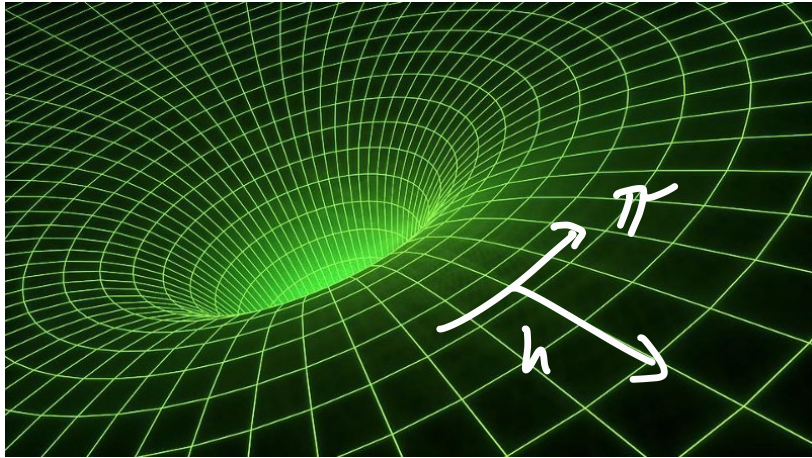


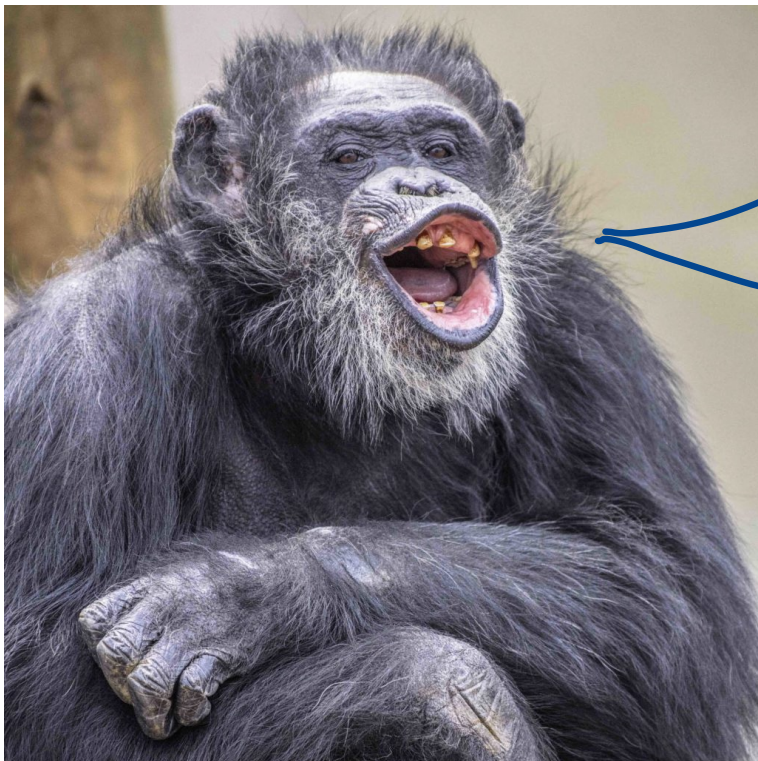
# Non-decoupling New Particles and EFT Geometry at FCC



July 5, 2023

Tim Cohen  
CERN/EPFL/UOregon  
w/ Ian Banta  
Nathaniel Craig  
Xiaochuan Lu  
Dave Sutherland

FCC Phenomenology Workshop



Build  
the  
FCC!

Where's  
the  
new Physics?

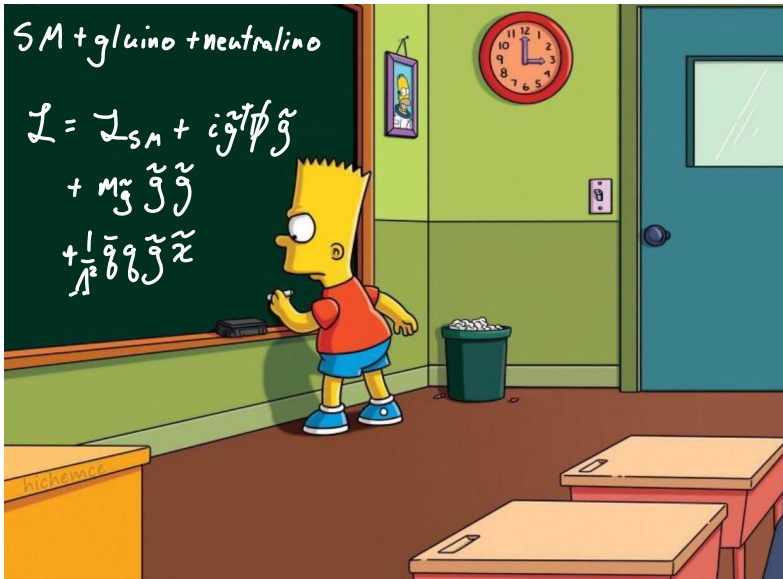
How  
best to  
search?

Can we  
be systematic?

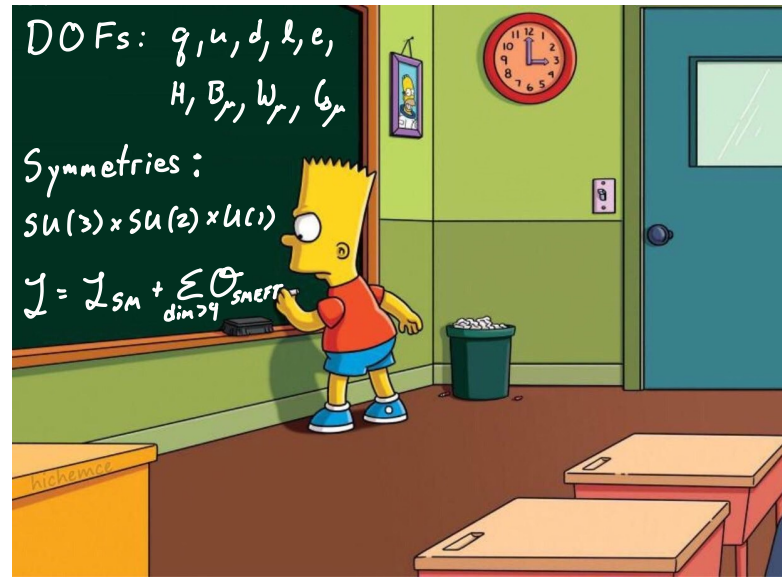


# How to organize BSM predictions?

## Simplified Models



## Effective Field Theory



# The Standard Model as EFT

"Heavy physics decouples"

- Only SM dofs
- Symmetries: Lorentz +  $SU(3) \times SU(2) \times U(1)$

Realize electroweak symmetry

linear or non-linearly  
↑ ↑  
SMEFT HEFT

# SMEFT ( $v=0$ )

Let  $\vec{\phi}$  be an  $O(4)$  vector

$$\vec{\phi} \rightarrow O \vec{\phi}$$

I identify  $H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_4 + i\phi_3 \end{pmatrix}$

s.t.  $\langle H \rangle \neq 0 \Leftrightarrow \langle \phi_4 \rangle \neq 0$

# SMEFT ( $v=0$ )

$$\mathcal{L}_{\text{SMEFT}} = A(|H|^2) |\partial H|^2 + \frac{1}{2} B(|H|^2) \left[ \partial(|H|^2) \right]^2 - \tilde{V}(|H|^2) + \mathcal{O}(\partial^4)$$

$A, B, \tilde{V}$  are real analytic @ origin  $|H|=0$

$\Rightarrow$  have convergent Taylor expansion @  $|H|=0$   
( $A(0)=1 \Rightarrow$  canonical norm)

Think of  $\varphi_i$  as Cartesian coords on manifold

$\vec{\varphi}=0$  is invariant point under  $O(4)$  transformations

# HEFT ( $v \neq 0$ )

Non-linearly realized Sym breaking

$O(4)/O(3)$  Calan, Coleman, Wess, Zumino (1969)

Polar coordinates:  $h$  (physical Higgs)  
 $\vec{n}$  (Goldstone bosons)

$$\vec{\varphi} = (v_0 + h) \vec{n} \quad \omega /$$
$$\vec{n} = \begin{pmatrix} n_1 = \pi_1/v \\ n_2 = \pi_2/v \\ n_3 = \pi_3/v \\ n_4 = \sqrt{1 - n_i^2} \end{pmatrix}$$
$$\vec{n} \in S^3 \quad \vec{n} \cdot \vec{n} = 1$$



# HEFT ( $v \neq 0$ )

$O(4)$  transformation:  $h \rightarrow h$ ,  $\vec{n} \rightarrow O\vec{n}$

$\Rightarrow \vec{n}$  in non-linear rep

$$\mathcal{I}_{\text{HEFT}} = \frac{1}{2} [\mathbb{K}(h)]^2 (\partial h)^2 + \frac{1}{2} [v F(h)]^2 (\partial \vec{n})^2 - V(h) + \mathcal{O}(\partial^4) \quad \langle h \rangle = 0$$

( $\mathbb{K}(0) = 1$  is canonical norm)

# HEFT ( $v \neq 0$ )

Does HEFT know that  $\langle H \rangle = v$ ?

Alonso,  
Jenkins,  
Manohar  
(AJM)  
1605.03602

$\Rightarrow$  There might be special place  
on manifold  $h_* = -v$  where  
 $O(4)$  symmetry is manifest

Determined by  $F(h_*) = 0$

If  $h = h_*$  exists  $\Rightarrow$

HEFT  $\rightarrow$  SMEFT possible

HEFT  $\rightarrow$  SMEFT?

Map:  $|H|^2 = \frac{1}{2} \vec{\phi} \cdot \vec{\phi} = \frac{1}{2} (v+h)^2$

$$|\partial H|^2 = \frac{1}{2} (\partial \vec{\phi})^2 = \frac{1}{2} (\partial h)^2 + \frac{1}{2} (v+h)^2 (\partial \vec{n})^2$$

$$(\partial |H|^2)^2 = (\vec{\phi} \cdot \partial \vec{\phi})^2 = (v+h)^2 (\partial h)^2$$

Naively:

$$\mathcal{L}_{\text{HEFT}} = \frac{v^2 F}{2 |H|^2} |\partial H|^2 + \frac{1}{2} (\partial |H|^2)^2 \frac{1}{2 |H|^2} \left( \mathbb{K}^2 - \frac{v^2 F^2}{2 |H|^2} \right) + \tilde{V}(|H|^2) + \mathcal{O}(\partial^4) \quad \text{Analytic @ } |H|=0?$$

# Light BSM and Analyticity

Given  $\mathcal{L}_{UV}$  integrate out heavy State

$\Rightarrow \mathcal{L}_{LPI}$ . Expand and truncate  $\Rightarrow \mathcal{L}_{EFT}$

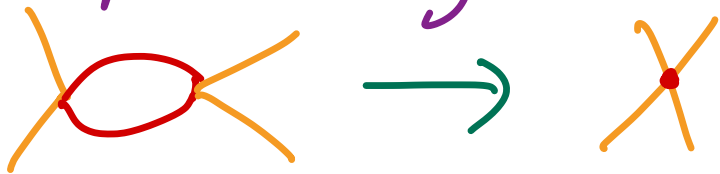
We will use functional methods

1) tree exchange

2 examples:



2) loop exchange



\* see also  
arXiv: 2011.02484  
by TC, Lu, Zhang

# Tree Exchange

Extend SM with singlet scalar  $S'$

$$\mathcal{L}_{BSM} = \frac{1}{2} S' (-\partial^2 - m^2 - \lambda |H|^2) S' - a |H|^2 S' + b S'^3$$

Integrate out  $S'$  using EOM (neglect  $\partial$ ):

$$S[H] = \frac{1}{b} \left( -m^2 + \lambda |H|^2 + \sqrt{(m^2 + \lambda |H|^2)^2 - 2ab |H|^2} \right)$$

$$\Rightarrow V_{ILPI} = V_{SM} - \frac{1}{3b^2} (m^2 + \lambda |H|^2) \left[ (m^2 + \lambda |H|^2)^2 - 3ab |H|^2 \right] + \frac{1}{3b^2} \left[ (m^2 + \lambda |H|^2) - 2ab |H|^2 \right]^{3/2}$$

Tree Exchange  $\Rightarrow$  SMEFT

Expand

$$V_{ILPI} = V_{SM} - \frac{1}{3b^2} (m^2 + \lambda |H|^2) \left[ (m^2 + \lambda |H|^2)^2 - 3ab |H|^2 \right] \\ + \frac{1}{3b^2} \left[ (m^2 + \lambda |H|^2) - 2ab |H|^2 \right]^{3/2}$$

in  $|H|^2/m^2 \Rightarrow$  Local SMEFT expansion

Tree Exchange  $\Rightarrow$  HEFT

But if  $m^2 = 0 \dots$

$$V_{ILPT} = V_{SM} + \frac{1}{3b^2} \left[ 3ab\lambda |H|^4 - \lambda^3 |H|^6 + \right. \\ \left. (-2ab|H|^2 + \lambda^2 |H|^4)^{3/2} \right]$$

is non-analytic about  $|H| = 0$

$\Rightarrow$  HEFT (will revisit)

Similar story for corrections to  $|aH|^2 + (a|H|^2)^2$

# Loop Exchange

Singlet extension w/  $a=b=0$

$$\mathcal{L}_{\text{BSM}} = \frac{1}{2} \bar{\psi} (-\partial^2 - m^2 - \lambda |H|^2) \psi$$

Leading BSM correction at one loop

Use functional matching (Coleman-Weinberg  
for  $V_{\text{LPI}}$ )

$$m_S^2 = m^2 + \frac{\lambda}{2} v^2 \Rightarrow \int d^4x \mathcal{L}_{\text{LPI}}^{1\text{-loop}} = i \log \det(\partial^2 + m^2 + \lambda |H|^2)$$



# Loop Exchange

$$\mathcal{L}_{\text{LPI}} = |\partial H|^2 - \mu_h^2 |H|^2 + \frac{1}{2} \lambda_h |H|^4$$

$$+ \frac{1}{16\pi^2} (m^2 + \lambda |H|^2)^2 \left( \ln \frac{\mu_h^2}{m^2 + \lambda |H|^2} + \frac{3}{2} \right)$$

$$+ \frac{1}{16\pi^2} \frac{1}{6} \frac{\lambda^2}{m^2 + \lambda^2 |H|^2} (\partial |H|^2)^2$$

When  $m^2 \neq 0 \Rightarrow$  expand to derive local SMEFT  
but  $m^2 = 0 \Rightarrow \log \left( \mu_h^2 / \lambda |H|^2 \right)$  Non-Analytic!  
 $\Rightarrow$  need HEFT  
(will revisit)

# Is Analyticity Enough?

Does requiring  $A, B, \tilde{V}$  be analytic  
ensure SMEFT?

Field redefinitions do not change physics  
(must be analytic and include linear term)

Cannot remove non-analyticity w/  $H$  redef  
What about using redefs of  $h$ ?

# Field Redefinitions of $h$

Let

$$\begin{aligned}\mathcal{L} &= \frac{1}{2} \left(1 + \frac{h}{2v}\right)^2 (\partial h)^2 + \frac{1}{2} (v+h)^2 \left(\frac{3}{4} + \frac{h}{4v}\right)^2 (\partial \vec{n})^2 - V \\ &= \frac{1}{4} \left(1 + \frac{\sqrt{2|H|^2}}{v} + \frac{|H|^2}{2v^2}\right) (\partial H)^2 \\ &\quad + \frac{1}{4v^2} \left(\frac{v}{\sqrt{2|H|^2}} + \frac{3}{4}\right) \frac{1}{2} (\partial |H|^2)^2 - \tilde{V}\end{aligned}$$

w/  $V = V(h)$   
 $V'(0) = 0$   
 $V$  analytic

Looks like no SMEFT expansion...

# Field Redefinitions of $h$

But let  $h_1 = h + \frac{1}{4\nu} h^2$  (no shift in min of  $V$ )

$$\Rightarrow \partial_\mu h_1 = \left(1 + \frac{h}{2\nu}\right) \partial_\mu h$$

and  $(\nu_1 + h_1)^2 = (\nu + h)^2 \left(\frac{3}{4} + \frac{h}{4\nu}\right)$   $\nu_1 = \frac{3}{4}\nu$

$$\begin{aligned} \Rightarrow \mathcal{I} &= \frac{1}{2} (\partial_\mu h_1)^2 + \frac{1}{2} (\nu_1 + h_1)^2 (\partial \vec{n})^2 + V \\ &= |\partial H_1|^2 + \tilde{V} \Rightarrow \text{SMEFT!} \end{aligned}$$

## Field Redefinitions of $h$

We learned that analytic field redefs of  $h$  can obscure analyticity in terms of  $H$ .

Field redefs within HEFT can obscure SMEFT

When should/must one use HEFT?

# Practical Criterion

One should match onto HEFT when integrating out a state whose mass is near (or below) the electroweak scale

Check radius of convergence for EFT expansion

# Practical Criterion

Radius of convergence:

$$\mathcal{I} \supset \sum_m \frac{C_m}{\Lambda^{2m-4}} |H|^{2m} \supset \lambda_h^2 h^3$$

$$= \sum_m \frac{2^{1-m}}{3} m(m-1)(2m-1) C_m \left(\frac{v}{\Lambda}\right)^{2m-4} v h^3$$

$\Rightarrow$  if  $\Lambda \sim v$  and  $C_m \sim 1$

$\Rightarrow$  SMEFT does not converge

# Applying the Practical Criterion

## Revisit singlet loop example

$$V^{1\text{-loop}} = \frac{1}{16\pi^2} (m^2 + \lambda |H|^2)^2 \left( \ln \frac{\Lambda^2}{m^2 + \lambda |H|^2} + \frac{3}{2} \right)$$

SMEFT: expand in  $\Sigma_{\text{SMEFT}} = \frac{\lambda |H|^2}{m^2}$

HEFT: expand in  $\Sigma_{\text{HEFT}} = \frac{\lambda (|H|^2 - \frac{1}{2} v^2)}{m^2 + \frac{1}{2} \lambda v^2}$

so that  $m^2 (1 + \Sigma_{\text{SMEFT}}) = m_{\text{phys}}^2 (1 + \Sigma_{\text{HEFT}})$



# Applying the Practical Criterion

$$V^{1\text{-loop}} = \frac{1}{16\pi^2} (m^2 + 2d|H|^2)^2 \left( \ln \frac{\Lambda_n^2}{m^2 + 2d|H|^2} + \frac{3}{2} \right)$$

$$V_{\text{SMEFT}} \supset \dots + \sum_{k=3}^{k_{\text{max}}} \frac{2(-1)^k}{k(k-1)(k-2)} \underline{\Sigma}_{\text{SMEFT}}^k$$

$$V_{\text{HEFT}} \supset \dots + \sum_{k=3}^{k_{\text{max}}} \frac{2(-1)^k}{k(k-1)(k-2)} \underline{\Sigma}_{\text{HEFT}}^k$$

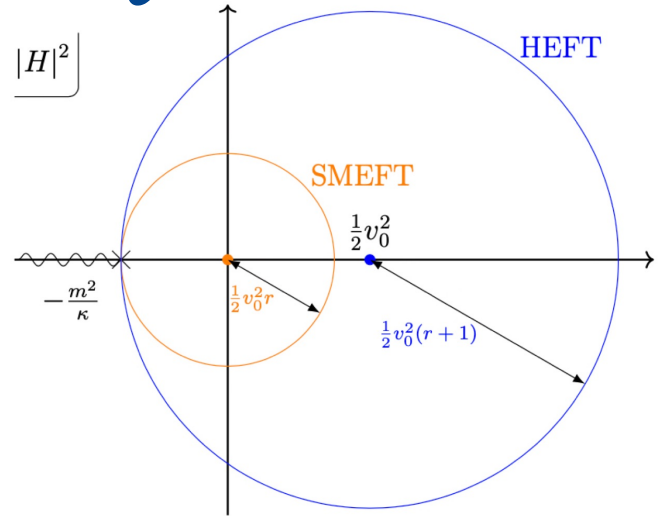
# Radius of Convergence

Define  $r = \frac{m^2}{2\alpha v^2/2}$

S.t.  $r \rightarrow \infty$  as  $m^2 \rightarrow \infty$

Then  $\Sigma_{\text{SMEFT}} \sim \frac{1}{r}$

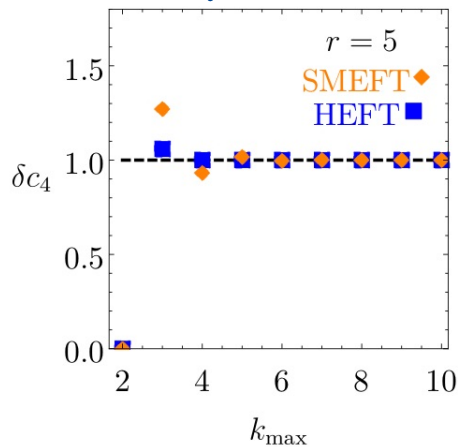
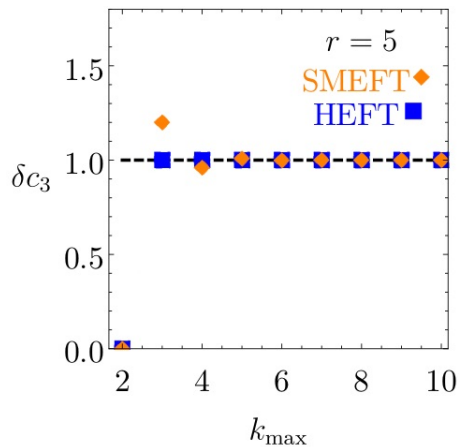
$\Sigma_{\text{HEFT}} \sim \frac{1}{r+1}$



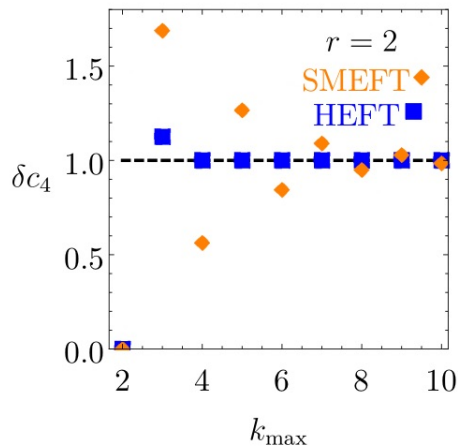
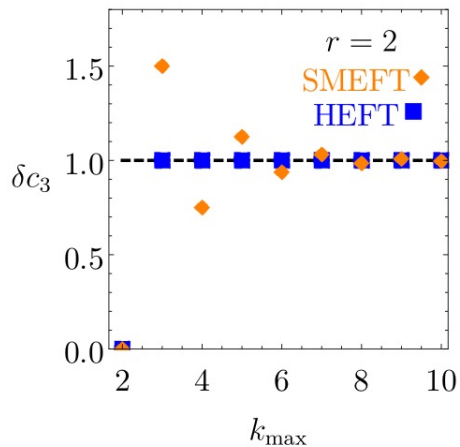
$r > 1$ : both converge  
 $0 < r \leq 1$ : SMEFT does not converge  
 $r = 0$ : SMEFT does not exist

# Rate of Convergence

Correction  
to  
 $h^3$



Correction  
to  
 $h^4$



# Curvature Invariants

We saw examples where particle gets all of its mass from Higgs  
 $\Rightarrow$  SMEFT does not exist

But could be obscured by field redef

Want basis independent check

Goal classify UV Theories that require HEFT

# Curvature Invariants

(AJM)

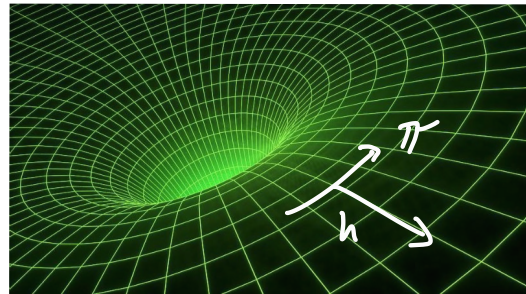
Analog w/ GR: define metric on moduli space

Note  $(\partial \vec{n})^2 = \left( \delta_{ij} + \frac{n_i n_j}{1 - n^2} \right) (\partial^\mu n_i) (\partial_\mu n_j)$

$$\Rightarrow \mathcal{I}_{\text{HEFT}} \supset \frac{1}{2} [\mathbb{K}(h)]^2 (\partial h)^2 + \frac{1}{2} [V F(h)]^2 (\partial \vec{n})^2$$

$\Rightarrow$  metric

$$g_{hh} = \mathbb{K}^2$$
$$g_{ij} = V F^2 \left( \delta_{ij} + \frac{n_i n_j}{1 - n^2} \right)$$



# Curvature Invariants

metric

$$g_{hh} = \mathbb{K}^2$$

$$g_{ij} = v F^2 \left( \delta_{ij} + \frac{n_i n_j}{1-n^2} \right)$$

Ricci scalar

$$R = - \frac{2 N_\varphi}{\mathbb{K}^2 F} \left[ (\partial_h^2 F) - (\partial_h \mathbb{K}) \left( \frac{1}{\mathbb{K}} \partial_h F \right) \right]$$

$$+ \frac{N_\varphi (N_\varphi - 1)}{v^2 F^2} \left[ 1 - \left( \frac{v}{\mathbb{K}} \partial_h F \right)^2 \right]$$

HEFT is a Black Hole

Conjecture: Checking finiteness of  $R$  &  $V$   
is sufficient.

Two classes of models need HEFT:

Conical singularity: BSM state gets  
all of its mass from  $H$

Horizon: BSM sources of symmetry  
breaking

# Conical Singularity

Ex: Singlet w/  $S^1/H^2 + S^3 \Rightarrow$  tree level

$$\Rightarrow R(h = -v) = \frac{a^2}{m^4} N_\varphi (N_\varphi + 1)$$

finite w/  $m^2 \neq 0$  but diverges as  $m^2 \rightarrow 0$

Ex: Singlet w/  $S^2/H^2 \Rightarrow$  loop level

$$\Rightarrow R(h = -v) = \frac{1}{192\pi^2} \frac{\lambda}{3m^2} N_\varphi (N_\varphi + 1)$$

but  $R|_{m^2=0} = \frac{N_\varphi(N_\varphi - 1)}{(v+h)^2} \frac{\lambda}{96\pi^2 + \lambda} \xrightarrow{h \rightarrow -v} \infty$

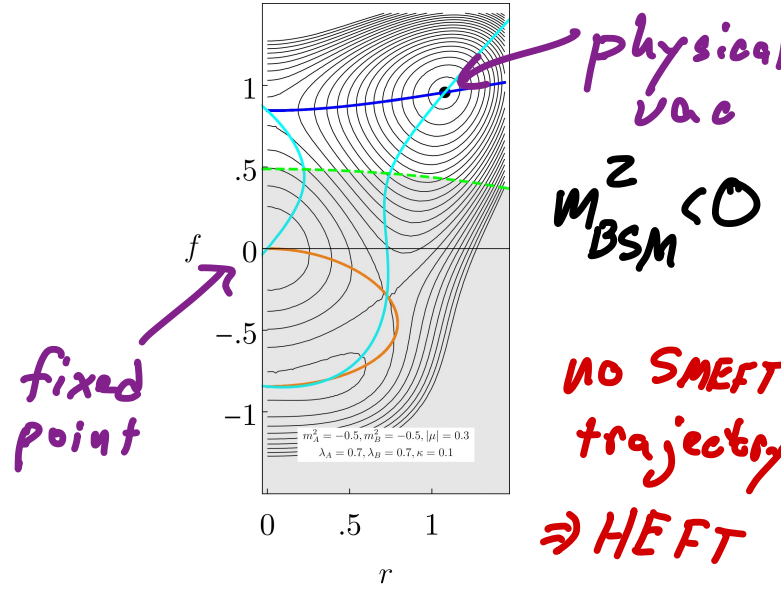
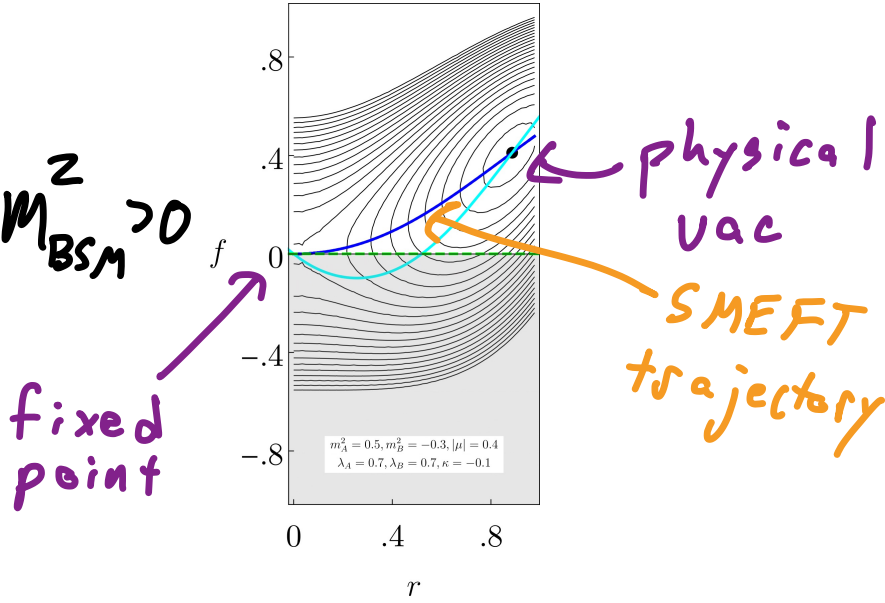


# Horizon (See our recent application to the 2HDM in arXiv:2304.09884)

We provide three examples in paper.

Rely on "EFT submanifold" picture

Ex: Abelian toy model w/ vevs  $f$  &  $r$



# Non-Decoupling New Particles

Does HEFT have experimentally viable UV completions? "Loryons"

We characterize all Loryons satisfying

- Color singlets have integer E/m charge
- Those possessing E/m charges decay promptly
- Fermionic Loryons introduced in pairs so we can write custodially symmetric Yukawa couplings

# Lorion Catalog

Notation

- Custodial irrep  $[L, R]_Y$
- SM charges  $(C, L)_Y \leftarrow U(1)_Y$   
 $\begin{matrix} \nearrow SU(3)_C & \nearrow SU(2)_L \end{matrix}$

$\nwarrow SU(2)_{L/R}$

Must get at least half their  
mass from the Higgs  $\Rightarrow$  Candidate  
UV completion of HEFT

# Loryon Catalog

## SCALARS

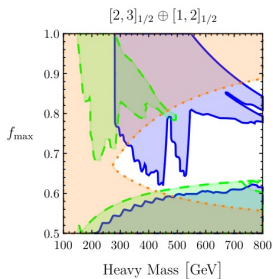
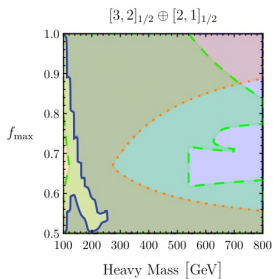
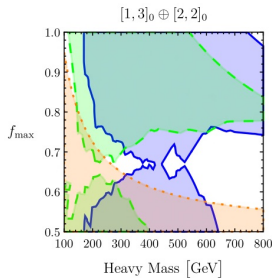
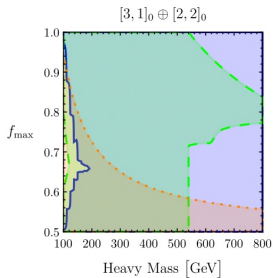
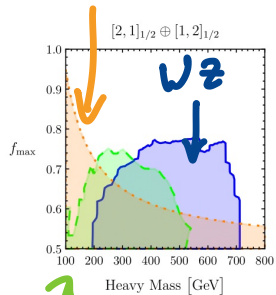
SM Reps	$(1, 1)_Y$	$(1, 2)_Y$	$(1, 3)_Y$	$(1, 4)_Y$	$(1, L)_Y$	$(3, 1)_Y$	$(3, 2)_Y$
Field	$S_Y$	$\Phi_{2Y}$	$\Xi_Y$	$\Theta_{2Y}$	$X_{L,Y}$	$\omega_{ 3Y }$	$\Pi_{ 6Y }$

	$R = 1$	2	3	4	5	6	7	8
$L = 1$	$ Y_{max}  = 3$	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$\times$
2	$\frac{7}{2}$	4	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	5
3	3	$\frac{7}{2}$	4	$\frac{9}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$
4	$\frac{7}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	5
5	3	$\frac{7}{2}$	4	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$
6	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{9}{2}$	4
7	3	$\frac{7}{2}$	3	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$
8	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$\times$	$\times$	$\times$	$\times$



# Fermionic Longyons

LEP



Open parameter space!

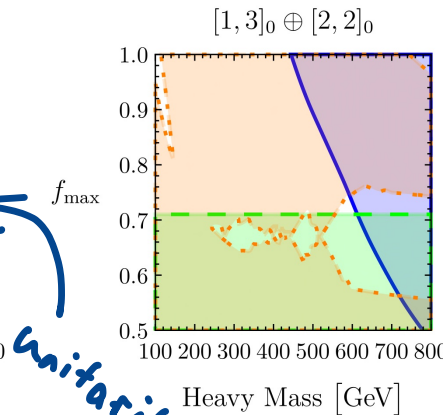
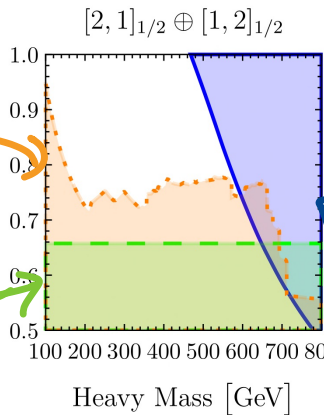
WW



Direct search

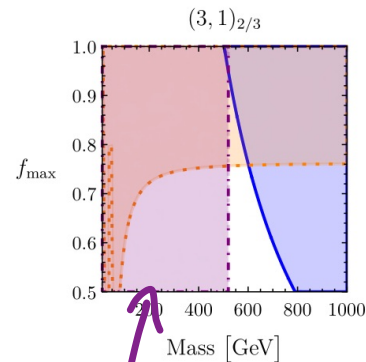
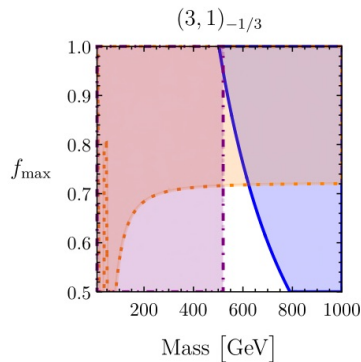
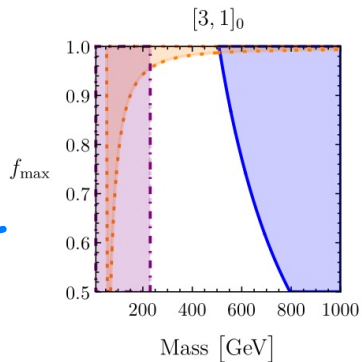
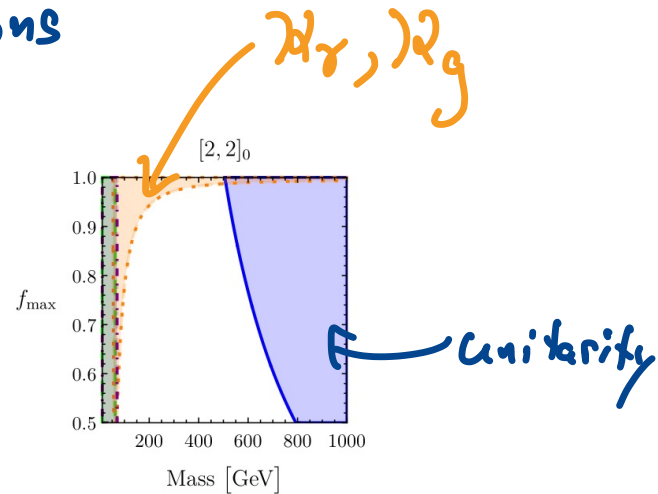
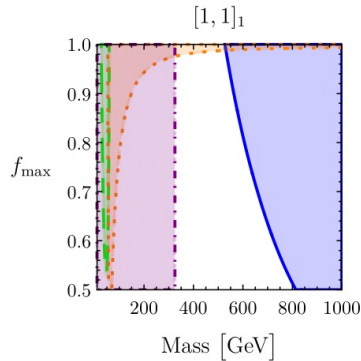
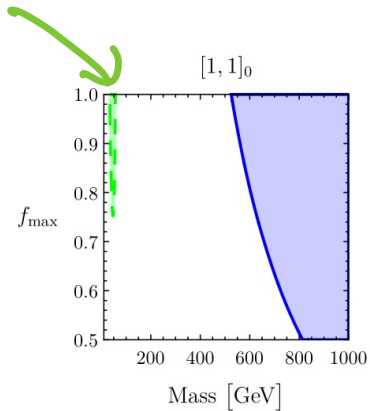
Precision EW

(Double use of colors ... 😬)



# Scalar Longons

Higgs decay



Open Parameter Space!

direct searches

# Future Prospects

- HL-LHC Higgs coupling improvements on  $\mathcal{R}_g$  and  $\mathcal{R}_\gamma$  rules out colored triplet scalars.
- Improved precision on  $\mathcal{R}_{Z\gamma}$  will also have impact

Rep	$[1, 1]_1$	$[3, 1]_0$	$[1, 3]_0$	$[2, 2]_0$	$[3, 3]_0$	$[4, 2]_0$	$[2, 4]_0$	$[2, 3]_{-1/2}$	$[2, 1]_{1/2} \oplus [1, 2]_{1/2}$	$[1, 3]_0 \oplus [2, 2]_0$
$\frac{\sum \eta_i C_i^{Z\gamma}}{\sum \eta_i Q_i^2}$	-0.12	.38	-0.12	.13	.13	.30	-0.032	-0.008	-0.019	-0.019

- Higgs wavefunction deviation has minimal impact

$$\mathcal{O}_H \equiv \frac{C_H}{\Lambda^2} \frac{1}{2} (\partial^\mu |H|^2)^2$$

projected constraint  $\frac{\Lambda}{\sqrt{C_H}} \sim 1.4 \text{ TeV}$

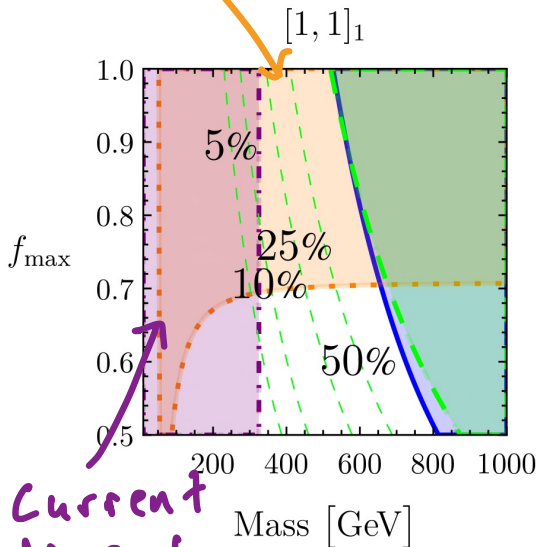


# Future Prospects

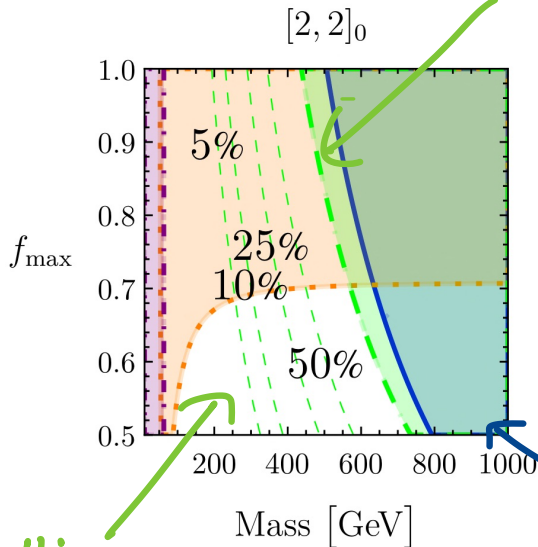
Higgs cubic coupling constraint helps

## Scalar Longons

$W\gamma + \gamma\gamma$   
@ HL-LHC



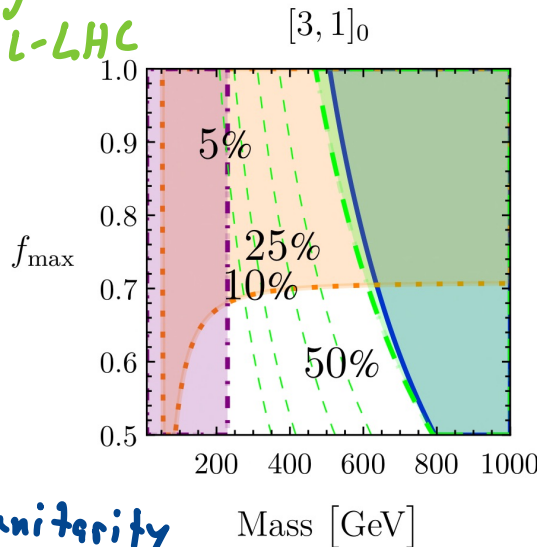
Current direct searches



Higgs Cubic Improvements

Higgs cubic HL-LHC

Unitarity



Can FCC determine if electroweak symmetry can be linearly realized around scales of  $O(m_h)$ ?

- Dedicated study needs to be done
- Probe remaining Loryon parameter space?
- Explore models with extra electroweak symmetry breaking (see our 2HDM example)
- Unitarity violation @  $4\pi v$  in Goldstone + Higgs scattering  
See TC, Craig, Lu, Sutherland [arXiv:2108.03240]