QUANTUM INFORMATION TOOLS AT THE INTERFACE BETWEEN QUANTUM THEORY AND GRAVITY

Flaminia Giacomini

ETH Zürich

Image credits: J. Palomino

Basics of Quantum Gravity
22-25 May 2023

ETH zürich

Swiss National Science Foundation

LECTURE 1

NONCLASSICAL SPACETIME WITH QUANTUM INFORMATION TOOLS

LECTURE 2

GRAVITATIONAL QUANTUM PHYSICS

LECTURE 3

QUANTUM CLOCKS AS PROBES OF NONCLASSICAL SPACETIME

LECTURE 4

QUANTUM REFERENCE FRAMES

LECTURE 1: INTRODUCTION

- What is nonclassical spacetime?
- Quantum interferometers
- Bell's theorem
- Generalized Probabilistic Theories
- Process matrices and indefinite causality

WHERE SHALL WE LOOK FOR QUANTUM EFFECTS IN GRAVITY?

HIGH ENERGIES:
PLANCK-SCALE
PHYSICS

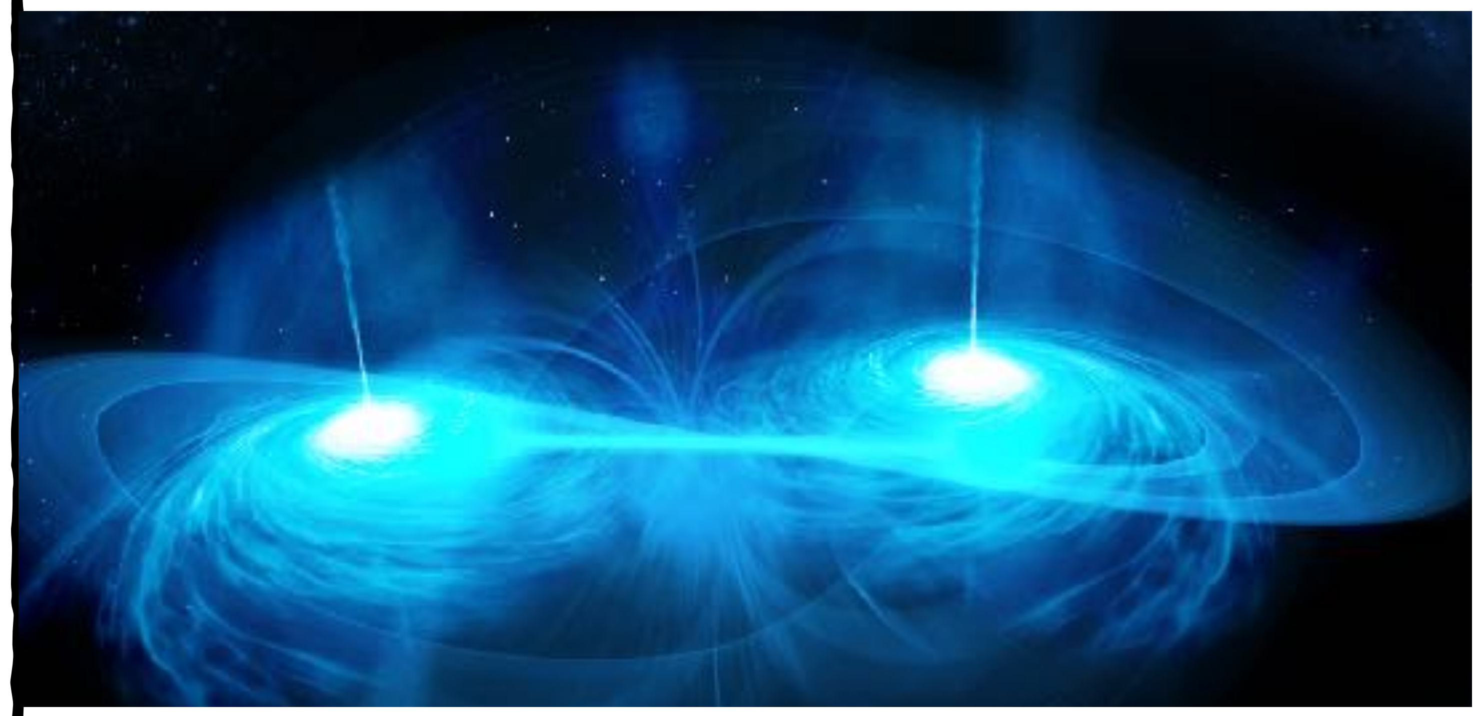
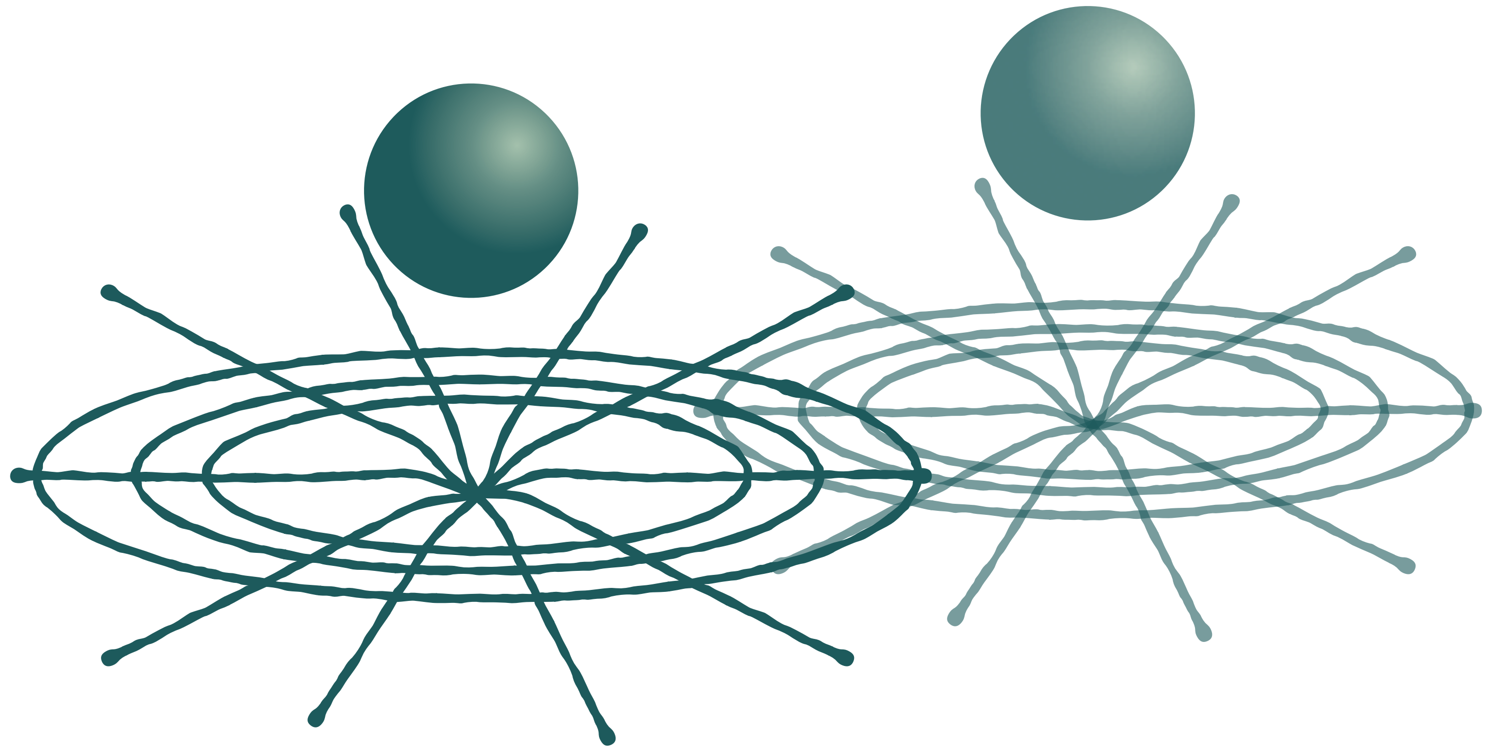


Image credits: Perimeter Institute

QUANTUM SPACETIME "FUZZINESS"

- Black Holes, spin foams, LQG
- String Theory
- Modified dispersion relations
- (...)

LOW ENERGIES:
PERTURBATIVE GRAVITY
QUANTUM PARTICLES



NONCLASSICAL SPACETIME

THESE LECTURES!

- Quantum Time and quantum clocks
- Indefinite causal structures
- Lack of classical reference frames
- (...)

Concrete scenarios with immediate physical meaning

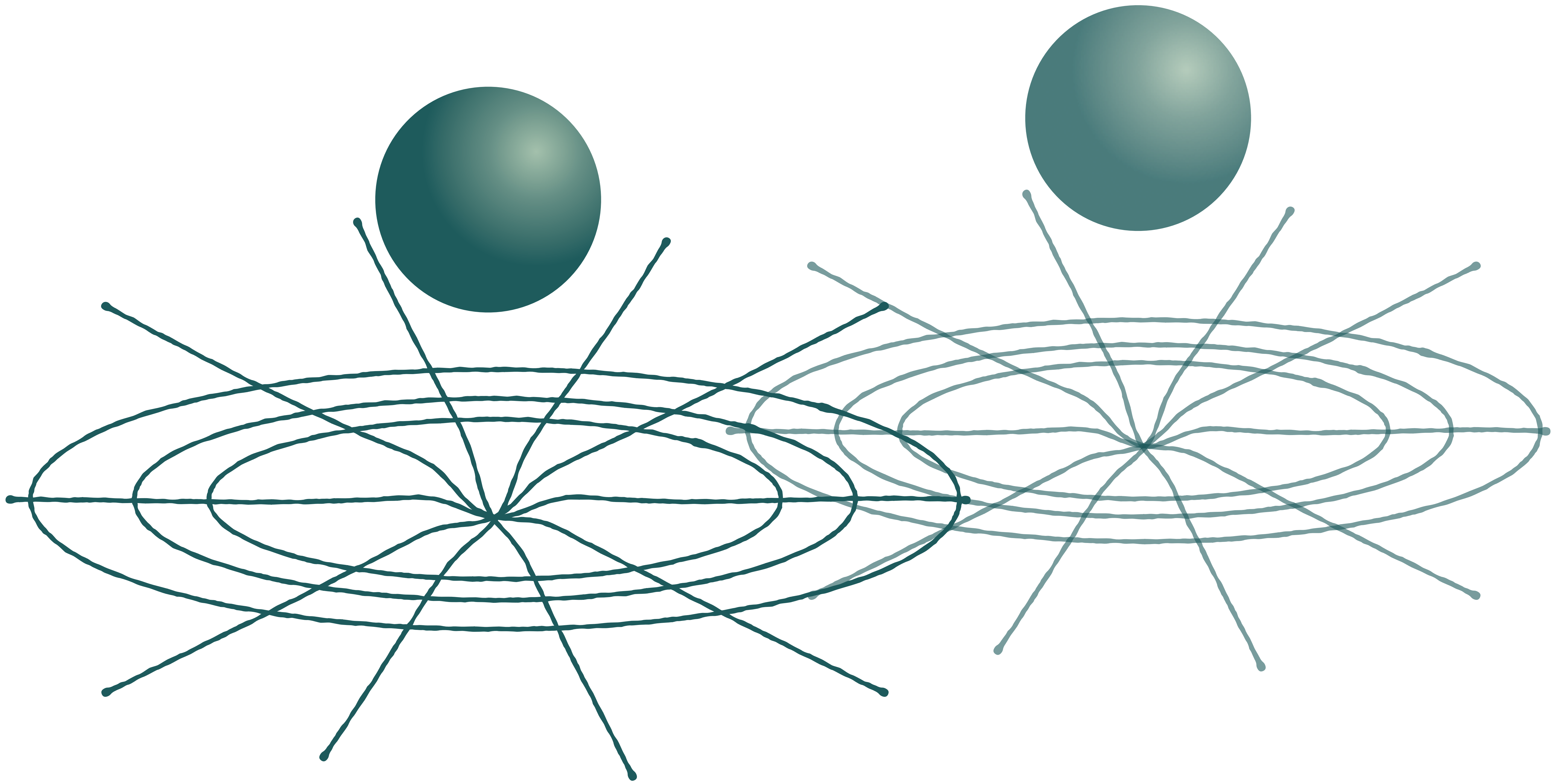
NONCLASSICAL SPACETIME FROM A QUANTUM SOURCE

GENERAL RELATIVITY

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

QUANTUM THEORY

$$T_{\mu\nu} \to \hat{T}_{\mu\nu}$$



WHAT IS THE ROLE OF (QUANTUM) PERTURBATIONS?

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} \rightarrow \hat{h}_{\mu\nu}$$

Why is this not just classical spacetime?

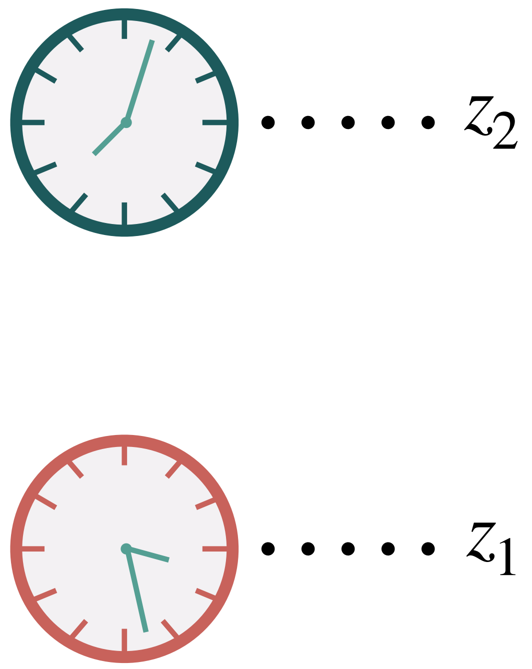
The perturbation contributes to spacetime too!

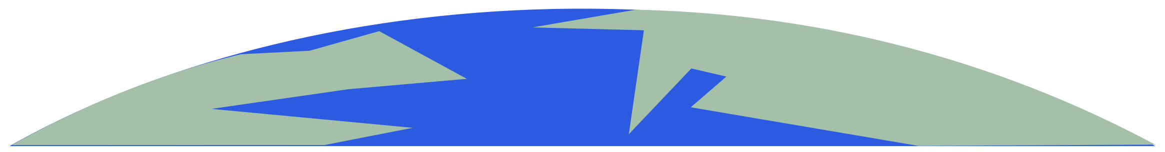
CLASSICAL EXAMPLE 1

"Lower is slower"

$$\tau_2 = \int_{t_0}^{t} dt' \sqrt{1 + h_{00}(z_2)}$$

$$\tau_1 = \int_{t_0}^{t} dt' \sqrt{1 + h_{00}(z_1)}$$





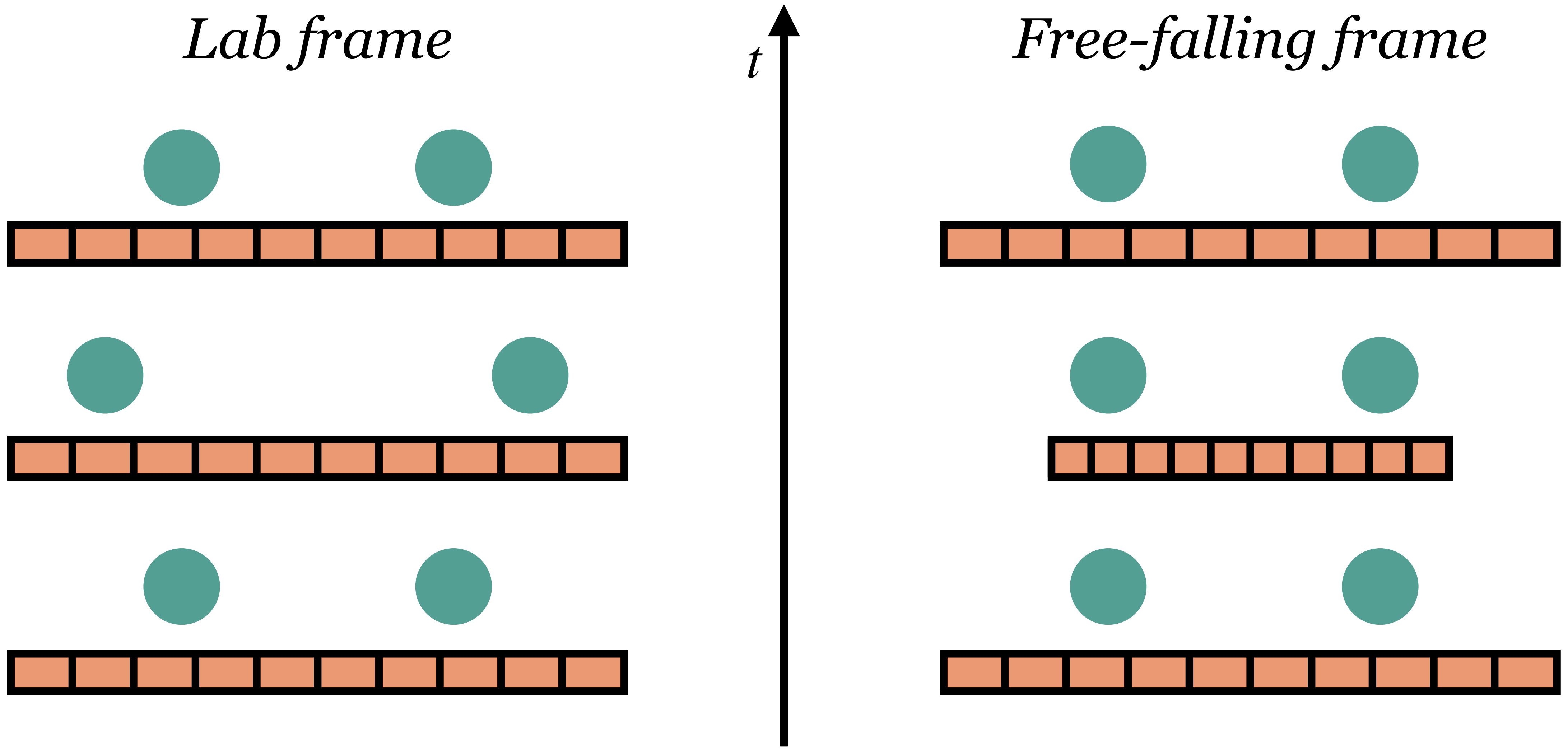
Article | Published: 16 February 2022

Resolving the gravitational redshift across a millimetre-scale atomic sample

Tobias Bothwell, Colin J. Kennedy, Alexander Aeppli, Dhruv Kedar, John M. Robinson, Eric Oelker, Alexander Staron & Jun Ye

Nature **602**, 420–424 (2022) | Cite this article

CLASSICAL EXAMPLE 2



$$s(t) = \int_{x_1}^{x_2} dx \sqrt{1 + h_+(x - ct)}$$

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration)
Phys. Rev. Lett. **119**, 161101 – Published 16 October 2017

NONCLASSICAL SPACETIME FROM A QUANTUM SOURCE

Article | Published: 10 March 2021 **LIGHTEST GRAVITY SOURCE:** $90\,mg$

Measurement of gravitational coupling between millimetre-sized masses

Tobias Westphal, Hans Hepach, Jeremias Pfaff & Markus Aspelmeyer

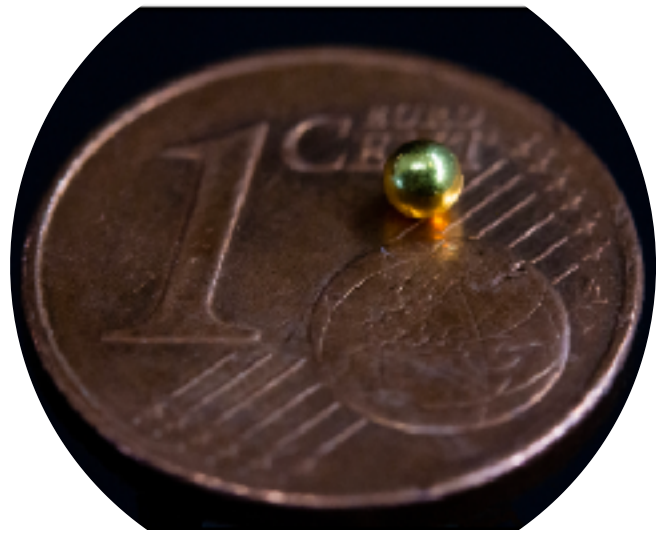
Nature **591**, 225–228 (2021) | Cite this article

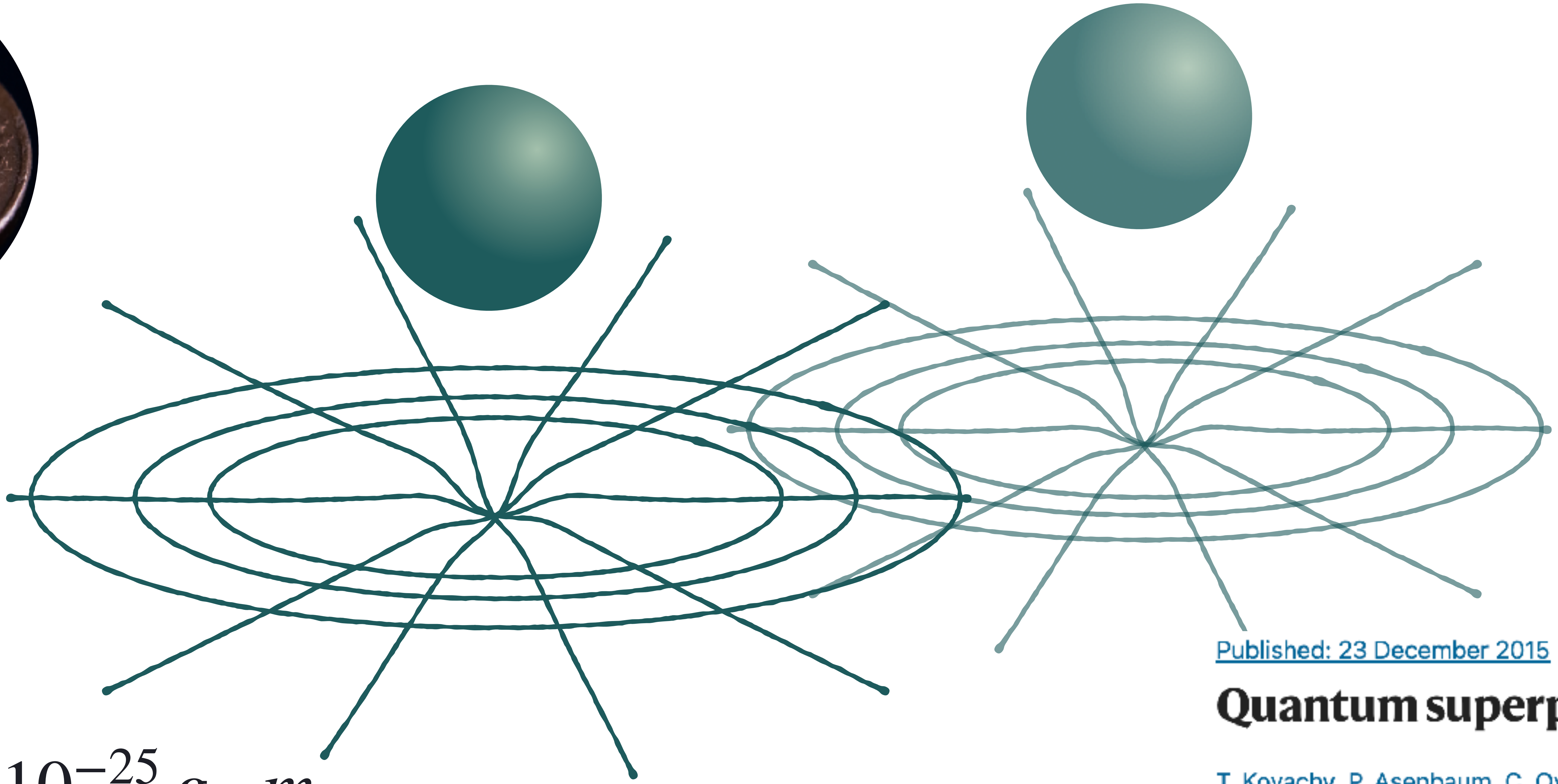
Letter | Published: 23 September 2019 **SUPERPOSED MASS:** $10^{-20}g$

Quantum superposition of molecules beyond 25 kDa

Yaakov Y. Fein, Philipp Geyer, Patrick Zwick, Filip Kiałka, Sebastian Pedalino, Marcel Mayor, Stefan Gerlich & Markus Arndt

Nature Physics **15**, 1242–1245 (2019) | Cite this article





Published: 23 December 2015 **LARGEST SUPERPOSITION:** $0.5\,m$

Quantum superposition at the half-metre scale

T. Kovachy, P. Asenbaum, C. Overstreet, C. A. Donnelly, S. M. Dickerson, A. Sugarbaker, J. M. Hogan & M. A. Kasevich

Nature **528**, 530–533 (2015) | Cite this article

$$m \cdot \Delta x \approx 10^{-25}\, g \cdot m$$

M. Aspelmeyer, 2203.05587 (2022)

WHY IS THIS INTERESTING?

LEVEL 1: We do NOT know which observation would prove in a compelling way that gravity has quantum features.

Good news: There will be experimental guidance!

LEVEL 2: Open questions in quantum gravity show up in this regime
(e.g. lack of a classical spacetime, quantum time, indefinite causality, relationalism, partition of Hilbert space into local algebras/subsystems, etc)

LEVEL 3: First-principle approach:
How do we reconcile the principles of GR and QT?
Internal consistency of GR and QT can be tested in thought experiments

NB: quantum information is not tied to a specific regime

STATES, MEASUREMENTS, INTERFERENCE

SOME DEFINITIONS

VECTOR STATES
(more restrictive)

$$|\psi\rangle \in \mathscr{H} \rightarrow \hat{\rho} \in \mathscr{L}(\mathscr{H})$$

STATES = DENSITY MATRICES
(more general)

$$\hat{\rho} \geq 0$$

$$\hat{\rho} = \hat{\rho}^*$$

$$\mathbf{Tr}\,\hat{\rho}^2 \leq \mathbf{Tr}\,\hat{\rho} = 1$$

One quantum system (and more)

PURE STATES

$$\hat{\rho} = |\psi\rangle\langle\psi|$$

MIXED STATES

$$\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$\sum_i p_i = 1$$

Two quantum systems (and more)

SEPARABLE STATES

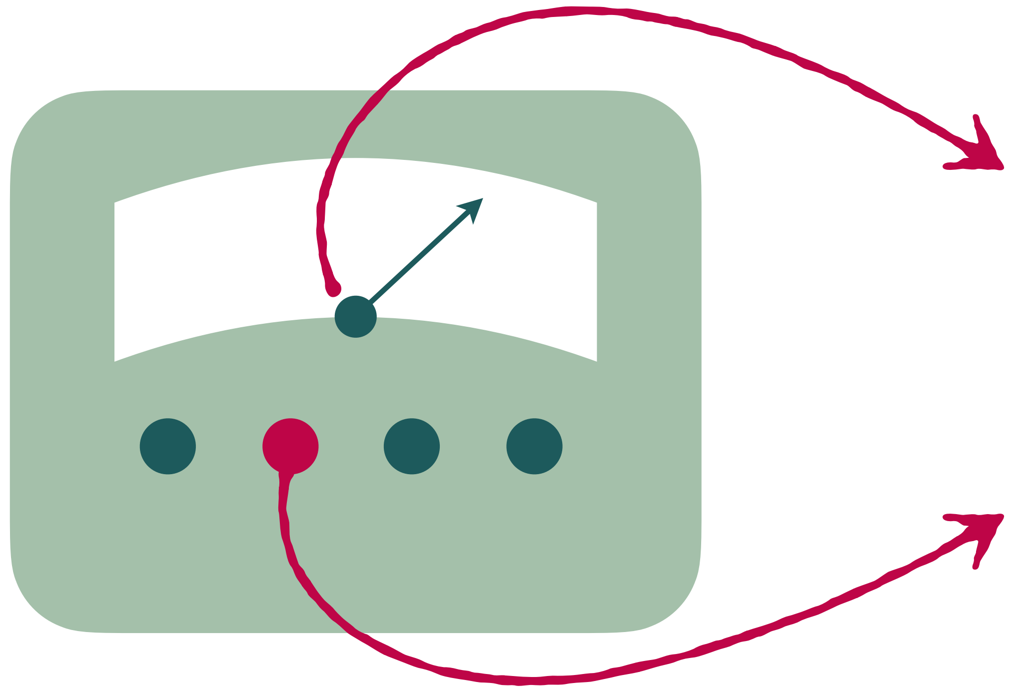
$$\hat{\rho}_{12} = \sum_i p_i\, \hat{\rho}_1^i \otimes \hat{\rho}_2^i$$

ENTANGLED STATES

$$\hat{\rho}_{12} \neq \sum_i p_i\, \hat{\rho}_1^i \otimes \hat{\rho}_2^i$$

- **Global phases are NOT observable**
- **Relative phases are observable**

MEASUREMENTS



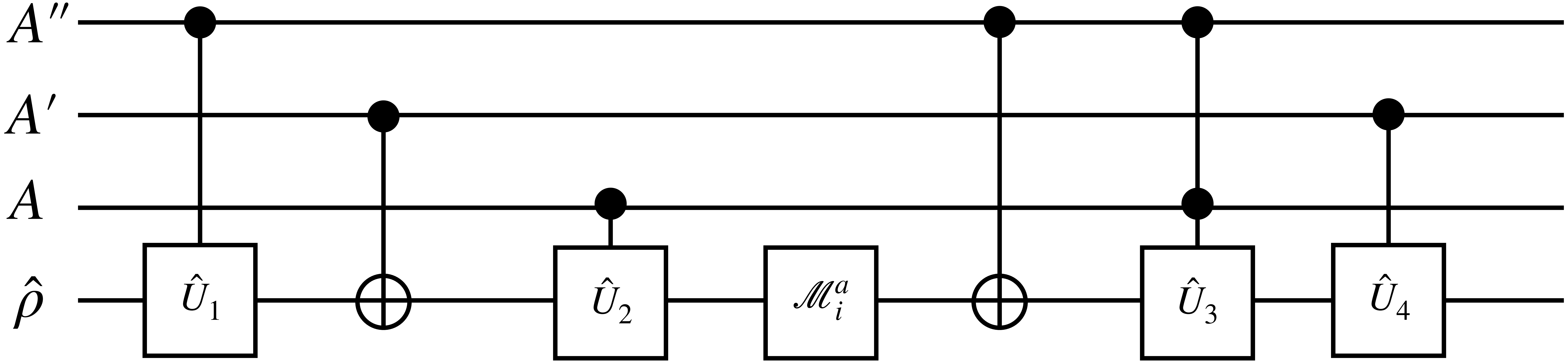
i: OUTPUT

a: INPUT

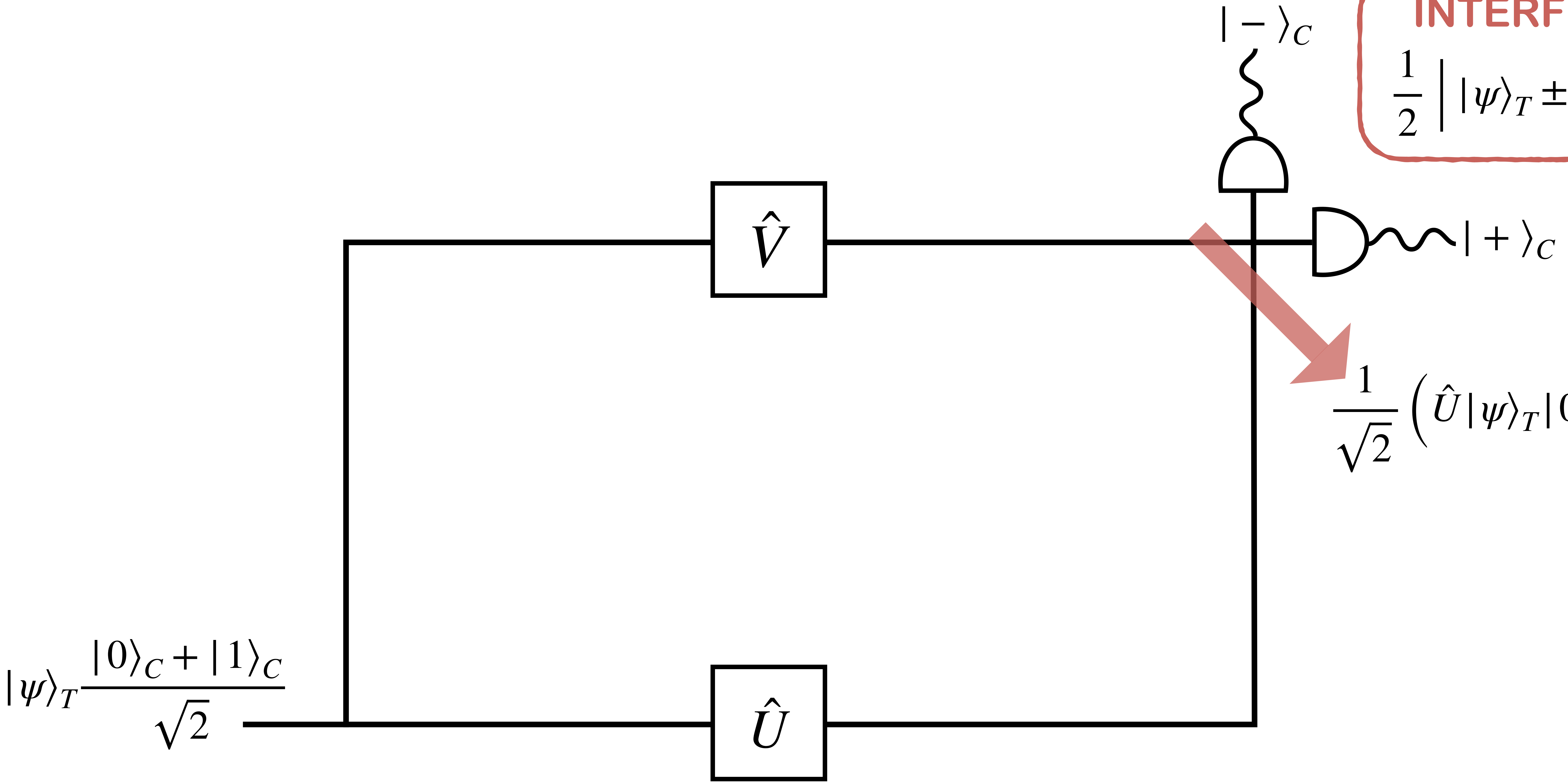
$\mathscr{M}_i^a$ is a Completely Positive (CP) trace non-increasing map

$\{\mathscr{M}_i^a\}_{i=1}^N$ is a QUANTUM INSTRUMENT

$$p(i \,|\, a, \rho) = \mathbf{Tr}[\mathscr{M}_i^a(\hat{\rho})]$$



EXAMPLE: A MACH-ZEHNDER INTERFEROMETER



INTERFERENCE

$$\frac{1}{2}\left| |\psi\rangle_T \pm \hat{U}^\dagger \hat{V} |\psi\rangle_T \right|^2$$

$$\frac{1}{\sqrt{2}}\left(\hat{U}|\psi\rangle_T|0\rangle_C + \hat{V}|\psi\rangle_T|1\rangle_C\right)$$

$$|+\rangle_C = \frac{1}{\sqrt{2}}\left(|0\rangle_C + |1\rangle_C\right)$$

$$|-\rangle_C = \frac{1}{\sqrt{2}}\left(|0\rangle_C - |1\rangle_C\right)$$

QI TOOLS: DEVICE-INDEPENDENT THINKING

SOME GOOD NEWS…

"First, some good news: quantum field theory is based on the same quantum mechanics that was invented by Schrödinger, Heisenberg, Pauli, Born and others in 1925-26 and has been used ever since in atomic, molecular, nuclear, and condensed matter physics."

Steven Weinberg, Quantum Field Theory
Chapter 2

1) Physical States are rays in a Hilbert space
2) Observables are hermitian operators
3) Born rule

WHAT DOES IT MEAN TO BE "QUANTUM"?

1) Quantum superposition or entanglement
2) Action in path integral
3) Expectation values in Heisenberg picture
4) Emission of quantised radiation
5) Measurements do not commute

Underlying common structure?

OPERATIONAL APPROACH

A theory is characterised by the set of probabilities
No ontological commitment

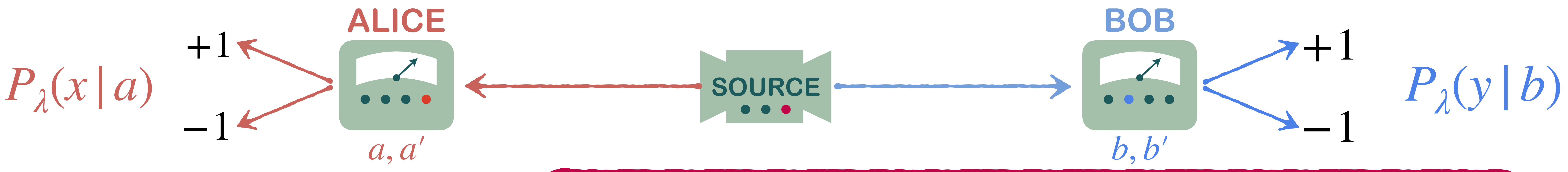
V. Scarani, Bell Nonlocality (book)

1) The rules of the game are known in advance
2) The players can think of a common strategy to win the game
3) The players cannot communicate during the game: no-signalling resources

*"**Bell locality** means that the process by which each player generates the output does not take into account the other player's input. In other words, all correlations between the players' outputs is due to the shared resource"*

LOCAL CAUSALITY/LOCAL REALISM

$$P(x, y \mid a, b) = \int d\lambda Q(\lambda)\, P_\lambda(x \mid a)\, P_\lambda(y \mid b)$$



$x = y$ **if** $(a, b) = \{(0,0), (0,1), (1,0)\}$

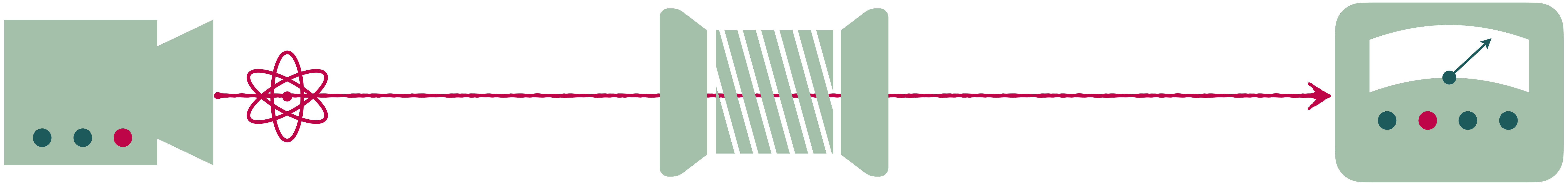
$x = -y$ **if** $(a, b) = (1,1)$

CHSH INEQUALITY (Clauser, Horne, Shimony, Holt 1969)

$$S = \langle x_0 y_0 \rangle + \langle x_0 y_1 \rangle + \langle x_1 y_0 \rangle - \langle x_1 y_1 \rangle < 2 < 2\sqrt{2}$$

Local Realism   Quantum Theory

THE OPERATIONAL APPROACH AND DEVICE INDEPENDENCE



PREPARATION

Knobs change the state

TRANSFORMATION

Device acting on system

MEASUREMENT

Obtain classical outcome

$$P(a \mid P, T, M)$$

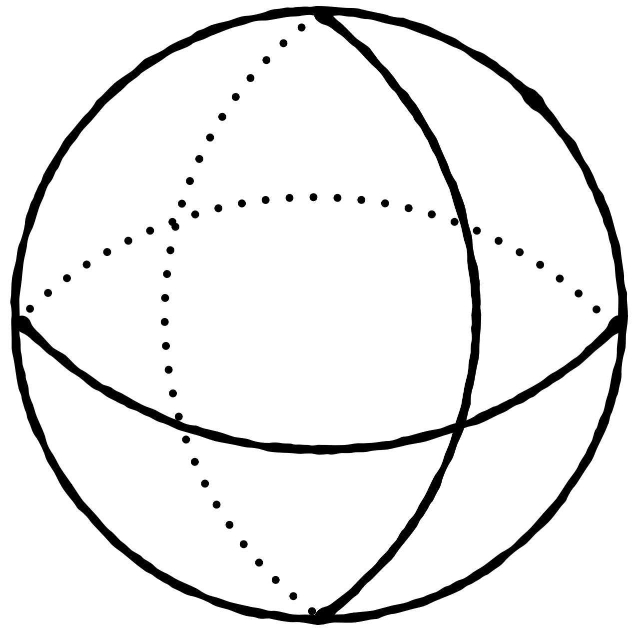
K (degrees of freedom):
minimum number of measurements needed to determine the state

N (dimension):
maximum number of states that can be perfectly distinguished

CAREFUL!
With normalisation
K -> K-1

EXAMPLE 1: BIT
N = K = 2

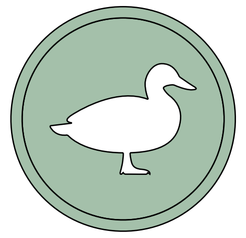
EXAMPLE 2: QUBIT
N =2, K = 4

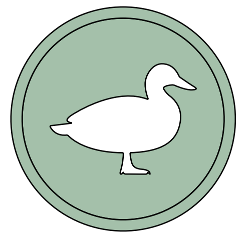


L. Hardy, arXiv:0101012 (2001)
M. Müller, arXiv:2011.01286 (2020)

REMOVE REDUNDANCY: EQUIVALENCE

PREPARATION 1

Throw a coin  

 **Prepare** $|\uparrow\rangle$

 **Prepare** $|\downarrow\rangle$

$$\hat{\rho} = \frac{1}{2}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\right)$$

PREPARATION 2

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle_{12} + |\downarrow\downarrow\rangle_{12}\right)$$

Discard 2

$$\hat{\rho} = \frac{1}{2}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\right)$$

Preparation 1 is EQUIVALENT to Preparation 2

The set of states is CONVEX (comes from probabilistic description)

$$P(a\,|\,\omega, M) = \sum_i p_i P(a\,|\,\omega_i, M) \qquad \omega = \sum_i p_i \omega_i$$

GENERALISED PROBABILISTIC THEORIES (GPTs)

KINEMATICS

Convex state space $\omega \in \Omega$

Measurements $f \in \mathscr{F}$

$$\sum_i f_i(\omega) = 1 \qquad \forall \omega \in \Omega$$

PURE STATES: extremal states of the set
MIXED STATES: convex combinations of pure states

TRANSFORMATIONS

$$\mathcal{T}\left(\sum_{i=1}^{n} p_i \left|\psi_i\right\rangle\left\langle\psi_i\right|\right) = \sum_{i=1}^{n} p_i \mathcal{T}\left(\left|\psi_i\right\rangle\left\langle\psi_i\right|\right)$$

COMPOSITION (related to locality)

Rules to embed states, measurements, and transformations
Rules to obtain reduced states

Compose spaces A and B
$\star : V_A \times V_B \to V_C$

independent preparations $V_A \otimes V_B \subseteq V_C$ joint space

= Tomographic Locality
(valid in classical and quantum theory)

In QT: space of density matrices (not vector states)

Galley, F.G., Selby, Quantum (2022)

SO, WHAT IS QUANTUM?

N-outcome classical probability theory

$$\Omega_A = \left\{ \omega = (p_1, \cdots, p_n) \in \mathbb{R}^N \,|\, p_i \geq 0, \sum_i p_i = 1 \right\}$$

$$\mathcal{F} = \{0 \leq f(\omega) \leq 1 \,|\, \omega \in \Omega_A\}$$

N-outcome quantum probability theory

$$\Omega_A = \left\{ \rho \in H_N(\mathbb{C}) \,|\, \rho \geq 0, Tr(\rho) = 1 \right\}$$

REMEMBER THIS FOR LATER!

$$\mathcal{F} = \{0 \leq f(\rho) \leq 1 \,|\, \rho \in \Omega_A\}$$

EXAMPLE in QT: Completely Positive maps $\{M_i^{A,B}\}_{i=1}^N$

$\sum_i M_i^{A,B}$ **also trace-preserving**

Set of measurements can be fully characterised from this definition.

EXAMPLE 1: NONLINEAR QUANTUM MECHANICS

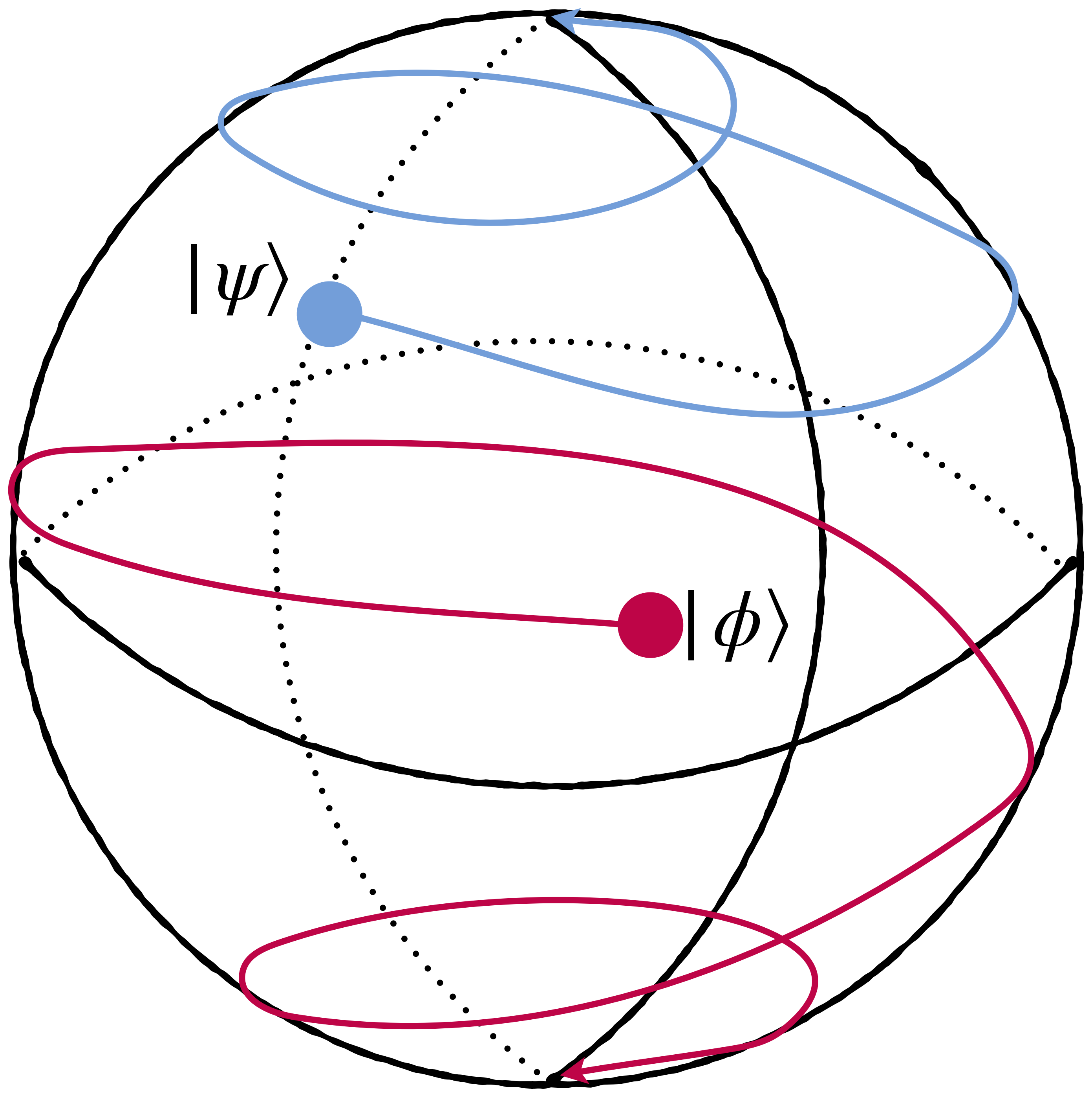
$$i\frac{\partial\psi}{\partial t} = -\nabla^2\psi + \epsilon f(|\psi|^2) + V\psi$$

e.g. Schrödinger-Newton equation

Arbitrary pure states $|\psi\rangle, |\phi\rangle$

It is possible to devise a procedure to distinguish perfectly any two states

Mobility cones



The theory acquires CLASSICAL FEATURES

Nonlinear dynamics changes the kinematics of the theory

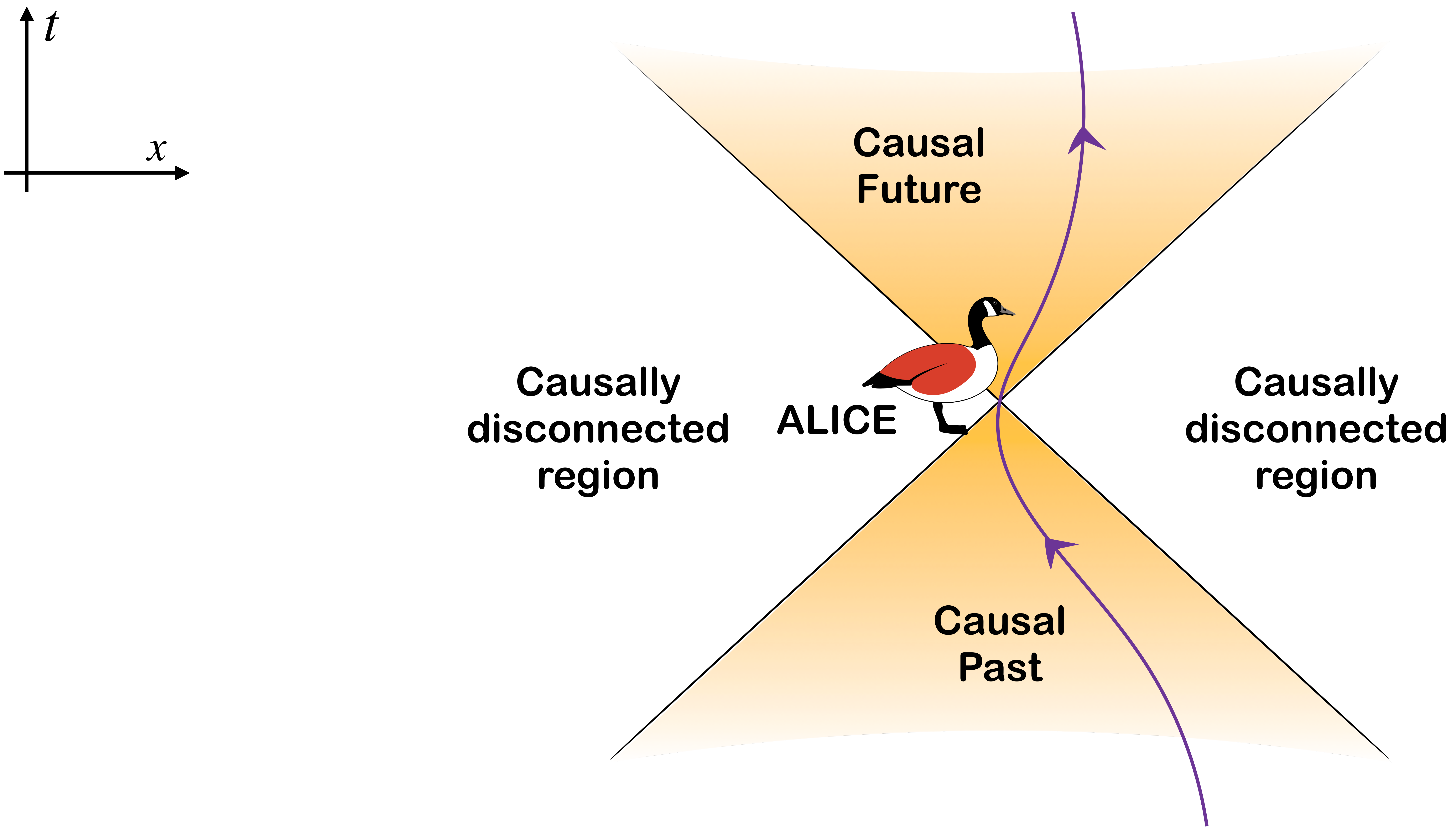
COMPOSITION?

Mielnik, Comm Math Phys (1974)
Mielnik, J Math Phys (1980)

Can we use theory-independent methods to talk about spacetime?

CAUSALITY WITHOUT SPACETIME

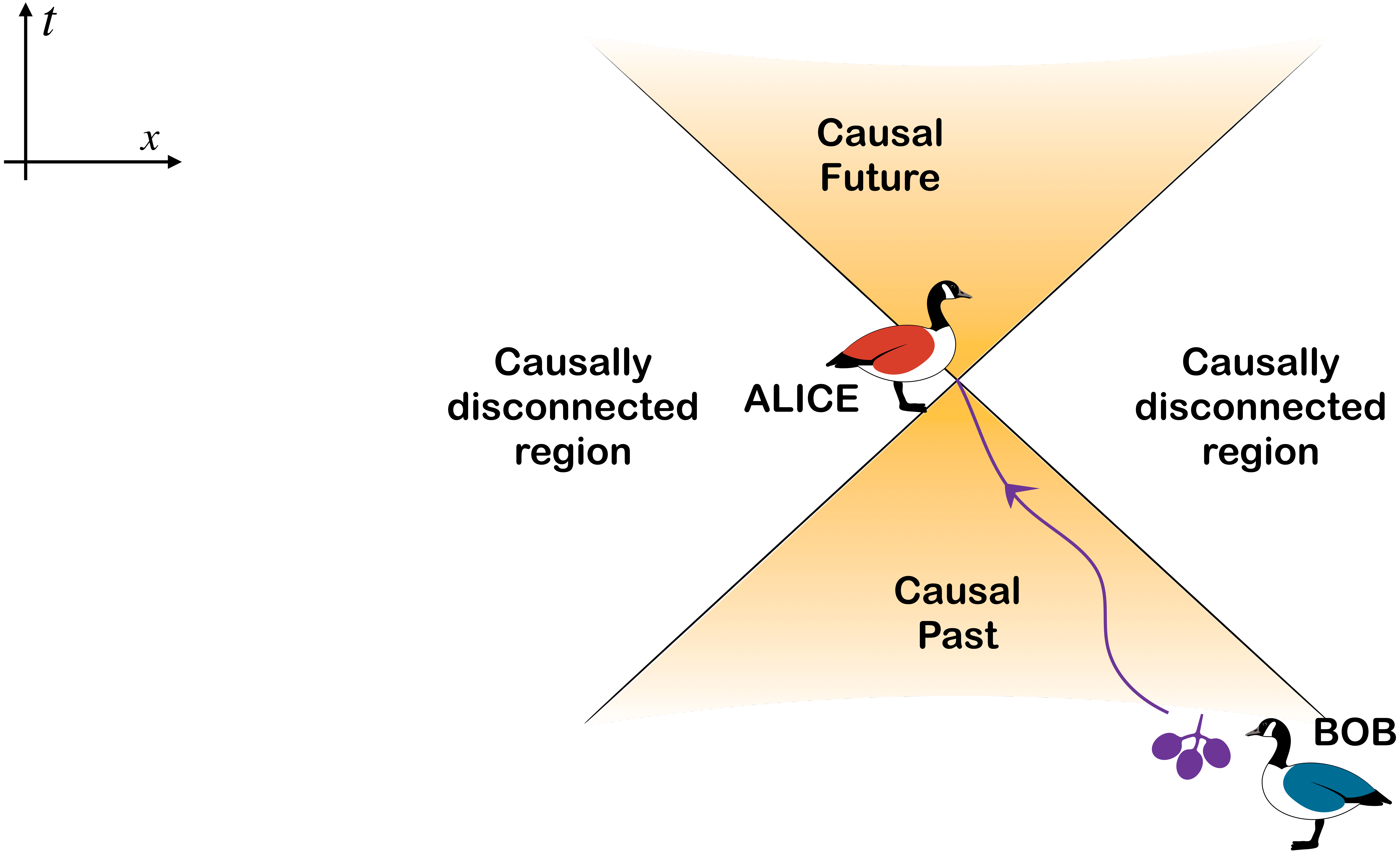
CAUSALITY IN THE OPERATIONAL APPROACH



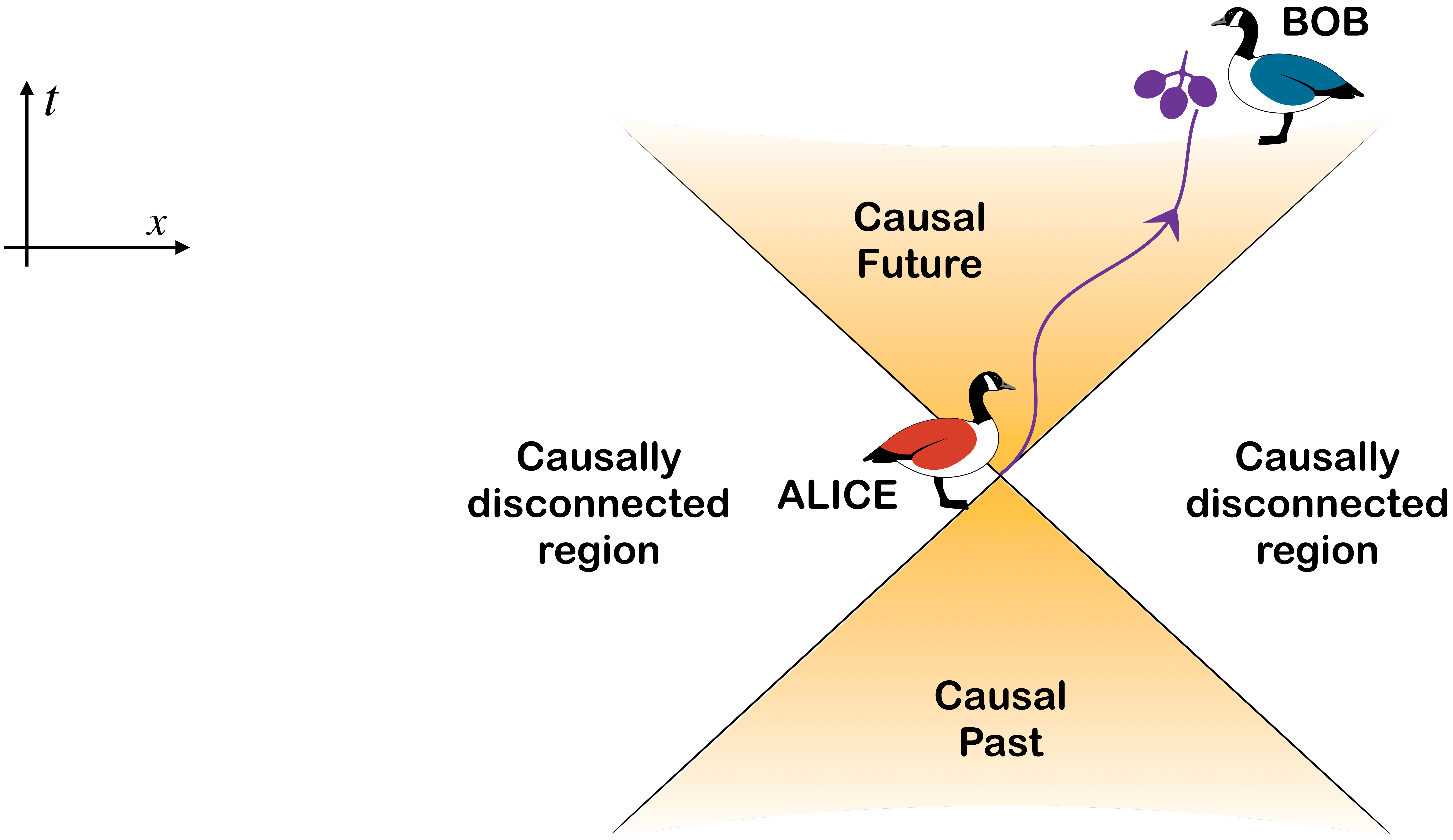
Nothing travels faster than light

CAUSALITY IN THE OPERATIONAL APPROACH

Bob sends a berry to Alice…



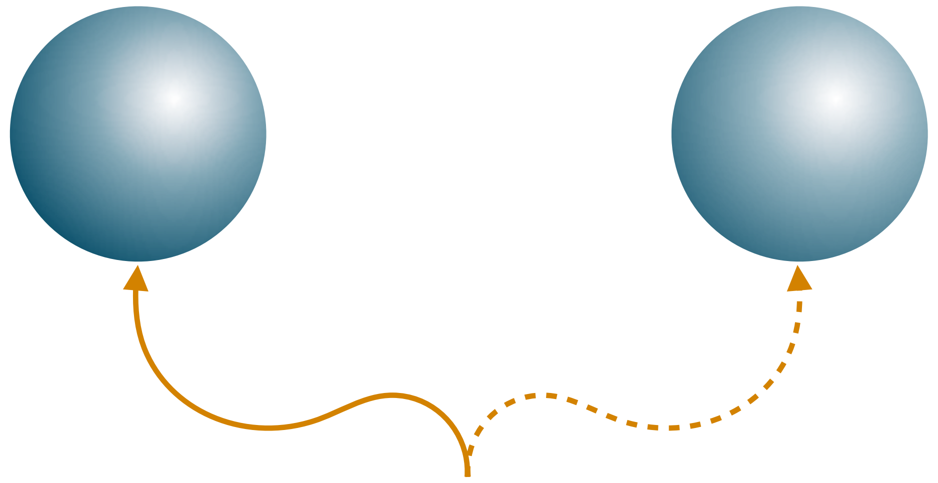
CAUSALITY IN THE OPERATIONAL APPROACH



...or Alice sends it to Bob

WHAT HAPPENS TO CAUSALITY IF GRAVITY IS QUANTUM?

QUANTUM THEORY

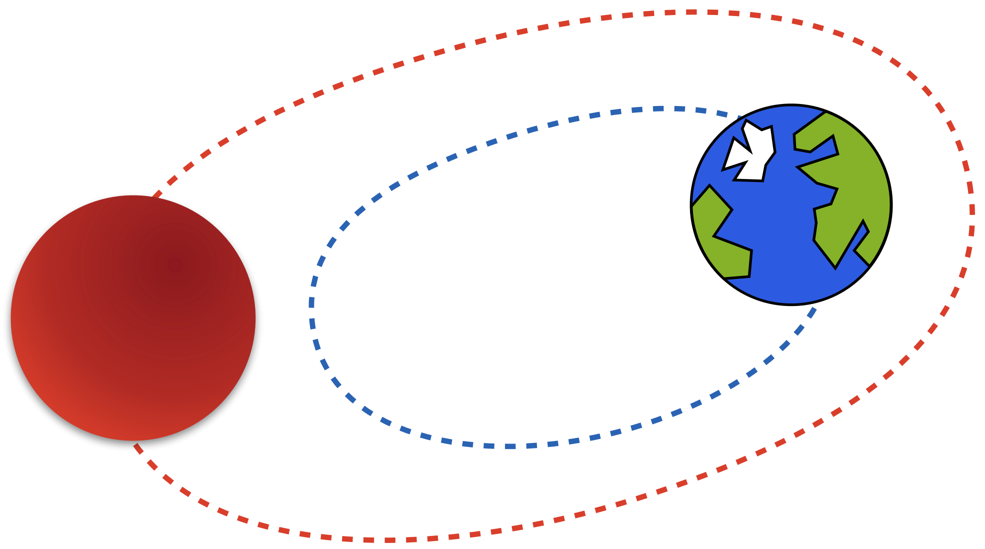


Entanglement, superposition…

Spacetime is the stage in which events happen: causal structure is a priori fixed.

The theory is probabilistic.

GENERAL RELATIVITY



Gravitating objects determine the causal structure

Spacetime is the actor: causal structure is dynamical.

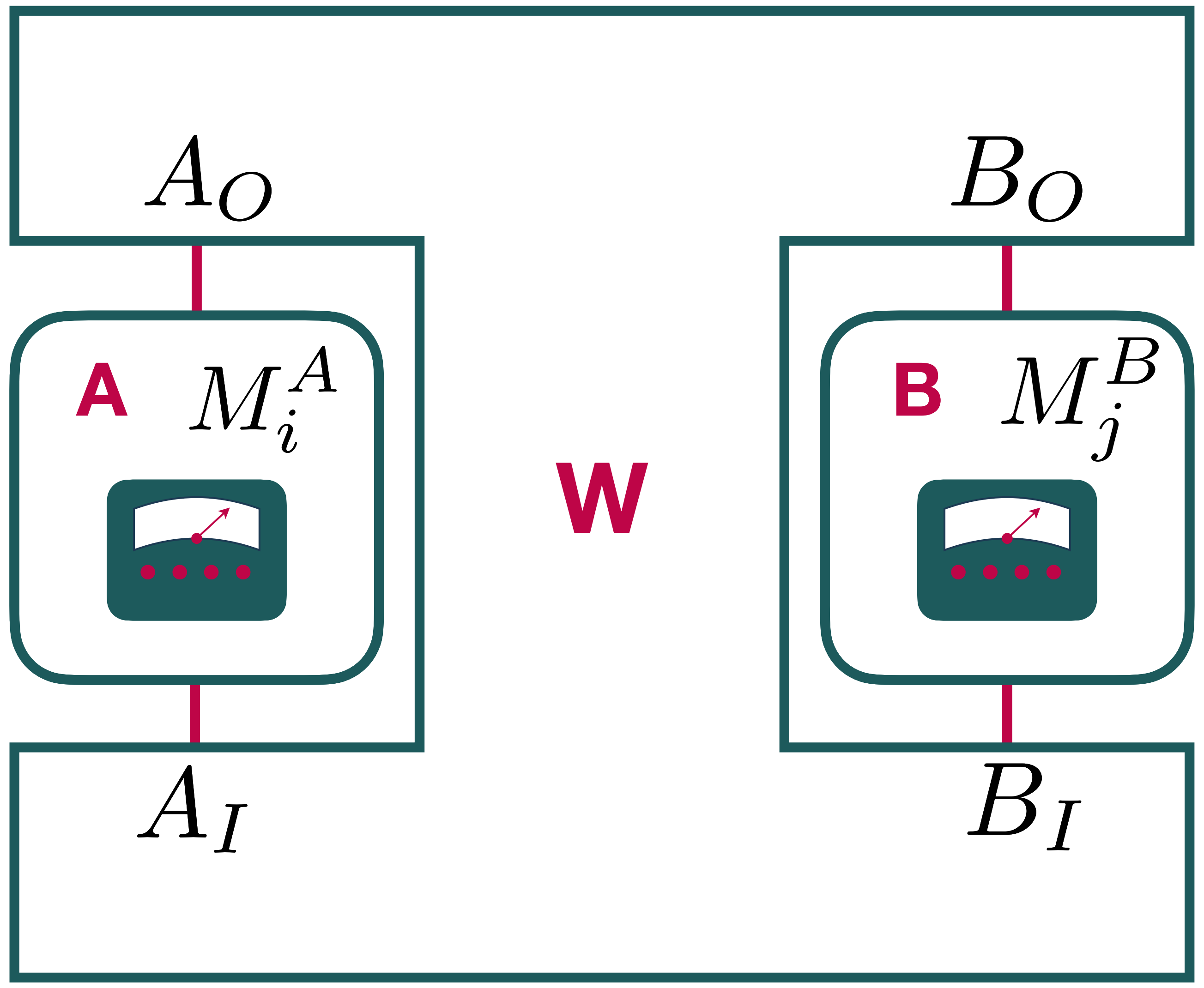
The theory is deterministic.

Probabilistic theory on indefinite causal structures.

(L. Hardy, 2005)

PROCESS MATRIX FORMALISM

Operational definition of causality: possibility of signalling



Remember?

QUANTUM LOCAL OPERATIONS

Completely Positive maps $\{M_i^{A,B}\}_{i=1}^N$

$\sum_i M_i^{A,B}$ **also trace-preserving**

OPPOSITE GAME: Fix measurements, Derive states

- **Positivity of probabilities:**

$$\mathrm{Tr}\, M_i^A \otimes M_j^B W \geq 0$$

- **Normalisation of probabilities:**

$$\sum_{ij} \mathrm{Tr}\, M_i^A \otimes M_j^B W = 1$$

W matrices:
1. **are positive operators** $\quad W \geq 0$
2. **are normalised** $\quad \mathrm{Tr}_I W = \mathbb{1}_O$
3. **live on a subspace of the total Hilbert space** $\quad P W = W$

Oreshkov, Costa, Brukner, Nat. Commun. (2012)

TAKE-HOME MESSAGE

Process matrices specify the signalling properties between the local laboratories.

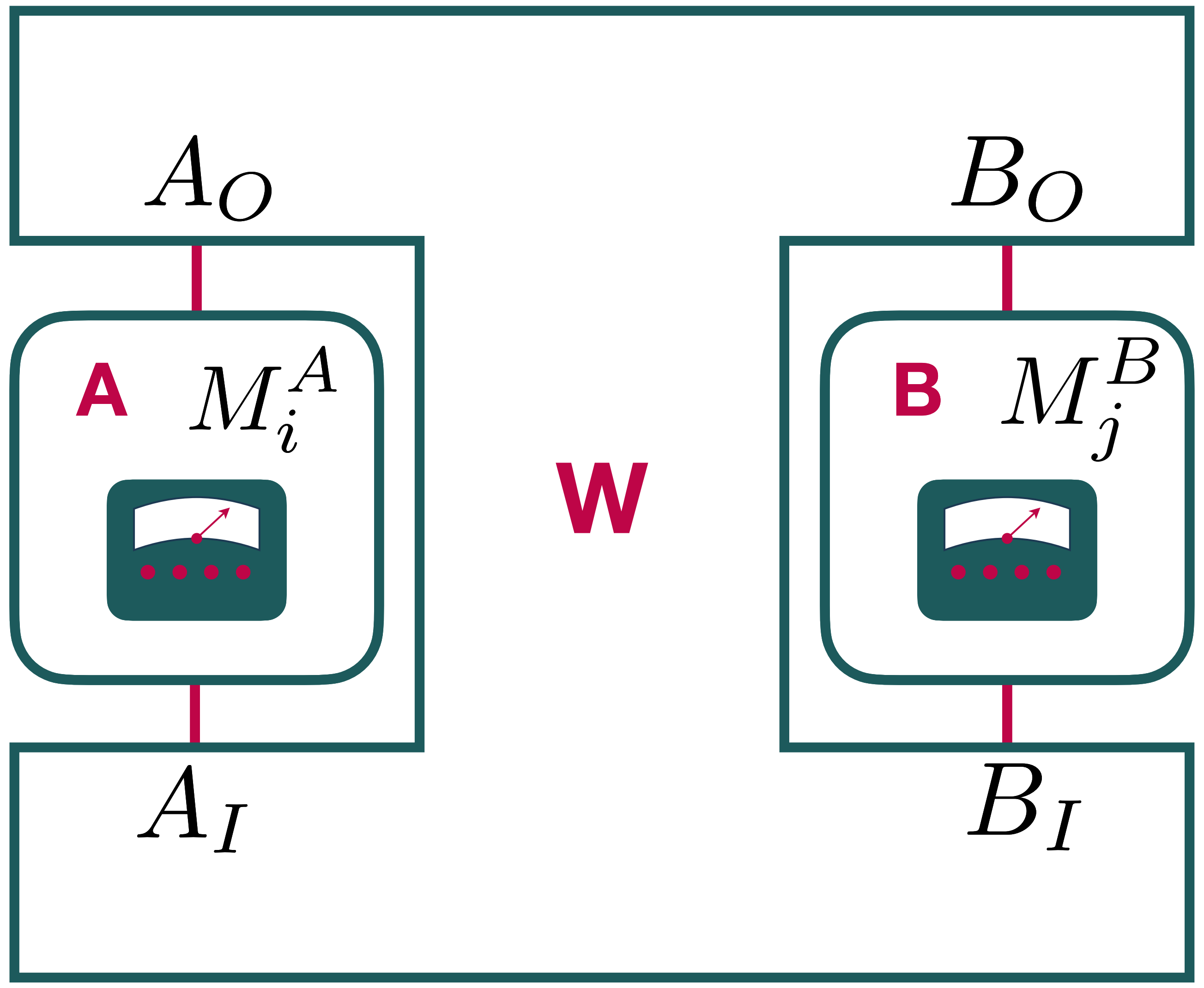
No-signalling from B to A

$$\sum_j p(i,j|x,y) = p(i|x) \qquad \forall i,x$$

No-signalling from A to B

$$\sum_i p(i,j|x,y) = p(j|y) \qquad \forall j,y$$

- Scenarios in which the order (signalling) between A and B is not definite
- No logical paradoxes



Does not rely on a spacetime structure!

Oreshkov, Costa, Brukner, Nat. Commun. (2012)

LEVELS OF INDEFINITE CAUSALITY

INDEFINITENESS ↑

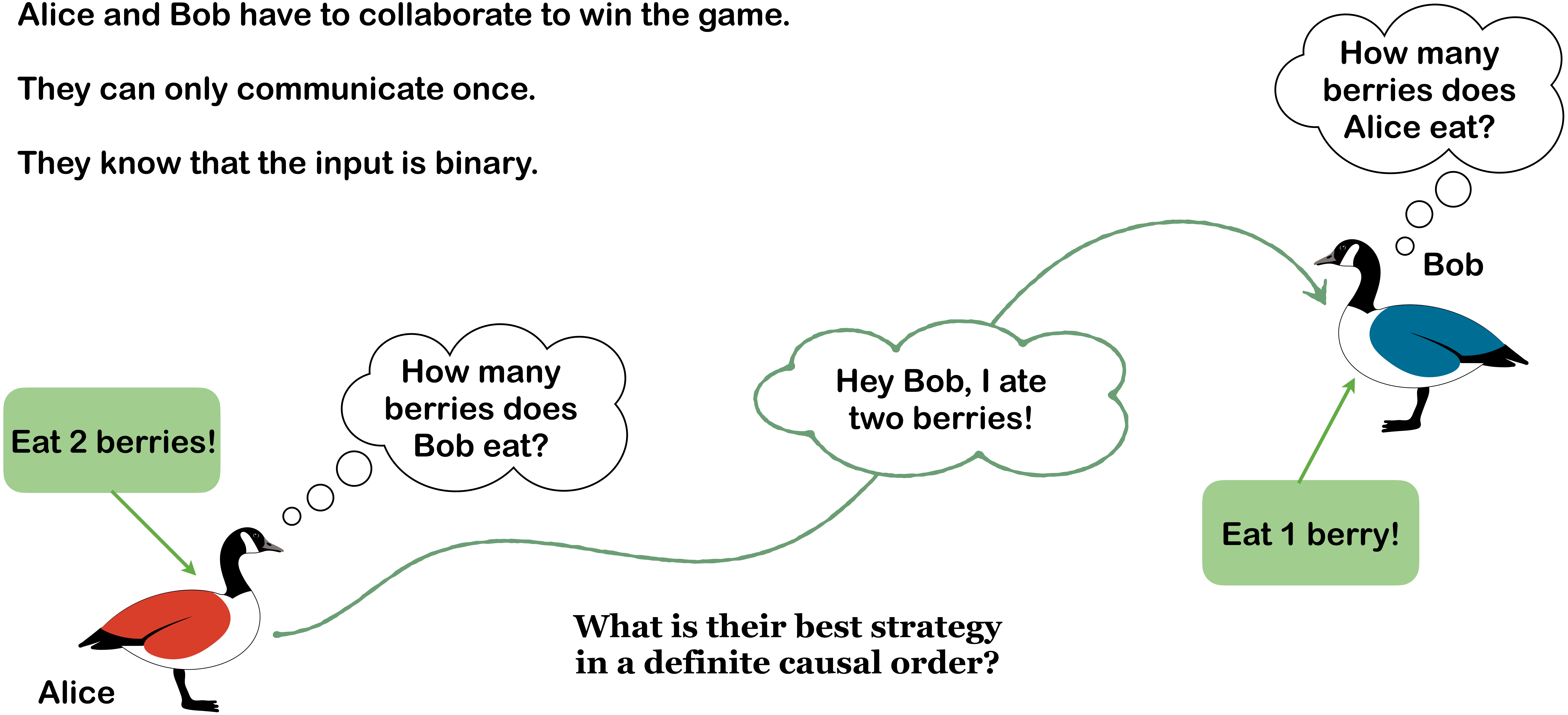
Violate causal inequalities	Do not have an intuitive explanation. Open problem: are they physical?
Causally nonseparable	A and B happen in a quantum superposition of causal orders (e.g. quantum switch)
Causally separable	Causal structure is determined by the throw of a (classical) coin, A and B are probabilistically ordered (classical).
Causally ordered	Our usual description of the world, A and B can be ordered.

THE SIMPLEST CAUSAL GAME

Alice and Bob have to collaborate to win the game.

They can only communicate once.

They know that the input is binary.

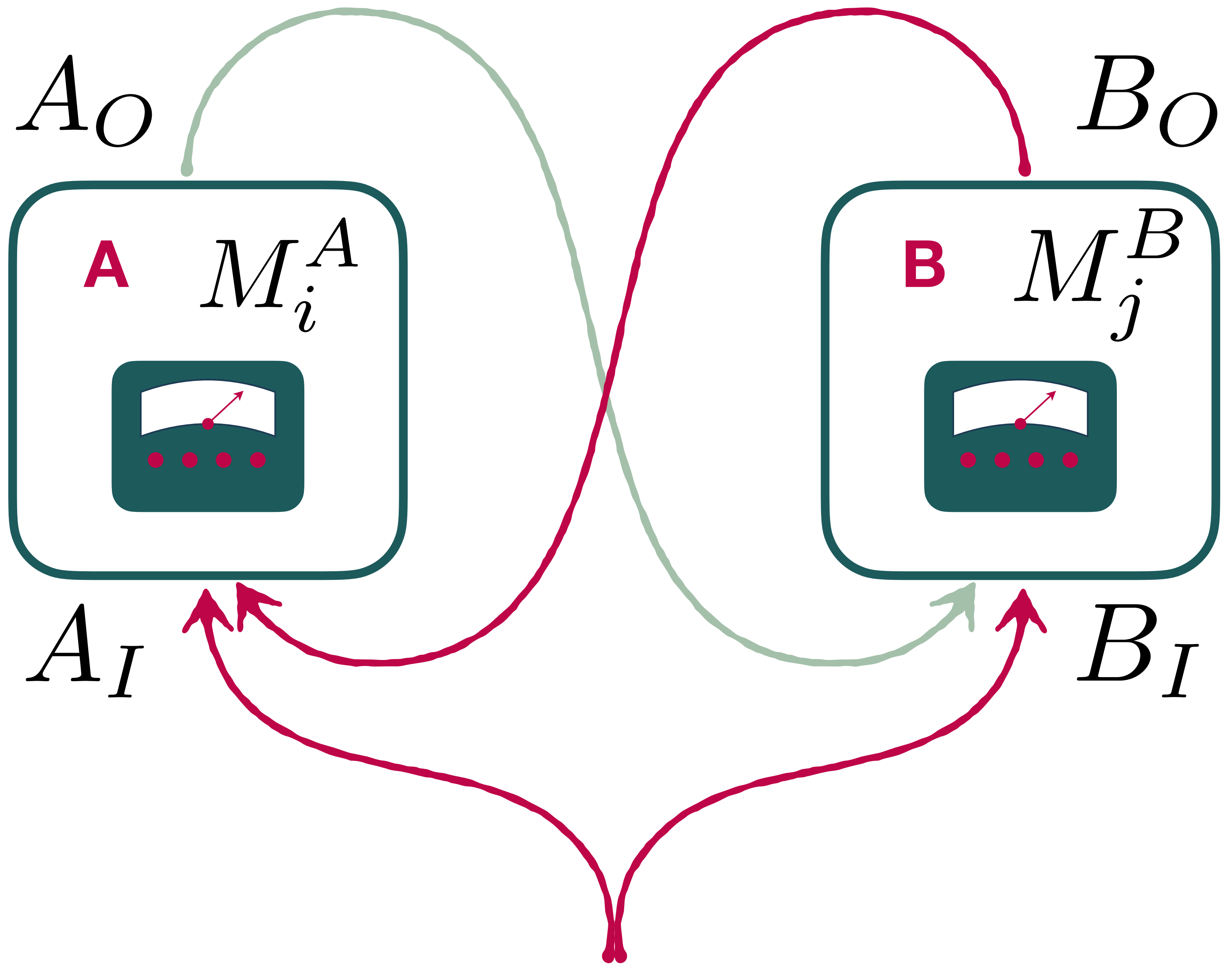


What is their best strategy in a definite causal order?

O. Oreshkov, F. Costa, C. Brukner, Nat. Commun. 3, 1092 (2012)
C. Branciard et al New J. Phys. 18, 013008 (2016)

A DIFFICULT CASE

$$W_{OCB} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}(Z_{A_O} Z_{B_I} + Z_{A_I} X_{B_I} Z_{B_O})\right]$$



OPEN QUESTION (intensely investigated):
Which process matrices can be physically realised?

SUMMARY

Map physical description onto algebraic properties

Identify set of principles and rules that give the result

Characterise the most general theory that is logically consistent

Relax or restrict assumptions

Go back to physics!

ADVANTAGES AND DISADVANTAGES

FULLY THEORY INDEPENDENT ARGUMENTS ARE VERY HARD!

More theory independence often means less details
Not obvious how to “traslate” notions

DEVICE-INDEPENDENT THINKING

↓

Generalised Probabilistic Theories (GPTs)

↓

Process matrices and indefinite causality

Internal consistency

Hybrid models

Systematic characterisation of physical properties

Robustness of the results independently of the theory (see Bell’s theorem)

THANK YOU!

Swiss National Science Foundation

ETH zürich