Lecture 4

Holographic cosmology

Top-down nonconformal gauge/gravity duality

•**Example** (Itzhaki, Maldacena, Sonnenschein, Yankielowicz, 1998): N D2-branes giving 2+1 dim. $SU(N)$ $\mathcal{N}=4$ SYM theory

.

$$\begin{aligned} ds^2_{\text{string}} &= H_2^{-1/2}(-dt^2+dx^2+dy^2)+H_2^{1/2}(dr^2+r^2 d\Omega_6^2) \\ H_2(r) &= 1+d_2\frac{g_{YM}^2 M\alpha'^2}{r^5} = 1+d_2\frac{g_{YM}^2 N}{\alpha'^2 U^5} \\ e^{\phi} &= H_2^{1/4} \end{aligned}$$

•decoupling limit: $r\to 0, \alpha'\to 0$, $U = r/\alpha'$ fixed, and $g_{YM}^2 = g_s/\sqrt{\alpha'}$. Then, drop the 1 in H_2, so 3+1 dim. $+\Omega_6$

$$\begin{aligned} \frac{ds^2_{\text{string}}}{\alpha'} &= \frac{U^2}{R^2}(-dt^2+dx^2+dy^2)+R^2\frac{dU^2}{U^2}+R^2 d\Omega_6^2 \\ R^2 &= \alpha'\sqrt{d_2\frac{g_{YM}^2 N}{U}} \end{aligned}$$

•duality is *holographic* as well (d+1 gravity $\to$ d field theory)

Phenomenological (bottom-up) gauge/gravity duality: cosmology

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•Usually: assume ∃ holography in AdS space, write perturbative phenomenological gravity th. in AdS space (gravity + other fields) ⇒ by holographic map, dual nonperturbative field theory has desired properties.

•BUT: we can also imagine opposite map: define perturbative field theory phenomenologically. Then, by holographic map, nonperturbative (quantum) gravity is defined implicitly.

•Cosmology: 3 spatial directions (x, y, z, with fluctuations $h_{ij}(x, y, z; t)$) + time t.

•But: double Wick rotation needed: $(t, x, y) \to (x, y, z)$; $r \to t$.

•Then, inverse RG flow in momentum $U \leftrightarrow r$ evolution → time t evolution.

•Inflation (exponential expansion $a(t) \propto e^{Ht}$, or power law $a(t) \propto t^n$, $n > 1$) is considered almost a "Standard Model" of cosmology, since it agrees with data (CMBR fluctuations) and solves a set of classic "puzzles" of Hot Big Bang cosmology

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•But there is an extension of inflation into the strong gravity domain, where it can be dealt with holographically (in AdS/CFT or gauge/gravity duality): holographic cosmology

•Model by P. Mc Fadden and K. Skenderis (2009) offers a phenomenological set-up in this extended paradigm: use 2+1d theories with "generalized conformal structure" and fix parameters from CMBR data.

•Different parametrical fitting than Λ−CDM with inflation, but fit to CMBR is as good (χ^2 of 0.5 difference, 824.0 vs. 823.4)

•Could be improved by lattice calculation at intermediate coupling (Skenderis et al., in progress)

•Besides, the classic puzzles of Hot Big Bang cosmology solved by inflation are also solved in holographic cosmology

Holographic cosmology (McFadden, Skenderis, 2009)

•Wick rotated cosmology ("cosmology/domain wall correspondence"), for $t \to z$

$$\begin{aligned} ds^2 &= +dz^2 + a^2(z)[\delta_{ij} + h_{ij}(z,\vec{x})]dx^i dx^j \ , \\ \Phi(z,\vec{x}) &= \phi(z) + \delta\phi(z,\vec{x})a \ , \end{aligned}$$

with $\bar{q} = -iq$, $\bar{\kappa}^2 = -\kappa^2$. h_{ij} and $\delta\phi \to$ fluctuations.

•This has a (phenomenological!) gravity dual; Wick rotation implies $\bar{q} = -iq$, $\bar{N} = -iN$.

•CMBR observations: power spectra of perturbations γ_{ij} and ζ (gauge inv. combinations of h_{ij} and $\delta\phi$)

$$\begin{aligned} \Delta_S^2(q) &\equiv \frac{q^3}{2\pi^3}\langle\zeta(q)\zeta(-q)\rangle \\ \Delta_T^2(q) &\equiv \frac{q^3}{2\pi^3}\langle\gamma_{ij}(q)\gamma_{ij}(-q)\rangle. \end{aligned}$$

•If $a(z) \propto e^{Hz} \leftarrow a(t) \propto e^{Ht}$: inflation, approx. de Sitter: treated by Maldacena, 2002, via a type of Wick rotation from Anti-de Sitter (AdS).

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•The AdS Witten prescription $Z_{\text{CFT}}[\phi_0] = Z_{\text{AdS}}[\phi_0] = e^{-S_{\text{sugra}}[\phi[\phi_0]]}$ becomes the **dS Maldacena prescription (map)**

$$Z_{\text{CFT}}[h_{ij}, \phi] = \Psi[h_{ij}, \phi]$$

for the CFT partition function Z_{CFT} (with 3d sources h_{ij}, ϕ) vs. the wavefunction of the Universe Ψ (path integral up to surface with 3-metric h_{ij} and ϕ, at time t).

•But, prescription can be extended to nonconformal theories (Skenderis et al. works) $\rightarrow a(z) \propto z^n \leftarrow a(t) \propto t^n$. Moreover, as for usual AdS/CFT, *assume* it is valid at any coupling, including **strong (nonperturbative quantum) gravity**.

•Then, new model: CMBR perturbations generated during a strong gravity (non-geometrical) cosmological phase

•A holographic (strong gravity $\rightarrow$ perturbative field theory) calculation, either direct, or based on Maldacena's map $Z[\Phi] = \Psi[\Phi]$, extended to this case, gives

$$\Delta_S^2(q) = -\frac{q^3}{16\pi^2 \text{Im} B(-iq)}$$
$$\Delta_T^2(q) = -\frac{2q^3}{\pi^2 \text{Im} A(-iq)}$$

(we used $\bar{\kappa}^2 = -\kappa^2$, $\bar{q} = -iq$), where

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl}$$
$$\Pi_{ijkl} = \pi_{i(k}\pi_{l)j} - \frac{1}{2}\pi_{ij}\pi_{kl} \, , \quad \pi_{ij} = \delta_{ij} - \frac{\bar{q}_i \bar{q}_j}{\bar{q}^2}$$

•Euclidean field theory is super-renormalizable $SU(N)$ gauge theory, with $A_i = A_i^a T_a$, $\phi^M = \phi^{aM} T_a$, $\psi^L = \psi^{aL} T_a$ and "generalized conformal structure" $\rightarrow$ dimensions contained in q only, and through $g_{\text{eff}}^2 = \frac{g^2 N}{q}$.

•Action is phenomenological (most general super-renormalizable, with "generalized conformal structure")

$$
\begin{aligned}
S_{\text{QFT}} &= \int d^3x \text{Tr}\left[\frac{1}{2}F_{ij}F^{ij} + \delta_{M_1M_2}D_i\Phi^{M_1}D^i\Phi^{M_2} + 2\delta_{L_1L_2}\bar{\psi}^{L_1}\gamma^i D_i\psi^{L_2}\right. \\
&\quad \left. +\sqrt{2}g_{YM}\mu_{ML_1L_2}\Phi^M\bar{\psi}^{L_1}\psi^{L_2} + \frac{1}{6}g_{YM}^2\lambda_{M_1\ldots M_4}\Phi^{M_1}\ldots\Phi^{M_4}\right] \\
&= \frac{1}{g_{YM}^2}\int d^3x \text{Tr}\left[\frac{1}{2}F_{ij}F^{ij} + \delta_{M_1M_2}D_i\Phi^{M_1}D^i\Phi^{M_2} + 2\delta_{L_1L_2}\bar{\psi}^{L_1}\gamma^i D_i\psi^{L_2}\right. \\
&\quad \left. +\sqrt{2}\mu_{ML_1L_2}\Phi^M\bar{\psi}^{L_1}\psi^{L_2} + \frac{1}{6}\lambda_{M_1\ldots M_4}\Phi^{M_1}\ldots\Phi^{M_4}\right]
\end{aligned}
$$

•Then, calculate in field theory

$$
\begin{aligned}
A(q,N) &= q^3N^2 f_T(g_{\text{eff}}^2)\,, \quad B(q,N) = \frac{1}{4}q^3N^2 f(g_{\text{eff}}^2) \\
f(g_{\text{eff}}^2) &= f_0\left[1 - f_1 g_{\text{eff}}^2 \ln g_{\text{eff}}^2 + f_2 g_{\text{eff}}^2 + \mathcal{O}(g_{\text{eff}}^4)\right] \\
f_T(g_{\text{eff}}^2) &= f_{T0}\left[1 - f_{T1} g_{\text{eff}}^2 \ln g_{\text{eff}}^2 + f_{T2} g_{\text{eff}}^2 + \mathcal{O}(g_{\text{eff}}^4)\right]
\end{aligned}
$$

which implies the phenomenological parametrization ($g, q_*, \beta, g_T, \beta_T$ depend on $g_{\text{YM}}^2, N, N_s, N_f$ and λ's, μ's)

$$
\Delta_S^2(q) = \frac{\Delta_0^2}{1 + \frac{gq_*}{q}\ln\left|\frac{q}{\beta g q_*}\right| + \mathcal{O}\left(\frac{gq_*}{q}\right)^2}, \quad \Delta_T^2(q) = \frac{\Delta_{0T}^2}{1 + \frac{g_T q_*}{q}\ln\left|\frac{q}{\beta_T g_T q_*}\right| + \mathcal{O}\left(\frac{g_T q_*}{q}\right)^2}
$$

•Why is spectrum almost flat (like for inflation)? Generalized conformal structure! Only $g^2_{\rm eff} = g^2N/q$ quantum corrections allowed, so if $g^2_{\rm eff}$ is small (perturbative QFT, so nonperturbative gravity), only corrections appear as above.

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•Comparison to $\Lambda - CDM$ + inflation:

$$\Delta^2_S(q)(\text{infl}) = \Delta^2_0 \left(\frac{q}{q_*}\right)^{n_s - 1 + \frac{\alpha_s}{2}\ln\frac{q}{q_*}}$$

with $n_s - 1 \ll 1$. Then $\Delta^2_S(q)(\text{infl}) \propto q^{n_s-1} \sim 1 - (n_s - 1)\ln q$ and $\Delta^2_s(q)(\text{holo.cosmo.}) \propto 1/[1 + A\ln q] \simeq 1 - A\ln q$ as well.

•Yet fit to data sufficiently complex that it can distinguish them.
•Nevertheless, fit to data is as good as Λ−CDM with inflation, χ^2 of 824.0 vs. 823.5, and fixes parameters $(N, g^2_{\rm eff}$, and simplified couplings).

•Find that $g^2_{\rm eff}$ is not perturbative for $l < 30 \Rightarrow$ exclude it from the fit. To put it back: need lattice calculation (in progress).
(Afshordi, Coriani, Delle Rose, Gould, Skenderis, 2017)

•Another quantity needed here: global symmetry current correlators, giving

$$\langle j_i^A(q) j_k^B(-q)\rangle = N^2 q \delta^{AB} \pi_{ik} f_J(g_{\rm eff}^2)$$

where again

$$f_J(g_{\rm eff}^2) = f_{J0}\left[1 - f_{J1} g_{\rm eff}^2 \ln g_{\rm eff}^2 + f_{J2} g_{\rm eff}^2 + \mathcal{O}(g_{\rm eff}^4)\right]$$

•For that, we need a global symmetry (restrict the phenomenological model)

•Will be related to monopole perturbations

Hot Big Bang puzzles and their solutions in inflation

1. Smoothness and horizon: Universe is smooth, and ∃ correlations > horizon: observed correlation size $2r_H$/ horizon distance d_H at ls (last scattering), today gives

$$N = \frac{2r_H(t_0)}{d_H(t_0)} \simeq 2(1+z_{\rm ls})^{1/2} \simeq 72$$

Inflation: expansion with $a(t) \propto t^n$, $n > 1$ or e^{Ht} ⇒ scales expand exponentially and $d_H(t_{ls}) \propto e^{N_e}$,

2. Flatness problem:

$$\Omega(t) - 1 = \frac{k}{a(t)^2 H(t)^2} \propto \left(\frac{t}{a(t)}\right)^2 \propto t^{2(1-p)}$$

needs $p > 1$ or $a(t) \propto e^{Ht}$ **(inflation)** to decrease to RD era, then increase until now:

$$\Omega_0 - 1 = (\Omega(t_{bi}) - 1)e^{-2N_e}\left(\frac{a(t_I)H_I}{a_0 H_0}\right)^2$$

3. Relic and monopole problem

-Monopoles: direct searches: $\exists < 10^{-30}$ monopoles/nucleon $\Rightarrow$ $< 10^{-30}$ monopoles per volume dilution (at phase transition, the Kibble mechanism gives ~ 1 mon./nucleon) $\Rightarrow$ need dilution by $N_e > \ln 10^{10} \simeq 23$ e-folds (for phase transition, before the end of inflation).

-Relics: Not over close the Universe $\Rightarrow < 10^{-11}$ reduction in volume since phase transition (when $\exists \simeq 1$ relic/nucleon)

4. Entropy problem: $S_H(t_{\rm BBN}) \sim 10^{63}$, but at phase transition, $\sim 1/$horizon. Inflation: large growth of entropy during reheating, and exponential expansion increases entropy in horizon.

5. Perturbations problem: CMBR pert. are *classical*, and were super-horizon in the past. Inflation: scales $\propto e^{Ht}$, but $H \simeq$ const. $\Rightarrow$ scales get out of horizon.

6. Baryon asymmetry problem: $(N_B - N_{\bar{B}})/N_B \sim 10^{-9}$. Its creation needs interactions out of equilibrium. Inflation $\rightarrow$ true (fast expansion) and 10^{-9}: $S_1 \sim 10^9$.

Solution of puzzles in holographic cosmology

1. Smoothness and horizon problem

•$\exists$ nongeometric phase, but at the end - geometrical.
•Holographic map nonlocal, even though field theory is causal and local $\rightarrow$ generates apparent nonlocality.
•Field theory finite in the IR (small cosmo times) (Skenderis et al. proof of old conjecture in 3d), so correlators are nonzero over large distances, there is no cosmo singularity, and light-cones coming from different regions are correlated: solution!
•Suppose it's not, define $g^2_{\rm eff} \sim 1$ as beginning $\Rightarrow$ constraint on RG flow.
•More precisely, RG flow (UV to IR) dual to *inverse* time evolution: AdS geodesic, joining x and y at spatial distance $L \Rightarrow$ $L = cR^2/r_0$, where $r_0 =$ minimum radial distance in AdS. But $r \rightarrow e^{-t/R}$, so $L = cRe^{-t/R}$, so $k = \frac{H}{c}e^{Ht}$, where k is mom. scale.
•Then, constraint on N_e becomes constraint on amount of RG flow $\Rightarrow$ an amount of 10^{-54} in k^2 (or 63 e-folds) for $T_I \sim 10^{16} GeV$: in order to avoid the large fluctuations.

2. Flatness problem

•Again RG flow $\leftrightarrow$ inverse time evolution. We want to see then that (grav.) perturbations decrease along the inverse RG flow (from IR to UV).

•For $g^2_{\text{eff}} = \frac{g^2 N}{q} \ll 1$ (late times), we find

$$f(g^2_{\text{eff}}) = f_0 \left(1 - f_1 g^2_{\text{eff}} \ln g^2_{\text{eff}} + f_2 g^2_{\text{eff}} + \mathcal{O}(g^2_{\text{eff}})\right)$$

where $f_1 < 0$ (for best fit, and most of the theor. parameter space) and f_1 dominates over f_2. But since

$$f(g^2_{\text{eff}}) \propto q^{2\delta} \sim 1 + 2\delta \ln q \sim 1 - 2\delta \ln g^2_{\text{eff}} + \ldots$$

we have $2\delta \simeq f_1 g^2_{\text{eff}} < 0 \Rightarrow T_{ij}$ is marginally relevant, $\langle TT \rangle \sim q^3 f(g^2_{\text{eff}}) \sim q^{3+2\delta} \Rightarrow \Delta = 3+\delta$. Then $S = S_{\text{QFT}} + \int d^3x \Lambda^{3-\Delta} \delta h_{ij} T^{ij}$.

•CFT terminology, but only generalized conf. structure, yet same results: $\delta < 0 \Rightarrow$ dilution along inverse RG flow.

•Quantitatively, same cond.: at least 10^{-54} of RG flow in k^2 (63 e-folds) for $T_I \sim 10^{16} GeV$.

4. Entropy problem: inflation $\rightarrow$ reheating.

•Now $\rightarrow \exists$ period corresponding to reheating. But, in field theory: *obvious*: dual field theory has grav. modes + SM modes: transfer of energy from one to the other. Entropy larger in the UV (late times) than IR (initial times) $\rightarrow$ # of d.o.f. decreases along RG flow $\Rightarrow$ arrow of time!!. Large entropy $\rightarrow$ large N. $S_1 \sim 10^9$ (UV) to $S_1 \sim 1$ (IR) is a constraint. So is the fact that $S \sim 10^{88} \ll 10^{121}$ (S_{1BH}). $S_1 \sim 1$ in the IR is natural.

5. Perturbations problem

•Also easier: classical $\langle h_{ij} h_{kl} \rangle$ perturbations in CMBR are dual to quantum $\langle T_{ij} T_{kl} \rangle \rightarrow$ usual QFT perturbations. But now, no assumptions (like QFT in curved space and Bunch-Davies vacuum) $\rightarrow$ initial conditions: vacuum is unique perturbative QFT vacuum.

6. Baryon asymmetry problem. Same solution. But now: reactions out of thermal equilibrium: no thermal equilibrium along the RG flow. Nr. of d.o.f. changes rapidly

Relic and monopole problem, and (toy) models

•$\nexists$ geometry. But monopole defined by topology: abstractly.

•Monopole in the bulk $\rightarrow$ vortex (top. and magn. charge) on the boundary. AdS/CFT: True case: "'t Hooft monopole" $\rightarrow$ "true vortex", but approx. case: "Dirac monopole" $\rightarrow$ "Dirac vortex".

•Constraint: dilution of monopole current $\tilde{j}_i^a$ perturbations in the bulk $\rightarrow$ in inverse RG flow, of 10^{-10} in linear size. $\Rightarrow$ need $\delta(\tilde{j}_i^a) < 0$. For relics, coupling to T_{ij}, need dilution of T_{ij} pert. along the RG flow of 10^{-4} $\rightarrow$ same, and less stringent, as for flatness problem.

•But: A_μ^a (gauge) in bulk $\rightarrow j_i^a$ (global) in QFT. Moreover, magnetic $\tilde{j}_i^a$ replaced by electric j_i^a. Since QFT is phenomenological, no definite $j_i^a \rightarrow$ need toy model.

•Toy model (though in fact, a posteriori: *calculation valid for all relevant models* HN+U.Portugal, 2020): $SU(N)$ gauge symm., $SO(3)$ global, allowing for vortex solutions. A_μ and 6 complex scalars ϕ_i^a, $i=1,2$ and $a=1,2,3$ for $\underline{3}$ of $SO(3)$, all in $SU(N)$. Potential (scalar self-int.)

$$V = \lambda \mathrm{Tr}\, |\vec{\phi}_1 \times \vec{\phi}_2|^2$$

Then the Euclidean action is

$$S = \int d^3x \mathrm{Tr} \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{i=1,2} |D_\mu \vec{\phi}_i|^2 + \lambda |\vec{\phi}_1 \times \vec{\phi}_2|^2 \right]$$

and the $SO(3)$ global currents are

$$j_\mu^a = \sum_{i=1,2} i\epsilon^{abc} \phi_i^{b,*} D_\mu \phi_j^c + h.c$$

where $D_\mu^{AB} = \partial_\mu \delta^{AB} - ig(T_C)^{AB} A_\mu^C$.

•Two loop calculation in dim. reg.: $\exists$ divergences, but removed $\to$ only p dependence in finite piece. Find (one-loop plus 2-loop):

$$\langle j^a_\mu(p) j^b_\nu(-p)\rangle = N^2\frac{p}{4}\delta^{ab}\left[\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right) - 4\cdot 16\frac{g^2 N}{p}J_0\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)+\text{finite}\right]$$

where $J_0 \simeq -\frac{1}{32\pi^2}\frac{1}{\epsilon}$+finite. But: generalized conf. structure $\to$

$$\langle j^a_\mu(p) j^b_\nu(-p)\rangle = \frac{N^2 p}{4}\pi_{\mu\nu}[1+cg^2_{\text{eff}}\ln g^2_{\text{eff}}+\ldots] = \frac{N^2 p}{4}\pi_{\mu\nu}[1-cg^2_{\text{eff}}\ln p+\ldots]$$

•But definining anomalous dimension as before,

$$\langle j^a_\mu(p) j^b_\nu(-p)\rangle \propto N^2\pi_{\mu\nu}p^{1+2\delta} \simeq N^2 p\pi_{\mu\nu}[1+2\delta\ln p+\ldots]$$

gives $2\delta = -cg^2_{\text{eff}}$. Finally, we obtain

$$\delta_j = \frac{2}{\pi^2}g^2_{\text{eff}} > 0$$

so j^a_i is irrelevant: grows in the UV.

•But: need *vortex* current. In Abelian-Higgs model,

$$j^{\mu}_{\text{vortex}} = \frac{1}{K}\epsilon^{\mu\nu\rho}\partial_\nu j_\rho$$

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Then the correlators are related as

$$\langle j_\mu(p) j_\nu(-p)\rangle = f\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \Rightarrow \langle j^{\mu}_{\text{vortex}}(p) j^{\nu}_{\text{vortex}}(-p)\rangle = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{p^2}{K^2} f$$

•But, more precisely (Witten; Herzog, Kovtun, Sachdev, Son) conformal structure in 2+1d $\Rightarrow$ (t replaced by K^{ab} in the nonabelian case)

$$\langle j_i(p) j_j(-p) = \left(p^2\delta_{ij} - p_i p_j\right)\frac{t}{2\pi\sqrt{k^2}} + \epsilon_{ijk}p_k \frac{w}{2\pi}$$

•Then implies for the magnetic current

$$\langle \tilde{j}_i(p)\tilde{j}_j(-p)\rangle = \frac{p^2\delta_{ij} - p_i p_j}{2\pi\sqrt{p^2}}\frac{t}{t^2+w^2} - \frac{\epsilon_{ijk}p_k}{2\pi}\frac{w}{t^2+w^2}$$

•For $w = 0 \Rightarrow t \to 1/t$ in Abelian case and $K_{ab} \to (K^{-1})_{ab}$ in the nonabelian case.

•In both cases, S duality $\to$ Maxwell duality in bulk. Acts the same for us.

•Conf. structure or generalized conf. structure $\to$ same form of correlators.

•Then, inversion $\Rightarrow$ $1 + 2\delta \ln p \to\simeq 1 - 2\delta \ln p$, so $\delta(\tilde{j}) = -\delta(j)$. Then $\delta(\tilde{j}) < 0$ and $\tilde{j}$ is relevant, as we wanted.

•Must $\exists$ vortex. Here: Abelian Dirac vortex. $\exists U(1) \subset SO(3)$ with

$$j_\mu = i \sum_{i=1,2} \vec{\phi}_i D_\mu \vec{\phi}_i + h.c.$$

under which $\phi_1^a \to e^{i\alpha}\phi_1^a$, $\phi_2^a \to e^{i\alpha}\phi_2^a$.

•Then, $\exists$ vortex ansatz tht keeps $V = 0$,

$$\phi_1^a = \phi_1(r) f^a e^{i\alpha}\,,\ \ \phi_2^a = \phi_2(r) f^a e^{i\alpha}$$

•Sol. of eq. of m. with ansatz $\to$ vortex nr. $\to$ vortex current.

•Thus, monopole solution also solved. All problems with Big Bang also solved, and CMBR fit as well as inflation!

•Reheating model: sketch of one available (HN, 2020), but more precise needed. Needs to reverse direction of flow of coupling: gravity is becoming stronger, but must eventually become weaker in order to transition to radiation domination.

•Lattice field theory calculation: test the matching to CMBR at low l ($l < 30$), and see whether inflation or holographic cosmology is better. Stay tuned!