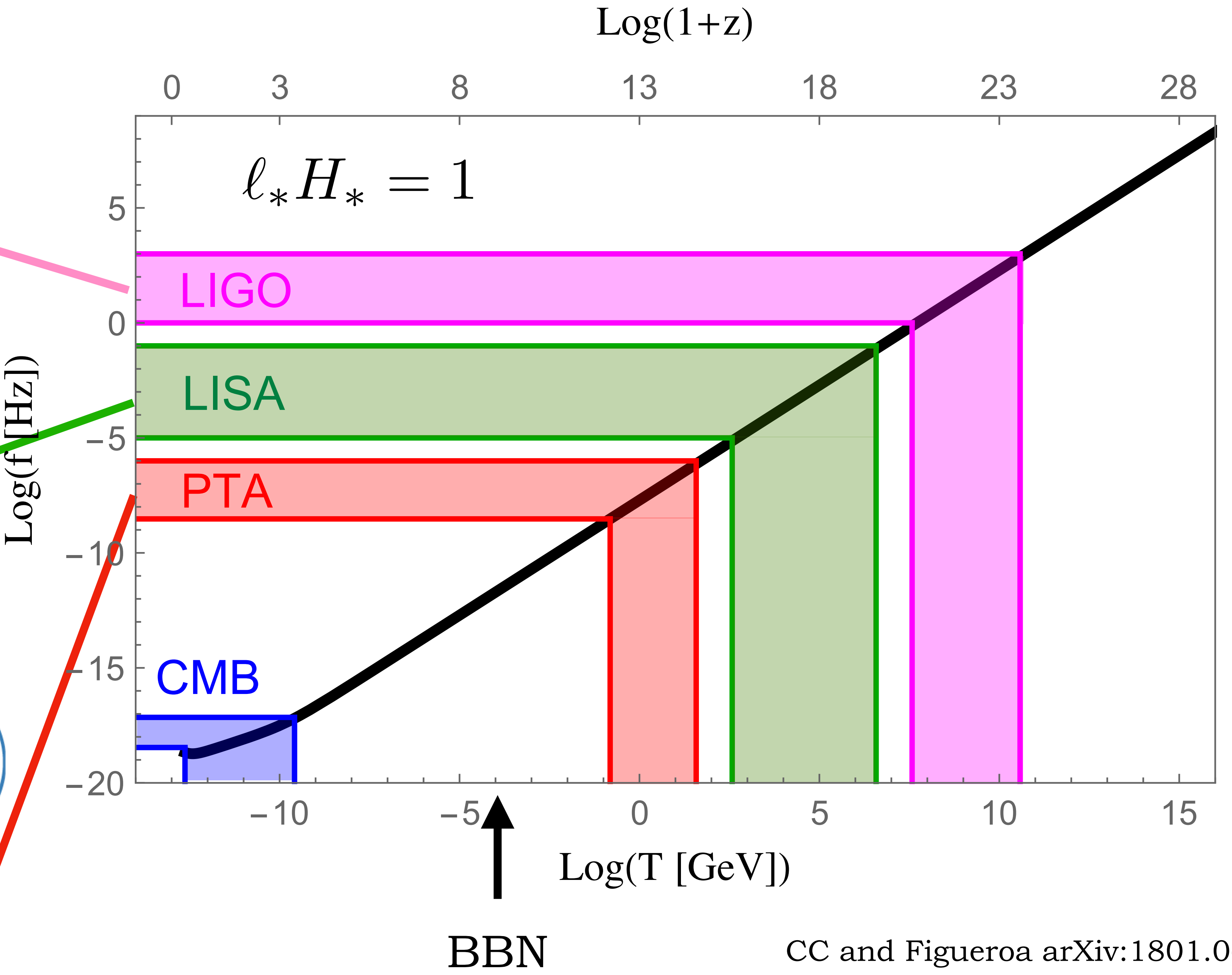
$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-5}}{\ell_* H_*} \left(\frac{g(T_*)}{100}\right)^{1/6} \frac{T_*}{100\mathrm{GeV}}\,\mathrm{Hz}$$

Cosmological gravitational waves

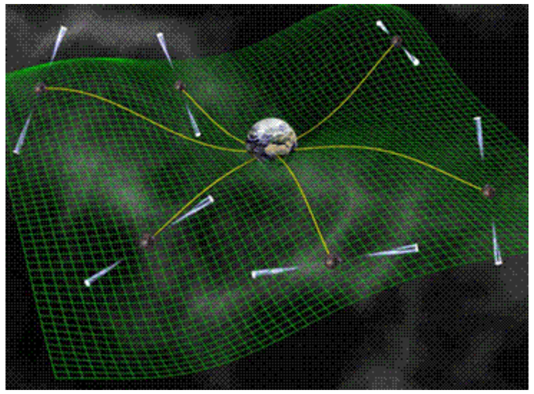
$1\,\mathrm{Hz} < f < 1000\,\mathrm{Hz}$



$10^{-5}\,\mathrm{Hz} < f < 0.1\,\mathrm{Hz}$

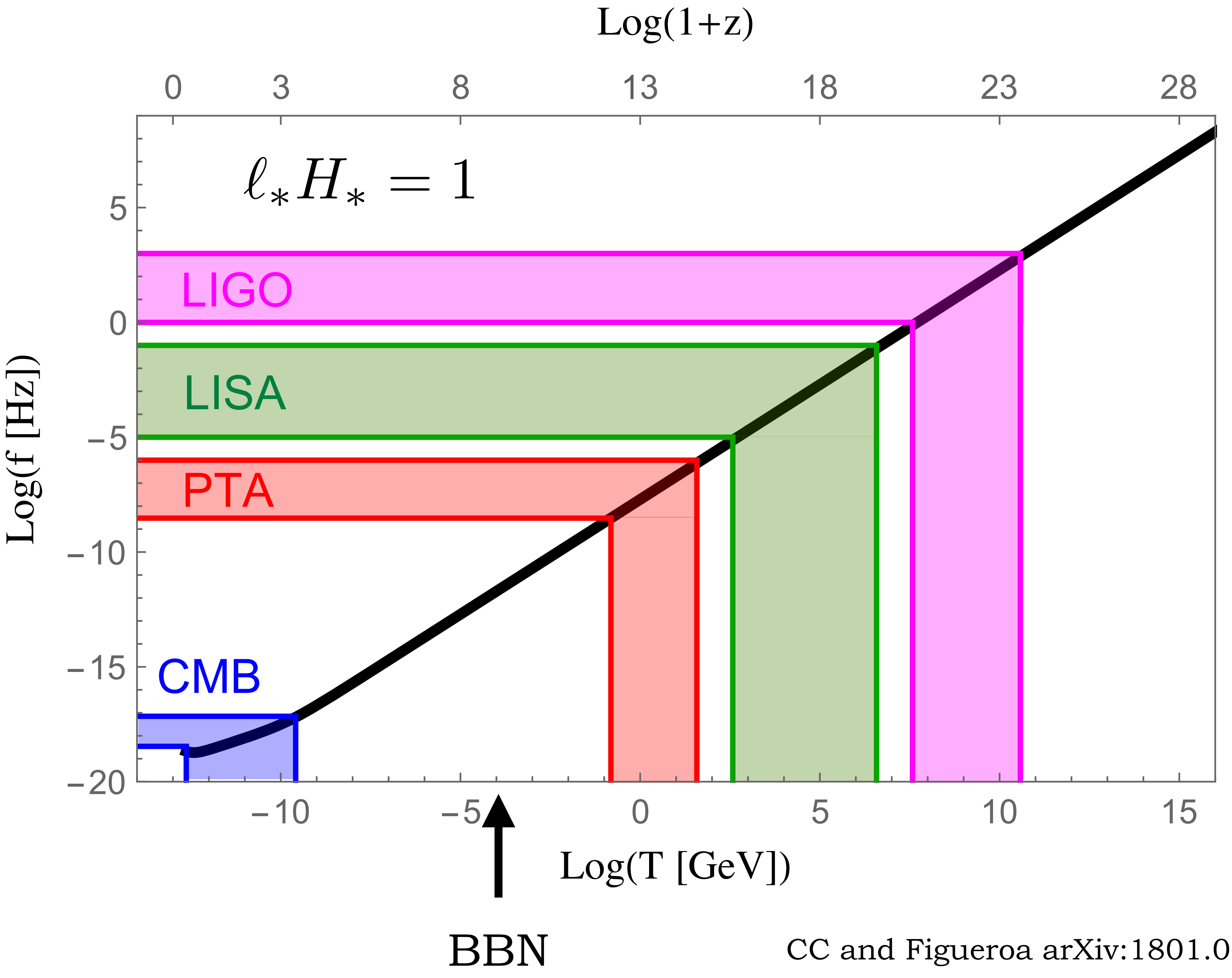


CC and Figueroa arXiv:1801.04268



$10^{-9}\,\mathrm{Hz} < f < 10^{-7}\,\mathrm{Hz}$

Cosmological gravitational waves



CC and Figueroa arXiv:1801.04268

$$T_{\rm QCD} \sim 100\,{\rm MeV} \qquad \ell_* H_* \sim 1 \qquad \longrightarrow \qquad f \sim 10\,{\rm nHz} \quad \text{PTA}$$

$$T_{\rm EW} \sim 100\,{\rm GeV} \qquad \ell_* H_* \sim 0.01 \qquad \longrightarrow \qquad f \sim {\rm mHz} \quad \text{LISA}$$

Cosmological gravitational waves

GW generating processes in the early universe?

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G\,(\bar{T}_{\mu\nu} + \delta T_{\mu\nu})$$

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = \boxed{16\pi G\,\Pi_{ij}^{TT}}$$

GW SOURCE

tensor anisotropic stress

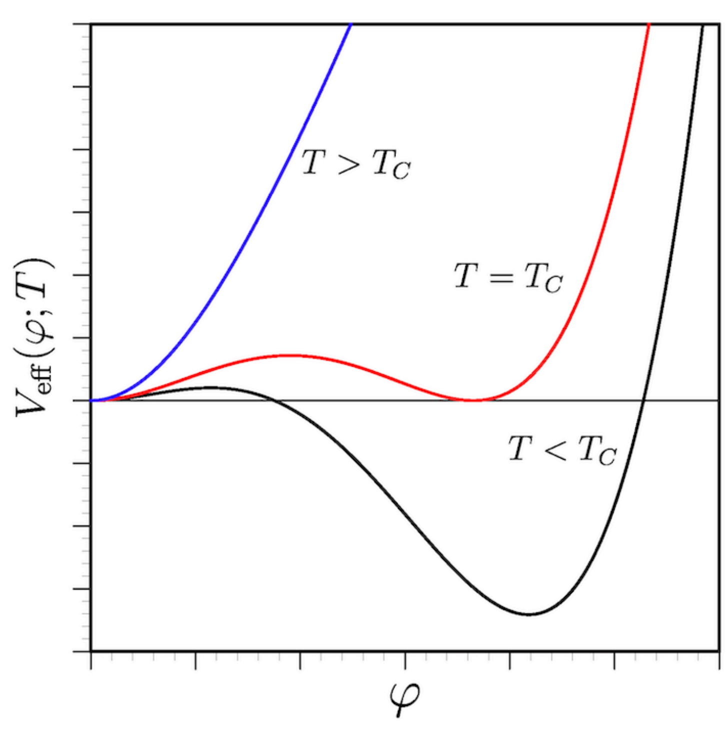
The source cannot break the observed homogeneity and isotropy of the universe: either it is weak (1st order in cosmological perturbation theory), or strong but short (and at high energy to have the time to thermalise by Nucleosynthesis)

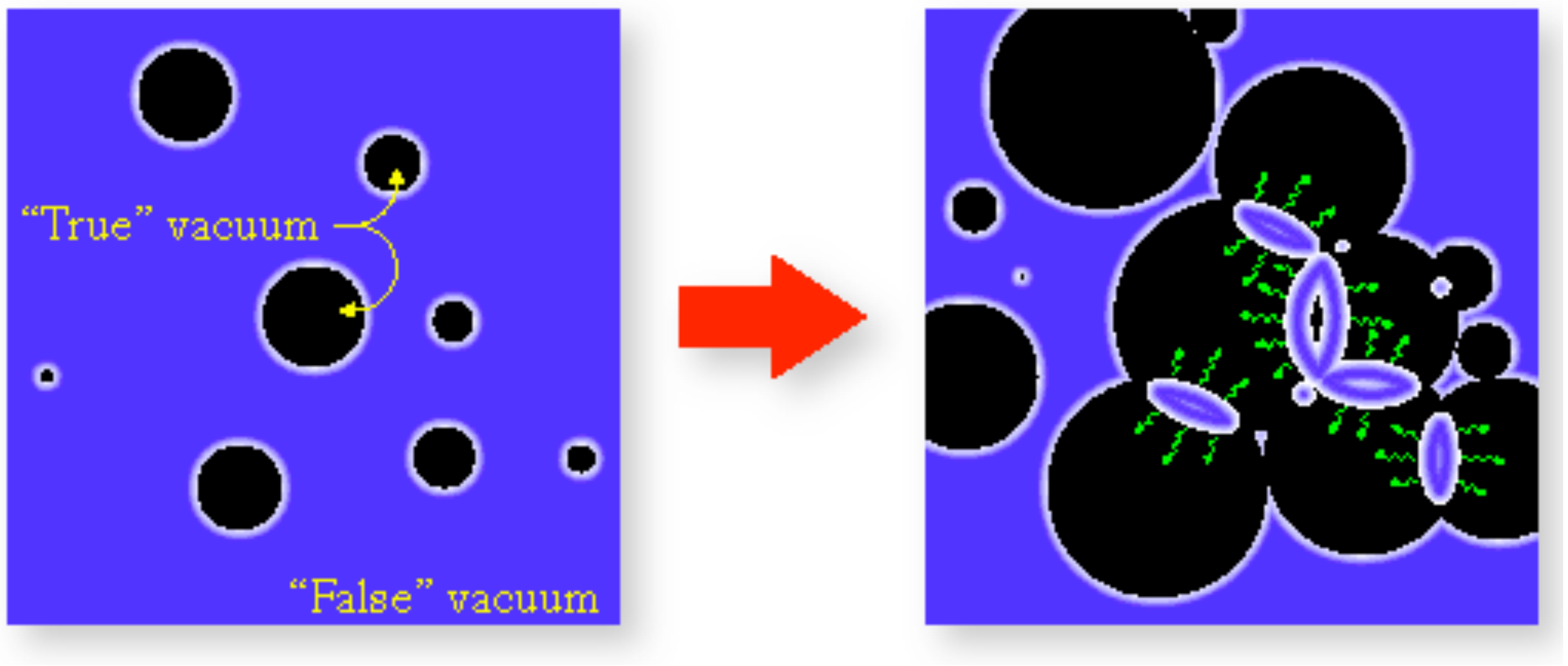
The SM plasma in thermal equilibrium generates a GW background, but very weak and peaking at the GHz: not observable in the near future

Ghiglieri and Laine
arXiv:1504.02569

Gravitational waves from first order phase transitions

Fast out-of-equilibrium process in the early universe? A first order PT!





Sources of tensor anisotropic stress at a first order PT:

Several processes, rich phenomenology!

- Bubble collision (scalar field gradients)

$$\Pi_{ij}^{TT} \sim [\partial_i\phi\partial_j\phi]^{TT}$$

- Bulk fluid motion

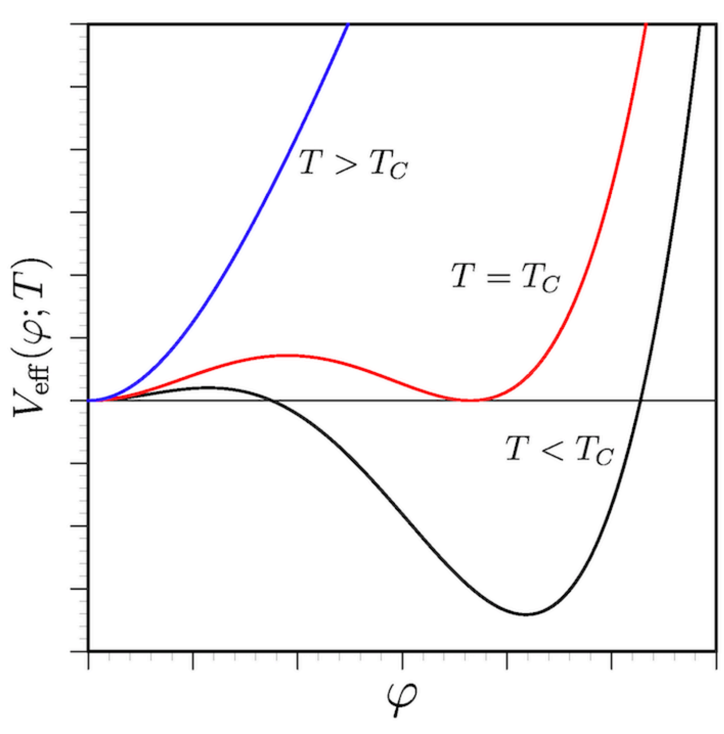
$$\Pi_{ij}^{TT} \sim [\gamma^2(\rho+p)v_iv_j]^{TT}$$

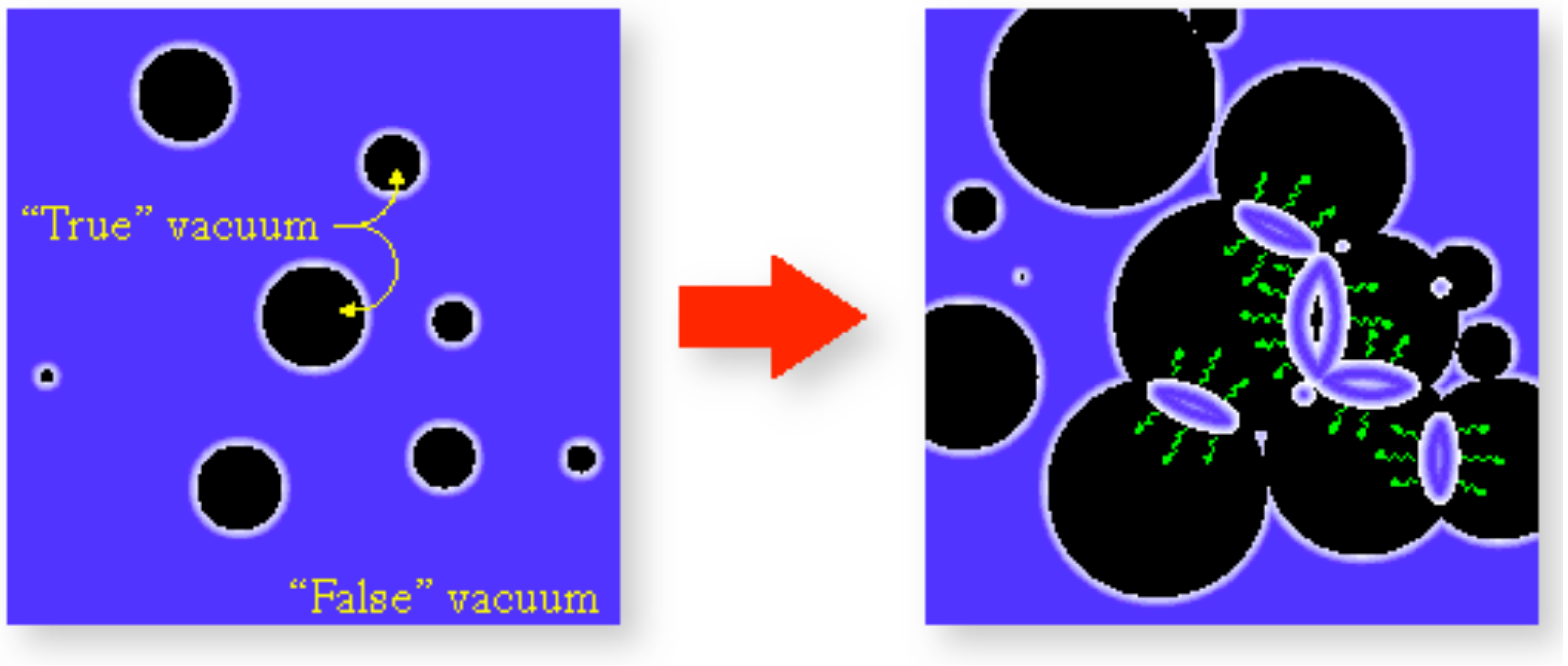
- Electromagnetic fields

$$\Pi_{ij}^{TT} \sim [-E_iE_j - B_iB_j]^{TT}$$

Gravitational waves from first order phase transitions

Fast out-of-equilibrium process in the early universe? A first order PT!





One can exploit the coincidence between energy scales and detectors sensitivity!

$T_{\rm QCD} \sim 100\,{\rm MeV}$ $\quad \ell_* H_* \sim 1$ $\longrightarrow$ $f \sim 10\,{\rm nHz}$ PTA

$T_{\rm EW} \sim 100\,{\rm GeV}$ $\quad \ell_* H_* \sim 0.01$ $\longrightarrow$ $f \sim {\rm mHz}$ LISA

EWPT: possible connections with baryon asymmetry, dark matter candidates…

Gravitational waves from first order phase transitions

Scaling of the GW energy density with the source parameters:

$$\delta G_{ij} = 8\pi G\, \delta T_{ij} \qquad k^2 h \sim 16\pi G\, \Pi \qquad \frac{\rho_{\rm GW}}{\rho^*_{\rm tot}} \sim \frac{\dot h^2}{32\pi G\, \rho^*_{\rm tot}} \sim \frac{8\pi G}{k^2}\frac{\Pi^2}{\rho^*_{\rm tot}}$$

k: characteristic scale of variation of the tensor perturbation

Π: anisotropic stress

$\frac{\rho_{\rm GW}}{\rho^*_{\rm tot}}$: fractional GW energy density

setting $k \sim \frac{1}{\ell_*}$

$$\boxed{\Omega^*_{\rm GW} \sim (\ell_* H_*)^2 \left(\frac{\Pi}{\rho^*_{\rm tot}}\right)^2}$$

$$\Omega^{\rm today}_{\rm GW} \sim 10^{-5}\,\Omega^*_{\rm GW} \gtrsim 10^{-11}$$

More or less the sensitivity of LISA

$$(H_*\ell_*)\left(\frac{\Pi}{\rho^*_{\rm tot}}\right) \gtrsim 10^{-3}$$

A lot of anisotropic stress needed: strong, slow first order PT

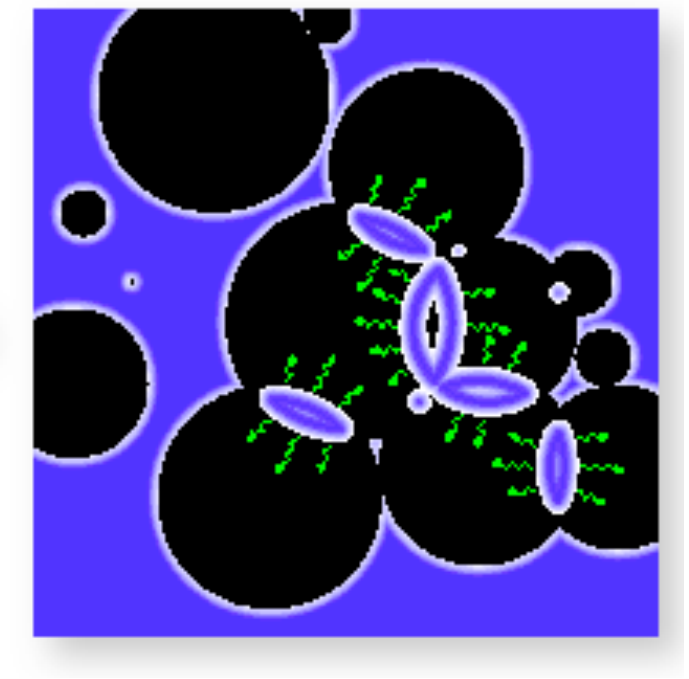
$\ell_* H_* \leq 1$ and $\frac{\Pi}{\rho^*_{\rm tot}} < 1$

Parameters entering the GW signal

- **the characteristic scale of the source (anisotropic stresses):** $\ell_* H_* \leq 1$

Size of the bubbles at collision (towards the end of the PT)

$$\ell_* \sim R_* \sim \frac{v_w}{\beta}$$



- Transition rate parameter ("duration" of the PT):

$$\beta = \frac{d}{dt} \ln \mathcal{P}(t) \Big|_{t_*}$$

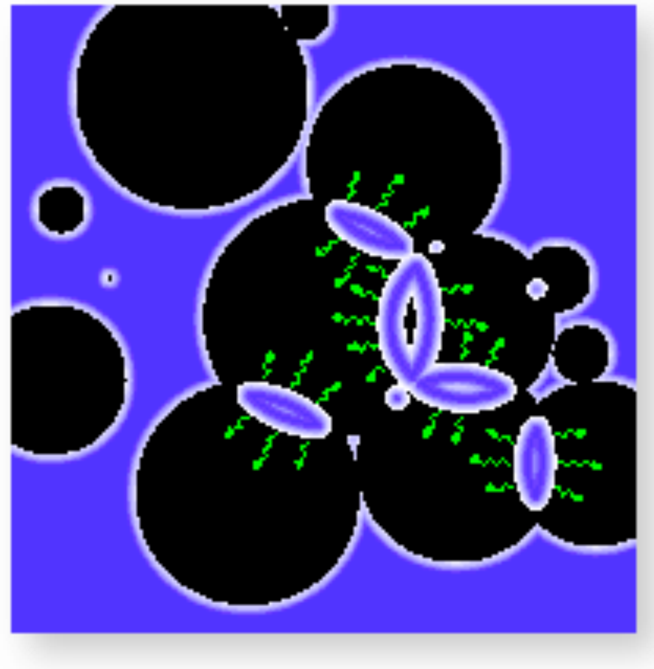
Rate of bubble nucleation per unit volume and time

$$\mathcal{P}(t) = A(t) e^{-S_c(t)}$$

Parameters entering the GW signal

- **the characteristic scale of the source (anisotropic stresses):** $\ell_* H_* \leq 1$

Size of the bubbles at collision (towards the end of the PT) $\ell_* \sim R_* \sim \frac{v_w}{\beta}$



- Transition rate parameter ("duration" of the PT): $\boxed{\beta} = \frac{d}{dt} \ln \mathcal{P}(t) \Big|_{t_*}$

Rate of bubble nucleation per unit volume and time $\mathcal{P}(t) = A(t) e^{-S_c(t)}$

- bubble wall velocity $\boxed{v_w}$

Difficult to estimate!
Thermal PT: terminal wall velocity (steady state bubble) given by the balance among the driving force (pressure difference) and the friction force (interaction of the wall with particles in the surrounding plasma)
Often used is a phenomenological description introducing a friction parameter (hopefully covering several particle theory models)

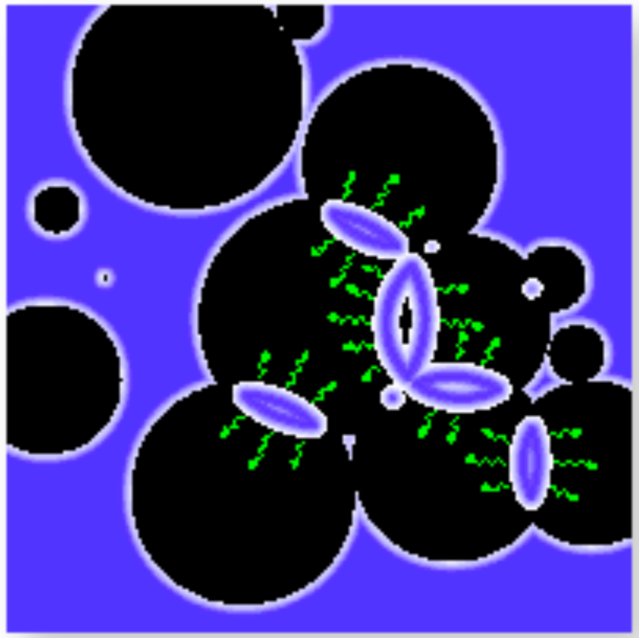
Huber and Sopena
arXiv:1302.1044

Parameters entering the GW signal

- **the characteristic scale of the source (anisotropic stresses):** $\ell_* H_* \leq 1$

Size of the bubbles at collision (towards the end of the PT)

$$\ell_* \sim R_* \sim \frac{v_w}{\beta}$$



- Transition rate parameter ("duration" of the PT): $\beta = \frac{d}{dt} \ln \mathcal{P}(t) \Big|_{t_*}$

Rate of bubble nucleation per unit volume and time $\mathcal{P}(t) = A(t) e^{-S_c(t)}$

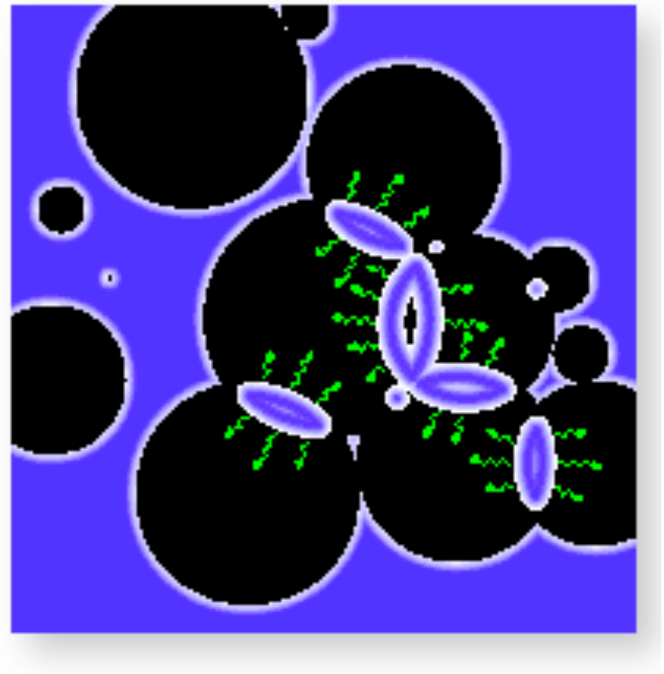
- bubble wall velocity v_w

- redshift to get the characteristic frequency of the GW signal today: temperature scale of the PT

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-5}}{\ell_* H_*} \left(\frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{100\mathrm{GeV}} \; \mathrm{Hz}$$

Parameters entering the GW signal

- **the anisotropic stress energy fraction:** $\frac{\Pi}{\rho^*_{\rm tot}}$



1. The colliding bubble walls source anisotropic stresses

   -> gradient energy in the scalar field

2. The coupling with the surrounding fluid sets it into motion
   bulk fluid motion sources anisotropies stress via

   - *Sound waves* (compressional mode, linear)
   - *Turbulence* (vortical mode, non-linear)

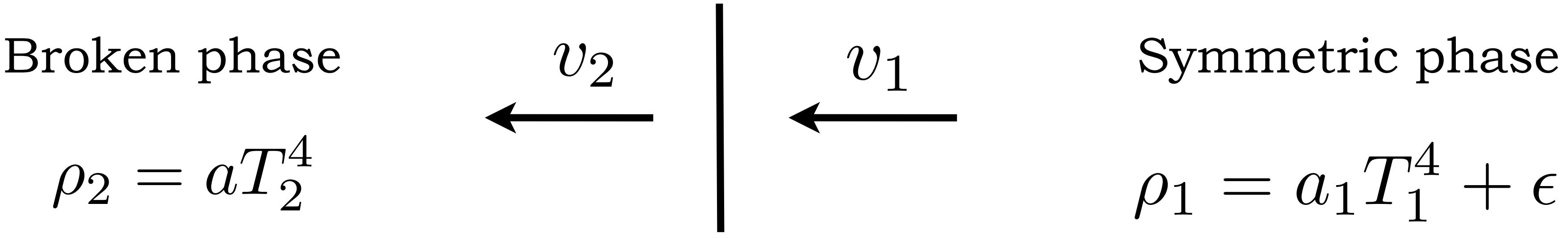
$$\tau_{\rm nl} \sim \frac{\ell_*}{v_{\rm rms}} \leq H_*^{-1} \quad \text{and} \qquad \mathrm{Re} = \frac{v_{\rm rms}\,\ell_*}{\nu} > 1$$

3. If turbulence, then it is accompanied by a *magnetic field* (MHD)

   -> kinetic (MHD) energy in the bulk fluid motion

Parameters entering the GW signal

Hydrodynamics of the bubble growth at late time (steady state)



Broken phase

v_2

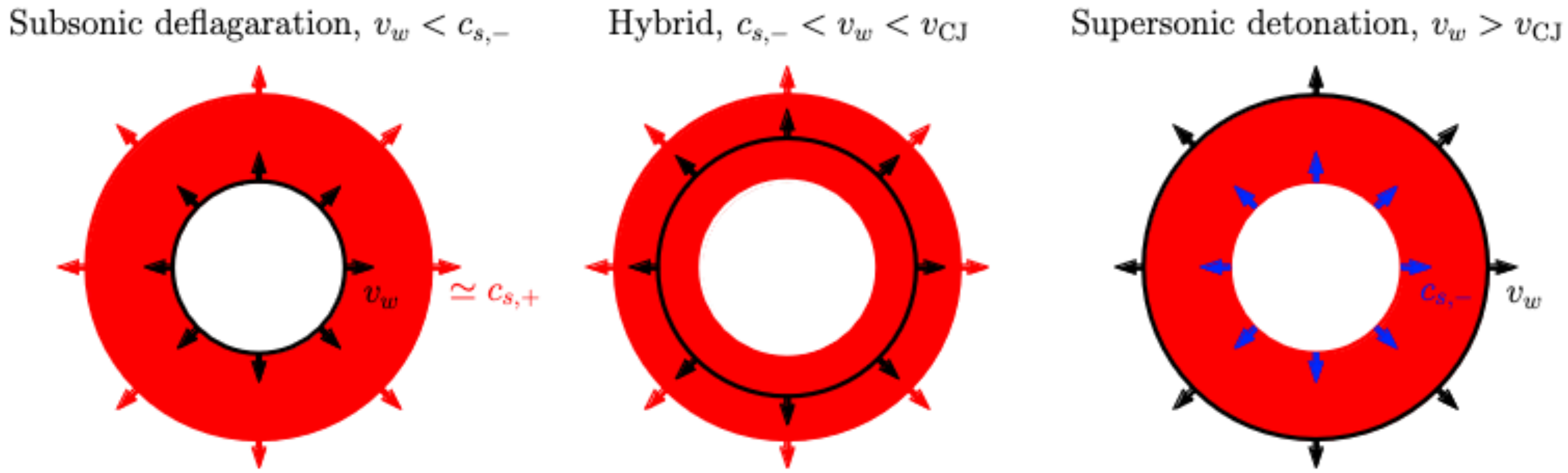
v_1

Symmetric phase

$\rho_2 = aT_2^4$

$\rho_1 = a_1 T_1^4 + \epsilon$

J. Espinosa et al
arXiv:1004.4187



Subsonic deflagaration, $v_w < c_{s,-}$

Hybrid, $c_{s,-} < v_w < v_{\rm CJ}$

Supersonic detonation, $v_w > v_{\rm CJ}$

P. Athron et al arXiv:2305.02357

+ non-stationary
solution:
runaway bubbles?

Bodeker and Moore
arXiv:1703.08215

Given the PT strength parameter $\boxed{\alpha =}\dfrac{\epsilon}{a_1 T_1^4}$ and the bubble wall velocity

the outcome of this evaluation provides the fluid velocity profile, and therefore the kinetic energy

$$\boxed{\kappa_v} = \frac{3}{\epsilon\, v_w^3} \int d\xi\, \xi^2 w\, \gamma^2 v^2 \equiv \frac{\rho_{\rm kin}}{\rho^*_{\rm vac}}$$

Parameters entering the GW signal

To summarise, the following parameters enters in the GW signal:

$T_*, \ \alpha, \ \dfrac{\beta}{H_*}$ → Determined by the effective potential

$v_w(\alpha, \eta), \ \kappa(\alpha, \eta)$ → Determined by the bubble expansion hydrodynamics

If the PT is strong and non-linearities in the bulk fluid develop, another parameter adds: the fraction of kinetic energy which is in turbulent motions

$$\varepsilon = \frac{\kappa_{\mathrm{turb}}}{\kappa_v}$$

Most are known (at least in principle) given a PT model

However, this was just the GW energy density scaling: the proper determination of the efficiency of anisotropic stress production, and of the shape of the GW power spectrum often requires **numerical simulations**

Spectral shape of the GW signal

$$\Omega^*_{\rm GW}(f) = \tilde{\Omega}\,(\ell_* H_*)^2 \left(\frac{\Pi}{\rho^*_{\rm tot}}\right)^2 S(f)$$

How much kinetic energy is in anisotropies stresses?

What is the spectral shape of the GW signal as a function of frequency?

numerical simulations

- SCOTTS code (Helsinki group): coupled dynamics of the field-fluid system, relativistic, no expansion of the universe, friction parameter

$$T^{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{g^{\mu\nu}}{2}(\partial\phi)^2 + (\rho + p)U^\mu U^\nu + g^{\mu\nu} p$$

Helsinki/Sussex group,
M. Hindmarsh et al, arXiv:1304.2433 and following

$$\partial_\mu T^{\mu\nu}_\phi = -\partial_\mu T^{\mu\nu}_f = -\eta\, U^\mu \partial_\mu\phi\partial^\nu\phi$$

Spectral shape of the GW signal

$$\Omega^*_{\mathrm{GW}}(f) = \tilde{\Omega}\,(\ell_* H_*)^2 \left(\frac{\Pi}{\rho^*_{\mathrm{tot}}}\right)^2 S(f)$$

How much kinetic energy is in anisotropies stresses?

What is the spectral shape of the GW signal as a function of frequency?

numerical simulations

- Pencil code (Nordita group): simulates MHD turbulence (present in the initial conditions or induced by adapted forcing), relativistic up to order v², expansion of the universe

$$\begin{aligned} T^{\mu\nu} &= (\rho + p) U^\mu U^\nu + g^{\mu\nu} p \\ &+ \frac{1}{4\pi}\left(F^{\mu\sigma}F^{\nu}{}_{\sigma} - \frac{1}{4} g^{\mu\nu} F_{\lambda\sigma}F^{\lambda\sigma}\right) \end{aligned}$$

A. Roper Pol et al, arXiv:1903.08585
and other works by the Nordita group

$$\nabla_\nu T^{\mu\nu}_f = -\nabla_\nu T^{\mu\nu}_{\mathrm{EM}} = F^{\mu\nu} J_\mu$$

Spectral shape of the GW signal

$$\Omega^*_{\rm GW}(f) = \tilde{\Omega}\,(\ell_* H_*)^2 \left(\frac{\Pi}{\rho^*_{\rm tot}}\right)^2 S(f)$$

How much kinetic energy is in anisotropies stresses?

What is the spectral shape of the GW signal as a function of frequency?

numerical simulations

- Higgsless simulations (DESY group):
  only fluid, initialised with random initial vacuum energy regions (bag EoS), relativistic, no expansion of the universe, fast

$$T^{\mu\nu} = (\rho + p)U^\mu U^\nu + g^{\mu\nu} p$$

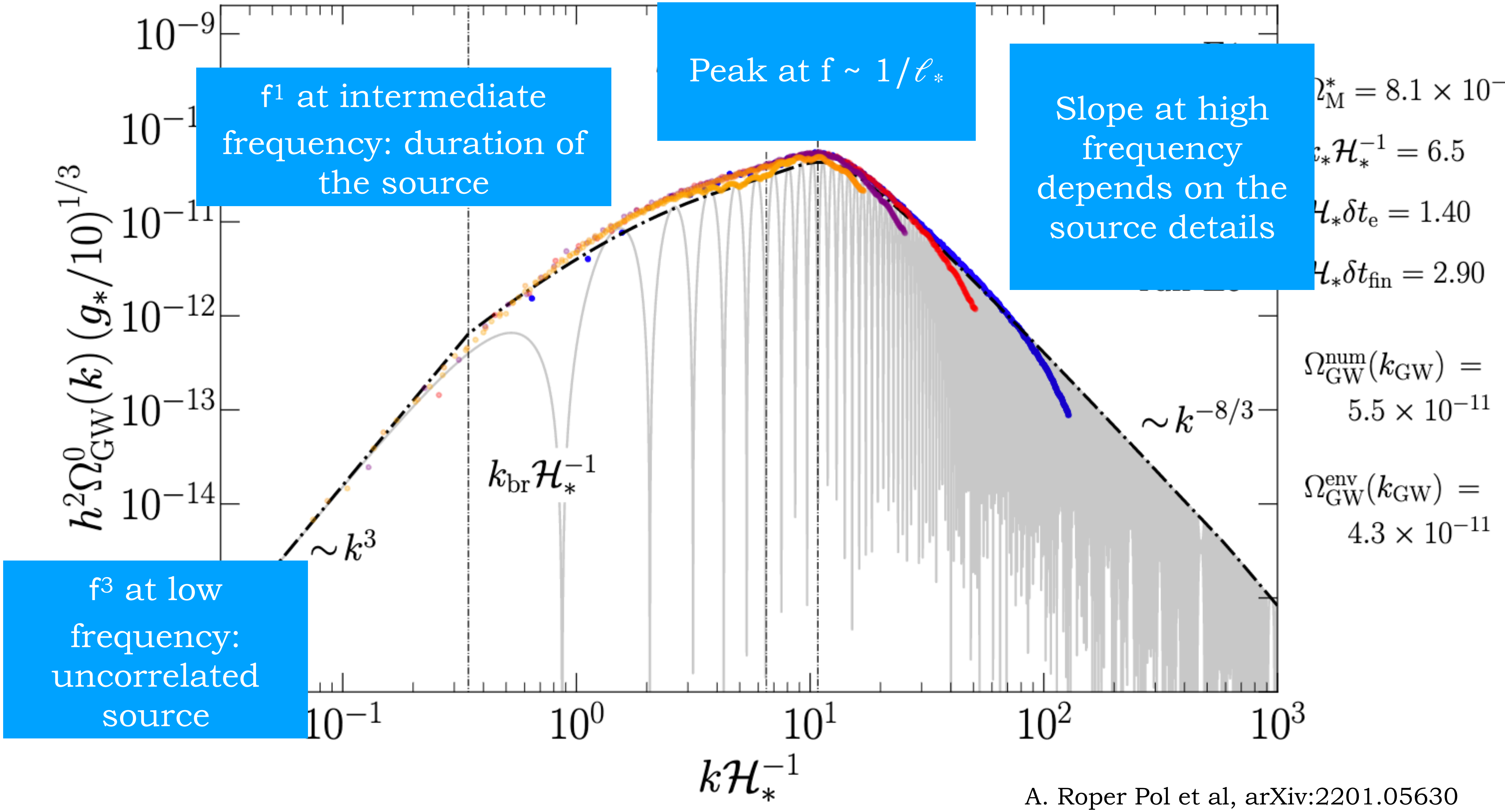
$$\partial_\mu T^{\mu\nu}_f = 0$$

$$p = \frac{1}{3}aT^4 + \epsilon, \quad w = \frac{4}{3}aT^4$$

R. Jinno et al, arXiv:2209.04369

Spectral shape of the GW signal

One example of GW signal from MHD turbulence, obtained from a simulation with the Pencil code, together with an analytical evaluation



Peak at f ~ $1/\ell_*$

f^1 at intermediate frequency: duration of the source

Slope at high frequency depends on the source details

f^3 at low frequency: uncorrelated source

A. Roper Pol et al, arXiv:2201.05630

Spectral shape of the GW signal

The spectral shape depends also on the relative contribution of the three GW sourcing processes

CC et al arXvi:1512.06239

- Bubble wall collisions (gradient energy from the scalar field) dominate the PT signal for:

very strong PTs with large supercooling $\boldsymbol{\alpha} \gg 1$, negligible fluid coupling and thereby no bulk motion, bubble move practically at the speed of light

- Sound waves (kinetic energy of the bulk motion) dominate the PT signal for:

weakly first order PTs $\boldsymbol{\alpha} \sim 0.001$ to 0.1, simulations by the Helsinki group, linear fluid motion, sound waves remain in the fluid long after the symmetric phase has disappeared

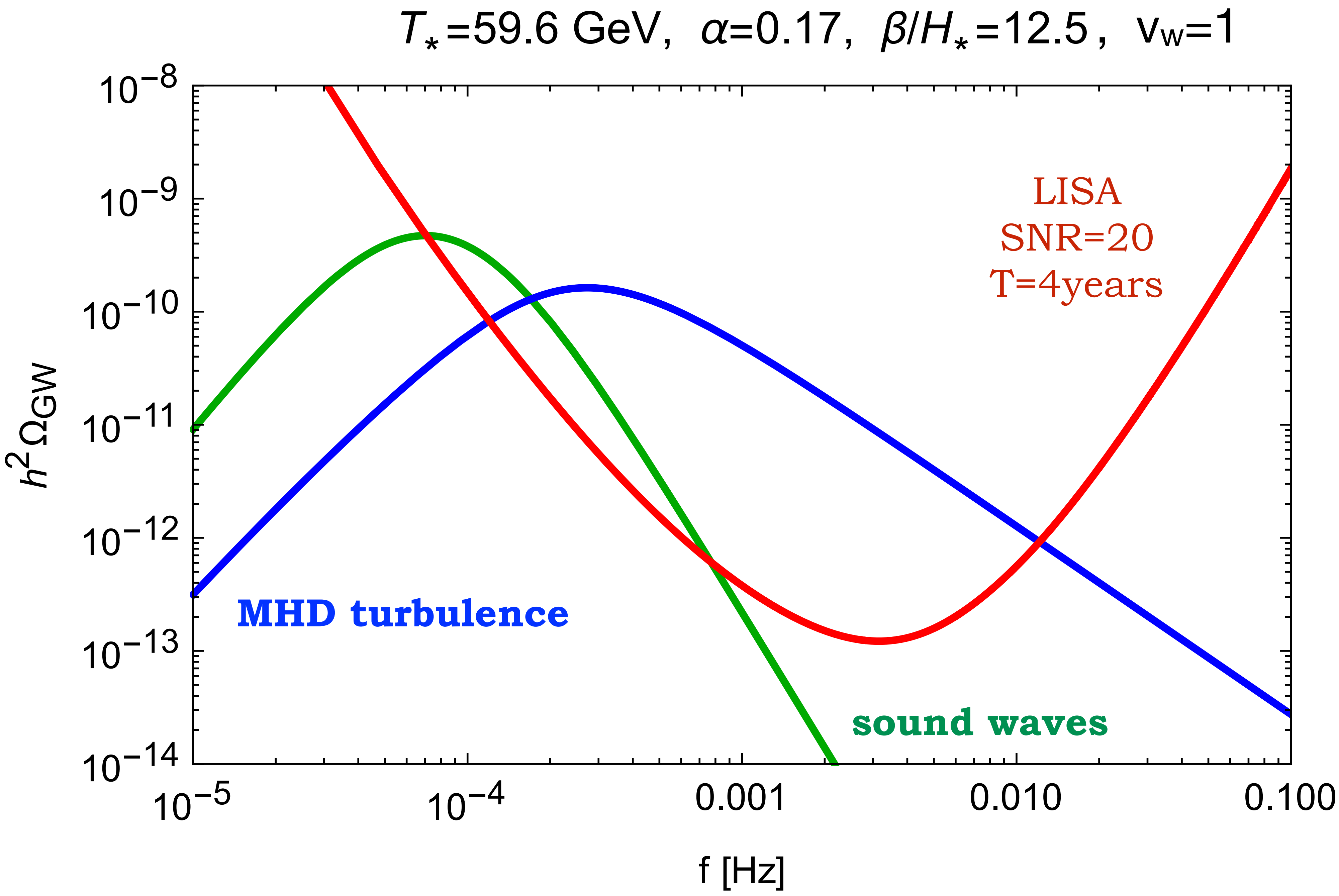
- MHD turbulence (kinetic energy of the bulk motion and magnetic energy) dominate the PT signal for:

(probably) moderately strong PTs $\boldsymbol{\alpha} \gtrsim 1$, non-linearities develop, simulations with the Pencil code, no onset of the turbulence observed so far but put in the initial conditions, turbulence remains in the fluid long after the symmetric phase has disappeared

Examples of detectable signal from the EWPT

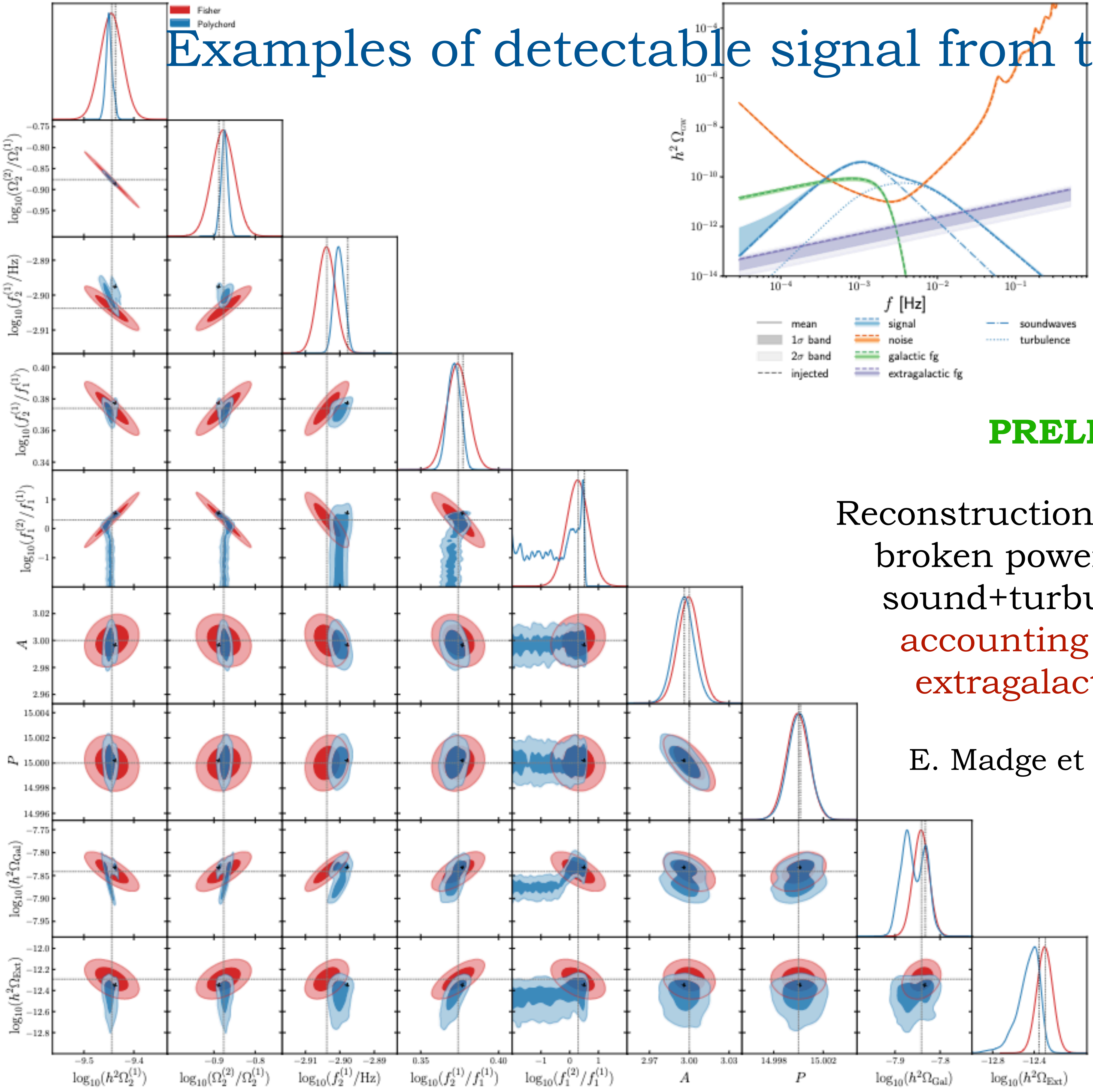
Examples of detectable signal from the EWPT

Just indicative: benchmark point from CC et al arXvi:1512.06239, singlet SM extension



$$\tau_{\rm nl} \sim \frac{\ell_*}{v_{\rm rms}} = \frac{0.54}{\mathcal{H}_*} \qquad \langle v^2 \rangle_{\rm turb} = 0.25\, \langle v^2 \rangle_{\rm sound}$$

Examples of detectable signal from the EWPT

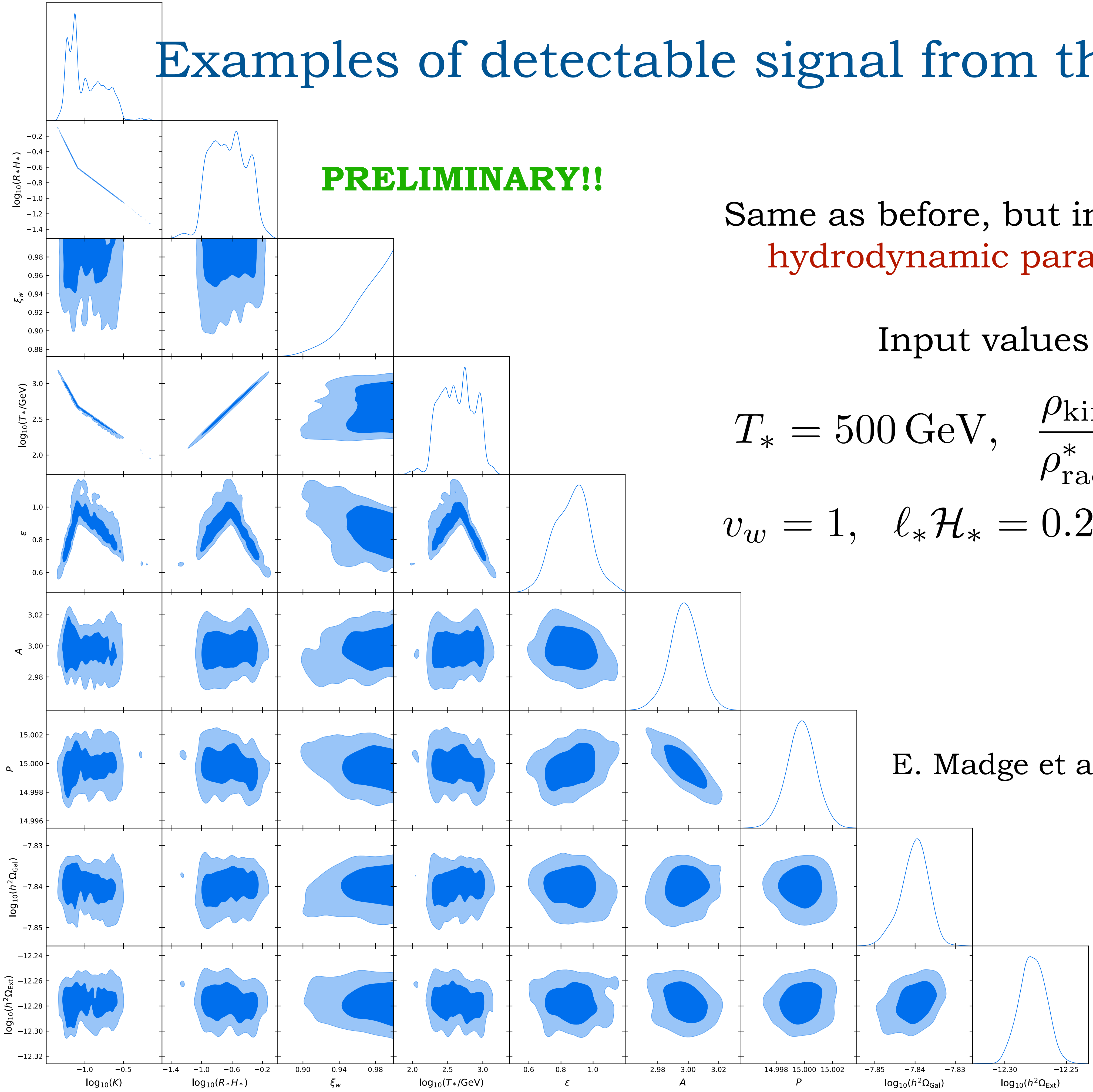


PRELIMINARY!!

Reconstruction of a generic double broken power law (inspired by sound+turbulence) with LISA accounting for galactic and extragalactic foregrounds

E. Madge et al, in preparation

Examples of detectable signal from the EWPT



PRELIMINARY!!

Same as before, but in terms of
hydrodynamic parameters

Input values:

$$T_* = 500\,\mathrm{GeV}, \quad \frac{\rho_{\rm kin}}{\rho^*_{\rm rad}} = 0.08,$$

$$v_w = 1, \quad \ell_* \mathcal{H}_* = 0.25, \quad \varepsilon = 1$$

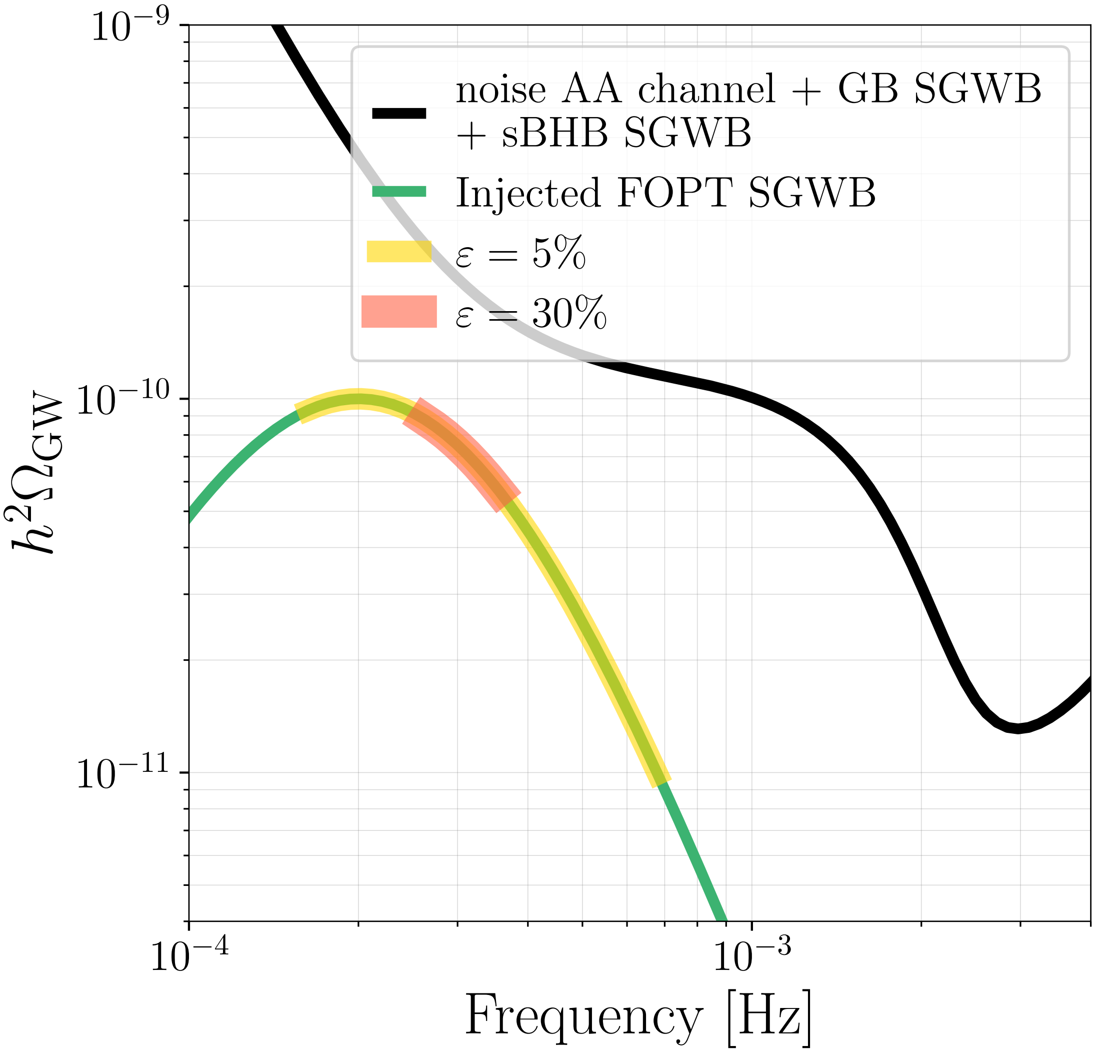
E. Madge et al, in preparation

Examples of detectable signal from the EWPT

Is it possible to reconstruct the GW signal spectral shape, to identify that the source is a FOPT?

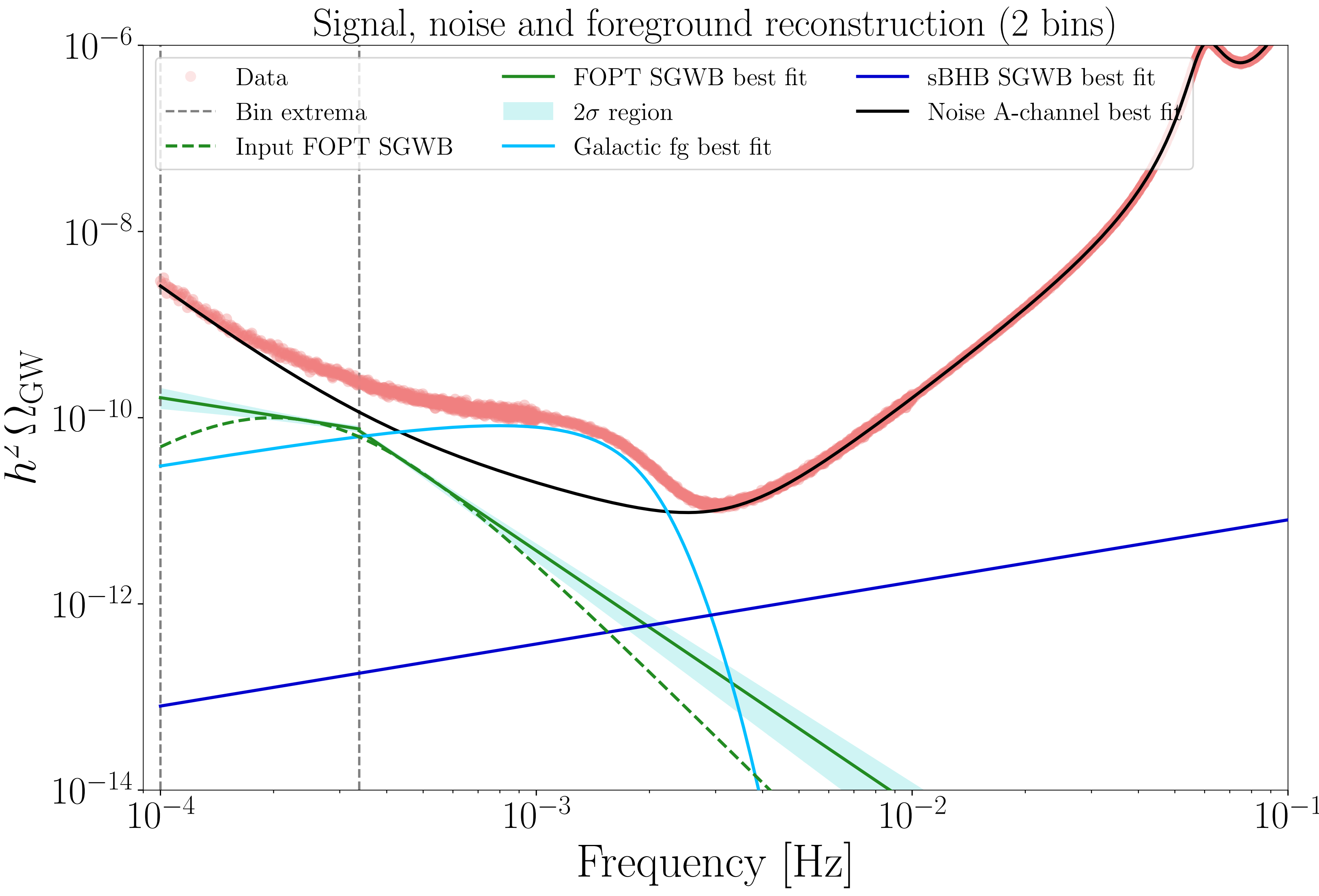
Signal from a singlet
extension of SM setting

$$m_s = 0.94\,\text{GeV},\ \lambda_s = 1,\ \lambda_{hs} = 0.92$$



N. Karnesis, arXiv:1906.09027

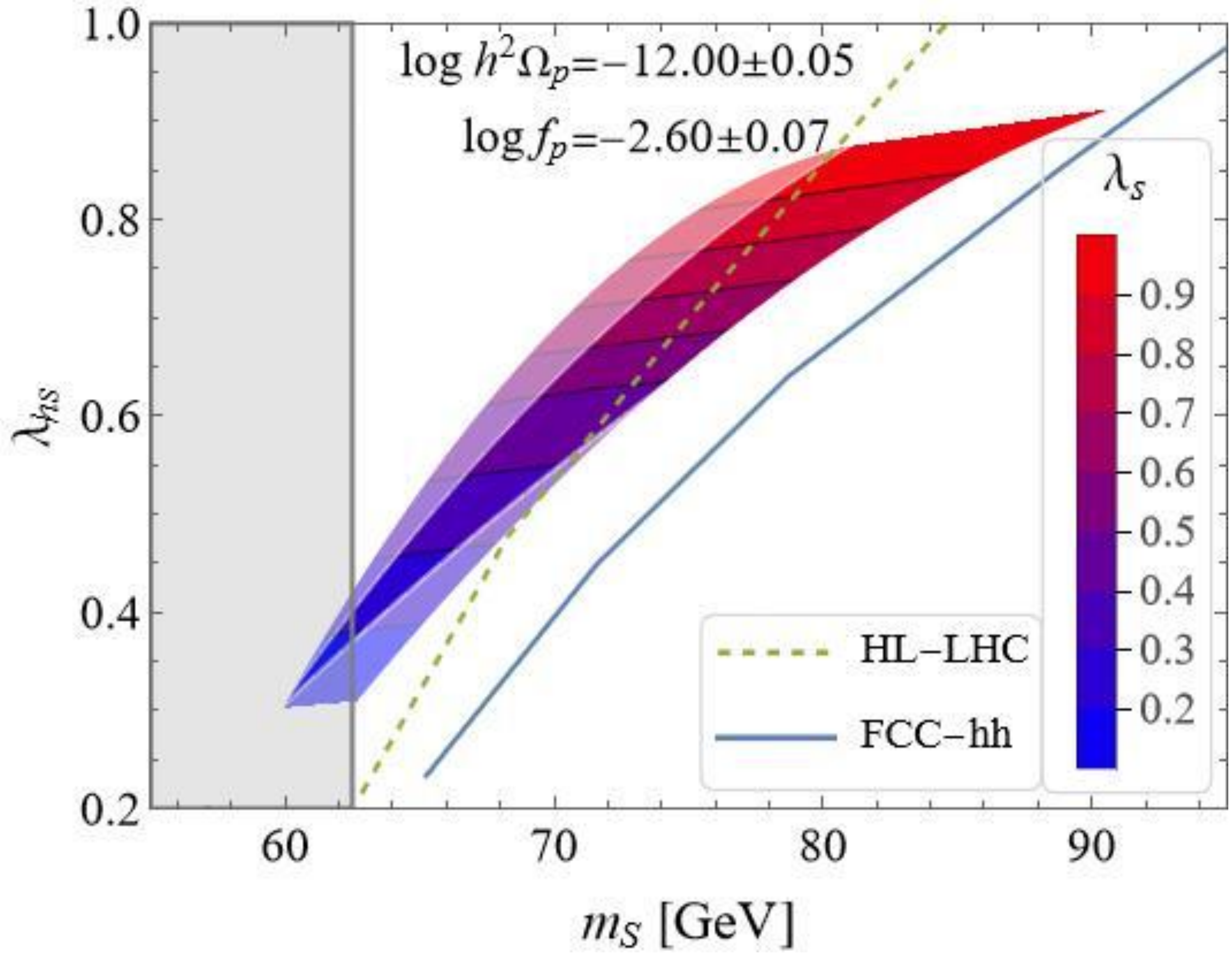
CC et al, arXiv:1906.09244, Flauger et al arXiv:2009.11845


Signal, noise and foreground reconstruction (2 bins)

Examples of detectable signal from the EWPT

Several model parameter values can correspond to the same GW signal, here assumed to be a single broken power law

E. Madge et al, in preparation



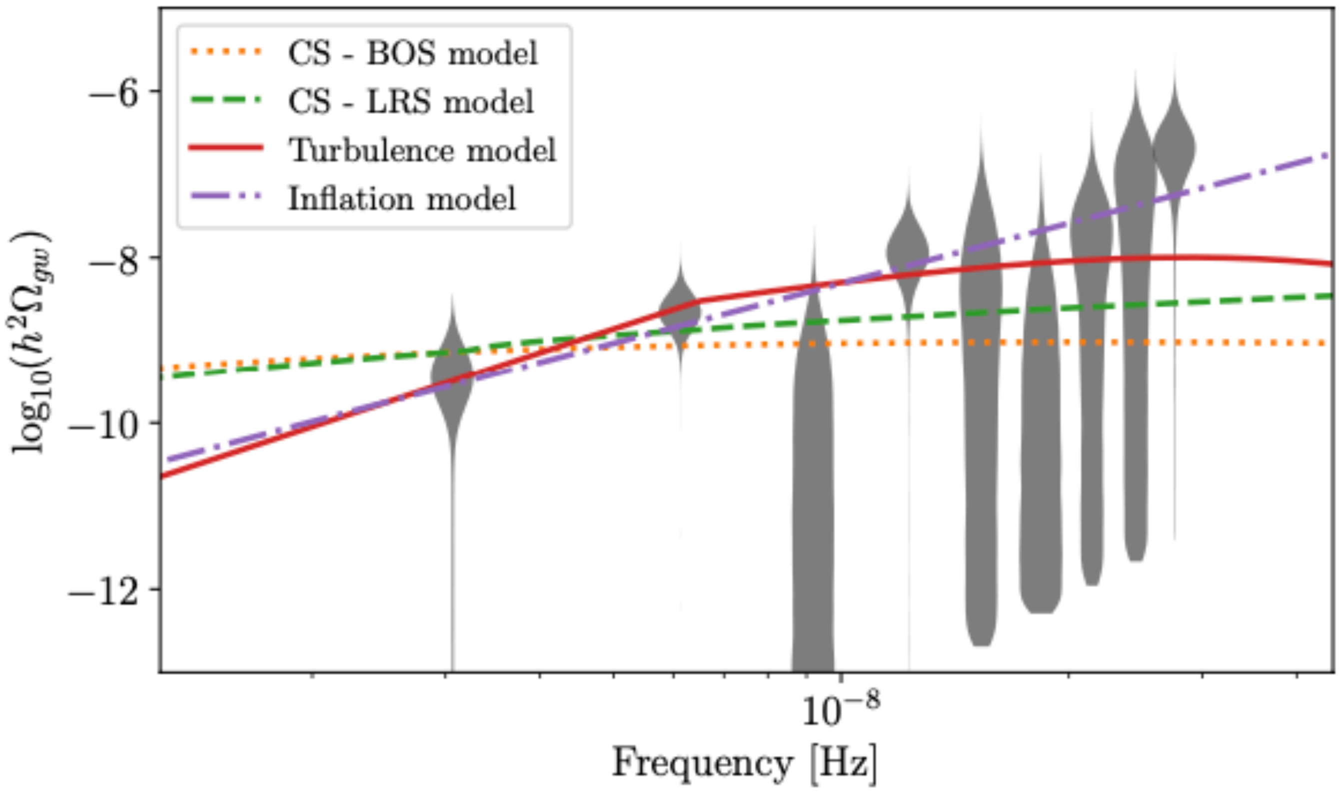
An example of possible detection at PTA

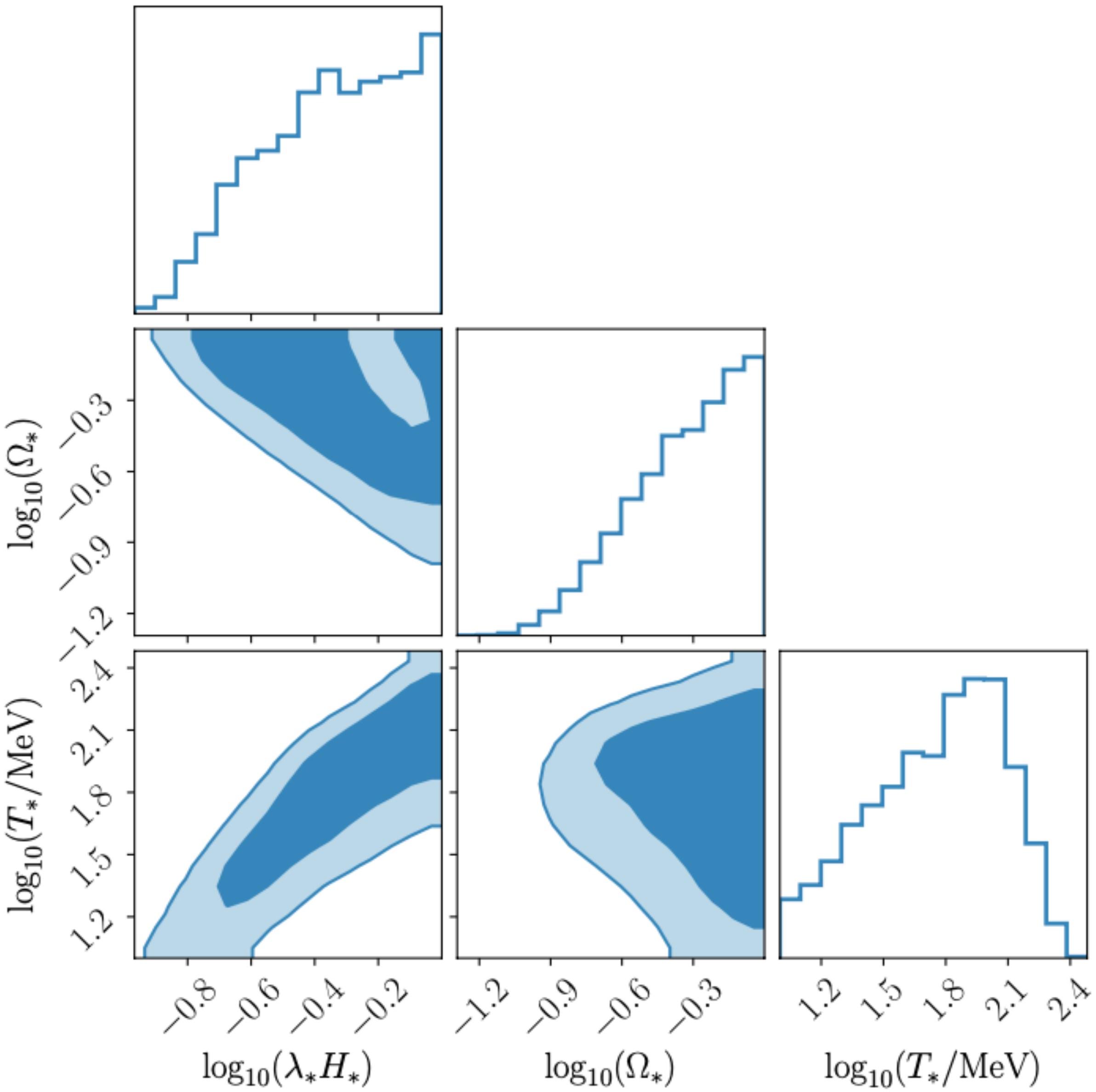
Pulsar Timing Arrays (nHz) have measured a common stochastic GW signal

They are sensitive to energy scales around the **QCD scale**, so they can probe physical processes connected to the QCDPT **IF** it is first order

D. Schwarz and Stuke, arXiv:0906.3434
M. Middeldorf-Wygas et al, arXiv:2009.0003

The signal is compatible with GWs generated by MHD turbulence at the QCD scale





J. Antoniadis et al, arXiv:2306.16227
A. Neronov et al, arXiv:2009.14174
A. Roper Pol et al, arXiv:2201.05630

To summarise:

- Stochastic GW backgrounds from the early universe form a **fossil radiation** which can provide interesting information on high energy physics

- GW sources are processes possessing a **strong anisotropic stress component**: an appropriate example is a **first order phase transition**

- This is particularly interesting since the **LISA frequency band corresponds to the EW scale in the early universe**, and there are BSM scenarios in which the EW symmetry breaking can become first order

- There are three possible GW sources linked to the first order phase transition dynamics: **bubble wall collision, sound waves and magnetohydrodynamic turbulence** in the fluid surrounding the bubbles

- The GW signal is determined by the PT temperature, its strength, its duration, the bubble radius at collision, the bubble wall velocity, the fraction of vacuum energy which gets converted into kinetic energy of the bulk fluid motion, and the efficiency of turbulence production

- The precise derivation of the GW signal spectral shape requires **numerical simulations** of the hydrodynamics of the coupled system of scalar field, fluid, and possibly electromagnetic field

- Simple extension of the standard model can provide signals detectable at LISA: we are start exploring **how to constrain and/or detect this signal at LISA**, and how to interpret a possible detection