General Relativity as a Perturbative Quantum Field Theory

- ISQG series

John Donoghue
June 8, 2023

ISQG Presents

Basics of Quantum Gravity

16 May 2023 to 16 November 2023 | Online via Zoom

The Basics of Quantum Gravity online school is a series of lectures on the key concepts and techniques involved in the research of Quantum Gravity. It is dedicated to young researchers who are tackling this great challenge in theoretical physics. The goal is to bridge several paths and approaches to quantum gravity and provide young researchers with the tools to work on all of these frameworks and utilize various formalisms.

The launching lecture for this event will take place on May 16th. The event will run throughout Europe's spring and summer of 2023, featuring lectures on String Theory, Loop Gravity, the perturbative and non-perturbative renormalization flow of General Relativity, and quantum information in Gravity. Additional lectures on other topics will be planned for the fall and winter.

June + August

Note apologies:

QUANTUM THEORY OF GRAVITATION*

By R. P. Feynman

(Received July 3, 1963)

My subject is the quantum theory of gravitation. My interest in it is primarily in the relation of one part of nature to another. There's a certain irrationality to any work in gravitation, so it's hard to explain why you do any of it; for example, as far as quantum effects are concerned let us consider the effect of the gravitational attraction between an electron

tion? But since I am among equally irrational men I won't be criticized I hope for the fact that there is no possible, practical reason for making these calculations.

read Feynman's paper (on web site)

Infrared Photons and Gravitons*

Steven Weinberg†
Department of Physics, University of California, Berkeley, California
(Received 1 June 1965)

It would be difficult to pretend that the gravitational infrared divergence problem is very urgent. My reasons for now attacking this question are:

(1) Because I can. There still does not exist any satisfactory quantum theory of gravitation, and in lieu of such a theory it would seem well to gain what

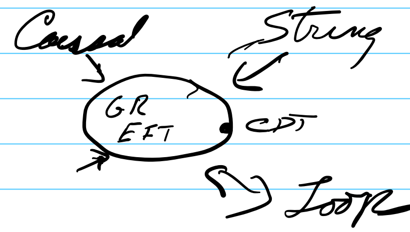
(2) Because something might go wrong, and that would be interesting. Unfortunately, nothing does go

No more apologies   QG is one pillar of theoretical physics

Gravity exists
Mysteries – still
QFT interesting

Why perturbative?

- Foundation
- QG at "ordinary scales"
- reliable predictions
- displays limitation



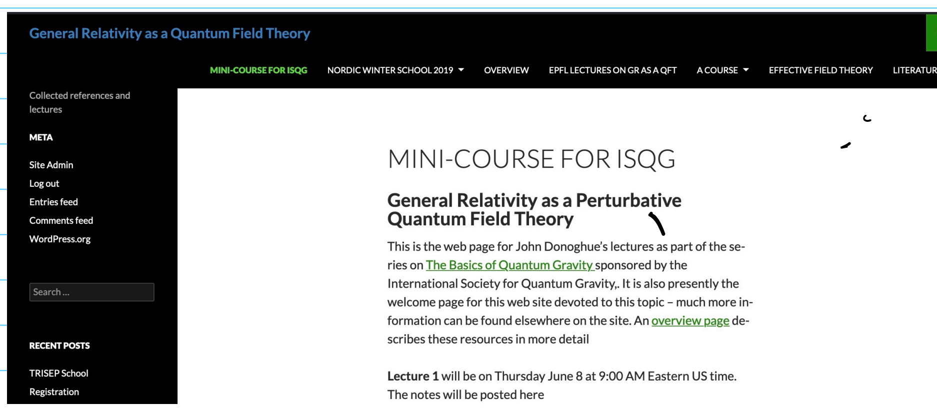
Philosophic goal
- gravity like other theories at "ordinary energies"

$$Z = \int [d\psi\, dA\, dg_{\mu\nu}]\, e^{i\int dV\left[-\frac{1}{4}F^2 + \bar{\psi} i \not{D}\psi + \frac{2}{\kappa^2}R + c_1 R^2 + \dots\right]}$$

Λ limitation

Book keeping:

Course web site


General Relativity as a Quantum Field Theory

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General Relativity as a Perturbative Quantum Field Theory

This is the web page for John Donoghue's lectures as part of the series on The Basics of Quantum Gravity sponsored by the International Society for Quantum Gravity,. It is also presently the welcome page for this web site devoted to this topic – much more information can be found elsewhere on the site. An overview page describes these resources in more detail

Lecture 1 will be on Thursday June 8 at 9:00 AM Eastern US time. The notes will be posted here

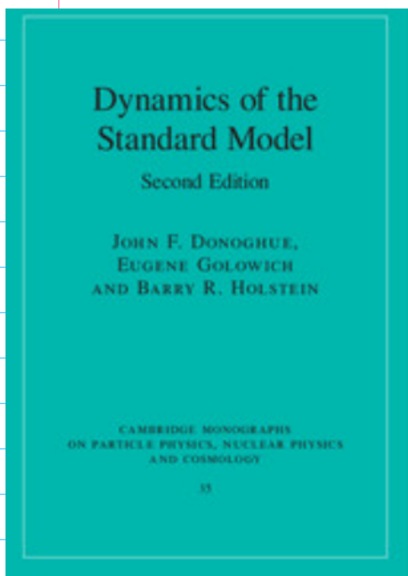
Metric $(+,-,-,-)$

EPFL Lectures on General Relativity as a Quantum Field Theory

John F. Donoghue[a], Mikhail M. Ivanov[b,c,d] and Andrey Shkerin[b,c]

2017

link is on web page



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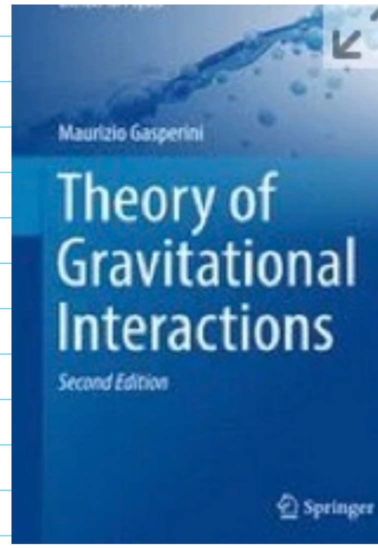
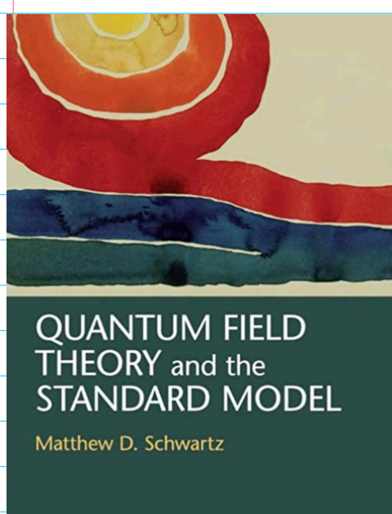
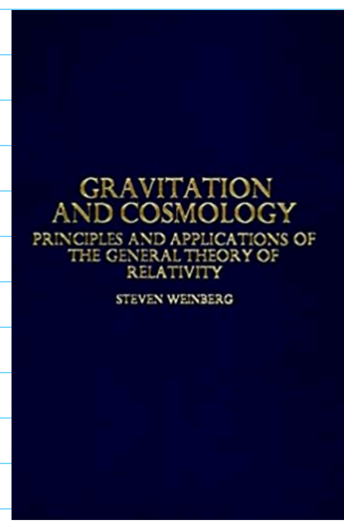


same metric convention

fermions



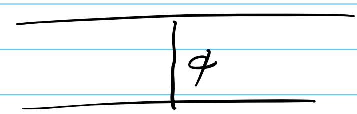


Theory for Gravity — think like a particle physicist

- not spin 1 — opposite charges
- spin 0 — attractive ✓

$$\mathcal{L} = -\frac{\kappa}{2} m \bar{\Psi}\Psi \phi$$

1, 2, ϕ



$$= -\overbrace{\frac{\kappa^2}{4}}^{G}\frac{m_1 m_2}{r}$$

Fails:

- Eotvos — Binding energies Equiv. princ.
- Bending of light $\phi F^2 \Rightarrow$ vanishes $\Rightarrow$ no light bending

$\Rightarrow$ Total energy needed

Relativistic $\Rightarrow E, p$

$\Rightarrow T_{\mu\nu}$

Space + time translations are symmetries leading to $T_{\mu\nu}$

$$x_\mu \to x_\mu' = x_\mu + a_\mu$$

Noether theorem $\Rightarrow$ $T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - \eta_{\mu\nu}\mathcal{L}$

Conservation $\partial_\mu T^{\mu\nu} = 0$

Example: **Scalar field:**

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2\right]$$

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{\eta_{\mu\nu}}{2}\left(\partial_\lambda \phi \partial^\lambda \phi - m^2 \phi^2\right)$$

drop in Feynman rules

$$\langle p' | T_{\mu\nu} | p \rangle = \frac{1}{\sqrt{2E\,2E'}}\underbrace{\left[(p_\mu p'_\nu + p'_\mu p_\nu) - \frac{\eta_{\mu\nu}}{2}(p\cdot p' - m^2)\right]}_{\text{goes into Feynman rules}}$$

$$\langle p' | p \rangle = 2E\,\delta^3(\vec{p} - \vec{p}\,')$$

Want $T_{\mu\nu}$ as source of gravity?
– how do we do this

Gauge theory construction

QED current J_μ

invariance $\psi \to e^{i\theta}\psi$

$$J_\mu = e\bar{\psi}\gamma_\mu \psi$$

Turn into local invariance

$$\underline{\psi \to e^{i\theta(x)}\psi}$$

$$\mathcal{L} = \bar{\psi}(i\cancel{\partial} - m)\psi \longrightarrow \bar{\psi}(i\cancel{D} - m)\psi \quad \text{is invariant}$$

$$D_\mu = \partial_\mu + ieA_\mu \leftarrow \text{gauge field}$$

$$\underline{D_\mu \psi \to e^{i\theta(x)} D_\mu \psi}$$

$$A_\mu \to A'_\mu = A_\mu + \frac{1}{g}\partial_\mu \theta$$ gauge inv.

Forming Field strength invariant

invariant $[D_\mu, D_\nu]\psi = ie F_{\mu\nu}\psi$

$$F_{\mu\nu} = \underbrace{\partial_\mu A_\nu - \partial_\nu A_\mu}_{\text{invariant}}$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi$$ invariant QED

Non abelian notation — closer to gravity case

$SU(N)$ $\quad \psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}$, $\quad \psi = U\psi$

$$U = \exp\left[-i\alpha^a \frac{\lambda^a}{2}\right]$$

Make this local

$$\psi \to U(x)\psi$$

$$D_\mu \psi \to U(x) D_\mu \psi$$

$$D_\mu = \partial_\mu + ig\frac{\lambda^a}{2}A^a_\mu$$ gauge fields

$$= \partial_\mu + ig\,\underline{\underline{A_\mu}}$$ matrix $[\;]^{ab}$ 3 indices

Invariance

$$\underline{\underline{A_\mu}} \to \underline{A'_\mu} = U\underline{\underline{A_\mu}}U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

$$[D_\mu, D_\nu]\psi = ig\underline{\underline{F_{\mu\nu}}}\psi$$

$$\underline{\underline{F_{\mu\nu}}} = \partial_\mu \underline{A_\nu} - \partial_\nu \underline{\underline{A_\mu}} + ig[\underline{\underline{A_\mu}}, \underline{A_\nu}]$$

Invariance $F'_{\mu\nu} = U F_{\mu\nu} U^{-1}$

$$\mathcal{L} = -\frac{1}{2} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} = \text{invariant}$$

Lesson:

Construct theory - gauging spacetime translations

Plan: $x'^{\mu} = x^{\mu} + a^{\mu} \Longrightarrow x'^{\mu} = x^{\mu} + a^{\mu}(x) = x'^{\mu}(x)$

$\Longrightarrow D_\mu$, - other fields $g_{\mu\nu}$, A_μ^{ab}

$$[D_\mu, D_\nu] \sim R_{\mu\nu\ldots}$$

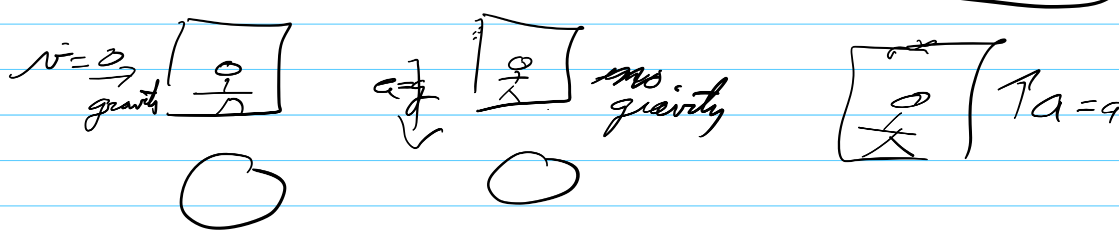
Actions R

Hopefully eq of motion $G_{\mu\nu} \sim T_{\mu\nu}$

$$\boxed{\begin{aligned} x'^{\mu} &= \Lambda^{\mu}{}_{\nu} x^{\nu} + a^{\mu} \\ &\Rightarrow \underbrace{\Lambda^{\mu}{}_{\nu}(x) x^{\nu} + a^{\mu}(x)}_{x'^{\mu}(x)} \end{aligned}}$$

Seems crazy

"Einstein elevator"



Equivalence of change of frame & gravity

This idea has a chance of working

Equiv. princ.

Locally No gravity in some coordinates $\eta_{\mu\nu}$ ← flat space

$$ds^2 = \eta_{ab}\, dy^a\, dy^b$$

Change coord

$$dy^a = e^a_\mu(x)\, dx^\mu$$

$$ds^2 = \underbrace{e^a_\mu\, e^b_\nu\, \eta_{ab}}_{g_{\mu\nu}(x)\ \text{metric}}\, dx^\mu\, dx^\nu$$

General Coord chang

$$dx'^\mu = J^\mu{}_\nu(x)\, dx^\nu \qquad J^\mu{}_\nu = \frac{\partial x'^\mu}{\partial x^\nu}$$

$$J^{-1\mu}{}_\nu = \frac{\partial x^\mu}{\partial x'^\nu}$$

Invariance

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu = g'_{\mu\nu}\, dx'^\mu\, dx'^\nu$$

$$g'_{\alpha\beta} = (J^{-1})^\mu{}_\alpha (J^{-1})^\nu{}_\beta\, g_{\mu\nu}$$

$$\left.\begin{aligned} x' &= J x \\ g' &= J^{-1} g J^{-1} \end{aligned}\right] \times$$

Technical details $\dfrac{g^{\mu\nu}}{\sqrt{-g}}$ written at the end of these notes

Invariant action

Scalar field $\phi(x) \to \phi'(x') = \phi(x)$

$$S = \int \underbrace{d^4x \sqrt{-g}}_{\text{invariant volume}} \left[\underline{g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} - \underline{\underline{m^2 \phi^2}} \right]$$

$$\phi \to \phi$$
$$\partial_\mu \phi \to J^{-1} \phi$$
$$g^{\mu\nu} \Rightarrow J \, g \, J$$

Immediate success

$$S = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2 \right]$$

$$\frac{\partial S_\phi}{\partial g^{\mu\nu}} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \left[g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - m^2 \phi \right]$$

$$\propto T_{\mu\nu}$$

Then gravity eq.

$$S = S_g + \underbrace{S_\phi}$$

$$\frac{\delta S}{\partial g_{\mu\nu}} = \frac{\partial S_g}{\partial g_{\mu\nu}} + T_{\mu\nu}$$

$$\Longrightarrow \frac{\delta S_g}{\partial g_{\mu\nu}} \propto \underline{\underline{T_{\mu\nu}}} \quad \checkmark$$

Technology

$$g^{\mu\nu} g_{\nu\alpha} = \delta^{\mu}_{\alpha}$$

$$\delta(g^{\mu\nu} g_{\nu\alpha}) = 0$$

$$\delta g_{\mu\nu} = -g_{\mu\rho} g_{\nu\sigma} \delta g^{\rho\sigma}$$

$$\frac{\delta \sqrt{-g}}{\delta g^{\mu\nu}} = -\sqrt{-g}\ \frac{1}{2} g_{\mu\nu}$$

Matter eq of motion

$$S_\phi = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - m^2 \phi^2 \right]$$

$$\frac{\delta S_\phi}{\delta \phi} = \underbrace{\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \right) \phi + m^2 \phi}_{} = 0$$

$$= (D^2 + m^2)\phi$$

covariant deriv

Preview second success: – looks like gravity

Need $g_{00} = 1 + 2\phi_g(x)$

grav potential

next lecture

$$\phi_g(\vec{x}) = -\frac{GM}{|\vec{x} - \vec{x}_0|}$$

$$= gz$$

Schroedinger Eq in grav field

Non rel reduction

$$\phi(x) = \underline{\underline{e^{-imt}}}\, \psi(x,t)$$

$$\left[\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\right) + \underline{\underline{m^2}}\right]\phi = 0$$

$$= e^{-imt}\left[g^{00}\left[-m^2 - 2im\partial_t + \partial_0^2 + (\partial g)\right] - \vec{g}\,\vec{\nabla} + m^2\right]\psi(x,t)$$

need to keep $m^2\phi_g$ biggest term

$$\left[-m^2 2\phi_g - 2im\partial_t - \underbrace{\nabla^2}_{\text{drop small}}\right]\psi(x,t) = 0$$

$$i\partial_t\psi(x,t) = \left[-\frac{\nabla^2}{2m_I} + m_g\phi_g\right]\underline{\underline{\psi}} \qquad \underline{\text{Schr. eq}}!$$

<u>Equiv princ</u> $m_g = m_I$

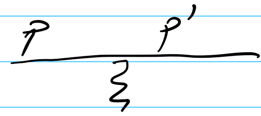
Also Force law:

$$H = \frac{p^2}{2m} + m\phi_g$$

Then $\dot{\vec{p}} = -i[H, \vec{p}\,] = -m_g\vec{\nabla}\phi$ ✓

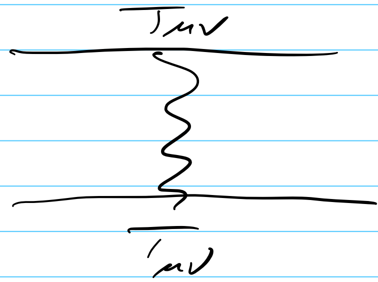
$$p = m_a v \qquad \dot{p} = m_I\vec{a} \implies \vec{a} = \underbrace{\vec{\nabla}\phi}_{\substack{\text{no } m\\ \text{if } m_g = m_I}}$$

Preview third success

ϕ  $= \frac{K}{2}\langle T_{\mu\nu}\rangle$

$\{p_\mu p'_\nu + p'_\mu p_\nu + \cdots\} \to 2E^2$ as $p \to p'$

Need $= \frac{P_{\mu\nu\alpha\beta}}{q^2}$ to be found later $\quad P_{0000} = \frac{1}{2}$



$$= \left(\frac{K}{2}\right)^2 \frac{1}{2E_1\, 2E_2} \frac{2E_1^2\; 2E_2^2}{q^2}$$

$$\xrightarrow[\text{NR} + \text{FT}]{} \underbrace{\frac{K^2}{32\pi}}_{G} \frac{m_1 m_2}{r}$$

Real QFT

Looking like a success!

At this stage we see that by gauging spacetime transformation we can arrive at a theory which looks like gravity

- $T_{\mu\nu}$ as source
- Schroedinger eq in grav field – equiv princ.
- Force law
- Gravitational potential from graviton exchange

Need to complete the theory + get Feynman rules!

↑ next time

Some technical details which I skipped over

Fundamental def of x^μ is with upper index x^μ

Then:

down index $\longrightarrow \partial_\mu = \frac{\partial}{\partial x^\mu}$ such that $\partial_\mu x^\nu = \delta_\mu^\nu$

$g_{\mu\nu}$ defined with down indices $ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$

There is assumed to be an inverse

$$g^{\mu\nu} \sim [g_{\mu\nu}]^{-1} \quad + \quad g^{\mu\nu} g_{\mu\alpha} = \delta^\nu_\alpha$$

so that $g^{\mu\nu}\partial_\nu = \partial^\mu$ etc

$g^{\mu\nu}$ transforms as

$$g'^{\mu\nu} = J^\mu{}_\alpha\, g^{\alpha\beta}\, J^\nu{}_\beta$$

The invariant volume:

– using locally flat coord $dV = d^4y$

In other coordinates $dy^a = e^a_\mu\, dx^\nu$

$$dV = d^4y = d^4x \left|\frac{\partial y}{\partial x}\right| = d^4x \det e$$

We normally use $\det e = \sqrt{-g}$

$$\det g_{\mu\nu} = \det\left(e^a_\mu\, e^b_\nu\, \eta_{ab}\right) = (\det e)^2 \underbrace{\det \eta}_{-1}$$

$$\Rightarrow \int dV = \int d^4x \sqrt{-g(x)} \quad \text{is invariant}$$

Shorthand notation

- while it is ultimately important to be careful with indices, writing them out sometimes gets in the way of the important message

- so sometimes I drop them

$dx' = J dx$ is like $dx'^{\mu} = J^{\mu}{}_{\nu} dx^{\nu}$

In this way

$$g_{\mu\nu} \to J^{-1} g J^{-1}$$

$$g^{\mu\nu} \to J \tilde{g} J$$

$$\partial_{\mu} \to J^{-1} \partial$$

etc -

upper indices transform with J,
lower " " " J^{-1}