

The Basics of Loop Quantum Gravity

Discussion 5: Frontiers of Quantum Gravity

International Society of
Quantum Gravity
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Bard College

Review question 1: What is the kinematical Hilbert space that LQG is built on?

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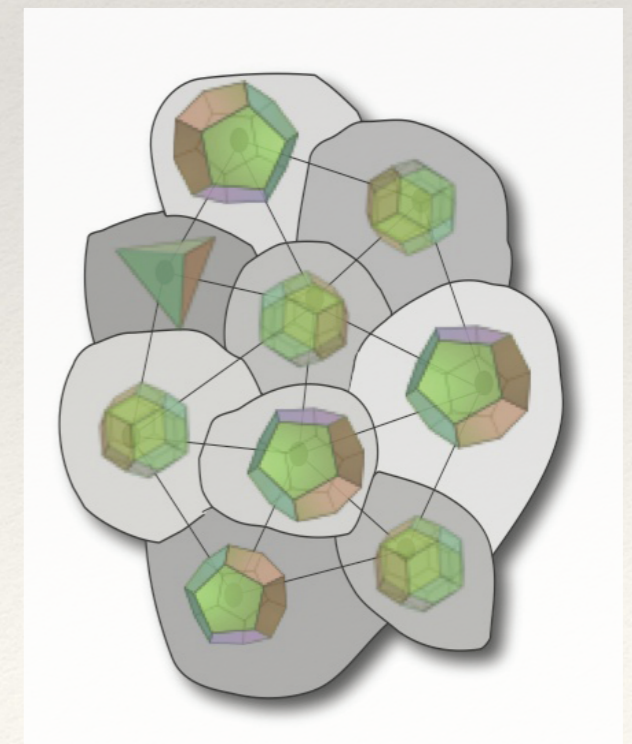
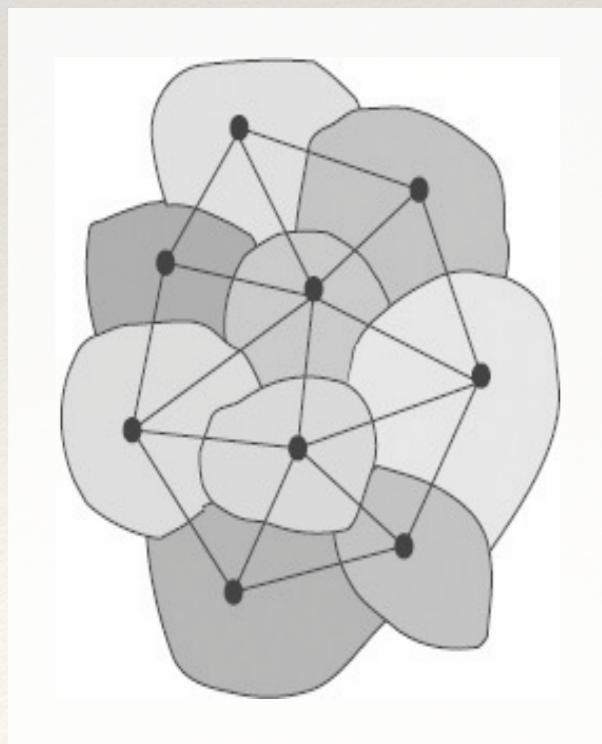
The loop approach has uncovered a remarkable kinematical Hilbert space. For a fixed graph Γ it is

$$\mathcal{H}_\Gamma = L^2[G^L/G^N, \mu_H],$$

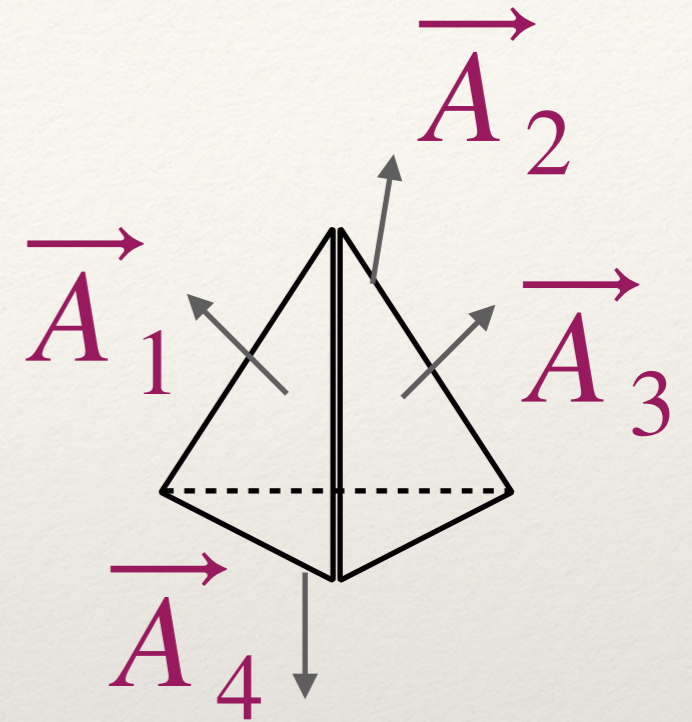
which consists of cylindrical functions:

$$f(A) = \Psi_\Gamma(\{h_\ell\}).$$

polyhedral
interpolation



Review question 2: Where was the ‘cat’s paw print’?



The gauge invariant parameters, $p_{tt'}^\tau$ and $p_{t't''}^\tau$, needed to complete our gauge invariant description of the shape of the tetrahedron (shape coords.) *do not commute*:

$$\{p_{tt'}^\tau, p_{t't''}^\tau\} = \gamma \vec{A}_t \cdot (\vec{A}_{t'} \times \vec{A}_{t''}) = \pm \gamma \frac{9}{2} \text{Vol}_\tau^2.$$

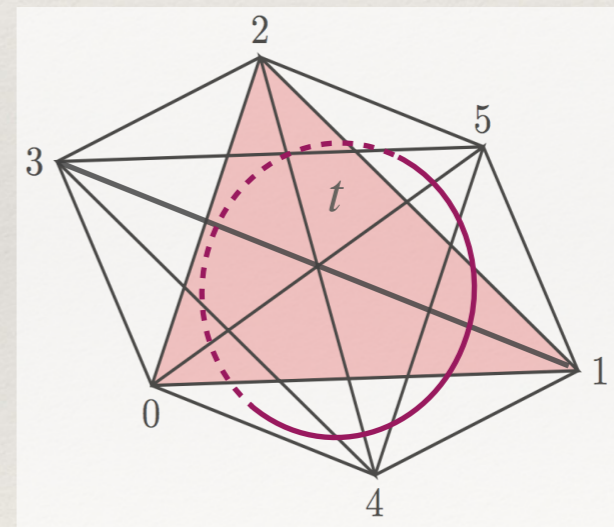
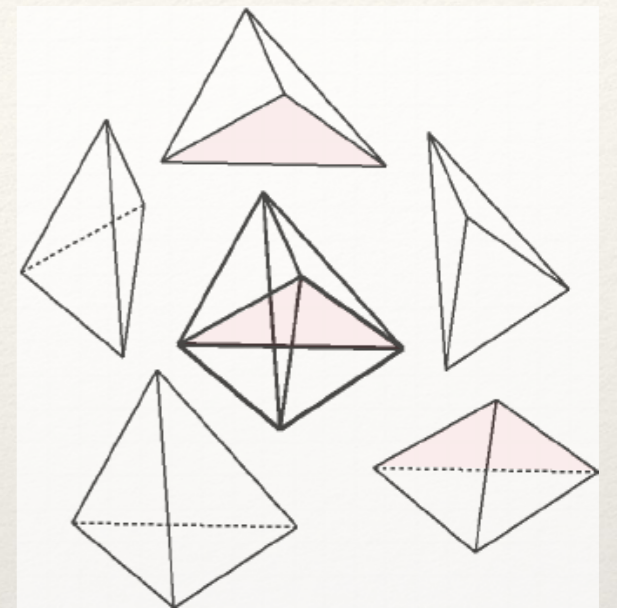
Summary point: Area Regge calculus is possible, but requires weak constraints!

In 4D the bones are 2D triangles t :

The area of the triangle t is conjugate to the curvature angle around the bone. This curvature angle is compact, indicating the areas will be quantized.

The choice of area variables is harmonious with LQG, and the focus of the discrete geometry path integrals of spin foams.

5 tetrahedra
glue into a
4D simplex



Prologue

Today I want to discuss the prospects and challenges of connecting quantum gravity to experiment.

I will also touch on some ideas and open problems in Loop Quantum Gravity.

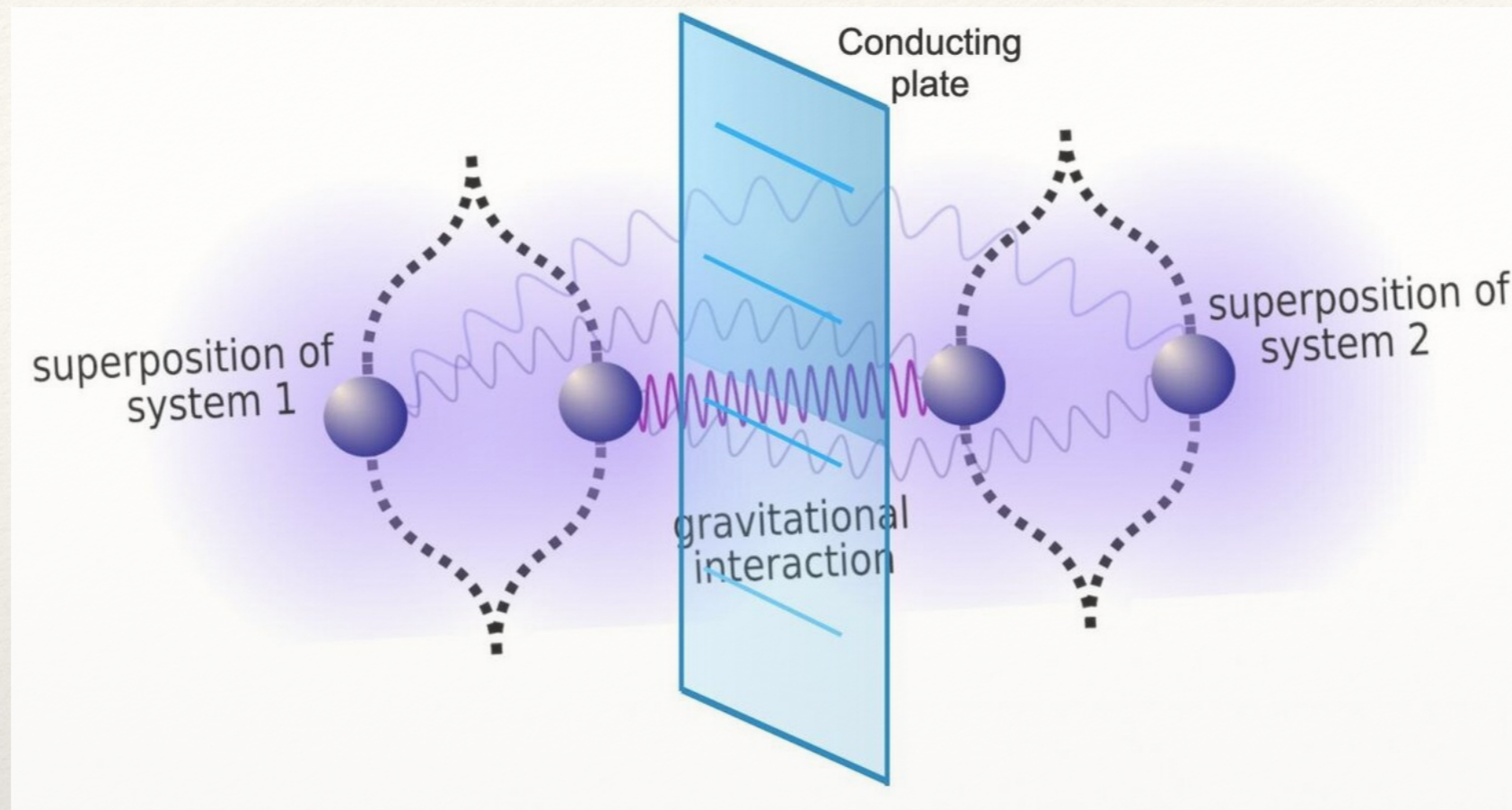
Planck Units

Name:	Dimension	Expression	Value (SI)
Planck length	Length (L)	$l_P = \sqrt{\hbar G/c^3}$	1.616×10^{-35} m
Planck mass	Mass (M)	$m_P = \sqrt{\hbar c/G}$	2.176×10^{-8} kg
Planck time	Time (T)	$t_P = \sqrt{\hbar G/c^5}$	5.391×10^{-44} s
Planck temp.	Temperature (Θ)	$T_P = \sqrt{\hbar c^5/Gk_B^2}$	1.416×10^{32} K
Planck area	Area (L^2)	$a_P = \hbar G/c^3$	2.612×10^{-70} m ²
Planck energy	Energy ($L^2M T^{-2}$)	$E_P = \sqrt{\hbar c^5/G}$	1.956×10^9 J
Planck force	Force ($LM T^{-2}$)	$F_P = c^4/G$	1.210×10^{44} N
Planck density	Density ($M L^{-3}$)	$\rho_P = c^5/\hbar G^2$	5.155×10^{96} kg/m ³
Planck power	Power ($L^2M T^{-3}$)	$F_P = c^5/G$	3.628×10^{52} W

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Gravity-Induced Entanglement



Three steps:

1. Systems 1 and 2, of equal masses m , are put in a superposition of positions, L and R : $(|R\rangle_1 + |L\rangle_1) \otimes (|R\rangle_2 + |L\rangle_2) = |R, R\rangle + |R, L\rangle + |L, R\rangle + |L, L\rangle$
2. In one branch (the $|R, L\rangle$ one here), the masses are a small distance d apart for a time t .
3. Gravitational time dilation in the close branch leads to a change in phase

$$\delta\phi = \frac{Gm^2t}{\hbar d}$$

$$S(M, J) = k \frac{A(M, J)}{4 \hbar G / c^3}$$

Are black holes simple or complex?

Pure classical solutions to GR:

Mass M , Spin J

Huge entropy:

$$S(M, J) = \frac{A(M, J)}{4\ell_P^2}$$



vs.



Today's Discussion

1. The Observational Challenge: Black Hole Spins
2. Matter and Λ : Massive and Curved 'Cats'
3. Quantum Gravitational Symmetries and Topology Change

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1. The Observational Challenge: Black Hole Spins
2. Matter and Λ : Massive and Curved 'Cats'
3. Quantum Gravitational Symmetries and Topology Change

Gravitational thermodynamics and statistical mechanics are atypical

- ❖ Gravity is long-range and cannot be shielded
- ❖ Strongly gravitating systems, like black holes, do not have additive energies (non-linearity)
- ❖ \rightsquigarrow Both of these facts undermine the basic notion of thermodynamic extensivity and much of the usual formalism

On the other hand, thermo-statistical arguments would appear to be the best chance we have to confirm a quantum Nature for the gravitational field.

This atypicality arises in practice: self-gravitating gases are odd

A gas of self-gravitating particles satisfies the virial theorem

$$U_{\text{pot}} = -2U_{\text{kin}},$$

and, hence,

$$U_{\text{tot}} = U_{\text{kin}} + U_{\text{pot}} = -U_{\text{kin}}.$$

By equipartition

$$U_{\text{tot}} = -\frac{3}{2}NkT \quad \text{and} \quad C = \left(\frac{\partial U}{\partial T} \right) = -\frac{3}{2}Nk.$$

Self-gravitating gasses have negative heat capacity. They break up into a core and halo, with the core condensing and heating and the halo accepting heat, expanding & cooling.

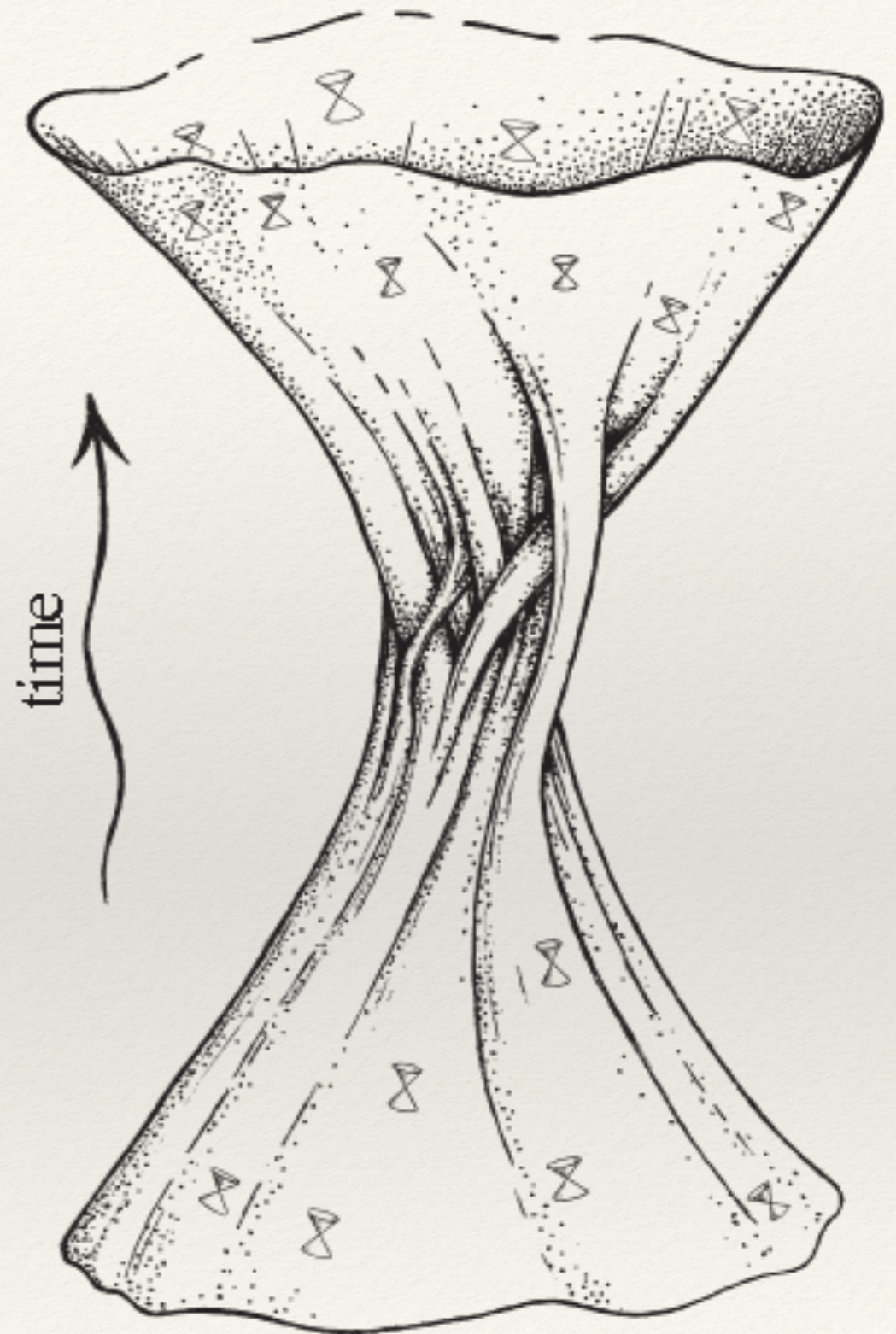


Roger Penrose in Berkeley, CA, 1978

As we've discussed...half of 2020
Physics Nobel prize

“for the discovery that black hole
formation is a robust prediction of
the general theory of relativity”

Is collapse avoidable?

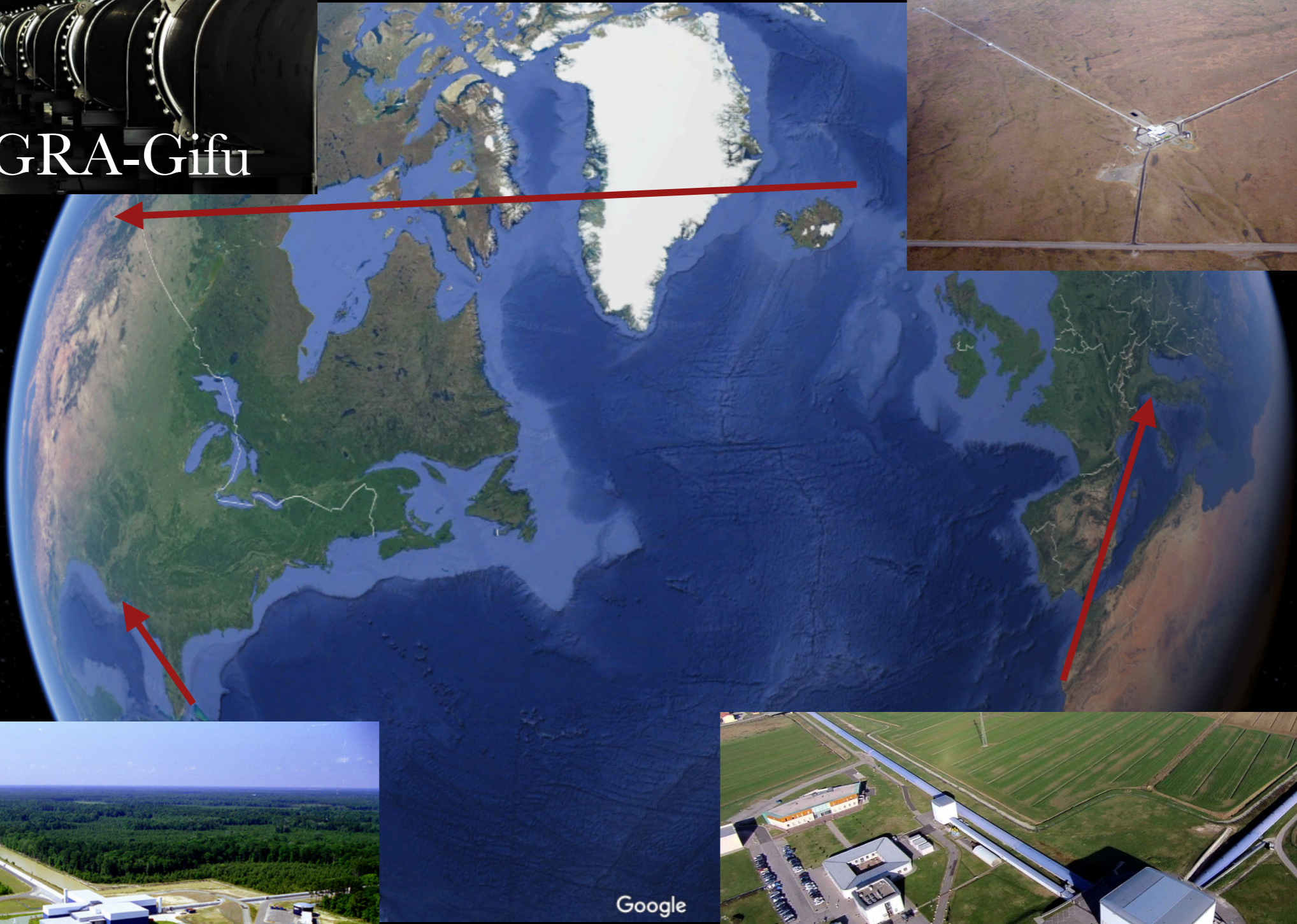




KAGRA-Gifu



LIGO-Hanford



Google

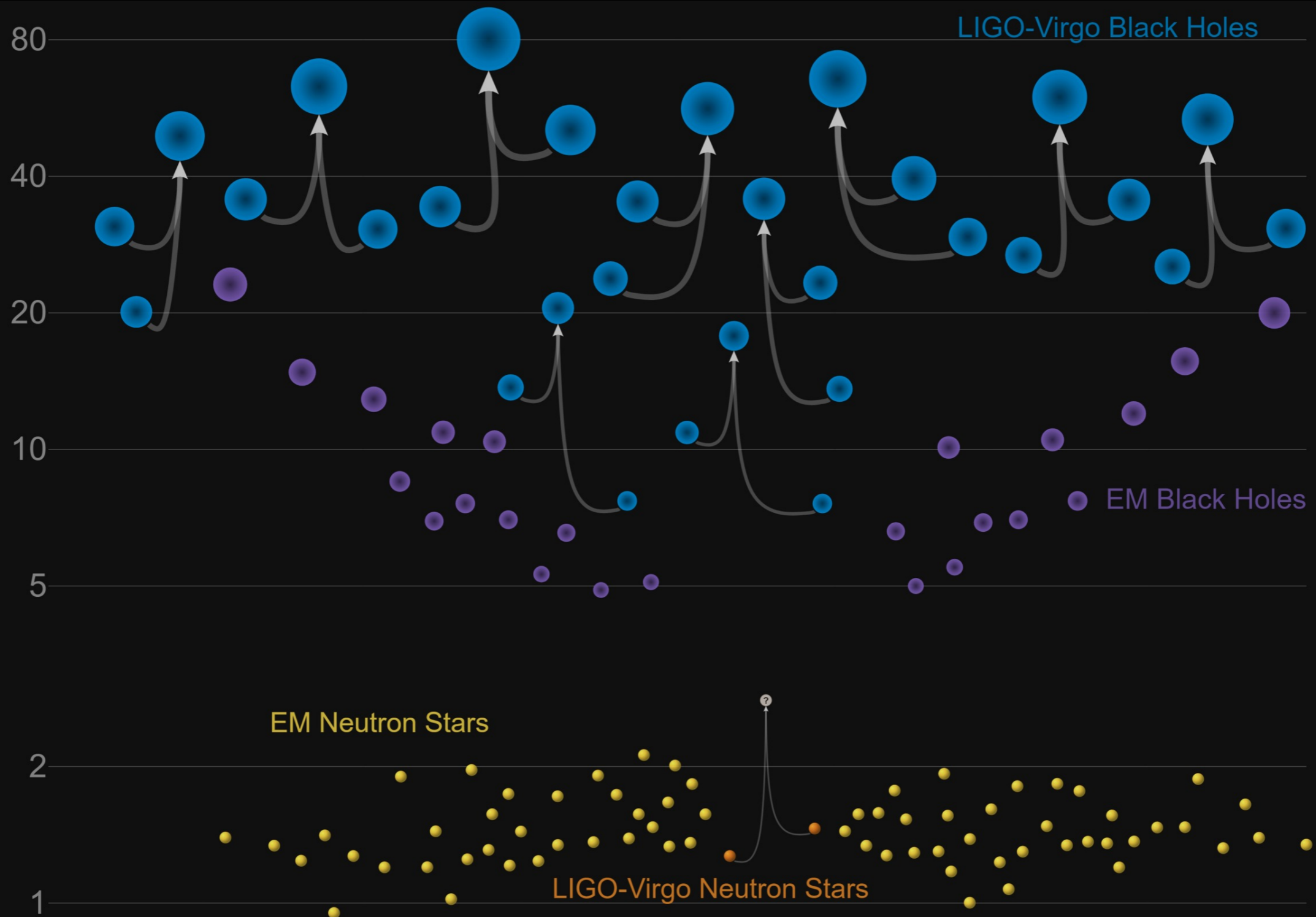


LIGO-Livingston



VIRGO-Cascina

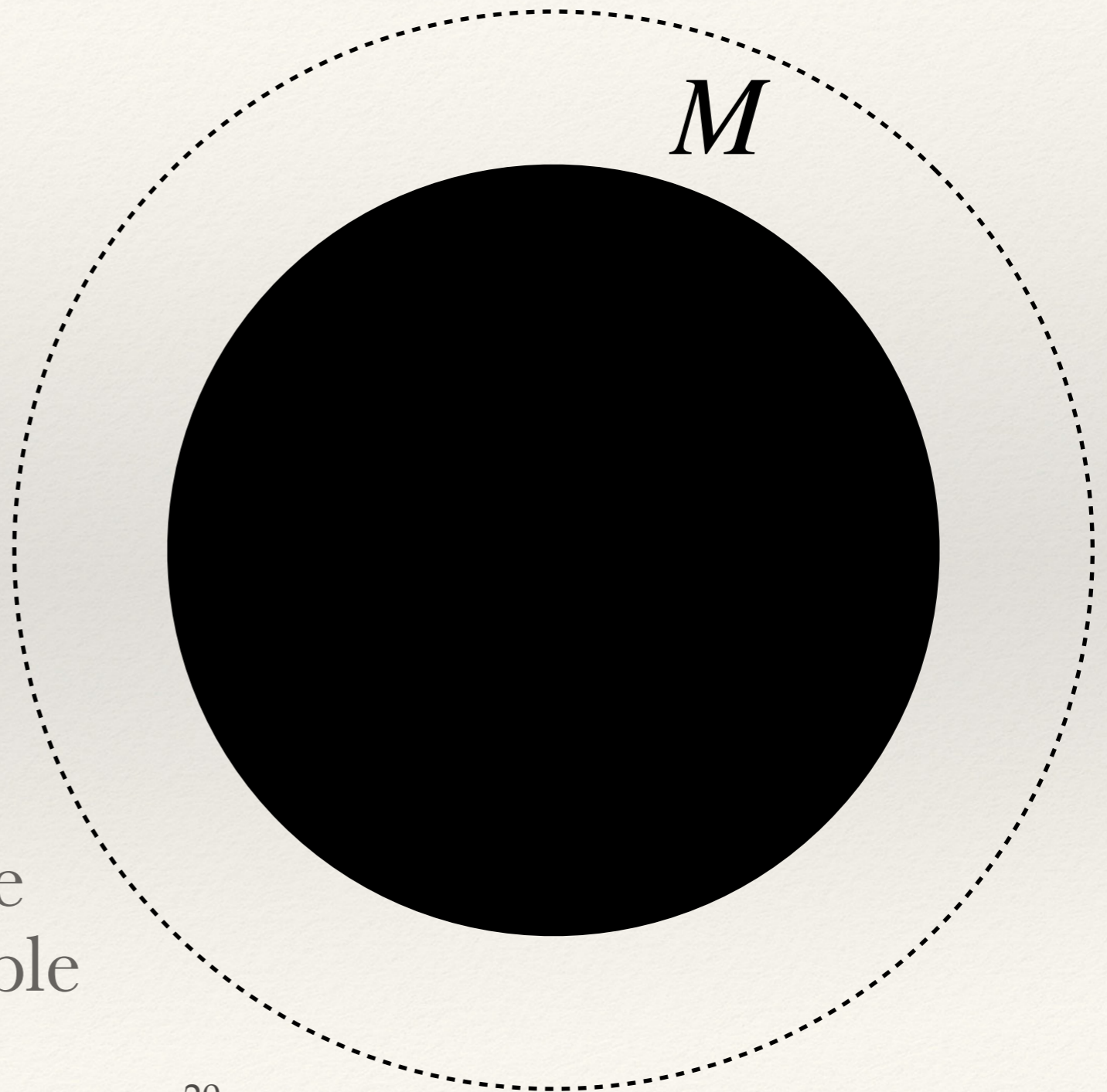
In solar masses



Black hole entropy suggests that we fix a thermodynamical setting:

Fix the mass-energy content of the dashed region to be M ...

...this corresponds to what we usually call the microcanonical ensemble



At fixed mass, rotating black holes have a smaller entropy

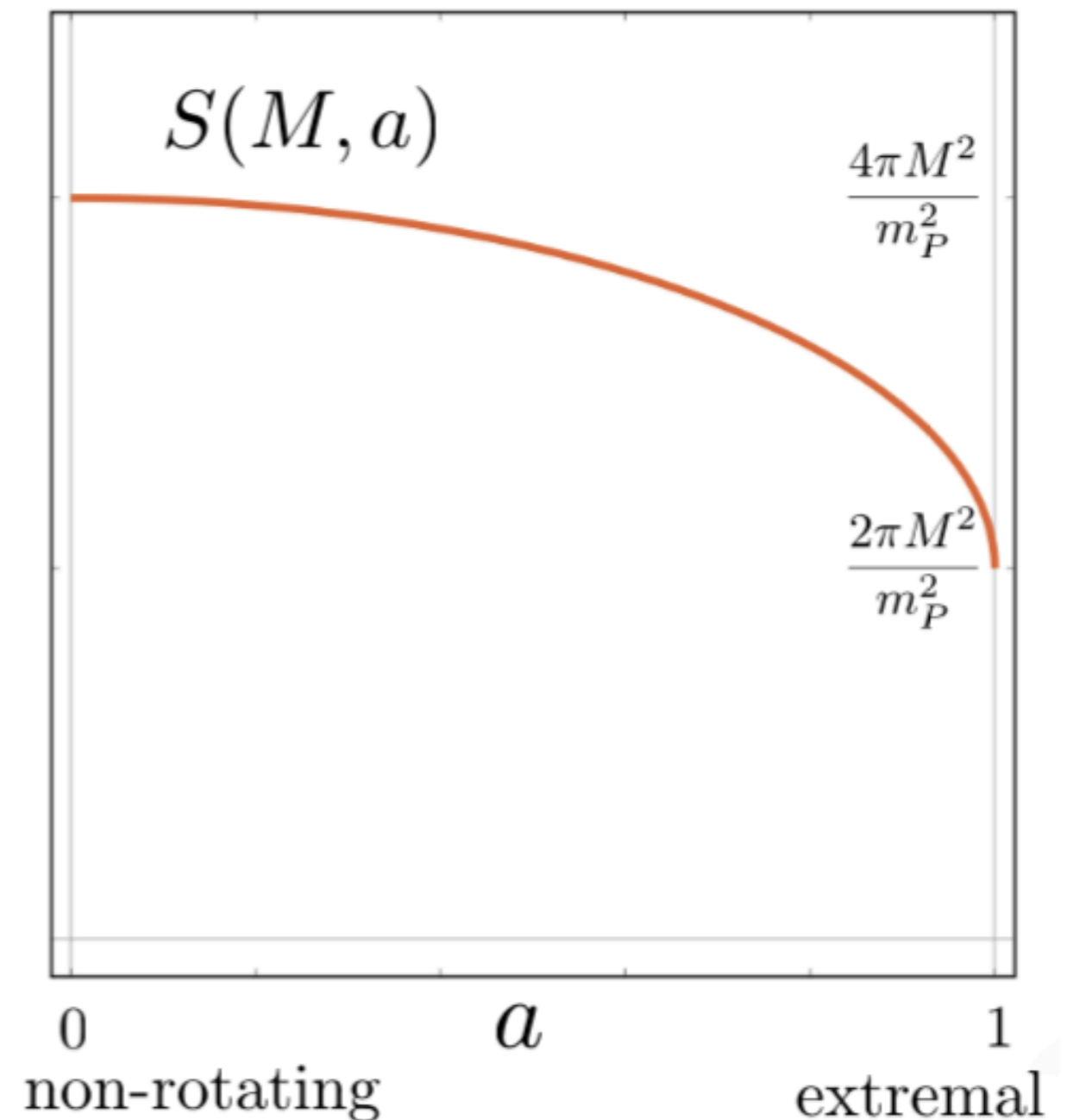
Dimensionless spin parameter

$$a = \frac{|\vec{J}|}{GM^2/c},$$

$$a \in [0, 1].$$

Bekenstein-Hawking entropy

$$S(M, a) = (1 + \sqrt{1 - a^2}) \frac{2\pi M^2}{m_P^2}$$



Interpreting the black hole entropy as a result of microstates, we can introduce the probability of different spins

Number of microstates at fixed (M, a) :

$$\mathcal{N} \sim e^{S(M,a)}$$

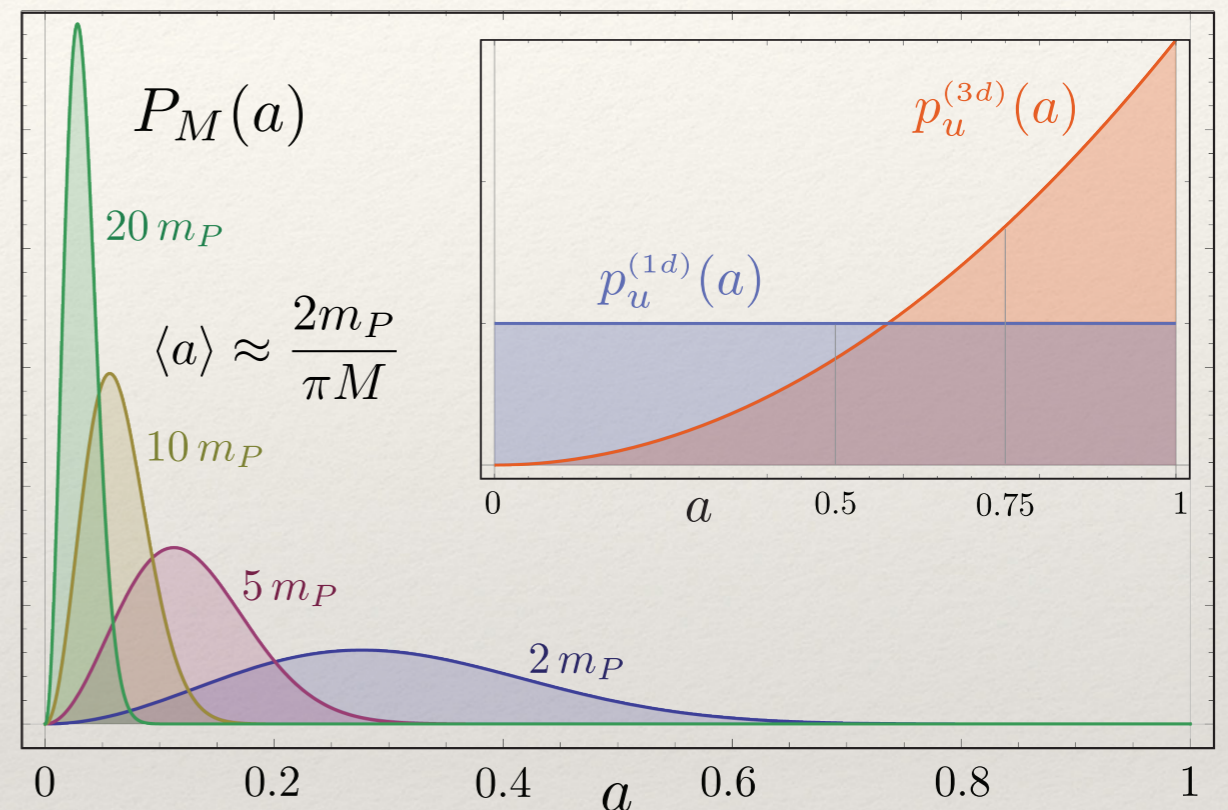
Number of microstates at fixed M :

$$\mathcal{N} \sim \int_0^1 e^{S(M,a')} a'^2 da'$$

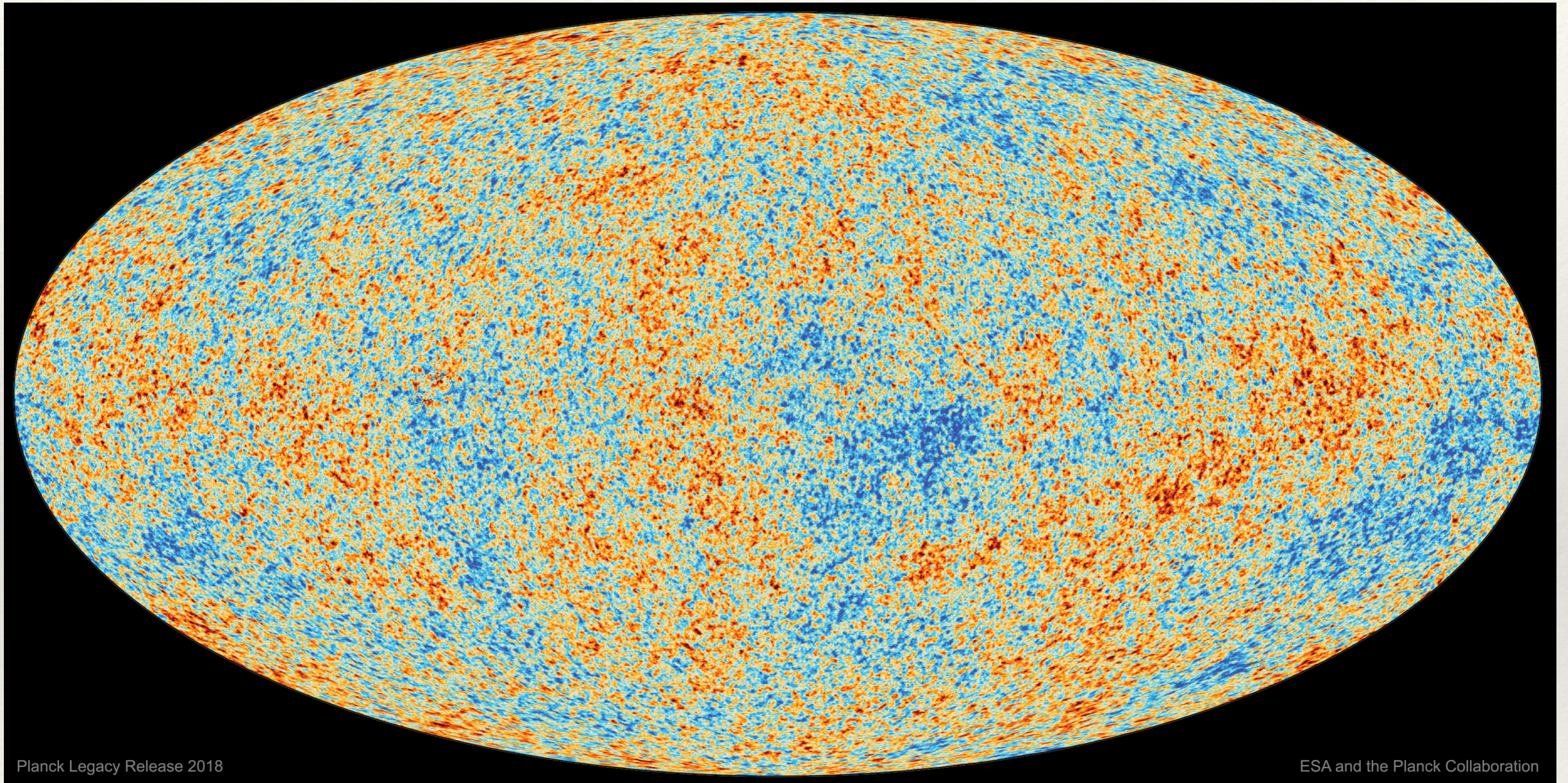
The ratio gives a probability:

$$P_M(a) = \frac{e^{A(M,a)/4\ell_P^2} a^2}{\int_0^1 e^{A(M,a')/4\ell_P^2} a'^2 da'}$$

Microcanonical BHs have small spins, $\langle a \rangle(M_\odot) \approx 10^{-38}$

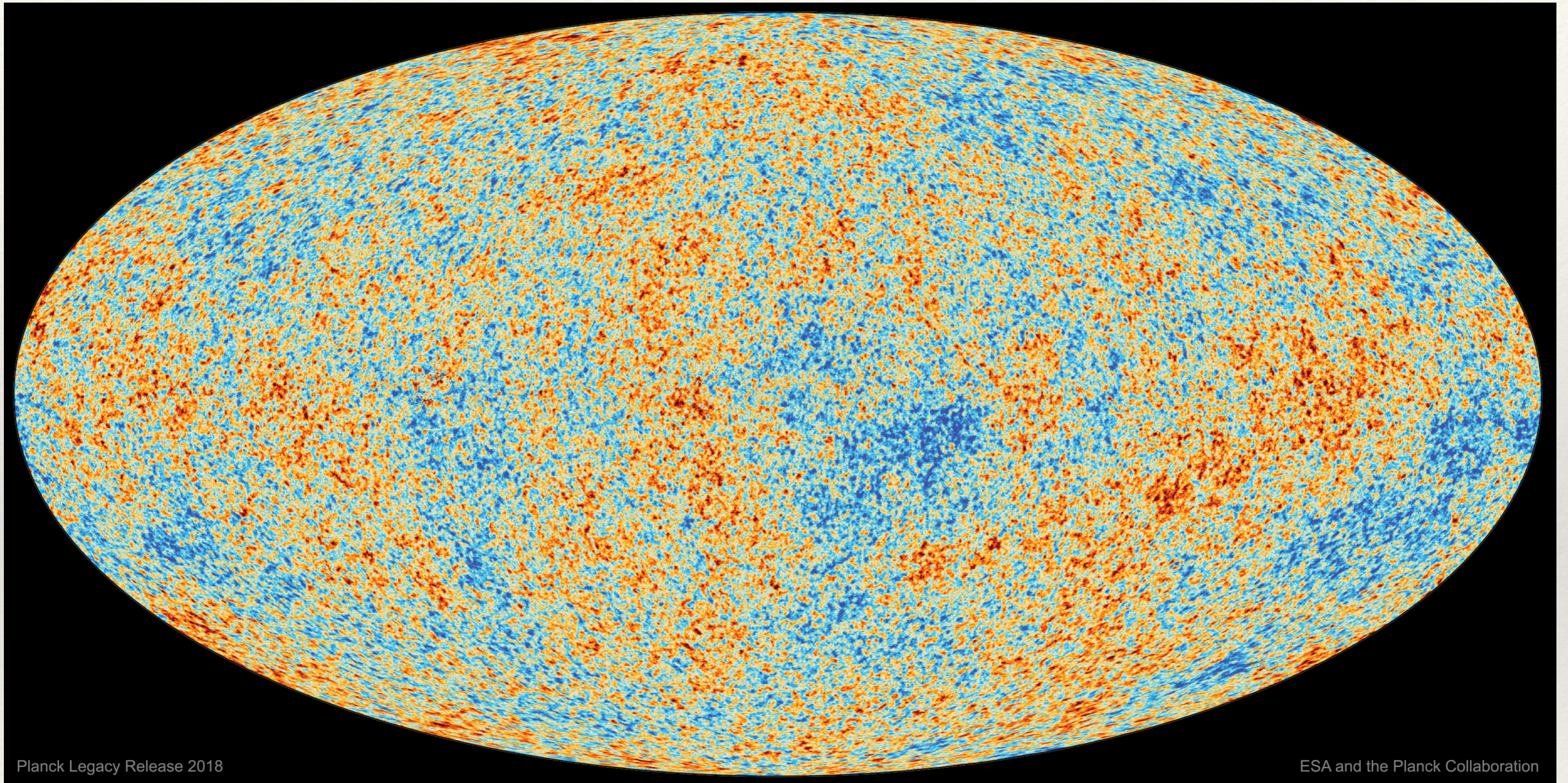


Inflationary models provide an intriguing explanation of the cosmic microwave background



Thermal fluctuations alone are too small to explain the temperature anisotropies...

Inflationary models provide an intriguing explanation of the cosmic microwave background



...but, these anisotropies are well described by a gaussian statistics of quantum curvature perturbations

What about large amplitude curvature perturbations?

We are taking the intriguing hypothesis that large amplitude perturbations are described by a Boltzmann statistics in the microcanonical ensemble:

$$P_B(g_{\mu\nu} | M) = \frac{\delta(G_{\mu\nu})\delta(\mathcal{H} - M)}{\int \mathcal{D}g_{\mu\nu} \delta(G_{\mu\nu})\delta(\mathcal{H} - M)}.$$

Why?

What about large amplitude curvature perturbations?

Why? Well, a quantum state for the geometry of the universe would lead to a probability $P_\Psi(g_{\mu\nu})$ and a microcanonical probability

$$P_\Psi(g_{\mu\nu} | M) = \frac{P_\Psi(g_{\mu\nu})\delta(\mathcal{H} - M)}{\int \mathcal{D}g_{\mu\nu} P_\Psi(g_{\mu\nu})\delta(\mathcal{H} - M)}.$$

If $P_\Psi(g_{\mu\nu})$ is sufficiently constant over the mass shell, then this exactly reproduces the Boltzmann distribution of the last slide(!) and we are doing Gen. Rel. statistical mechanics. This leads to all the BH entropy results:

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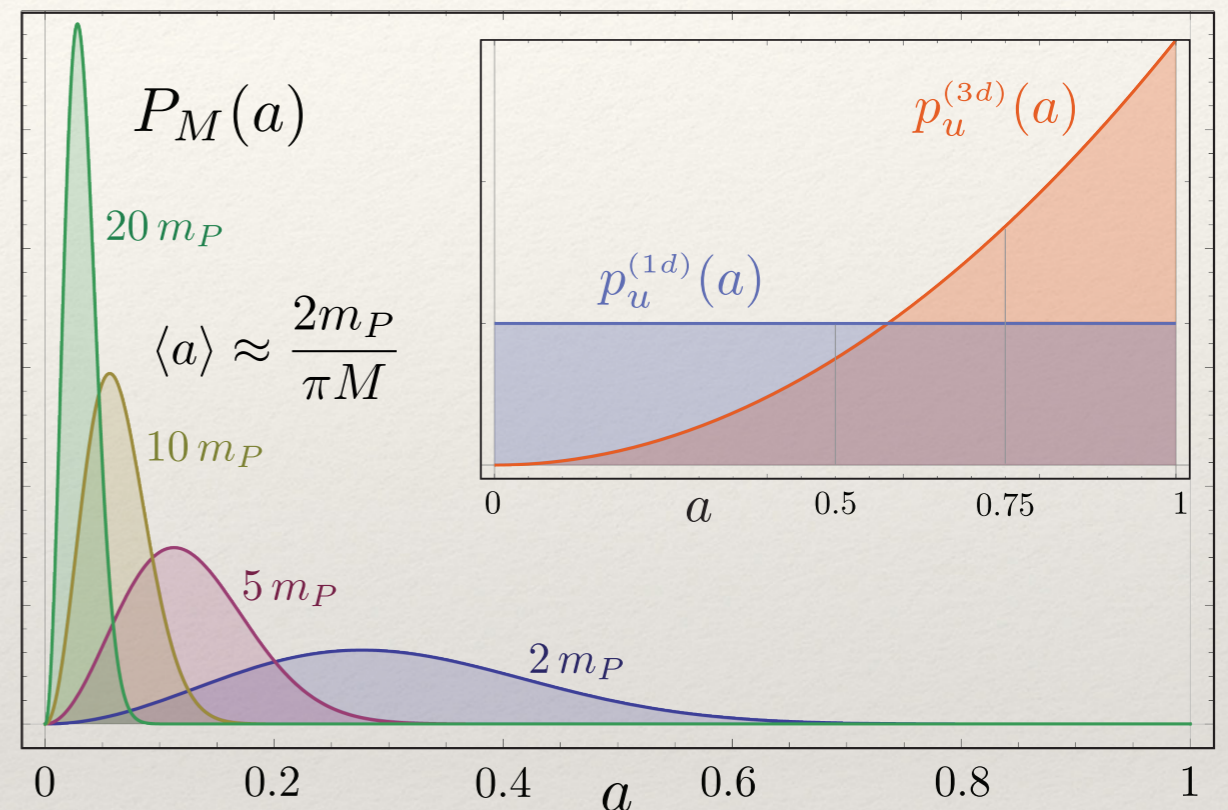
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What are the spins of black holes that form via collapse?

$$0 \leq a \equiv \frac{|\vec{J}|}{GM/c^2 \cdot Mc} = \frac{|\vec{J}|}{GM^2/c} \leq 1$$

Table 1 The masses and spins, measured via continuum-fitting, of ten stellar black holes^a.

System	a_*	M/M_\odot	References
Persistent			
Cyg X-1	> 0.95	14.8 ± 1.0	Gou et al. 2011; Orosz et al. 2011a
LMC X-1	$0.92^{+0.05}_{-0.07}$	10.9 ± 1.4	Gou et al. 2009; Orosz et al. 2009
M33 X-7	0.84 ± 0.05	15.65 ± 1.45	Liu et al. 2008; Orosz et al. 2007
Transient			
GRS 1915+105	$> 0.95^b$	10.1 ± 0.6	McClintock et al. 2006; Steeghs et al. 2013
4U 1543–47	0.80 ± 0.10^b	9.4 ± 1.0	Shafee et al. 2006; Orosz 2003
GRO J1655–40	0.70 ± 0.10^b	6.3 ± 0.5	Shafee et al. 2006; Greene et al. 2001
XTE J1550–564	$0.34^{+0.20}_{-0.28}$	9.1 ± 0.6	Steiner et al. 2011; Orosz et al. 2011b
H1743–322	0.2 ± 0.3	$\sim 8^c$	Steiner et al. 2012a
LMC X-3	$< 0.3^d$	7.6 ± 1.6	Davis et al. 2006; Orosz 2003
A0620–00	0.12 ± 0.19	6.6 ± 0.25	Gou et al. 2010; Cantrell et al. 2010

Are some black holes born spinless?

Is zero spin evidence for the statistical nature of black hole entropy?

What comes of confronting these ideas with gravitational wave data?

Observation of black-hole spins in GW events

$$(M_1, \vec{a}_1) + (M_2, \vec{a}_2) + \vec{L} \longrightarrow (M_f, \vec{a}_f) + GW$$

Dimensionless spin

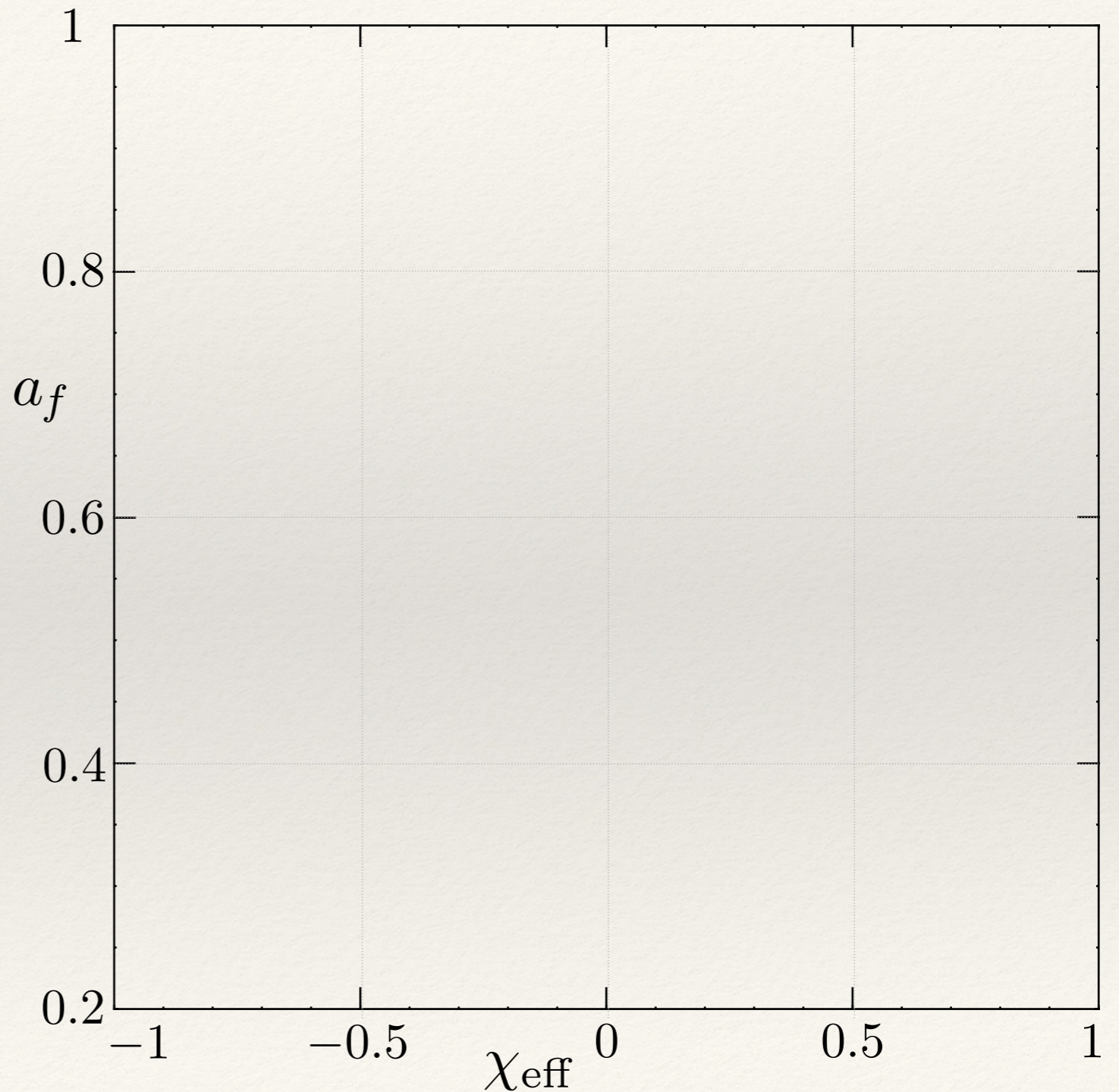
$$a = \frac{|\vec{S}|}{GM^2/c}$$

$$a \in [0, 1].$$

Effective initial spin

$$\chi_{\text{eff}} = \frac{M_1 \vec{a}_1 + M_2 \vec{a}_2}{M_1 + M_2} \cdot \frac{\vec{L}}{|\vec{L}|},$$

with $\chi_{\text{eff}} \in [-1, 1]$.



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▲ GW150914 ▼ GW151012 ◀ GW151226

Dimensionless spin

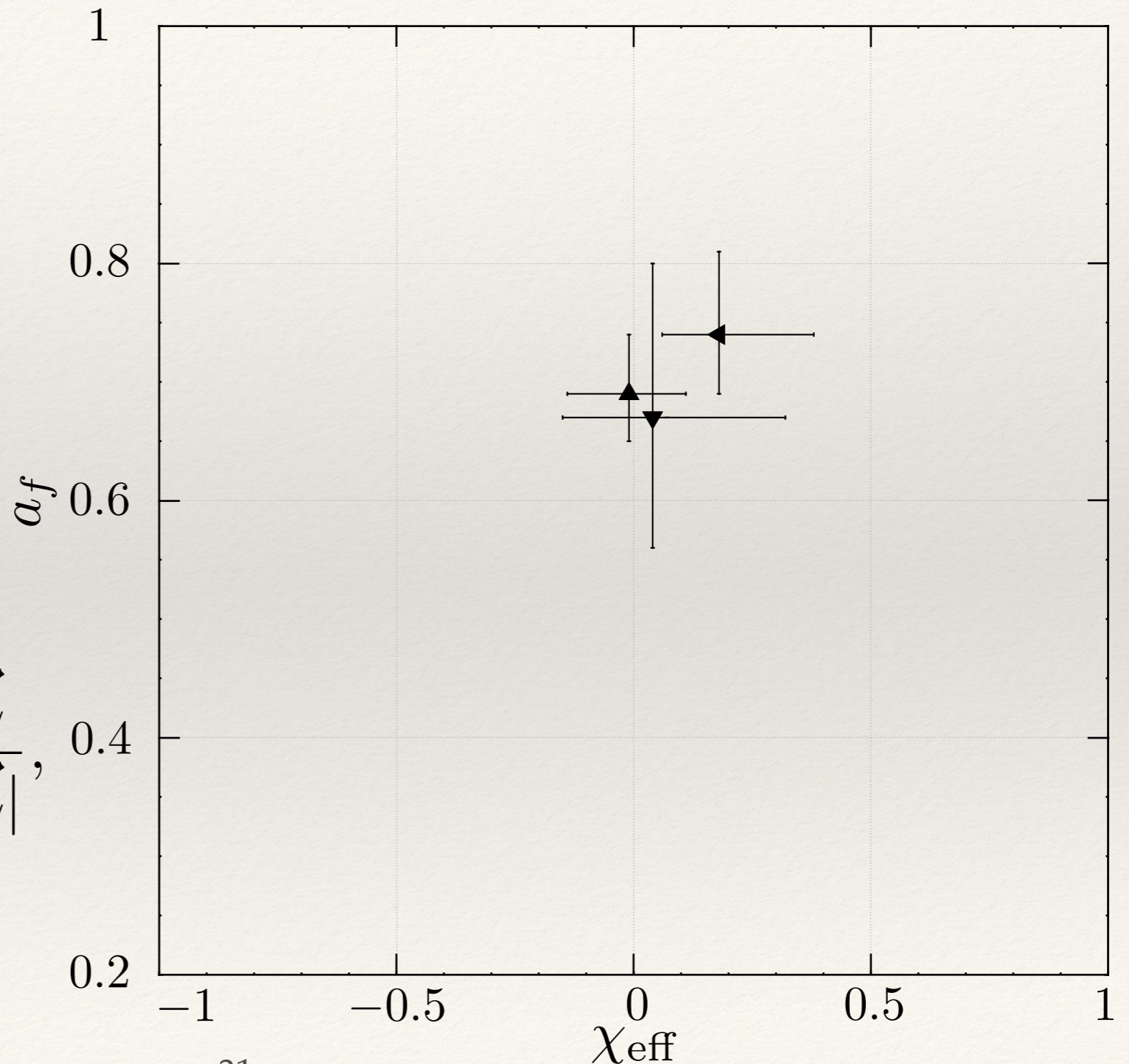
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- | | | | | |
|------------|------------|------------|------------|------------|
| ▲ GW150914 | ▼ GW151012 | ◄ GW151226 | ► GW170104 | ◆ GW170608 |
| ■ GW170729 | ◆ GW170809 | ✕ GW170814 | ★ GW170818 | ★ GW170823 |

Dimensionless spin

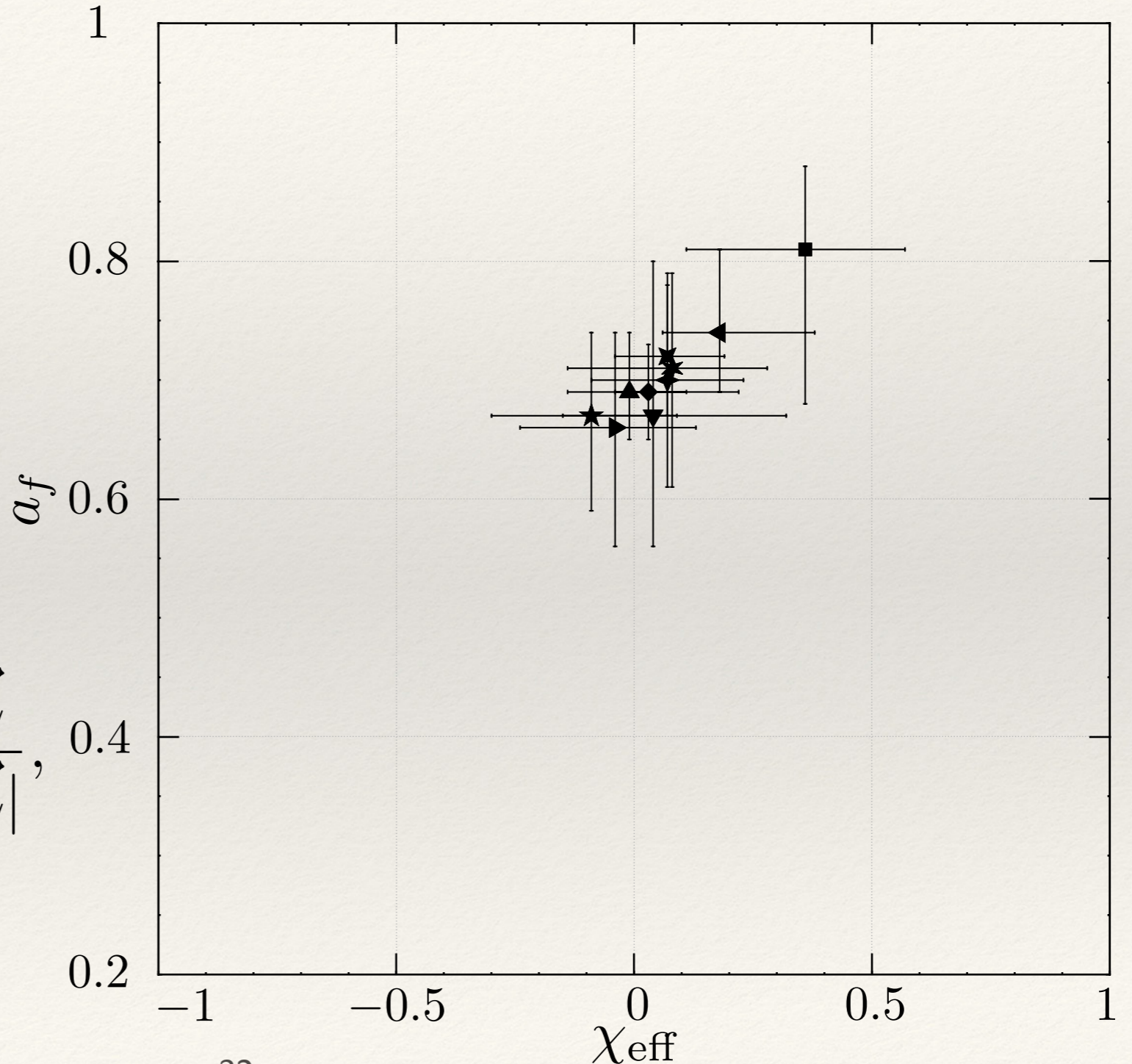
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Comparison of microcanonical ensemble to gravitational waves

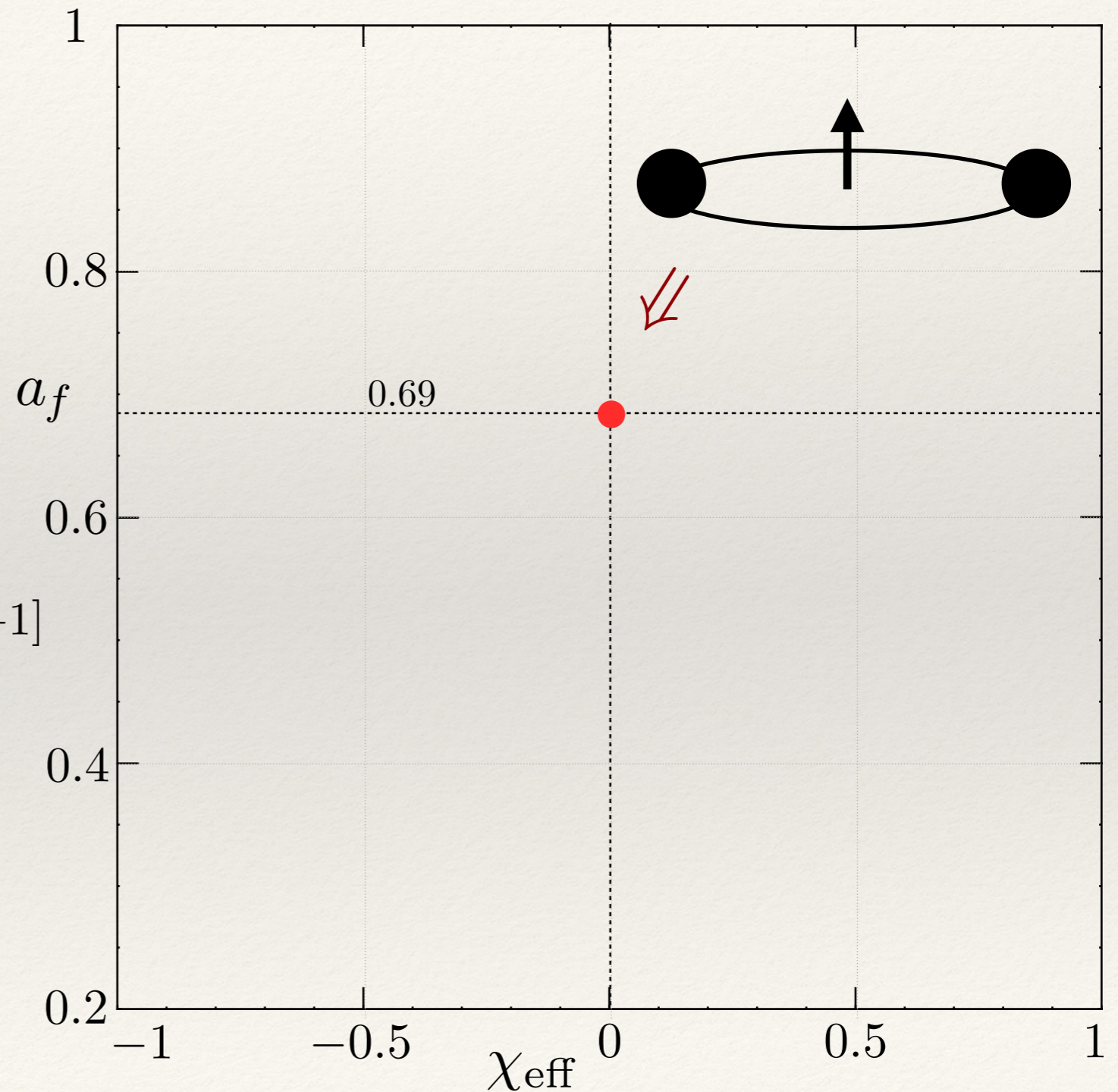
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Final spin

$$a_f \in [0, 1]$$

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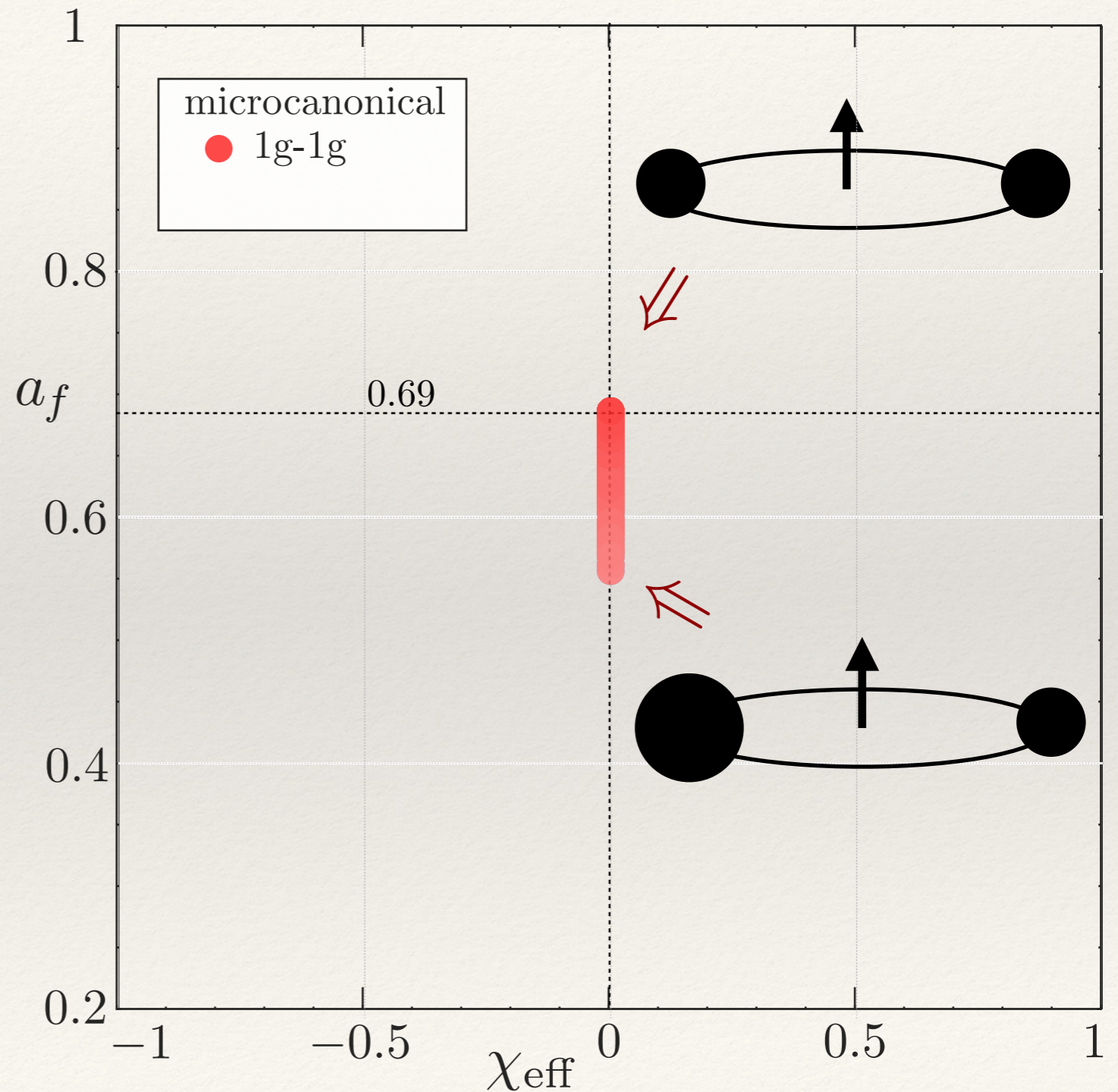
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A population with final spin computed from fit of numerical-relativity [LIGO-T1600168]

$$a_f \simeq 0.69 - \left(\frac{M_1 - M_2}{M_1 + M_2}\right)^2 \times 0.56$$



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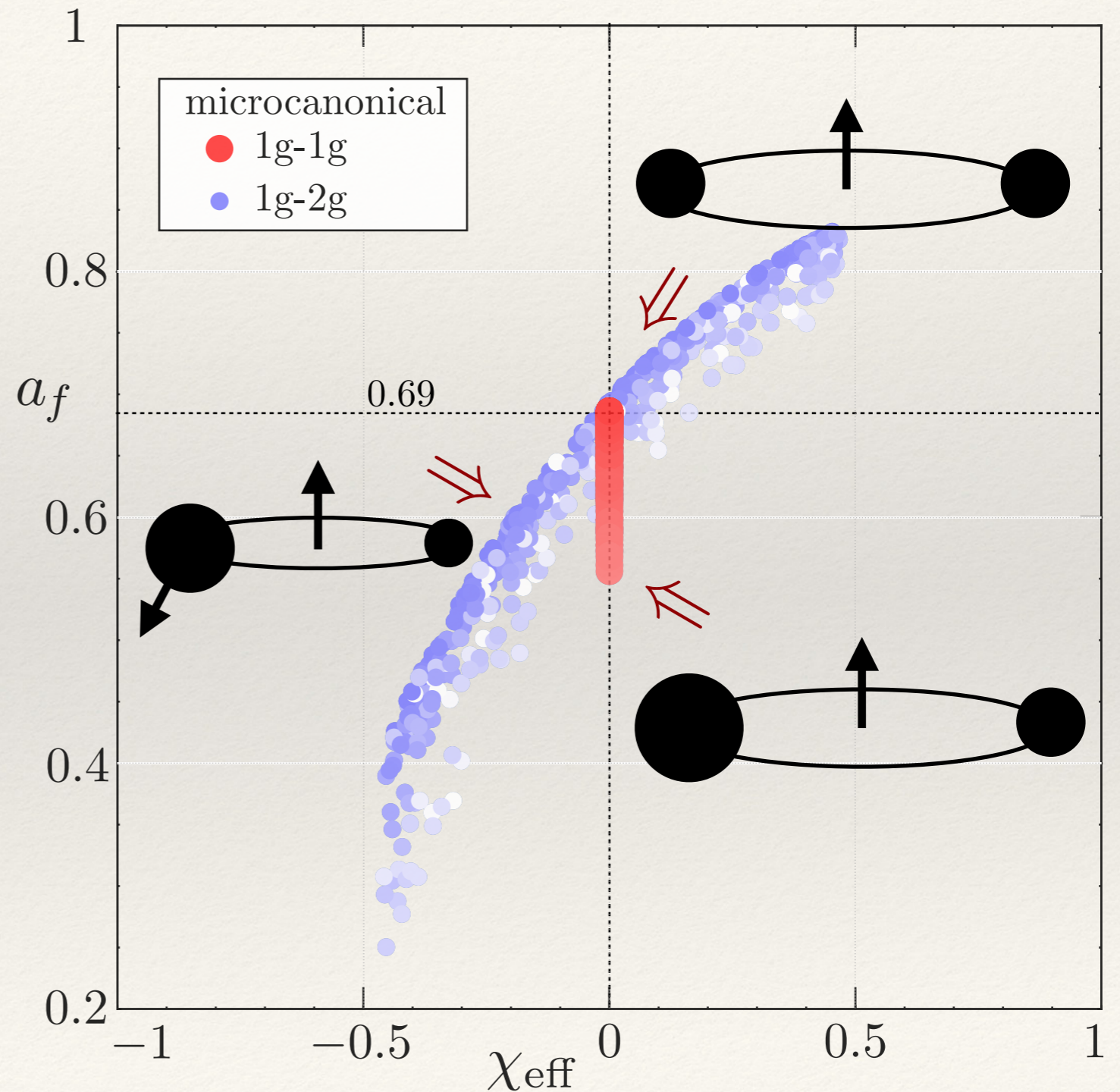
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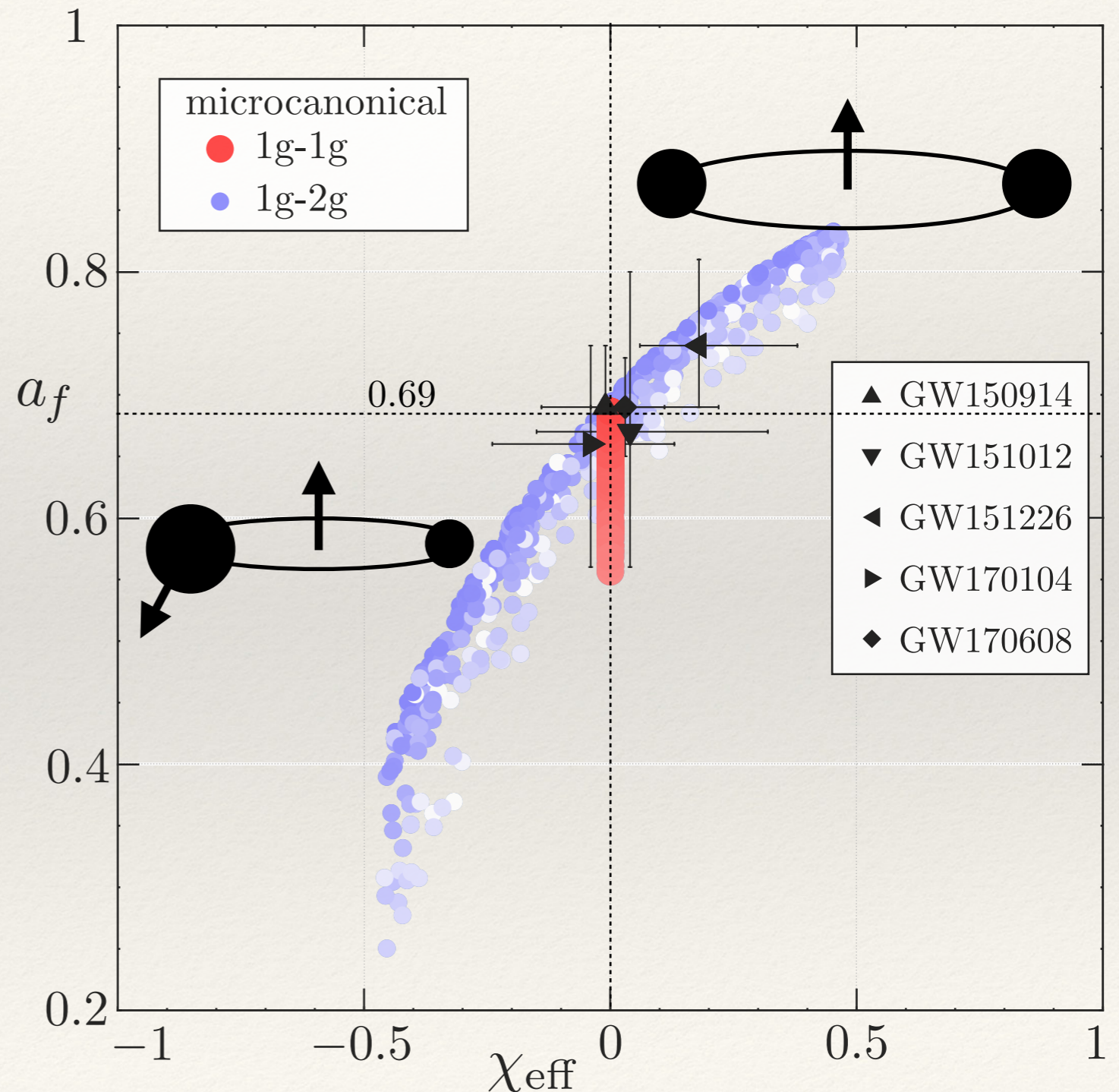
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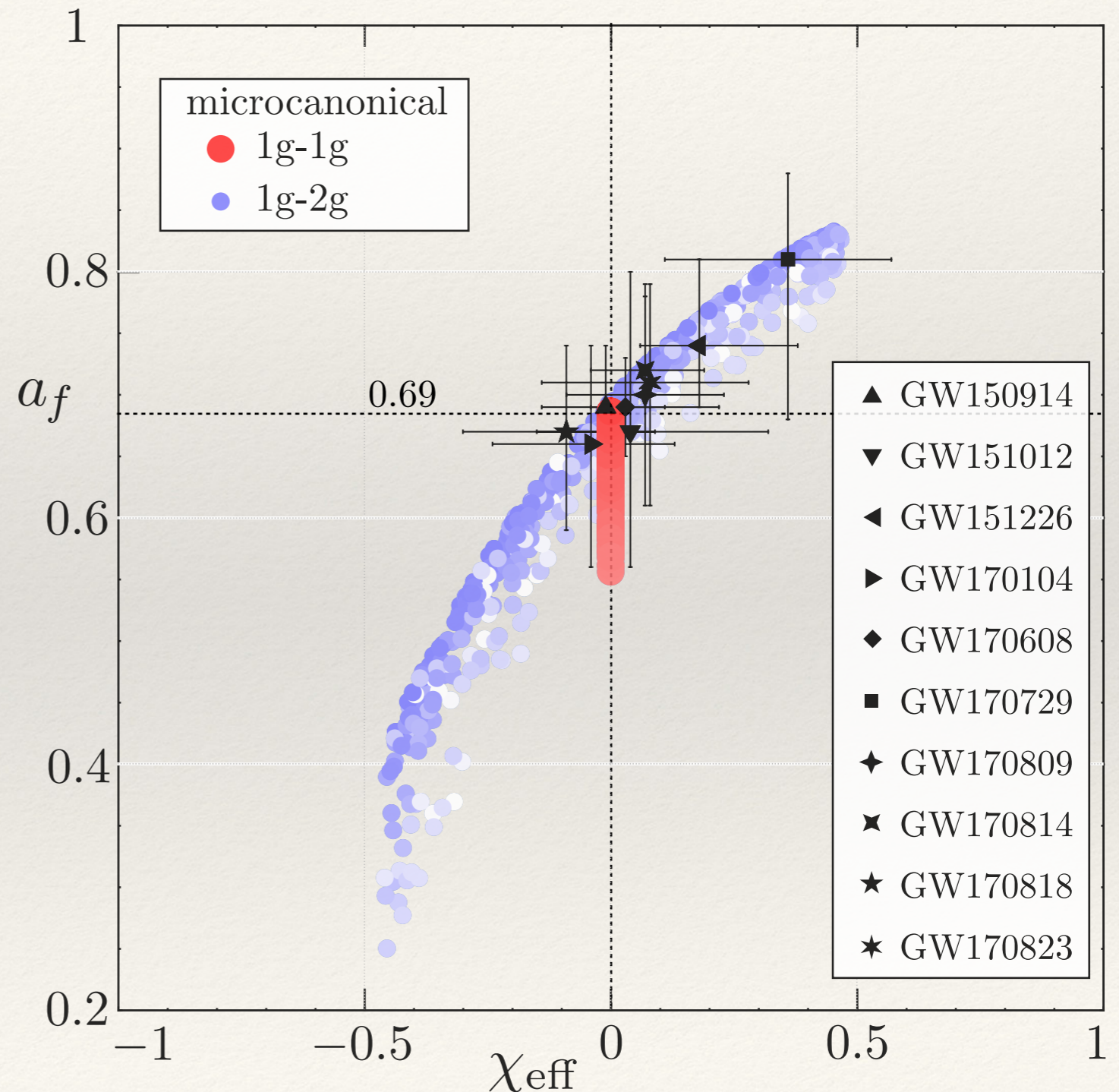
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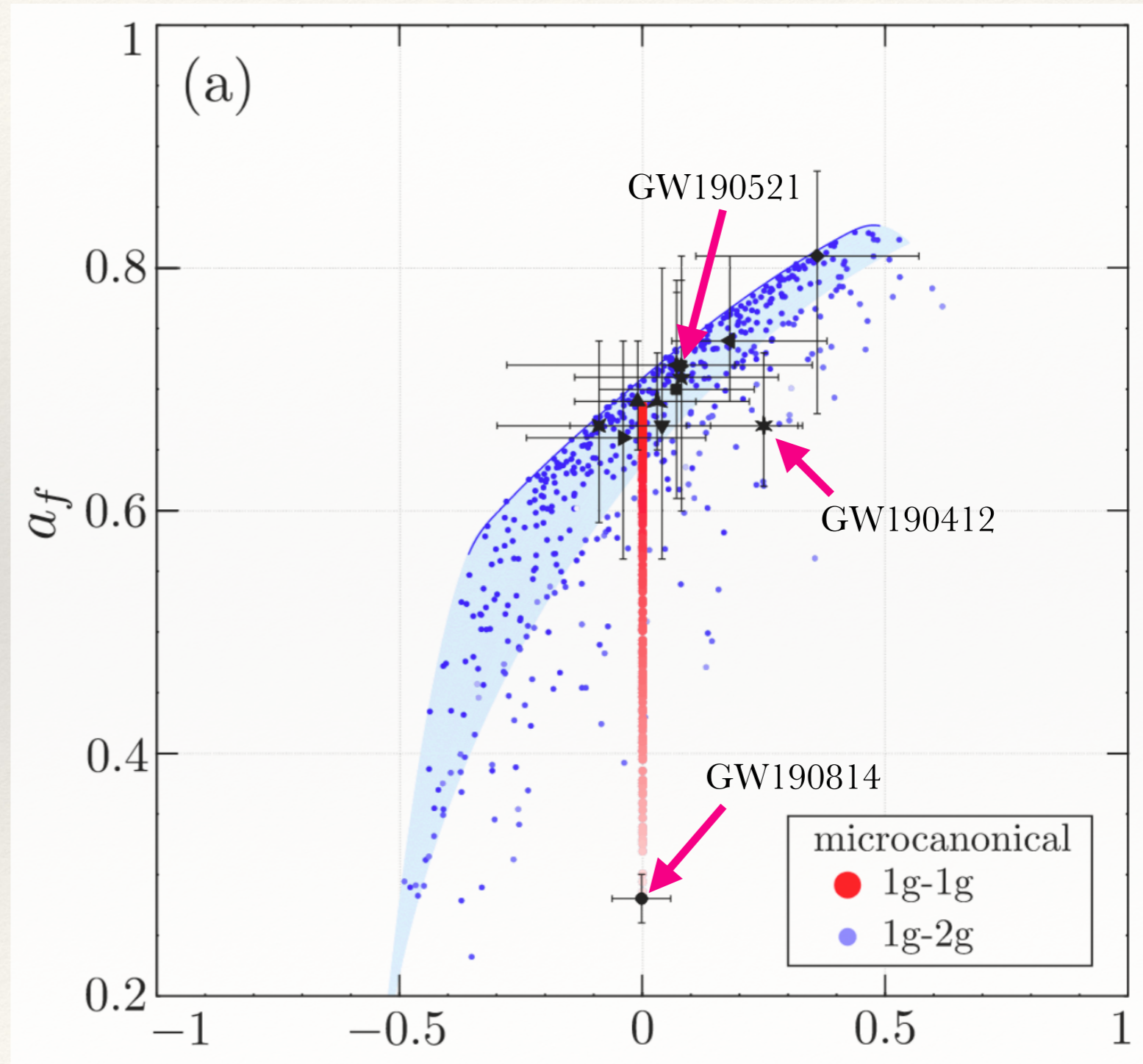
The first 3 events from
run O3:

GW190412

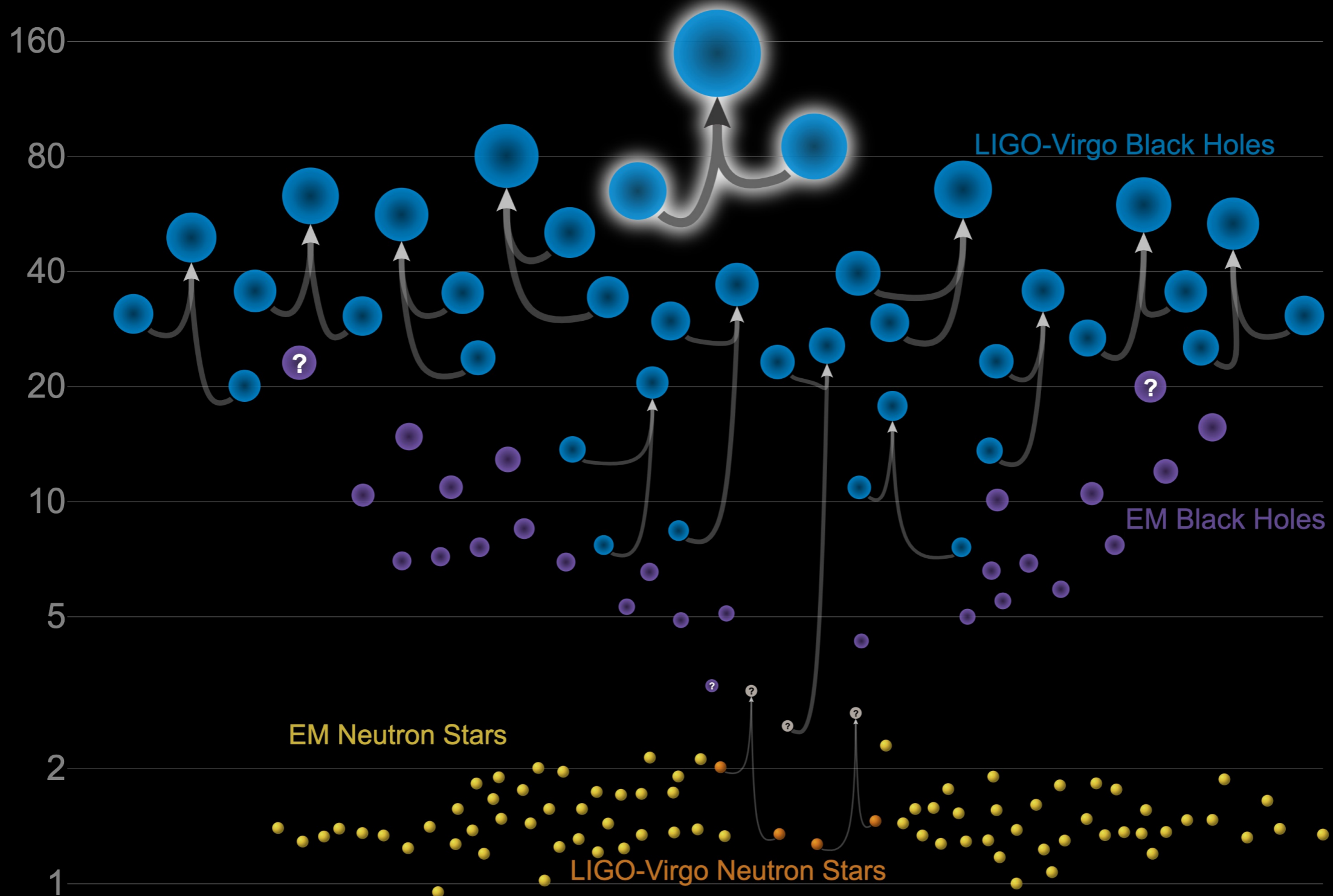
GW190814

GW190521

GW190814 has a mass
ratio of $q \sim 10$



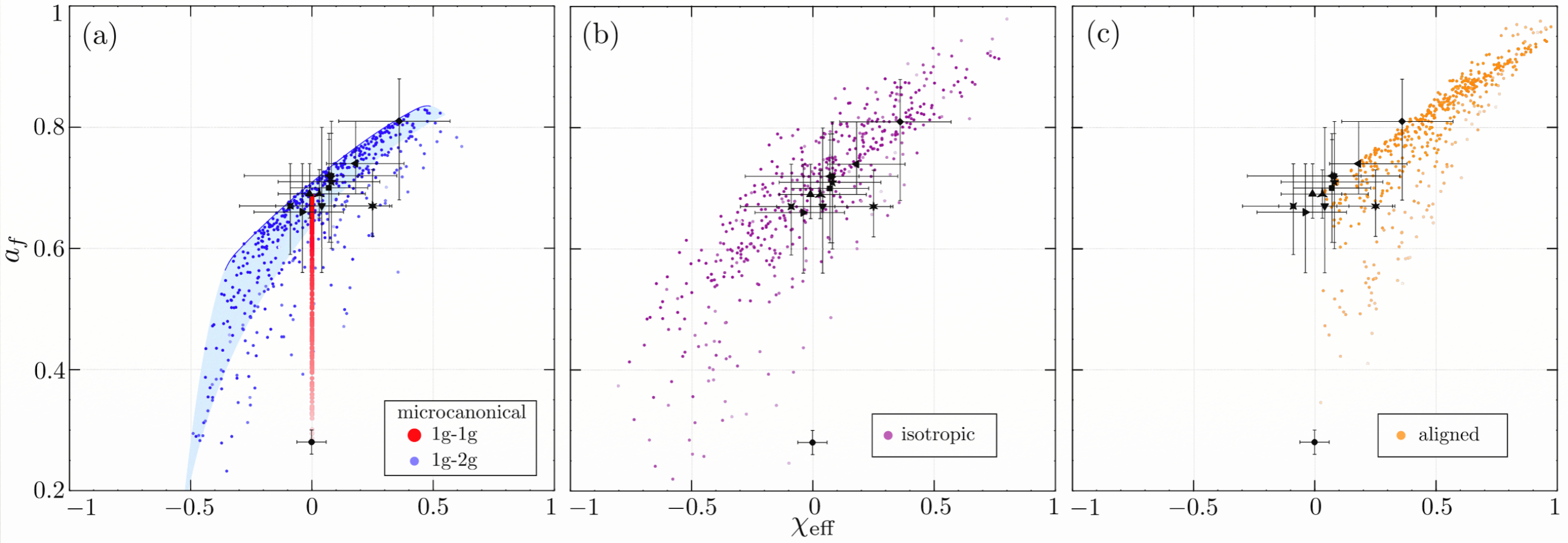
In solar masses



Updated 2020-09-02

LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

▲ GW150914 ▼ GW151012 ◀ GW151226 ▶ GW170104 ▲ GW170608 ◆ GW170729 ■ GW170809 + GW170814 * GW170818 * GW170823 * GW190412 • GW190521 • GW190814

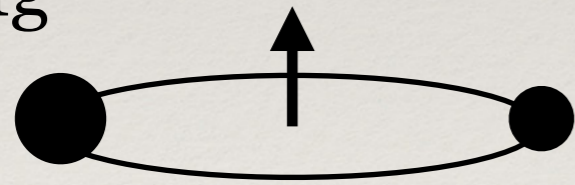


microcanonical

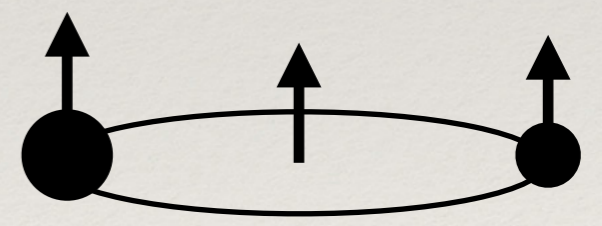
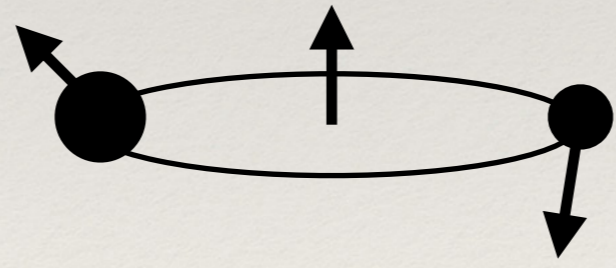
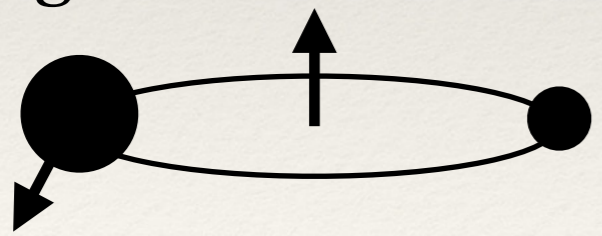
isotropic

aligned

1g-1g



1g-2g

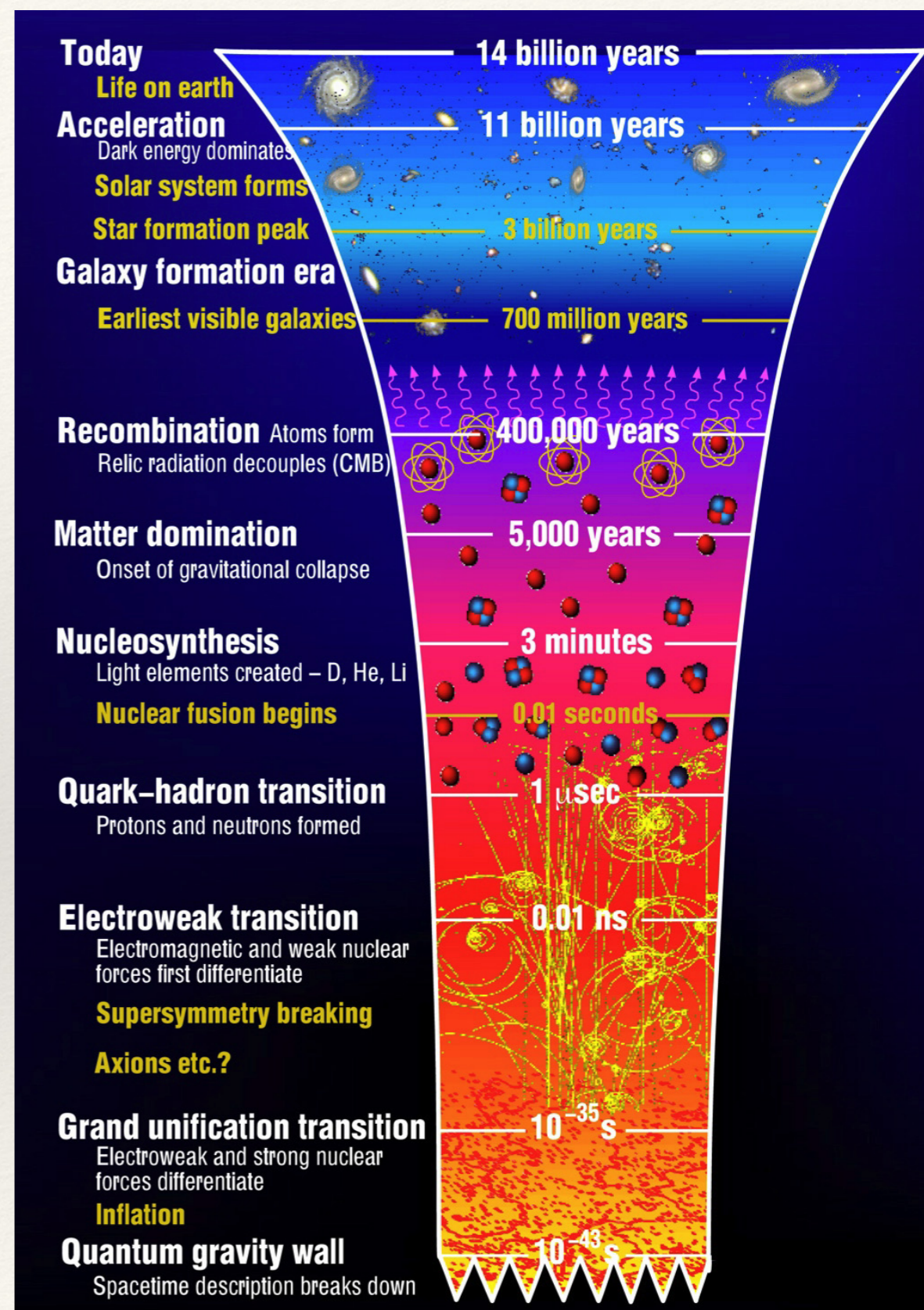
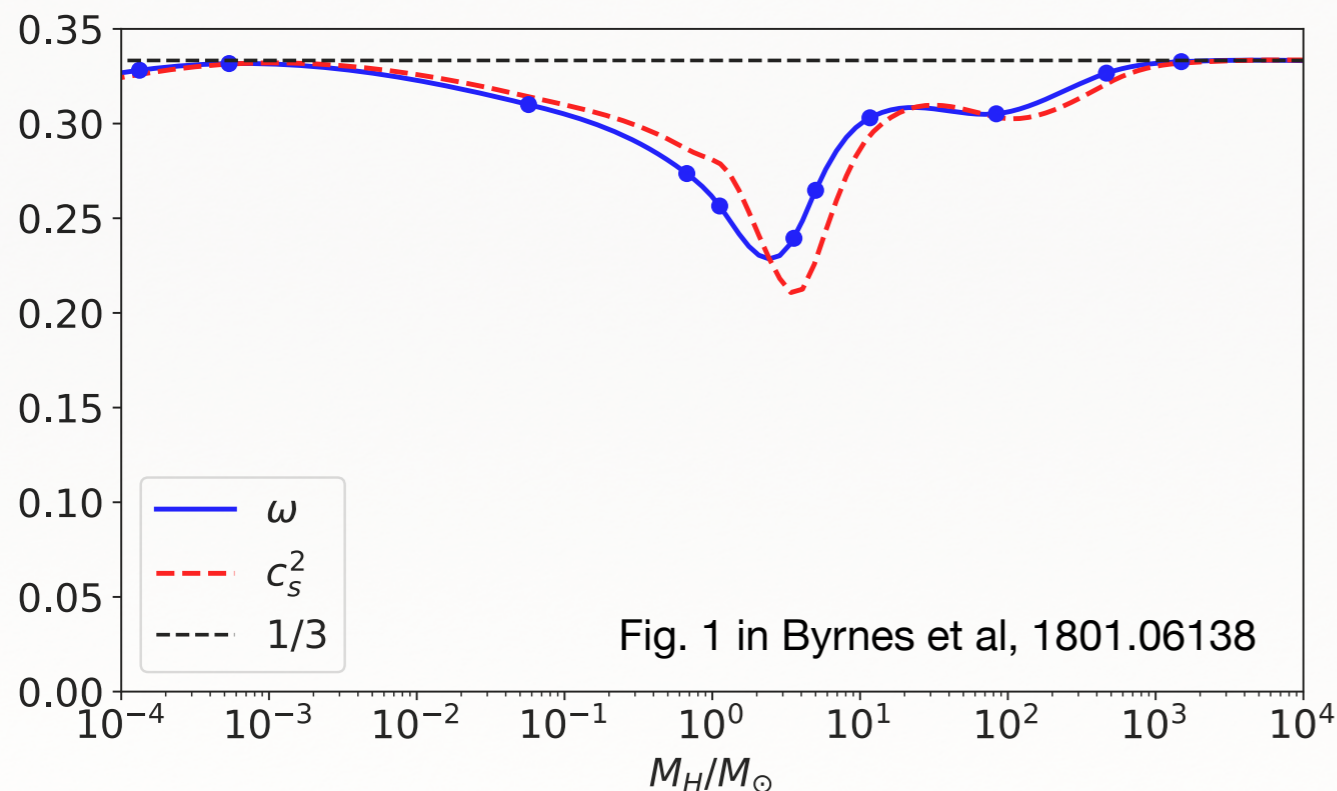


Bianchi, Gupta, HMM, & Sathyaprakash, [1812.05127](#)

Farr et al. "Distinguishing Spin-Aligned and Isotropic Black Hole Populations With Gravitational Waves," Nature 548 (2017) 426
 Belczynski et al., "The origin of low spin of black holes in LIGO/Virgo mergers," 1706.07053
 Rodriguez et al., "Illuminating Black Hole Binary Formation Channels with Spins in Advanced LIGO," Astrophys. J. 832 (2016)

LSC may be observing primordial black holes

In the early Universe during the QCD transition, the pressure drops and collapse of overdense regions is amplified:



LSC may be observing primordial black holes

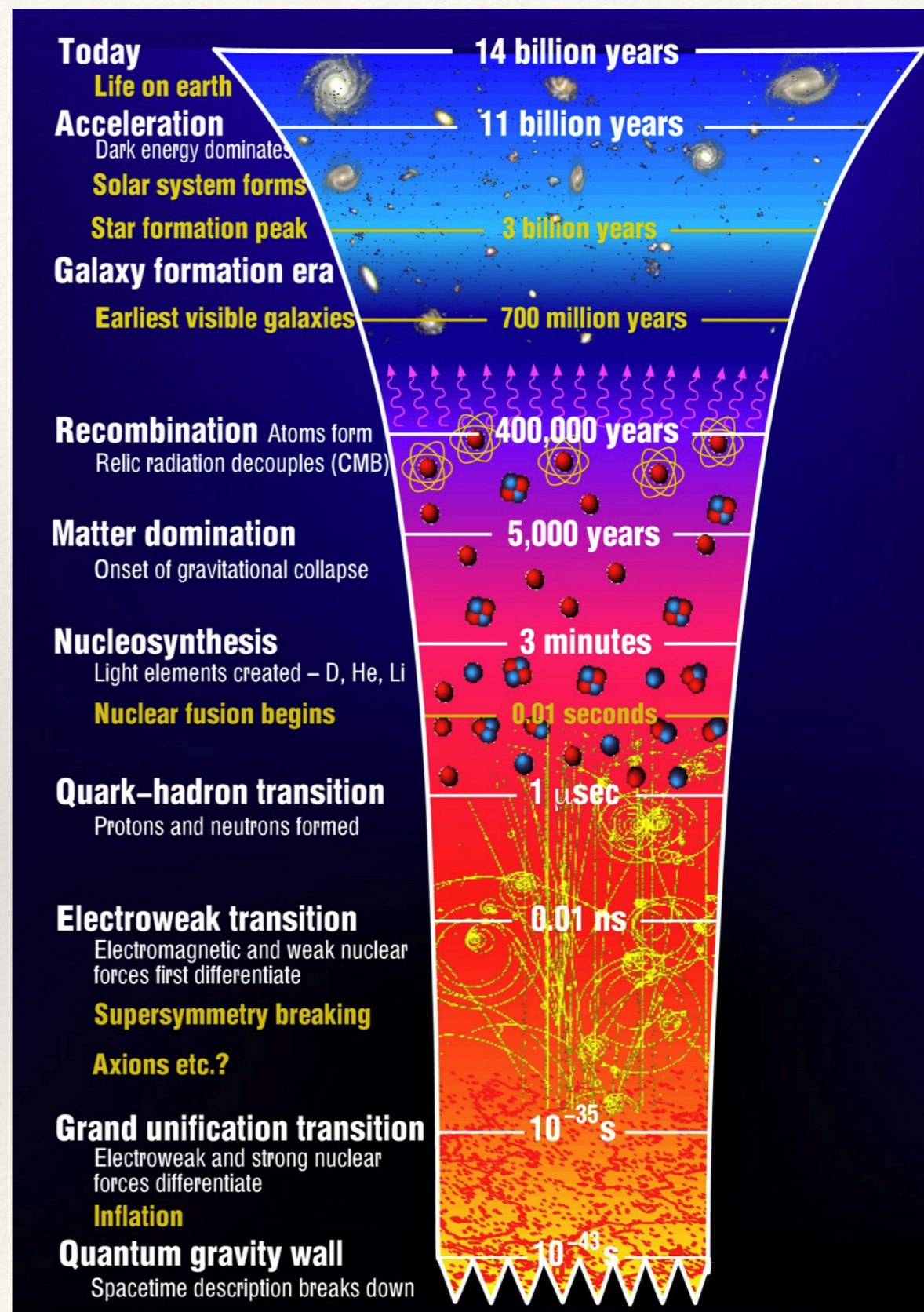
A quick estimate of the BH mass range gives:

$$\rho_0 = M_0/r_S^3 \quad r_S = 2GM_0/c^2$$

$$\rightsquigarrow M_0 = \frac{c^3}{2\sqrt{2}G^{3/2}} \frac{1}{\sqrt{\rho_0}}$$

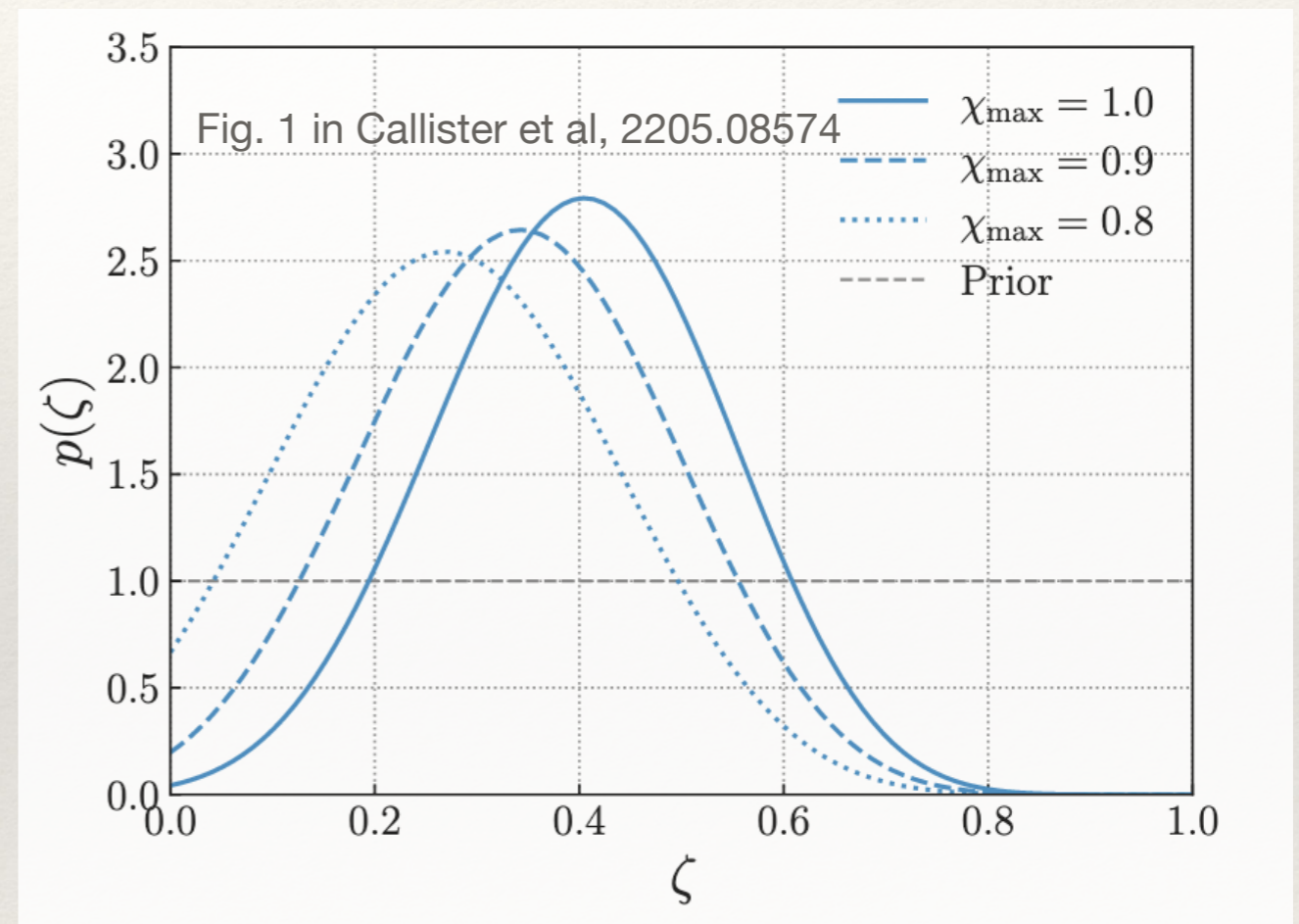
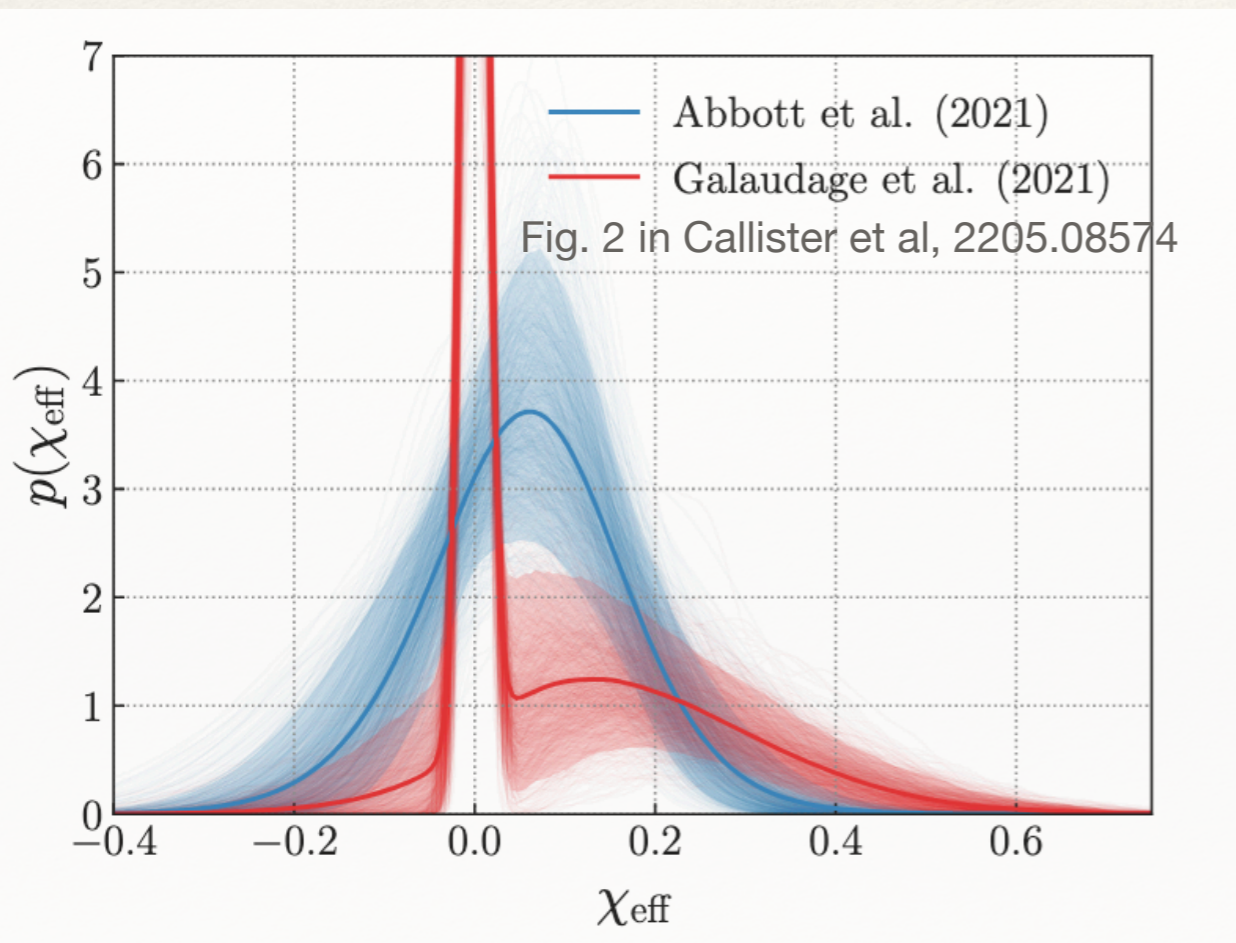
And $\rho_0 \sim (150 \text{ MeV})^4 / \hbar^3 c^5$
gives $M_0 \sim 25M_\odot$!

Lattice QCD simulations give
a mass range: $0.1 - 100 M_\odot$



What does the evidence say about a subpopulation of zero spin black holes?

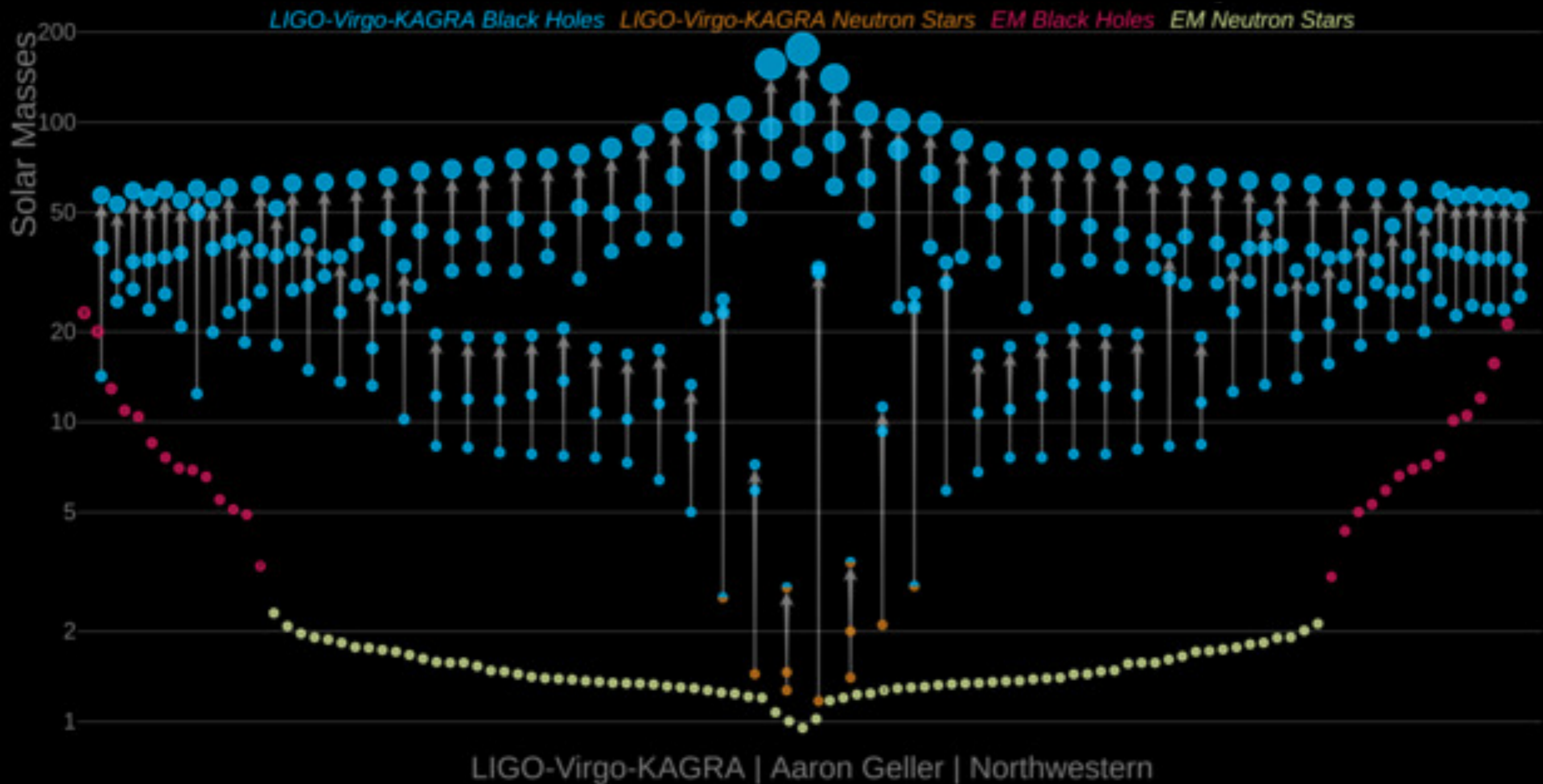
This is a topic of active discussion: Galaudage et al modeled individual spins and mixed in a zero spin population...

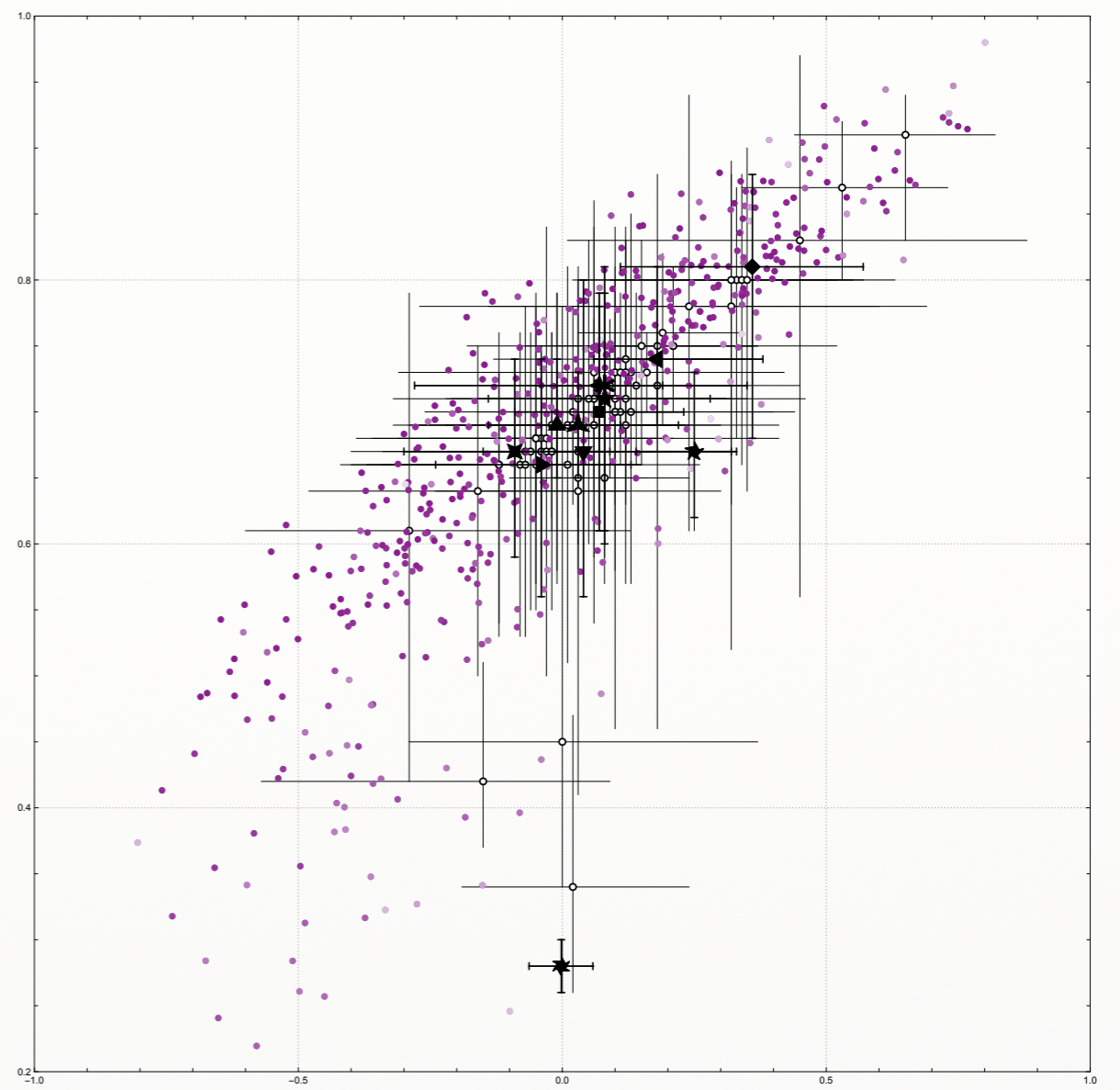
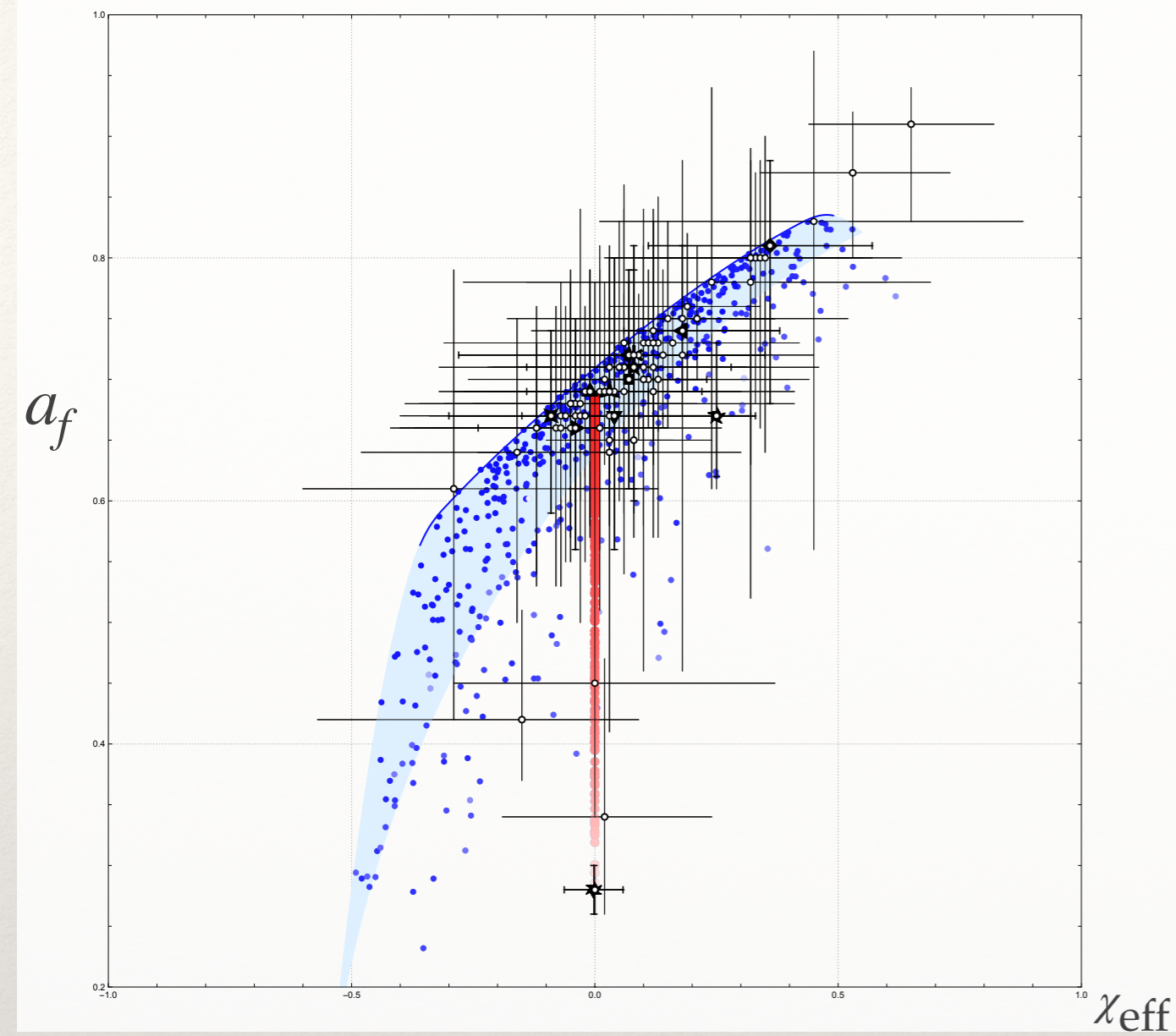


...Callister et al model the zero spin population more flexibly & find a smaller population, but their fraction of zero spin binaries, ζ , is still peaked in the range 0.25-0.4.

Full released catalog GWTC-3

83 BBH, 7 NS/BH Mergers

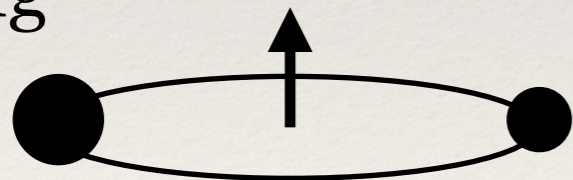




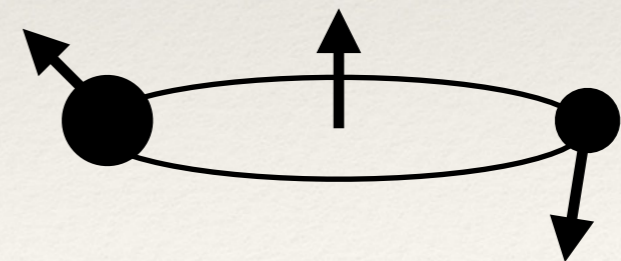
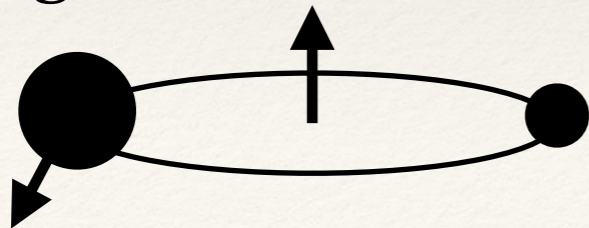
microcanonical

isotropic

1g-1g



1g-2g



The observational challenge

What can we further correlate with mass and spin to pin down a population of massive primordial black holes?

Are there more ways to leverage the advent of gravitational wave observations to investigate quantum gravity?

Calcagni, [2012.08251](#); Parikh, Wilczek, & Zahariade, [PRL 127, 081602](#); ...

Can we observe gravity-mediated entanglement?

Bose, Mazumdar, Morley, Ulbricht, Toroš, Paternostro, Geraci, Barker, Kim, & Milburn, [PRL 119, 240401](#); Marletto & Vedral [PRL 119 240402](#); Christodoulou, Di Biagio, Aspelmeyer, Brukner, Rovelli, & Howl, [2202.03368](#); ...

Are there signs of quantum gravity in the CMB or elsewhere in cosmology?

Ashtekar, Gupt, & Sreenath, [Frontiers Ast. & Spc. Sci., 8, 685288](#); Agullo, Kranas, Sreenath, [ibid](#); Weltman et al, [Publ. Astron. Soc. Aust. 37](#);

More?

Today's Discussion

1. The Observational Challenge: Black Hole Spins
2. Matter and Λ : Massive and Curved 'Cats'
3. Quantum Gravitational Symmetries and Topology Change

Question

I am excited about all the developing work that includes matter into quantum gravity—fermions on spin networks, resolution of fermion doubling, ...—and I would like to better understand matter's impact on the gravitational Gauss law...

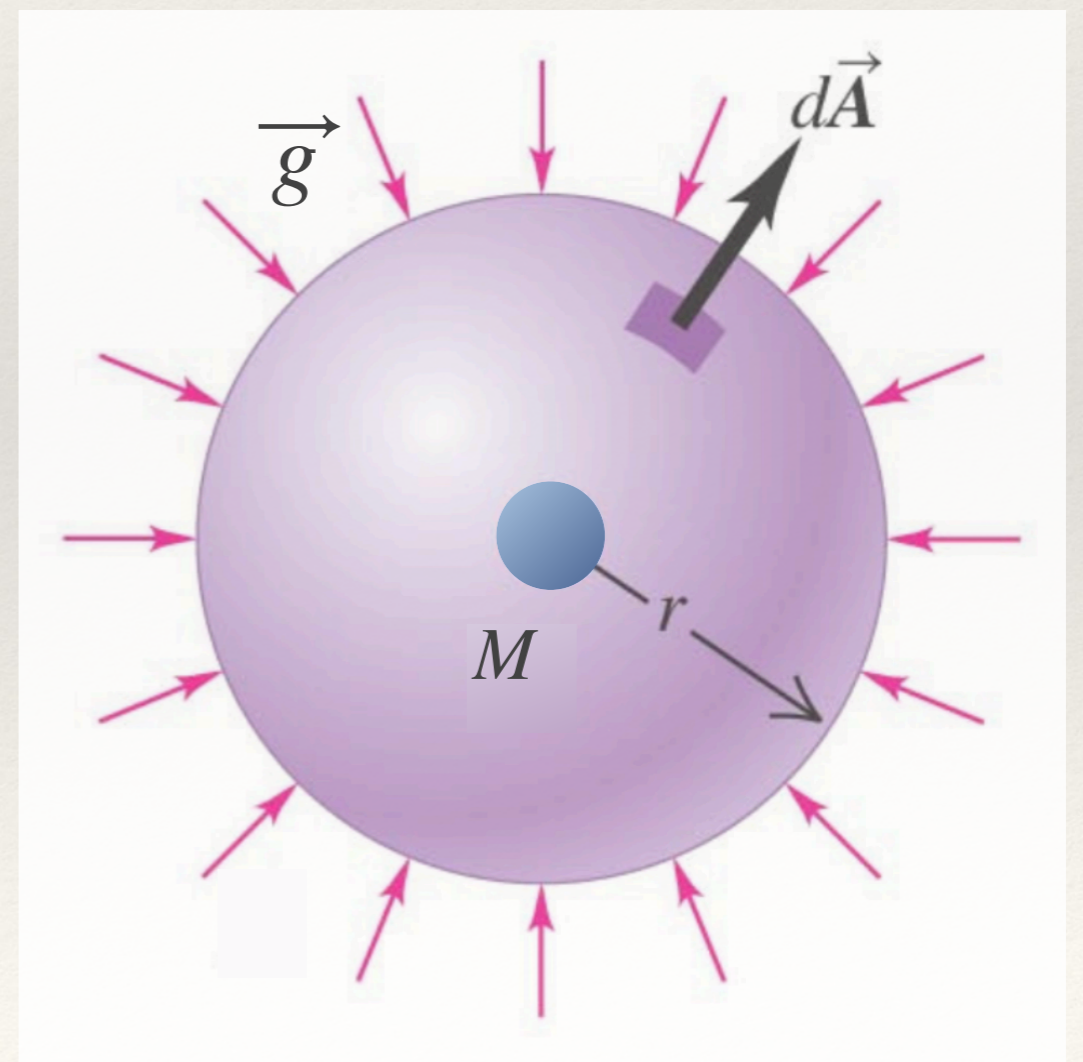
Bianchi, Han, Rovelli, Wieland, Magliaro, & Perini, [CQG 30, 235023](#); Lewandowski & Zhang, [2112.08865](#); Gambini & Pullin, [Phys. Lett. B 749, 374](#); Zhang, Liu, & Han [2205.12208](#); ...

For a static weak field $\vec{g} = -\vec{\nabla}\Phi$ and we have the intriguing

$$\oint \vec{g} \cdot \vec{dA} = -4\pi GM$$

that carries information about the mass content of the region out to its boundary.

Where is this in spin networks?



Question

In other words, what is a massive cat?



Ideas and challenges I

...this is one inroad to the beautiful work that is being done on corner symmetries, edge modes, gauge theories, and soft theorems:

Freidel, Geiller, & Pranzetti, [I](#), [II](#), [III](#); Donnelly, Freidel, Moosavian, & Speranza, [2012.10367](#); Riello [2104.10182](#); Riello & Gomes, [SciPost Phys. 10, 130](#); Kabel & Wieland, [2206.00029](#); Chen, [JHEP 4, 011](#); Chen, Chua, Liu, Speranza, de S. L. Torres, [PRL 125, 241302](#)...

In gauge theories there is a foundational, though oft unstated, ‘co-rotation’ [c.f. Riello Hopfmüller, & Gomes, [Nucl. Phys. B 941, 249](#)] of gauge and matter fields

$$\mathcal{A}_\mu \rightarrow \mathcal{A}_\mu + \partial_\mu \lambda \quad \text{and} \quad \psi \rightarrow e^{-iq\lambda(x)/\hbar c} \psi.$$

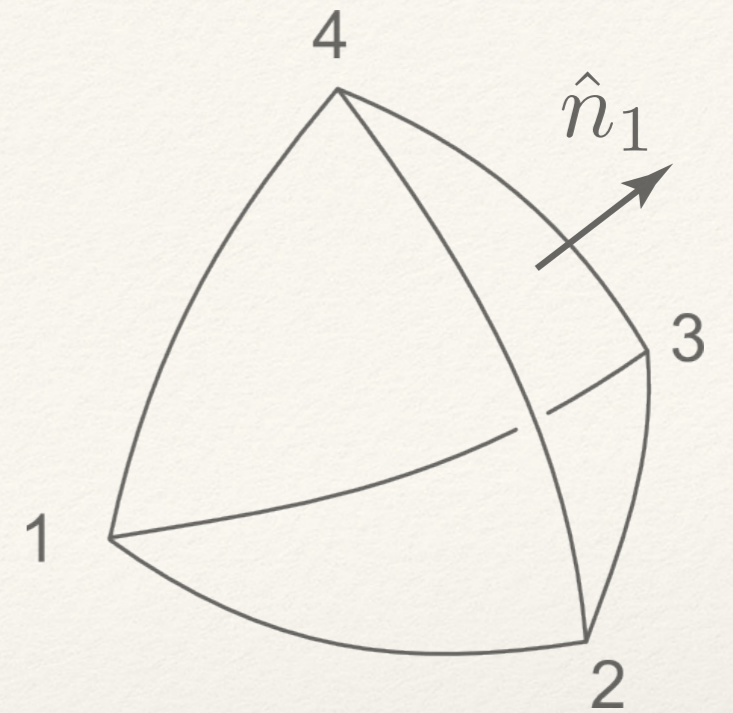
This leaves me quite interested in extensions

$$\oint \vec{dA} = 0 \quad \longrightarrow \quad \oint \vec{g} \cdot \vec{dA} = -4\pi GM$$

and beyond!

Ideas and challenges II

The generalization to spaces with cosmological constant, or constant curvature geometries, is also pregnant with ideas:



The Lie algebra closure becomes a group closure

$$\vec{A}_1 + \vec{A}_2 + \vec{A}_3 + \vec{A}_4 = 0 \quad \longrightarrow \quad O_1 O_2 O_3 O_4 = \mathbb{1}$$

with $O_\ell = \exp \left\{ \frac{A_\ell}{r^2} \hat{n}_\ell \cdot \vec{J} \right\} = \exp \left\{ \Lambda A_\ell \hat{n}_\ell \cdot \vec{J} \right\}$. Even at the classical level the non-commutativity of the O_ℓ leads to rich subsystem composition related to Poisson-Lie groups, r matrices, and q -deformation. The similarities of these structures with the statistics of particle exchange is mysterious and intriguing!

Bonzom, Dupuis, Girelli, & Pan, [2205.13352](#); Dupuis, Girelli, Livine, [GRG 46, 11](#); HMMH, Han, Kaminski, & Riello, [Phys. Lett. B 752, 257](#); Haggard, Han, & Riello, [AHP 17, 2001](#); ...

Question

In other words, what is a spherical cat?



Today's Discussion

1. The Observational Challenge: Black Hole Spins
2. Matter and Λ : Massive and Curved 'Cats'
3. Quantum Gravitational Symmetries and Topology Change

Ideas and challenges III

Challenge: perhaps the single most common and insidious error that infects papers on quantum gravity is too easily making the subsystem split

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2.$$

The trouble is that this split is seldom done in a way that respects the constraints: the gravitational Gauss law (discussed above), and the diffeomorphism constraints. Indeed, the \otimes may even need to be generalized.

Donnelly & Freidel, [JHEP 09, 102](#); Arrighi, Durbec, & Wilson, [2110.10587](#); See Bianchi et al...

Ideas and challenges IV

The Quantum Gravity community has a remarkable idea: that spectral discreteness of spacetime can be completely consistent with Lorentz invariance through spacetime superpositions.

Rovelli & Speziale, [PRD 67, 064019](#)

$$|\chi_+^{(x)}\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle)$$

What is a concrete, specific realization of this idea? For example, the transformation of the area and volume spectra of a single tetrahedral grain.

I think that having such an explicit model of spacetime superposition and its consequences would be a valuable resource for interesting people in quantum gravity.

Ideas and challenges V

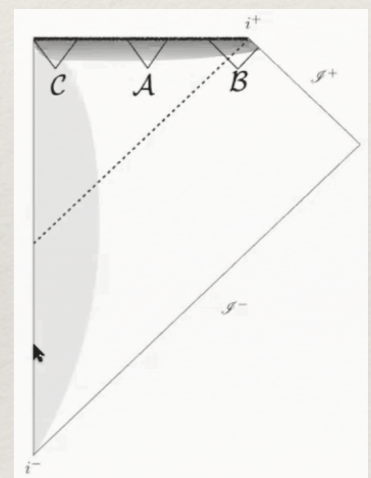
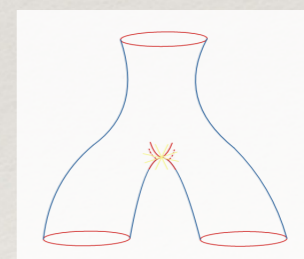
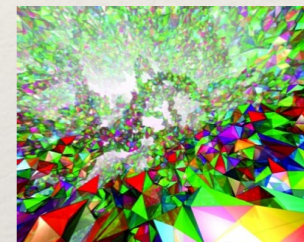
I am delighted by the examples of work in Lorentzian Regge calculus and spin foams that I have seen in recent years.

My thinking about the different roles for causality in spin foams and loop quantum gravity is muddier than I would like.

How should we organize our approaches to Lorentz signature, causal structure, and topology changing spacetimes?

The answer to this question has impact on:

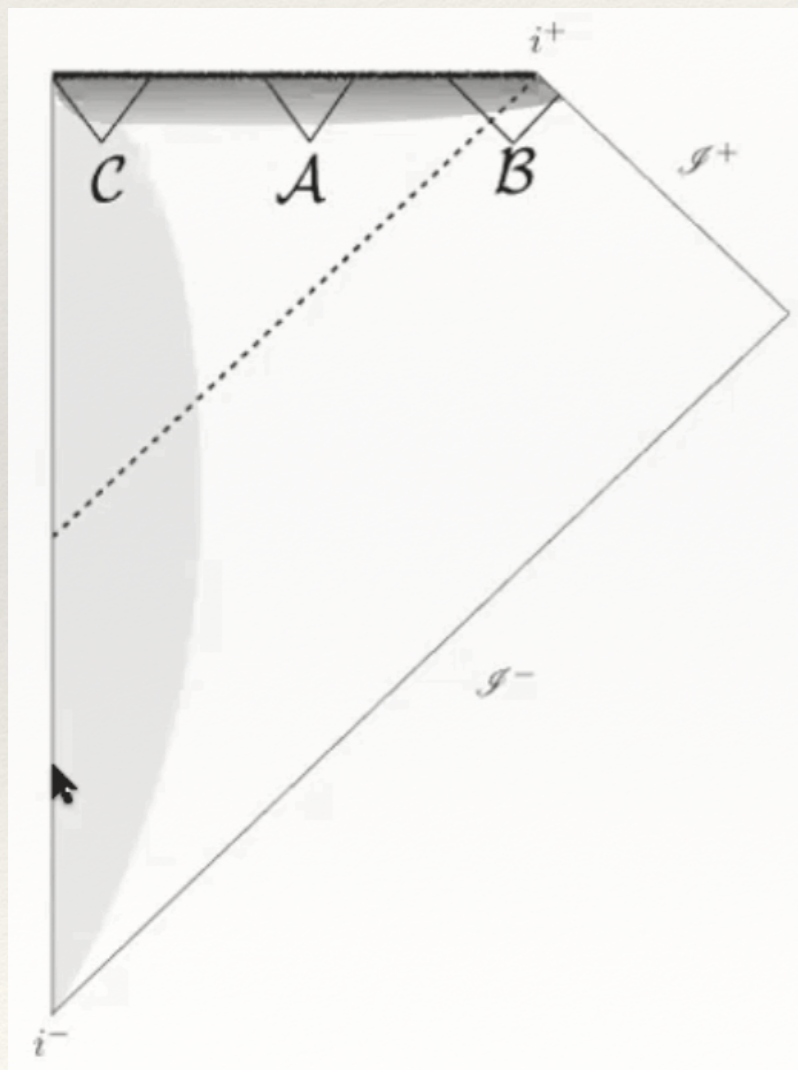
- The variables we use to describe spin foams
- Causal structure of evaporating black holes
- Geometries that can contribute to the sum over states ...



Asante, Dittrich, & Padua-Argüelles [2112.15387](#); Livine & Oriti [0210064](#); Bianchi & Martin-Dussaud, [2109.00986](#); Soltani, Rovelli & Martin-Dussaud, [PRD 104, 066015](#); HMH & Rovelli [PRD 92, 104020](#); Feldbrugge, Lehnert & Turok, [PRD 95, 103508](#);...

In the questions period there were several questions about black-to-white hole transitions. A selection of references that discuss this follow, from which you can find many more:

Rignon-Bret & Rovelli [PRD 105, 086003](#), Soltani, Rovelli & Martin-Dussaud, [PRD 104, 066015](#); Bianchi, Christodoulou, d'Ambrosio, HMH, Rovelli [CQG 35, 225003](#), HMH & Rovelli [PRD 92, 104020](#), Ambrus & Hajicek, [PRD 72 064025](#), Hajicek & Kiefer [IJMPD 10, 775](#).



The central point is that there are quantum effects that are relevant for black holes in all three of the regions, A , B , and C , indicated in the Penrose diagram at left.

The End



I am hugely grateful to the Quantum Information Structure of Spacetime (QISS) Project and to the Perimeter Institute for Theoretical Physics for their support of my work.

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