AdS/CFT and Quantum Gravity

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Lectures in the

Basics of Quantum Gravity series
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Plan:

- Lecture 1. Supergravity and String Theory
- Lecture 2. AdS/CFT and its nontrivial tests
- Lecture 3. Gauge/gravity dualities, the pp wave correspondence and spin chains
- Lecture 4. Holographic Cosmology

Lecture 1

Supergravity and String Theory

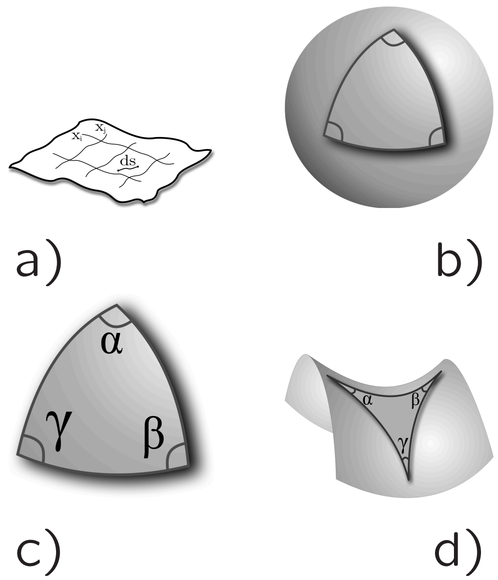
•A natural evolution: special relativity $\rightarrow$ general relativity $\rightarrow$ supergravity $\rightarrow$ string theory $\rightarrow$?

•We are still missing a *nonperturbative* (fundamental) version of quantum gravity, that is self-consistent, and predicts a complete set of observables, that we can test.

•M-theory: a set of recipes for various corners in parameters space, not yet a general definition.

•But, use string theory, alternatively, as a *duality* : AdS/CFT: classical strings (supergravity) for nonperturbative gauge theory. But, also hints of: perturbative gauge theory for non-perturbative strings: quantum gravity.

•Phenomenologically defined AdS/CFT applied to cosmology: holographic cosmology: strongly coupled (nonperturbative) quantum gravity.



xi
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a)
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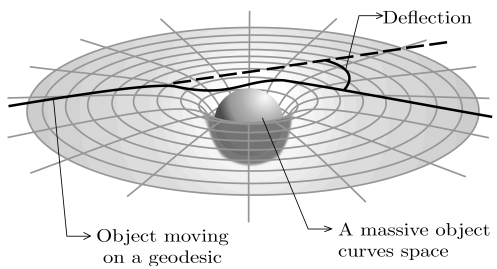

Deflection
Object moving
on a geodesic
A massive object
curves space

Einstein's theory of general relativity:

- A1: Gravity is geometry: matter follows geodesic in curved space, and to us it appears as gravity.

- A2: Matter sources gravity: matter curves space $\Rightarrow$ Princ.:

•1.Physics is invariant under general coordinate transformations:

$$x'_i = x'_i(x_j) \Rightarrow ds^2 = g_{ij}(x)dx^i dx^j = g'_{ij}(x')dx'^i dx'^j$$

•2.Equivalence principle: there is no difference between acceleration and gravity

$$m_i = m_g, \text{where} \quad \vec{F} = m_i\vec{a}(Newton) \quad \vec{F}_g = m_g\vec{g}(gravity)$$

•Dynamics of gravity: Einstein's eqs.

•Define: inverse metric $g^{\mu\nu} = g^{-1}_{\mu\nu}$, and then Christoffel symbol:

$$\Gamma^{\mu}{}_{\nu\rho} = \frac{1}{2} g^{\mu\sigma}(\partial_\rho g_{\nu\sigma} + \partial_\nu g_{\sigma\rho} - \partial_\sigma g_{\nu\rho}) \ ,$$

from $D_\mu g_{\nu\rho} = \partial_\mu g_{\nu\rho} - \Gamma^{\sigma}{}_{\mu\nu} g_{\sigma\rho} - \Gamma^{\sigma}{}_{\mu\rho} g_{\nu\sigma} = 0$, and Riemann tensor

$$R^{\mu}{}_{\nu\rho\sigma}(\Gamma) = \partial_\rho \Gamma^{\mu}{}_{\nu\sigma} - \partial_\sigma \Gamma^{\mu}{}_{\nu\rho} + \Gamma^{\mu}{}_{\lambda\rho} \Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\mu}{}_{\lambda\sigma} \Gamma^{\lambda}{}_{\nu\rho}$$

•$\Gamma^{\mu}_{\nu\rho} \sim$ gauge field of gravity. $R^{\mu}{}_{\nu\rho\sigma} \sim$field strength. Indeed, analogous to field strength of $SO(d-1,1)$ gauge group,

$$F^{A}_{\mu\nu} = \partial_\mu A^{A}_{\nu} - \partial_\nu A^{A}_{\mu} + f^{A}{}_{BC}(A^{B}_{\mu} A^{C}_{\nu} - A^{B}_{\nu} A^{C}_{\mu}),$$

•Then $R^{\mu}{}_{\nu\rho\sigma} \rightarrow$ tensor, as are $R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}$, $R = R_{\mu\nu} g^{\mu\nu}$. R is coordinate invariant $\rightarrow$ true measure of curvature of space.

•The simplest choice for action for gravity is correct (compatible with experiment): integral of scalar with invariant measure

$$S_{\text{gravity:E-H}} = \frac{1}{16\pi G_N} \int d^d x \sqrt{-\det(g_{\mu\nu})}\, R$$

$\Rightarrow$ Einstein's equation

$$8\pi G\Big[\frac{\delta S_{\text{gravity}}}{\sqrt{-g}\delta g^{\mu\nu}}+\frac{\delta S_{\text{matter}}}{\sqrt{-g}\delta g^{\mu\nu}}\Big]=0\Rightarrow R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=8\pi G_N T_{\mu\nu}$$

•With $1/(16\pi G_N)=M_{\text{Pl}}^2/2$, and $[R]=2$, usual QFT logic of constructing actions: in increasing mass dimension. R=the first after 1, gives equation $g_{\mu\nu}=0$. Next term is $\sim [R^2]=4$ but, since maximum observed radius is $10km\sim[10^{-10}eV]^{-1}$,

$$\frac{R}{M_{\text{Pl}}^2}\sim\frac{1}{([10km]M_{\text{Pl}})^2}\sim[10^{-29}]^2.$$

•Yet $R\times(R/M_{\text{Pl}}^2)=R\times(8\pi G_N R)$ appears in Quantum Gravity and String Theory effective actions: allowed by general relativity! Only unobservable.

•How? Loop (and string α') effects.

•Perturbation in $\kappa_N = \sqrt{8\pi G_N}$ for *variations* in $g_{\mu\nu}$: $\eta_{\mu\nu} + 2\kappa_N h_{\mu\nu}$.
•The Einstein-Hilbert action becomes the *Fierz-Pauli action*

$$S_{\text{E-H}} \simeq S_{\text{F-P}} = \int d^4x \left[-\frac{1}{2}(\partial_\mu h_{\nu\rho})^2 + h_\mu^2 - h^\mu \partial_\mu h + \frac{1}{2}(\partial_\mu h)^2 \right].$$

•In de Donder gauge ($\partial^\mu(h_{\mu\nu} - \eta_{\mu\nu} h/2) = 0$), just a KG action:

$$S_{\text{F-P,de D}} = \int d^4x \left[-\frac{1}{2}(\partial_\mu \bar{h}_{\nu\rho})^2 + \frac{1}{4}(\partial_\mu h)^2 \right].$$

•Consider also vertices by expanding $S_{\text{E-H}}$ to higher orders, e.g. $\mathcal{O}(\kappa_N h^3)$ $\Rightarrow$ **gravity (+matter) is one-loop nonrenormalizable!** $[\kappa_N] = -1 < 0$, so effective coupling is $(\kappa_N E) \to \infty$ as $E \to \infty$.
•Fundamental problem?: not really, one just needs to add more and more terms to the action, e.g. $\frac{1}{M^2}R^2, \frac{1}{M^4}R^3$, etc., at increasing loop order, and renormalize at each loop order.
•But suggests more fundamental (perturbative) Quantum Gravity might exist: renormalizable? finite? : supergravity? string theory? First attempt: supergravity: cancelation of some loop divergences between bosons and fermions. Then, string theory: $\alpha' \sim \frac{1}{M^2}$. All $\frac{1}{M^{2n}}R^{n+1}$ terms from the start.

Vielbein-spin connection formulation of GR: 1st vs. 2nd order

•Any space is locally flat: tangent space: Lorentz invariance that is local (at any point).

•**Vielbein** e^a_μ: "square root" of metric, making local Lorentz invariance manifest:

$$g_{\mu\nu}(x) = e^a_\mu(x) e^b_\nu(x) \eta_{ab}$$

$\to e^a_\mu \to \Lambda^a{}_b e^b_\mu$.

•Covariant derivative acting on tensors (bosons): with $\Gamma^\mu{}_{\nu\rho}$

$$D_\mu T^\rho_\nu \equiv \partial_\mu T^\rho_\nu + \Gamma^\rho{}_{\mu\sigma} T^\sigma_\nu - \Gamma^\sigma{}_{\mu\nu} T^\rho_\sigma$$

•Covariant derivative acting on spinors (fermions): with **spin connection** ω^{ab}_μ, multiplying the generator of the Lorentz group in the spinor representation, $\frac{1}{4}\Gamma_{ab}$,

$$D_\mu \Psi = \partial_\mu \Psi + \frac{1}{4} \omega^{ab}_\mu \Gamma^{ab} \Psi$$

•Second order formulation: $\omega_\mu^{ab}=\omega_\mu^{ab}(e)$ satisfies "vielbein postulate", or "no torsion constraint" ($T^a_{\mu\nu}$=torsion),

$$T^a_{[\mu\nu]}=D_{[\mu}e^a_{\nu]}=\partial_{[\mu}e^a_{\nu]}+\omega^{ab}_{[\mu}e^b_{\nu]}=0$$

(if there are no fundamental fermions; if there are, there are extra terms). Equivalently,

$$D_\mu e_\nu^a\equiv\partial_\mu e_\nu^a+\omega_\mu^{ab}e_\nu^b-\Gamma^\rho{}_{\mu\nu}e^a_\rho=0$$

•The solution is

$$\omega_\mu^{ab}(e)=\frac{1}{2}e^{a\nu}(\partial_\mu e^b_\nu-\partial_\nu e^b_\mu)-\frac{1}{2}e^{b\nu}(\partial_\mu e^a_\nu-\partial_\nu e^a_\mu)-\frac{1}{2}e^{a\rho}e^{b\sigma}(\partial_\rho e_{c\sigma}-\partial_\sigma e_{c\rho})e^c_\mu\ .$$

•Define the field strength of ω_μ^{ab} (=$SO(1,d-1)$ gauge field)

$$R^{ab}_{\mu\nu}(\omega)=\partial_\mu\omega_\nu^{ab}-\partial_\nu\omega_\mu^{ab}+\omega_\mu^{ab}\omega_\nu^{bc}-\omega_\nu^{ab}\omega_\mu^{bc}\ .$$

•Then we have

$$R^{ab}_{\rho\sigma}(\omega(e)) = e^a_\mu e^{-1,\nu b} R^\mu{}_{\nu\rho\sigma}(\Gamma(e)) \; , \quad R = R^{ab}_{\mu\nu} e_a^{-1\;\mu} e_b^{-1\;\nu}$$

so that the Einstein-Hilbert action *in second order formulation* ($\omega = \omega(e)$) is

$$S_{EH} = \frac{1}{16\pi G_N} \int d^d x (\det e) R^{ab}_{\mu\nu}(\omega(e)) e_a^{-1,\mu} e_b^{-1,\nu}.$$

•But then: first order formulation: ω_μ^{ab} = independent variable in the same action, rewritten as

$$\begin{aligned} S_{EH} &= \frac{1}{16\pi G_N} \frac{1}{4} \int d^4 x \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} R^{ab}_{\mu\nu}(\omega) e^c_\rho e^d_\sigma \\ &= \frac{1}{16\pi G_N} \int \epsilon_{abcd} R^{ab}(\omega) \wedge e^c \wedge e^d \end{aligned}$$

•Then the ω_μ^{ab} equation of motion is

$$T^a_{[\mu\nu]} \equiv 2 D_{[\mu} e^a_{\nu]} = 0$$

so solving it, we are back at the second order formulation.

Supersymmetry

•Bose-fermi symmetry. e.g. 2d: 1 Majorana spinor Ψ + 1 real scalar ϕ. **On-shell supersymmetry:** 1 bose degree of freedom, 1 fermi d.o.f.

$$S = -\frac{1}{2}\int d^2x[(\partial_\mu\phi)^2 + \bar{\Psi}\partial\!\!\!/\Psi]$$

•Dimensions: $[\phi] = 0, [\Psi] = 1/2$. Fermi-bose $\Rightarrow$ start as

$$\delta\phi = \bar{\epsilon}\Psi \Rightarrow [\epsilon] = -1/2 \Rightarrow$$
$$\delta\Psi = \partial\!\!\!/\phi\epsilon$$

•Action is on-shell invariant.

•**Off-shell supersymmetry**: Ψ has 2 d.o.f. $\Rightarrow$ need to add 1 auxiliary field

$$S = -\frac{1}{2}\int d^2x[(\partial_\mu\phi)^2 + \bar{\Psi}\partial\!\!\!/\Psi - F^2]$$
$$\delta F = \bar{\epsilon}\partial\!\!\!/\Psi; \quad \delta\Psi = \partial\!\!\!/\phi\epsilon + F\epsilon; \quad \delta\phi = \bar{\epsilon}\Psi$$

•Also *algebra must be satisfied off-shell (without e.o.m.)*

Supergravity

•Supergravity = supersymmetric theory of gravity, OR: theory of local supersymmetry.

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•Local supersymmetry $\Rightarrow \epsilon^\alpha(x) \Rightarrow \exists$ "gauge field of supersymmetry", "$A^\alpha_\mu(x)$" $\rightarrow$ gravitino $\Psi_{\mu\alpha}(x)$: supersymmetric partner of $e^a_\mu(x)$.

•$\mathcal{N} = 1$ supergravity in 4d: $\{e^a_\mu, \Psi_{\mu\alpha}\}$. Supersymmetry laws:

$$\delta e^a_\mu = \frac{\kappa_N}{2}\bar{\epsilon}\gamma^a\Psi_\mu$$
$$\delta\Psi_\mu = \frac{1}{\kappa_N}D_\mu\epsilon; \quad D_\mu\epsilon = \partial_\mu\epsilon + \frac{1}{4}\omega^{ab}_\mu\gamma_{ab}\epsilon$$

•Action:

$$\begin{aligned} S &= S_{E-H}(\omega, e) + S_{RS}(\Psi_\mu) \\ &= \frac{1}{16\pi G}\int d^dx(\det e)R^{ab}_{\mu\nu}(\omega)e_a^{-1\mu}e_b^{-1\nu} - \frac{1}{2}\int d^dx(\det e)\bar{\Psi}_\mu\gamma^{\mu\nu\rho}D_\nu\Psi_\rho \end{aligned}$$

•Second order formalism: $e_\mu^a, \psi_{\mu\alpha}$ indep., ω_μ^{ab} dependent. However, $\exists$ dynamical fermions, so $\omega_\mu^{ab} = \omega_\mu^{ab}(e) + \psi\psi$ terms, obtained by varying action with respect to ω_μ^{ab} (as in first order formalism) $\Rightarrow \omega_\mu^{ab}(e,\psi)$.

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•First order formalism: $e_\mu^a, \psi_{\mu\alpha}, \omega_\mu^{ab}$ independent.

•In 4d, maximal susy (for multiplets of spins ≤ 2) is $\mathcal{N} = 8$. It has graviton e_μ^a, 8 gravitini $\psi_{\mu\alpha}^i$, 28 vectors A_μ^{IJ}, 56 fermions χ_{ijk}^α and 35 scalars forming a matrix ν.

•It is the dimensional reduction of an $\mathcal{N} = 1$ supergravity multiplet in 11 dimensions, with graviton e_μ^a, gravitino $\psi_{\mu\alpha}$ and 3-index antisymmetric tensor $A_{\mu\nu\rho}$.

•11d=maximal supergravity, best candidate for a fundamental supergravity. But: potentially non-renormalizable at 7-loops ($\exists$ new super-invariant that one can write at 7-loop order). So no good. Something else? String theory: finite at each loop!!!

String theory

•Nambu-Goto action for bosonic string = area of "worldsheet" spanned by string × string tension. Generalization of particle action: area of worldsheet. $X^\mu(\sigma,\tau)$= coordinates in spacetime. $\xi^a = (\sigma,\tau)$ = intrinsic coordinates on worldsheet.

$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d\sigma d\tau\sqrt{\det(h_{ab})}$$

where h_{ab} =metric induced on worldsheet (pullback)

$$\begin{aligned} ds^2_{ind} &= dx^\mu dx^\nu g_{\mu\nu}(X) = d\xi^\mu d\xi^\nu h_{ab}(\xi) \Rightarrow \\ h_{ab}(\sigma,\tau) &= \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}(X) \end{aligned}$$

•Is worldsheet diffeomorphism (gen. coord., or reparametrization) invariant.
•(Quantum) parameters: α', $[\alpha'] = -2$, so $\alpha' \propto G_N$, and string coupling g_s (VEV, not parameter).

•First order form: again introduce auxiliary field = independent worldsheet metric.

•⇒ Polyakov action. In flat spacetime,

$$S_P[X,\gamma] = -\frac{1}{4\pi\alpha'}\int d\sigma d\tau \sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

•Symmetries:

-Spacetime Poincare invariance

-Worldsheet diffeomorphism invariance: $X'^\mu(\sigma',\tau') = X^\mu(\sigma,\tau)$

-Worldsheet Weyl invariance: $\gamma'_{ab} = e^{2\omega(\sigma,\tau)}\gamma_{ab}$

•Use them to fix conformal (unit) gauge: $\gamma_{\alpha\beta} = \eta_{\alpha\beta}$.

•Action becomes

$$S = -\frac{T}{2}\int d^2\sigma \eta^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

$\rightarrow$ action for free massless scalars in 2d: **conformally invariant** (conf. inv. = residual gauge invariance: dependence on $\sigma + \tau$ only), with equations of motion

$$\Box X^\mu = \left(\frac{\partial^2}{\partial\sigma^2} - \frac{\partial^2}{\partial\tau^2}\right)X^\mu = 0 \Rightarrow X^\mu(\sigma,\tau) = X^\mu_R(\sigma-\tau) + X^\mu_L(\sigma+\tau)$$

•Boundary term: gives string types:

$$-\frac{1}{2\pi\alpha'}\int d\tau\sqrt{-\gamma}\delta X^\mu \partial_\sigma X_\mu \Big|_{\sigma=0}^{\sigma=l} = 0 \Rightarrow$$

•Closed strings (periodic): $X^\mu(\tau,l) = X^\mu(\tau,0)$; $\gamma_{ab}(\tau,l) = \gamma_{ab}(\tau,0)$.

•Neumann open strings (free endpoints, $v = c$): $\partial^\sigma X^\mu(\tau,0) = \partial^\sigma X^\mu(\tau,l)$.

•Dirichlet open strings (fixed endpoints): $\delta X^\mu(\tau,0) = \delta X^\mu(\tau,l) = 0$.

•(Virasoro) Constraints: equations of motion of γ_{ab} (fixed to unit) $\equiv T_{ab} = 0$

$$T_{ab} = -\frac{1}{4\pi}\frac{1}{\sqrt{-\gamma}}\frac{\delta S_P}{\delta\gamma^{ab}}\Big|_{\gamma_{ab}=\eta_{ab}} = \frac{1}{\alpha'}\left(\partial_a X^\mu \partial_b X_\mu - \frac{1}{2}\eta_{ab}\partial_c X^\mu \partial^c X_\mu\right) \Rightarrow$$

.
$$\alpha' T_{01} = \alpha' T_{10} = \dot{X}\cdot X' \,, \quad \alpha' T_{00} = \alpha' T_{11} = \frac{1}{2}(\dot{X}^2 + X'^2).$$

•Closed strings: expand $X_R^\mu(\tau-\sigma)$ and $X_L^\mu(\tau+\sigma)$ in Fourier modes $\alpha_n^\mu, \tilde{\alpha}_n^\mu$,

$$X^\mu(\sigma,\tau) = x^\mu + \alpha' p^\mu \tau + i\frac{\sqrt{2\alpha'}}{2}\sum_{n\neq 0}\frac{1}{n}\left[\alpha_n^\mu e^{-in(\tau-\sigma)} + \tilde{\alpha}_n^\mu e^{-in(\tau+\sigma)}\right].$$

•Neumann open strings: identify $\alpha_n^\mu = \tilde{\alpha}_n^\mu$.
•Fourier modes $L_m, \bar{L}_m$ of constraints T_{--}, T_{++} are $L_m, \bar{L}_m$, for closed strings

$$L_m = \frac{1}{2}\sum_{n=-\infty}^{+\infty}\alpha_{m-n}^\mu \alpha_n^\mu \,, \quad \bar{L}_m = \frac{1}{2}\sum_{n=-\infty}^{+\infty}\tilde{\alpha}_{m-n}^\mu \tilde{\alpha}_n^\mu.$$

and $H = L_0 + \bar{L}_0 = 0$ (closed) or $H = L_0$ (open) give (classically)

$$M^2_{\text{closed}} \equiv -p_\mu p^\mu = \frac{2}{\alpha'}\sum_{n\geq 1}(\alpha_{-n}^\mu \alpha_n^\mu + \tilde{\alpha}_{-n}^\mu \tilde{\alpha}_n^\mu)\,, \quad M^2_{\text{open}} \equiv -p_\mu p^\mu = -\frac{\alpha_0^2}{2\alpha'} = \frac{1}{\alpha'}\sum_{n\geq 1}\alpha_{-n}^\mu \alpha_n^\mu$$

•What does the string action represent? Particle action: is first quantized: Need to also define vertices and propagators. String action: defines the propagator; vertex is unique!!

•**Quantization**: $\alpha^\mu_{-n},\tilde{\alpha}^\mu_{-n}$: creation operators. More precisely, $\alpha^\mu_m=\sqrt{m}a^\mu_m$, $\alpha^\mu_{-m}=\sqrt{m}a^{\dagger\mu}_m$ for $m>0$.

•But ∃ gauge inv.: easiest in light-cone gauge. $X^\pm$ auxiliary, X^i physical. Then $H=p^-$ and the open string mass spectrum is

$$M^2\equiv 2p^+p^- - p^ip^i=\frac{1}{\alpha'}(N-a)\ ,\ \ N=\sum_{n\geq 1}\alpha^i_{-n}\alpha^i_n=\sum_{n\geq 1}na^{\dagger i}_na^i_n\ ,$$

where

$$a=-\sum_{i=1}^{D-2}\sum_{n\geq 1}\frac{n}{2}=-\frac{D-2}{2}\sum_{n\geq 1}n=\frac{D-2}{24}=1\Rightarrow D=26.$$

•Bosonic closed string spectrum is similar, but with N and $\bar{N}$,

$$\sim a^{i_1}_{n_1}...a^{i_k}_{n_k}\tilde{\alpha}^{\tilde{i}_1}_{m_1}...\tilde{\alpha}^{\tilde{i}_j}_{m_j}|0\rangle\ ,$$

with the constraint $P=L_0-\bar{L}_0=0$, so $N=\bar{N}$. Spectrum → different fields ⇒ **String theory = field theory of infinite number of different kinds of fields.**

•Massless fields: $A_{\mu\nu} = \alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0>= \{A_{((\mu\nu))} = g_{\mu\nu}, A^{[\mu\nu]} = B^{\mu\nu}, \phi = A^{\mu\mu}\}$ $\rightarrow$ spacetime fields.

•Thus: metric $g_{\mu\nu}$ is among quantum modes of the string $\Rightarrow$ **String theory is a theory of quantum gravity!**

•Massless fields create a spacetime background for string

•**Superstring**: Supersymmetric string. In Green-Schwarz formulation, spacetime susy + "κ symmetry". (Fix a gauge for κ symmetry $\Rightarrow$ worldsheet susy). Introduce θ^A = spacetime spinors and worldsheet scalars. Replace $\partial_a X^\mu$ with spacetime susy invariant

$$\Pi_a^\mu = \partial_a X^\mu - i\bar{\theta}^A \Gamma^\mu \partial_a \theta^A$$

invariant under

$$\delta X^\mu = -\bar{\epsilon}^A \Gamma^\mu \partial_a \theta^A, \quad \delta\theta^A = \epsilon^A$$

$$S = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma \sqrt{-\gamma}\gamma^{ab}\Pi_a^\mu \Pi_b^\nu g_{\mu\nu} + \int d\tau d\sigma \epsilon^{ab}\Pi_a^M \Pi_b^N B_{MN}$$

• flat space:

$$B \equiv \epsilon^{ab}\Pi_a^M \Pi_b^N B_{MN} = -idX^\mu \wedge (\bar{\theta}^1\Gamma_\mu d\theta^2 - \bar{\theta}^2\Gamma_\mu d\theta^1) + \bar{\theta}^1\Gamma^\mu d\theta^1 \wedge \bar{\theta}^2\Gamma_\mu d\theta^2$$

• Kappa symmetry,

$$\delta_\kappa \theta^A = -2\Gamma_\mu \Pi_a^\mu \kappa^{Aa} \,, \quad \delta_\kappa X^\mu = -\bar{\theta}^A \Gamma^\mu \delta\theta^A \,, \quad \ldots$$

is fixed by the condition (together with lightcone gauge for bosons)

$$\Gamma^+\theta^1 = \Gamma^+\theta^2 = 0 \,, \quad \Gamma^\pm = (\Gamma^0 \pm \Gamma^9)/\sqrt{2}$$

and $\theta^{A\alpha}$ are regrouped as 2-comp. Majorana worldsheet spinors S^m, m spinor of $SO(8)$ (now, $d = 10$, so $SO(8)$=little group),

$$S_{\rm lc} = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left[\partial_a X^i \partial^a X^i + 2\alpha' \bar{S}^m \gamma^a \partial_a S^m\right].$$

• **Supersymmetry means tachyons (and other states) are out of the spectrum**. Vacuum: massless states $A^{\mu\nu} = \{g^{\mu\nu}, B^{\mu\nu}, \phi\}+$ others

• No divergences at any loop: Feynman diagrams involve worldsheets: no point singularities: interactions spread out over sheet

•**T-duality of closed and open strings**: symmetry of string perturbation theory on compact spaces.
•For a string winding m times around X^{25}, bound. cond.

$$X^{25}(\tau,\sigma+2\pi) = X^{25}(\tau,\sigma) + 2\pi\alpha' w.$$

•The classical solution is

$$X^{25}(\tau,\sigma) = X_L + X_R = x_0 + \alpha' p\tau + \alpha' w\sigma + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}(\alpha_n e^{in\sigma} + \tilde{\alpha}_n e^{-in\sigma})\ ,$$

where $p = n/R$ and $w = mR/\alpha'$. The constraint is now $L_0 - \tilde{L}_0 = \alpha' pw + N^\perp - \tilde{N}^\perp$ and gives the spectrum

$$\begin{aligned} M^2_{\text{compact}} &= p^2 + w^2 + \frac{2}{\alpha'}(N^\perp + \tilde{N}^\perp - 2) \\ &= \left(\frac{n}{R}\right)^2 + \left(\frac{mR}{\alpha'}\right)^2 + \frac{2}{\alpha'}(N^\perp + \tilde{N}^\perp - 2). \end{aligned}$$

•We observe the T-duality symmetry of the spectrum

$$M^2(R;n,m) = M^2(\tilde{R};m,n).$$

extended to

$$x_0 \leftrightarrow q_0; \quad p \leftrightarrow w; \quad \alpha_n \leftrightarrow -\alpha_n; \quad \tilde{\alpha}_n \leftrightarrow \tilde{\alpha}_n\ ,$$

•This T-duality exchanges then:

$$X^{25}(\tau,\sigma)X_L(\tau+\sigma)+XR(\tau-\sigma) \leftrightarrow X'^{25}(\tau,\sigma) = X_L(\tau+\sigma)-X_R(\tau-\sigma)$$

•**T-duality of open strings:** Do the same exchange for the open string solution. Obtain

$$X'^{25}(\tau,\sigma) = X_L^{25}(\tau+\sigma) - X_R^{25}(\tau-\sigma) = q_0^{25} + \sqrt{2\alpha'}\alpha_0^{25}\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^{25}}{n}e^{-in\tau}\sin n\sigma\,,$$

$$\alpha_0^{25} = \frac{1}{\sqrt{2\alpha'}}\frac{x_2^{25}-x_1^{25}}{\pi}.$$

•But then the boundary condition changes from Neumann to Dirichlet and vice versa,

$$\partial_\alpha X^{25} = \epsilon_{\alpha\beta}\partial_\beta X'^{25}.$$

•**Reminder:** Vary Polyakov action ⇒ equations of motion, and boundary term

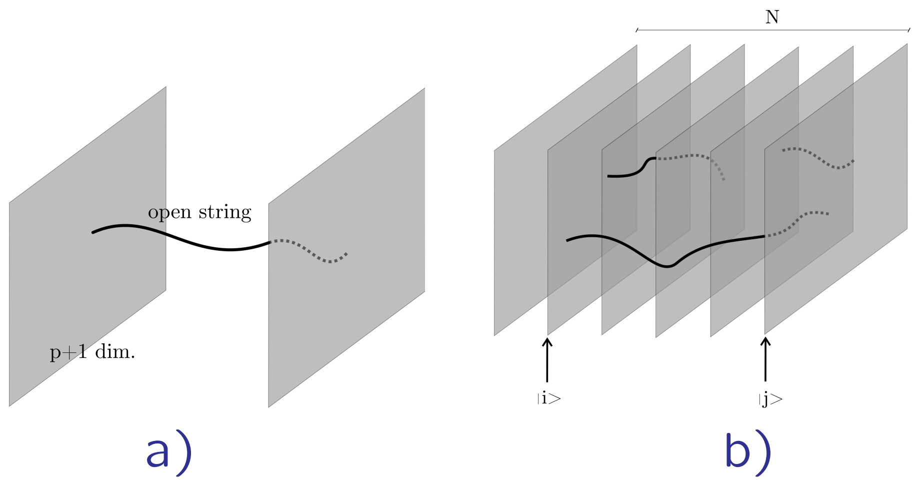
$$\delta S_{P,bd.} = -\frac{1}{2\pi\alpha'}\int d\tau\sqrt{-\gamma}\delta X^\mu \partial^\sigma X_\mu|_{\sigma=0}^{\sigma=\pi} \Rightarrow$$

•Neumann boundary condition: $\partial^\sigma X_\mu = 0|_{\sigma=0,\pi}$ ⇒ endpoints of string move at the speed of light: usual.

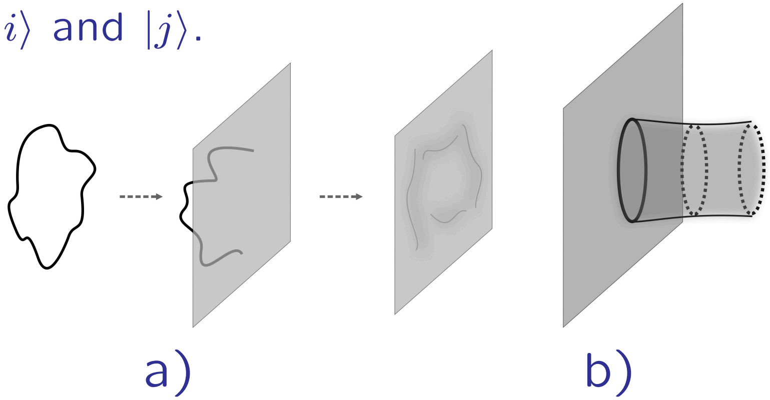
•Dirichlet boundary condition: $\delta X^\mu = 0|_{\sigma=0,\pi}$. ⇒ $X^\mu =$ constant at $\sigma = 0, \pi$. → endpoints fixed.

•We can have Neumann for $p+1$ coordinates and Dirichlet for $D-p-1$ ⇒ "Dp-brane".

•Spacetime fields can excite coordinates X^μ transverse to the Dp-brane (Dirichlet directions) → fluctuations ⇒ this is Dp-brane is a dynamical object.



a) Open string between two D-p-branes ($p+1$ dimensional "walls"). b)The endpoints of the open string are labelled by the D-brane they end on (out of N D-branes), here $|i\rangle$ and $|j\rangle$.



a) Closed string colliding with a D-brane, exciting an open string mode and making it vibrate b) String worldsheet corresponding to it, with a closed string tube coming from infinity and ending on the D-brane as an open string boundary. Allows us to calculate the D-brane action and couplings.