QUANTUM INFORMATION TOOLS AT THE INTERFACE BETWEEN QUANTUM THEORY AND GRAVITY

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Image credits: J. Palomino

Basics of Quantum Gravity
22-25 May 2023

LECTURE 4: QUANTUM REFERENCE FRAMES

- Superposition and entanglement depend on the QRF
- Covariance of physical laws and extended symmetry transformations
- Group structure of Galilean quantum reference frames
- Perspective neutral approach and relationalism
- Coherent and incoherent group average: what is the difference?
- QRFs in spacetime

Final remarks and connections between the topics of the lectures

QUANTUM REFERENCE FRAMES - THE BASICS

Image credits: C. Murzek

How do we deal with the lack of a classical reference frame?

A HISTORICAL COINCIDENCE…

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Quantum Theory of Gravity. I. The Canonical Theory

Bryce S. DeWitt
Phys. Rev. **160**, 1113 – Published 25 August 1967

PHYSICAL REVIEW JOURNALS ARCHIVE
Published by the American Physical Society

Journals Authors Referees Browse Search Press

Charge Superselection Rule

Yakir Aharonov and Leonard Susskind
Phys. Rev. **155**, 1428 – Published 25 March 1967

Which QRFs?

Diversity of approaches:
Quantum Information/Communication
Quantum Foundations
Quantum Gravity

(sometimes the distinction is not sharp…)

Reference frames associated to physical systems

What are QRFs?

Measurement

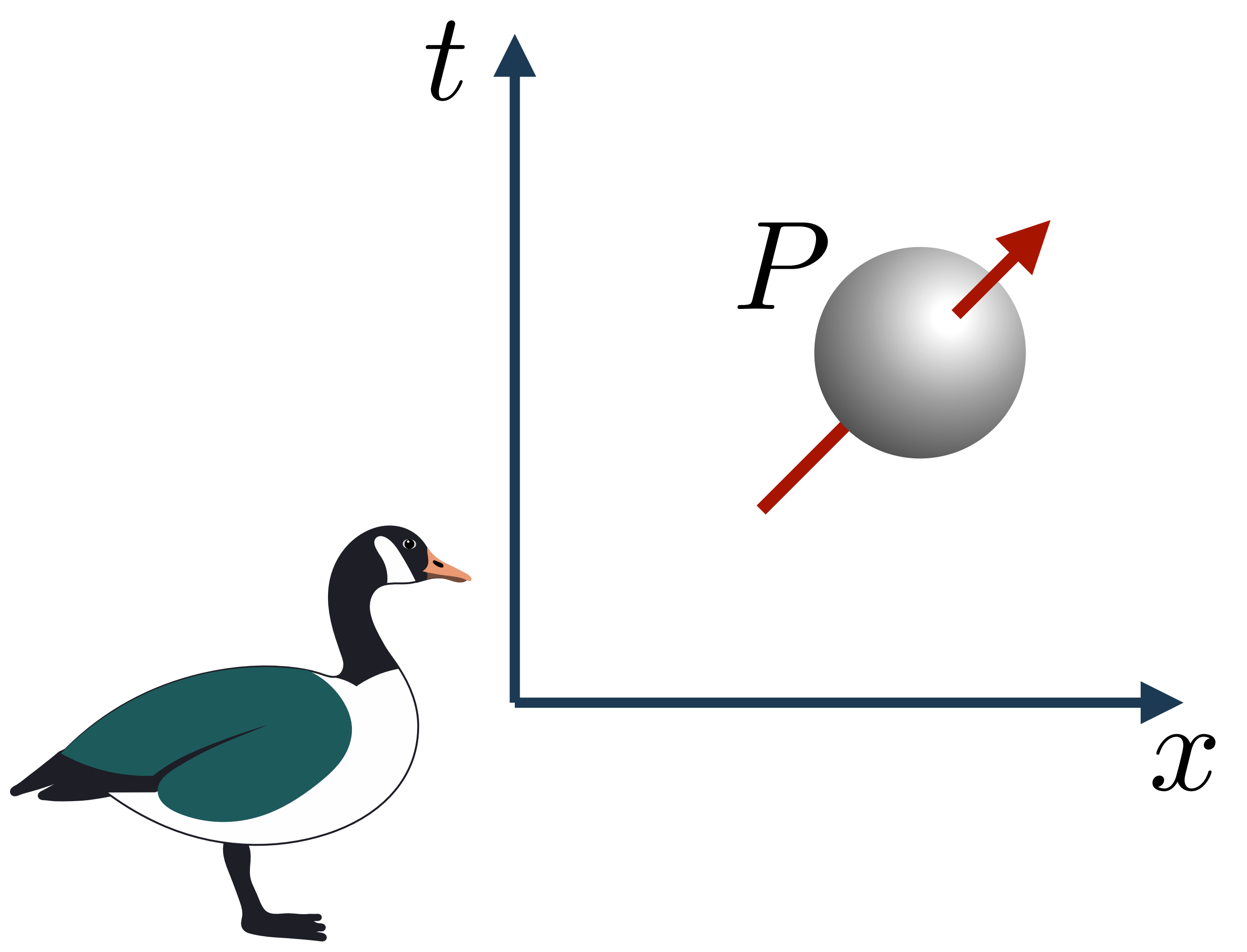
Relational Information

Reference Frames/Observers

Resources: Bounded/Unbounded

One perspective is not enough!

All reference frames are equally good for describing the laws of physics.



Translation $\hat{U}_T = e^{\frac{i}{\hbar}X_0\hat{p}}$

Galilean boost $\hat{U}_B = e^{\frac{i}{\hbar}v\hat{G}}$ $\hat{G} = \hat{p}t - m\hat{x}$

⋮

The reference frame enters the transformation as a parameter.

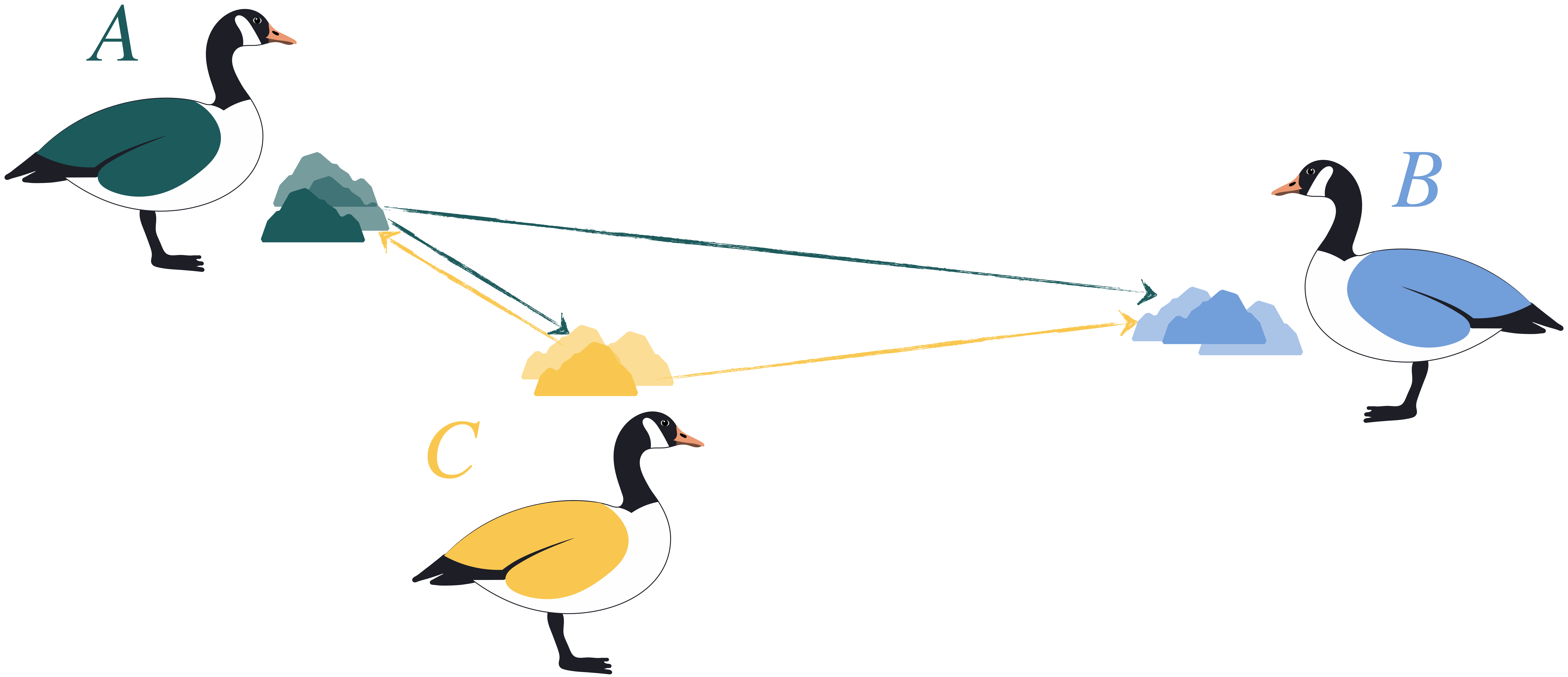
Covariance of physical laws

$$\hat{H}' = \hat{U}\hat{H}\hat{U}^\dagger + i\hbar\frac{d\hat{U}}{dt}\hat{U}^\dagger$$

Symmetry

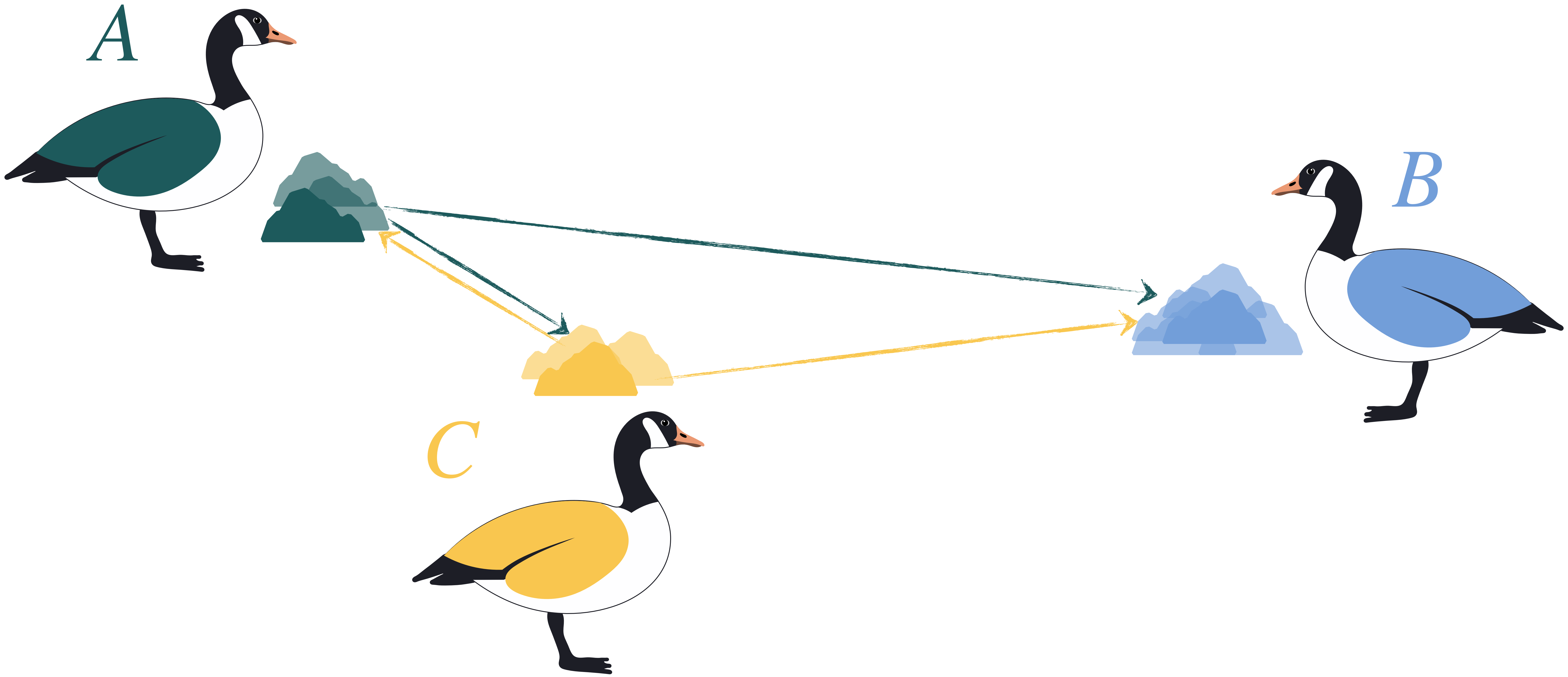
$$\hat{H}' = \hat{H}$$

QUANTUM REFERENCE FRAMES (QRFs)



Can we "attach" a reference frame to an object whose state is in a superposition of classical states (in some basis)?

QUANTUM REFERENCE FRAMES (QRFs)



QUANTUM REFERENCE FRAME TRANSFORMATIONS

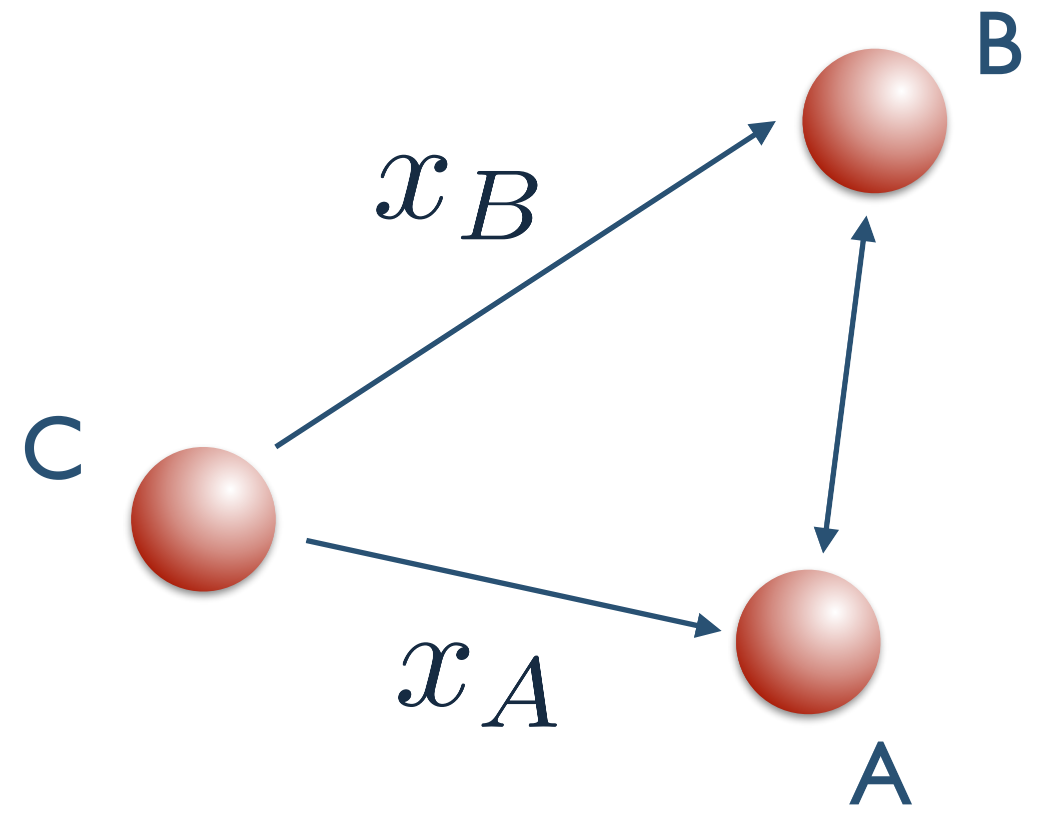
The simplest case: Transformation to relative coordinates

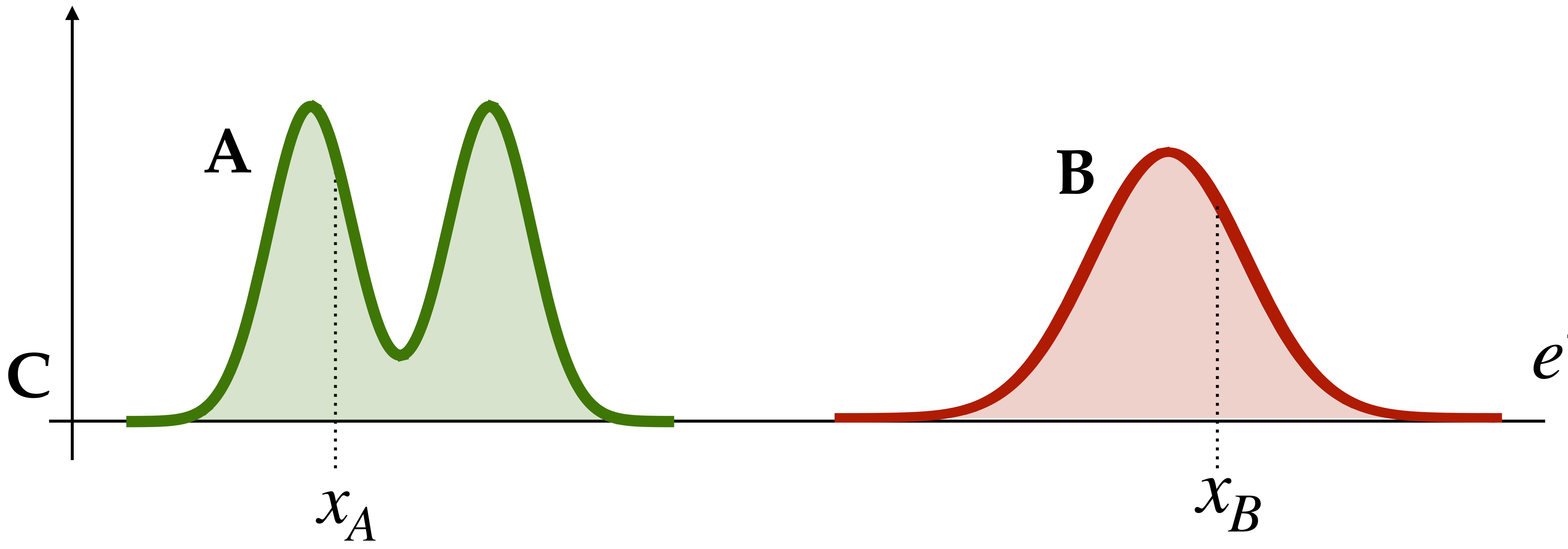
Giacomini, Castro-Ruiz, Brukner, Nat. Commun. (2019)

$$x_A \mapsto -q_C$$

$$x_B \mapsto q_B - q_C$$

$$e^{\frac{i}{\hbar}\alpha \hat{p}_B}|x\rangle_B = |x-\alpha\rangle_B$$





$$e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}|\phi\rangle_A|\psi\rangle_B$$

$$\boxed{\hat{S}_x = \mathscr{P}_{AC}e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}}$$

$$\rho_{BC}^{(A)} = \hat{S}_x\rho_{AB}^{(C)}\hat{S}_x^\dagger$$

$$\mathscr{P}_{AC}\hat{x}_A\mathscr{P}_{AC}^\dagger = -\hat{q}_C$$

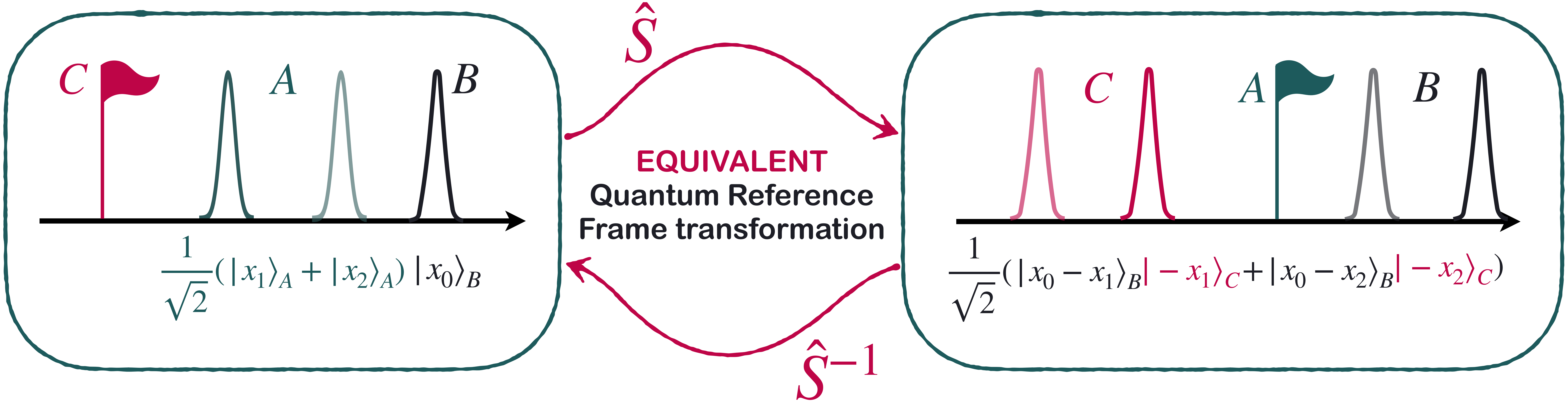
parity-swap operator

QUANTUM REFERENCE FRAME TRANSFORMATIONS

The simplest case: spatial translations in 1D

Giacomini, Castro-Ruiz, Brukner, Nat. Commun. (2019)

$$\hat{S} = \mathscr{P}_{AC}\, e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$



1. Quantum-controlled translation on state of A
2. QRF has a Hilbert space assigned to it

QUANTUM REFERENCE FRAMES (QRFs)

Entanglement and superposition are QRF dependent

Giacomini, Castro-Ruiz, Brukner, Nat. Commun. (2019)
Vanrietvelde, Höhn, Giacomini, Castro-Ruiz, Quantum (2020)
Ahmad, Galley, Höhn, Lock, Smith, PRL (2022)
Castro-Ruiz, Oreshkov, arXiv (2021)

Extended transformations: superpositions of transformations

Giacomini, Castro-Ruiz, Brukner, Nat. Commun. (2019)
de la Hamette, Galley, Quantum (2020)
Ballesteros, Giacomini, Gubitosi, Quantum (2021)
de la Hamette, Galley, Höhn, Loveridge, Müller, arXiv (2021)

$$\hat{S} = \sum_i \hat{U}_{\alpha_i} \otimes |\alpha_i\rangle\langle\alpha_i|$$

standard reference frame transformation

- Spatial translation
- Galilean boost
- Accelerated frame
- Lorentz boost
- Diffeomorphism (GR)

state of the quantum reference frame

- Position (space and spacetime)
- Velocity
- Effective acceleration

$$\hat{U} = \Pi_i e^{i\alpha_i \hat{O}_i}$$

$$\hat{O}_i \in \mathscr{H}_B$$

$$\hat{S} = \mathscr{P}_{AC} \Pi_i e^{i\alpha_i \hat{O}_i}$$

$$\hat{O}_i \in \mathscr{H}_A \otimes \mathscr{H}_B$$

Algebra $\{\hat{O}_i\}_{i=1,\cdots,N}$

$$[\hat{O}_i, \hat{O}_j] = i \sum_k c_{ijk} \hat{O}_k$$

Galilei group of inertial transformations

$$\hat{P}_B = \hat{p}_B$$

$$\hat{G}_B = \hat{p}_B t - m_B \hat{x}_B$$

$$\hat{H}_B = \frac{\hat{p}_B^2}{2m_B}$$

$$\hat{M}_B = m_B \mathbb{I}_B$$

Group of QRF inertial transformations

$$\hat{P}_{AB} = \hat{x}_A \otimes \hat{p}_B \qquad \hat{K}_{AB} = \frac{\hat{p}_A}{m_A} \otimes \hat{G}_B$$

$$\hat{D}_A = \frac{1}{2}(\hat{x}_A \hat{p}_A + \hat{p}_A \hat{x}_A) \otimes \mathbb{I}_B \qquad \hat{Q}_A = \frac{\hat{p}_A^2}{2m_A} \otimes \mathbb{I}_B$$

$$\hat{D}_B = \mathbb{I}_A \otimes \frac{1}{2}(\hat{x}_A \hat{p}_B + \hat{p}_B \hat{x}_B) \qquad \hat{Q}_B = \mathbb{I}_A \otimes \frac{\hat{p}_B^2}{2m_B}$$

$$\hat{T} = \hat{p}_A \otimes \hat{p}_B$$

Ballesteros, Giacomini, Gubitosi, Quantum (2021)

EXTENDED COVARIANCE AND SYMMETRIES OF DYNAMICAL LAWS

A: new reference frame
B: quantum system
C: old reference frame

Schrödinger equation in C's reference frame

$$i\hbar \frac{d\hat{\rho}^{(C)}_{AB}}{dt} = [\hat{H}^{(C)}_{AB}, \hat{\rho}^{(C)}_{AB}]$$

To change to the frame of A we apply the unitary transformation $\hat{S}$

$$i\hbar \frac{d\hat{\rho}^{(A)}_{BC}}{dt} = [\hat{H}^{(A)}_{BC}, \hat{\rho}^{(A)}_{BC}]$$

The evolution in the new reference frame is unitary.

$$\hat{H}^{(A)}_{BC} = \hat{S}\hat{H}^{(C)}_{AB}\hat{S}^\dagger + i\hbar \frac{d\hat{S}}{dt}\hat{S}^\dagger$$

$$\hat{\rho}^{(A)}_{BC} = \hat{S}\hat{\rho}^{(C)}_{AB}\hat{S}^\dagger$$

We define an extended symmetry transformation as:

$$\hat{S}\hat{H}^{(C)}_{AB}(\{m_i, \hat{x}_i, \hat{p}_i\}_{i=A,B})\hat{S}^\dagger + i\hbar \frac{d\hat{S}}{dt}\hat{S}^\dagger = \hat{H}^{(A)}_{BC}(\{m_i, \hat{q}_i, \hat{\pi}_i\}_{i=B,C})$$

Valid for:
- Superposition of translations
- Superposition of Galilean boosts

Giacomini, Castro-Ruiz, Brukner, Nat. Commun. (2019)

EXAMPLE: SUPERPOSITION OF TRANSLATIONS

The new QRF is described by system A at time 0.

$$|\Psi_0\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|x_1\rangle_A + |x_2\rangle_A\right)|\phi_0\rangle_B$$

$$\hat{H}^{(C)}_{AB} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B}$$

Centre the QRF at the position of A at $t = 0$

$$\hat{S}_T = \exp\left(-\frac{i}{\hbar}\overbrace{\frac{\hat{\pi}_C^2}{2m_C}}^{\hat{Q}_C}t\right)\underbrace{\hat{\mathscr{P}}^{(x)}_{AC}\exp\left(\frac{i}{\hbar}\overbrace{\hat{x}_A\hat{p}_B}^{\hat{P}_{AB}}\right)}_{\hat{S}_x}\exp\left(\frac{i}{\hbar}\overbrace{\frac{\hat{p}_A^2}{2m_A}}^{\hat{Q}_A}t\right)$$

$\hat{S}_x$ **Transformation to relative coordinates**

NB: inertial transformation represented as product of group elements

$$\hat{H}^{(A)}_{BC} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C}$$

The free hamiltonian is symmetric under generalised translations.

Giacomini, Castro-Ruiz, Brukner, Nat. Commun. (2019)

A STEP BACK: WHAT ARE WE DOING HERE?

Start from Galilean quantum systems (no gravity)

Devise QRF transformations which:

a) Generalise standard RF transformations

b) Connect operationally to quantities that are well-defined in the lab

c) Generalise to gravitational scenarios

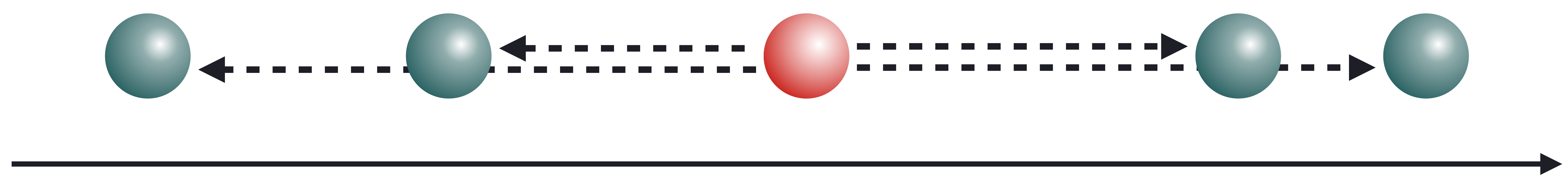
Needed in QG!

Hard to do in full QG

RELATIONALISM IN QRF TRANSFORMATIONS

1D model

$$\hat{H} = \sum_i \frac{\hat{p}_i^2}{2m_i} + \sum_{i<j} V(\hat{x}_i - \hat{x}_j) + \lambda \hat{P} \qquad \left(\hat{P} = \sum_i \hat{p}_i \approx 0\right)$$



$$\hat{P}|\Psi\rangle^{ph}_{ABC} = 0 \longrightarrow |\Psi\rangle^{ph}_{ABC} = \frac{1}{2\pi}\int d\alpha\, e^{\frac{i}{\hbar}\alpha\hat{P}}|\phi\rangle_{ABC}$$

State with zero total momentum
= coherent group averaging

$$\mathcal{T}_{A|BC} = e^{\frac{i}{\hbar}\hat{x}_A(\hat{p}_B+\hat{p}_C)}$$

$$\mathcal{T}_{A|BC}|\Psi\rangle^{ph}_{ABC} = |p_A = 0\rangle_A |\psi\rangle^{(A)}_{BC}$$

Map constraint on QRF Hilbert space

$$|\psi\rangle^{(A)}_{BC} = \int dp'_A \langle p'|\mathcal{T}_{A|BC}|\Psi\rangle^{ph}_{ABC}$$

Reduced state in the relational variables

$$\hat{S}_x = \mathcal{P}_{AC}\, e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}$$

Remember?

Vanrietvelde, Höhn, Giacomini, Castro Ruiz, Quantum (2020)
Vanrietvelde, Höhn, Giacomini, accepted in Quantum

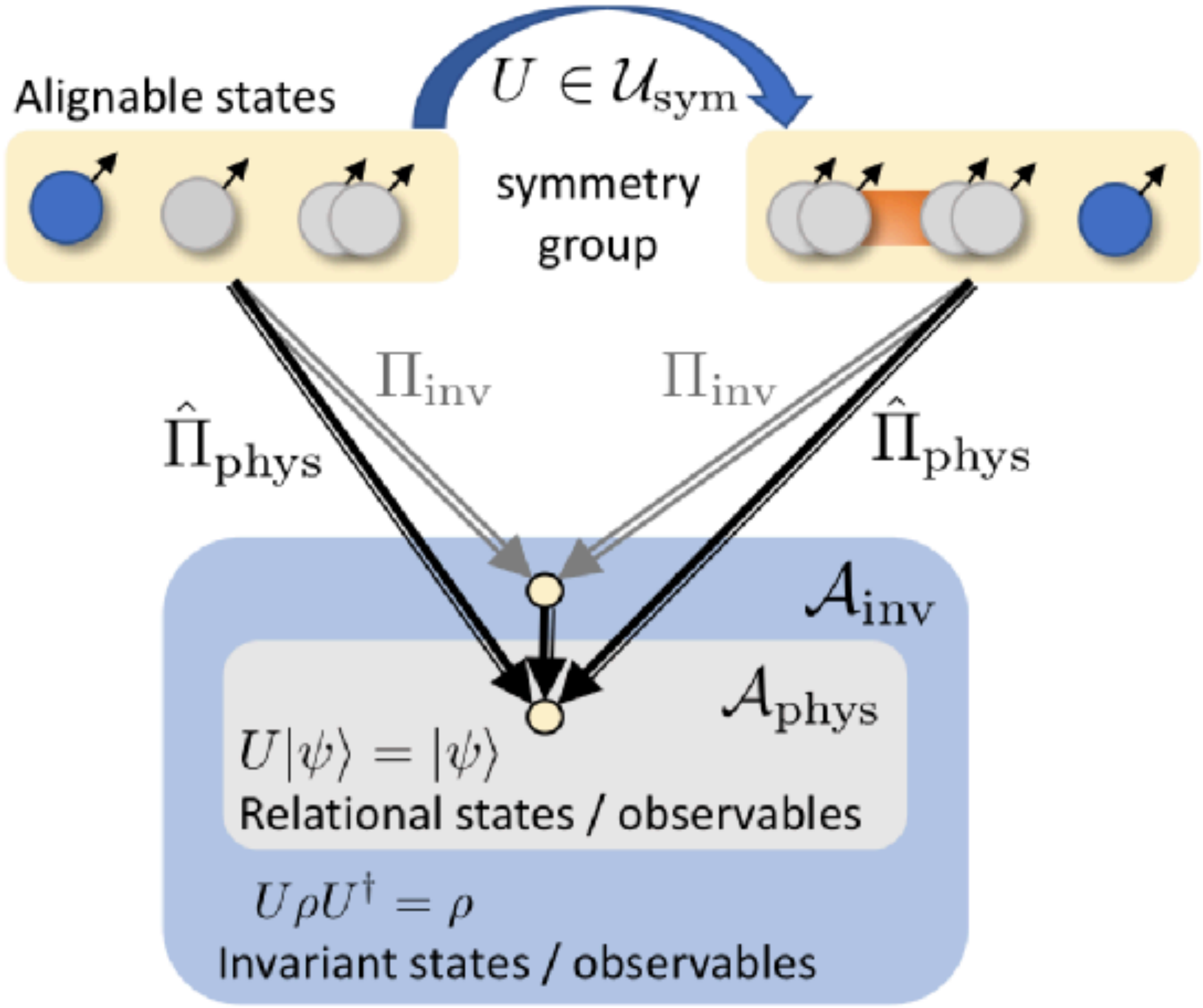
COHERENT VS. INCOHERENT GROUP AVERAGING

Invariant observables/states

$$\hat{U}\hat{A}\hat{U}^\dagger = \hat{A}$$

$$\hat{U}\hat{\rho}\hat{U}^\dagger = \hat{\rho}$$

E.g., invariant under global translations



Relational observables/states

$$\hat{U}|\psi\rangle = |\psi\rangle$$

$$\hat{P}_{tot}|\Psi\rangle_{ABC} = 0$$

$$\mathscr{A}_{inv} = \left\{ A_{phys} \oplus \left(\oplus_{p\neq 0} A_p \right) \right\}$$

States with zero total momentum

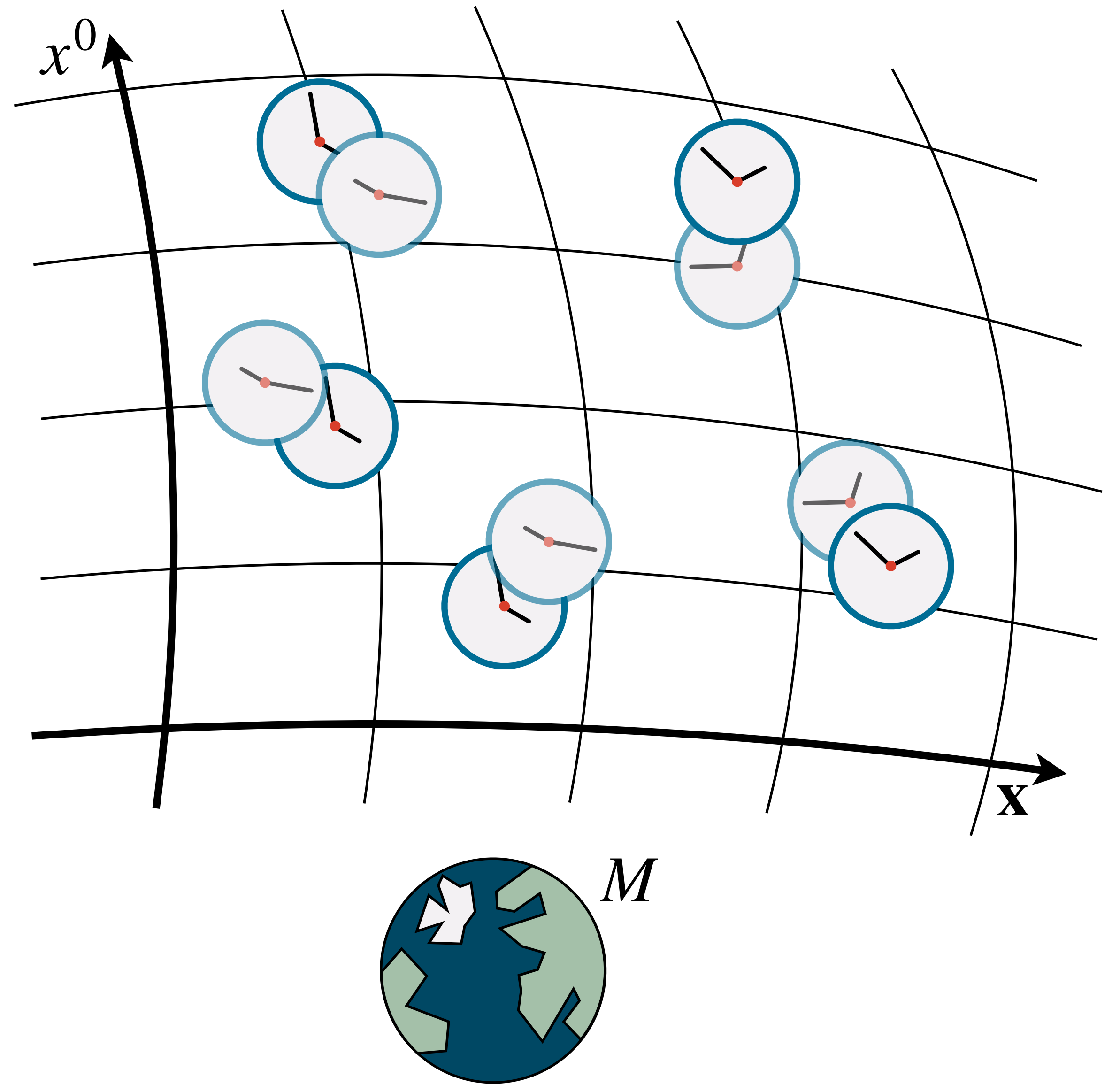
States with total momentum different to zero

$$\begin{pmatrix} \left(\mathscr{A}_{phys}\right) & & & \\ & \bullet & & \\ & & \bullet & \\ & & & \bullet \end{pmatrix}$$

Krumm, Höhn, Müller, Quantum (2021)

FROM SPACE TO SPACETIME

Quantum clocks: external and internal d.o.f.



$$\hat{C}_i = \sqrt{g^{00}(\hat{x}_i - \hat{x}_M)\hat{p}_0^i} - \frac{\hat{p}_i^2}{2m_i}$$ **Dispersion relation of the single clock**

$$\begin{cases} \hat{f}^0 = \sum_i \left(\hat{p}_0^i + \sqrt{g_{00}(\hat{x}_i - \hat{x}_M)}\frac{\hat{H}_i}{c} \right) + \hat{p}_0^M \\ \hat{f} = \sum_i \hat{p}^i + \hat{p}^M \end{cases}$$

→ **See Page-Wootters**

→ **See relational construction of** Vanrietvelde, Höhn, Giacomini, Castro Ruiz, Quantum (2020)

Conservation of the 4-momentum

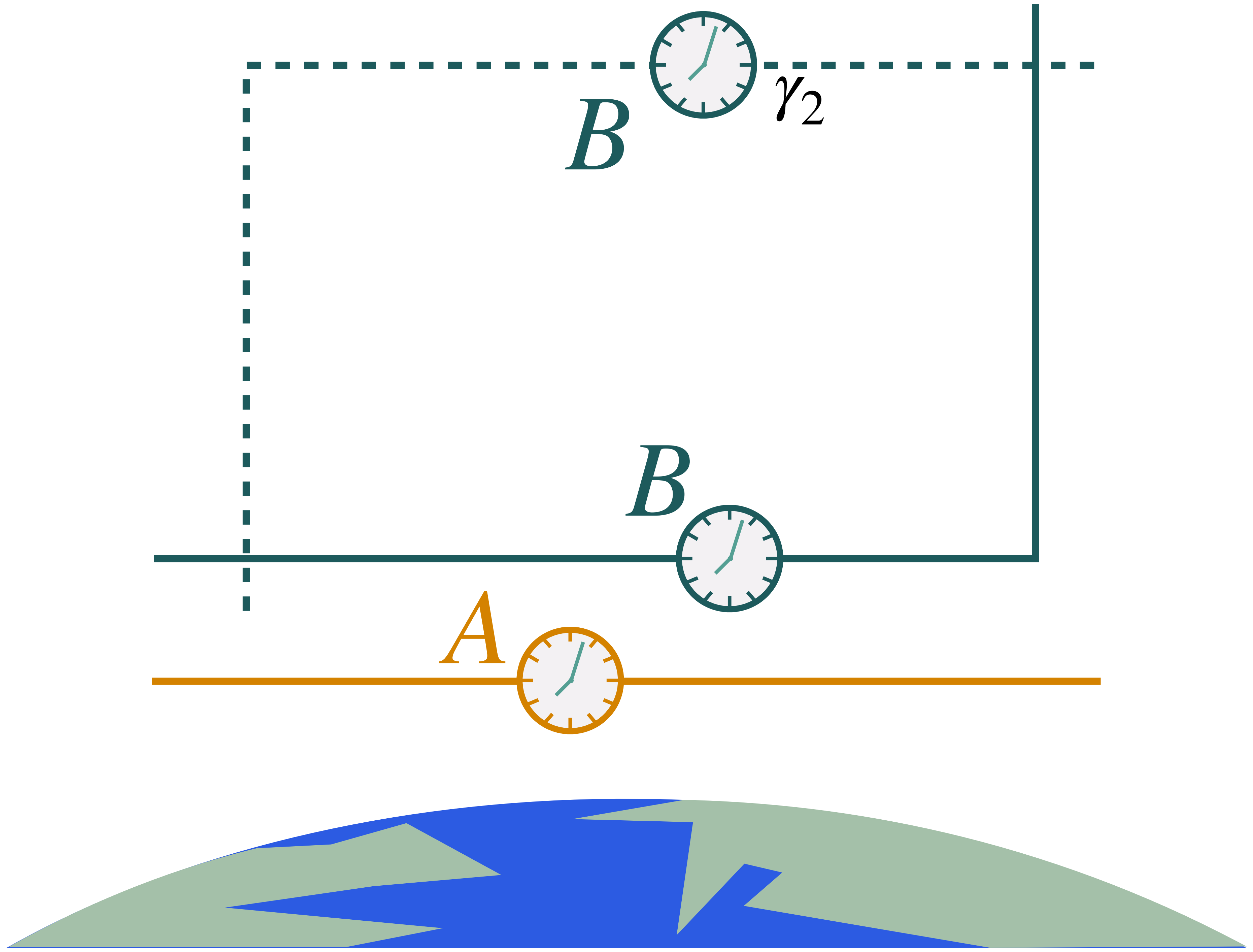
$$i\hbar \frac{d|\psi(\tau_i)\rangle^{(i)}}{d\tau_i} = \sum_{j\neq i} \left[\hat{H}_j^{ext} + \Delta(\hat{x}_j, \hat{x}_M)\hat{H}_j \right] |\psi(\tau_i)\rangle^{(i)}$$

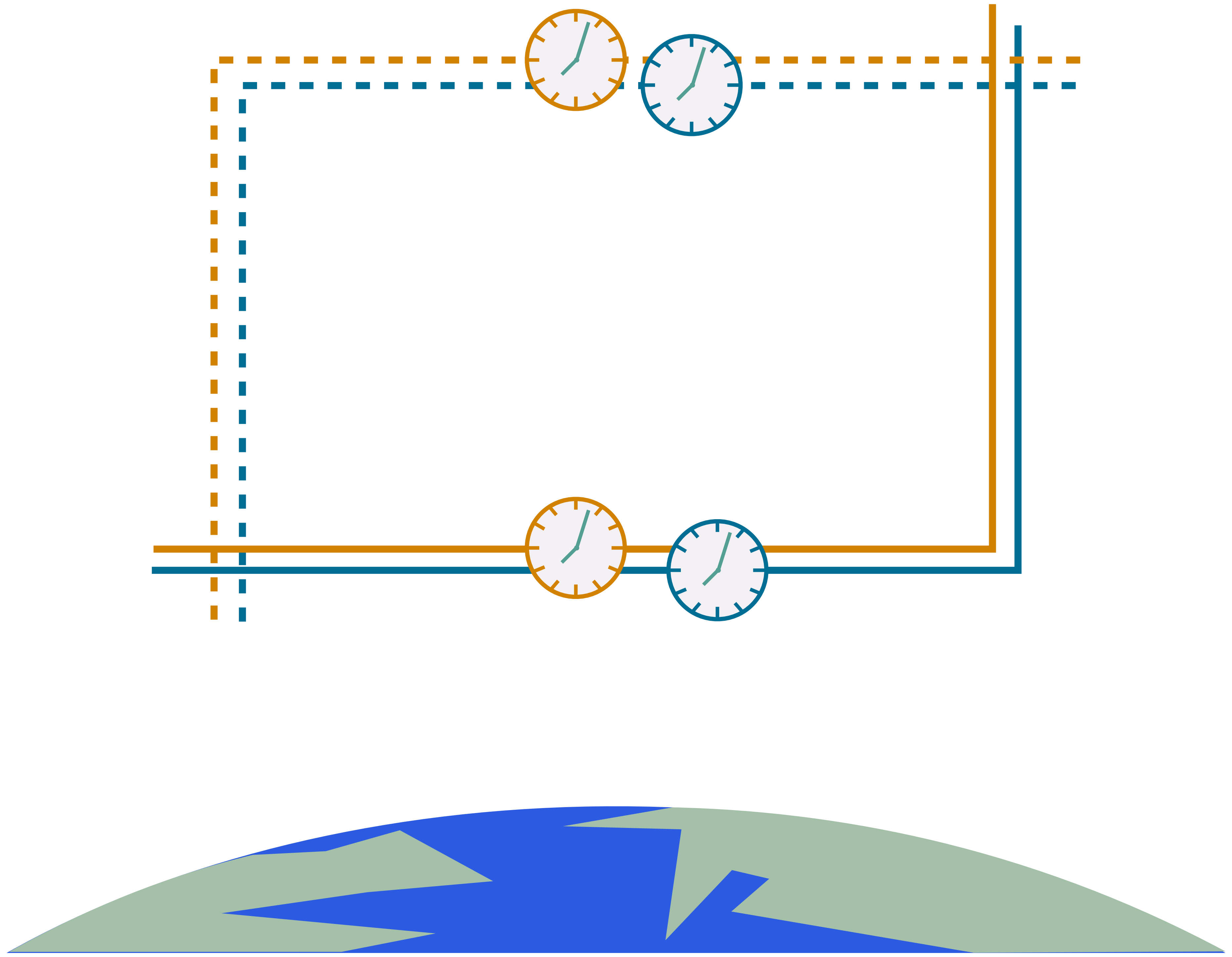
Proper time of the i-th clock

Gravitational time dilation $\Delta(\hat{x}_j, \hat{x}_M) = 1 + \frac{\Phi(\hat{x}_j - \hat{x}_M) - \Phi(\hat{x}_M)}{c^2}$

Giacomini, Quantum (2021)

QRF IN TIME - RELATIVE LOCALISATION OF EVENTS





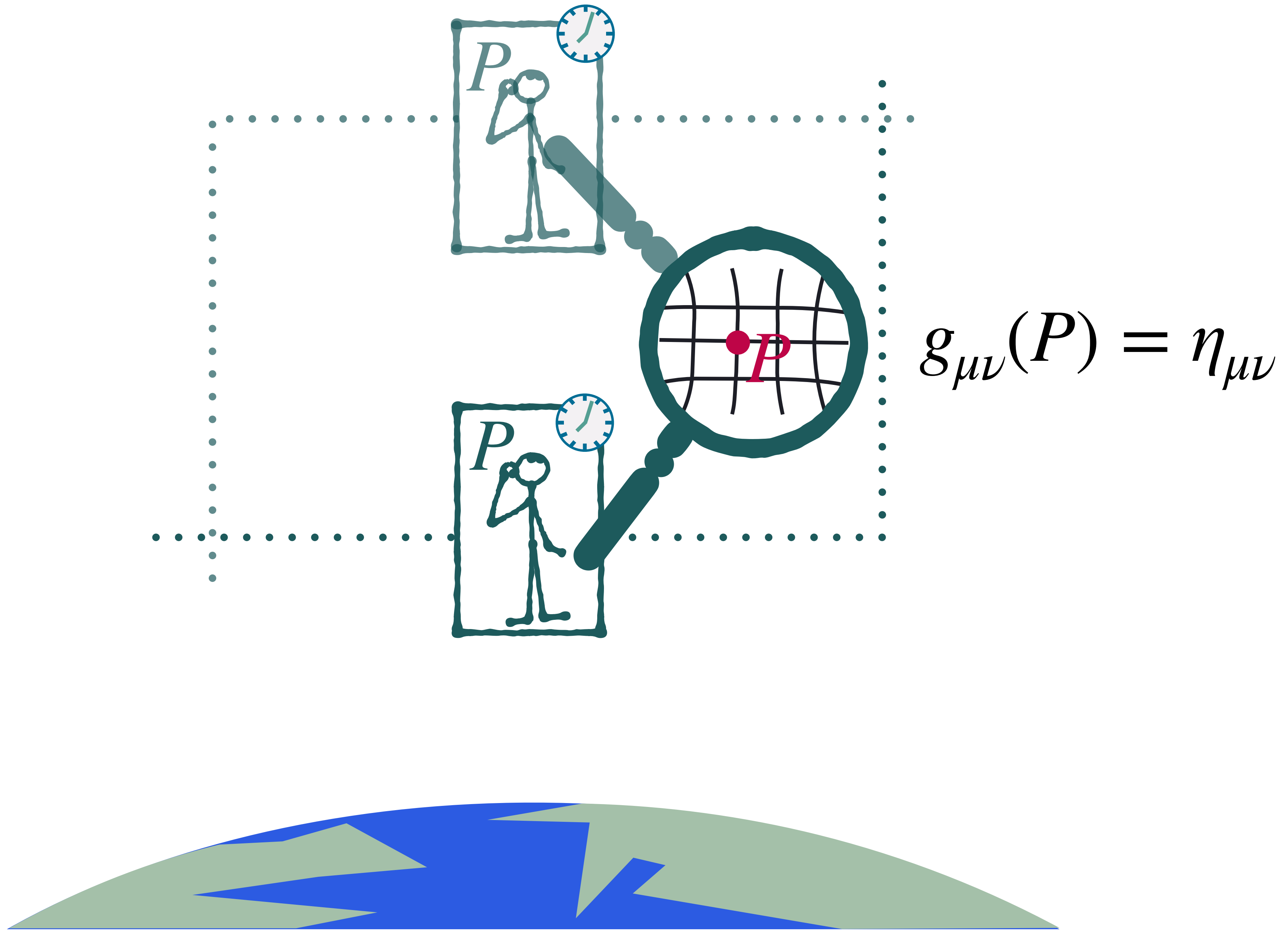
B in a superposition from A
A in a superposition from B

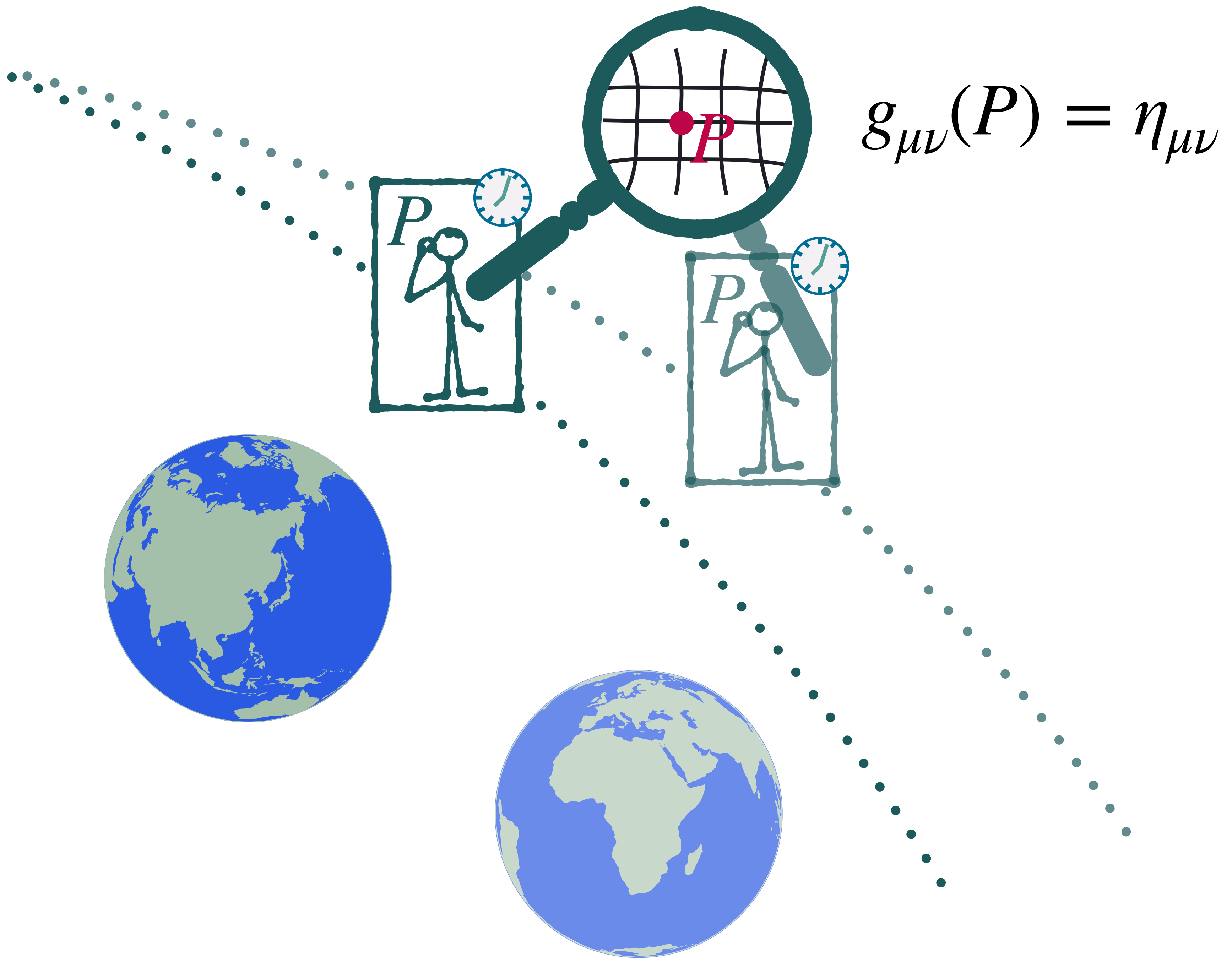
A and B tell the same time;
Sharp from each other's perspective

Giacomini, Quantum (2021)
Cepollaro, Giacomini, 2112.03303 (2021)

EINSTEIN'S EQUIVALENCE PRINCIPLE FOR QUANTUM REFERENCE FRAMES

Giacomini, Brukner, 2012.13754 (2020)
Giacomini, Brukner, AVS Quantum Science (2022)
Cepollaro, Giacomini, 2112.03303 (2021)





Test of EEP for QRFs in atom interferometer with quantum clocks

If EEP for QRF not valid, it is not possible to define time evolution in a QRF

Cepollaro, Giacomini, 2112.03303 (2021)

Reconciliation of EEP and Principle of Linear Superposition

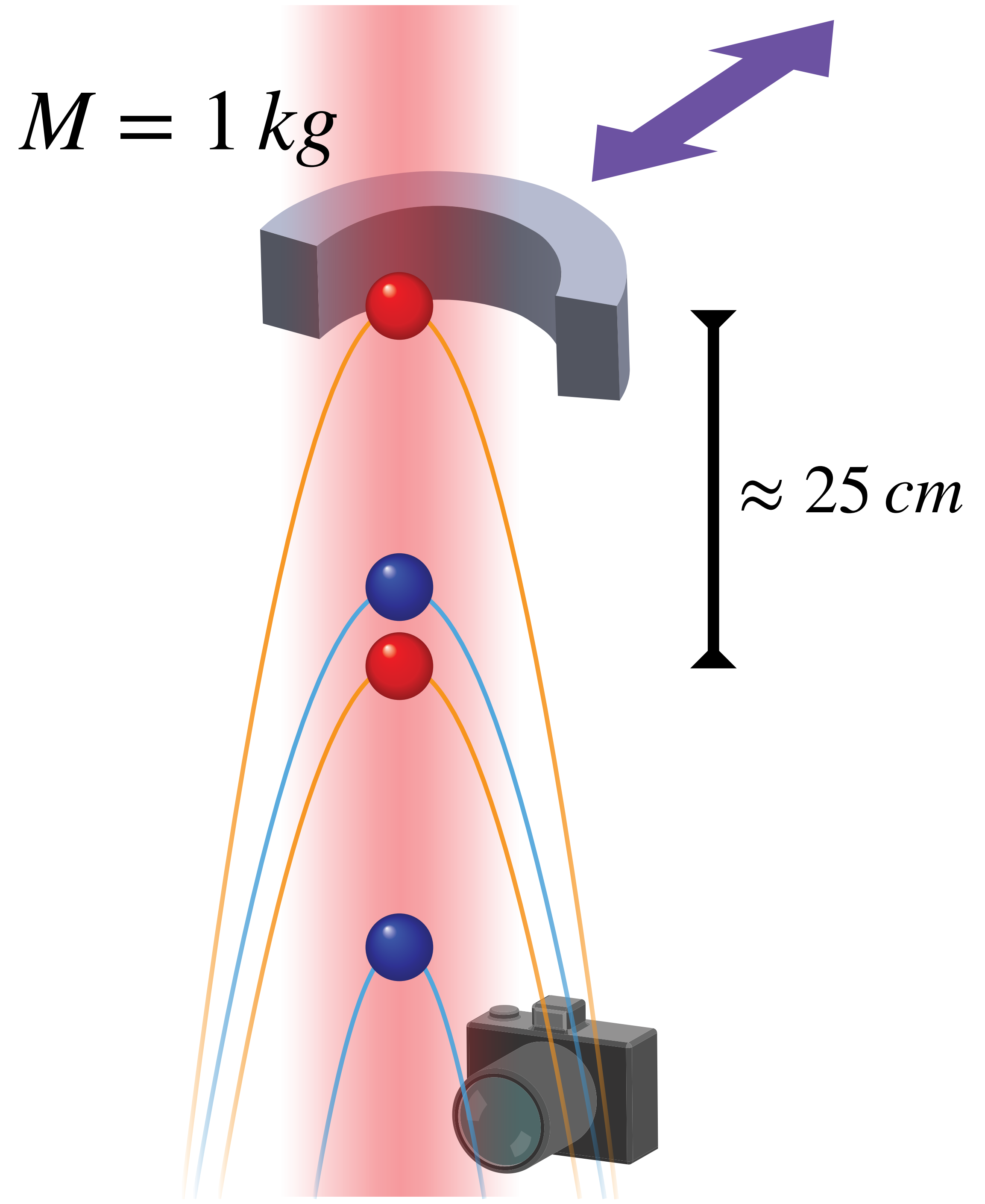
Overcomes Penrose's spontaneous state reduction

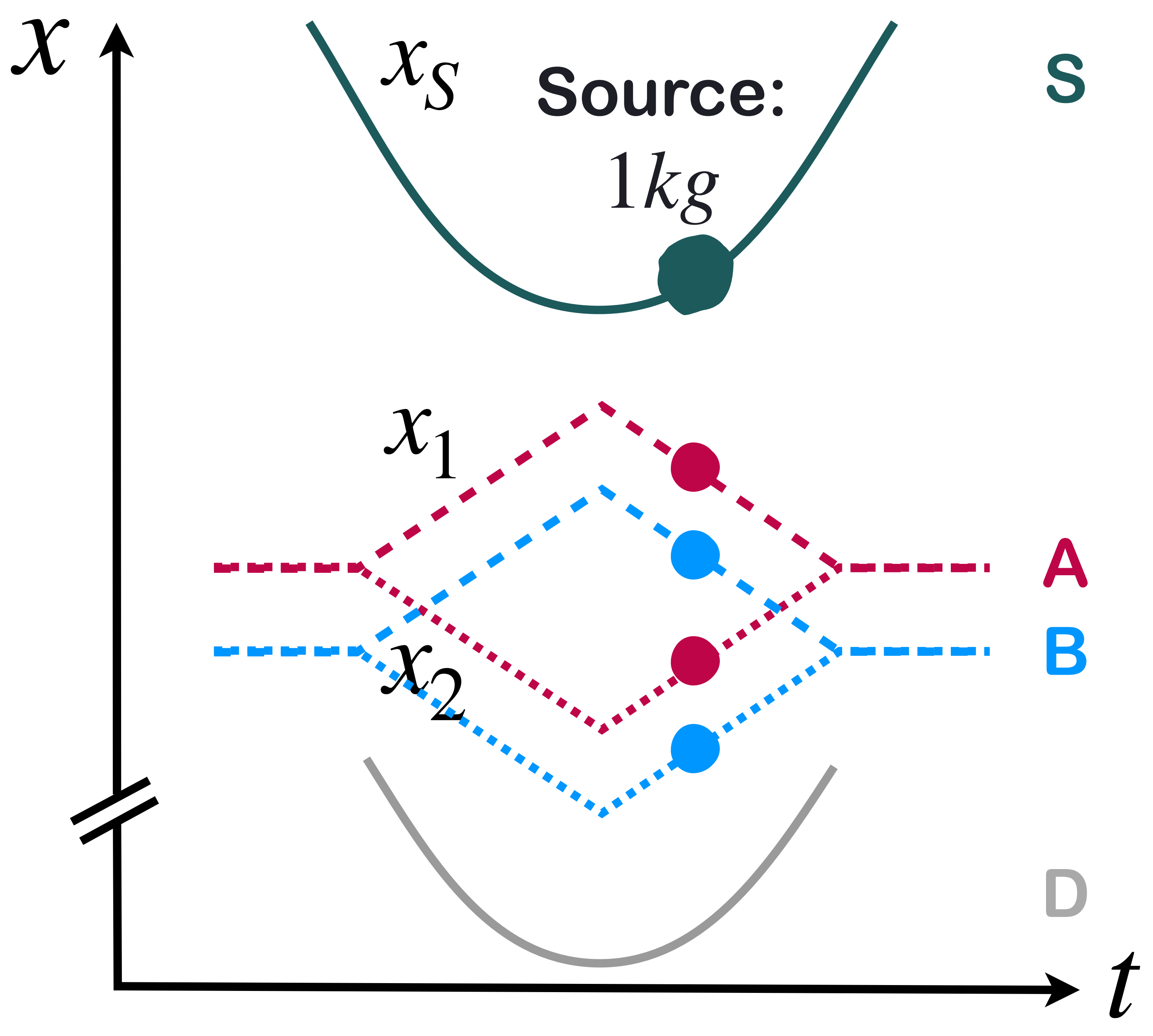
Giacomini, Brukner, AVS Quantum Science (2022)

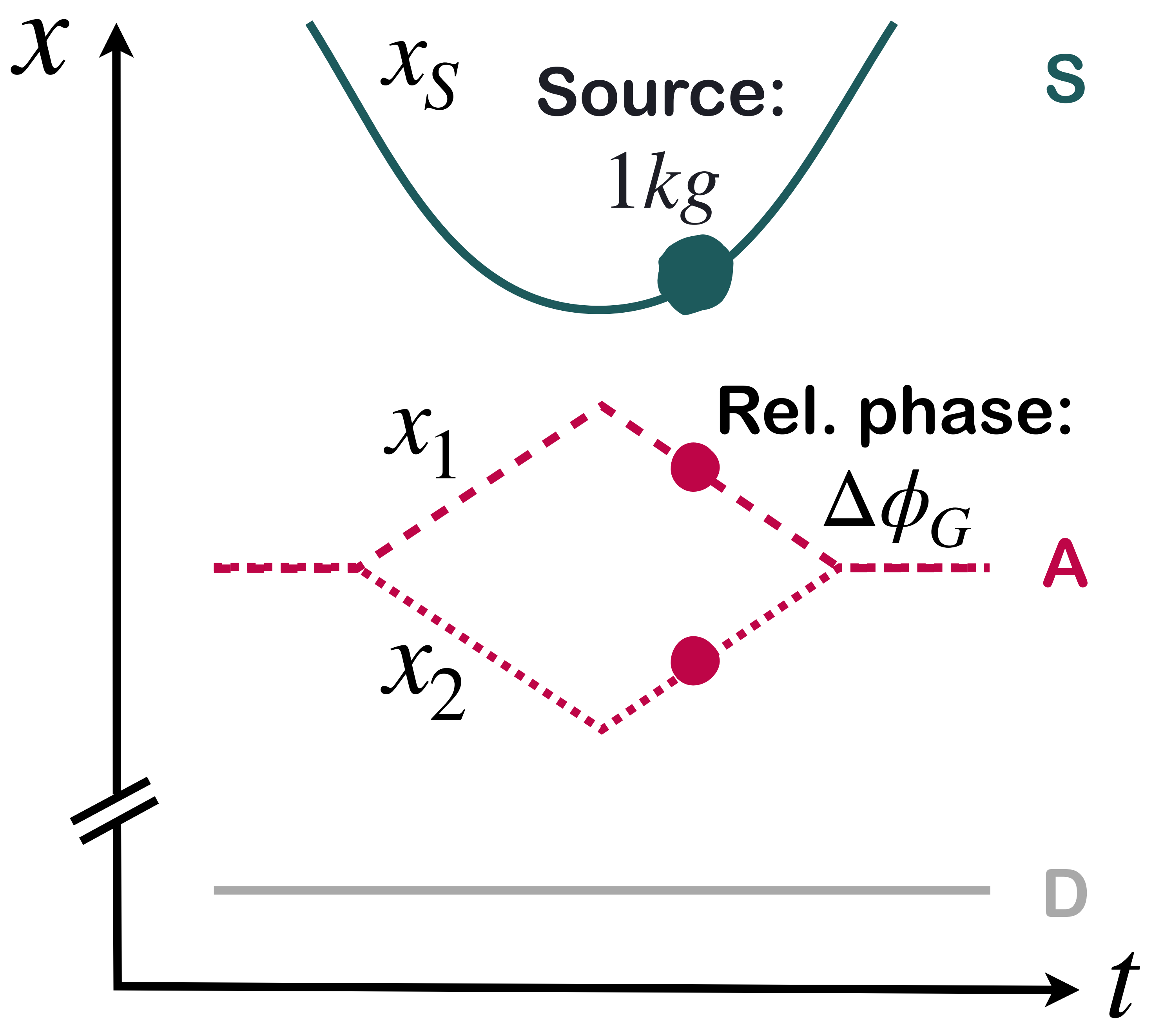
IS THE SUPERPOSITION OF GRAVITATIONAL FIELDS A RELATIVE CONCEPT?

GRAVITATIONAL AHARONOV-BOHM EXPERIMENT

Overstreet, Asenbaum, Curti, Kim, Kasevich, Science (2022)







Gravitational action difference: $\Delta\phi_G = \frac{1}{\hbar}\int_0^T dt[V(x_2 - x_S) - V(x_1 - x_S)]$

Phase shift beyond linear regime

NB: Earth factors out from the description

WHAT WE KNOW FROM CURRENT EXPERIMENTS

- Localised masses as light as $90\,mg$ source a gravitational field

  Westphal, Hepach, Pfaff, Aspelmeyer, Nature (2021)

- Equivalence Principle is valid up to experimental resolution (10^{-15})

  MICROSCOPE mission, PRL (2022)

- Existence of gravitational waves

  B. Abbott et al., PRL (2016)

- Gravitational phase shift between different paths

- Classical gravity (tungsten) and Quantum Theory (atom) are compatible beyond the linear regime

What are we missing to have a superposition of gravitational fields?

QRF DESCRIPTION OF THE EXPERIMENT

CLASSICAL GRAVITATIONAL FIELD

SUPERPOSITION OF GRAVITATIONAL FIELDS

1. EXISTENCE OF GRAVITATIONAL FIELDS
2. QUANTUM RELATIVITY PRINCIPLE

Overstreet, Asenbaum, Curti, Kim, Kasevich, Giacomini 2209.02214 (2022)

SUMMARY OF THIS LECTURE

Operational and relational formalism for quantum reference frames:
associate a reference frame to a quantum system.

In quantum mechanics:

Frame-dependence of entanglement and superposition

Generalisation of covariance

Generalisation of the weak equivalence principle

Operational definition of the rest frame of a quantum system (relativistic spin)

In gravity:

Generalisation of the Einstein Equivalence Principle

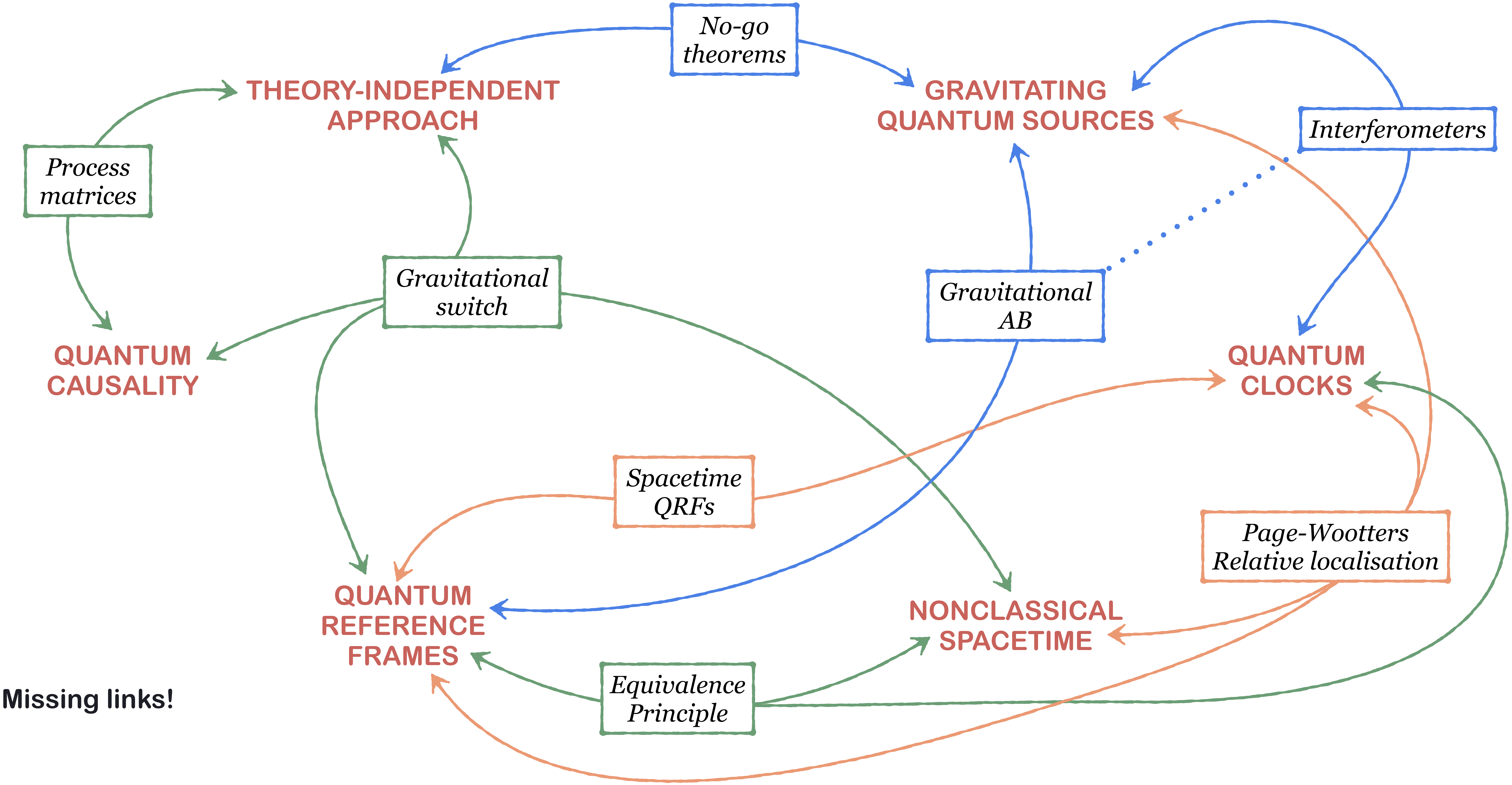
Penrose decoherence

In quantum gravity, we cannot have a classical reference frame.
Quantum reference frames can help us!

The intersection between QT and GR is still vastly unexplored,
many open questions (a lot of work to do!)

RELATIONS BETWEEN THE TOPICS

HOW IS THIS ALL CONNECTED?



Missing links!

SUMMARY AND CONCLUSIONS

Method outlined characterises broadly the field of quantum foundations:

1. *Establish connection between the first principles and the mathematical structure*
2. *Develop tools to systematically test the internal consistency of the theory*
3. *Operationally define the elements of the theory in terms of laboratory procedures*

What limits the approach?
Connections/integration with traditional QG research?

THANK YOU!

Swiss National Science Foundation

ETH zürich