GR as a Perturbative QFT

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Predictions, Lessons + Limitations

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- Tying it all together

GR — EFT

- know low E d.o.f + interaction
- don't know "high energy"

$\Longrightarrow$ IR effects

In practice

- UV physics $\rightarrow$ "local"
- IR physics $\rightarrow$ "non local"

In calc.

$$\frac{1}{q^2 - M^2} \rightarrow \frac{-1}{M^2} + \frac{q^2}{M^4} + \cdots$$

$$\rightarrow \frac{\delta(x)}{M^2} + \frac{\Box\,\delta(x)}{M^4} + \cdots$$

$\log q^2 \sim$ non analytic $\rightarrow$ non local

Reframe the Program

- symmetries, $g_{\mu\nu}(x)$
- Energy/derivative/curvature expansion

$$S=\int d^4x\sqrt{-g}\left[-\Lambda+\frac{2}{\kappa^2}R+c_1R^2+c_2R_{\mu\nu}R^{\mu\nu}+\dots\right]$$

$R\sim\partial^2$
$R^2\sim\partial^4$

Note: Gauss Bonnet
$$E=R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-4R_{\mu\nu}R^{\mu\nu}+R^2$$
- total deriv in 4D

$$\chi=\int d^4x\sqrt{-g}\,E$$

$\Longrightarrow$ don't need $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$

Match or measure coefficients

$$\Lambda\sim(10^{-3}eV)^4\;;\quad\kappa^2=32\pi G$$

Bounds on c_1, c_2

Roughly $(\nabla^2+\kappa^2c_1\nabla^4)\phi=\kappa^2\rho$

$$\frac{1}{q^2-\kappa^2c_1q^4}=\frac{1}{q^2}-\frac{1}{q^2-1/\kappa^2c_1}\leftarrow m^2$$

$$V(r)\sim-\cancel{GmM}\left[1-e^{-Mr}\right]$$

$\Longrightarrow c_1, c_2\lesssim 10^{+65}$

* Ostrogradsky comment: 1850

$$\begin{cases} q_1 = \phi \quad , \quad q_2 = \dot{\phi} \\ \pi \sim \frac{\Box + m^2}{m^2} \dot{\phi} \quad , \quad \pi_2 = -\frac{\Box}{m^2}\phi \end{cases}$$

$$\mathcal{L} = \phi \Box \phi + \phi \frac{\Box^2}{m^2} \phi + V(\phi)$$

$\rightarrow \quad H = \underbrace{\pi_1 \phi_2}_{} + \ldots$

not pos definite $\Rightarrow$ instability

But not problem in EFT:

QED

$$\mathcal{L} = F^2 + F \frac{\Box}{m^2} F$$

small

no instability when treated as perturbation

Renormalization

- EFT is wrong in UV
- UV physics is local — local $\mathcal{L}$
- renormalize an "unrenormalizable" theory
- general local $\mathcal{L}$,
- if symmetry preserve → parameter in local $\mathcal{L}$

Background Field Method

't Hooft Veltman

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}(x)$$

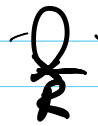
Lesson 3

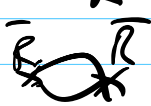
$$\mathcal{L} = \frac{-2}{\kappa^2} R = \sqrt{-\bar{g}}\Big[\frac{-2}{\kappa^2}\bar{R} - \frac{1}{\kappa}[h^\alpha_\alpha \bar{R} - 2\bar{R}^\alpha_\nu h^\nu_\alpha]$$
$$+ \frac{1}{2}\bar{D}_\alpha h_{\mu\nu}\bar{D}^\alpha h^{\mu\nu} - \frac{1}{2}\bar{D}_\alpha h^\lambda{}_\lambda \bar{D}^\alpha h^\sigma{}_\sigma$$
$$+ \bar{D}_\nu h^\lambda{}_\lambda \bar{D}^\beta h^\nu_\beta - \bar{D}_\nu h_{\alpha\beta}\bar{D}^\alpha h^{\nu\beta}$$
$$- \bar{R}\Big[\frac{1}{2}(h^\lambda{}_\lambda)^2 - \frac{1}{2}h^\alpha_\beta h^\beta_\alpha\Big] + h^\lambda_\lambda h^\alpha_\nu \bar{R}^\nu_\alpha + 2h^\nu_\beta h^\beta_\alpha \bar{R}^\alpha_\nu \Big]$$

gauge invariant wrt background

⟹ symmetry preserved ✓

Like massless scalar (Lesson 4)

 $\sim \int \frac{d^d k}{k^2} \to 0$

 $\sim R \int d^d k \frac{1}{k^2}\frac{1}{(k-q)^2} \to R\underbrace{\Big[\frac{1}{\epsilon} - \ln q^2\Big]}_{\text{Barvinsky Vilkovisky}} R$

$\Rightarrow R^2$

Use heat kernel $t\,H - V$

$$\mathcal{L} = \frac{1}{16\pi^2}\,\frac{1}{\varepsilon}\left[\frac{1}{120} R^2 + \frac{7}{20} R_{\mu\nu} R^{\mu\nu}\right]$$

$\Longrightarrow$ shift in $c_1\, c_2$ (absorb into "measured values")

"<u>Pure gravity is one loop finite</u>"

- <u>pure gravity</u> satisfies $R_{\mu\nu} = 0$
  only graviton $\leftarrow *$
- but matter exists

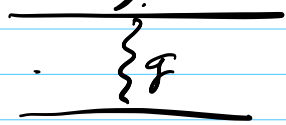
But, unreliable

$\Longrightarrow$ shift to low E

Gravitational potential

- Scattering potential $\Longrightarrow$ NR $\Longrightarrow$ F.T.

Leading



$$\xrightarrow{NR} \frac{\kappa^2 M m}{q^2} \xrightarrow{FT} -\frac{GMm}{r}$$

Next order local terms

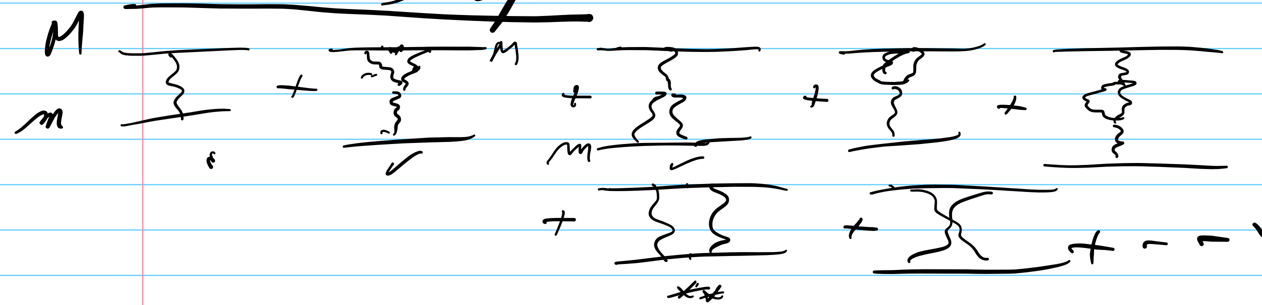
$c_1 R^2,\ c_2 R_{\mu\nu}R^{\mu\nu}$

$$c\, q^4 \sim \text{const} \xrightarrow{FT} \delta^3(x)$$

Coeff (+div) $\sim \delta^3(x)$

$$\frac{e^{-mr}}{r} \longrightarrow \frac{1}{m^2}\delta^3(x)$$ rep of δ function

No do loops:



$$\mathcal{M} = \frac{GMm}{q^2}\left[1 + \frac{\kappa^2\sqrt{q^2 m^2}}{T} + \kappa^2 q^2 \ln q^2 + \kappa^2 q^2\right]$$

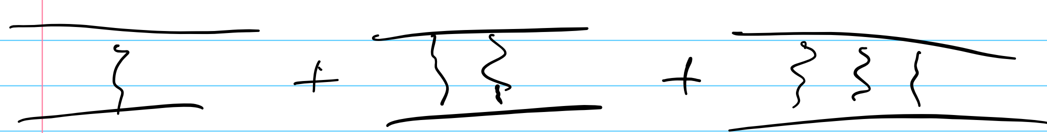
$\downarrow$ FT

quantum

$$V(r) = \frac{GMm}{r}\left[1 + \frac{G(M+m)}{rc^2} + \frac{G\hbar}{r^2 c^3}\right] + \left(\delta^3(x)\right)$$

Calculate:

$$V(r) = -\frac{GMm}{r}\left[1 + 3\frac{G(M+m)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right] \checkmark$$



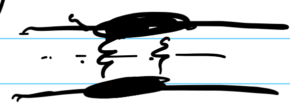
$\Rightarrow$ "Coulomb" phase
"Kbutas"

$\}$ exponentiates
Weinberg

$$\left|e^{i\theta}\mathcal{M}_0\right|^2$$

Recalculated "Unitarity" Techniques

— Feynman's tree theorem



~ trees, bubbles ...

GR ~ $(YM)^2$ (Compton amplitudes)

Low energy thm

Bending of Light — massless particle

$\Longrightarrow$ scattering amps

$\Longrightarrow$ bending angle "Eikonal"

classical quantum

$$\theta = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8c_b - 47 + 64\log(2r_0/b)}{\pi}\frac{G^2M\hbar}{b^3}$$

$\leftarrow$ impact parameter

where $c_b = (371/120,\ 113/120,\ -29/8)$ for scalars, photons and gravitons

not universal

Graviton graviton

$\times$

Lessons:

1) Potential is universal
   - Ross + Holstein
   - understood "unitarity" technique
   - Amps are universal $\Rightarrow$ One loop amp is universal

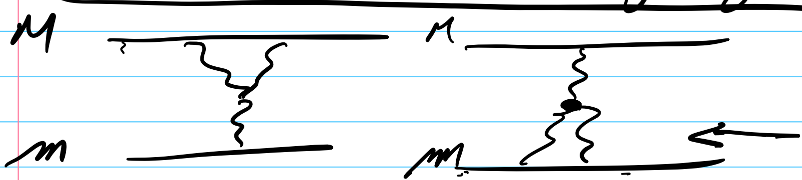
2) Classical effects from loops
   - "loop expansion is $\hbar$ expansion" is false

$$\bar{\psi}(i\hbar\partial\!\!\!/ - m)\psi \to \hbar\,\bar{\psi}\Big(i\partial\!\!\!/ - \underbrace{\frac{m}{\hbar}}_{\text{hidden}}\Big)\psi$$

$$\hbar\sqrt{q^2\frac{m^2}{\hbar^2}} \sim \text{Classical}$$

* Become subfield - gravity waves from QFT

3) No "test mass limit" for quantum effect



Class. $M\sqrt{q^2}$ $\quad m\sqrt{q^2}$

Quantum $q^2\ln$ $\quad q^2\ln$

4) No "quantum metric"

- no quant corr to Schw.
- Kirilin not invariant under field redef
  - ⇒ Haag's theorem does not apply
- "off shell" partial amplitude
- not a good quantum question
  - not Δ"$g_{\mu\nu}$"

5) Massless particle not universal

6) Geodesics are ill defined:

- quantum ~ "tidal"

propagate long distances

- small in EFT limit
- uncontrolled away from limit

7) G, Λ are not running parameters

- in physical processes

Limits of EFT:

High energy / curvature

$$\mathcal{M} \sim \frac{1}{q^2}\left[1 + \underline{\kappa^2 q^2 \ln + \kappa^4 q^4 \ln \ldots}\right]$$

$\Longrightarrow$ need some UV complete theory

IR issues:

- grav effect build up
  - eg BH

$$g_{\mu\nu}^{(x')} = \eta_{\mu\nu} + \frac{1}{3} R_{\mu\nu\alpha\beta}(x) \underline{(x'-x)^\alpha (x'-x)^\beta}$$

- technical
- quantum effects "feel" long distances
  - $t\ 1/r^3$
- Propagating past BH
- ...

Our core theory

$$Z = \int [d\bar\psi\, d\psi\, dA\, dg]_{\Lambda} \exp i \int d^4x \sqrt{-g}\left[\mathcal{L}_{SM} - \Lambda + \frac{2}{\kappa^2} R + \underline{c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu}}\right]$$

limit of knowledge

$+ \mathcal{L}_{BSM}$

GR forms a QFT
- EFT type