Quantum systems as manipulators of information:

Entanglement, state complexity, and information scrambling

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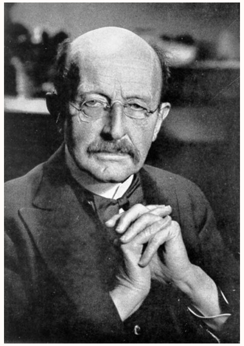
Max Planck Institute of Quantum Optics, Germany

Online talk@ NCSR Demokritos, *Institute of Nuclear and Particle Physics*

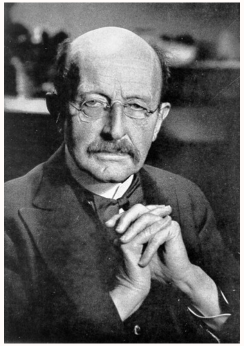
Friday, April 28, 2023

Quantum Information Science



Quantum Theory

Quantum Information Science



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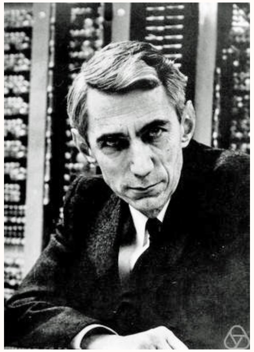

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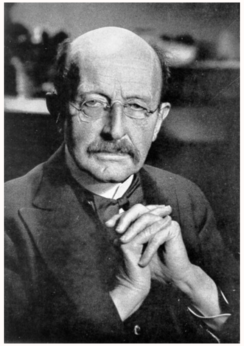

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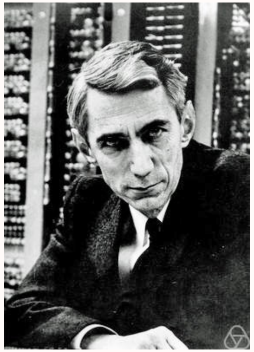

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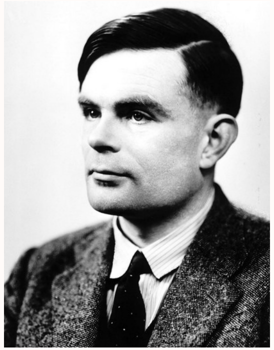

Computer Science

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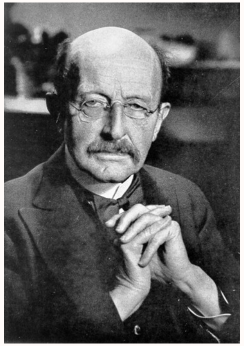

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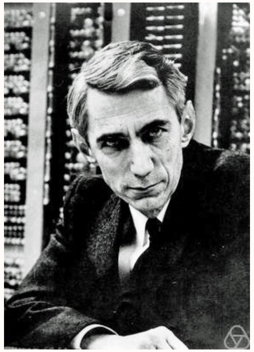

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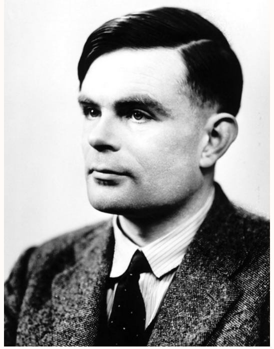

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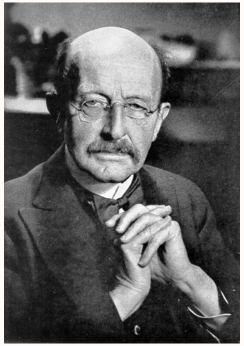

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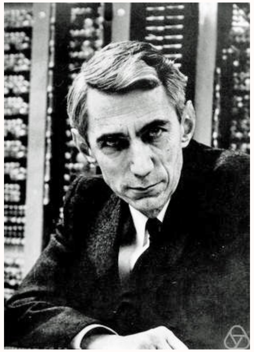

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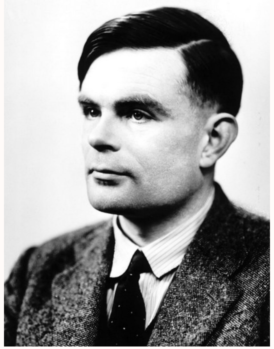

Quantum Theory + *Information Theory* + *Computer Science*

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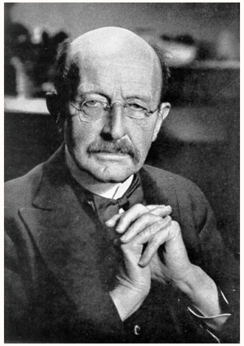

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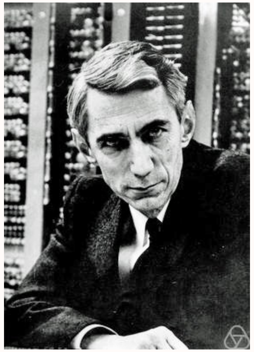

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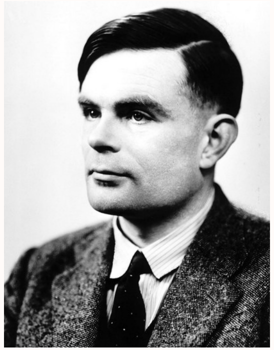

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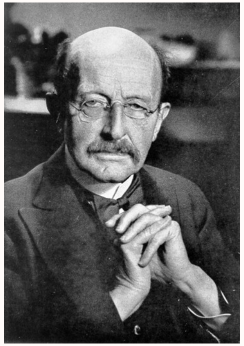

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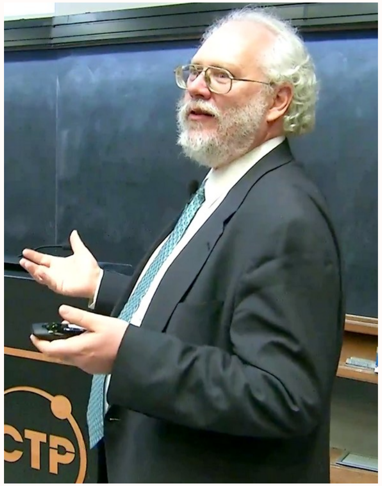

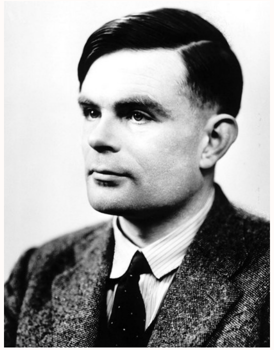

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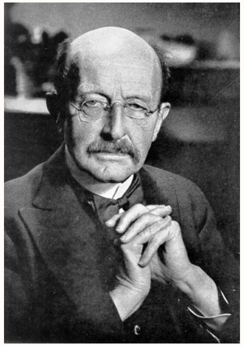

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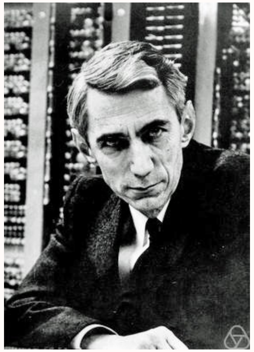

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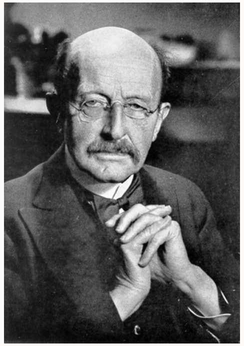

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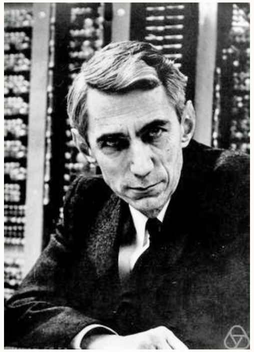

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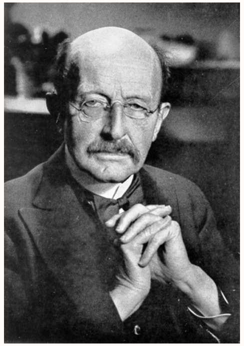

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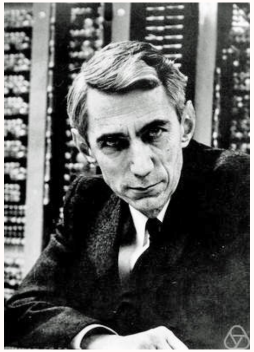

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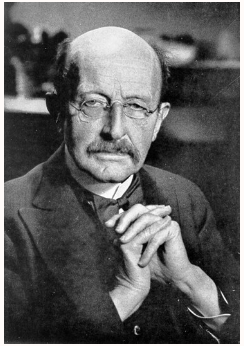

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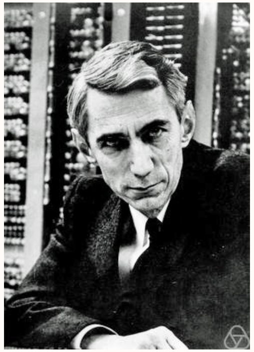

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- *Teleportation*: Shared ***entangled*** pair + 1 ***classical bit*** ⟶ Teleportation 1 classical bit !

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- **Correlations of quantum matter at T = 0 can be very complex!**

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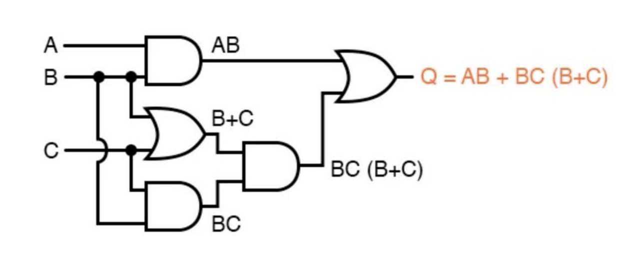

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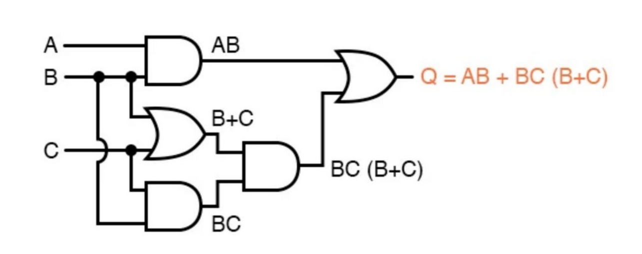

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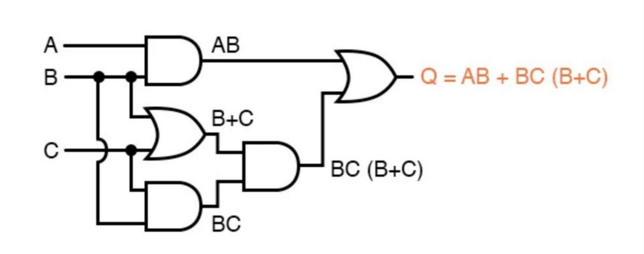

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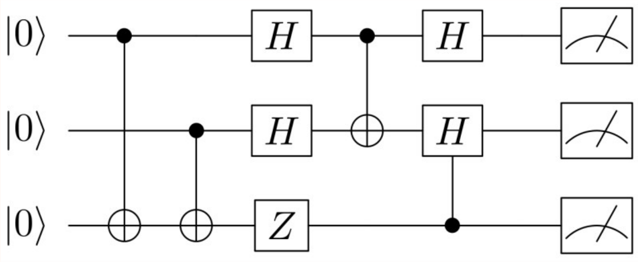

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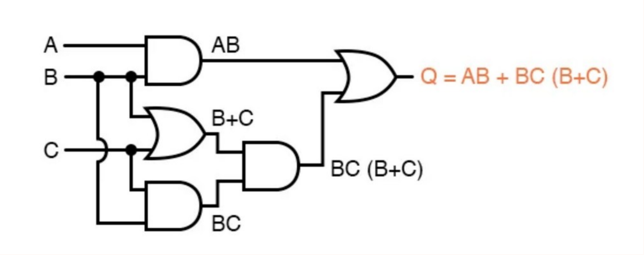

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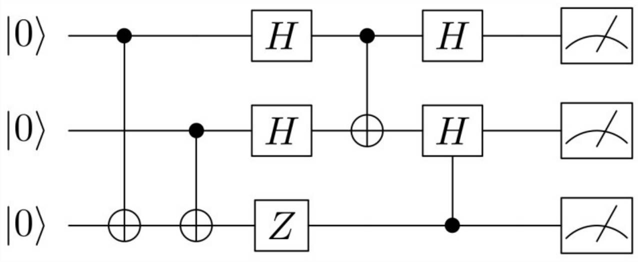

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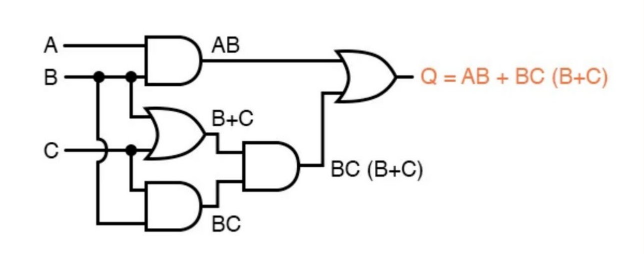

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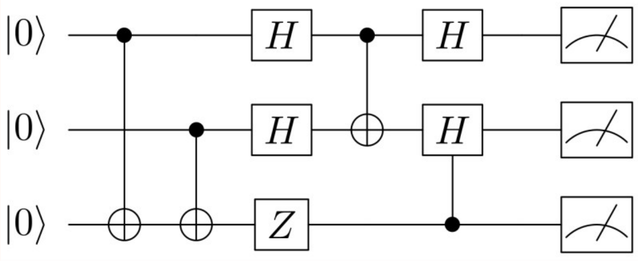

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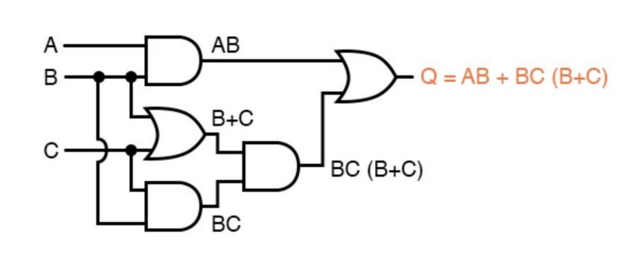

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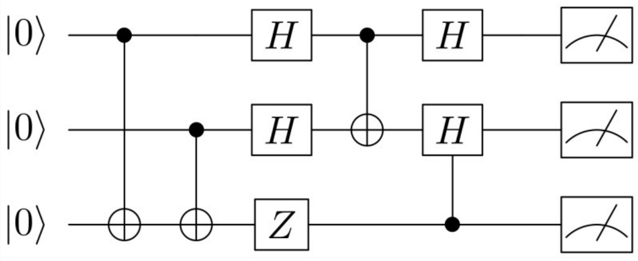

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<ins>**Complexity of a state:**</ins> Complexity of unitary generating target from a ***product state.***

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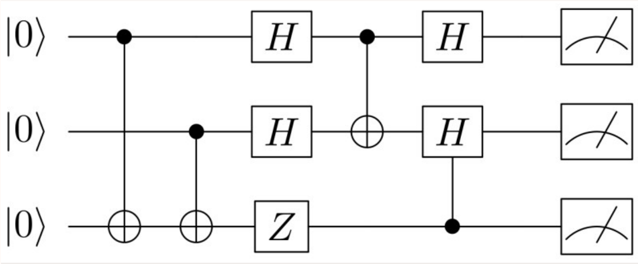

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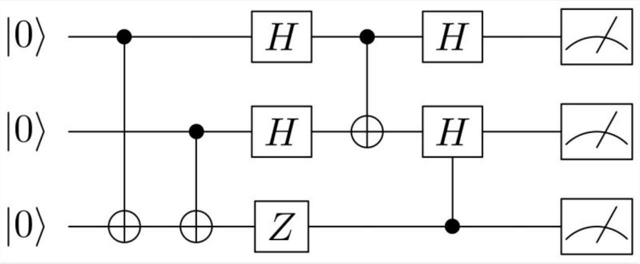

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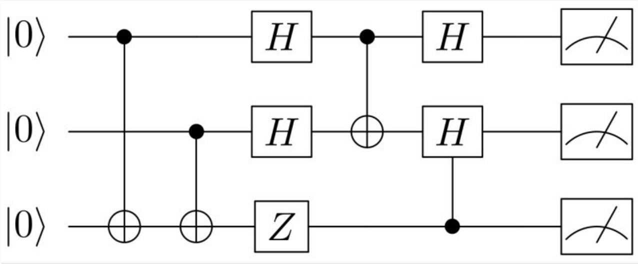

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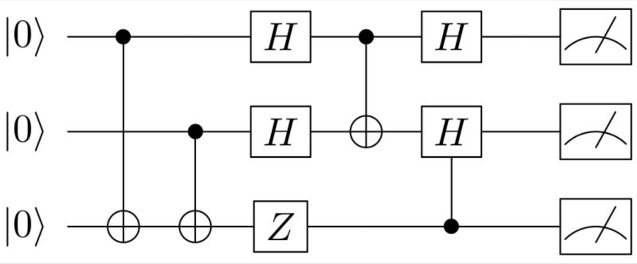

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Complexity is a statement about the structure of entanglement!

Entanglement as a Resource

In what sense is shared entanglement useful?

$$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right)$$

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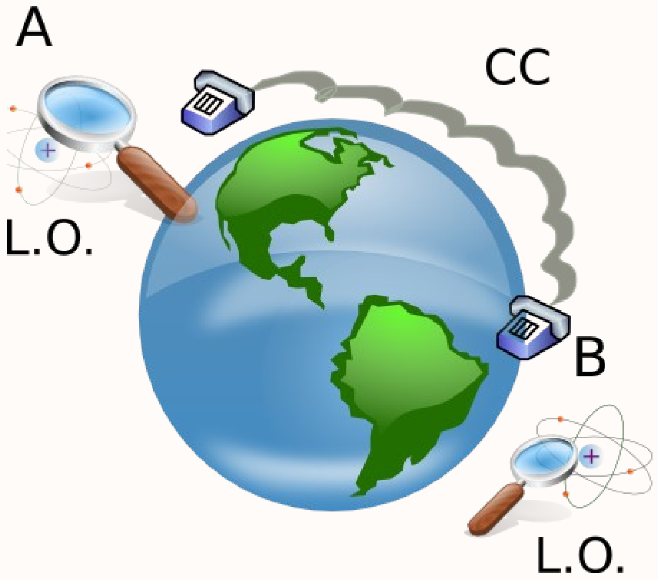

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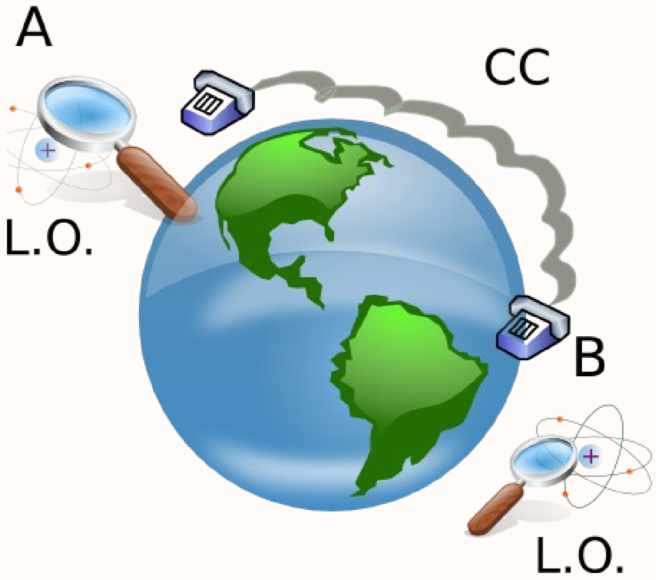

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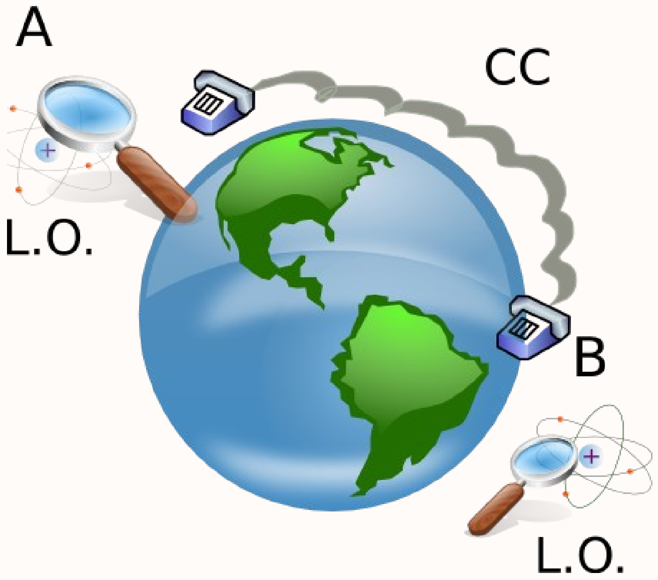

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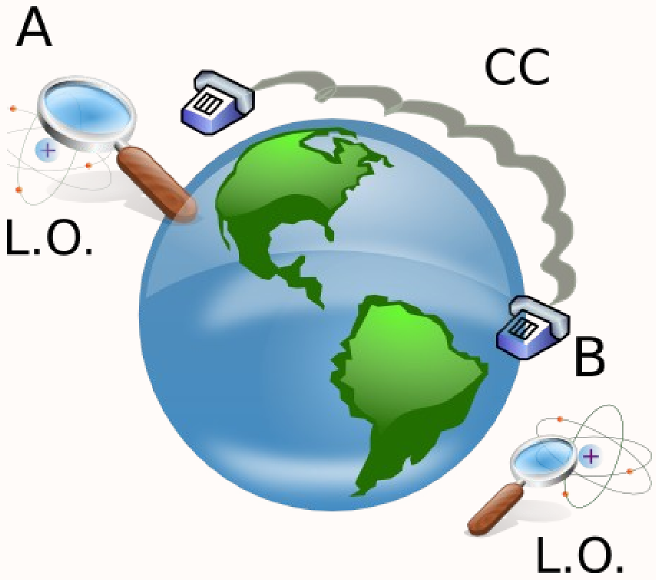

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- **Entanglement cannot be created from product states using these operations alone.**

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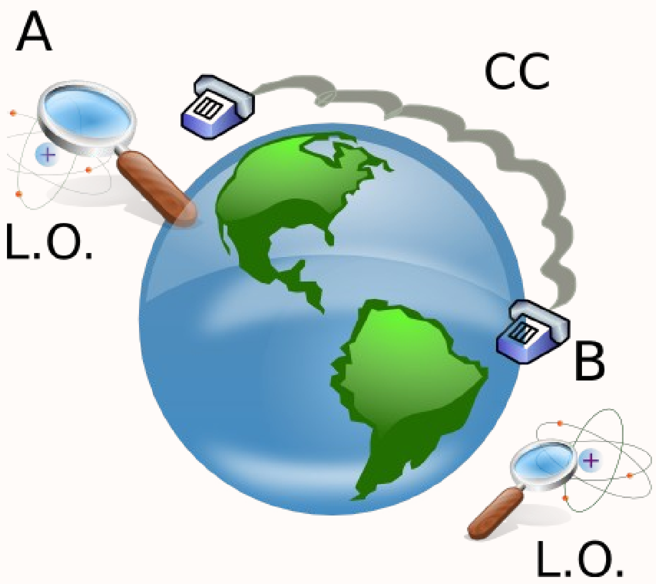

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Entanglement as a Resource

In what sense is shared entanglement useful?

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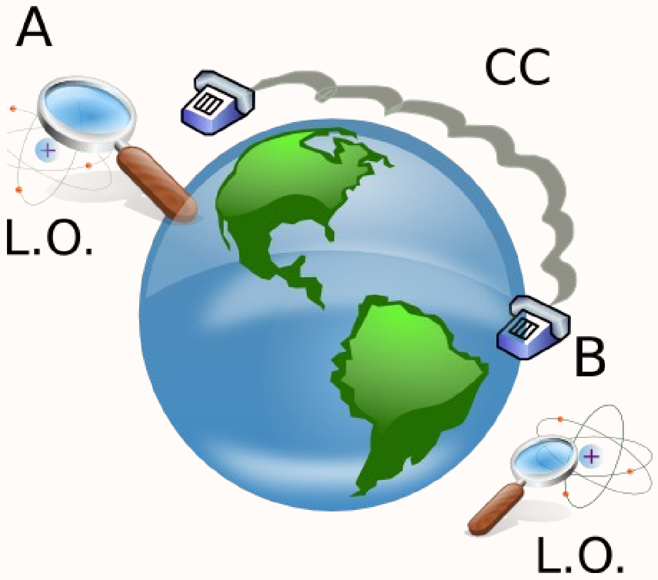

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- *Framework for teleportation, dense coding, ...*

Topological Phases:

Interplay of entanglement, complexity and locality

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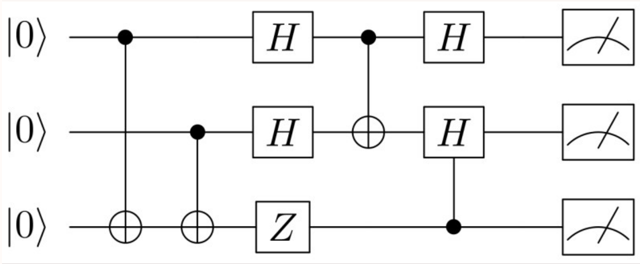

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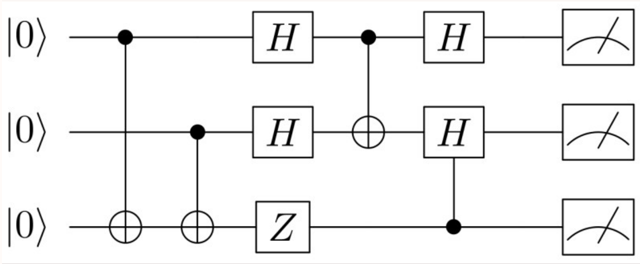

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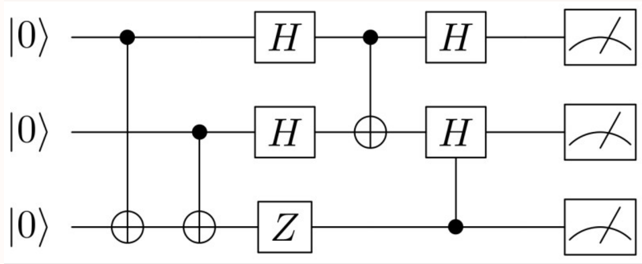

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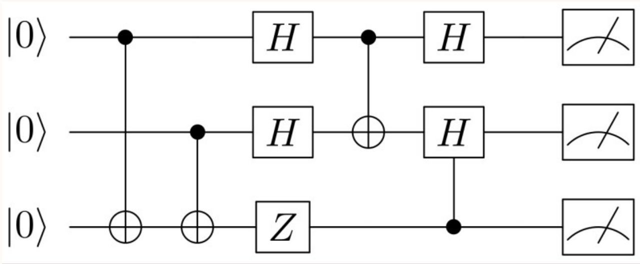

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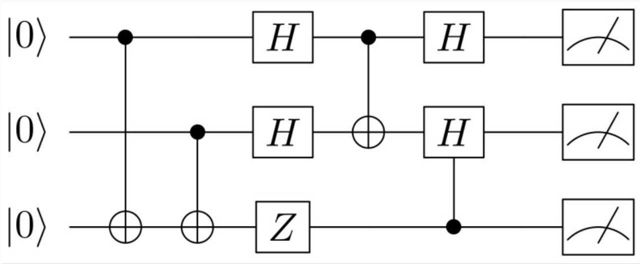

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- What is an easy operation? **<u>Constant depth local quantum circuit!</u>**

Classifying Topological Phases of Quantum Matter

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T = 0 Phase Transition

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- Measurements make possible to *overcome light cone of finite depth local circuits*
- In 2D, *some phases trivialize, others do not*! (*Tantivasadakarn et al. '21 & Bravyi et al '22*)

Entanglement and Scrambling of Quantum Dynamics

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Initial Product State → Unitary Evolution (interactions) → Entanglement

Unitary evolution retains all initial information, but it becomes locally inaccessible!
"Information scrambling"

Information Scrambling and the OTOC

- The *Out-of-Time-Order Correlator* (OTOC) (Kitaev '15) probes **scrambling**:

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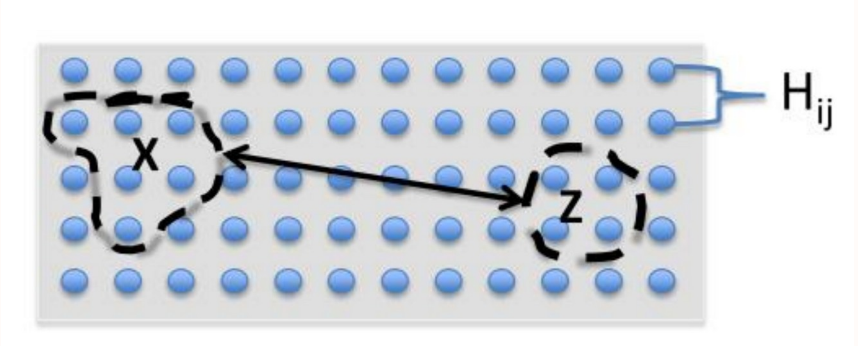

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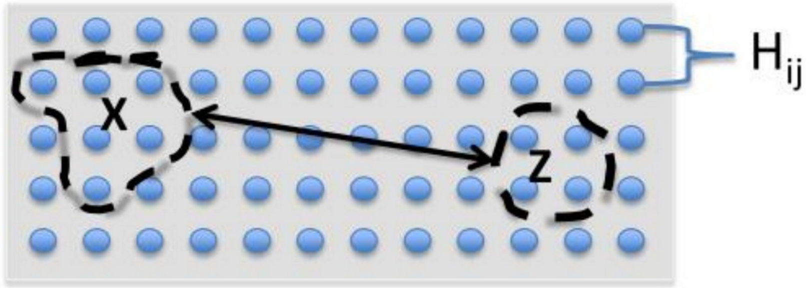

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- **Scrambling**: For "*typical*" and **local operators** *V* and *W*, *V(t)* will spread within the light cone and thus may fail to commute with *W(0).*

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Quantum Chaology, Not Quantum Chaos

Michael Berry

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Received September 15, 1988 accepted October 25, 1988

Scrambling in Bipartitions = Operator Entanglement

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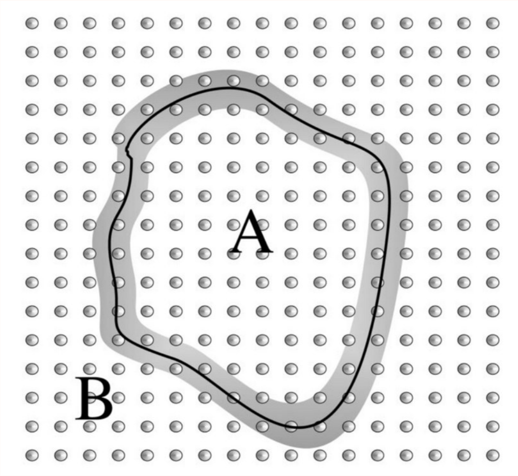
  Pick a region A and let B be its complement.



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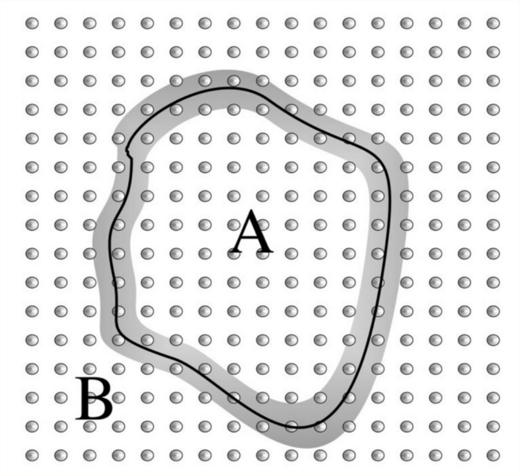

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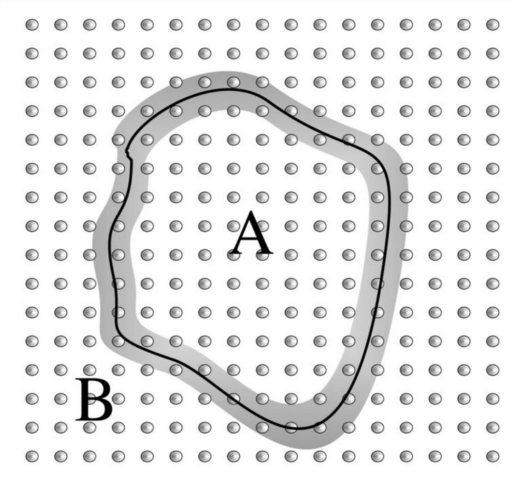

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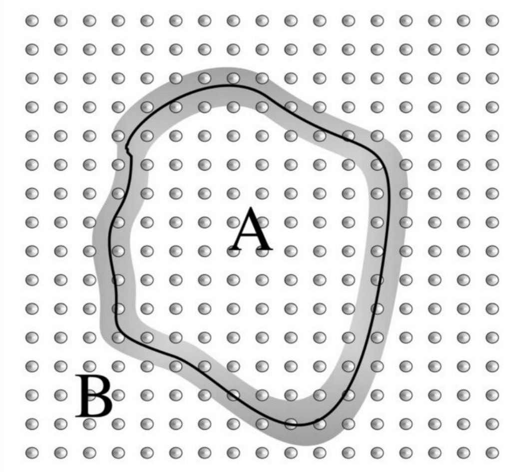

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Theorem (GS, Anand, Zanardi '21): The average OTOC is exactly equal to the operator entanglement of the evolution U. Deviations from the average value are exponentially suppressed.

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