The Basics of Loop Quantum Gravity

Discussion 1: Why Quantum Gravity?

International Society of
Quantum Gravity
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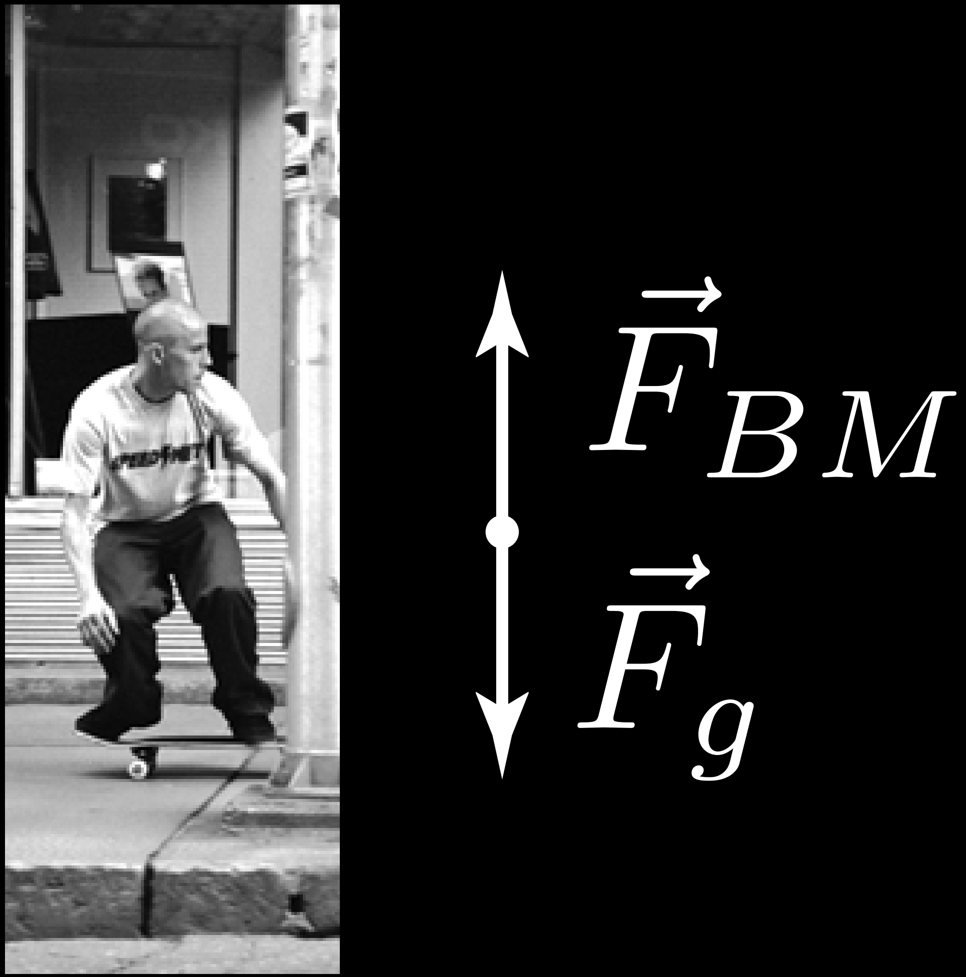
Bard College

Prologue

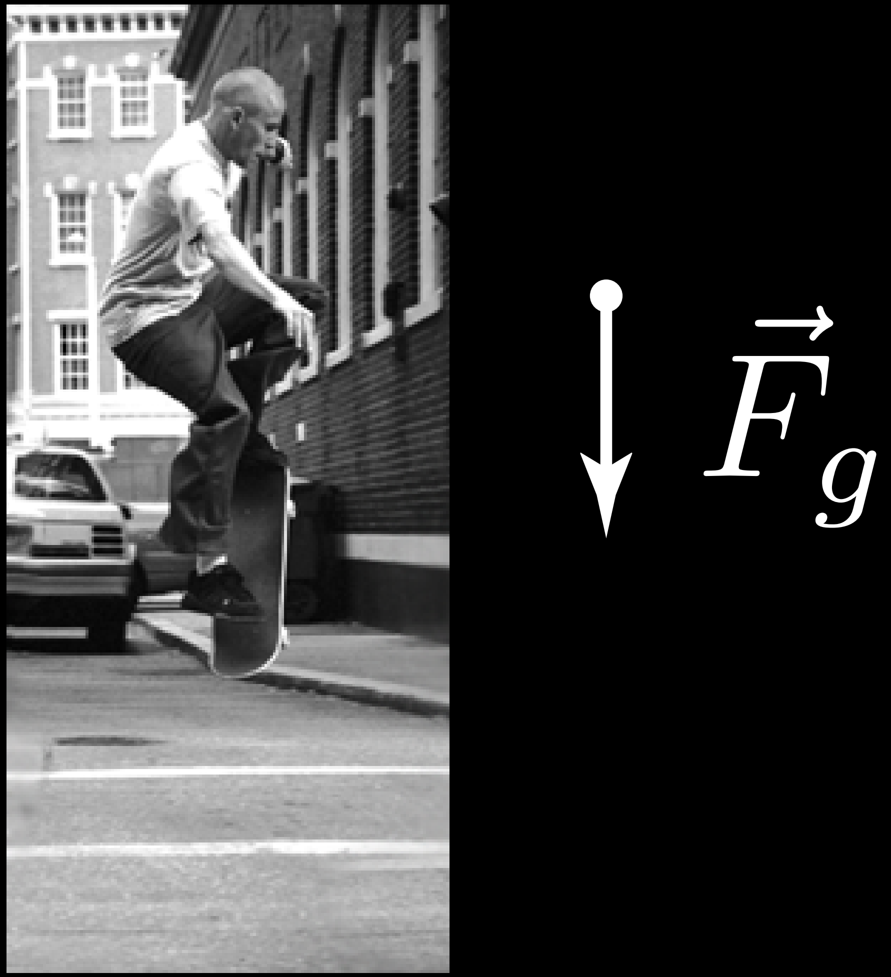
photo courtesy of Thrasher Magazine

Forces are contact forces



$$\vec{F}_{BM}$$

$$\vec{F}_g$$



$$\vec{F}_g$$

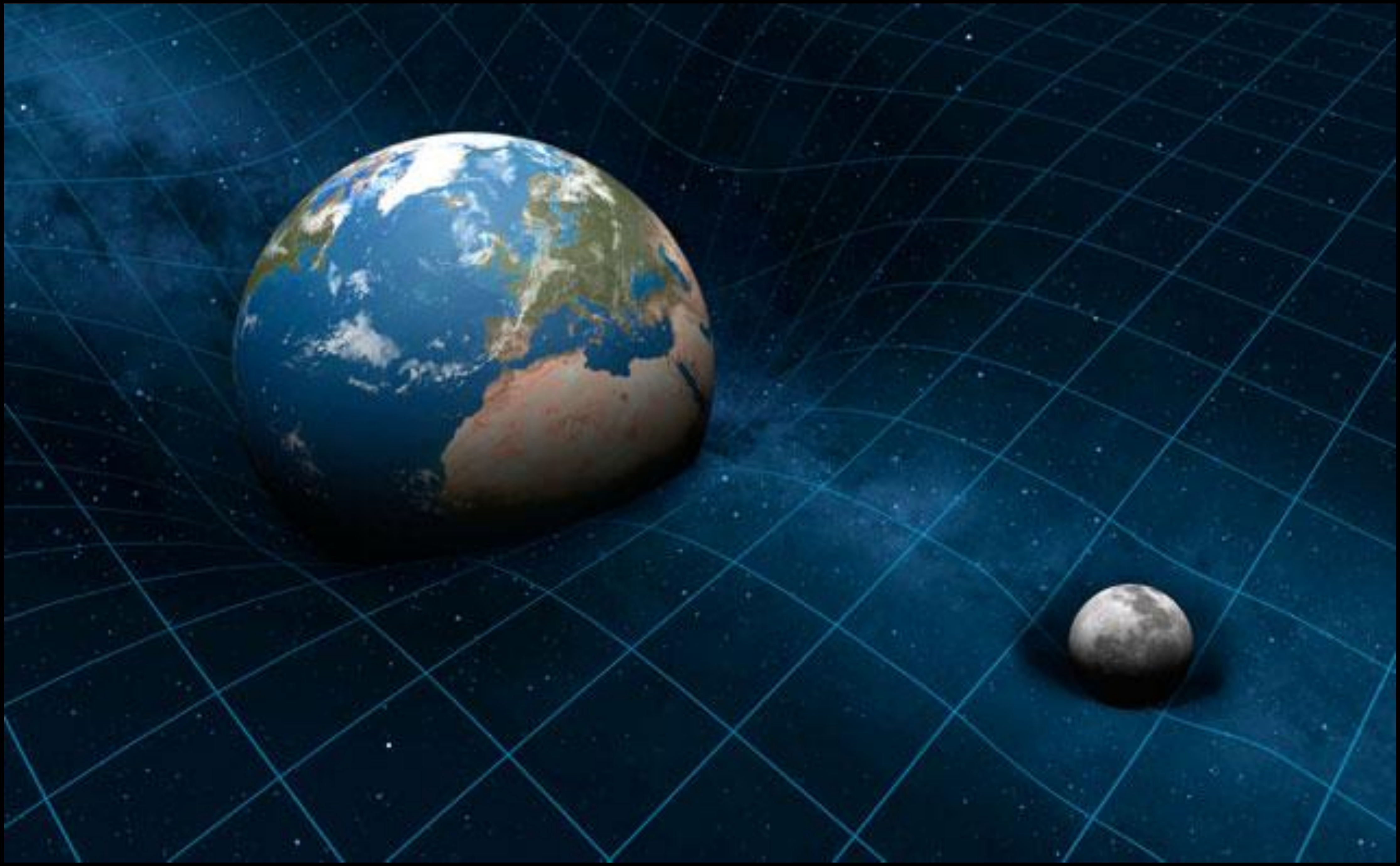
except for the force of gravity (and E & M).

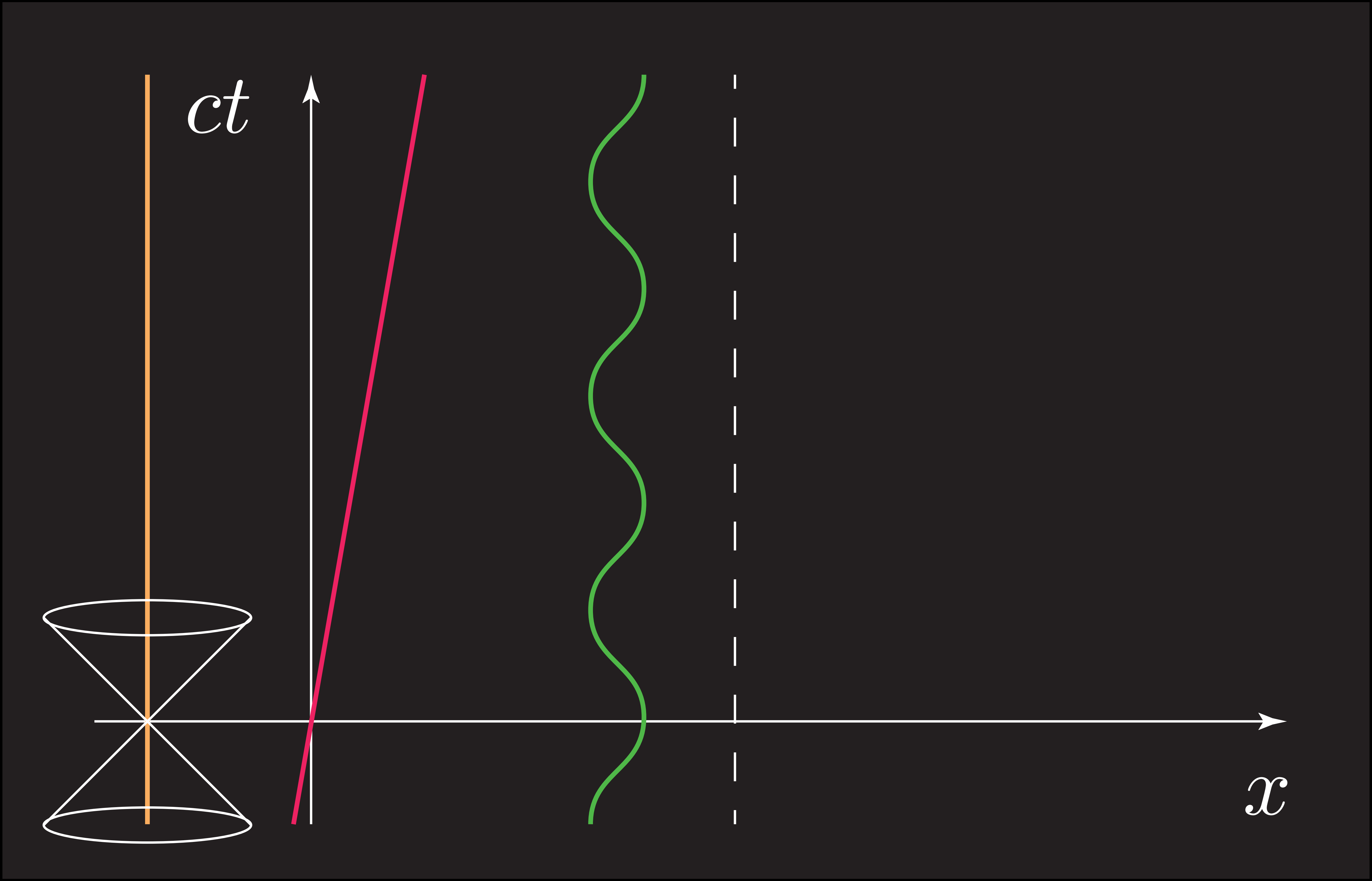
Action at a distance

$$F = G\frac{mM}{r^2}$$

> "That one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one another, is to me so great an absurdity that, I believe, no man who has in philosophic matters a competent faculty of thinking could ever fall into it."

–Isaac Newton

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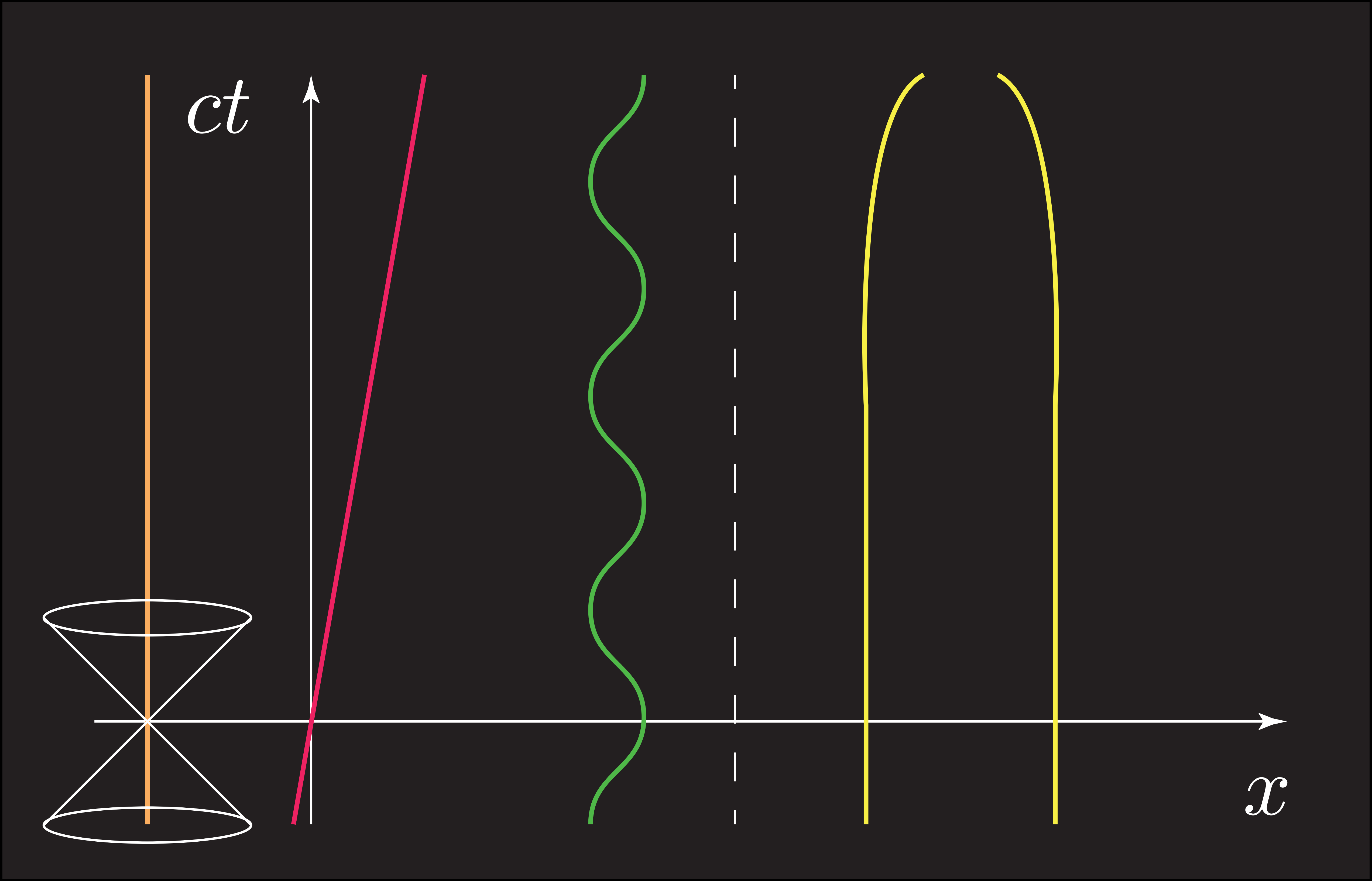
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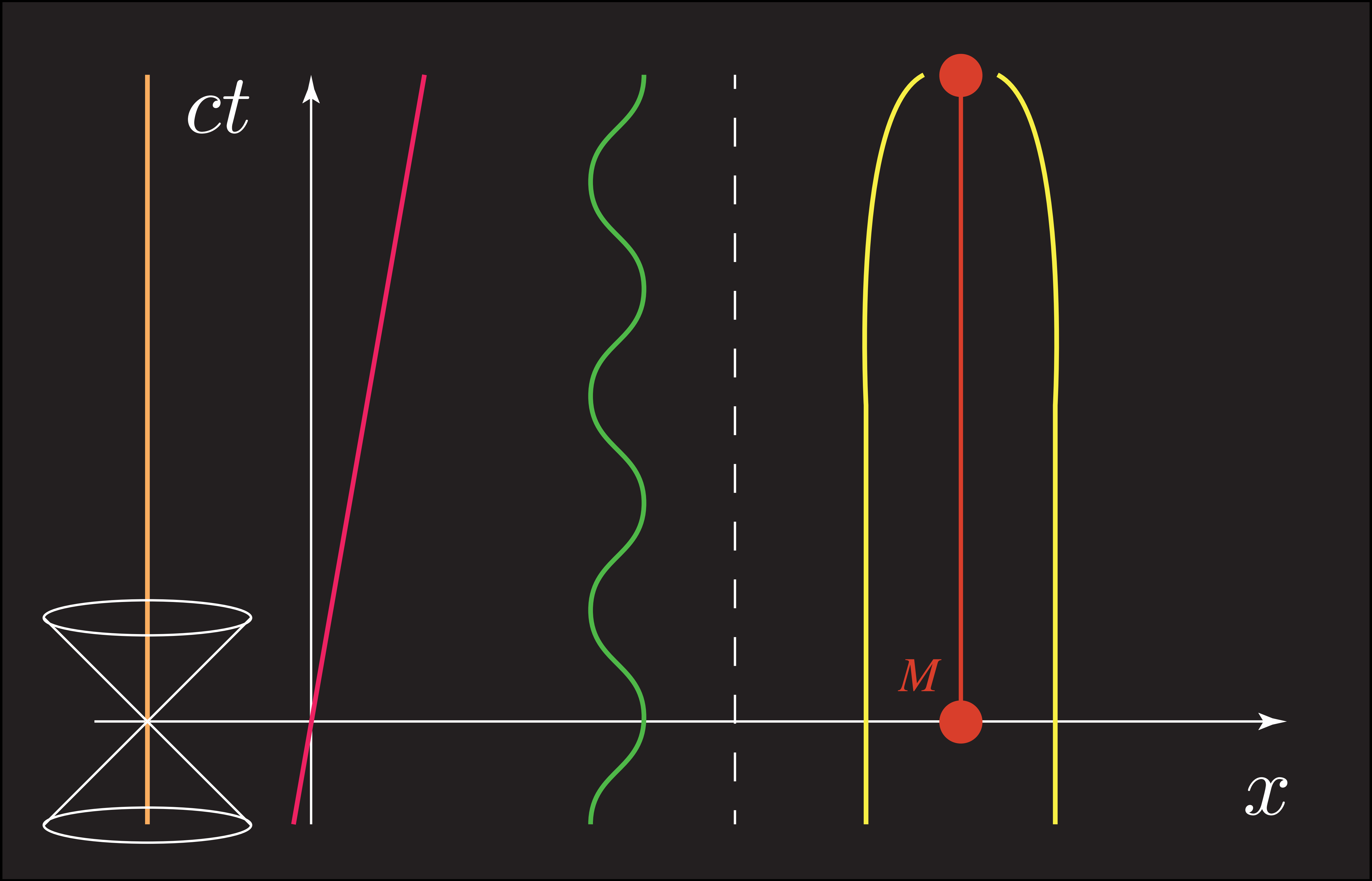
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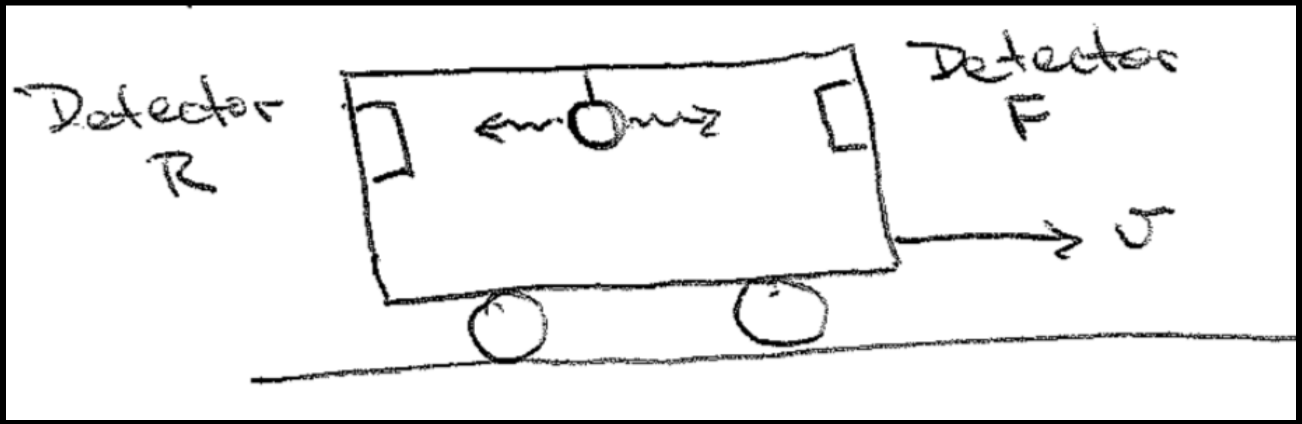
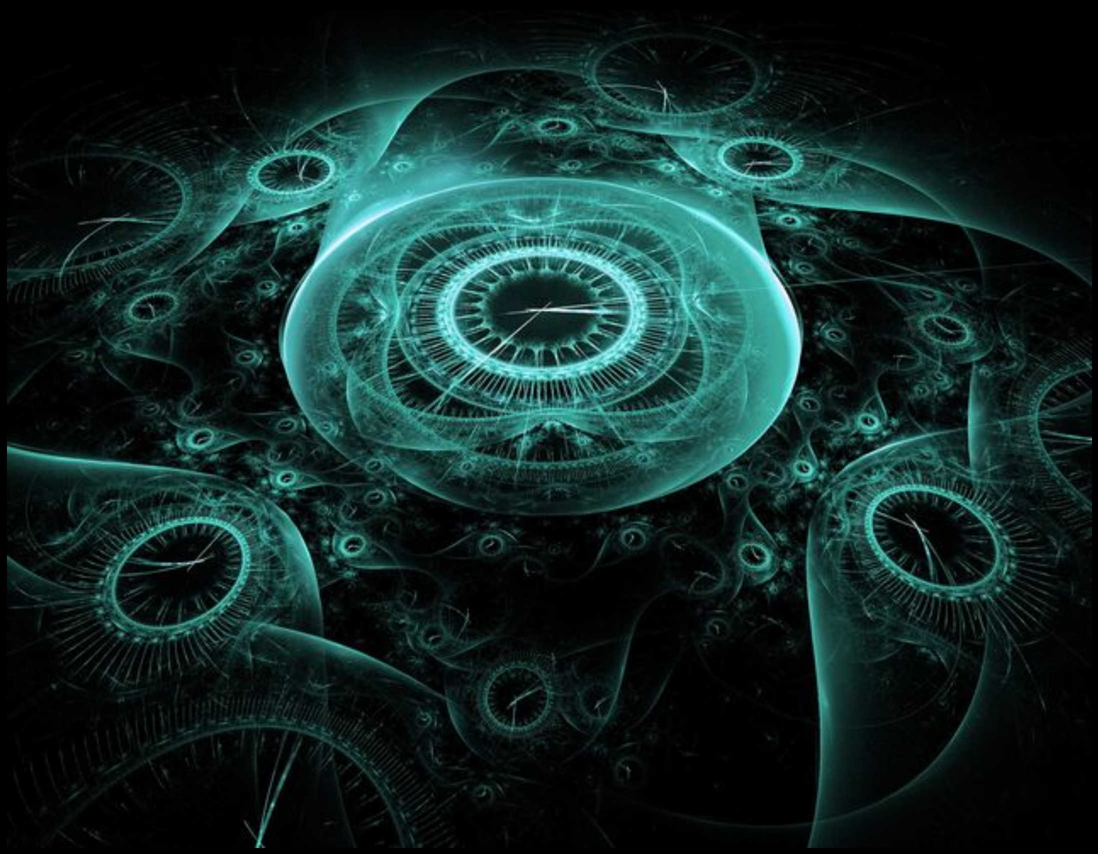
Since I was an undergraduate and first learned the train argument for the relativity of simultaneity, I have been delighted and intrigued with the insights that relativity has developed into the nature of time. Often subtle, these insights completely transform how we think about time, observations and reality.





I hope and expect—can even see glimpses of some of the ways in which—quantum gravity will bring as deep a transformation in our conceptions of time moving forward.

Today's Discussion

1. Why Quantum Gravity?
2. Some of the Reasons that Quantum Gravity is Difficult
3. A Few Features of the Loop Approach to Quantum Gravity
4. Overview of the Course

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Photo: Cirone-Musi, Festival della Scienze via W

Roger Penrose

2020 Nobel Citation

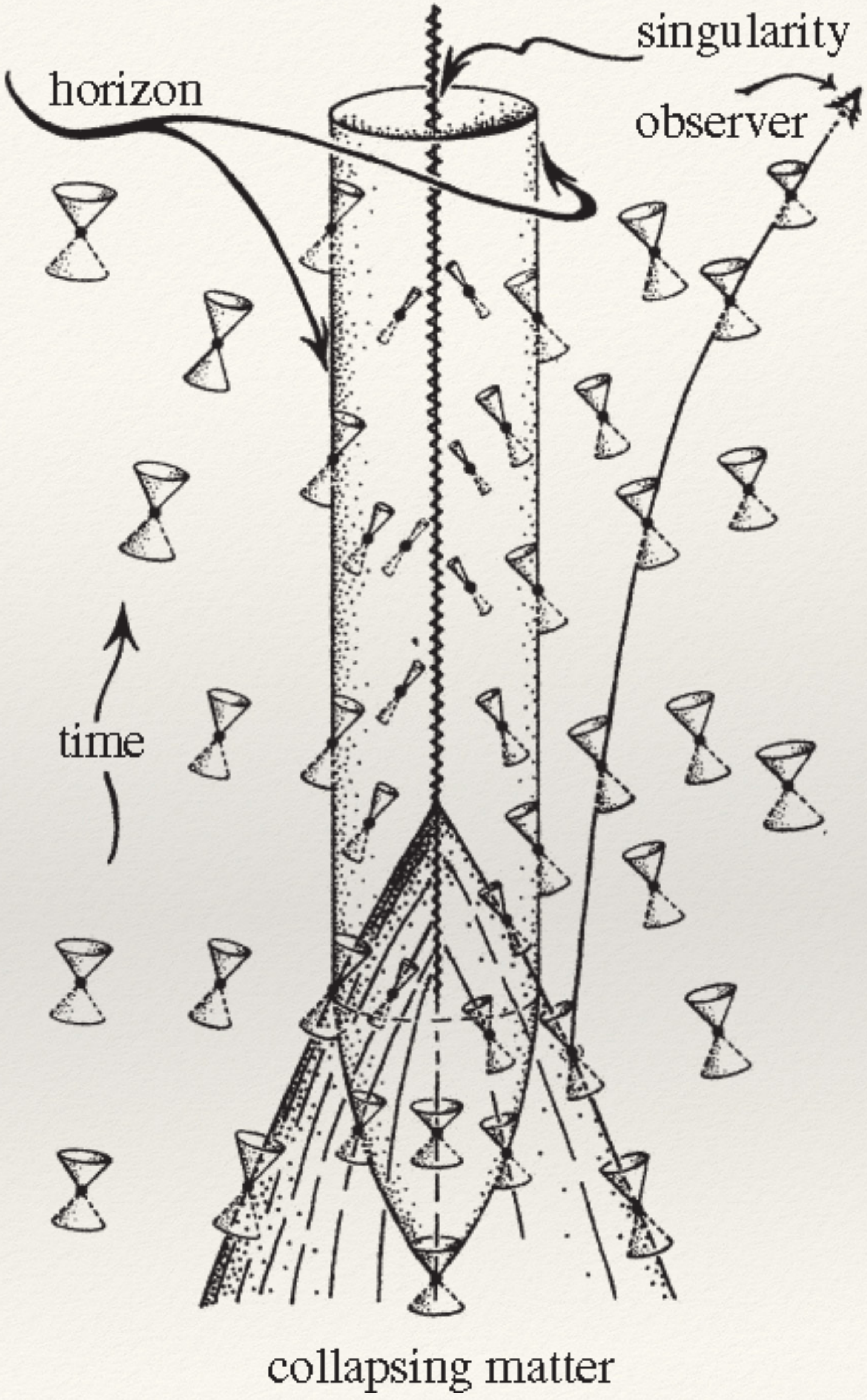
“for the discovery that black hole formation is a robust prediction of the general theory of relativity”

In Berkeley, CA, 1978

1. Black Holes and Cosmology:

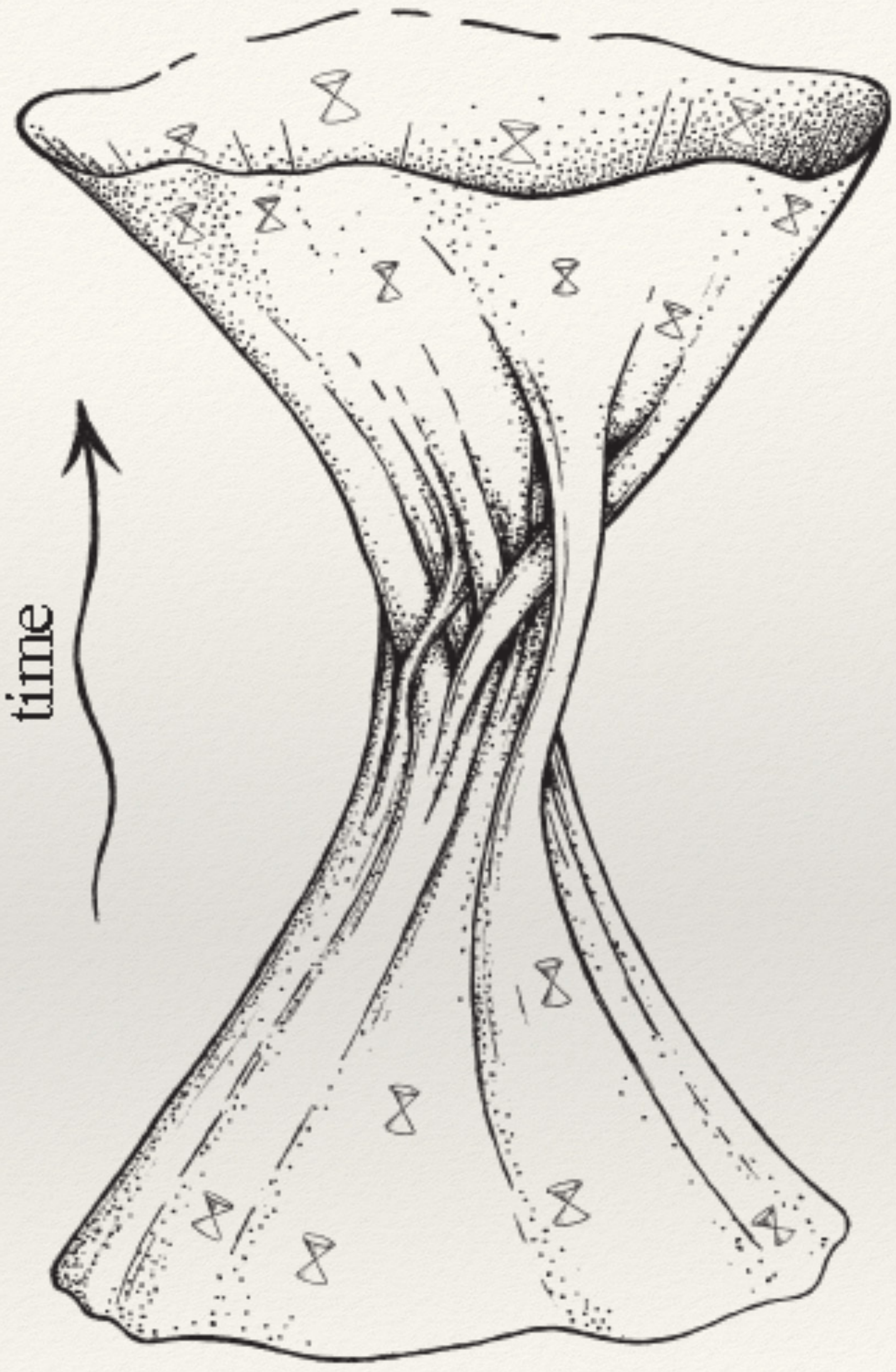
General Relativity Predicts Its Own Demise

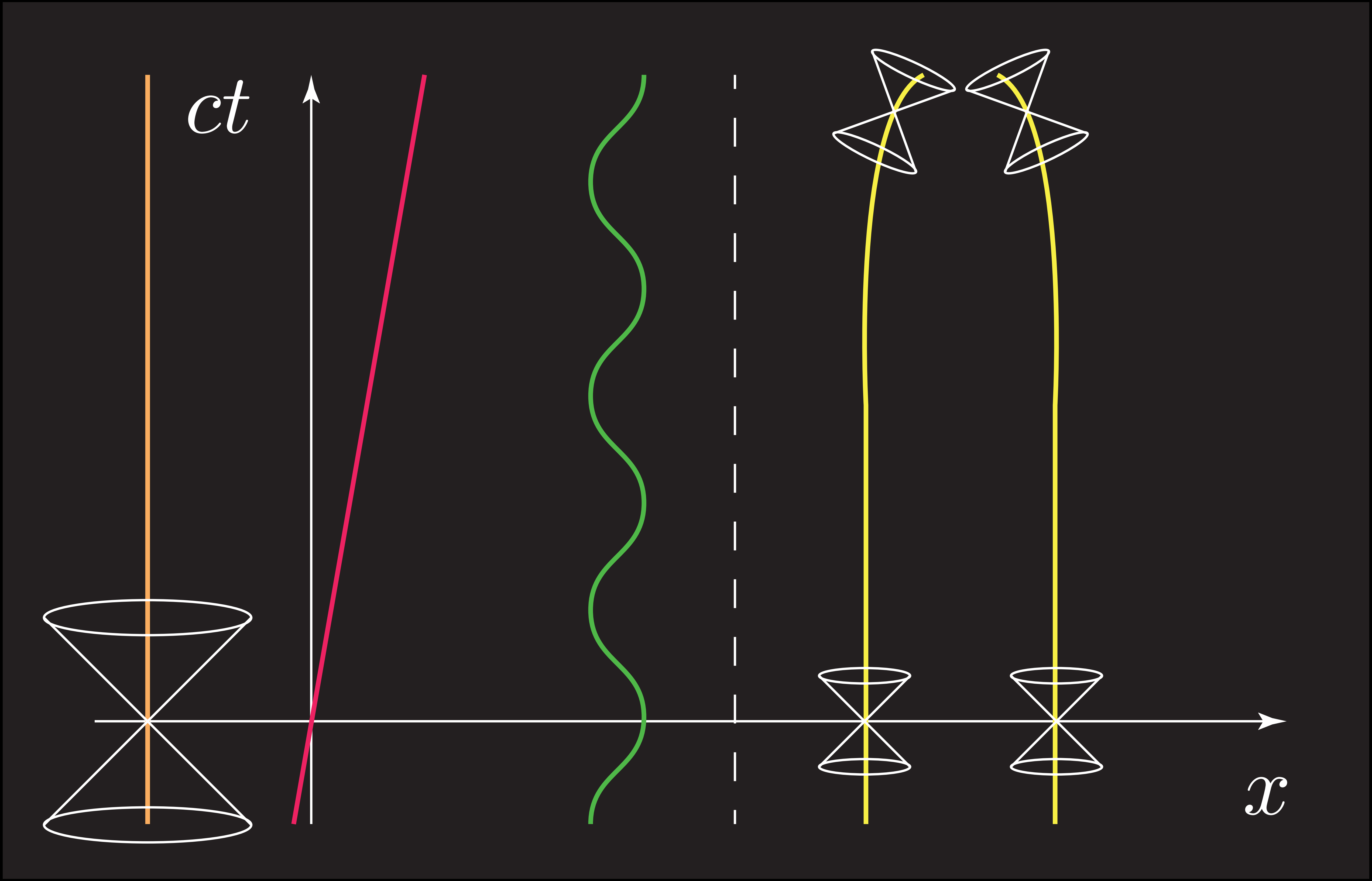
In 1939 Oppenheimer and Schneider realized that a spherical clump of gravitationally collapsing matter would form a singularity:



Researchers started to ask:

If the assumptions on spherical symmetry are relaxed, could it be that singularities are avoided?

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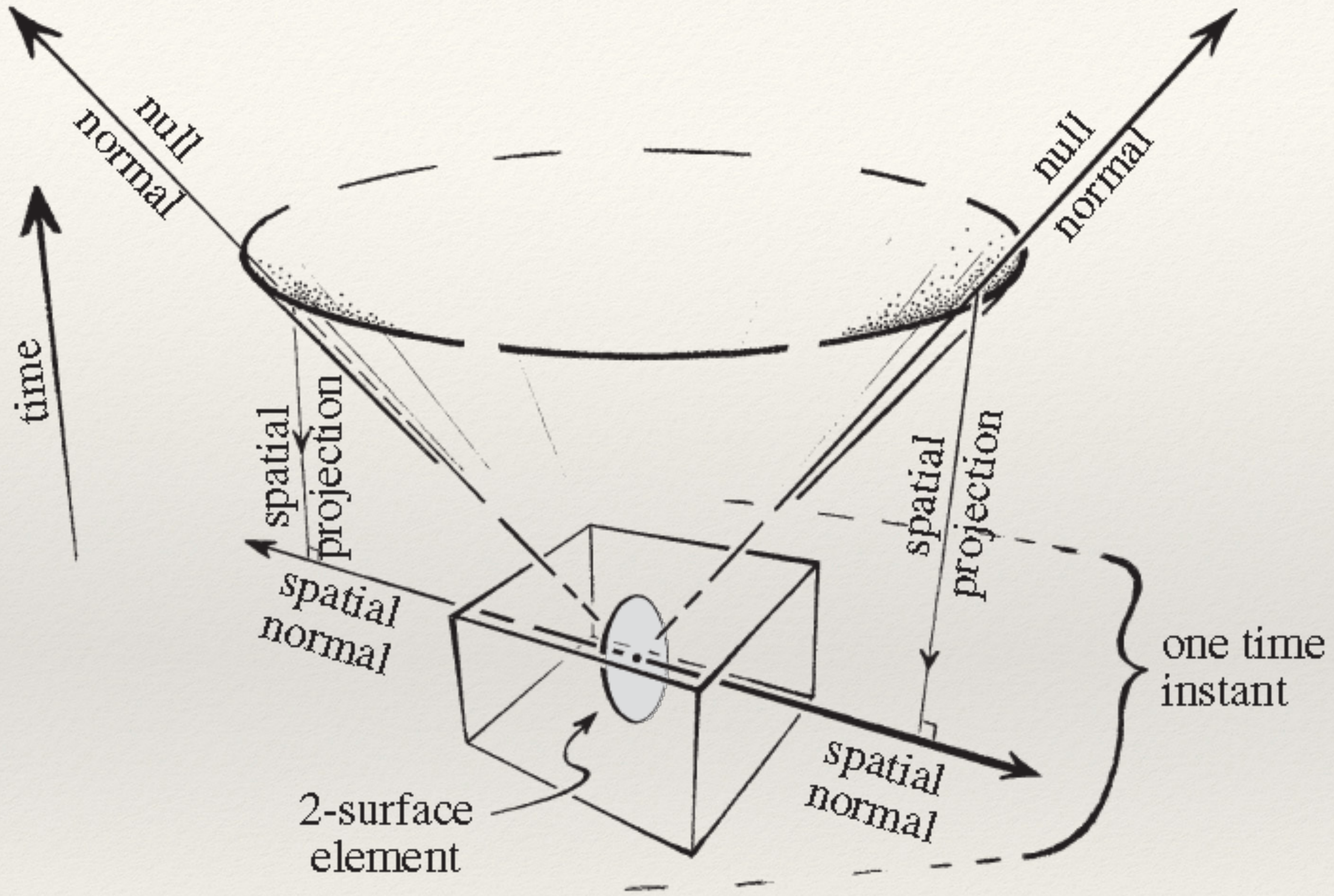
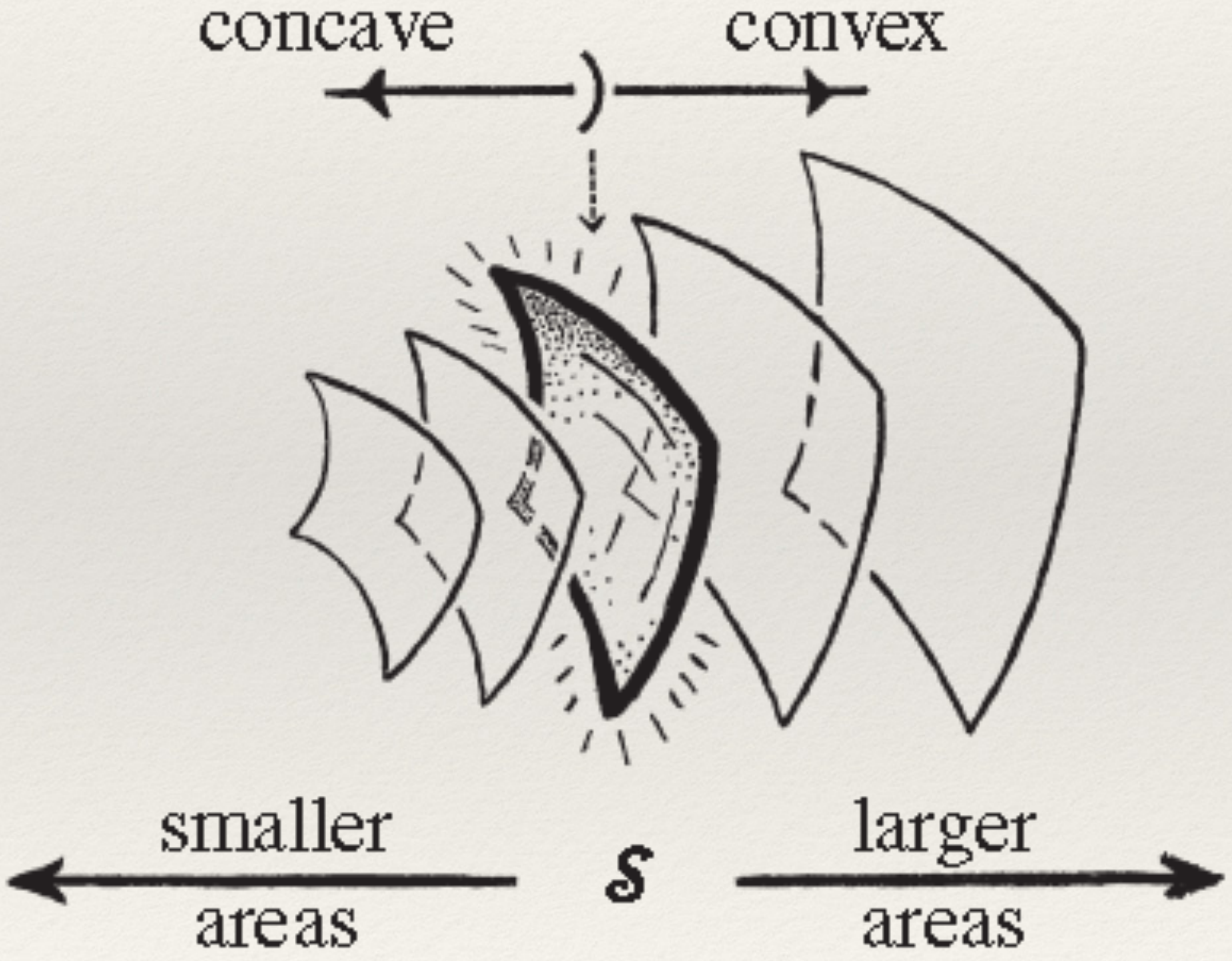
Penrose was captured by this characterization of curvature and introduced *trapping surfaces*



He begins with a sheet of light—you could imagine a whole collection of tiny lightbulbs everywhere along a 2D surface…

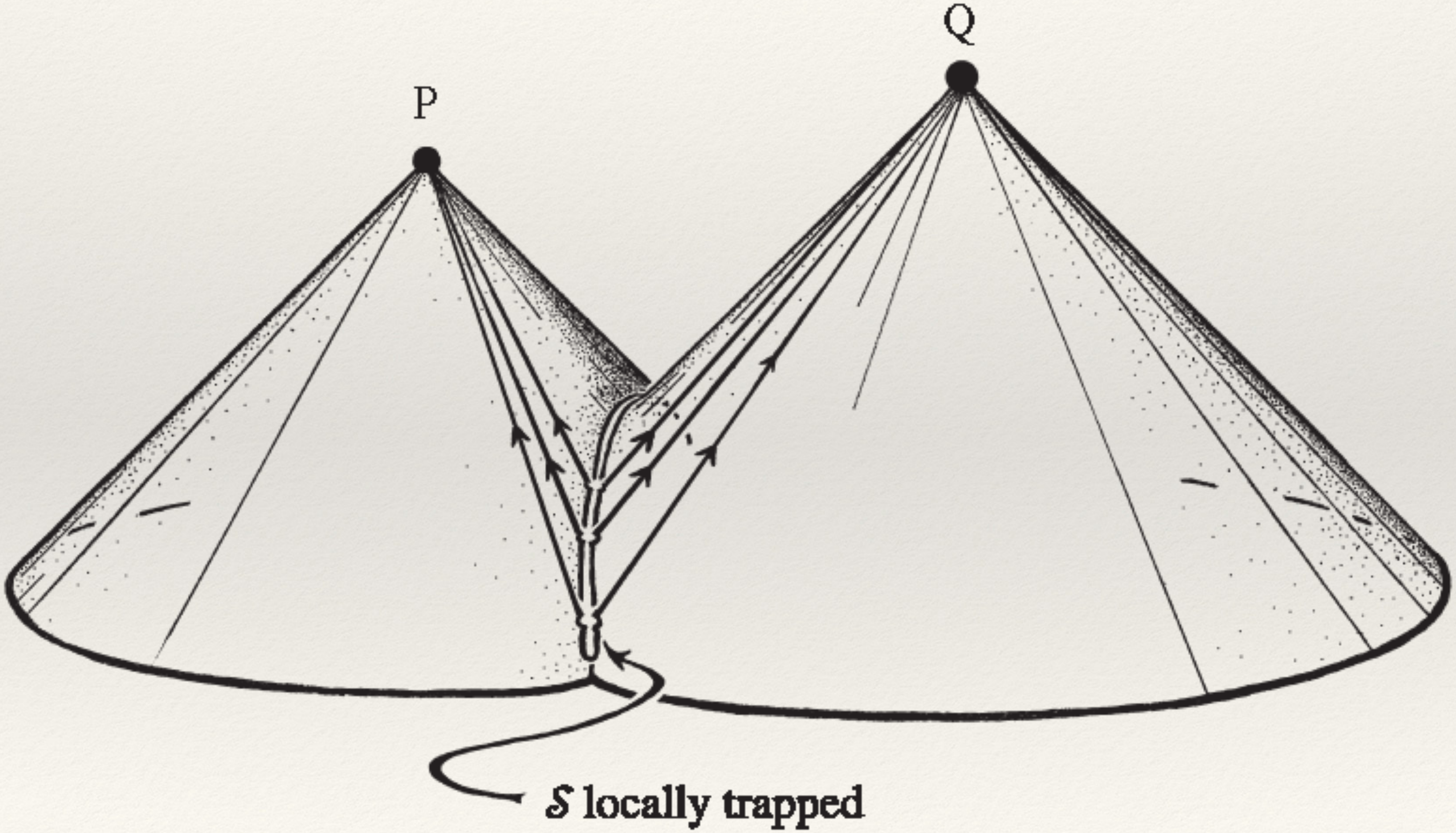
Trapping surfaces

…if the sheet is curved, we expect the light sheet on one side to contract down and focus and that on the other side to expand outwards…



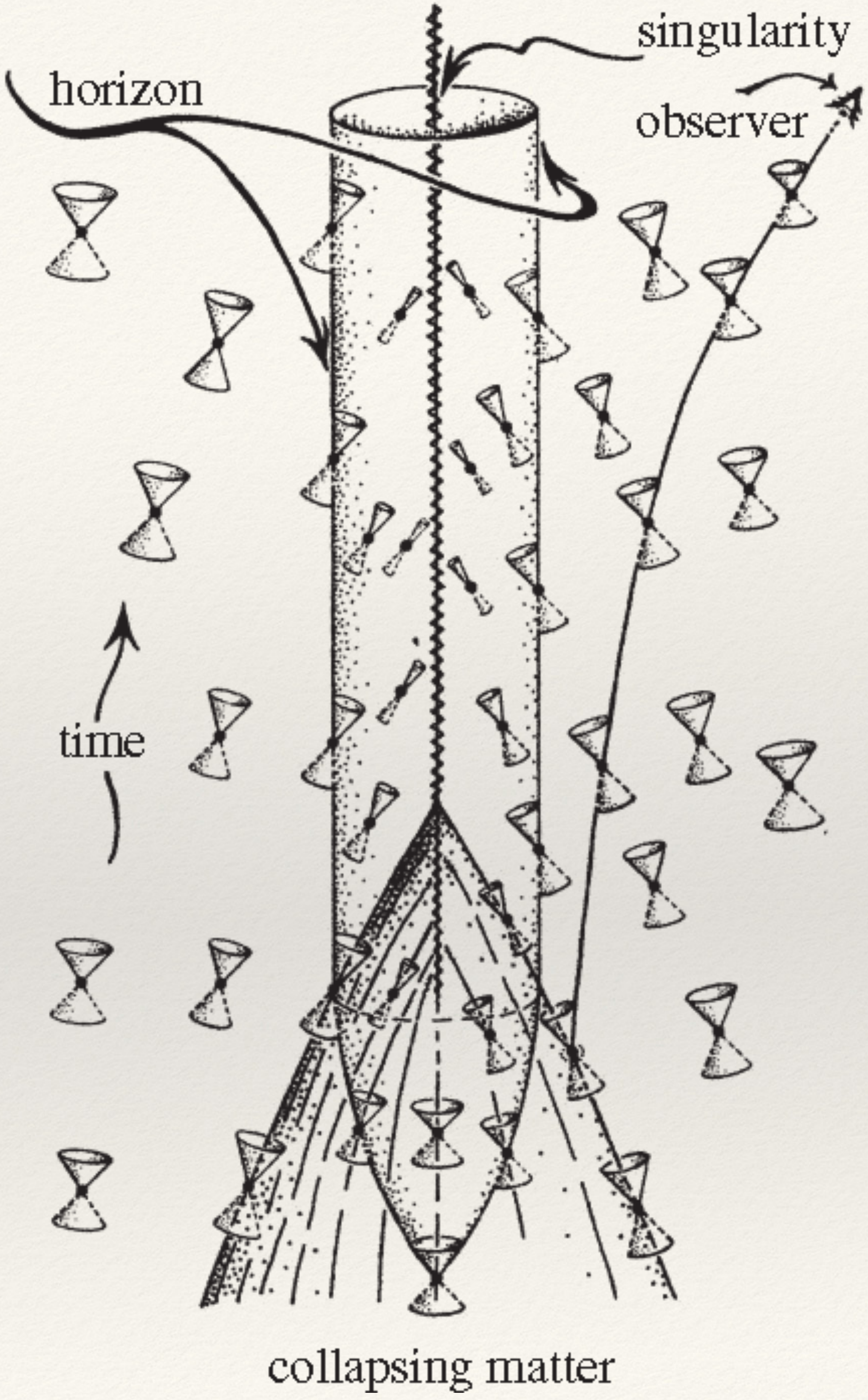
Trapping surfaces

…however, there are special surfaces in spacetime, trapping surfaces, for which the light rays on both sides focus towards smaller areas.



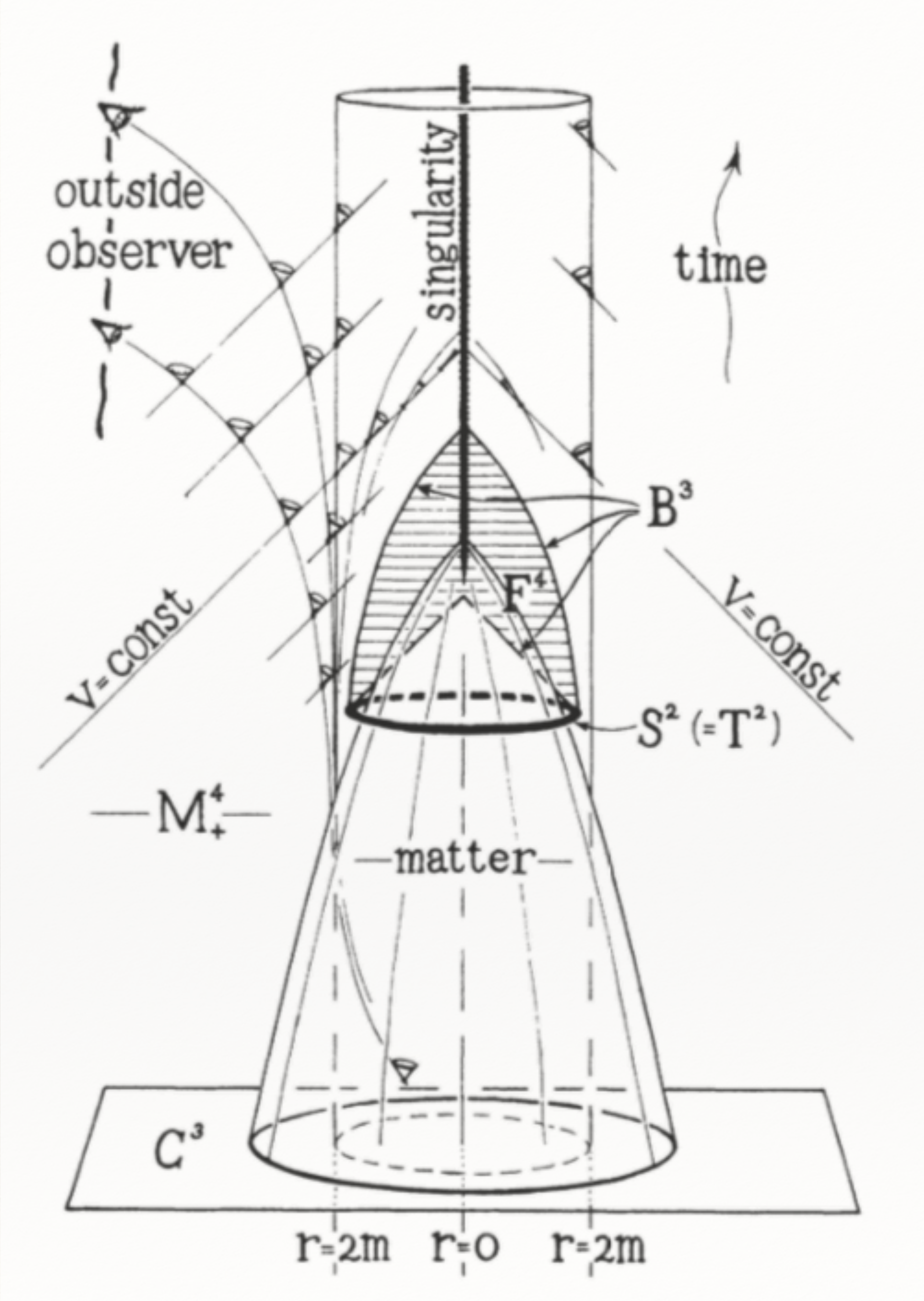
This is precisely what happens in the formation of a black hole. Light cones tip over and both ingoing and outgoing light is focused towards the singularity.

In 1964 Penrose proved, under very weak assumptions, that collapsing matter generically leads to compact trapping surfaces and that geodesics within these trapped regions were eventually inextendible—this is strong evidence that the region contains a singularity.

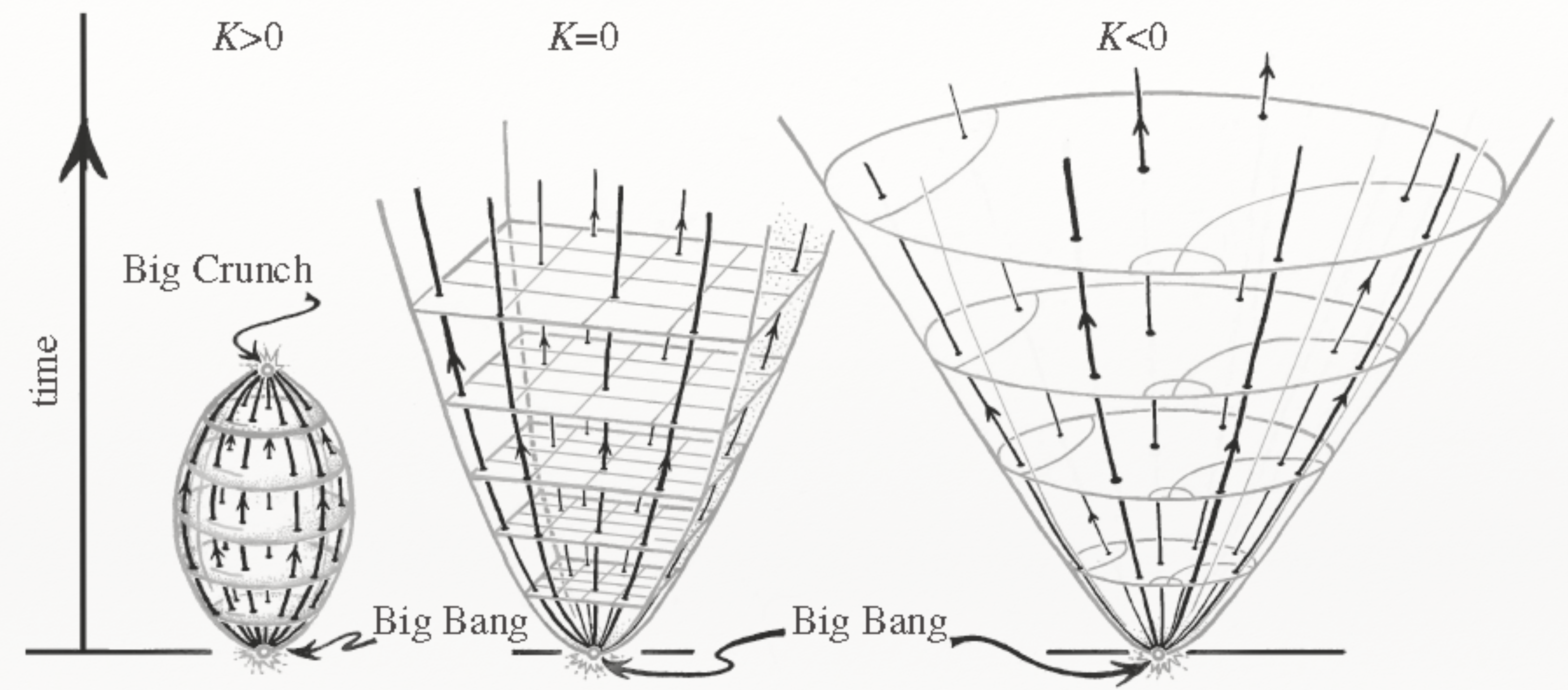


Assumptions in the theorem: (i) Einstein's equations hold, (ii) for a matter distribution that satisfies reasonable energy conditions ($T_{\mu\nu}n^\mu n^\nu \geq 0$), (iii) the spacetime away from the collapsing matter is spatially unbounded.

With these assumptions Penrose showed that spacetime is inextendible.



Different spatial geometries for an expanding universe: spherical, flat, and hyperbolic



Expanding on Penrose's work, in 1966 Stephen Hawking showed, under different energy assumptions, that the backwards evolution of cosmological models also generically results in inextendibility.

1. Black Holes and Cosmology:

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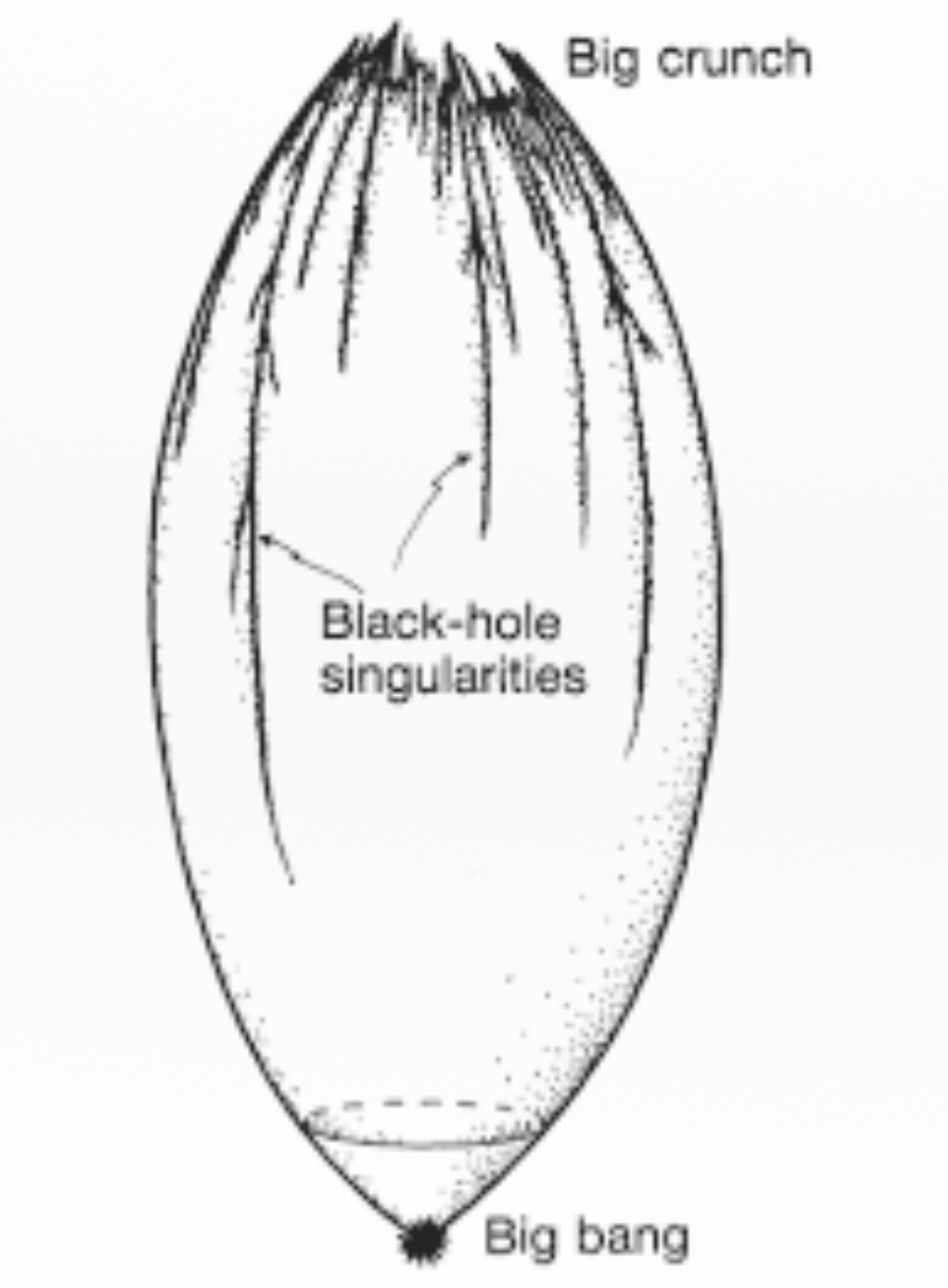


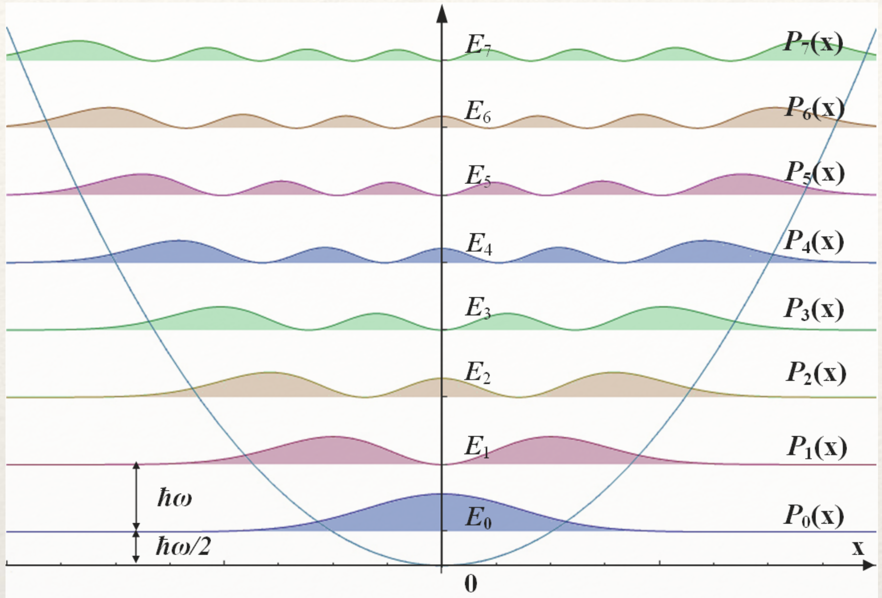
Most physicists take the deduction of singularities from General Relativity not as a prediction of the theory, but as a signal from within the theory that we have reached the limit of its validity.

Resolution of singularities is a compelling motivator for Quantum Gravity.

2. “Gravity Chops Hilbert space Down to Size”:

—Sean Carroll

Quantum Gravity is not a Quantum Field Theory (QFT)



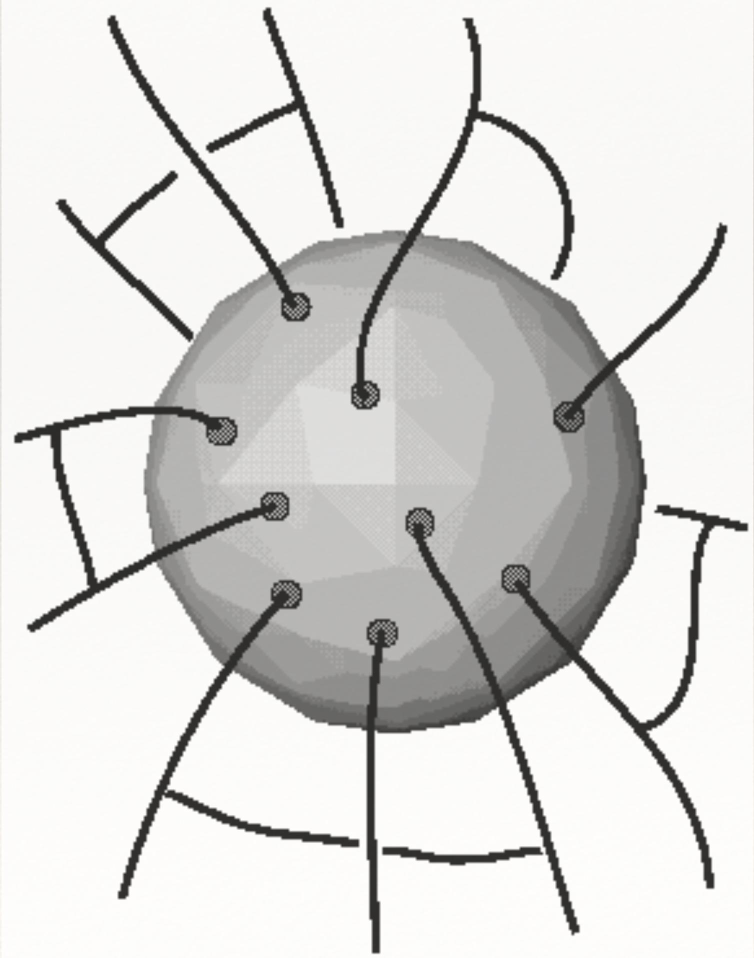
In QFT $\dim(\mathscr{H}) = \infty$ because each mode of the field accepts infinitely many quanta, and hence an infinite energy.

Gravity has a built in cutoff—put too much energy in a region and the region collapses to a black hole:

$$\dim(\mathscr{H}) \sim e^{A_{BH}/A_P} = \text{finite},$$

A_{BH} = black hole boundary area,

A_P = Planck area $= 10^{-70}\ \text{m}^2$.



3. Localization: The Planck Scale

"Here we should take into account a circumstance that reveals the fundamental distinction between quantum electrodynamics and the quantum theory of the gravitational field. Formal quantum electrodynamics that ignores the structure of the elementary charge does not, in principle, limit the density of ρ. When it is large enough we can measure the electric field's components with arbitrary precision. In nature, there are probably limits to the density of electric charge ... but formal quantum electrodynamics does not take these into account... The quantum theory of gravitation represents a quite different case: it has to take into account the fact that the gravitational radius of the test body ($\kappa\rho V$) must be less than its linear dimensions $\kappa\rho V < V^{1/3}$." —Matvei Petrovich Bronstein

[Here $\kappa = 8\pi G/c^4$.]

(Bronstein 1936b, p.217, transl. from Gorelik and Frenkel 1994, p.105 and Gorelik 1992, pp.376-377).

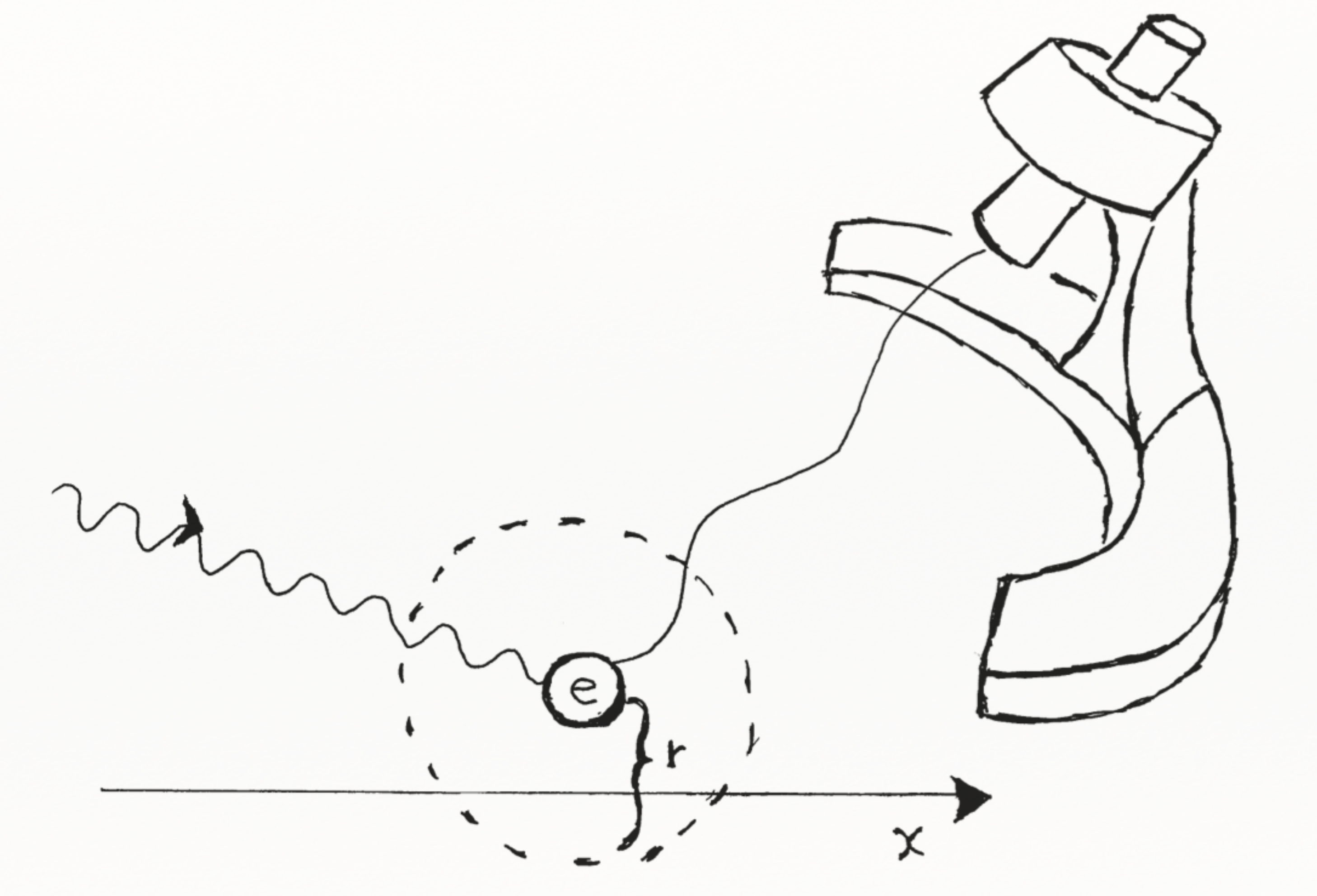
3. Localization: The Planck Scale

Heisenberg microscope—measure the position and momentum of, say, an electron, using a beam of light: resolve position to $\Delta x \approx \lambda$, de Broglie says $p_\gamma = 2\pi\hbar/\lambda$, so $\Delta p \approx \hbar/\lambda$ and $\Delta x \Delta p \gtrsim \hbar$

Now, give up on momentum: how well can we localize the electron?

Shorter wavelength probe is better localization, but carries more energy, which gravitates:

$$\Delta x = \max\left(\frac{\hbar}{\Delta p}, \frac{\hbar G}{c^3}\frac{\Delta p}{\hbar}\right)$$



Best localization possible: $r^2 \sim \hbar G/c^3 = A_P$.

3. Localization: The Planck Scale

> “The elimination of the logical inconsistencies connected with this result requires a radical reconstruction of the theory, and in particular, the rejection of a Riemannian geometry dealing, as we have seen here, with values unobservable in principle, and perhaps also rejection of our ordinary concepts of space and time, modifying them by some much deeper and nonevident concepts.”
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> —Matvei Petrovich Bronstein

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4. Matter: Quantum Particles

Matter sources the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}.$$

But, matter is quantum, and the stress-energy tensor is an operator $\hat{T}_{\mu\nu}$. Without a quantum theory of gravity in hand, how should the fields $g_{\mu\nu}$ and $\hat{T}_{\mu\nu}$ be combined?

One long-standing approach is to study Quantum Field Theory on Curved Spacetimes, for example, quantizing fields living on a fixed classical background metric $g^c_{\mu\nu}$ or/and considering $\langle \hat{T}_{\mu\nu} \rangle$ on the right hand side of the equations. Tricky mathematically: requires renormalization and careful choice of states.

4. Matter: Quantum Particles

In Fock space, there is a Poincaré invariant vacuum: $|0\rangle$
All inertial observers agree on the particle content of this state, i.e. none.

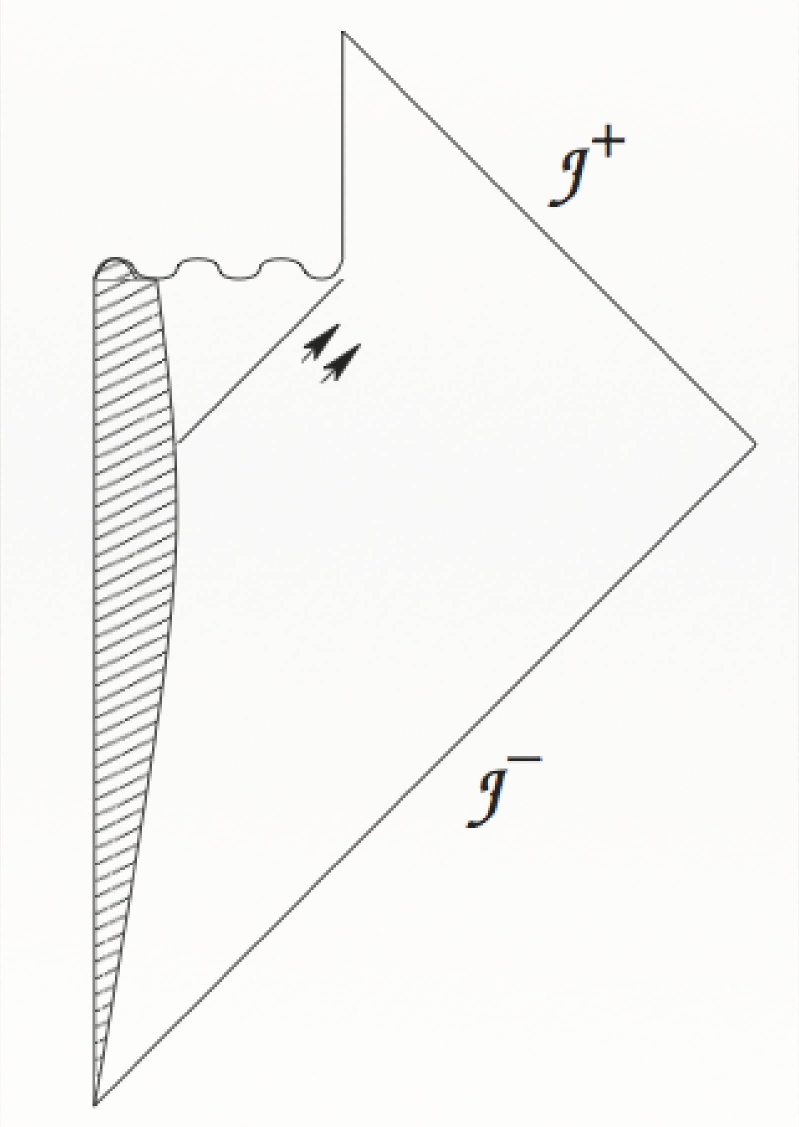
This is no longer true for changes of frame from inertial to non-inertial; e.g. accelerated or rotating observers will see a particle content in this state. [Although, see Klink & Wickramasekara, PRL 2013]

- Parker '66, Cosmology
- Hawking '74, Black Holes
- Unruh '73, Davies '74, Fulling '75, Accelerated observers (Rindler spacetimes)

Led to appreciation of $T_H = \dfrac{\hbar c^3}{8\pi G M k_B} \approx 10^{-8} \, \dfrac{M_\odot}{M} \, \mathrm{K}$

5. Black Hole Evaporation: Singularities cannot remain hidden

While many interesting questions about Hawking radiation remain, e.g. where it is generated, the prediction that black holes can evaporate seems to be robust.

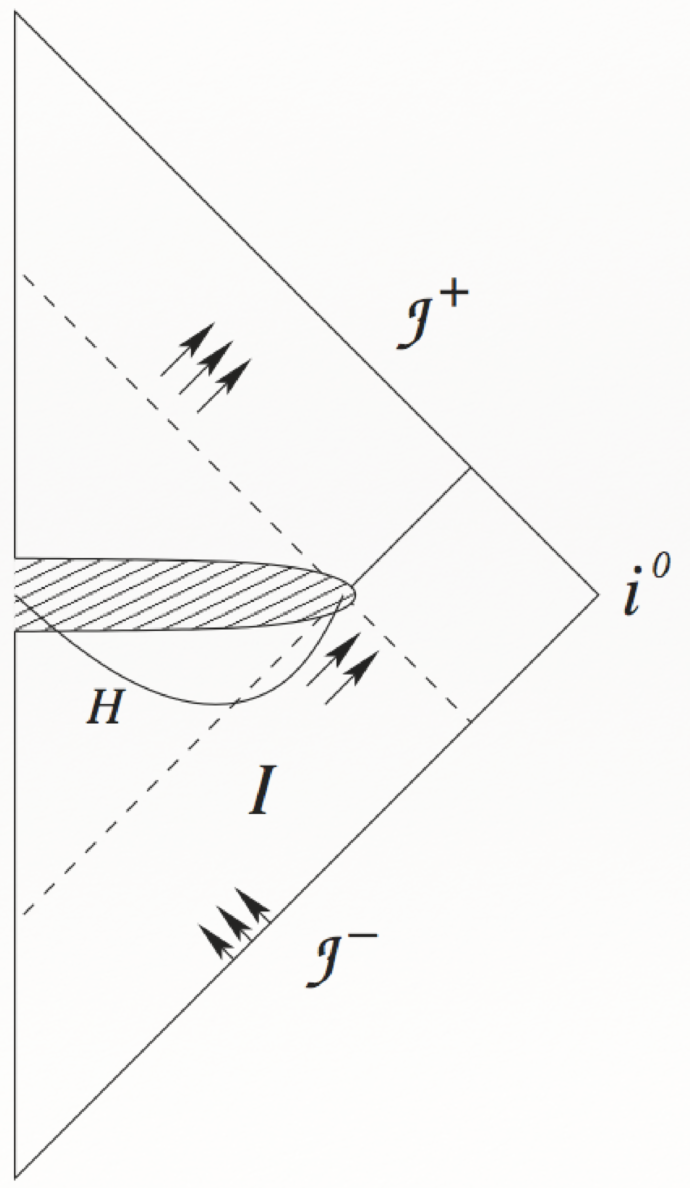


Black hole lifetime:

$$\tau_H \sim M^3 \approx 10^{67}\,\mathrm{yr}\left(\frac{M}{M_\odot}\right)^3$$

Age of the universe:

$$\tau_U \sim 10^{10}\,\mathrm{yr}$$



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Why is quantum gravity so difficult?

(i) We have few experiments to guide us.

You and me at roughly 1.5-2 m in height are closer to the size of the observable universe ($\sim 10^{26}$ m) than to the Planck scale ($\sqrt{\hbar G/c^3} \sim 10^{-35}$ m).

The Compton wavelength of the Higgs is $\lambda_H \approx 10^{-17}$ m.

Without experiments, it can be difficult to sort out what should be taken as key principles and what as wishful thinking.

Why is quantum gravity so difficult?

(i) We have few experiments to guide us.

(ii) Profound conceptual differences from QFT:

- In GR there is no universal notion of energy or of time

Suppose there were a tensorial gravitational energy, then

$$T^{(G)}_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\nu}}{\partial x^{\nu'}}T^{(G)}_{\mu\nu}.$$

But, by the equivalence principle, we can always locally transform away the effects of gravity, so $T^{(G)}_{\mu\nu} = 0$ in this coordinate system, and hence always vanishes. This is closely related to the lack of a preferred time, else this time would specify an energy via Noether's theorem.

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- There is no fixed background causal structure
- Microcausality, $\langle [O(x), O(y)] \rangle = 0$ if $(x-y)^2 > 0$ (i.e. if x & y spacelike separated; note $\eta_{\mu\nu} = \text{diag}(-+++)$), fails

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- The action $S[g_{\mu\nu}]$ is non-polynomial in the (basic) metric variables

Non-polynomial actions

In QFT we typically compute perturbation theory around a quadratic kinetic term:

$$Z[J] = \int \mathcal{D}\phi e^{\frac{i}{\hbar}S + \int J\phi d^4x},$$

with, for example

$$S[\phi] = S_0 + S_I = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{3!}g\phi^3.$$

We do perturbation theory by taking $S_I[\phi] \to S_I[\delta/\delta J]$.

The Einstein-Hilbert action

$$S[g_{\mu\nu}] = \frac{1}{2\kappa}\int (R - 2\Lambda)\sqrt{-g}\,d^4x,$$

doesn't depend on the metric components $g_{\mu\nu}$ in a polynomial fashion: there's both $\sqrt{-g}$ and the inverse $g^{\mu\nu}$ in R. This complicates things considerably.

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- The action $S[g_{\mu\nu}]$ is non-polynomial in the (basic) metric variables
- Perturbative quantum gravity is not renormalizable

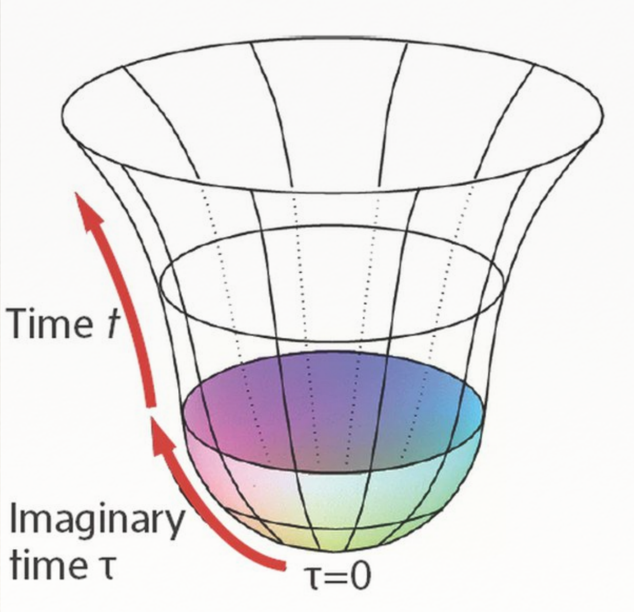
Why is quantum gravity so difficult?

(i) We have few experiments to guide us.

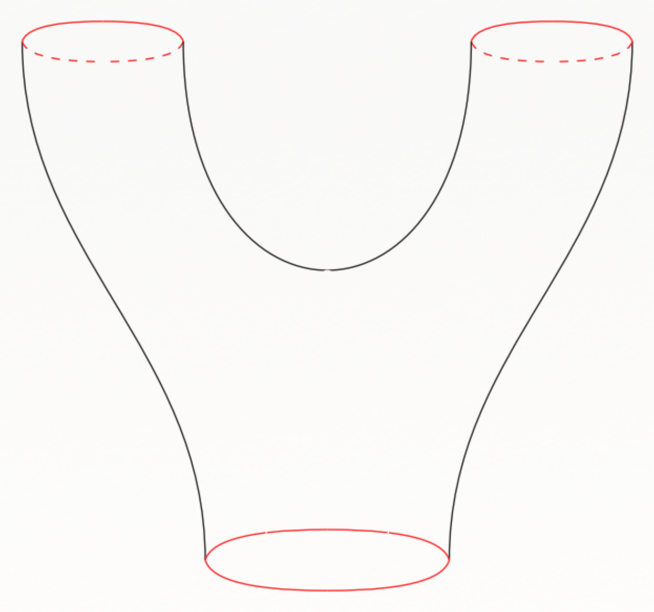
(ii) Profound conceptual differences from QFT.

(iii) Substantial mathematical and technical challenges

- You might like to characterize quantum gravity by a variable number of branches of the universe (like the variable number of particles in QFT):



Hartle-Hawking No Bndry



Topology Change

Why is quantum gravity so difficult?

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- You might like to characterize quantum gravity by a variable number of branches of the universe (like the variable number of particles in QFT).
- But, we don't even have a mathematical classification of the topologies of 4-manifolds.

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- You might like to characterize quantum gravity by a variable number of branches of the universe (like the variable number of particles in QFT).
- But, we don't even have a mathematical classification of the topologies of 4-manifolds.
- The group of diffeomorphisms is large and complicated. The classification of different smooth structures on a smoothable 4-manifold is largely open.

Why is quantum gravity so difficult?

(i) We have few experiments to guide us.
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(iii) Substantial mathematical and technical challenges.

I will return to experiments later in the course…

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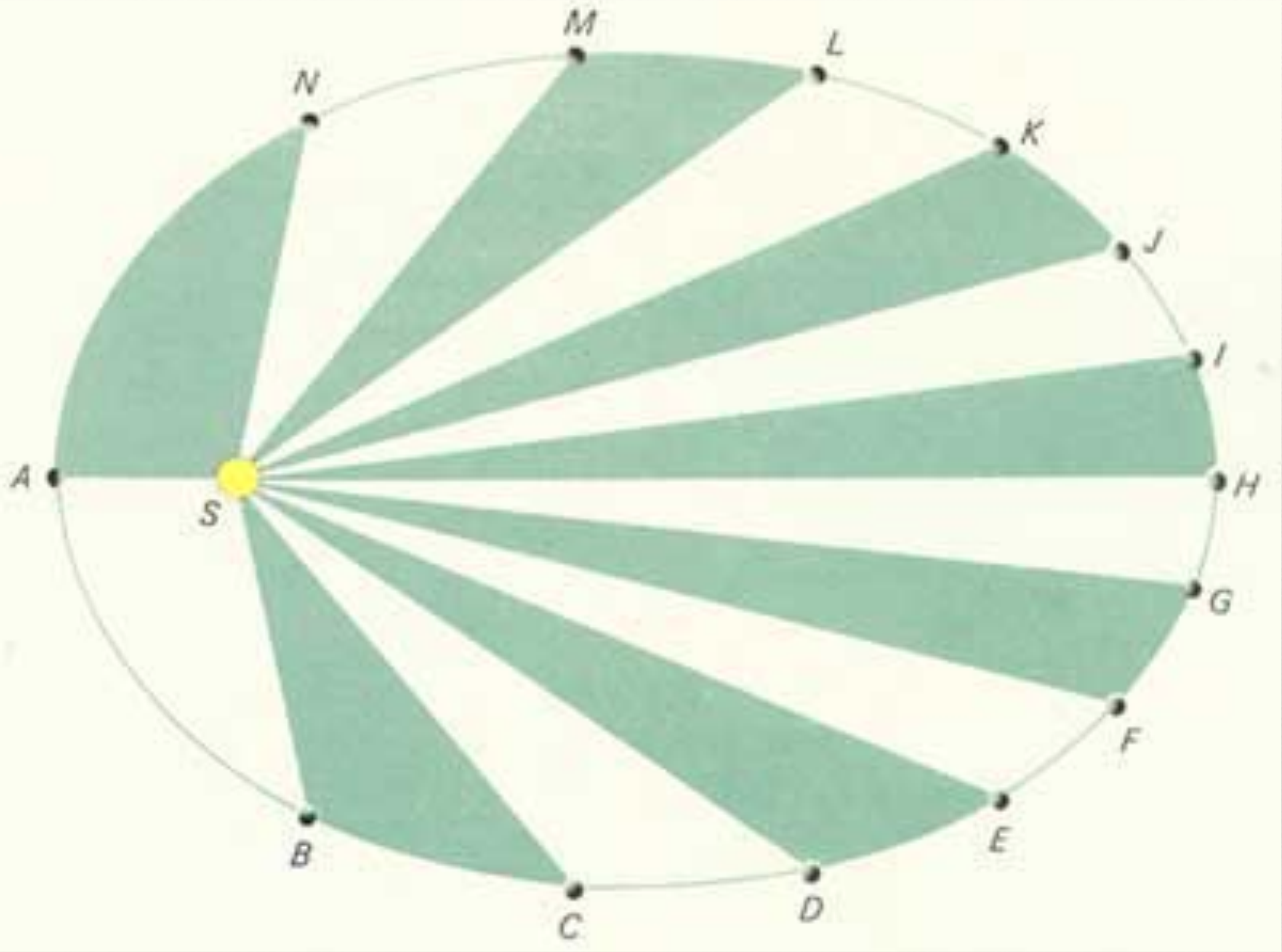
A naive point that I will take seriously: gravitational actions are naturally *areas*



Going all the way back to Kepler, the 2nd law states that the orbit of a planet sweeps out equal areas in equal times.

A naive point that I will take seriously: gravitational actions are naturally *areas*

The integrand of any gravitational path integral will have

$$e^{i\frac{S[g_{\mu\nu}]}{\hbar}},$$

with

$$\frac{S[g_{\mu\nu}]}{\hbar} = \frac{1}{2\hbar\kappa}\int(R-2\Lambda)\sqrt{-g}\,d^4x = \frac{c^4}{16\pi\hbar G}\int(R-2\Lambda)\sqrt{-g}\,d^4x$$

And we once again see the Planck area emerging

$$A_P = \frac{\hbar G}{c^3} \approx 10^{70}\ \mathrm{m}^2.$$

First feature of Loop Gravity: new variables

In 1986 Abhay Ashtekar discovered a new set of variables to describe general relativity. One of these variables, the Ashtekar Electric field, puts areas at center stage.

Electric field: $\tilde{E}^{ia}(x)$, (inverse densitized spatial triad)

- here $a = 1,2,3$ is an index for the coordinates of a spatial slice of spacetime
- $i = 1,2,3$ is an "internal index". The electric field is $\mathfrak{su}(2)$ -valued; fix a basis for this Lie algebra e_i.
- the electric field is naturally associated to a 2-form

$$\tilde{E}^i(x) = \tilde{E}^{ia}(x)\epsilon_{abc}dx^b \wedge dx^c.$$

First feature of Loop Gravity: new variables

While $\tilde{E}^{ia}(x)$ looks complex at first, it is just as naturally associated to areas as the metric is to lengths:

$$\text{Length}(\gamma) = \int_\gamma \left(\frac{\partial x^a}{\partial \tau} \frac{\partial x^b}{\partial \tau} q_{ab} \right)^{1/2} d\tau,$$

and

$$\text{Area}(\sigma) = \int_\sigma \left(\sum_i E^i E^i \right)^{1/2} = \int_\sigma ||E||.$$

Second feature of Loop Gravity: the new variables make gravity into rich gauge theory with polynomial action

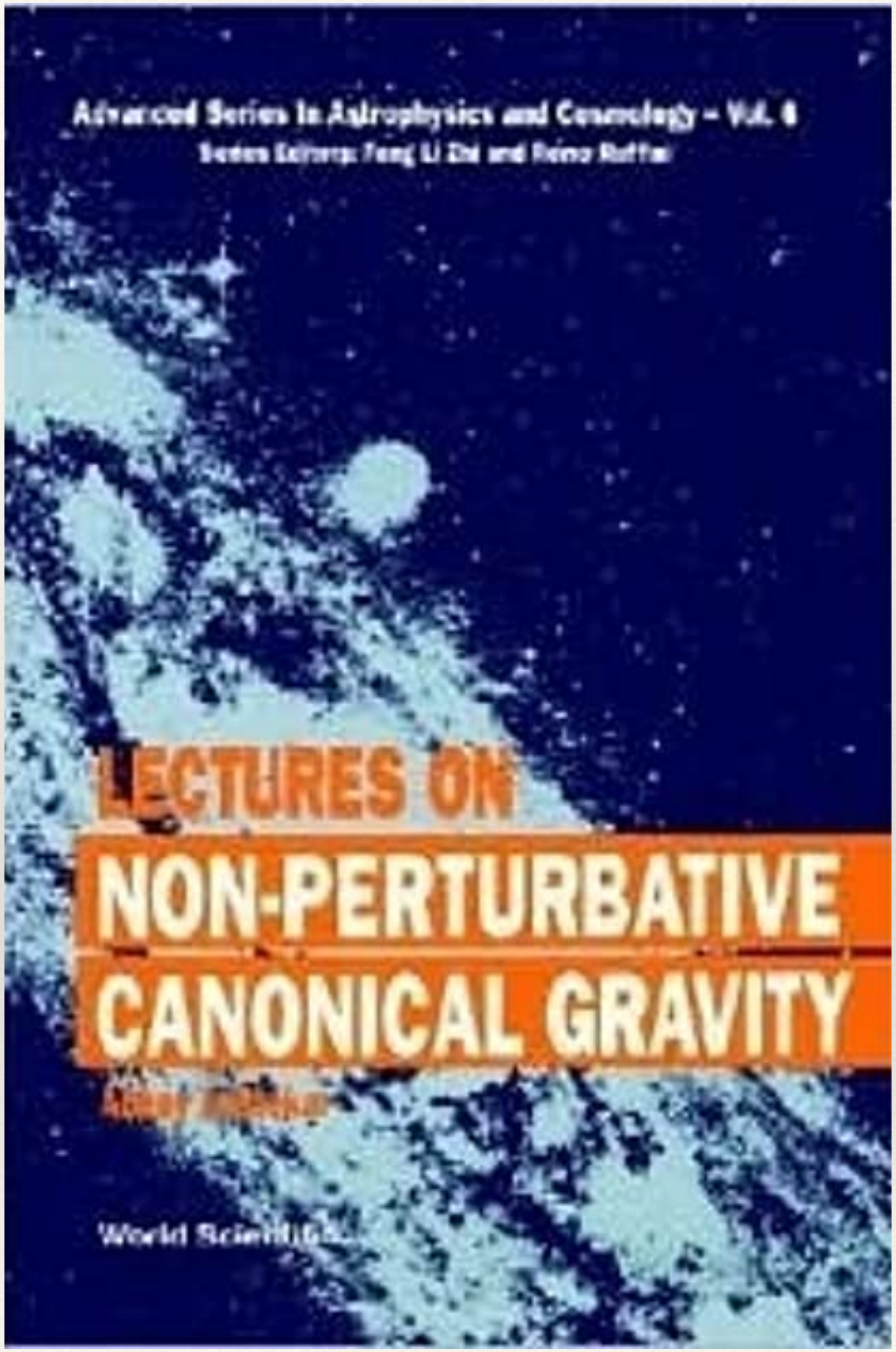
The canonically conjugate variable to the electric field $\tilde{E}^{ia}(x)$, is an $\mathfrak{su}(2)$-valued connection one-form $A^i_a(x)$ on the spatial 3-manifold Σ:

$$\{A^i_a(x), \tilde{E}^b_j(y)\} = \delta^b_a \delta^j_i \delta^3(x, y).$$

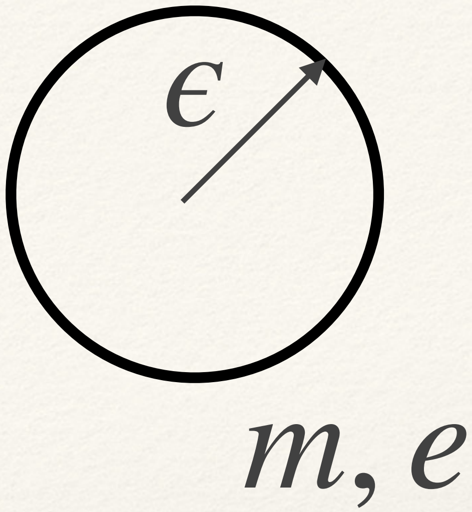
However, these gauge field have additional gauge beyond that of $SU(2)$ rotations; they also have the diffeomorphism group. This will be a central feature moving forward.

Third feature of Loop Gravity: There is an argument that begins A. Ashtekar’s book “Lectures on Non-Perturbative Canonical Gravity” that I’ve long found intriguing:

Are there features of classical GR that would indicate that non-perturbative quantum gravity is very different from perturbative quantum gravity? He proceeds to a simple, but insightful computation:



Consider the self-energy of a shell of charge e and uniform mass density as the radius, ϵ, goes to zero.



Ignoring gravity,

$$m(\epsilon) = m_0 + \frac{e^2}{\epsilon}.$$

For a Newtonian self interaction

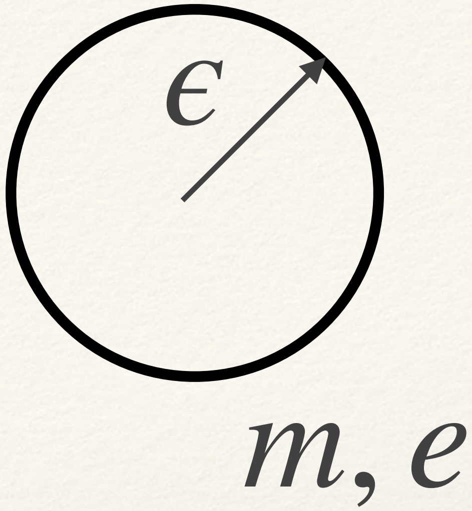
$$m(\epsilon) = m_0 + \frac{e^2}{\epsilon} - \frac{Gm_0^2}{\epsilon},$$

and in both cases the result diverges as $\epsilon \to 0$. In GR

$$m(\epsilon) = m_0 + \frac{e^2}{\epsilon} - \frac{G\,m^2(\epsilon)}{\epsilon} \quad \Longrightarrow \quad m(\epsilon) = \frac{-\epsilon}{2G} + \sqrt{\frac{1}{4G^2}\epsilon^2 + \frac{m_0}{G}\epsilon + \frac{e^2}{G}}$$

This has a finite limit as $\epsilon \to 0$, $m \to e/\sqrt{G}$!

Consider the self-energy of a shell of charge e and uniform mass density as the radius, ϵ, goes to zero.



But, if we expand around small G

$$m(\epsilon) = \frac{-\epsilon}{2G} + \frac{\epsilon}{2G}\sqrt{1 + \frac{4G}{\epsilon}\left(m_0 + \frac{e^2}{\epsilon}\right)}$$

$$= \left(m_0 + \frac{e^2}{\epsilon}\right) - \left(m_0 + \frac{e^2}{\epsilon}\right)^2 \frac{G}{\epsilon} + 2\left(m_0 + \frac{e^2}{\epsilon}\right)^3 \left(\frac{G}{\epsilon}\right)^2 + \cdots$$

Every term is divergent in the $\epsilon \to 0$ limit.

I take this cautionary tale seriously; beware of over interpreting perturbative divergences! Also

Perturbative divergences carry interesting information & structure $\rightsquigarrow$ known as resurgence

Today's Discussion

1. Why Quantum Gravity?
2. Some of the Reasons that Quantum Gravity is Difficult
3. Features of the Loop Approach to Quantum Gravity
4. Overview of the Course

Course overview

Discussion 2 (Wed, June 14th): Falling cats, GR as a gauge theory, and Quantum Tetrahedra

Discussion 3 (Wed, June 21st): Building space—Spin networks and Quantum Discreteness

Discussion 4 (Wed, June 28th): Spin foams—Discrete Geometry Path Integrals

Discussion 5 (Wed, July 5th): Frontiers of Loop Quantum Gravity—Experiments, Black Holes, and Open Questions

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