Quantum Gravity/Gravitization of the Quantum: Fundamental Questions and Challenges

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What is Quantum Gravity (QG)?

I will discuss various **theoretical approaches** to QG and list **fundamental questions and challenges** and **emphasize the need for empirical probes**. I will argue for "**gravitization of quantum theory**", with specific experimental signatures (**triple and higher-order interference**) at low energies. As an existing empirical probe I will discuss the **cosmological constant (perhaps the first measured quantum gravity phenomenon)** [arXiv:2202.06890 [hep-th]], [arXiv:2203.17137 [gr-qc]], [arXiv:2303.15645 [gr-qc]], [arXiv:2212.00901 [hep-th]], [arXiv:2212.06086 [hep-th]], [arXiv:2003.00318 [hep-th]], and references therein. (**Synergy between different approaches!**) (Work done with Freidel, Leigh (FLM), Kowalski-Glikman (FKLM), Berglund, Hübsch (BHM), Mattingly and Geraci.)

What is Quantum Gravity?

To **motivate the problem of quantum gravity** let us start with the fundamental constants ("magic cube of physics"): Newton's gravitational **G**, speed of light **c**, Planck constant $\hbar$.

1) **c** relates space and time - feature of spacetime "stuff" ("structure constant" of spacetime that converts from measurement of time to measurement of space) and of the causal structure in spacetime

2) **G** measures the response ("elasticity") of spacetime geometry to the presence of matter and makes causal structure dynamical (gravity tilts light cones)

3) $\hbar$ distinguishes between possible and actual (observed with some probability) histories of time evolution of matter degrees of freedom (also, atom of action - atomistic nature of matter)

What is Quantum Gravity?

Note the fundamental dichotomy: **spacetime is classical** (where events are observed), **matter is quantum** (atomistic).
(*Spacetime as "index set" for quantum matter.*)
Spacetime is a (dynamical) arena for physics; matter makes physics happen in spacetime.
Each fundamental constant defines an axis of the "cube of theories/frameworks".
(Eight fundamental theories/frameworks - six theories/frameworks realized so far.)

What is Quantum Gravity?

1) The origin of the cube - non-gravitational, non-relativistic, classical (non-quantum) physics (**Galilean physics**/Newtonian non-gravitational physics) - concepts of time and space, inertia assumed
2) **G** - **Newtonian gravity** (Newtonian physics with gravity) - deviation of Galilean geodesics (curvature in Newton-Cartan non-relativistic spacetime)

What is Quantum Gravity?

3) **c - (special) relativistic** field theory (like EM) - fields living in Minkowski spacetime with fixed causal structure (need inertial frames) - in the non-relativistic limit reduces to Galilean physics
4) **G, c - general relativity** (Einstein's theory of gravity) - GR (*dynamical spacetime - dynamical causal structure*) that in a non-relativistic limit gives the Newton-Cartan theory). The source - matter sector (which is fundamentally quantum):
$G_{ab} + \Lambda g_{ab} = 8\pi G T_{ab}$ (Λ - cosmological constant.)

What is Quantum Gravity?

5) $\hbar$ - **quantum mechanics** (non-relativistic) QM - theory of possibilities turned into actualities, that, in principle (Koopman-von-Neumann), reduces to its classical limit (needed for the definition of measurements?)

6) $\hbar, \mathbf{c}$ - **quantum field theory** (QFT - **relativity plus quantum theory** - *unitarity, locality, cluster decomposition*) - can be generalized to QFT in fixed but curved spacetime. Also the non-relativistic version of QFT is relevant for many body, statistical and condensed matter physics (CMP). (In QFT as we know it, **spacetime labels are classical, the fields are quantum** in the sense that they are "operator like", $\hat{\phi}(x)$, but they live on a classical spacetime x - flat or curved. Non-commutative QFT attempts to generalize that situation - possible issues with covariance.)

What is Quantum Gravity?

This is the fundamental physics we know.
1), 2), 3) and 4) are classical theories (one history out of many - via minimum action; not intrinsically probabilistic): $\delta S = 0$.
5) and 6) are quantum theories (sum over histories; intrinsically probabilistic; measurement problem?): $\int Dq e^{\frac{i}{\hbar} S(q)}$
In principle, classical theories emerge from their deeper quantum description ($\hbar \ll S$).

What is Quantum Gravity?

Empirically we see (currently) **NO departures from canonical QFT (including quantum mechanics) as well as GR (including special theory of relativity)**.
(*Is the discovery of dark energy (cosmological constant, Λ/vacuum energy) the first observable quantum gravity effect?*)
What is obviously missing in this list is a theory/framework (7, 8) defined with all three fundamental constants (and its non-relativistic contraction).

What is Quantum Gravity?

7) (and 8) $\mathbf{G}, \mathbf{c}, \hbar$ - **relativistic, quantum theory of gravity (QG)** and its non-relativistic limit/contraction $\mathbf{G}, \hbar$ - a non-relativistic quantum theory of gravity. (**QG - "under construction"**.)
This theory/framework should contain, in appropriate limits, QFT, quantum theory, classical field theory (like EM), Newtonian gravity and Galilean physics.
Cosmologically, QG represents the "original/initial" physics that gets diversified in various physical limits.
From this point of view, **everything in** the magic cube of **physics is a left-over of quantum gravity**: *Quantum spacetime, quantum matter, origin of space, time, inertia and the quantum!*

What is Quantum Gravity?

NB: Given the relevance of QFT for many body, condensed matter and statistical physics, it is natural to expect that the technology of quantum gravity will be important for problem of many body, condensed matter and statistical physics (such as complex systems?) that are beyond the reach of the techniques of QFT. (Example: AdS/CMP, but valid more generally.)

What is Quantum Gravity?

So what is that theory of quantum gravity (QG) and what phenomena does is describe? (History review - Rovelli)
Historically, the question was asked very early (Einstein mentions in quanta of gravitational waves - what we now call gravitons - in his early papers on gravitational waves), even before we understood how to quantize EM (QED). First thesis devoted to quantum gravity - M. P. Bronstein in the mid 1930s - already pointed out that quantizing gravity will be radically different than quantizing EM (or other fundamental fields). Many pioneering figures. Early quantum gravity conference, UNC, Chapel Hill, 1957.
Since then, thousands of papers, without any empirical motivation (vacuum energy?) (Many creative theoretical/mathematical ideas and techniques! Many relevant questions and puzzles raised.)

What is Quantum Gravity?

After the **singularity theorems** of Penrose, Hawking and others, at least a precise theoretical issue arose: *GR breaks down in black hole and cosmological singularities* - need QG (modified gravity?). Natural appearance of the Planck length $l_P = \sqrt{\hbar G/c^3}$ as the characteristic length (and Planck time t_P) as the natural scales for QG effects.
(What happens to matter at the spacetime singularity (equation of state, nature of matter) - a practical astrophysical question.
What happens to physics at the Big Bang singularity?)

What is Quantum Gravity?

However, as everyone learned in kindergarten:
$l_P = 10^{-35} m$, $t_P = l_P/c = 10^{-43} s$ and $E_P = \hbar/t_P = 10^{19} GeV$
Seems hopeless to probe empirically! (**Is QG irrelevant? NO!**)
Also black hole singularities are hidden by horizons (cosmic censorship) - not directly available for observation.
(There could be gravitational wave signals from the Big Bang (or inflationary phase, if such a phase really exists), or from the initial singularity. Cyclic cosmology - no gravitational waves...)

What is Quantum Gravity?

Similarly, since the development of of the modern (Wilsonian) picture of QFT (and effective field theory EFT) and in particular, since the development of **QFT in curved spacetime backgrounds** (especially with black holes) new motivations arose, such as the **black hole information puzzle** (*do pure states evolve into pure or mixed states? Is quantum theory universally valid?*). The issue of **gravitational thermodynamics** (*entropy of black holes, the meaning of black hole entropy, and the underlying degrees of freedom*; similarly for cosmological horizons) also became one of the motivating questions for quantum gravity.

What is Quantum Gravity?

Next, **the problem of vacuum energy (the cosmological constant, Λ, problem)** became one of the most important questions, especially after the discovery of the *accelerated expansion of the universe* (supernovae, CMB, structure) (as well as development of the **EFT techniques in cosmology** - think of inflationary models needed for *structure formation*, for example). Also, pre-Big Bang; cyclic models; loop cosmology.

What is Quantum Gravity?

The recent development of cosmology as a precision science motivated investigations of **dark energy (Λ?) and dark matter**, some in the context of quantum gravity (even though this was also a motivation for considering **theories of modified gravity**).
For many people (**string theory, asymptotic safety, twistors**, etc) apart from the invisible (but gravitating) - dark - sector, the problem of explaining the parameters in the **visible matter** sector (**masses of elementary particles** etc) also became part of the task of quantum gravity.

What is Quantum Gravity?

Finally, most recently, questions of **quantum foundations**, in the context of **quantum information science**, became part of the quantum gravity lore.
The **quantum measurement problem** is also a motivation for some researchers in quantum gravity. (LIGO/Virgo probes of the quantum domain.)
Interplay of quantum information, gravitational thermodynamics and quantum spacetime.

What is Quantum Gravity?

Possible practical/observable relevance of quantum gravity, or quantum theory of spacetime for the **resolution of the quantum measurement puzzle** - with possible empirical implications in everyday quantum physics, in all its realms.
The idea here is to resolve the **micro/(quantum) vs macro/(classical)** dichotomy that seems to be an inherent feature of quantum theory and the quantum measurement problem.

What is Quantum Gravity?

Still, *there are NO clear quantum gravity phenomena that we could speak of empirically, after all these years! (Λ/Vacuum energy?)*
However: Currently - perhaps the most exciting empirical times in gravitational physics! Hopes for experimental quantum gravity with clear observable phenomena!
Anyway, even without any direct empirical motivation, the theorists soldiered on!
Given the structure of the "magic cube" of physics theories/frameworks we can reach the QG triple point either from QFT by adding gravity of from the point of view of GR by adding "quantization".

What is Quantum Gravity?

If we start from **QFT (locality, unitarity, cluster decomposition)** in the context of gravity we get various approaches: *perturbative quantum gravity/effective field theory EFT approach, QFT in curved spacetime, asymptotic safety, supergravity, string theory, holography (AdS/CFT, dS/CFT, celestial, higher spin, edge modes/symmetries, tensor networks, spacetime/gravity from entanglement), Euclidean quantum gravity, modified/massive gravity, non-local theories of gravity, etc.*

What is Quantum Gravity?

If we start from the GR point by paying attention to GR features like **diffeo invariance, background independence, dynamical causal structure** etc (not part of the usual QFT game) and then quantize (maybe non-canonically - because what does it mean to quantize? surely, quantum physics was cosmologically before classical physics!) we get apparently different formulations: *canonical quantum gravity, quantum geometrodynamics/quantum cosmology, loop quantum gravity (Hamiltonian and covariant/spin foams), causal sets, dynamical/causal triangulations, random matrix models, group field theory, quantum graphity, topological/categorical quantum field theory (BF etc)...*

What is Quantum Gravity?

There are "more radical" approaches where one realizes that one needs to **quantize spacetime** in order to quantize gravity, and definitely spacetime is classical in the domain of all known quantum physics (quantum mechanics, QFT, relativistic or non-relativistic): *twistor theory, spacetime code, non-commutative geometry, causal sets, (metastring theory, gravitizing the quantum)...*
There is also **quantum gravity phenomenology** where one looks at possible deformations of the traditional spacetime symmetries and searches for experimental bounds on such deformations: (*doubly special relativity, relative locality, fractal spacetime, entropic gravity, general gravitational thermodynamics, etc*).

What is Quantum Gravity?

In a related vein, there are emerging approaches where one starts from a seemingly smaller set of symmetries and looks for **emergence** of covariant spacetime structures (*Horava-Lifshitz, analog gravity models, many-body models of emergent gravity, quantum graphity, various holographically inspired approaches*).

There are **numerical** approaches/**discrete** approaches (*lattice Einstein, lattice gauge theories of gravity, Regge calculus, (causal) dynamical triangulations*) based on simplicial/lattice techniques.

Modified classical gravity: *teleparallel,* $F(R)$, *tensor-vector-scalar, Weyl/conformal, susy-Einstein-Cartan, bimetric, shape dynamics...*

What is Quantum Gravity?

So, **there are at least two dozen (plus) approaches to QG**: *EFT approach, asymptotic safety, supergravity, string theory, holography (AdS/CFT and generalizations), Euclidean quantum gravity, topological (and categorical) quantum field theory, canonical quantum gravity, loop quantum gravity (Hamiltonian/spin networks and covariant/spin foams), causal sets, quantum cosmology, group field theory, emergent gravity in condensed matter, Regge calculus, (causal) dynamical triangulation, non-commutative geometry, twistor theory, Horava-Lifshitz, analog gravity models, modified/massive gravity, gauge theories of gravity, non-local theories of gravity, shape dynamics, entropic gravity, quantum gravity phenomenology, quantum graphity, gravitizing the quantum etc.*

What is Quantum Gravity?

Lots of ideas, lots of work/papers (and books), lots of hard-working people, but **NO existing clear quantum gravity phenomena** that would unify all these approaches! (Λ/**Vacuum energy?**) There are **thought experiments**, like the argument that gravity has to be quantized in order to reconcile Heisenberg uncertainty principle with minimal length, some **actual experiments** that probe quantum phenomena in the gravitational background (showing that the equivalence principle and quantum interference are empirically compatible - via neutron interferometry in a gravitational potential) and some **recent experimental proposals** to study non-classical (quantum) aspects of Newtonian interaction (like the non-classical entanglement of masses - long coherence times needed - 1-2 s or so).

What is Quantum Gravity?

But **NO explicit experimental puzzles! (Λ/Vacuum energy?)**
In spite of all this, I will talk about **quantum gravity as the center of fundamental (simple) physics!**
(Perhaps technically, or even conceptually, relevant for complex physics as well.)
Cosmologically quantum gravity comes first, and then quantum theory emerges (in the guise of QFT), and then the classical world of classical spacetime and GR, and its non-relativistic limits with a variety of complex phenomena at various temporal and spatial scales.

Question: *We talk about "quantization of gravity", but why don't we talk about "gravitization of quantum theory"?* (Penrose?)
I will argue that "quantum gravity" IS "gravitization of quantum theory" that implies the entire known physics structure (encapsulated by the magic cube of physics).
Thus, after the discovery of classical physics (including relativistic physics) and the discovery of quantum physics (including QFT), I expect that quantum gravity (with very specific phenomenology) should be the great third conceptual advance/revolution in physics!

What is Quantum Gravity?

Questions: Spacetime sector: *resolution of singularities (black hole and cosmological), quantum black holes (including BH entropy, information puzzle), astrophysics of the (resolved) BH singularity, cosmology of the (resolved) initial singularity, fine tuning of the initial state, early universe cosmology, relevance of quantum gravity for structure formation, quantum structure of spacetime and its phenomenology, the vacuum energy problem...*
Matter sector: *hierarchy of scales, origin of the Standard Models of particle physics and cosmology, masses and couplings of fundamental particles, dark matter and dark energy, baryogenesis...*,
Conceptual: *emergence of quantum theory, quantum/classical transition, quantum measurement problem, topology change, problem of time, CTCs, emergence of spacetime, inertia...*

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QG=Gravitization of Quantum Theory (GQ)

A tall order indeed! **To address these questions** we **need** theoretically **clear conceptual ideas** and methodology. Experimentally, we need **clear empirical phenomena**. (Relativity as a good example!) In what follows I will concentrate on a particular approach we call "gravitization of the quantum" (GQ), in which the fixed geometry of quantum theory is made dynamical. In the context of **QG=GQ** I will discuss specific empirical implications (Sorkin's triple interference etc). $QG = GQ$ synergizes with **string theory**, **loop quantum gravity** (via **quanta of spacetime**), **matrix models**, as well as **non-commutative geometry** and **twistor theory**, and the idea of **relative locality** and the program of **quantum gravity phenomenology** and **quantum foundations**. (Also, explains observed Λ.)

QG=Gravitization of Quantum Theory

Central intuition: **Quantum Relativity** in analogy with **Classical Relativity**

Classical relativity:

a) special relativity - motivated by EM - (*Minkowski spacetime/geometry, relativity of simultaneity*),

b) relativistic field theory (*reps of Lorentz/Poincare, particles/antiparticles*),

c) general relativity (*dynamical classical spacetime*)

Spacetime relativity - first (classical) relativity (both spacetime and matter classical).

QG=Gravitization of Quantum Theory

Quantum relativity: (FLM, '13, '14, '15, '16, '17)
A) QM from quantum spacetime (**modular spacetime, Born geometry, relative locality**),
B) QFT (**metafields/metaparticles**),
C) **gravitized quantum theory** (**dynamical quantum spacetime**, **dynamical Born geometry**, **metastrings**, metaparticles - dark matter, geometry of dual spacetime - dark energy)
QM/QFT - second (quantum) relativity (matter quantum, spacetime classical).
QG=GQ - third (QG) relativity (spacetime/matter quantum).
(Third relativity - Finkelstein; Wheeler)

QG=Gravitization of Quantum Theory

Main insight (FLM): suppose we define our physics on a lattice (lattice QFT). Continuum via Wilsonian RG (via path integral). Lattice is classical, physics is quantum. In quantum theory we need a **lattice (l) and a dual lattice ($\tilde{l}$)**! (as noticed by Zak)
Instead of considering the standard commutation relations between the position and momentum operators, $[q, p] = i\hbar$, take the generators of translations in *phase space*

$$\hat{U}_a = e^{\frac{i}{\hbar}\hat{p}a}, \quad \hat{V}_{\frac{2\pi\hbar}{a}} = e^{\frac{i}{\hbar}\hat{q}\frac{2\pi\hbar}{a}}, \quad \Longrightarrow [\hat{U}_a, \hat{V}_{\frac{2\pi\hbar}{a}}] = 0 \tag{1}$$

In terms of **modular variables** a la Aharonov et al,

$$[\hat{q}]_a \equiv \hat{q} \bmod a \quad [\hat{p}]_{\frac{2\pi\hbar}{a}} \equiv \hat{p} \bmod \frac{2\pi\hbar}{a} \quad \Longrightarrow [[\hat{q}]_a, [\hat{p}]_{\frac{2\pi\hbar}{a}}] = 0 \tag{2}$$

QG=Gravitization of Quantum Theory

Note that **modular variables are covariant** (modular energy, modular time as well).

Take fundamental length λ and energy ϵ, so that $\lambda\epsilon \equiv \hbar$. **Modular variables are non-local (but consistent with causality - origin of the uncertainty principle)**.

Contextuality: in a double slit experiment the parameters λ and ϵ are contextual to the experiment. This will be an important point when we discuss the cosmological constant problem! (FKLM)

QG=Gravitization of Quantum Theory

Explicit non-locality: Take $H = \frac{p^2}{2m} + V(q)$ and write the Heisenberg equation of motion for $e^{ipR/\hbar}$, or equivalently $[p]_R$ (R - contextuality parameter, such as the distance between two slits in the double slit interference experiment). (Aharonov et al)

$$\frac{d[p]_R}{dt} = ... \frac{V(q + R/2) - V(q - R/2)}{R} \tag{3}$$

Quantum mechanics=non-locality (from modular variables) plus causality (compatibility with Lorentz).
Now, reformulate quantum mechanics (QM) using (covariant) modular variables via modular spacetime. (**Quantum theory tells us something new about quantum spacetime!**)

QG=Gravitization of Quantum Theory

What is **modular space**? (FLM, '16)
Modular space is the space of all commuting subalgebras of the Heisenberg-Weyl algebra.
Note $[q, p] = i\hbar$ - Heiseinberg-Weyl algebra, whereas $[[q]_a, [p]_{2\pi\hbar/a}] = 0$ - commuting subalgebra of Weyl-Heisenberg.
Theorem of Mackey: the space of all commuting subalgebras of the Heisenberg-Weyl algebra is a self-dual phase space lattice lifted to Heisenberg-Weyl.
Use covariant modular variables - **modular spacetime** of d spacetime dimensions.

Note (FLM): **phase space - symplectic structure** $Sp(2d)$ - ω_{ab}.
Self-dual lattice (l plus $\tilde{l}$) - **doubly-orthogonal** $O(d,d)$ - η_{ab}.
To **define the vacuum** on this self-dual lattice - **need doubly metric structure** $O(2,2d-2)$ - H_{ab}.
ω, η, H **define Born geometry.** (FLM, '13, '14, '15, '16)
Their triple intersection gives the Lorentz group.
Thus **QM follows from non-locality (fundamental length/time) consistent with causality.**
Note: can be localized (local QFT possible!) in a particular phase space cell, but can't tell in which one (uncertainty principle)!

QG=Gravitization of Quantum Theory

How can fundamental length/time be consistent with Lorentz? (One of the main puzzles of QG.)
This is possible because of **relative (observer dependent) locality**. (Amelino-Camelia, Freidel, Kowalski-Glikman, Smolin)
Different observers see different spacetimes (slices of modular spacetime). **Different spacetimes are in linear superposition, and so fundamental length/time is consistent with Lorentz.**
(Similar to spin: the superposition of up and down spin gives the Bloch sphere which is consistent with rotation symmetry, even though spin is discrete.) (FLM)

QG=Gravitization of Quantum Theory

Generic quantum polarization (FLM, '16) - **modular polarization** (defined via the Zak transform). Given Schrödinger's $\psi_n(x)$

$$\psi_\lambda(x, \tilde{x}) \equiv \sqrt{\lambda} \sum_n e^{-2\pi i n \tilde{x}} \psi_n(\lambda(n + x)) \tag{4}$$

($x \equiv q/\lambda$, $\tilde{x} \equiv p/\epsilon$, so $[x, \tilde{x}] = i$, $\lambda\epsilon = \hbar$). Note, from the point of view of modular polarization, Schrödinger's polarization is very singular. Introduce $\mathbb{X}^A \equiv (x^a, \tilde{x}_a)^T$, so that $[\hat{\mathbb{X}}^a, \hat{\mathbb{X}}^b] = i\omega^{AB}$. We can write the translations operators in phase space covariantly $W_{\mathbb{K}} \equiv e^{2\pi i \omega(\mathbb{K}, \mathbb{X})}$, where $\mathbb{K}$ stands for the pair $(\tilde{k}, k)$ and $\omega(\mathbb{K}, \mathbb{K}') = k \cdot \tilde{k}' - \tilde{k} \cdot k'$. (Note W - **Aharonov-Bohm phases - prototypical example of modular variables**.)

QG=Gravitization of Quantum Theory

So far we have discussed covariant quantum phase space as an example of modular space, and so we are ready to discuss modular spacetime. Consider (FKLM, '18) a **metaparticle** (mp) propagating in a modular space defined by Born geometry - ω, η, H. The metaparticle world-line action $S_{mp} = \int d\tau L_{mp}$ (canonical particle - $\mu \to 0$ and $\tilde{p} \to 0$)

$$L_{mp} = p_\mu \dot{x}^\mu + \tilde{p}^\mu \dot{\tilde{x}}_\mu + \lambda^2 p_\mu \dot{\tilde{p}}^\mu - \frac{N}{2}\left(p_\mu p^\mu + \tilde{p}_\mu \tilde{p}^\mu - m^2\right) + \tilde{N}\left(p_\mu \tilde{p}^\mu - \mu\right), \tag{5}$$

where ω is in ("Berry-phase") $p_\mu \dot{\tilde{p}}^\mu$, and η in the diffeo constraint $p_\mu \tilde{p}^\mu = \mu$ and H in the Hamiltonian constraint $p_\mu p^\mu + \tilde{p}_\mu \tilde{p}^\mu = m^2$. **Dual spacetime** $\tilde{x}$, $[x, \tilde{x}] = i\lambda^2$, and **dual momentum space** $\tilde{p}$, $[p, \tilde{p}] = 0$. (Also, $[x, p] = i\hbar = [\tilde{x}, \tilde{p}]$.)

QG=Gravitization of Quantum Theory

The metaparticle can be understood also as follows: If one second quantizes Schrödinger's $\psi(x)$ one naturally ends up with a quantum field operator $\hat{\phi}(x)$. Similarly, the second quantization of the modular $\psi_\lambda(x,\tilde{x})$ would lead to a modular quantum field operator $\hat{\phi}_\lambda(x,\tilde{x})$ (**modular fields - metafields**)

$$\hat{\phi}(x) \to \hat{\phi}_\lambda(x,\tilde{x}). \tag{6}$$

with $[x,\tilde{x}] = i\lambda^2$ - covariant non-commutative field theory. (FLM, '17)
Classical spacetime label x of canonical QFT - choice of (classical spacetime) polarization in modular (quantum) spacetime with a contextuality parameter λ.

QG=Gravitization of Quantum Theory

Quanta of canonical quantum fields $\phi(x)$ - particles (and their antiparticles).

Quanta of modular quantum fields $\phi_\lambda(x, \tilde{x})$ - metaparticles.

First prediction of modular spacetime approach to quantum theory - metaparticles! (FKLM)

(We will argue that dual particles, correlated to visible particles, represent dark matter.)

Note that if we turn on backgrounds $p \to p + \phi$ and $\tilde{p} \to \tilde{p} + \tilde{\phi}$. Thus we have **"dark matter" fields** $\tilde{\phi}(x)$ in the effective classical spacetime x description (after integrating over the dual spacetime $\tilde{x}$).

QG=Gravitization of Quantum Theory

Note (FLM): *modular spacetime has double the dimension of spacetime*.
The modular cells are not simply connected (there is a unit flux through each cell - matter).
Modular wavefunctions are quasiperiodic.
Classical spacetime emerges from the process of **extensification** (imagine one unit length in dual direction and many, N, modular cells in the spacetime direction).
Spacetime emerges, in the large N limit, as a natural pointer basis in quantum theory. (Quantum measurement.)
Also, **spacetime and matter - "two sides of the same coin"**.
Cosmology: interplay between the visible and dual spacetimes.
(Singularities? Information paradox?...)

QG=Gravitization of Quantum Theory

Propagator for the metaparticle (FKLM, '18)

$$G(p,\tilde{p};p_i,\tilde{p}_i) \sim \delta^{(d)}(p-p_i)\delta^{(d)}(\tilde{p}-\tilde{p}_i)\frac{\delta(p\cdot\tilde{p}-\mu)}{p^2+\tilde{p}^2+m^2-i\varepsilon}. \quad (7)$$

Canonical particle: highly singular $\tilde{p}\to 0$ (and $\mu\to 0$) limit of this expression. Dispersion relation (in a particular gauge $\vec{\tilde{p}}=0$)

$$E_p^2+\frac{\mu^2}{E_p^2}=\vec{p}^2+m^2. \quad (8)$$

For each particle at energy E there exists a dual particle at energy $\frac{\mu}{E}$. (Analogous to the prediction of antiparticles in QFT.)
Dispersion relation - **quantum gravity phenomenology in the IR**! (FKLM, '21) Friedel-like static potential for metaparticles. Quasi-metaparticles (CMP).

QG=Gravitization of Quantum Theory

Dual "particles" (dual fields) - dark matter (to leading order in λ)

$$S_{eff} = -\int \sqrt{g(x)\tilde{g}(\tilde{x})}[R(x)+\tilde{R}(\tilde{x})+L_m(A(x,\tilde{x}))+\tilde{L}_{dm}(\tilde{A}(x,\tilde{x}))], \quad (9)$$

Here the A fields denote the usual Standard Model fields, and the $\tilde{A}$ are their duals, as predicted by the general (modular) formulation of quantum theory that is sensitive to the minimal length.
Note that we need to integrate over the dual space coordinates $\tilde{x}$ to get an effective description of **visible matter,** $A(x)$**, and dark matter,** $\tilde{A}(x)$, in classical x spacetime. (BHM, '21, '22)

QG=Gravitization of Quantum Theory

Dynamical geometry of dual spacetime - dark energy (to leading order in λ)

$$S_{eff} = -\int \sqrt{-g(x)}\sqrt{-\tilde{g}(\tilde{x})}[R(x) + \tilde{R}(\tilde{x}) + ...], \qquad (10)$$

In this leading limit, the $\tilde{x}$-integration in the first term defines the gravitational constant G_N, and in the second term produces a **positive cosmological constant constant**! (BHM, '21, '22)
In general, visible and dark matter degrees of freedom are correlated (via the minimal length λ) - origin of Milgrom's scaling (galaxies, clusters, superclusters) and Milgrom's acceleration $a_0 \sim cH/(2\pi)$. ($\Lambda \sim H^2$.)

QG=GQ - Explicit Realization

Explicit realization in terms of a chiral phase-space reformulation of the bosonic string, the "**metastring**," (FLM, '13, '14, '15) - also a non-perturbative proposal (BHM '21, '22) of QG (matrix model-like, time-asymmetric (?), $\partial_\sigma \cdot \equiv [\hat{\mathbb{X}}, \cdot]$, where $\hat{\mathbb{X}}$ matrix comes from modular world-sheet) - spacetime/matter quanta

$$S_{\rm str}^{\rm ch} = \int \mathrm{d}\tau \mathrm{d}\sigma \, \Big[\partial_\tau \mathbb{X}^a \big(\eta_{ab}(\mathbb{X}) + \omega_{ab}(\mathbb{X})\big) - \partial_\sigma \mathbb{X}^a H_{ab}(\mathbb{X})\Big] \partial_\sigma \mathbb{X}^b, \tag{11}$$

where $\mathbb{X}^a \equiv (X^a/\lambda, \tilde{X}_a/\lambda)^T$ are coordinates on phase-space like (doubled) target spacetime and η, H, ω are all dynamical. $x^a, \tilde{x}_a$ come from the left and right moving modes of the bosonic string,

$$x^a \equiv x_L^a + x_R^a, \quad \tilde{x}^a \equiv \tilde{x}_L^a - \tilde{x}_R^a \tag{12}$$

In the context of a flat metastring we have constant η_{ab}, H_{ab} and ω_{ab} (zero ω_{ab} - connection to double field theory)

$$\eta_{ab} = \begin{pmatrix} 0 & \delta \\ \delta^T & 0 \end{pmatrix}, \quad H_{ab} = \begin{pmatrix} h & 0 \\ 0 & h^{-1} \end{pmatrix}, \quad \omega_{ab} = \begin{pmatrix} 0 & \delta \\ -\delta^T & 0 \end{pmatrix}, \tag{13}$$

The standard Polyakov action is obtained when setting $\omega_{ab} = 0$ and integrating out the $\tilde{x}_a$,

$$S_P = \int \mathrm{d}\tau \mathrm{d}\sigma \gamma^{\alpha\beta} \partial_\alpha X^a \partial_\beta X^b \eta_{ab} + \dots \tag{14}$$

The triplet (ω, η, H) define the Born geometry (FLM, '13, '14) and the metastring propagates in a modular spacetime, a generic phase space like structure of quantum theory (FLM, '16). QFT in modular spacetime - **intrinsically non-commutative**. The *space* of commuting subalgebras of the Heisenberg algebra, $[\hat{x}, \hat{\tilde{x}}] = i\lambda^2$, becomes modular spacetime (FLM, '15, '16)

The new feature in the metastring formulation of the bosonic string is intrinsic non-commutativity and so there is a new Heisenberg algebra (vertex operators reps of Weyl-Heisenberg - no cocycles)

$$[\mathbb{X}^a, \mathbb{X}^b] = i l_s^2 \omega^{ab} \implies [X^a, \tilde{X}^b] = i\delta^{ab} l_s^2 \tag{15}$$

in addition to the standard ones, with $\Pi_b, \tilde{\Pi}_b$ the respective conjugate momenta to $X^a, \tilde{X}^a$,

$$[\mathbb{X}^a, \mathbb{P}_b] = i\hbar\delta^a_b \implies [X^a, \Pi_b] = i\delta^a_b\hbar, \quad [\tilde{X}^a, \tilde{\Pi}_b] = i\delta^a_b\hbar \tag{16}$$

Note, if Kalb-Ramond B_{ab} (axion) constant but non-zero, dual coordinates do not commute! In general - non-associativity (FLM,'17.) **Zero modes of the metastring - metaparticles** (FKLM, '18, '21) - little rigid strings. (Each Standard Model particle has a correlated "dual" - dark matter pheno (BHM, '20, '21).)

QG=GQ - Explicit Realization

The metastring has **dynamical Born geometry**, (FLM, '14, '15) $\omega_{ab}(\mathbb{X}), \eta_{ab}(\mathbb{X}), H_{ab}(\mathbb{X})$, but Born geometry is the geometry of the modular spacetime formulation of quantum theory.
Thus **by making Born geometry dynamical we can "gravitize quantum theory"** (that is, **make the geometry of quantum theory dynamical**)! (FLM)
Also, metastring is a theory of quantum gravity, and so we arrive at "QG = gravitized quantum theory".
This reasoning is "top-down".
(Note, classical GR gravitizes all of classical physics!)

QG=GQ - Explicit Realization

There is a "bottom-up" reason for "gravitization of quantum theory". (Minic, Tze)

Geometry of quantum theory (review by Ashtekar and Schilling) - maximally symmetric geometry of complex projective spaces (symplectic structure, compatible with the metric structure - the Born rule, and the product of the symplectic and metric structure gives complex structure - responsible for interference).

Quantum clock relates the Born rule (the Fubini-Study metric of complex projective spaces) to infinitesimal time (Aharonov and Anandan)

$$2\hbar ds_{FS} = \Delta E dt \tag{17}$$

where ΔE is the dispersion of energy defined by the Hamiltonian associated with the atomic clock.

QG=GQ - Explicit Realization

In the presence of quantum spacetimes (topology change) - no unique timelike Killing vector - and thus ΔE is state dependent which makes the geometry state dependent, and thus, dynamical. (Minic, Tze)

So - for quantum spacetimes - we should expect "gravitized quantum theory". That is, **the geometry of quantum theory is dynamical**. In general - topology change. (Thus the Bloch sphere becomes a Riemann surface of an arbitrary genus.)

In general, not a single Hilbert space, but **observer dependent Hilbert spaces**. Thus, dynamical Born rule and generalized kinematics with new experimental signatures (Sorkin's triple and higher order quantum interference). New observables - beyond the S-matrix etc. (Single Hilbert space = Born rule. Canonical observables, like the S-matrix.)

QG=GQ - Explicit Realization

What is the first experimental consequence of "gravitized quantum theory"? (Beglund, Geraci, Hübsch, Mattingly, Minic)

Triple and higher-order interference! (Experiment possible in the next few years.)

Note - the canonical quantum theory does not have intrinsic triple quantum interference (consequence of the Born rule and the fixed geometry of the complex projective space).

Current experimental bounds (photonic) - rather weak (10^{-3}).

Neutrino bounds expected to be surprisingly similar (and to be measured at JUNO)(Huber, Minakata, Minic, Pestes, Takeuchi).

QG=GQ - Explicit Realization

In more detail (Sorkin): Classically, we have addition of probabilities

$$P_n(A, B, C, \cdots) \;=\; P_1(A) + P_1(B) + P_1(C) + \cdots \;, \tag{18}$$

for any number of paths. Quantum mechanically, we have for two paths $P_2(A, B) = |\psi_A + \psi_B|^2$ or more explicitly

$$|\psi_A|^2 + |\psi_B|^2 + (\psi_A^*\psi_B + \psi_B^*\psi_A) \equiv P_1(A) + P_1(B) + I_2(A, B) \tag{19}$$

where the last term

$$I_2(A, B) = P_2(A, B) - P_1(A) - P_1(B) \tag{20}$$

is the “interference” of the two paths A and B. **Non-vanishing double-path interference, $I_2(A, B) \neq 0$, distinguishes quantum theory from the classical one.**

QG=GQ - Explicit Realization

The **Born rule** dictates that all the superimposed paths only interfere with each other in a pairwise manner. For instance, for three paths we have $P_3(A,B,C) = |\psi_A + \psi_B + \psi_C|^2$

$$P_2(A,B) + P_2(B,C) + P_2(C,A) - P_1(A) - P_1(B) - P_1(C), \qquad (21)$$

where only pairwise interferences between the pairs (A,B), (B,C), and (C,A) appear.
It is clear from the above that in order for there to be a non-linear correction in an interference pattern the Born rule must be relaxed.

Consider **a triple slit experiment**: Since only pairwise interferences between the pairs (A, B), (B, C), and (C, A) appear, it makes sense to define any deviation from this relation as the intrinsic triple-path interference $I_3(A, B, C)$ (Sorkin)

$$P_3(A,B,C)-P_2(A,B)-P_2(B,C)-P_2(C,A)+P_1(A)+P_1(B)+P_1(C). \tag{22}$$

(This can be easily generalized for the case of n-paths.) For both classical and quantum theory, this intrinsic triple-path interference is zero for any triplet of paths. **Experimental confirmation of** $I_3 = 0$ **would be a confirmation of the Born rule.**
Weak bounds were placed on the parameter ($\kappa \sim 10^{-3}$) in photonic experiments

$$\kappa = \frac{\varepsilon}{\delta}, \quad \varepsilon = I_3(A,B,C), \quad \delta = |I_2(A,B)| + |I_2(B,C)| + |I_2(C,A)|. \tag{23}$$

The claim of (Berglund, Geraci, Hubsch, Mattingly, Minic) is that **with quantum gravitational degrees of freedom turned on, one can get $I_3 \neq 0$,** but for that one needs gravitized quantum theory, with **observer dependent Hilbert spaces and dynamical Born rule**. Inspired by metastring theory, the generalized probability in this approach to quantum gravity is given by

$$P = g_{ab}(\psi)\,\psi_a\psi_b \equiv \delta_{ab}\,\psi_a\psi_b + \gamma_{abc}\,\psi_a\psi_b\psi_c + \ldots, \qquad (24)$$

where a, b, c are state-space indices and with (schematically)

$$\frac{\mathrm{d}\psi_a}{\mathrm{d}\tau} = \Gamma_{abc}\,\psi_b\psi_c, \qquad (25)$$

where τ is the appropriate evolution parameter. (Here Γ_{abc} is such that one has Schrodinger's evolution for a fixed Hilbert space.)

QG=GQ - Explicit Realization

Effective triple interference - possible in non-linear optical media! (Instead of ψ, non-linear waves; instead of probability P - non-linear/cubic energy density.)
Thus intrinsic triple interference with quantum gravity degrees of freedom - analogous to a non-linear "quantum spacetime medium". (*Note: non-linear quantum theory with fixed Hilbert spaces is NOT GQ!*)
Based on the discussion of the vacuum energy/cosmological constant (the first experimental QG effect?) in what follows we argue for low energy scales: $10^{-4}m$ **or** $10^{-19}m$**.**
Finally, we discuss the possible first experimental QG effect - the observed vacuum energy/cosmological constant.

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The CC Problem in QFT

Cosmological constant Λ (a parameter in Einstein's eqs $G_{\mu\nu} + \Lambda_{cc} g_{\mu\nu} = 8\pi G T_{\mu\nu}$) has been measured (supernovae, CMB, large scale structure). It corresponds to the (quantum) vacuum energy $\Lambda_{cc}/(8\pi G) \sim (10^{-3} eV)^4$. The natural Planckian value is 10^{124} times off $(10^{19} GeV)^4$ - **the cosmological constant problem**.

The measurement of Λ_{cc} IS the first QG observation!

Let us start with the QFT vacuum partition function (free scalar):

$$Z_{vac} = \int D\phi e^{-\int \frac{1}{2}\phi(-\partial^2+m^2)\phi} = \sqrt{\frac{\#}{\det(-\partial^2+m^2)}} \qquad (26)$$

which we can rewrite as

$$Z_{vac} = e^{-\frac{1}{2}\mathrm{Tr}\log(-\partial^2+m^2)} \qquad (27)$$

The CC Problem in QFT

In momentum space, $-\partial^2 = k^2$, and also

$$-\frac{1}{2}\log(k^2+m^2) = \int \frac{\mathrm{d}l}{2l} e^{-(k^2+m^2)l/2} \tag{28}$$

where the Schwinger parameter l is a worldline parameter associated with a **particle (quantum of the field ϕ)**.

The CC Problem in QFT

Note that after taking the trace we have

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \log(k^2 + m^2) = \int \frac{\mathrm{d}^{D-1} k}{(2\pi)^{D-1}} \frac{\omega_k}{2} \tag{29}$$

because

$$\int \frac{\mathrm{d}l}{2l} \int \frac{\mathrm{d}k^0}{2\pi} e^{-(k^2+m^2)l/2} = \frac{\omega_k}{2} \tag{30}$$

where $\omega_k^2 = k^2 + m^2$ with ω_k equivalent to k_0 on-shell.

The CC Problem in QFT

Thus, vacuum energy density in D spacetime dimensions becomes

$$\rho_0 = \int \frac{\mathrm{d}^{D-1}k}{(2\pi)^{D-1}} \frac{\omega_k}{2} \sim \Lambda_D \tag{31}$$

with Λ_D the volume of energy-momentum space.
This is a divergent expression that leads to the cosmological constant problem. (Weinberg's classic review)
The cosmological constant in 4d is (Einstein's equations) $\Lambda_{cc} \sim \rho_0 G_N \sim \rho_0 l_P^2$.

The CC Problem in QFT

Note that the **vacuum partition function** is also

$$Z_{vac} = \langle 0 | e^{-iH\tau} | 0 \rangle = e^{-i\rho_0 V_D} \tag{32}$$

where V_D is the volume of D-dimensional spacetime, and ρ_o is the vacuum energy density.
Furthermore $Z_{vac} = exp(Z_{S^1})$ where Z_{S^1} is the partition function on S^1 in the world-line formulation

$$Z_{s^1} = V_D \int \frac{\mathrm{d}^D k}{(2\pi)^D} \int \frac{\mathrm{d}l}{2l} e^{-(k^2+m^2)\frac{l}{2}} \tag{33}$$

Thus the **vacuum energy density** is given by (scaling as before)

$$\rho_0 = \frac{iZ_{S^1}}{V_d} \sim \Lambda_D \tag{34}$$

The CC Problem in QG/String Theory

For the case of a **bosonic string**, instead of one particle we have an infinite tower of particles with mass spectrum (Polchinski '86; Polchinski's String theory book) - graviton $(h = 1, \bar{h} = 1)$

$$m^2 = \frac{2}{\alpha'}(h + \bar{h} - 2) \tag{35}$$

Thus, summing over the physical string states

$$\sum_{\text{p.s}} Z_{S^1} = \sum_{h,\bar{h}} V_D \int \frac{\mathrm{d}l(2\pi l)^{-D/2}}{2l} \int \frac{\mathrm{d}\theta}{2\pi} e^{i(h-\bar{h})\theta} e^{-\frac{2}{\alpha'}(h+\bar{h}-2)\frac{l}{2}} \tag{36}$$

where we have imposed the level matching $h = \bar{h}$ (or $\delta_{h,\bar{h}}$).

The CC Problem in QG/String Theory

Define $\tau = \theta + i\frac{l}{\alpha'} \equiv \tau_1 + i\tau_2$.
We get the partition function of a bosonic string on T^2

$$Z_{T^2} = V_D \int \frac{\mathrm{d}\tau \mathrm{d}\bar{\tau}}{2\tau_2} (4\pi^2 \alpha' \tau_2)^{-D/2} \sum_h q^{h-1} \bar{q}^{\bar{h}-1} \qquad (37)$$

where $q \equiv e^{2\pi i \tau}$. This can be derived directly from the Polyakov path integral.

The CC Problem in QG/String Theory

Note that we can rewrite, with $l \equiv \alpha' \tau_2$,

$$(4\pi^2 \alpha' \tau_2)^{-D/2} = \int \frac{\mathrm{d}^D k}{(2\pi)^D} e^{-k^2 \frac{l}{2}} \tag{38}$$

Thus, as in QFT we can write $Z_{T^2} \equiv V_D \int \frac{\mathrm{d}^D k}{(2\pi)^D} f(k^2) \sim V_D \Lambda_D$ with

$$\Lambda_D \equiv \int \frac{\mathrm{d}^D k}{(2\pi)^D}; \quad f(k^2) \equiv \int_F \frac{\mathrm{d}^2 \tau}{2\tau_2} e^{-k^2 \alpha' \tau_2 / 2} \sum_h q^{h-1} \bar{q}^{h-1} \tag{39}$$

where F is the fundamental domain. Note that $f(k^2)$ is dimensionless, so it does not contribute to the scaling of Z_{T^2} and the vacuum energy $\rho_0 \sim Z_{T^2} / V_D$.

The CC Problem in QG/String Theory

The only difference is that in QFT the region of integration is

$$|\tau_1| < \frac{1}{2}, \quad \tau_2 > 0. \tag{40}$$

In string theory, because of modular invariance,

$$|\tau_1| < \frac{1}{2}, \quad |\tau| > 1. \tag{41}$$

So, the cosmological constant is UV finite(!) in string theory, but still $\rho_0 \sim Z_{T^2}/V_D \sim \Lambda_D$ (!) - **the cc problem persists in string theory**.

Unbroken SUSY - flat space or AdS. Broken SUSY - still $\rho_0 \sim \Lambda_D$.

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Quantum Gravity, Phase Space and Observation

Following FKLM [2212.00901], [2303.17495] (see also BHM, [2212.06086]) we return to Z_{S^1}, setting $m = 0$ and with p denoting the momentum,

$$Z_{S^1} = V_D \int \frac{\mathrm{d}\tau}{2\tau} \int \frac{\mathrm{d}^D p}{(2\pi)^D} e^{-\frac{p^2\tau}{2}} \tag{42}$$

But the spacetime volume is given by $V_D = \int \mathrm{d}^D q$. We therefore consider the **phase space** expression

$$Z_{S^1} = \int \frac{\mathrm{d}\tau}{2\tau} Z(\tau), \quad Z(\tau) = \int \frac{\mathrm{d}^D q}{(2\pi)^D} \int \mathrm{d}^D p e^{-\frac{p^2\tau}{2}} \equiv \mathrm{Tr} e^{-\frac{p^2\tau}{2}} \tag{43}$$

where Tr is in *phase space*.

In $D = 4$ we then get

$$Z(\tau) = \prod_{i=1}^{4} \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}q_i \int_{-\infty}^{\infty} \mathrm{d}p_i e^{-\frac{p_i^2 \tau}{2}} \tag{44}$$

or by discretizing phase space

$$Z(\tau) = \left(\frac{\lambda\epsilon}{2\pi} \sum_{k,\tilde{k} \in \mathbf{Z}} \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}\tilde{x} e^{-\frac{(\tilde{x}+k)^2 \epsilon^2 \tau}{2}} \right)^4 \tag{45}$$

where $p \to \epsilon\tilde{x}, q \to \lambda x$ with $\lambda\epsilon = \hbar$.

This is divergent but restrict the sum to finite range, **modular regularization** (Following papers on modular polarization in quantum theory and modular spacetime by FLM, '15, '16)

$$Z(\tau) = \left(\frac{\lambda\epsilon}{2\pi} \sum_{k=0}^{N_q-1} \sum_{\tilde{k}=0}^{N_p-1} \int_0^1 \mathrm{d}x \mathrm{d}\tilde{x} e^{-\frac{(k+\tilde{x})^2 \epsilon^2 \tau}{2}} \right)^4 \tag{46}$$

where N_q, N_p count the number of unit cells in the spacetime and momentum space dimensions, respectively.
Now, define

$$l \equiv N_q \lambda, \quad \text{and} \quad \Lambda \equiv N_p \epsilon \tag{47}$$

where then $l^4 \equiv V_4$ is the size (volume) of spacetime and Λ^4 is the size (volume) of energy-momentum space, and $N = (N_p N_q)^4 \in \mathbf{Z}$.

Thus,

$$l^4\Lambda^4 = N, \quad \text{or} \quad \Lambda^4 = \frac{N}{l^4} \tag{48}$$

But there is actually an **upper bound on** $\rho_0 \sim \Lambda^4 \leq \frac{N}{l^4}$ in $D = 4$ due to $exp(-p^2\tau/2) \leq 1$.
The same bound also holds in string theory following our earlier calculation of the partition function of the bosonic string on T^2 in $D = 4$. (*The same bound holds in QFT; cosmological phase transitions described by an effective potential included.)* (FKLM)

$$\rho_0 \leq \frac{N}{l^4} \tag{49}$$

Quantum Gravity, Phase Space and Observation

We now consider the **Bekenstein bound** (**holography!**) in a space with a cosmological horizon, ie assuming that the cosmological constant is positive and we have a dS spacetime. (*This is a feature of semiclassical gravity, and also of gravitational thermodynamics.*) In static coordinates, dS spacetime metric is

$$\mathrm{d}s_{dS}^2 = -(1 - \frac{r^2}{r_{CH}^2})dt^2 + \frac{dr^2}{(1 - \frac{r^2}{r_{CH}^2})} + r^2 \mathrm{d}\omega_{S^2}^2 \qquad (50)$$

where $l \equiv r_{CH}$, the cosmological horizon, is the size of the observed spacetime.

By identifying the above quantum number N with the gravitational entropy, the **Bekenstein bound ($S_{grav} = l_P^{-2} Area$)** becomes

$$N \leq \frac{l^2}{l_P^2} \tag{51}$$

(*Experimental consequence for black holes - gravitational wave "echoes" - in the quantum chaos phase, because N is large - quantum scars!*)
Combine the Bekenstein bound with the bound on ρ_0 ($\rho_0 \leq N/l^4$)

$$\rho_0 \leq \frac{1}{l^2 l_P^2} \tag{52}$$

and hence a **mixing of the UV (l_P) and the IR (l) scales**.

With the cosmological constant in $D = 4$ dimensions, $\Lambda_{c.c} = \rho_0 l_P^2$ we then get the bound

$$\Lambda_{c.c} \leq \frac{1}{l^2} \tag{53}$$

Thus, the natural energy scale, $\epsilon_{c.c}$ associated with the vacuum energy density,

$$\rho_0 = \epsilon_{c.c}^4 \sim \frac{1}{l^2 l_p^2} \tag{54}$$

the corresponding natural length scale, $l_{c.c} \simeq 1/\epsilon_{c.c}$, we get the **see-saw formula** (FKLM)

$$l_{c.c} \simeq \sqrt{l\, l_p} \tag{55}$$

Quantum Gravity, Phase Space and Observation

Note:

- with $l \sim 10^{28}m$ we get $l_{c.c} \simeq 10^{-4}$ m or 10^{-3} eV in agreement with observations!
- natural with $\rho_0 \to 0$ when $l \to \infty$, and l is the IR scale
- radiativelly stable since no UV dependence
- the cc is small because the universe is filled with stuff (large number of degrees of freedom (dof) $N \sim 10^{124}$)
- N is large because fluctuations scale as $\frac{1}{\sqrt{N}}$ - stability of the universe (Schrödinger's argument "Why are atoms small?")
- N_i (where i is t, x, y, z) is $N^{1/4} \sim 10^{31}$ (not so unreasonable if we recall the Avogadro number 10^{23}, for matter dof).

Note further the **contextuality** of the argument: the measurement of a quantum observable depends on which commuting set of observable are within the same measurement set of observable, ie quantum measurements depend on the *context*!

- First, ϵ is NOT a cut-off, as ϵ and λ can be arbitrary, though have to satisfy $\lambda\epsilon = \hbar$
- Second, ϵ^4 is replaced by N, which is the new quantum number, and the size of spacetime, $l = r_{CH}$
- N is determined by the Bekenstein bound–N is related to l and l_P, which is where gravity enters via $G_N \sim l_P^2$

Effective field theory (EFT) does not "see" N and in particular does NOT know about the Bekenstein bound. Vacuum energy cancels in the computation of EFT correlation functions. Also, EFT lives in classical spacetime.

We can repeat this argument for the modular space of the second Heisenberg algebra of the metastring and apply the logic of the cosmological constant computation to the relation between the Higgs mass and the cosmological constant (ξ and $\langle \mathcal{X} \rangle$ are defined in Abel & Dienes, and they are of order 1)

$$m_H^2 = \frac{\xi \Lambda^4}{M_P^2} - \frac{g_s^2 M_s^2}{8\pi^2} \langle \mathcal{X} \rangle, \tag{56}$$

We get a **seesaw formula for the Higgs mass** (BHM, '22)

$$m_H \sim \sqrt{M_\Lambda M_P} \sqrt{\frac{\langle \mathcal{X} \rangle}{8\pi^2}} \tag{57}$$

where M_Λ is the vacuum energy scale (10^{-3} eV) and M_P is the Planck energy (10^{19} GeV). The observed Higgs mass ($125\, GeV \to 10^{-19} m$) - if $\sqrt{\frac{\langle \mathcal{X} \rangle}{8\pi^2}} \sim 10^{-2}$

Quantum Gravity, Phase Space and Observation

The upshot of the two calculations: the calculation of the observed vacuum energy (cosmological constant) and the observed mass of the Higgs, is that these two *low energy (IR) scales* do know about quantum gravity.
Thus, in the discussions of triple (and higher order) interference we have mentioned that the scales of $10^{-4}m$ **and** $10^{-19}m$ should be taken as **natural scales of quantum gravity phenomenology**.
Is $I_3 \neq 0$ *in the context of QG=GQ at these scales?*
Experiment possible in the next few years!

Quantum Gravity, Phase Space and Observation

Extend this logic to the observed **masses of quarks and leptons** (Berglund, Hubsch, Minic).
Note, for matter degrees of freedom, entropy extensive l^3/l_{BZ}^3, with l_{BZ} - **Bjorken-Zeldovich scale**
Equate $N \sim l^2/l_P^2$ **to** l^3/l_{BZ}^3**, and find that** $l_{BZ}^3 \sim l l_P^2$**.**
$l_{BZ} \sim 10^{-14} m \to m_{BZ} \sim 10 MeV$. (Note: QCD scale $\sim 300 MeV$.)
All observed masses of quarks and leptons: seesaw-like expressions (like the ones for CC and the Higgs mass) involving m_{BZ} (Bjorken; Oxford twistor group). For example:
$m_c \sim \sqrt{m_{BZ} m_t}$, $m_s \sim \sqrt{m_{BZ} m_b}$, $m_\mu \sim \sqrt{m_{BZ} m_\tau}$, $m_e \sim m_{BZ}^2/m_\mu$,
Can be extended to neutrino masses.

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Outlook

Phenomenological Implications of QG=GQ and future work

- Cosmological constant (cc) as the first empirical quantum gravity phenomenon
- Metaparticles (zero modes of the metastring) and dark matter (entangled/correlated SM/dual (DM) particles) - probes?
- Dark energy (cc) as the curvature of the dual spacetime (naturally small by the above argument) - probes?
- Gravitational wave "echoes" - experimental probe of $N \sim l^2/l_P^2$ for black holes (quantum chaos - quantum scars)
- Dynamical Born geometry, "gravitizing the quantum," look at triple (and higher order) quantum interference (Sorkin) in the presence of gravity
- See-saw formulae for the CC and the Higgs mass and SM fermion masses (Bjorken/Oxford group).

Parting words

- Many theoretical approaches to quantum gravity
- Need synergy between different approaches
- Experimental input crucial!
- Exciting times for gravity (and quantum gravity) research!
- Join the fun and stay tuned!