QUANTUM INFORMATION TOOLS AT THE INTERFACE BETWEEN QUANTUM THEORY AND GRAVITY

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Image credits: J. Palomino

Basics of Quantum Gravity
22-25 May 2023

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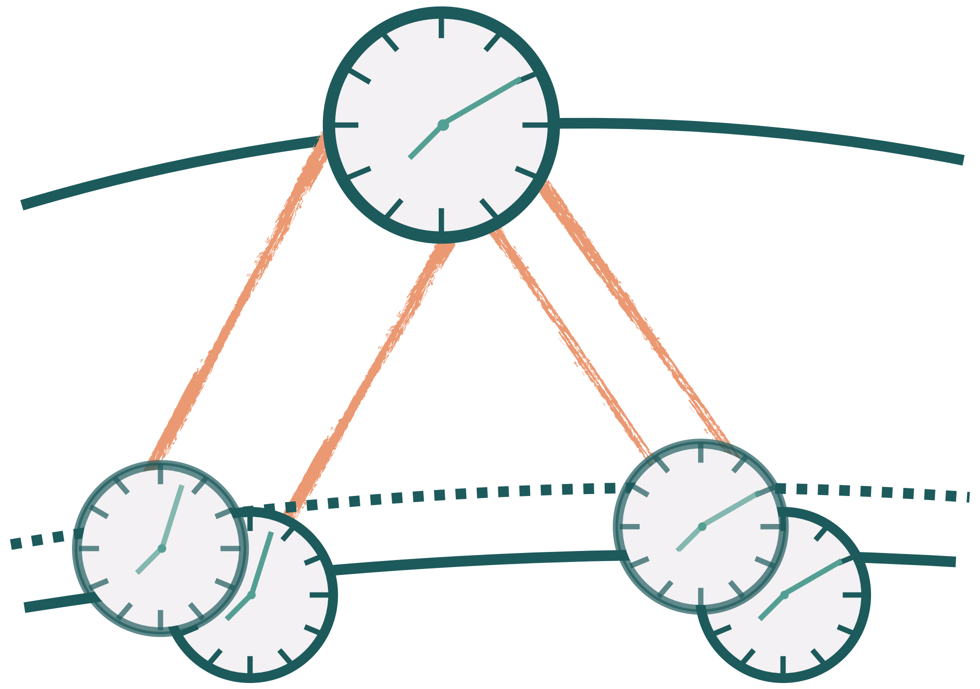
Swiss National Science Foundation

LECTURE 4: QUANTUM CLOCKS AS PROBES OF NONCLASSICAL SPACETIME

- Quantum clock: the basics
- Quantum clocks in classical spacetime
- Quantum clocks in nonclassical spacetime: limits to measurability and relative localization of events
- The gravitational quantum switch: quantum clocks and superposition of temporal order

Basics of Quantum Gravity
22-25 May 2023

QUANTUM CLOCKS IN CLASSICAL SPACETIME



How do quantum clocks encode information about spacetime?

WHAT IS A QUANTUM CLOCK?

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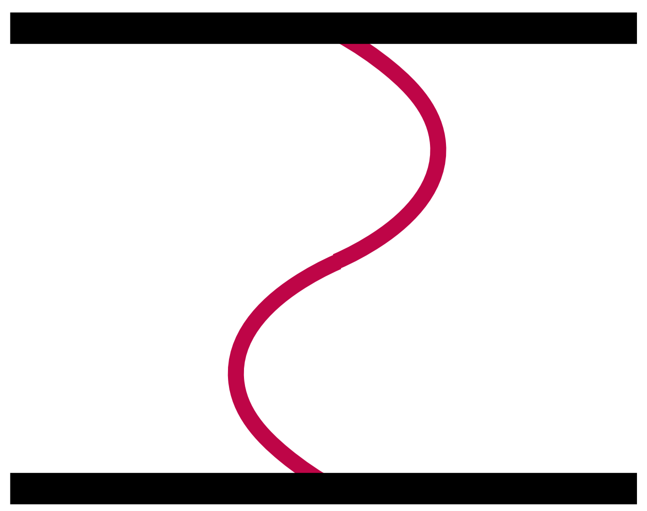


$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|E_0\rangle + |E_1\rangle)$$

$$H_C = E_0|E_0\rangle\langle E_0| + E_1|E_1\rangle\langle E_1|$$

$$|\psi_t\rangle = \frac{1}{\sqrt{2}}(|E_0\rangle + e^{\frac{i}{\hbar}(E_1-E_0)t}|E_1\rangle)$$

-1

$$\langle\psi_{t_\perp}|\psi_0\rangle = 0$$

ORTHOGONALISATION TIME

$$t_\perp = \frac{\pi\hbar}{(E_1 - E_0)}$$

WHAT IS A QUANTUM CLOCK?

$$E = mc^2$$

$$m \rightarrow m + \frac{\hat{H}_I}{c^2}$$

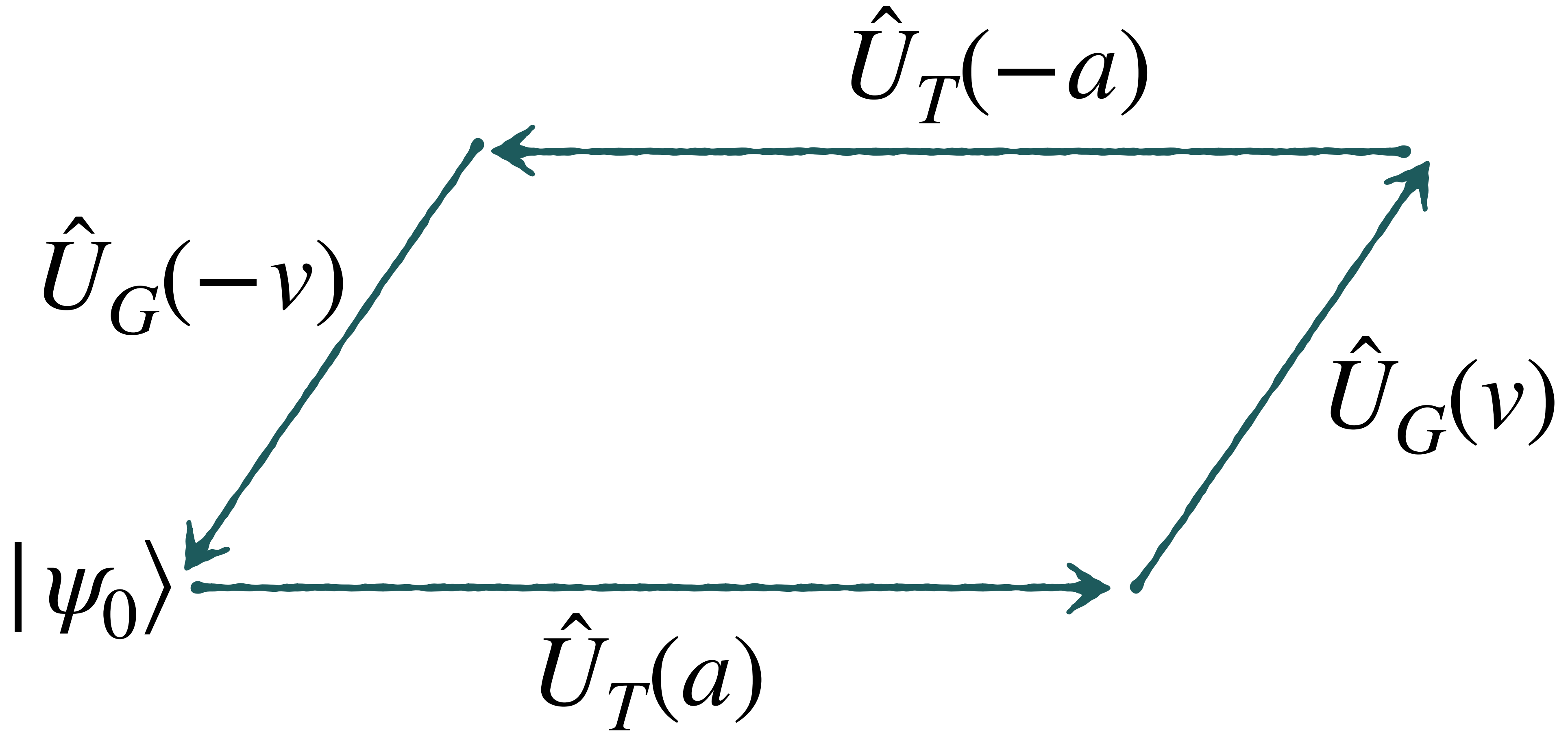
THE MASS BECOMES AN OPERATOR!

MASS AND PROPER TIME

$$g_{\mu\nu}p^{\mu}p^{\nu} = c^{-2}E_{rest}$$

$$E_{rest} = mc^2 + H_C$$

Total rest energy
(Mass +internal degrees of freedom)



Translation operator

$$\hat{U}_T(a) = e^{\frac{i}{\hbar}a\hat{p}}$$

Galilean boost operator

$$\hat{U}_G(v) = e^{\frac{i}{\hbar}v\hat{G}}$$

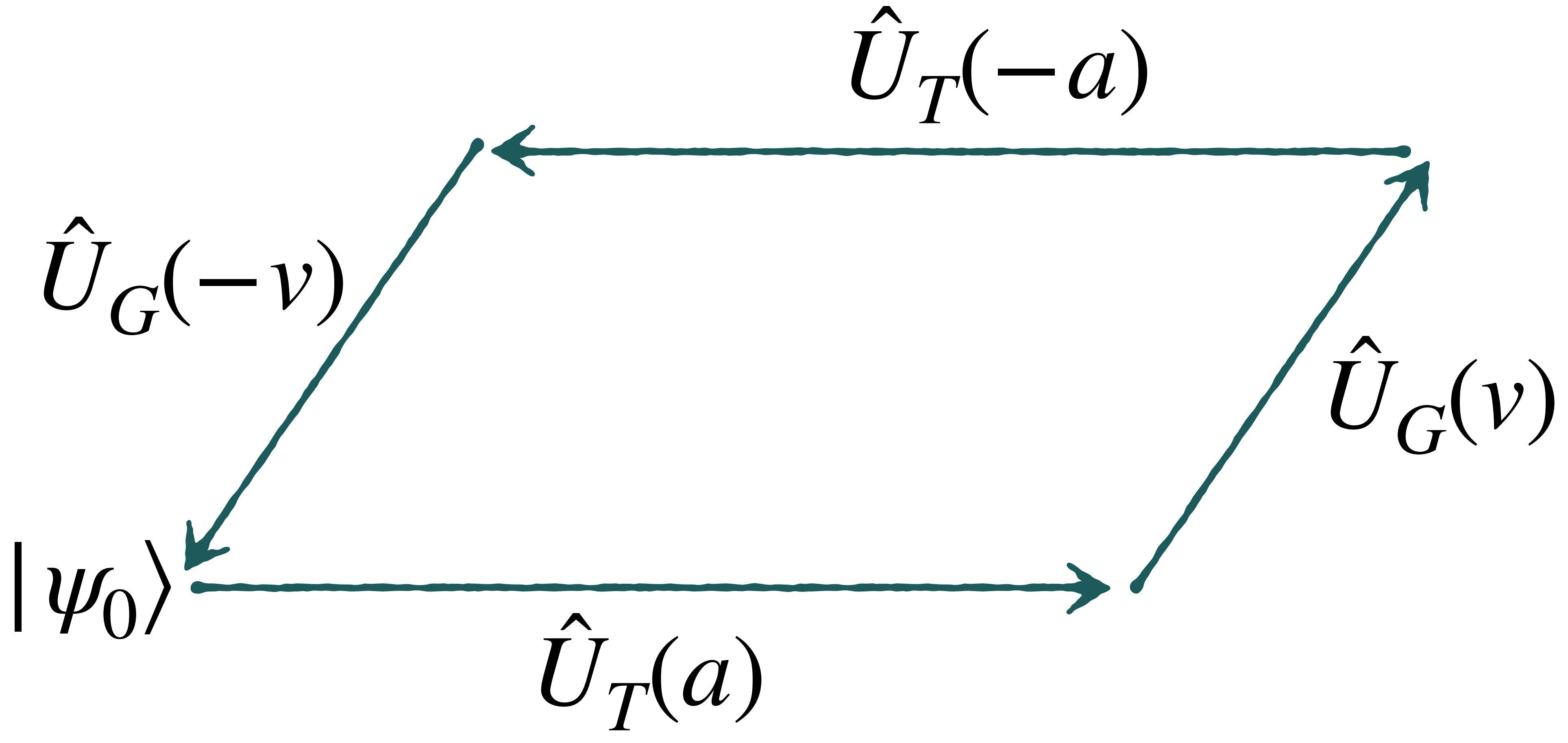
$$\hat{G} = \hat{p}t - m\hat{x}$$

MASS AND PROPER TIME

$$g_{\mu\nu}p^{\mu}p^{\nu} = c^{-2}E_{rest}$$

$$E_{rest} = mc^2 + H_C$$

Total rest energy
(Mass +internal degrees of freedom)



$$|\psi_0\rangle \rightarrow e^{\frac{i}{\hbar}mav}|\psi_0\rangle$$

$$e^{\frac{i}{\hbar}mav} = e^{\frac{i}{\hbar}\Delta\tau c^2 \Delta m}$$

Greenberger, PRL 87 (2001)

SUPERPOSITION OF MASSES?

$$|\psi_0^{(m_1)}\rangle + |\psi_0^{(m_2)}\rangle \rightarrow |\psi_0^{(m_1)}\rangle + e^{\frac{i}{\hbar}\Delta m \cdot av}|\psi_0^{(m_2)}\rangle$$

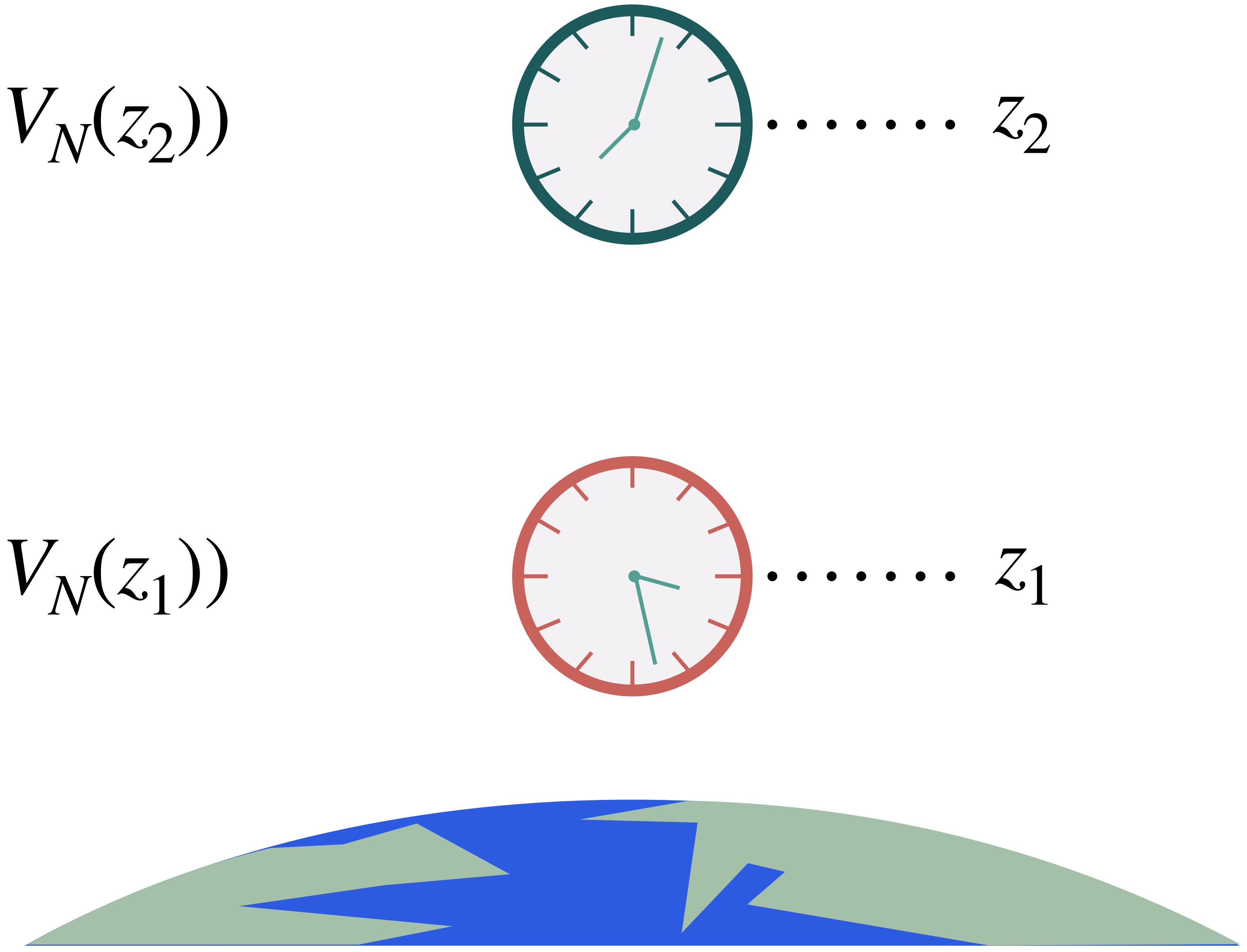
Bargmann: mass superselection rule

GRAVITY INFLUENCES HOW A CLOCK TELLS THE TIME

"Lower is slower"

$$\tau_2 = \Delta t(1 + V_N(z_2))$$

$$\tau_1 = \Delta t(1 + V_N(z_1))$$



Article | Published: 16 February 2022

Resolving the gravitational redshift across a millimetre-scale atomic sample

Tobias Bothwell, Colin J. Kennedy, Alexander Aeppli, Dhruv Kedar, John M. Robinson, Eric Oelker, Alexander Staron & Jun Ye

Nature **602**, 420–424 (2022) | Cite this article

QUANTUM CLOCKS AS PROBES OF SPACETIME

$$c^2 d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Line element in classical spacetime

REST FRAME $$\frac{d}{d\tau}|\psi\rangle_C = \hat{H}_C |\psi\rangle_C$$

LAB FRAME $$\frac{d}{dt}|\psi\rangle_C = \dot{\tau}\hat{H}_C |\psi\rangle_C$$

$$\dot{\tau} = \sqrt{g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}$$

Internal and external degrees of freedom

$$|\Psi_t\rangle = e^{-\frac{i}{\hbar}(\hat{H}_{ext}+\dot{\tau}\hat{H}_C)t}|\phi_0\rangle|\psi_0\rangle_C = \int D[x(t)]\phi_0(x_0)e^{\frac{i}{\hbar}\int ds(S_{ext}+\dot{\tau}\hat{H}_C)}|x(t)\rangle|\psi_0\rangle_C$$

semiclassical evolution for external d.o.f.

M. Zych, PhD thesis

QUANTUM CLOCKS AS PROBES OF SPACETIME

THE NEWTONIAN LIMIT

$$g_{00}(x) = 1 + \frac{2\Phi(x)}{c^2}, \qquad g_{0i}(x) = 0, \qquad g_{ij}(x) = -1.$$

$$c^2 d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Line element in classical spacetime

$$S = mc^2 \int d\tau = mc^2 \int dt \sqrt{g_{00} - \frac{|\dot{x}|^2}{c^2}}$$

$$\approx mc^2 \int dt \left\{ 1 - \frac{|\dot{x}|^2}{2c^2} + \frac{\Phi(x)}{c^2} + O(c^{-4}) \right\}$$

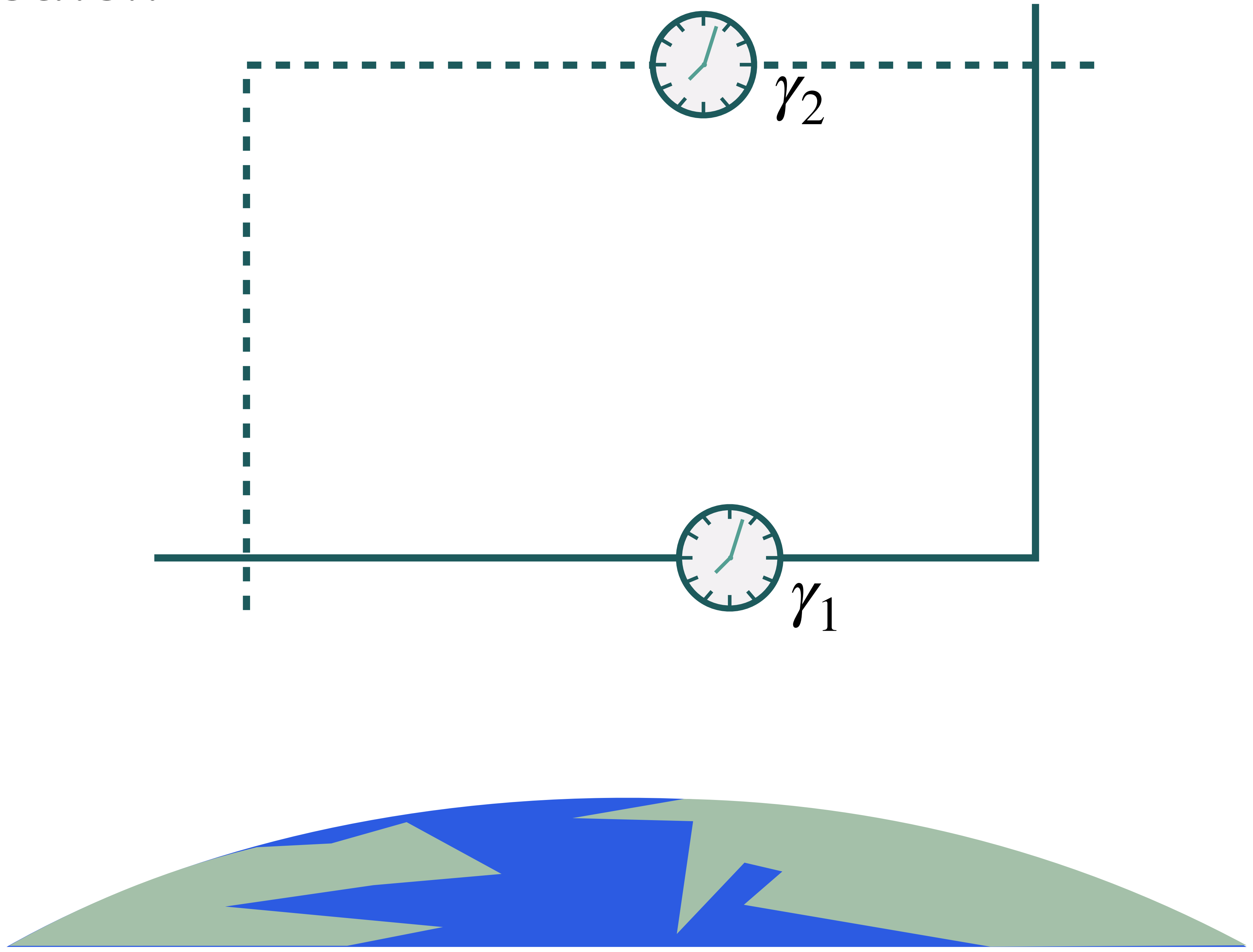
$$H = \frac{|p|^2}{2m} + m\Phi(x)$$

Internal energy contributes to the total mass

$$H = \frac{\hat{p}^2}{2}\left(m + \cancel{\frac{\hat{H}_C}{c^2}}\right)^{-1} + \left(m + \frac{\hat{H}_C}{c^2}\right)\Phi(\hat{x})$$

Coupling between internal and external degrees of freedom

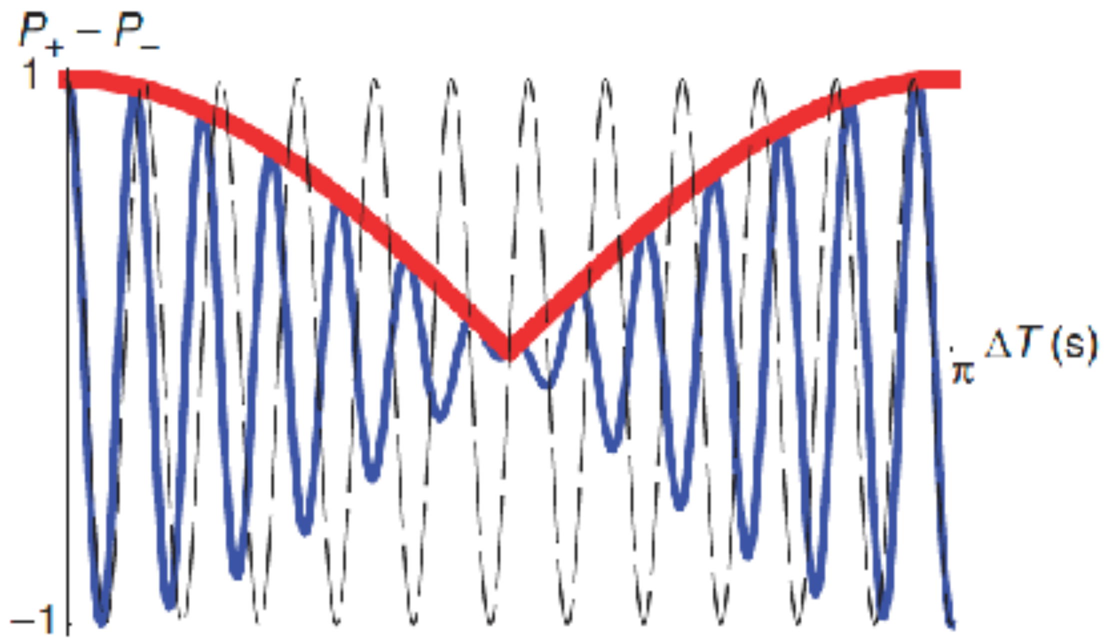
Quantum clocks test the principle of linear superposition and gravitational time dilation together.



"LOWER IS SLOWER"

$$|\tau_0\rangle = \frac{1}{\sqrt{2}}\left(|E_0\rangle + |E_1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|\gamma_1\rangle|\tau_1\rangle + e^{i\Delta\phi}|\gamma_2\rangle|\tau_2\rangle\right)$$



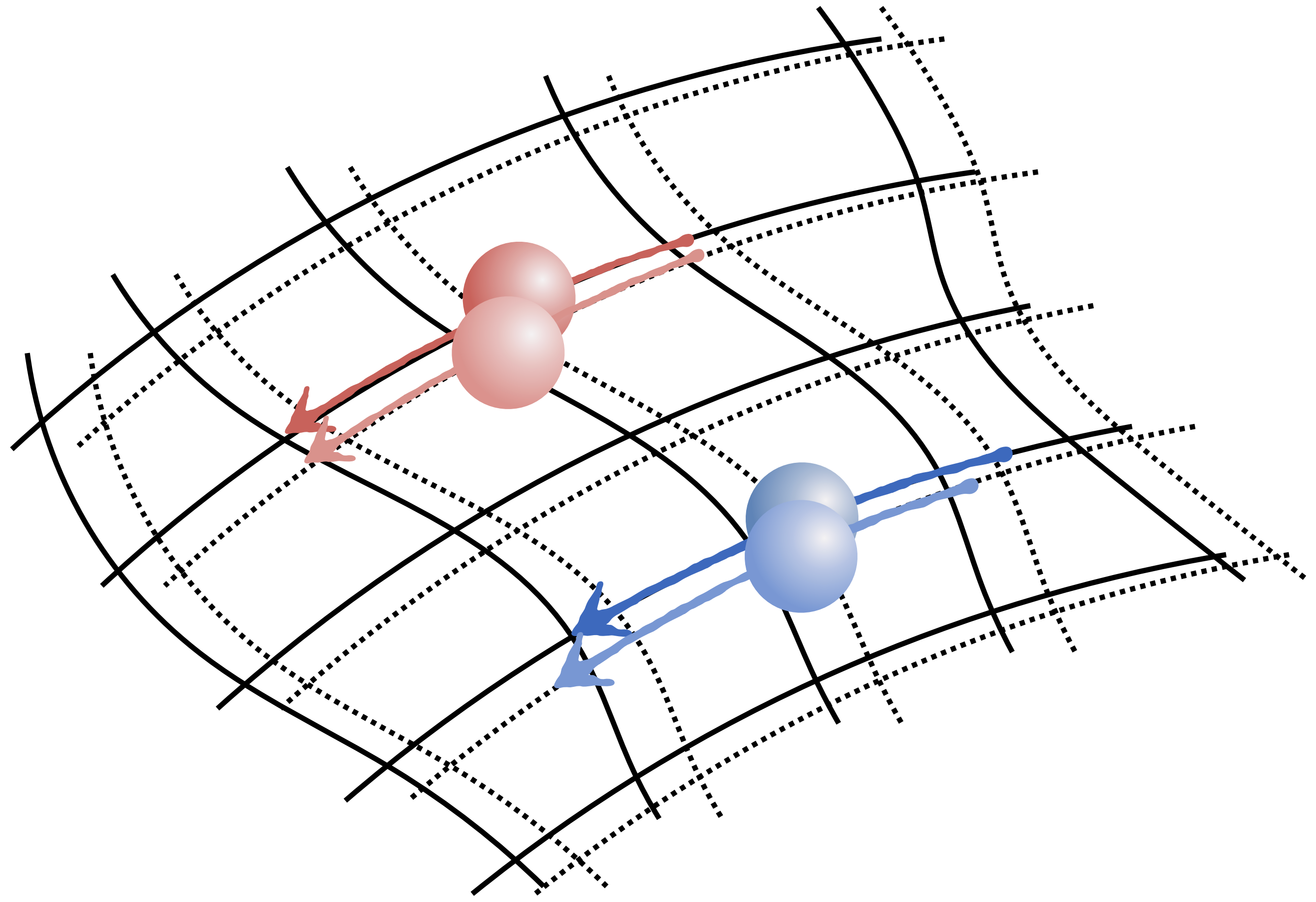
$$\mathscr{V} = |\langle\tau_1|\tau_2\rangle|^2$$ **Visibility (interference)**

$$\mathscr{D} = 1 - |\langle\tau_1|\tau_2\rangle|^2$$ **Distinguishability (which-way information)**

The clock carries "which-way" information in its proper time.

Zych, Costa, Pikovski, Brukner, Nat. Commun. (2011)

QUANTUM CLOCKS IN NONCLASSICAL SPACETIME



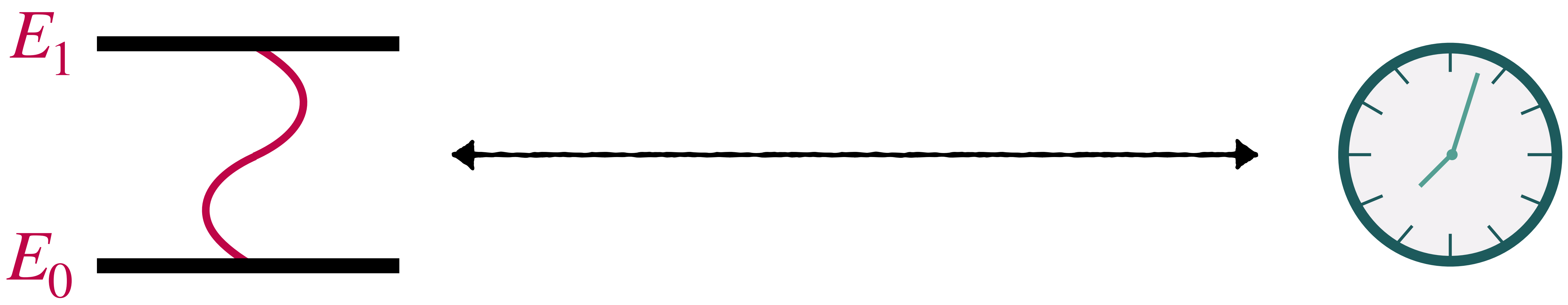
Can quantum clocks probe nonclassical spacetime?

ENTANGLEMENT OF QUANTUM CLOCKS THROUGH GRAVITY

$E_0 = 0$

$E_1 = \Delta E$

$$H = H_A + H_B - \frac{G}{c^4 x} H_A H_B$$



$$\frac{1}{\sqrt{2}}(|E_0\rangle + |E_1\rangle)$$

$$|\psi_t\rangle = e^{-\frac{i}{\hbar}\hat{H}t}|\psi_0\rangle$$

$$\propto |E_0\rangle_A(|E_0\rangle_B + e^{-\frac{i}{\hbar}\Delta E t}|E_1\rangle_B) + |E_1\rangle_A(|E_0\rangle_B + e^{-\frac{i}{\hbar}\left(1 - \frac{G\Delta E}{c^4 x}\right)\Delta E t}|E_1\rangle_B)$$

Difference between the ticks due to clock A

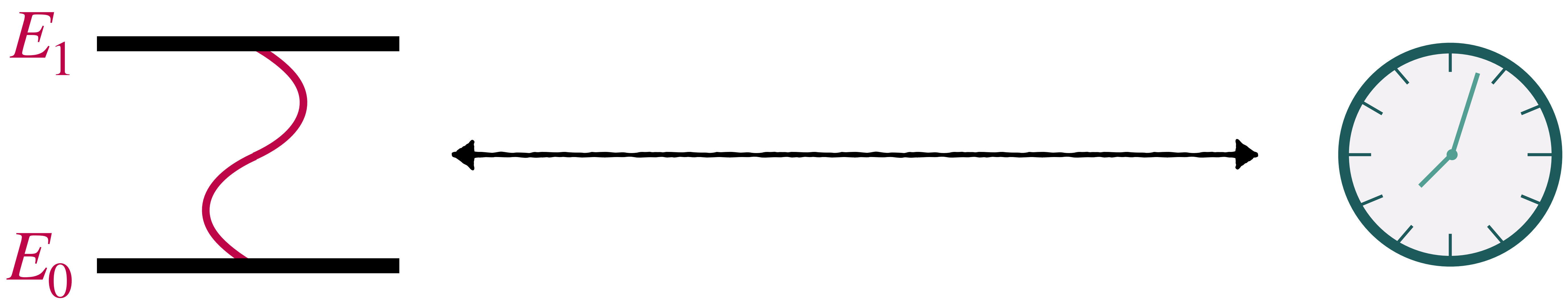
Ticking of clock B when A is in E_0

Ticking of clock B when A is in E_1

Castro Ruiz, Giacomini, Brukner, PNAS (2017)

ENTANGLEMENT OF QUANTUM CLOCKS THROUGH GRAVITY

$$H = H_A + H_B - \frac{G}{c^4 x} H_A H_B$$



$$\frac{1}{\sqrt{2}}(|E_0\rangle + |E_1\rangle)$$

Precision of clock A

$$t_\perp = \frac{\pi\hbar}{(E_1 - E_0)}$$

Disturbance to clock B due to clock A

$$\Delta t = \frac{G(E_1 - E_0)}{c^4 x} t$$

$$t_\perp \Delta t = \frac{\pi\hbar G t}{c^4 x}$$

Limitation to the measurability of time for nearby clocks

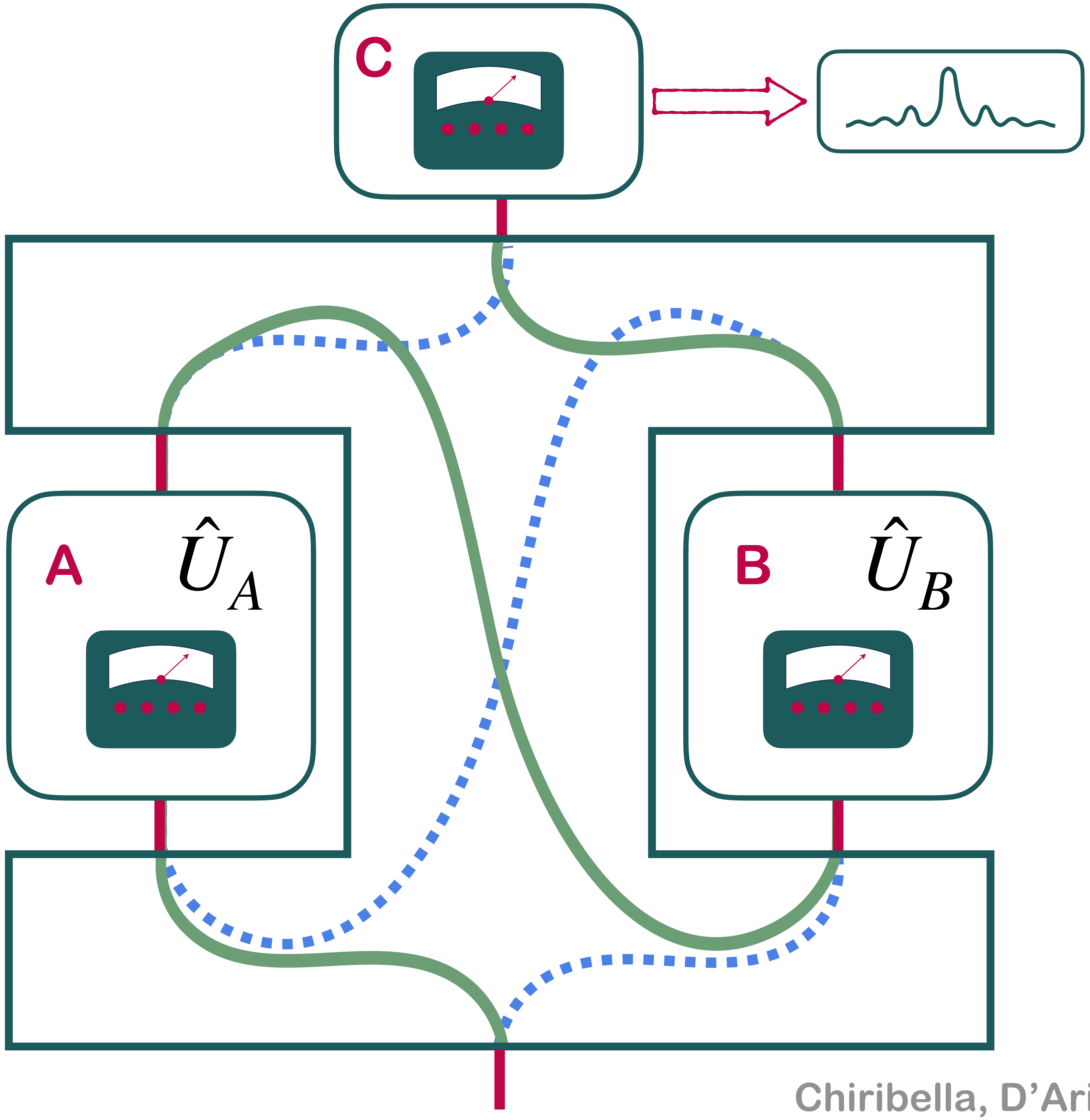
Castro Ruiz, Giacomini, Brukner, PNAS (2017)

THE QUANTUM SWITCH: SUPERPOSITION OF CAUSAL ORDERS

Process matrix with 3 parties: A, B, C

T: target qubit
C: control qubit

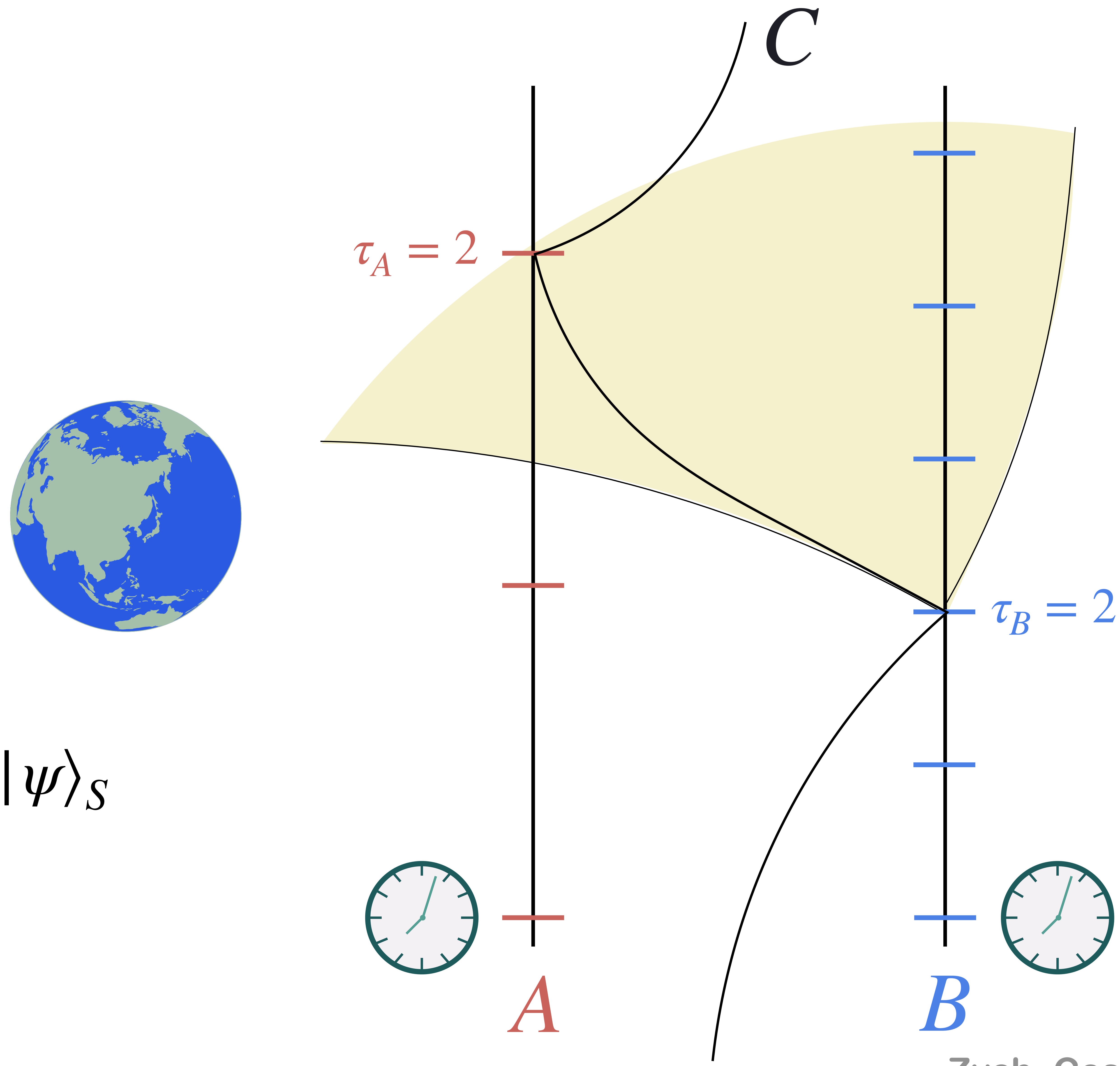
$$\frac{1}{\sqrt{2}}\Big(|0\rangle_C \hat{U}_B \hat{U}_A |\psi\rangle + |1\rangle_C \hat{U}_A \hat{U}_B |\psi\rangle\Big)$$



Measurement in $|\pm\rangle_C\langle\pm|$

Chiribella, D'Ariano, Perinotti, Valiron, PRA (2013)
Araujo, Costa, Brukner, PRL (2014)

THE GRAVITATIONAL QUANTUM SWITCH

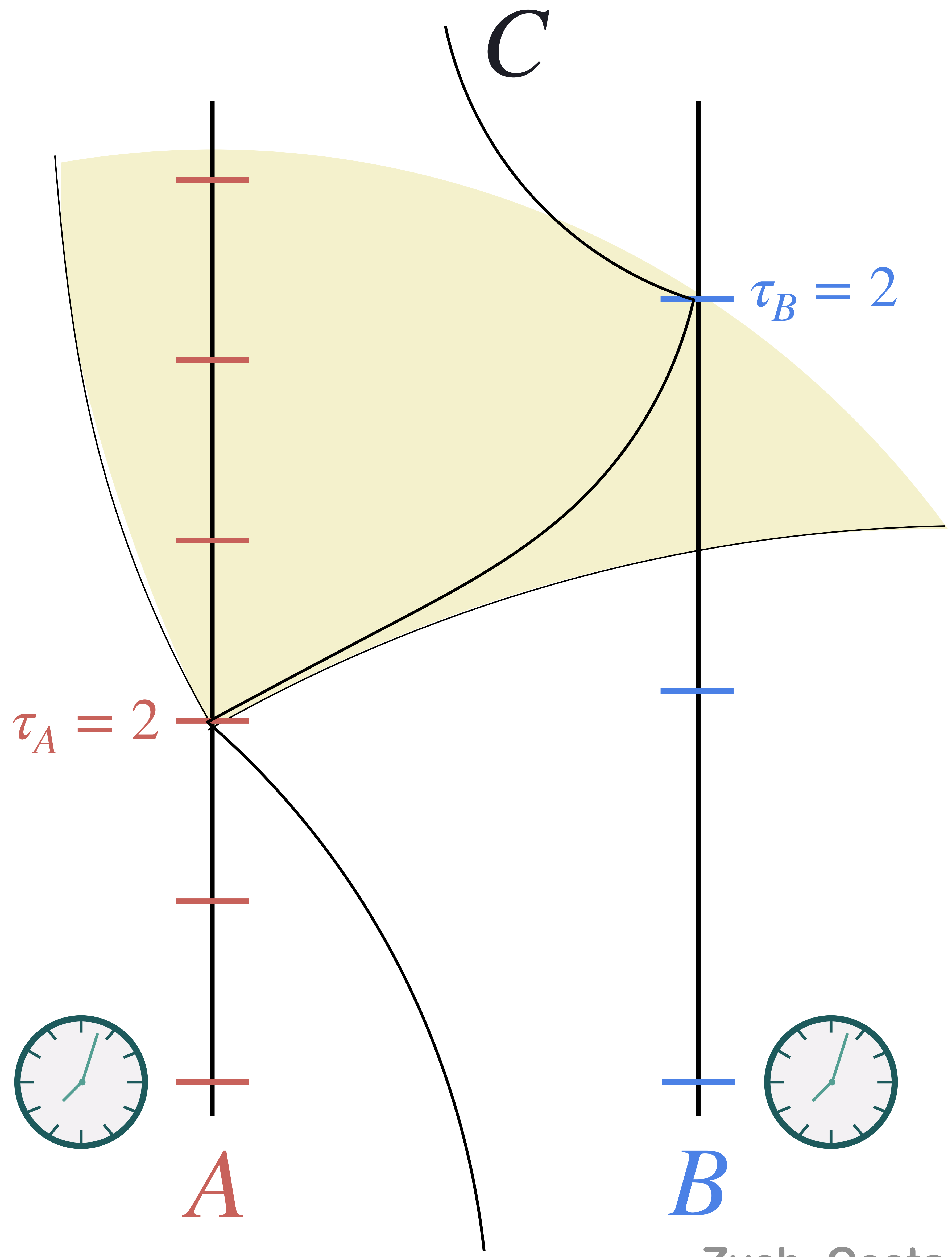


$$|L\rangle_M \hat{U}_A \hat{U}_B |\psi\rangle_S$$

Zych, Costa, Pikovski, Brukner, Nat. Commun. (2019)

THE GRAVITATIONAL QUANTUM SWITCH

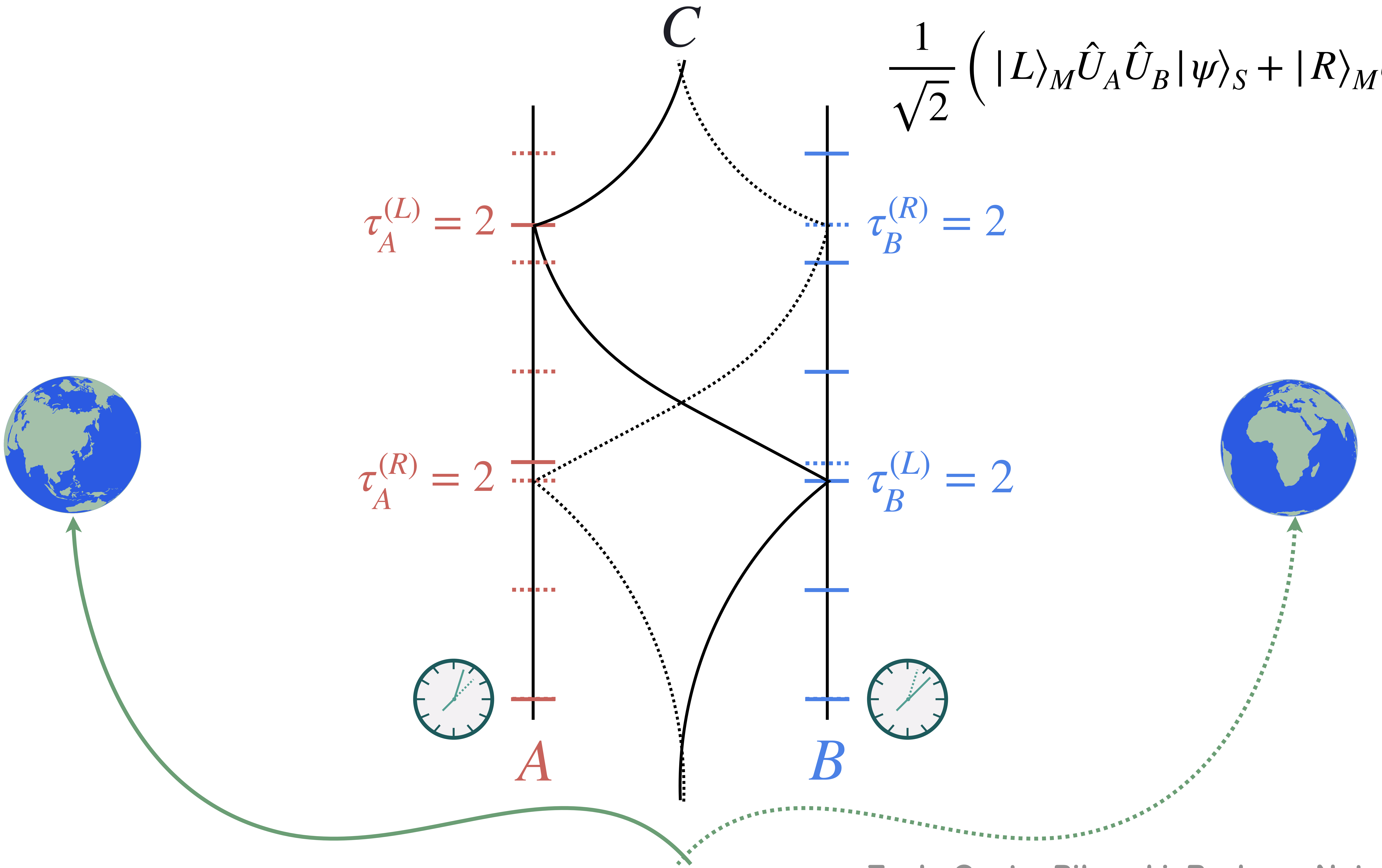
$$|R\rangle_M \hat{U}_B \hat{U}_A |\psi\rangle_S$$



Zych, Costa, Pikovski, Brukner, Nat. Commun. (2019)

THE GRAVITATIONAL QUANTUM SWITCH

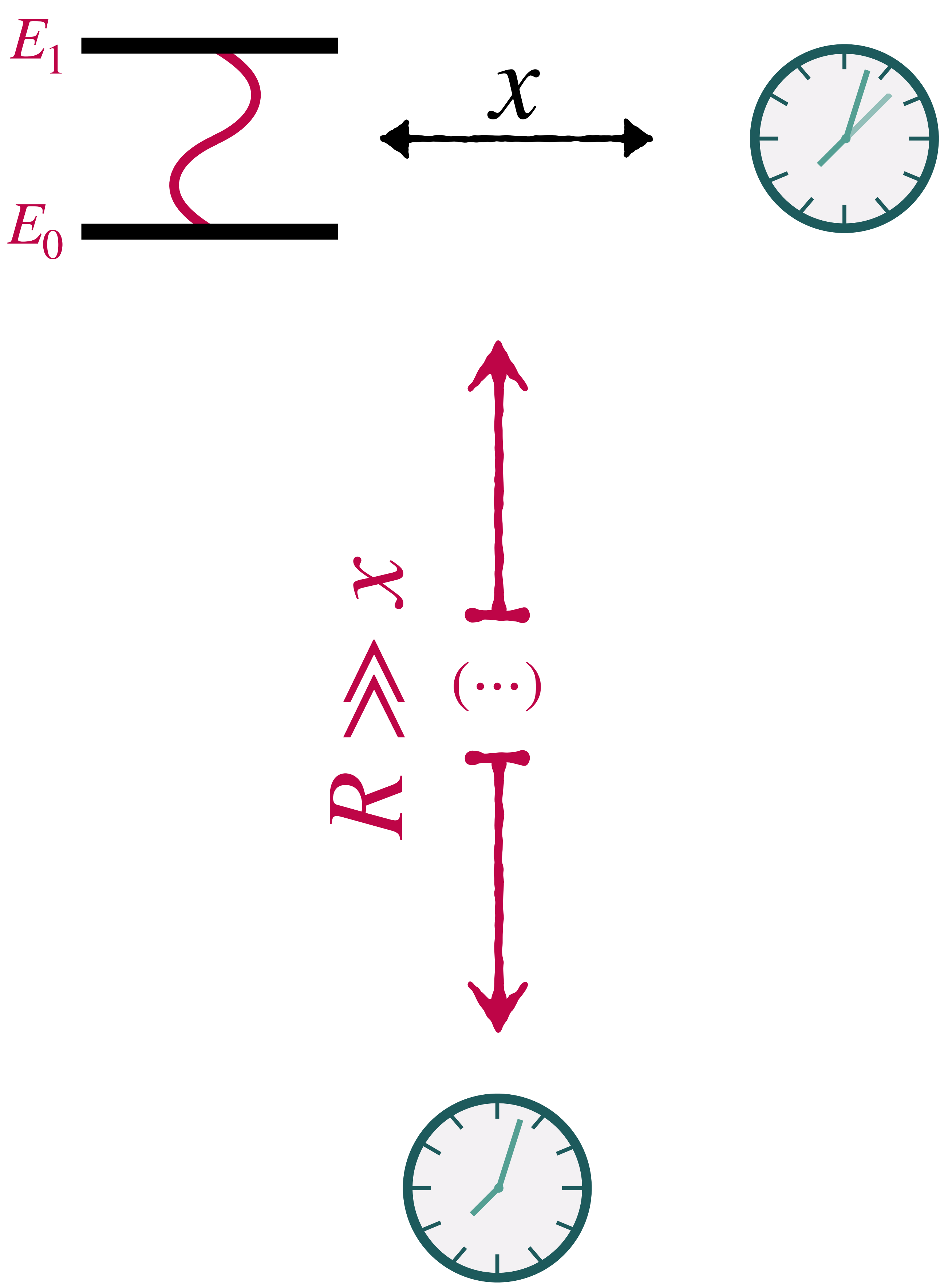
$$\frac{1}{\sqrt{2}}\left(|L\rangle_M \hat{U}_A \hat{U}_B |\psi\rangle_S + |R\rangle_M \hat{U}_B \hat{U}_A |\psi\rangle_S\right)$$



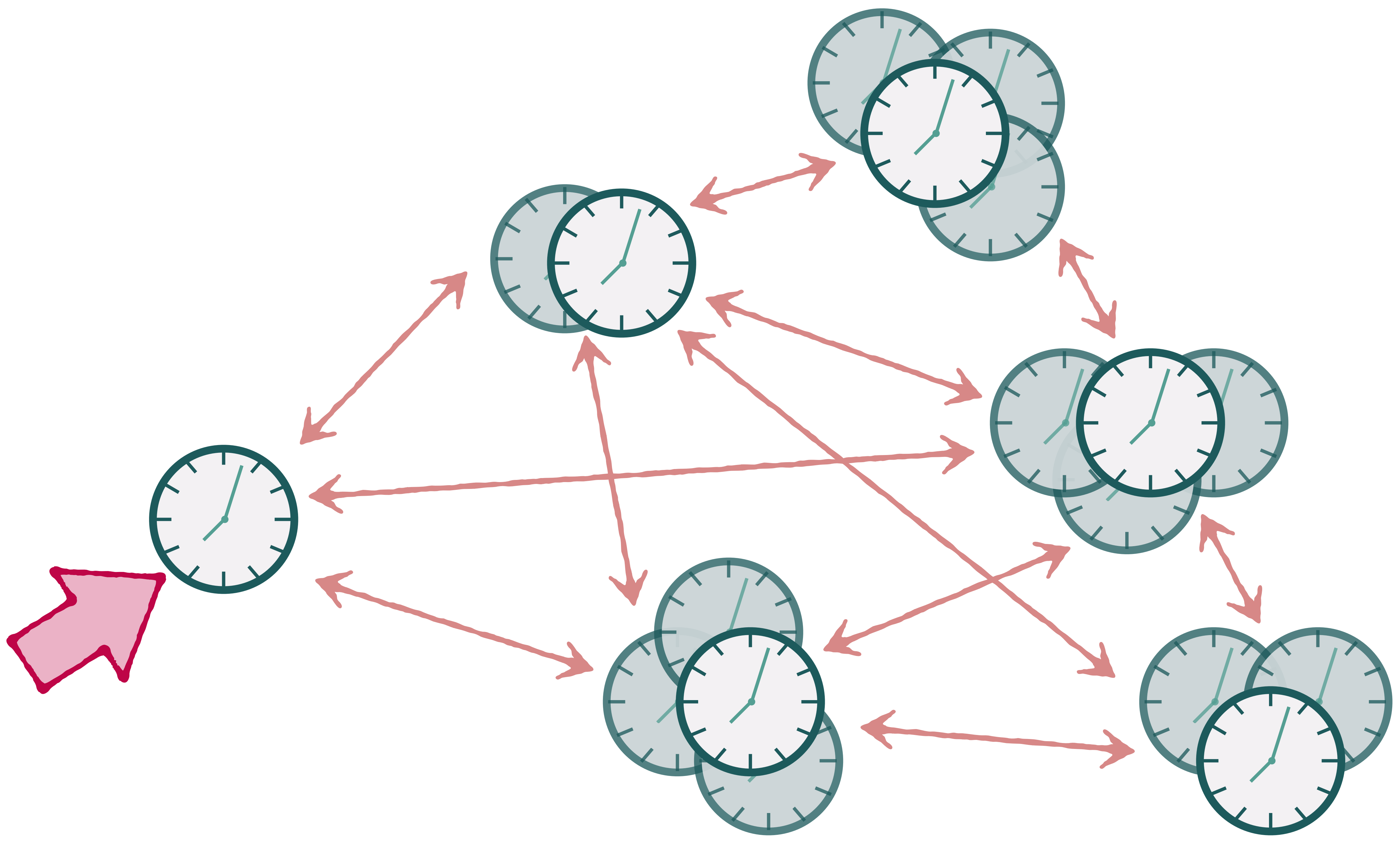
Zych, Costa, Pikovski, Brukner, Nat. Commun. (2019)

WHO WRITES THE QUANTUM STATE?

The far-away observer picture



The quantum clock "local" picture



Can we "stand" on different clocks and describe the quantum dynamics from their point of view?

HOW DO QUANTUM CLOCKS TELL THE TIME?

PAGE-WOOTTERS FORMALISM

$$\hat{C} = \hat{H}_A \otimes \mathbb{I}_B + \mathbb{I}_A \otimes \hat{H}_B$$

$$[\hat{T}_i, \hat{H}_i] = i\hbar \qquad \hat{C}|\Psi_{ph}\rangle = 0$$

$$|\Psi_{ph}\rangle = \int d\tau e^{\frac{i}{\hbar}\hat{C}\tau}|\phi\rangle_{AB}$$

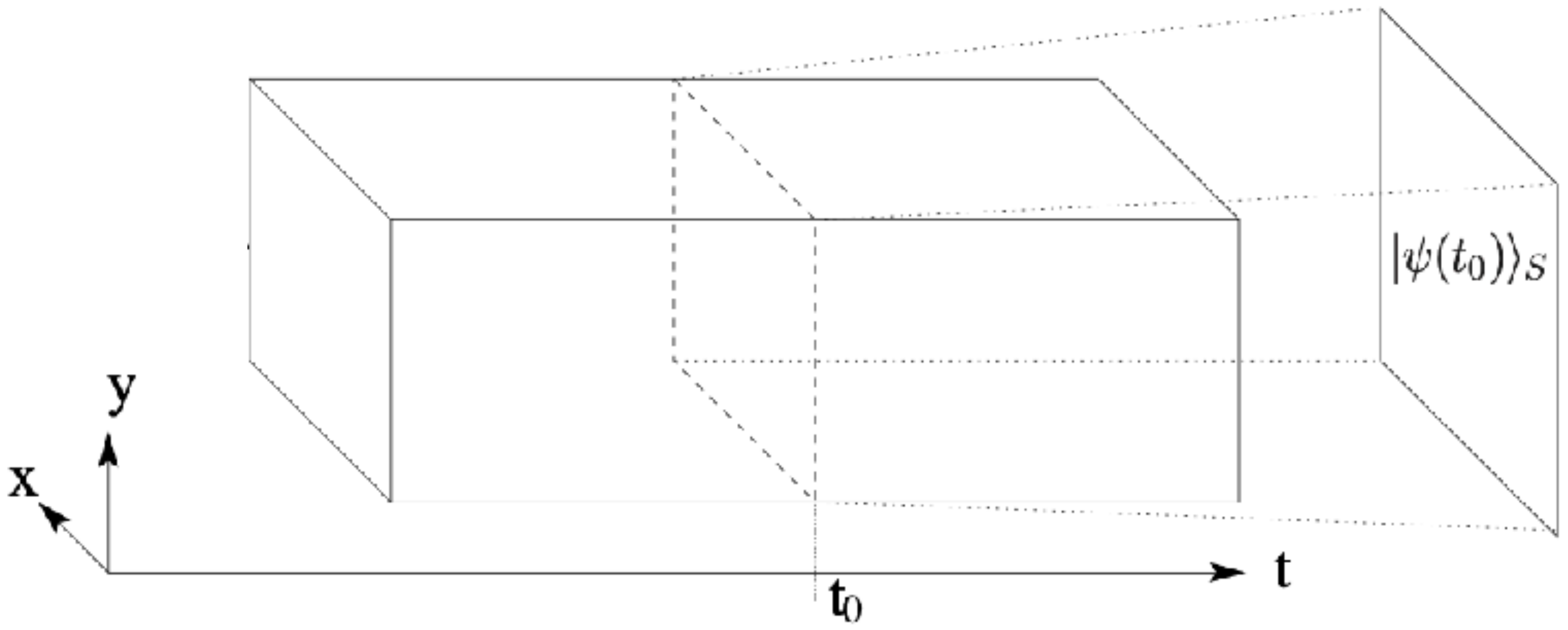


figure from Giovannetti, Lloyd, Maccone, PRD (2015)

Why is the state “timeless”?

$$e^{-\frac{i}{\hbar}\hat{C}\tau'}|\Psi_{ph}\rangle = \int d\tau e^{\frac{i}{\hbar}\hat{C}(\tau\_\tau')}|\phi\rangle_{AB} = |\Psi_{ph}\rangle$$

Page, Wootters, PRD (1983)
Giovannetti, Lloyd, Maccone, PRD (2015)

PAGE-WOOTTERS FORMALISM

$$\hat{C} = \hat{H}_A \otimes \mathbb{I}_B + \mathbb{I}_A \otimes \hat{H}_B$$

$$[\hat{T}_i, \hat{H}_i] = i\hbar \qquad \hat{C}|\Psi_{ph}\rangle = 0$$

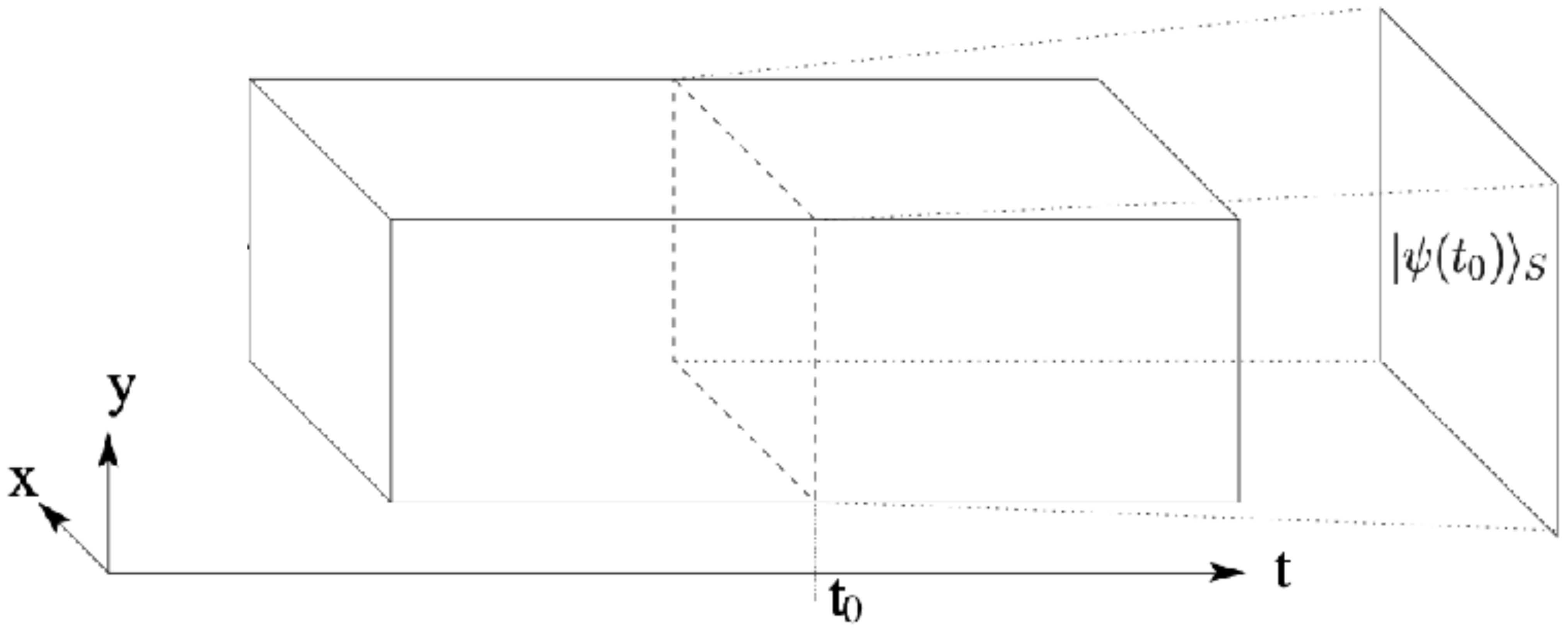


figure from Giovannetti, Lloyd, Maccone, PRD (2015)

How do we recover the dynamics?

$$_A\langle t|\Psi_{ph}\rangle = |\psi(t)\rangle_B$$

$$_A\langle t|(\hat{H}_A + \hat{H}_B)|\Psi_{ph}\rangle = 0$$

$$i\hbar\frac{d}{dt}|\psi(t)\rangle_B = \hat{H}_B|\psi(t)\rangle_B$$

Dynamics emerges from correlations between systems

$$|\Psi_{ph}\rangle = \int dt\,|t\rangle_A|\psi(t)\rangle_B$$

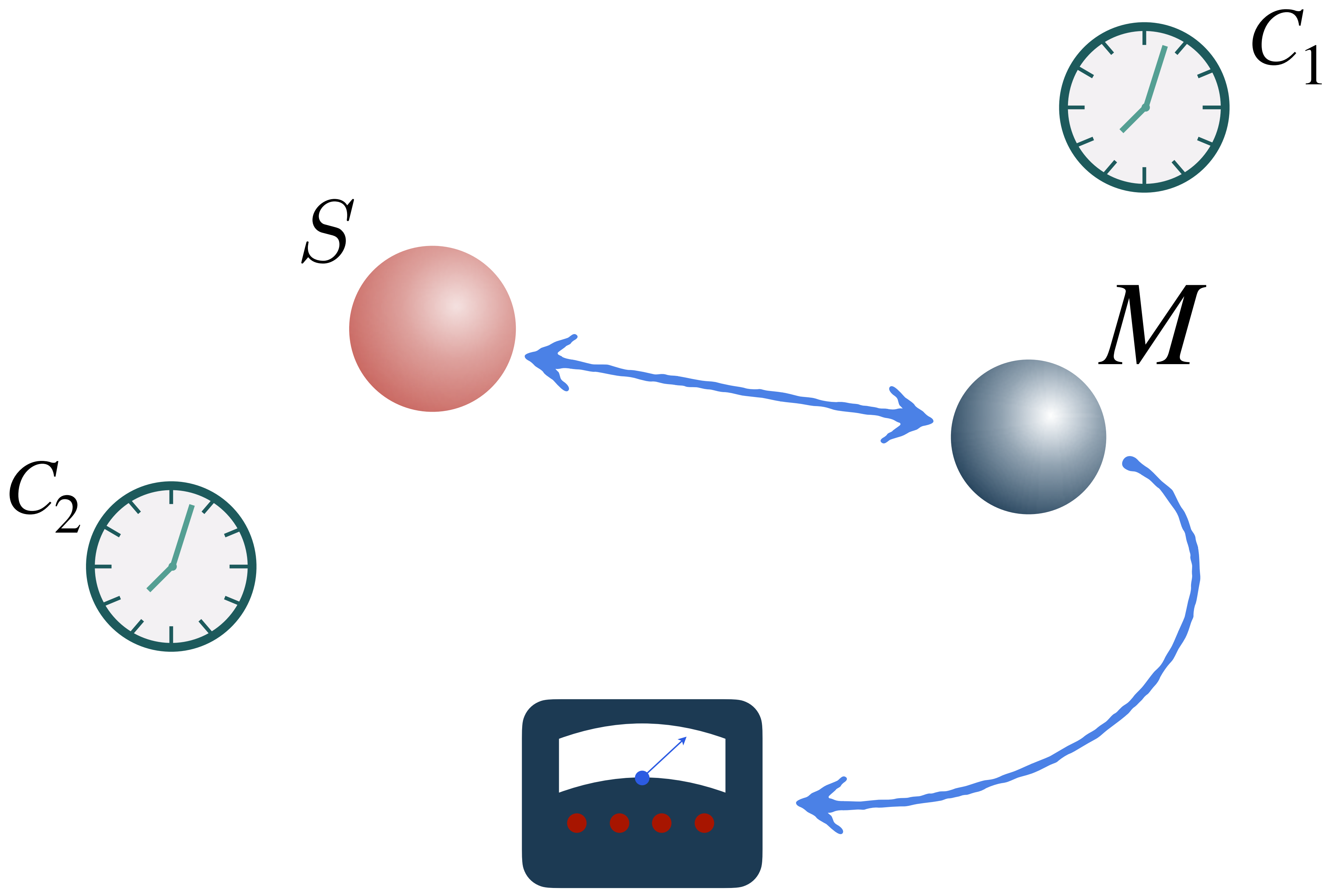
History state

Page, Wootters, PRD (1983)
Giovannetti, Lloyd, Maccone, PRD (2015)

MEASUREMENTS IN THE PAGE-WOOTTERS FORMALISM

$$[\hat{C}, \hat{O}] = 0$$

Non-evolving quantities?
Restriction of observables?



Solution: "Purify" the measurement

Clocks 1 and 2

System S

Measurement system M

Previous Hamiltonian

$$\hat{C} = \overbrace{\hat{H}_1 + \hat{H}_2 + \hat{H}_S}^{\text{Previous Hamiltonian}} + (1 + \lambda \hat{H}_1) \sum_i \delta(\hat{T}_2 - t_i) \hat{K}_i^{MS}$$

Time dilation factor due to clock 1

Time of measurement controlled by clock 2

Observable on S and M

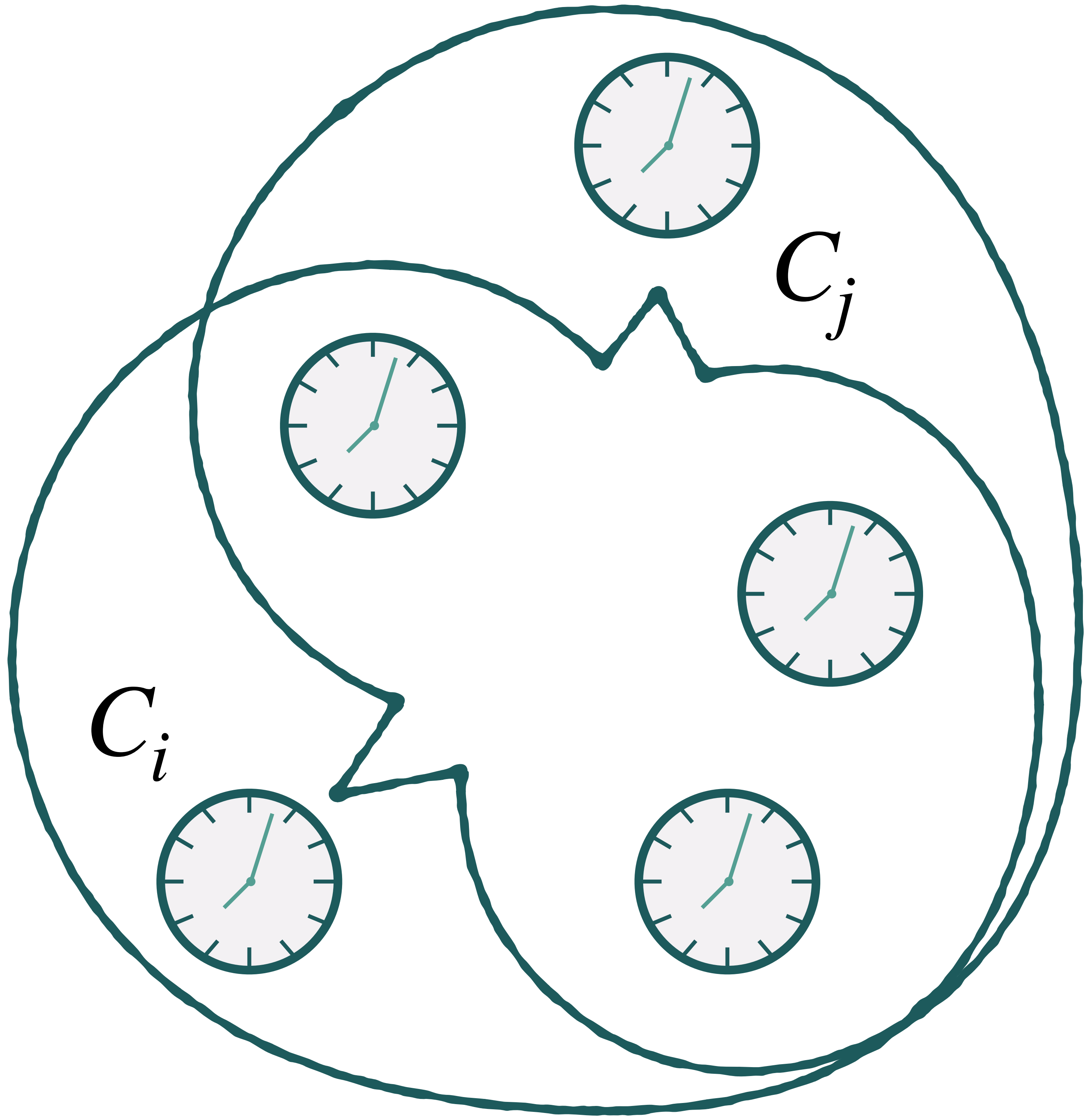
F Hellmann, M Mondragon, A Perez, C Rovelli PRD (2007)
V Giovannetti, S Lloyd, L Maccone, PRD (2015)

(GRAVITATIONALLY) INTERACTING QUANTUM CLOCKS

$$\hat{C}|\Psi\rangle_{ph} = 0$$

$$\hat{C} = \sum_{k=1}^{N} \hat{H}_k + \sum_{j<k} \lambda_{jk} \hat{H}_j \hat{H}_k$$

$$\lambda_{jk} = -\frac{G}{c^4 x_{jk}}$$



Perspective of clock i

$${}_i\langle t_i|\Psi\rangle_{Ph} = |\psi(t_i)\rangle^{(i)}$$

$$i\hbar\left(1 + \sum_{k\neq i} \lambda_{ik}\hat{H}_k\right)\frac{d|\psi(t_i)\rangle^{(i)}}{dt_i} = \left(\sum_{k\neq i}\hat{H}_k + \sum_{j<k}\lambda_{jk}\hat{H}_j\hat{H}_k\right)|\psi(t_i)\rangle^{(i)}$$

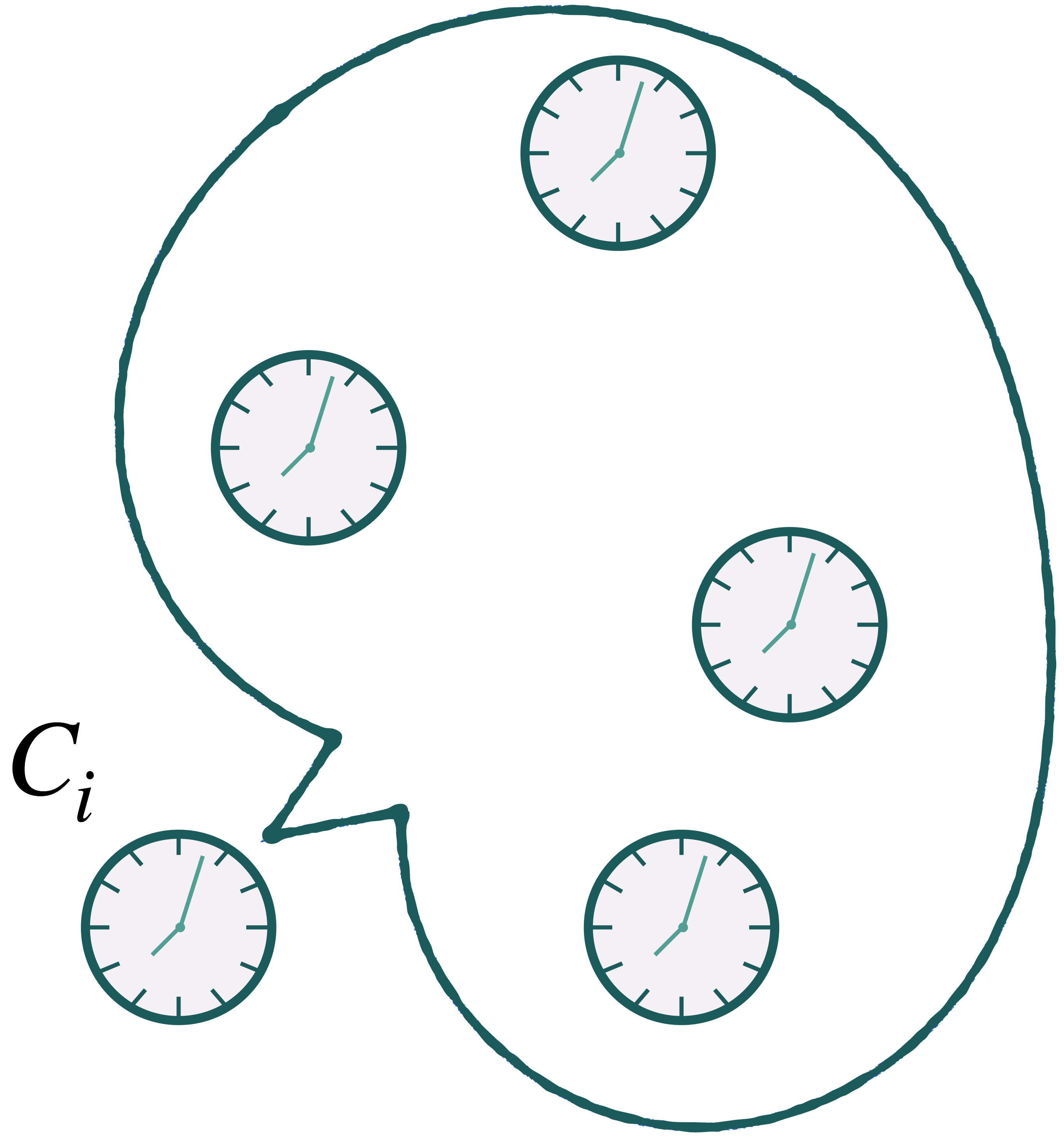
Check also
Smith Ahmadi, Quantum (2019)

Castro Ruiz, Giacomini, Belenchia, Brukner, Nat. Commun. (2020)

(GRAVITATIONALLY) INTERACTING QUANTUM CLOCKS

$$i\hbar \frac{d|\psi(t_i)\rangle^{(i)}}{dt_i} = \left(1 - \sum_{k \neq i} \lambda_{ik} \hat{H}_k\right)\left(\sum_{k \neq i} \hat{H}_k + \sum_{j<k} \lambda_{jk} \hat{H}_j \hat{H}_k\right) |\psi(t_i)\rangle^{(i)}$$

$$\lambda_{jk} = -\frac{G}{c^4 x_{jk}}$$



RESULTS

1. Recover clock Hamiltonian (far-away observer and clock description)

$$H = H_A + H_B - \frac{G}{c^4 x} H_A H_B$$

2. Relative localisation of events
3. Gravitational quantum switch

Castro Ruiz, Giacomini, Belenchia, Brukner, Nat. Commun. (2020)

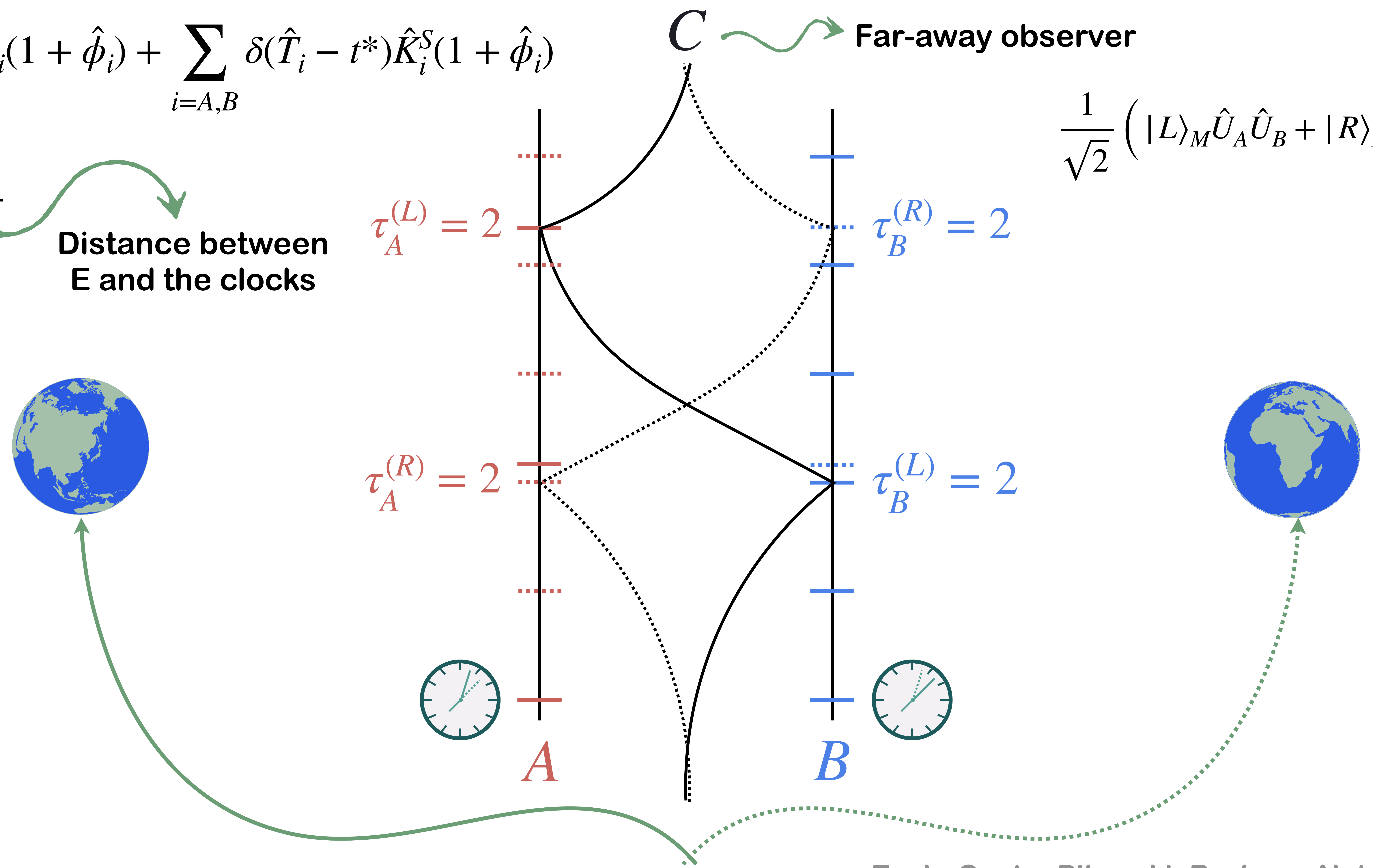
THE GRAVITATIONAL QUANTUM SWITCH

$$\hat{C} = \sum_{i=A,B,C} \hat{H}_i(1+\hat{\phi}_i) + \sum_{i=A,B} \delta(\hat{T}_i - t^*)\hat{K}_i^S(1+\hat{\phi}_i)$$

$$\hat{\phi}_i = -\frac{GM_E}{c^2\hat{x}_i}$$

Distance between E and the clocks

$$\frac{1}{\sqrt{2}}\left(|L\rangle_M \hat{U}_A \hat{U}_B + |R\rangle_M \hat{U}_B \hat{U}_A |\psi\rangle_S\right)$$



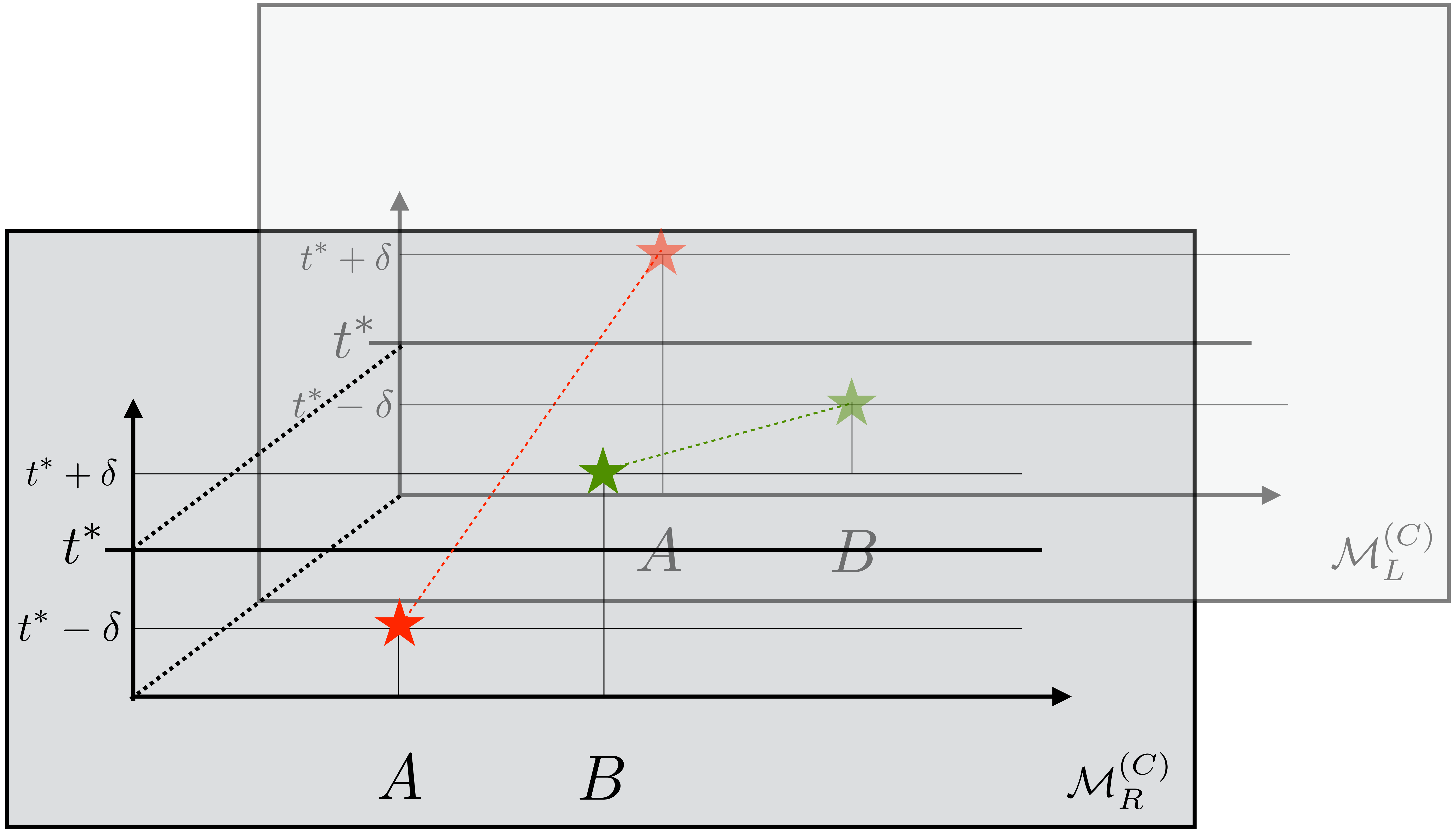
Zych, Costa, Pikovski, Brukner, Nat. Commun. (2019)

RELATIVE LOCALISATION OF EVENTS IN THE GRAVITATIONAL SWITCH

$$\hat{C} = \sum_{i=A,B,C} \hat{H}_i(1+\hat{\phi}_i) + \sum_{i=A,B} \delta(\hat{T}_i - t^*)\hat{K}_i^S(1+\hat{\phi}_i)$$

$$\hat{\phi}_i = -\frac{GM_E}{c^2\hat{x}_i}$$

From C's point of view



Castro Ruiz, Giacomini, Belenchia, Brukner, Nat. Commun. (2020)

RELATIVE LOCALISATION OF EVENTS IN THE GRAVITATIONAL SWITCH

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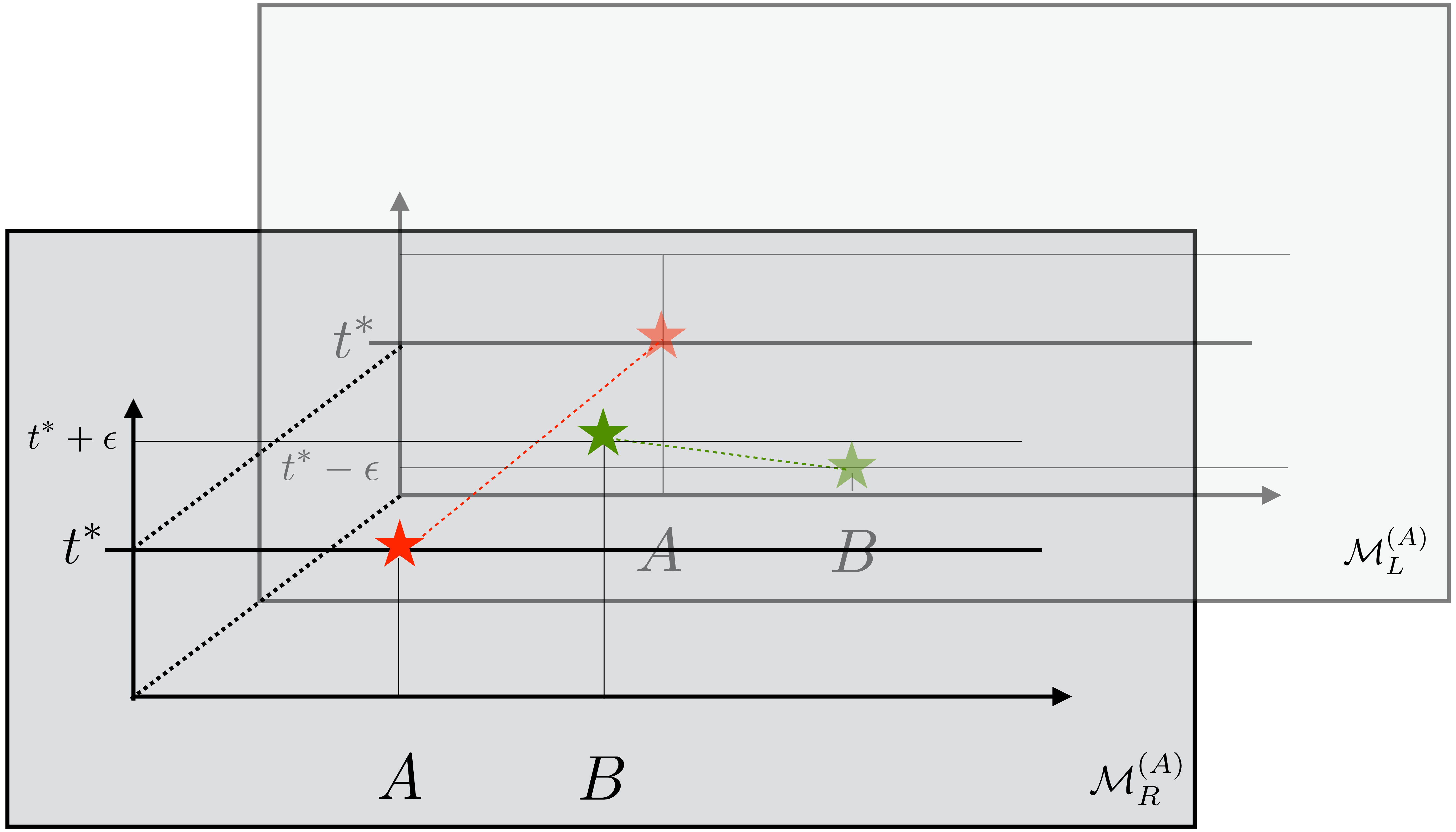
$$\hat{\phi}_i = -\frac{GM_E}{c^2\hat{x}_i}$$

From A's point of view



Castro Ruiz, Giacomini, Belenchia, Brukner, Nat. Commun. (2020)

SUMMARY

Quantum clocks are a powerful tool to test the interface between general relativity and quantum theory.

Simplest clock is a two-level system.

Fundamentally, mass-energy equivalence plays a role.

In classical spacetime, quantum clocks can encode information about their position.

In nonclassical spacetime:

- limits to measurability
- relative localisation of events
- relation to indefinite causal order

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