Magnetometer Theory

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Outlines

Introduction

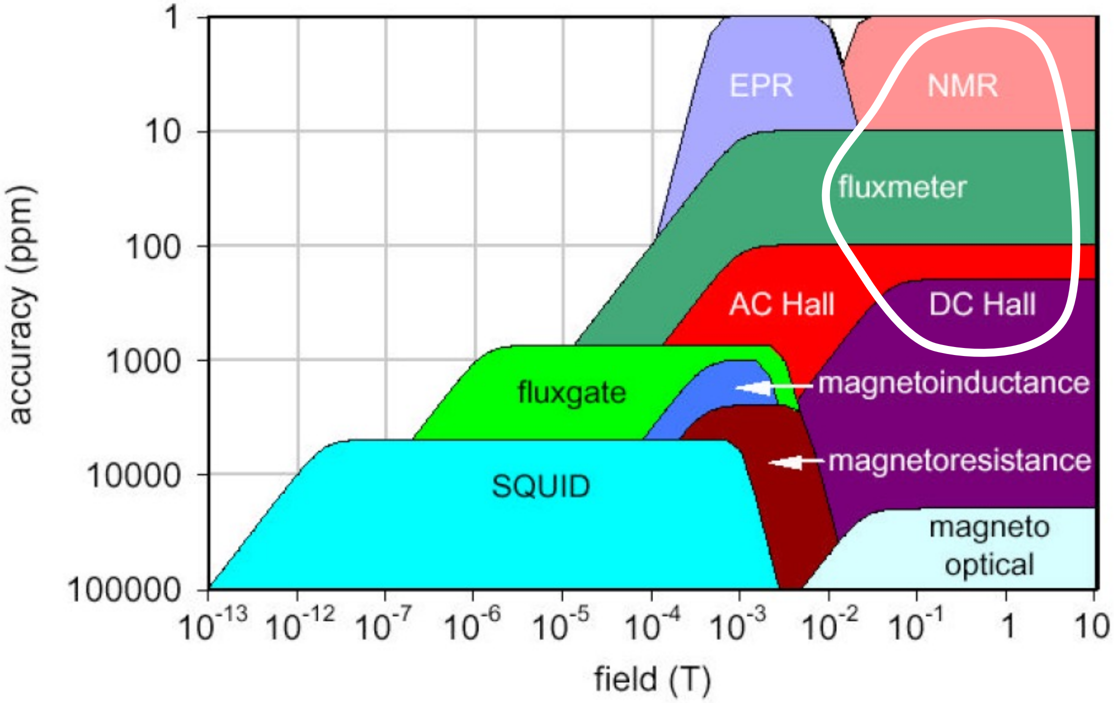
- Magnetic field measurement
  - ✓ Final diagnostic to verify the conformity to magnet requirements
    - ✓ Measurement at Room temperature, and Cryogenic temperature
      - ✓ Series production : might be done only at Room temperature
      - ✓ one-of-a-kind magnet : done at both R.T. and C.T.

- Magnetometer
  - – Device to measure the magnetic field (Amplitude and/or Direction)
    - Important and essential device for magnet diagnostic

✓ *No all-purpose magnetometer that can measure magnetic field amplitude and direction simultaneously with high accuracy and resolution*

Magnetometer vs Accuracy/Range

- Many types of magnetometers



L. Bottura (2002) Field Measurement Methods. Presentation at CERN Accelerator School (CAS) on Superconductivity, Erice, May 8–17, 2002.

- First step for magnetic field measurement
  - Select magnetometer according to purpose
    - Amplitude only or vector, how much accuracy, point-wise or average over volume,……

Outlines

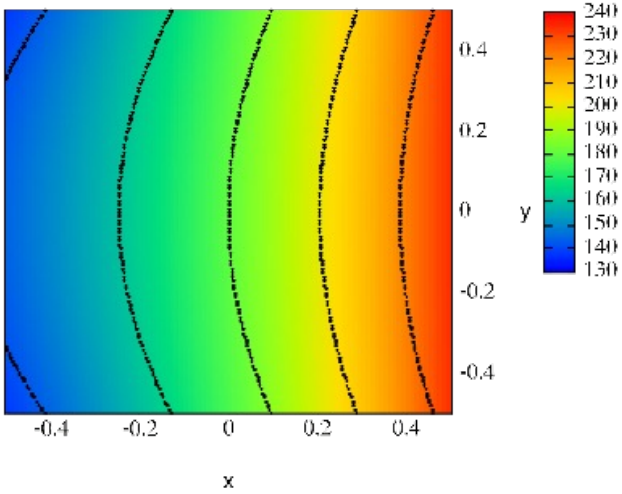
1. Introduction
2. 2D and 3D description of magnetic field
3. Typical magnetometers
4. Calibration of magnetometers
5. Summary

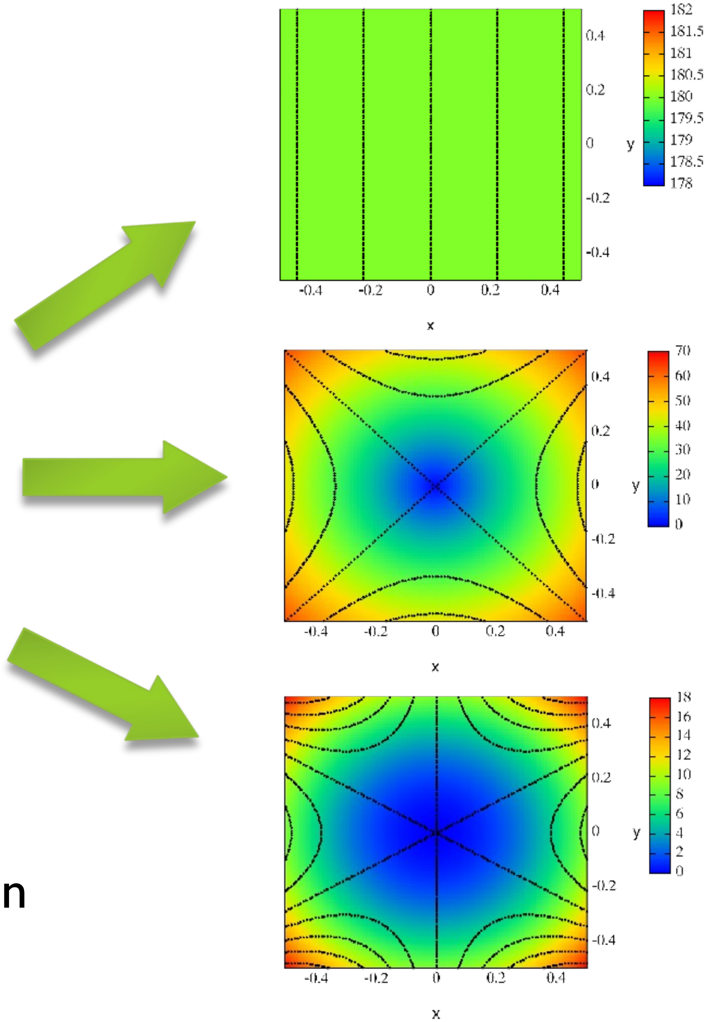
Description of magnetic field quality

- 2D : Multipole expansion
- 3D : Spherical harmonic expansion

<u>Easy to discuss/evaluate field quality</u>

ex) 2D field map





Dipole
(~bending)

Quadrupole
(~focusing)

Sextupole
(~chromaticity
correction)

✓ Decomposition of field distribution
  - Easy to find error field
  - Easy to discuss how to compensate error field

2D multipole expansion

- SC magnets for accelerator
  - Long compared with aperture -> 2D description is valid for most part of acc. magnet

Magnetic field component :

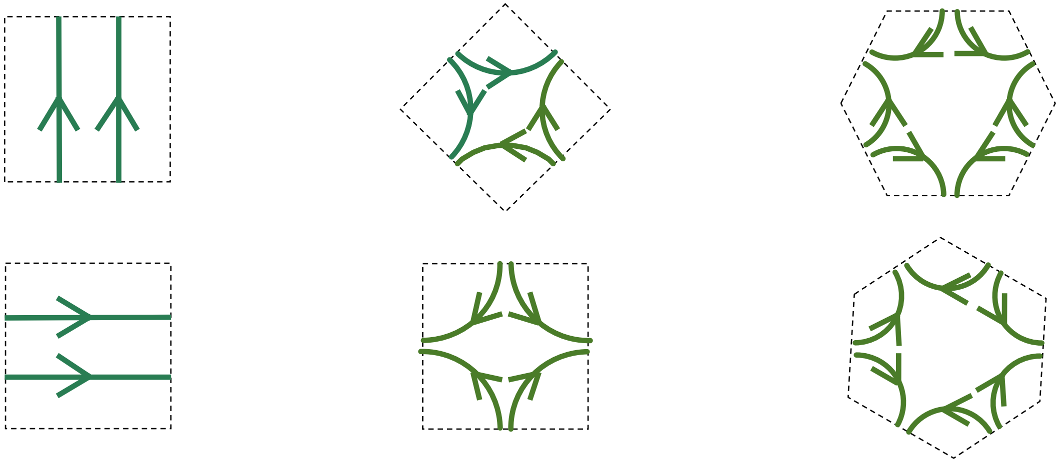
$$\boldsymbol{B} = B_y + iB_x = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{x + iy}{r_0} \right)$$

*No current in the region

expanded in Taylor series

CERN Accelerator school proceedings
https://cdsweb.cern.ch/record/503603/files/CERN-2004-008.pdf

	Dipole n=1	Quadrupole n=2	Sextupole n=3
Normal : B$_n$			
Skew : A$_n$			



3D spherical harmonic expansion

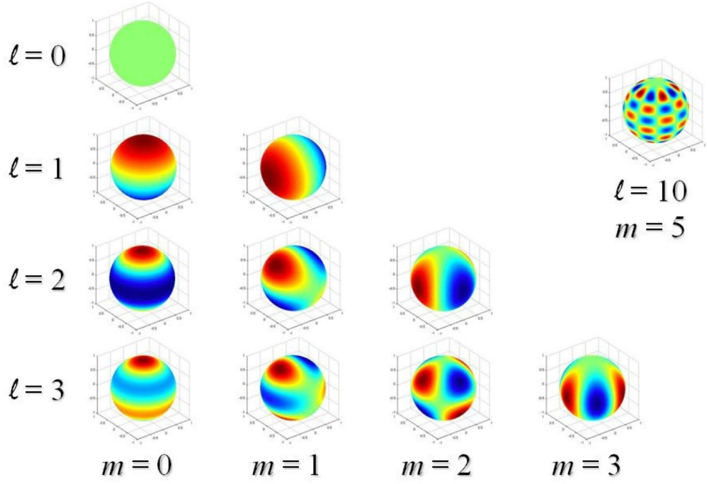
- Harmonic expansion in Spherical volume
- Magnet for Magnetic Resonance Imaging (MRI)
  - Need several ppm uniformity of magnetic field within 40~50 cm DSV (Diameter Spherical Volume)

- Description in polar coordinate

$$H_z(r,\theta,\varphi) = \sum_{n=1}^{\infty}\sum_{m=0}^{n-1} r^{n-1}(n+m)\mathrm{P}_{n-1}^{m}(u)(A_n^m \cos m\varphi + B_n^m \sin m\varphi)$$

- Description in cartesian coordinate

$$\begin{aligned}
H_z = {} & A_1^0 + 2A_2^0 z + \frac{3}{2}A_3^0(2z^2 - x^2 - y^2) + A_4^0 z(4z^2 - 6x^2 - 6y^2) + \\
& + 3A_2^1 x + 12A_3^1 xz \qquad + \frac{15}{2}A_4^1 x(4z^2 - x^2 - y^2) \\
& + 3B_2^1 y + 12B_3^1 yz \qquad + \frac{15}{2}B_4^1 y(4z^2 - x^2 - y^2) \\
& + 15A_3^2(x^2 - y^2) \qquad + 90A_4^2 z(x^2 - y^2) \\
& + 30B_3^2 xy \qquad + 180B_4^2 xyz \\
& \qquad + 105A_4^3 x(x^2 - 3y^2) \\
& \qquad + 105B_4^3 y(3x^2 - y^2)
\end{aligned}$$



(b) Spherical harmonic functions

I. Sahoo, et al., “A Test for Isotropy on Sphere using Spherical Harmonic Functions,” Statistica Sinica, **29**, pp.1253-1256, 2019. DOI:10.5705/ss.202017.0475

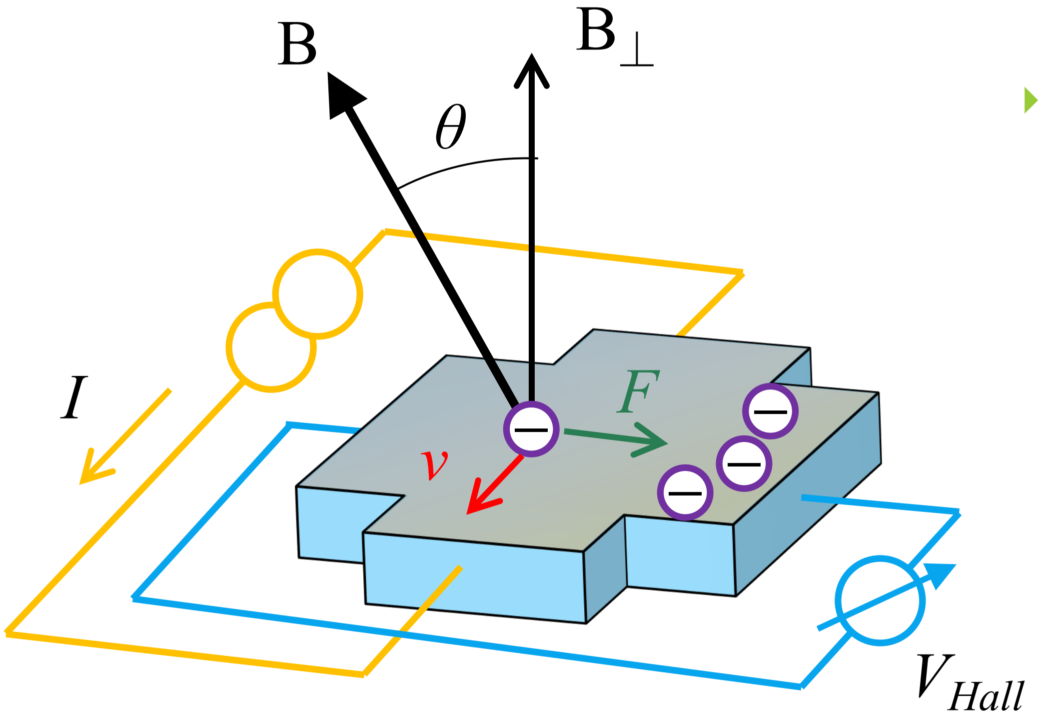
Typical magnetometers

1. Hall sensor
2. Fluxmeter
3. NMR magnetometer

Hall sensor ~ Principle

- **Utilize Hall effect** (discovered by E. Hall 1879)
  - electric current flows through a conductor in B
  - Transverse force on moving charge carriers
  - Build up of charge balances magnetic influence

Producing a measurable transverse voltage



- Conductor
  - Mostly Semiconductor
    - InSb
    - GaAs
    - InAs

very commonly used device

F : Lorentz force $= v \times B$

Hall sensor ~ Output voltage

$$V_{Hall} = G\, R_H I\, B\cos(\theta)$$

G : Geometry factor
I : Current
$B\cos(\theta)$: B perpendicular to the sensor surface
R_H : Hall coefficient depending on material

- Sensitivity
  - InSb > InAs > GaAs
- Temp. characteristic
  - GaAs > InAs > InSb

Hall sensor ~ Benefits/Limitation

- Benefits
  - Simple & robust solid-state device
    - easy to use
  - Detect both amplitude and direction of B
  - Small size : ~ 0.1 mm
  - Wide range of sensitivity
    - 1µT – higher than 10T
  - Wide temperature range
  - Faster measurement
    - DC – hundreds kHz
  - High availability
    - commercial accuracy : < 1 %

- Limitations
  - Non-linearity
    - depends on material and shape
  - Drift with time
    - aging effect
  - Temperature dependence
    - Quantum effect at cryo. temp.
  - DC offset
  - Planar Hall effect

Hall sensor ~ Quantum effect

- Shubnikov-de Haas effect
  - at cryogenic temperature
- Oscillation of Hall coefficient as a function of 1/B

depending on material

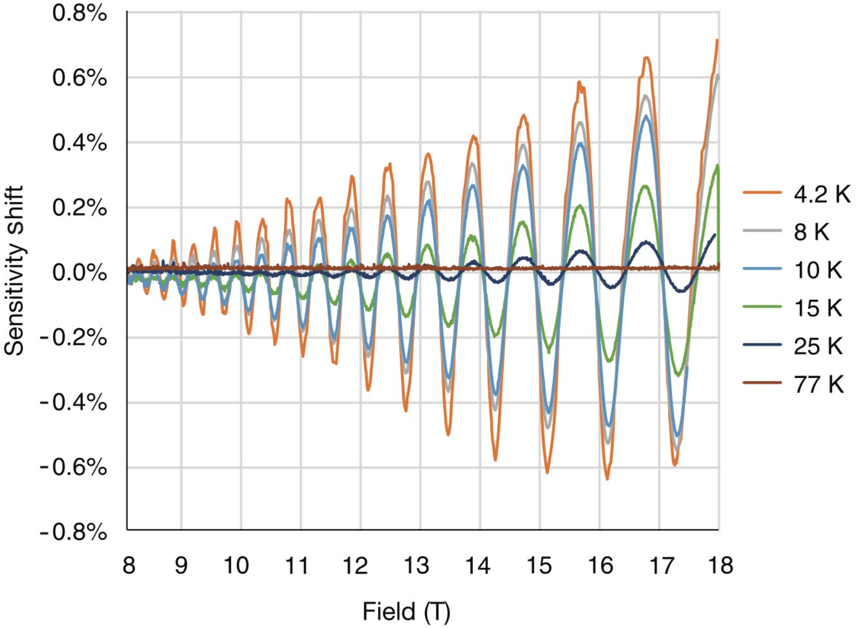


Figure 2: *The SdH effect causes quantum oscillations for high fields at cryogenic temperatures. The exact shape of these oscillations is difficult to predict, so they are not easily compensated for and should instead just be accounted for.*

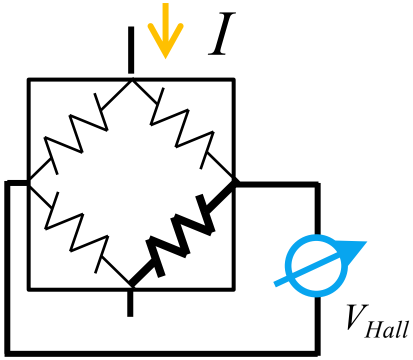
Lake Shore Cryotronics, Inc., Application note "Magnetic field measurement at cryogenic temperatures" : https://www.lakeshore.com/docs/default-source/240-measurelink/magnetic-field-measurement-at-cryogenic-temperatures.pdf

<u>Difficult to predict the exact shape</u>

✓ Need careful calibration at operation temperature and B

Hall sensor ~ DC offset

- Apparent voltage appears even in the zero magnetic field due to
  - Structural asymmetry (Shape error, material inhomogeneity etc..)
  - Alignment errors of the sensor contacts



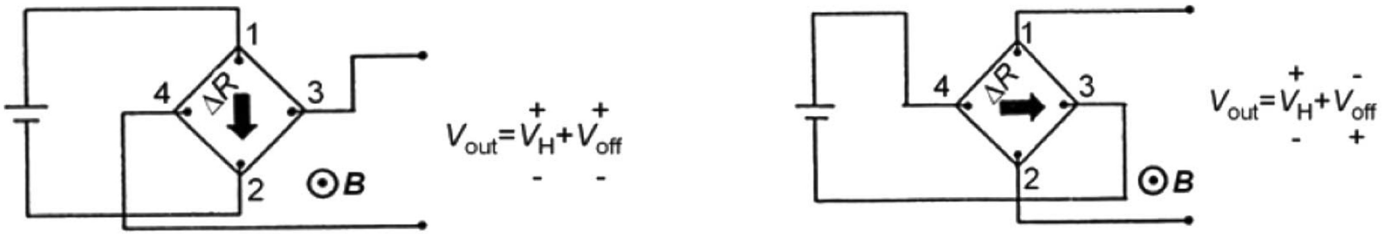
Unpredictable before using

- And also, offset will change with time and temperature

✓ Frequent calibration in a zero-field chamber

✓ Special compensation technique

◆ Spinning-current method



Phase 1

Phase 2

D. Popovic et al., "Three-axis teslameter with integrated Hall probe," IEEE Trans. Instrum. Meas., **56**, pp.1396-1402, 2007

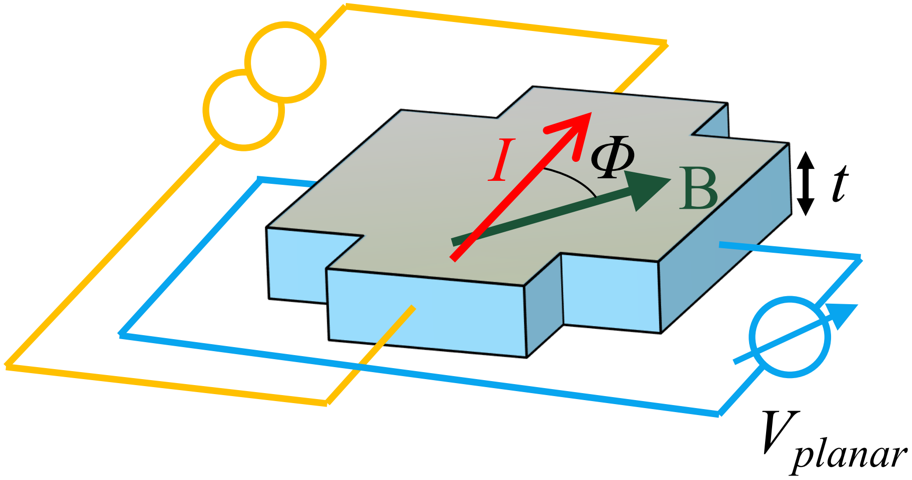
Hall sensor ~ Planar Hall effect (1)

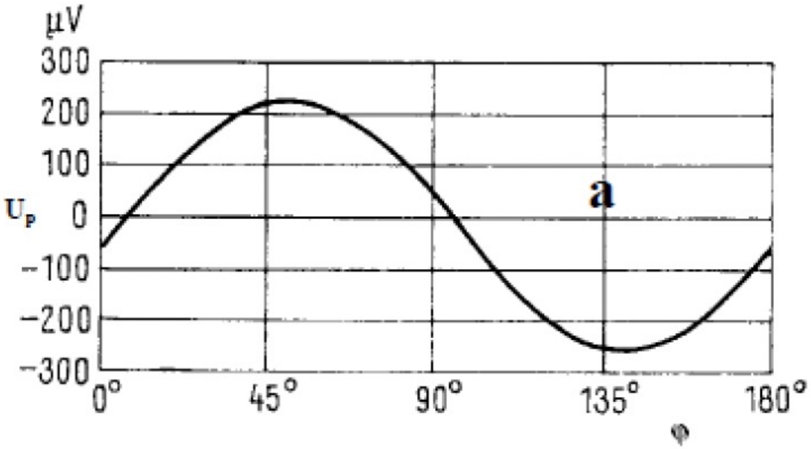
- Voltage when magnetic field is in-plane

C. Goldberg and R. E. Davis, New galvanomagnetic effect,
Phys. Rev. 94, pp. 1121–1125, 1954.

$$V_{planar} = \frac{P_H I B^2}{2t} \sin(2\emptyset)$$

P_H: planar coefficient



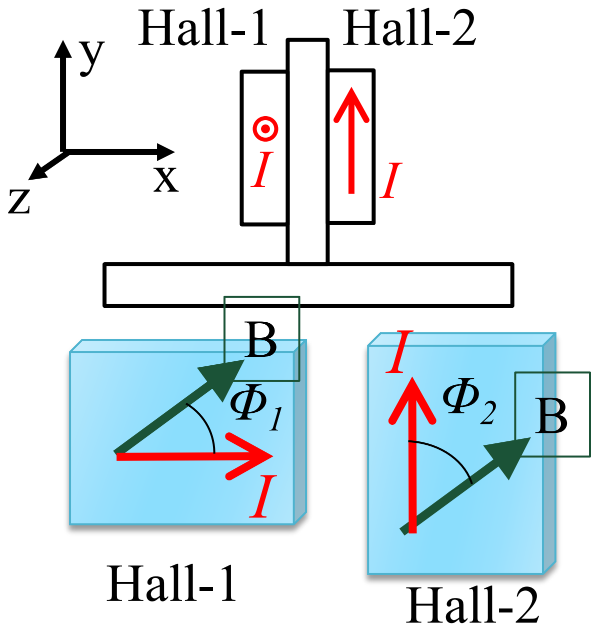


J. Kvitkovic, Hall generators, CERN Accelerator School on Measurement and Alignment of Accelerator and Detector Magnets, Anacapri, Italy, 1997, CERN-98-05, 233–249.

Hall sensor ~ Planar Hall effect (2)

- Introduce significant systematic error in the following meas.
  - Measurement of weak B field perpendicular to main strong B field
  - 3D field mapping using 3-axis Hall probe

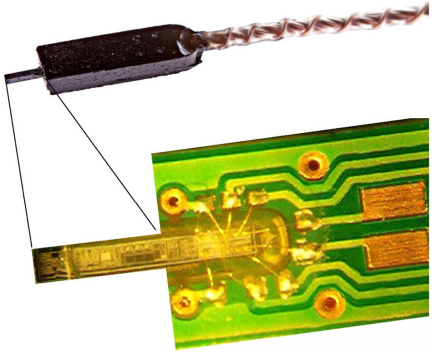
- Compensation method
  - Two sensors
  - Orthogonal directions of Hall currents
  - Add two outputs to cancel V_{planar}



$\Phi_2 = 90° - \Phi_1$

S. I. Redin et al., "Radial magnetic field measurements with a Hall probe device in the muon (g-2) storage ring magnet at BNL," Nuclear Inst. Meth. A, **473**, pp.260-268, 2001

✓ Recent commercial device has offset and planar effect cancelation.



Integrated 3D Hall probe

Standard accuracy : 0.1 %

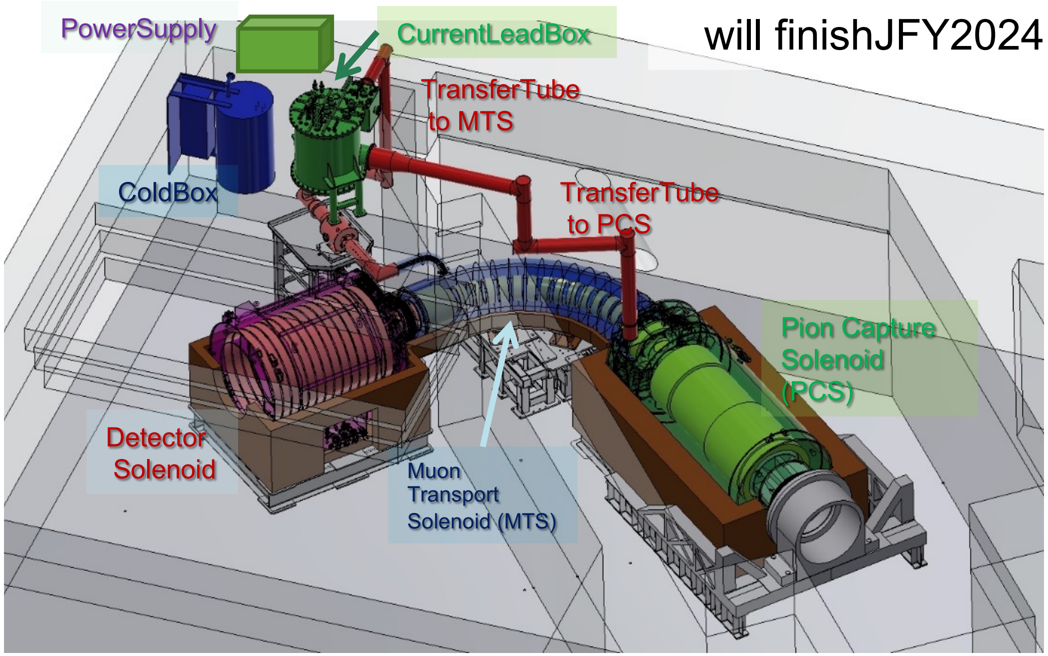
D. Popovic et al., Three-axis teslameter with integrated Hall probe, IEEE Trans. Instrum. Meas, Vol. 56, pp. 1396–1402, 2007

Hall sensor ~ Application

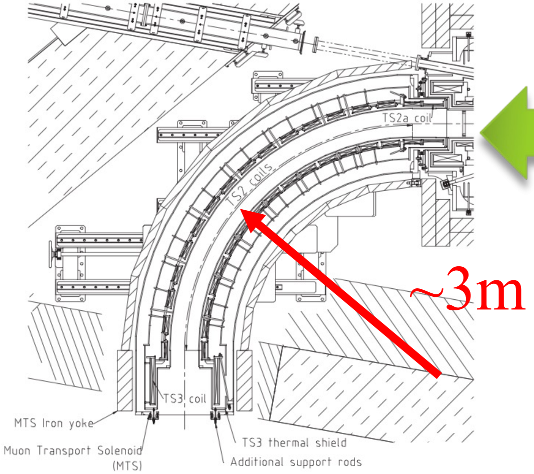
- Field mapping system for COMET magnets

	Bmax
Pion capture solenoid	: 5T
Muon transport solenoid	: 3T
Bridge solenoid	: 1.6T
Detector solenoid	: 1T

Under construction
will finishJFY2024



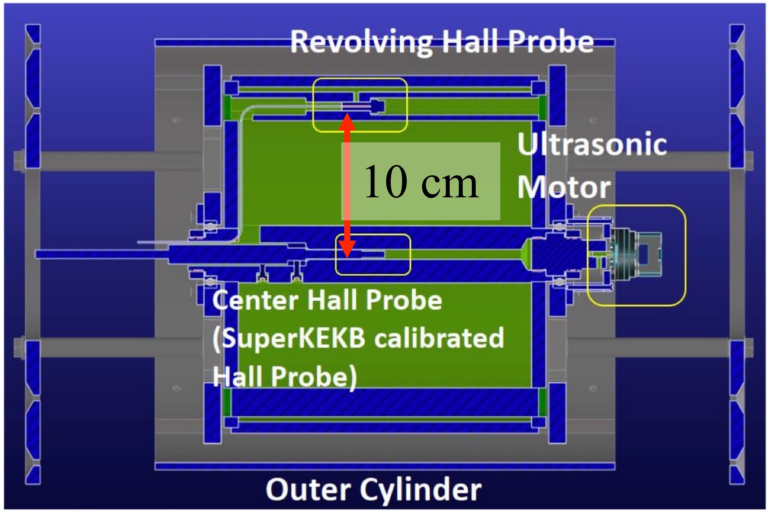
- Transport solenoid
  - Curved solenoid with dipole magnet
    - Solenoid field
      - ☐ 3.0 T at 210 A
    - Dipole field
      - ☐ 0.06 T at 175 A



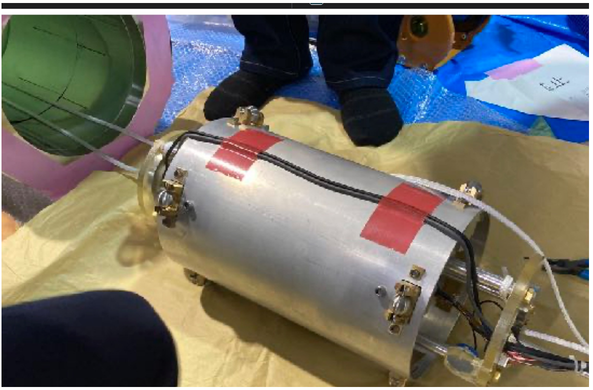
View from upstream

Hall probe unit for COMET muon transport solenoid

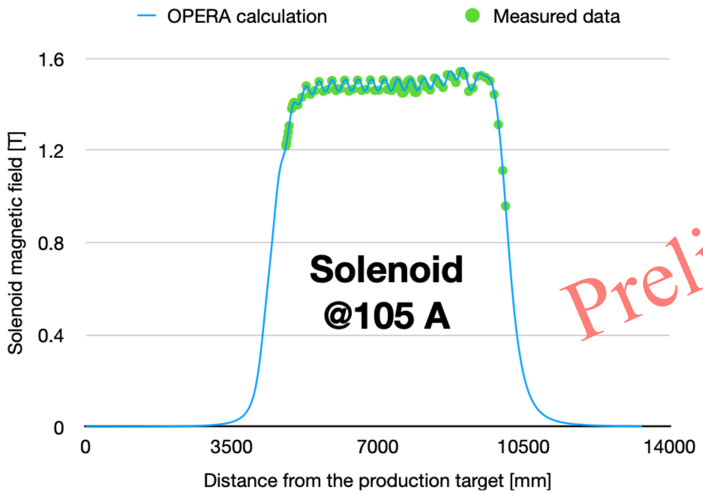
- Probe unit



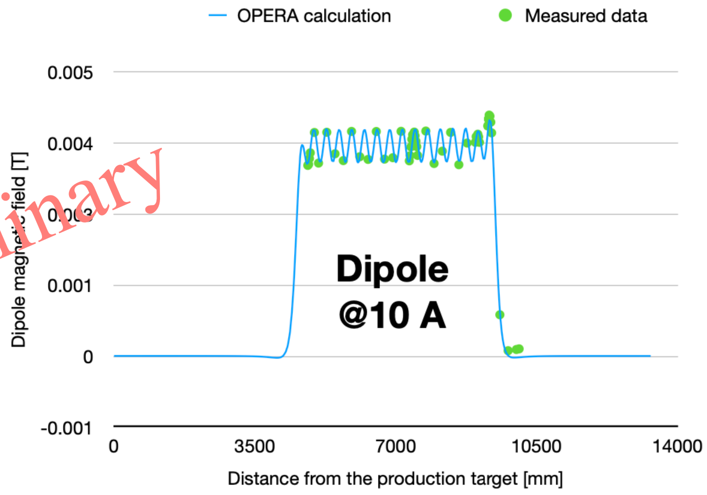
Hall probe unit



- Probe unit
  - Two 3-axis probes : on axis, off axis
  - Probes can rotate around z axis
  - Unit moves along magnet bore



Preliminary



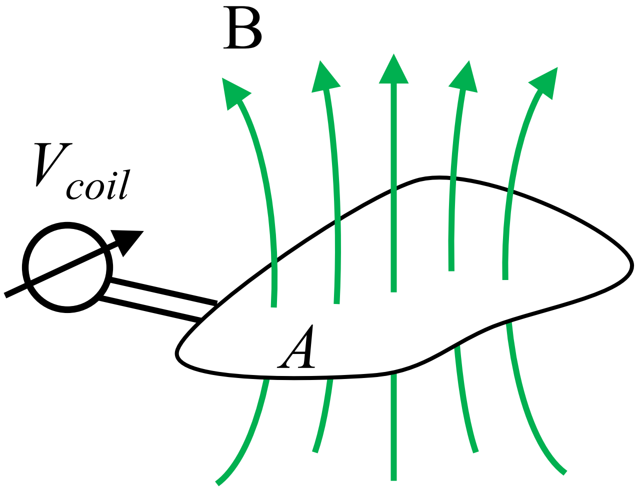
N. Sumi et al., IEEE Trans. Appl. Supercond., Vol. 33, 2023, 4500205. DOI: 10.1109/TASC.2023.3247679

Typical magnetometers

1. Hall sensor
2. Fluxmeter
3. NMR magnetometer

Fluxmeter ~ Principle

- Direct application of Faradays' low
  - Since 19 centuries
  - Commonly used even now
    - essential device for accelerator magnets



Coil induced voltage : $V_{coil} = -\frac{d\Phi}{dt}$,

A : effective area of coil

Total flux penetrating coil : $\Phi = \int_A \mathbf{B} dA$

Change in flux density : $\Delta\mathbf{B} = \frac{-\int_{t_{start}}^{t_{end}} V_{coil} dt}{A}$

- B : inversely proportional to *A*

Fluxmeter~ Benefits/Limitation

- Benefits
  - High flexibility
    - Coil geometry can be changed according to magnet geometry and magnetic field to be measured
  - Adjustable sensitivity
    - By changing the number of windings
  - High accuracy : 0.01 ~ 0.001 %
    - according to precision of coil geometrical factor (effective area)
  - Wide temperature range
    - from Cryogenic to very high temperatures
    - small temperature dependence

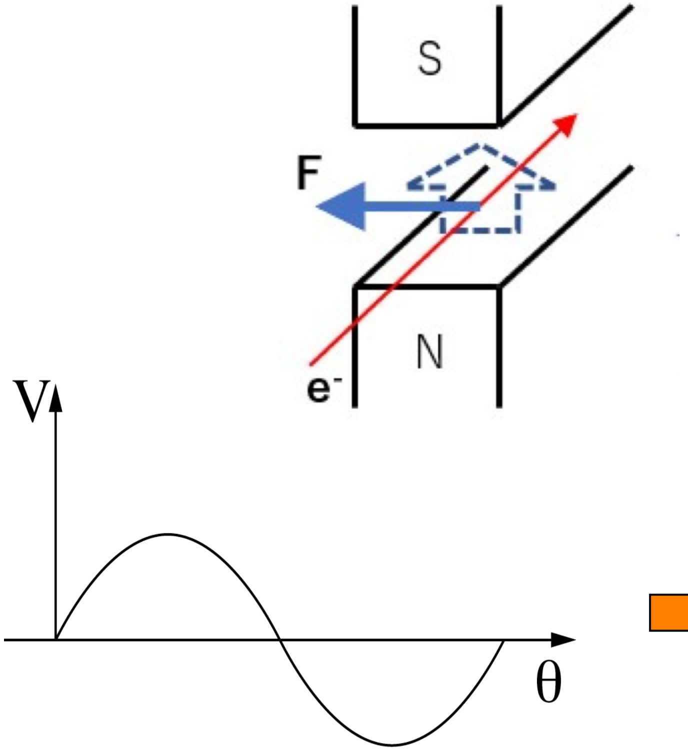
- Limitations
  - Must design the custom coil for each task or magnet geometry
  - Relative measurement
    - for absolute DC measurement
      - Offset calibration at B=0
      - Use symmetric motion
        - rotating coil or flip coil
  - Need high technological devices
    - voltage integrator
      - low noise, drift cancelation, ...
    - precise motion control of coil for DC field measurement

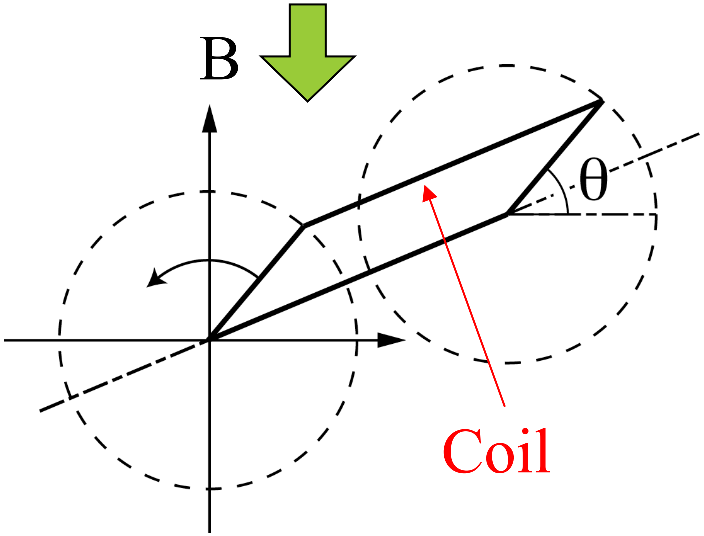
Fluxmeter ~ Application for accelerator magnets

Details and Practical application -> see Next Lecture by L. Fiscarelli

- Rotating coil ~ Harmonic coil
  - This method goes well with multipole field expansion
  - Standard method to check field quality of accelerator magnet

Dipole magnet



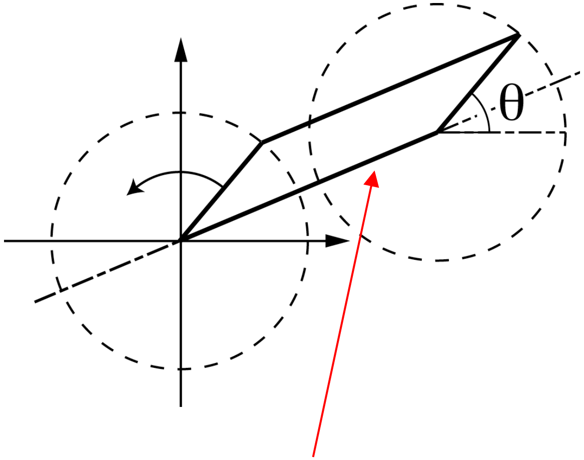


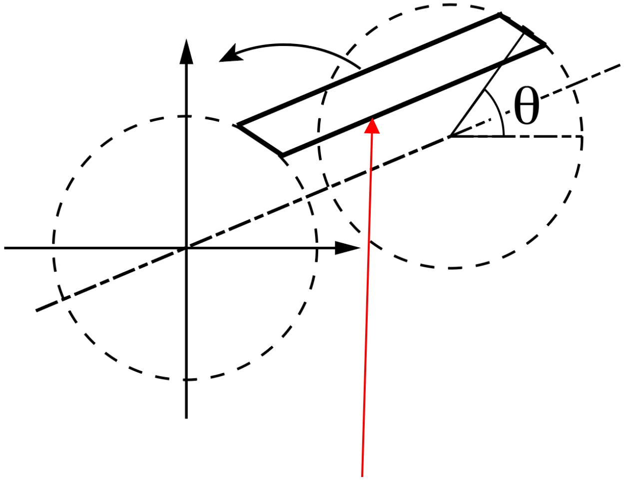
Integrate & Frequency analysis (FFT)

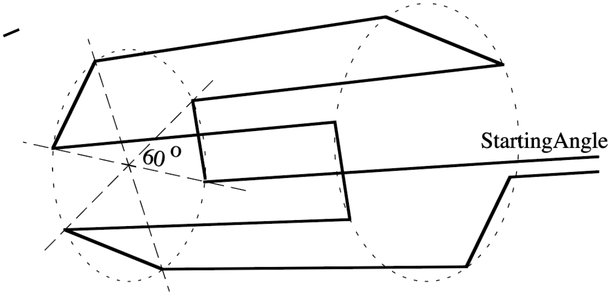
to get multipole components of B

Fluxmeter ~ Application for accelerator magnets

- Three typical winding patterns for harmonic coil







Sextupole coil

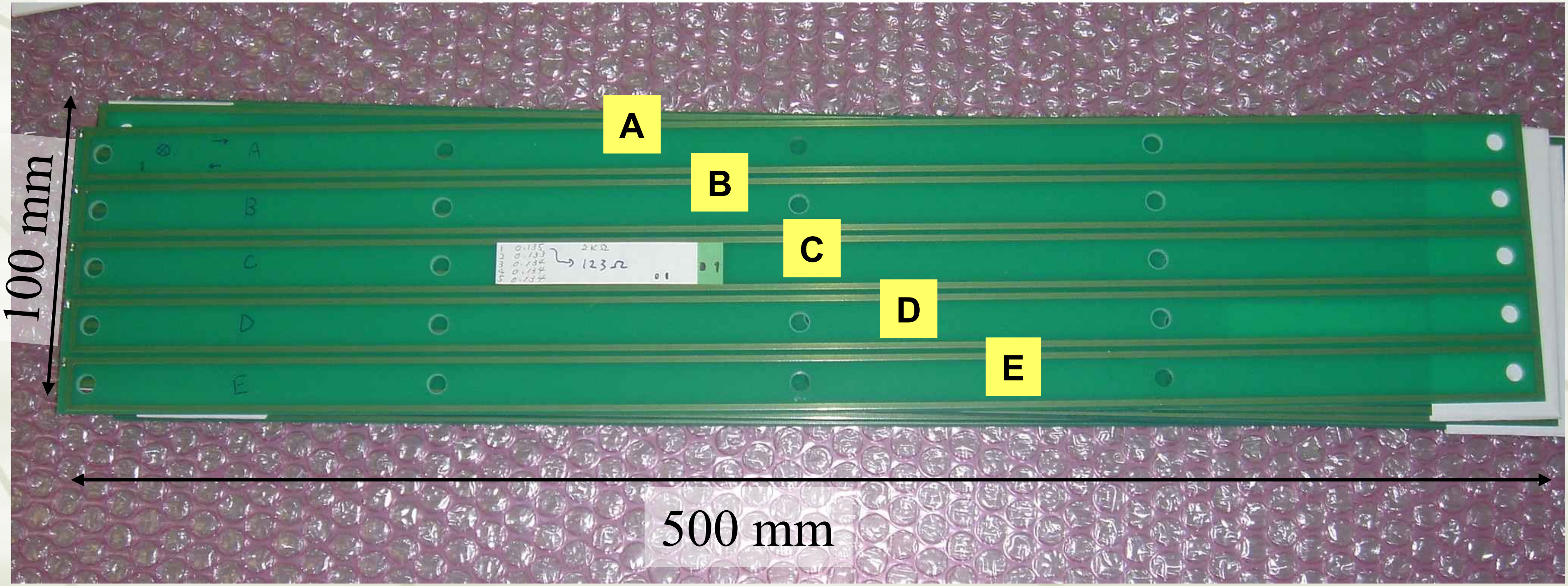
- ✓ Radial coil
- ✓ Tangential coil
- ✓ Morgan coil (Multipole coil)

- Harmonic coil for accelerator magnet consists of several coils
  - Main coil (radial or tangential)
    - measure all components of B
  - Bucking coils (radial or multipole)
    - cancel main component of B in order to increase sensitivity for higher order harmonics

Harmonic Coil example

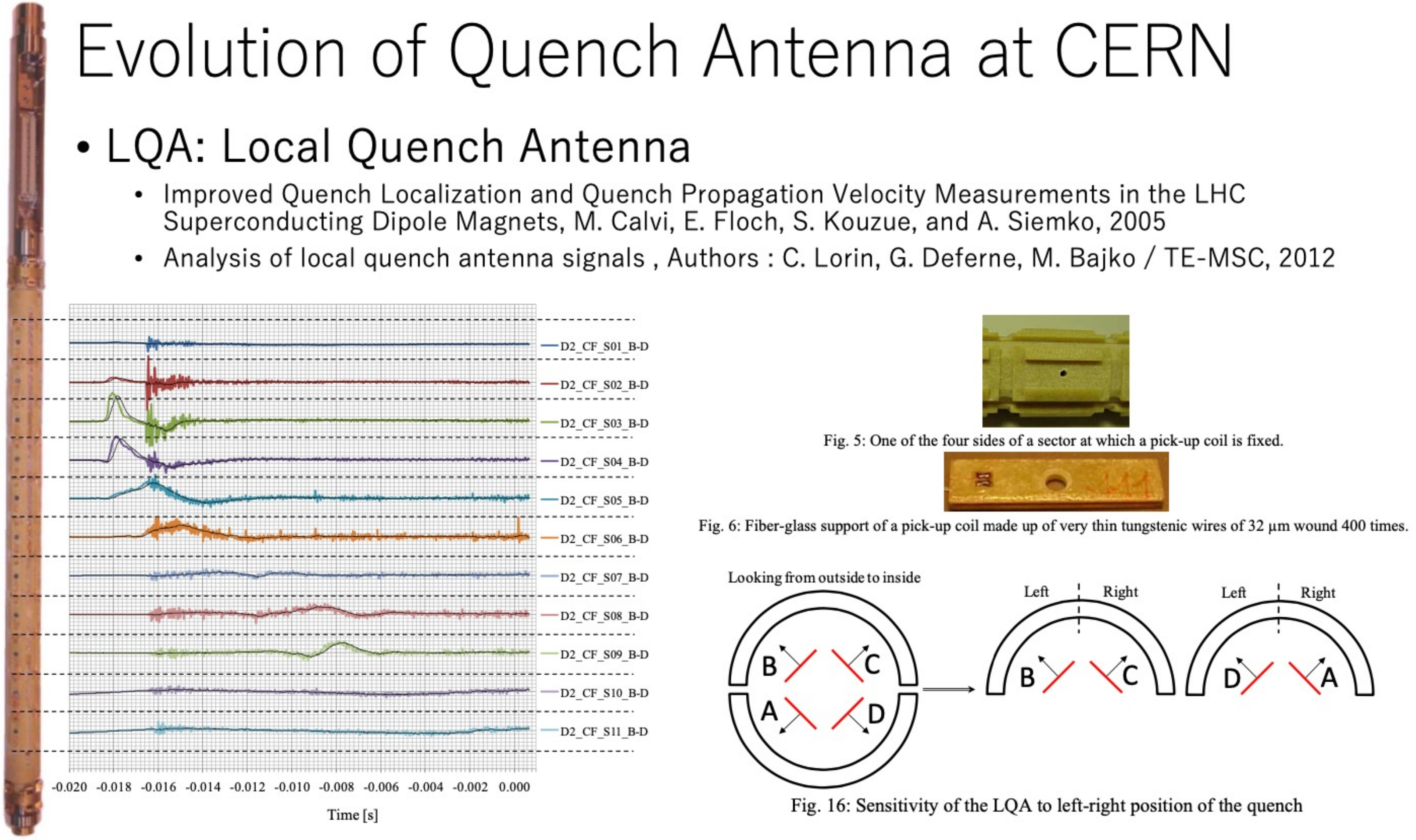
Radial coils for SC combined function magnet for J-PARC NU beam line

- 5 Radial Coils; Coil A E
  - Coil E+B-D-C for dipole and quadrupole bucked signal
  - Coil B-D for dipole bucked signal
  - Coil E and C are take as single coil
    - Use C for dipole, E is redundancy



Fluxmeter ~ Quench antenna

- Any diagnosis associated with field perturbation
  - Fixed coil, install inside magnet bore
  - Localization of quench position, Mechanical disturbance measurement



Evolution of Quench Antenna at CERN

- LQA: Local Quench Antenna
  - Improved Quench Localization and Quench Propagation Velocity Measurements in the LHC Superconducting Dipole Magnets, M. Calvi, E. Floch, S. Kouzue, and A. Siemko, 2005
  - Analysis of local quench antenna signals , Authors : C. Lorin, G. Deferne, M. Bajko / TE-MSC, 2012

Fig. 5: One of the four sides of a sector at which a pick-up coil is fixed.

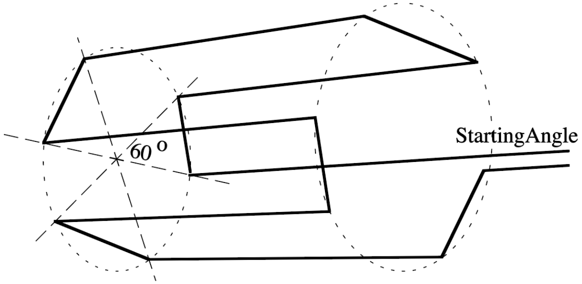
Fig. 6: Fiber-glass support of a pick-up coil made up of very thin tungstenic wires of 32 μm wound 400 times.

Fig. 16: Sensitivity of the LQA to left-right position of the quench

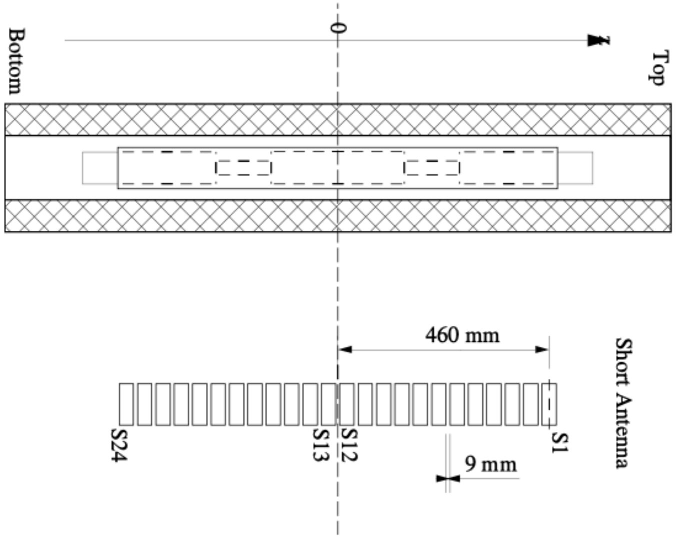
Courtesy of T. Ogitsu; https://indico.cern.ch/event/951071/

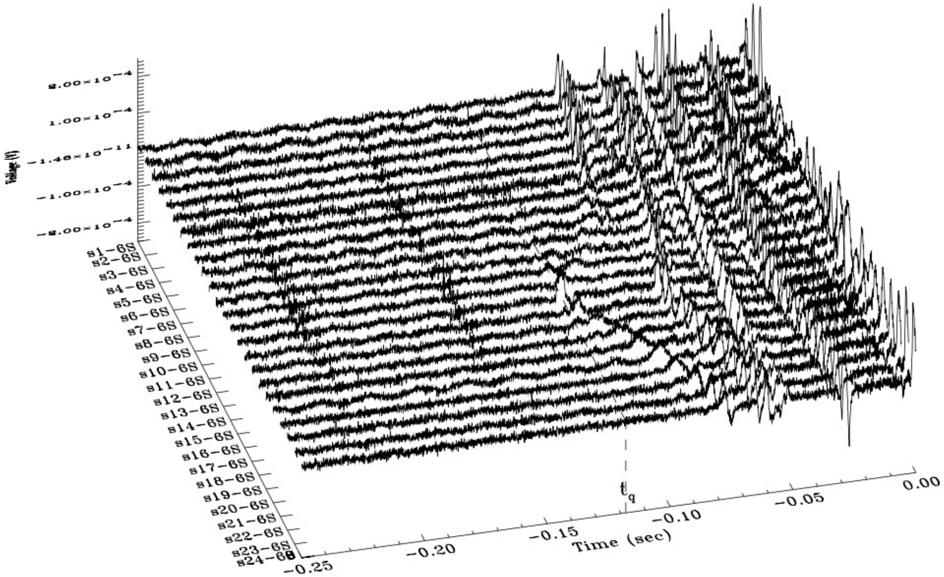
Fluxmeter ~ Quench antenna

- Any diagnosis associated with field perturbation
  - Fixed coil, install inside magnet bore
  - Localization of quench position, Mechanical disturbance measurement



Flexible printed circuit quench antenna





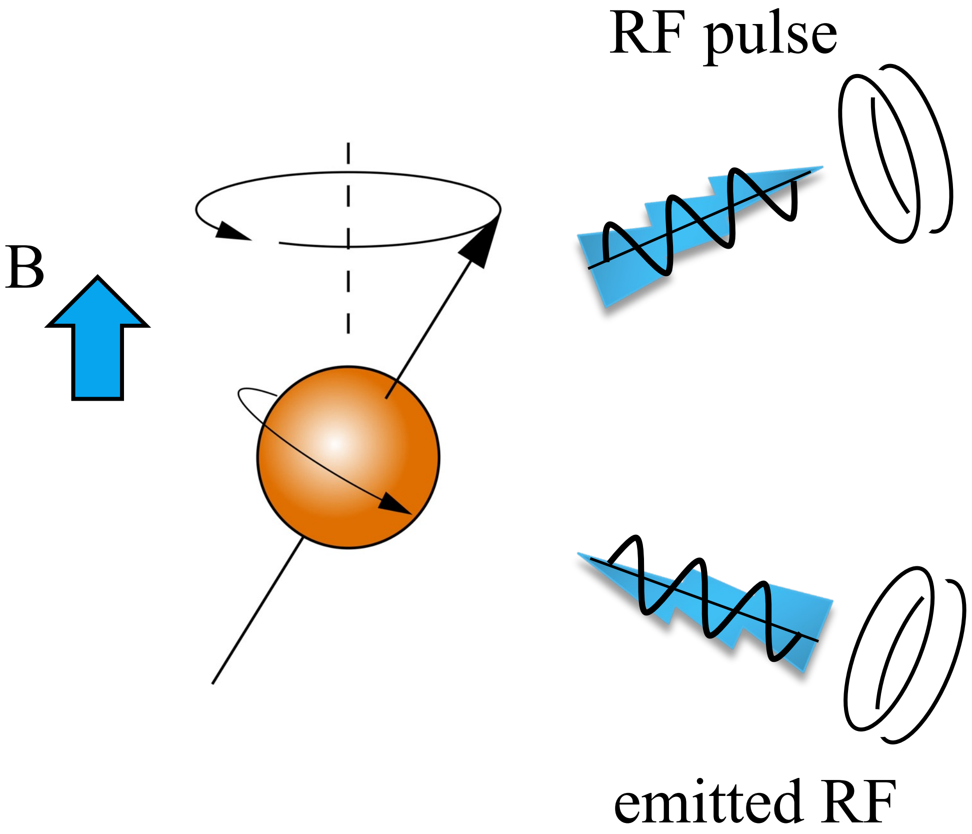
K. Sasaki et al., Study of quench propagation with quench antennas, NIM-A, 416, pp. 9-17, 1998.

Typical magnetometers

1. Hall sensor
2. Fluxmeter
3. NMR magnetometer

Nuclear Magnetic Resonance, NMR ~ Principle

- Utilize nuclear magnetic resonance of particle
  - Particle in a magnetic field interact with RF pulse with specific frequency



Resonant frequency $f = \gamma B$

γ : Gyromagnetic ratio

Frequency = f

NMR magneto meter ~ Principle

- NMR sample at room temperature : usually proton

	γ (MHz/T)	Range
e^-	28026.5	0.5〜3.2 mT
^{1}H	42.576396(3)	0.04 〜 2T
^{2}H	6.53569	2 〜 14T
^{3}He	32.4336	Cryogenic
^{27}Al	11.0942	Cryogenic

$$f = \gamma B$$

In principle,
accuracy : < 0.1 ppm

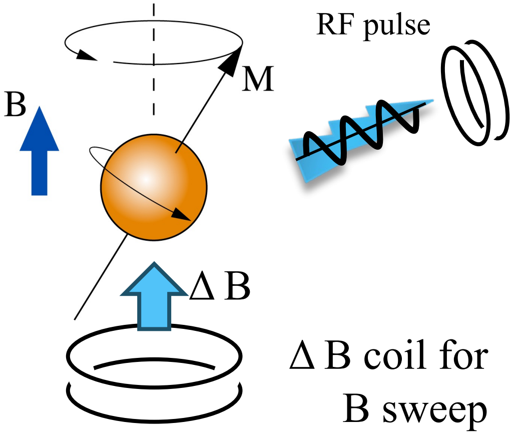
- Practical sample rich in hydrogen nuclei
  - usually use Water or Rubber in NMR magnetometer
  - sometimes water solution of para-magnetic material

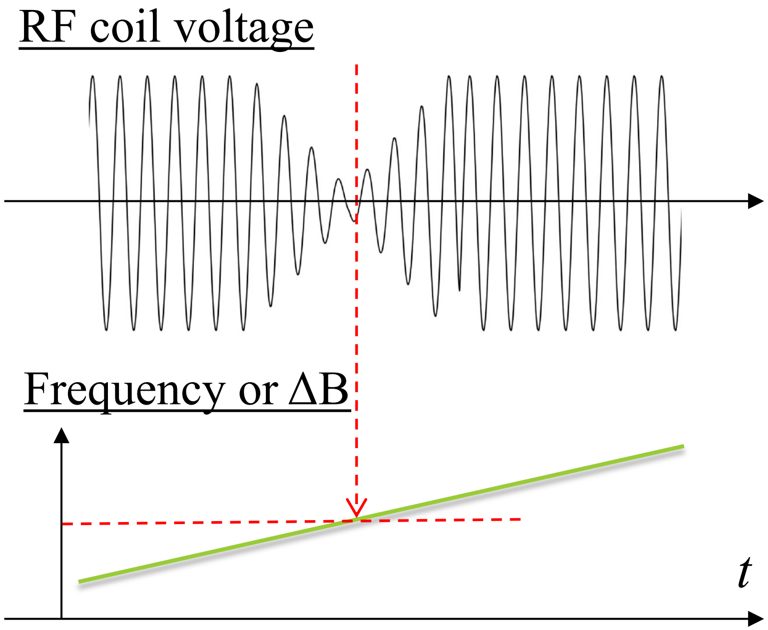
For ultimate accuracy : Ultrapure water

- Two types of NMR magnetometers
  - Continuous Wave(CW) NMR
  - FID NMR (Pulse NMR)

NMR magnetometer ~ CW NMR

- Continuous Wave(CW) NMR
  - Search RF frequency at which resonance absorption occurs
    - Two techniques
      - Frequency sweep
      - Magnetic field sweep



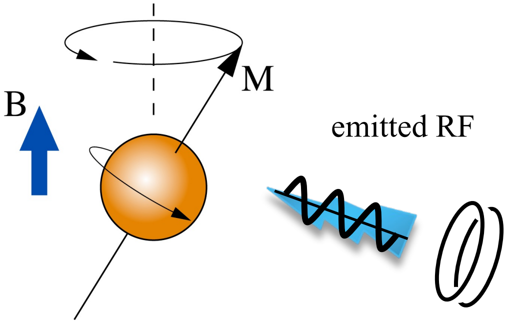


1. RF signal is applied to NMR sample continuously
2. Gradually change
   - Frequency of RF or Δ B
3. Find frequency or ΔB when resonant absorption occurs

M. Newton et al., “Advances in Electronics Prompt a Fresh Look at Continuous Wave (CW) Nuclear Magnetic Resonance (NMR)”, Electronics, 6, 89, 2017, DOI:10.3390/electronics6040089

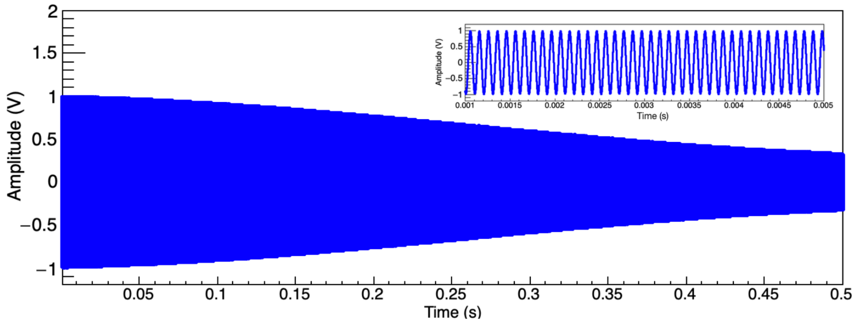
NMR magnetometer ~ FID NMR

- FID NMR
  - Measure and analyze Free induction decay (FID) signal

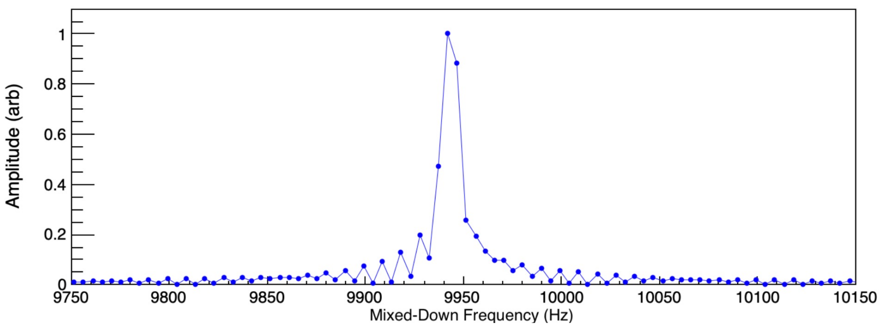


1. Apply short and wide-band pulsed RF signal near the Larmor frequency (resonant frequency)
2. Detect the emitted RF signal corresponding to precession frequency during the relaxation time (FID signal)
3. Extract frequency of FID signal by spectrum analysis

FID signal



Fourier transform of FID

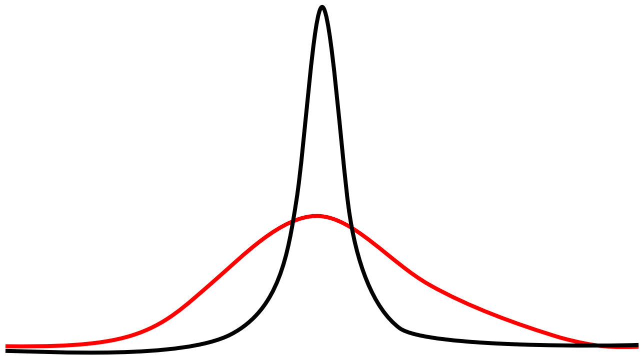


D. Flay et al., "High-accuracy absolute magnetometry with application to the Fermilab Muon g-2 experiment", JINST, 16, P12041, 2021, DOI:10.1088/1748-0221/16/12/P12041

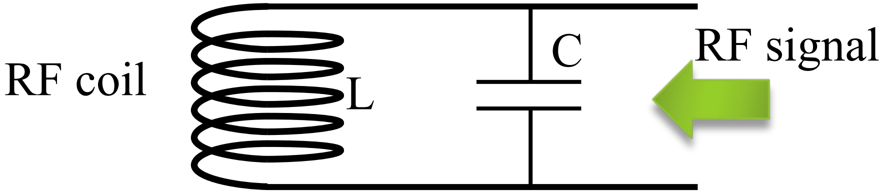
NMR magnetometer ~ Benefits/Limitation

- Benefits
  - High resolution
    - can be < 0.1 ~ 0.001 ppm
  - High accuracy
    - can be < 0.1 ppm
  - Very small temperature drift
    - < 0.01 ppb

✓ Typical NMR signal
- Uniform field
- Non-uniform field



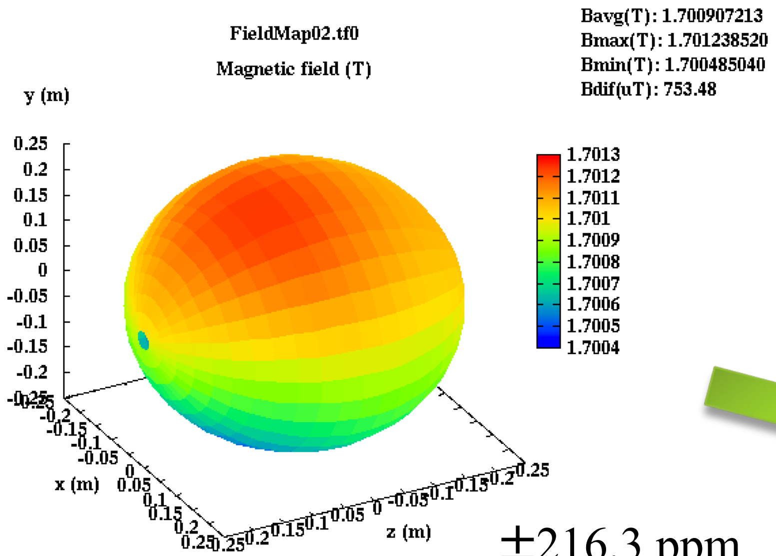
- Limitations
  - Spatially uniform field
    - < 100 ppm /mm
      - ☐ Resolution and accuracy depend on uniformity
  - Slow
    - depends on relaxation time of NMR sample
      - ☐ water : several seconds
  - Narrow measurable range / probe
    - use LC resonant circuit to increase the gain -> limited resonance range



- L and C have to be tuned to get clear signal

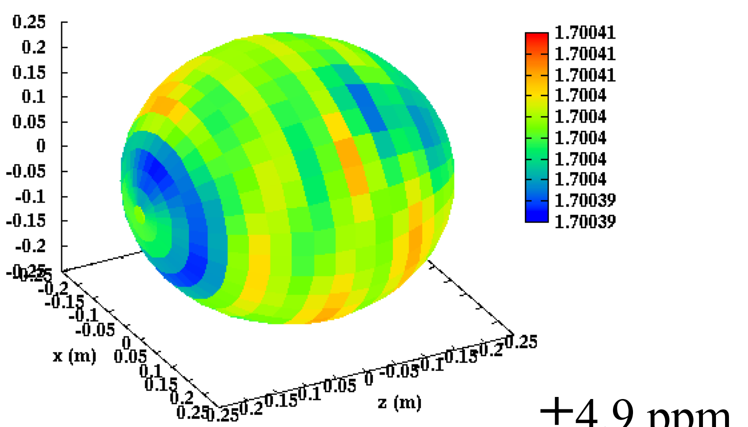
NMR magnetometer ~ Applications

- Field camera
  - Commercial device
    - 24 ch CW-NMR with frequency modulation
  - Commonly used for MRI magnet
    - Shimming to improve uniformity
      - ☐ Target : ± several ppm uniformity



±216.3 ppm

Meas. & Shimming × 4

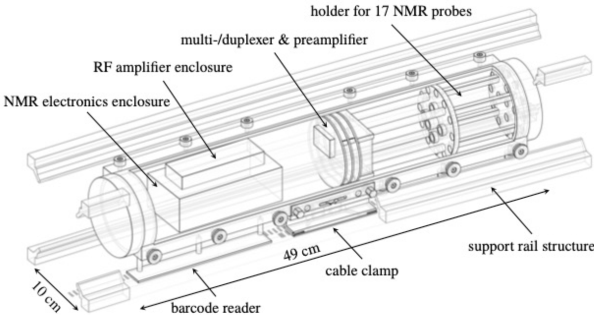


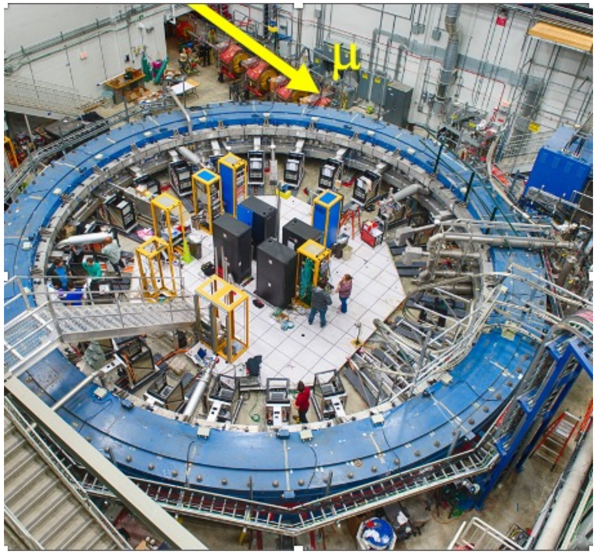
±4.9 ppm

＊ shimming : field adjustment to improve uniformity

NMR magnetometer ~ Applications

- Trolley field mapper for g-2 at FNAL
  - 17 ch pulsed NMR probes
    - Target : < 0.1 ppm accuracy

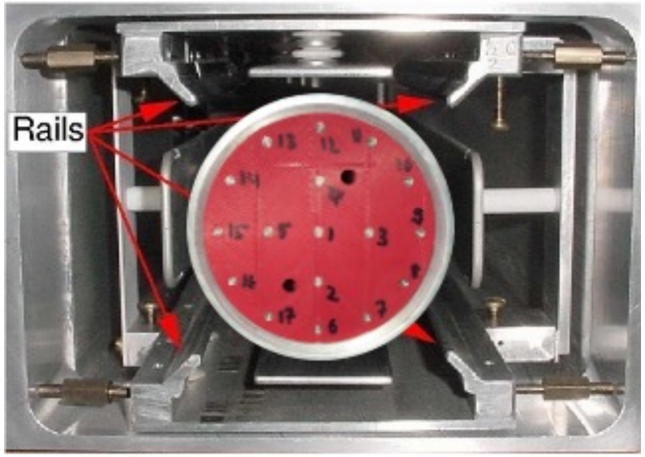


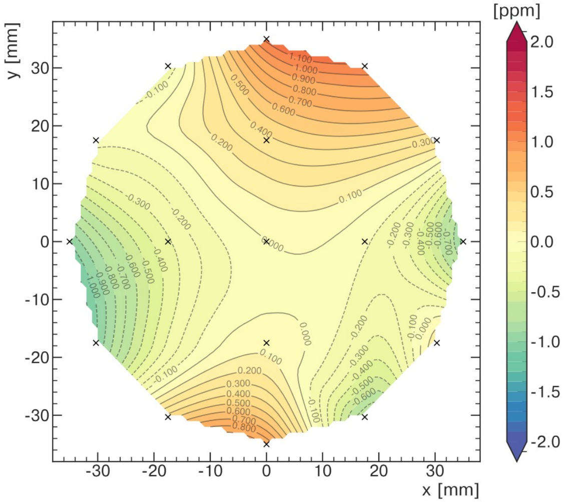


Absolute accuracy : ~0.030 ppm

<u>Superconducting coils as muon storage ring</u>

B: 1.45 T, R : 7.1 m





the field uniformity azimuthally averaged

S. Corrodi et al. Design and performance of an in- vacuum, magnetic field mapping system for the muon g-2 experiment. JINST, 15:P11008, 2020.

Outlines

Calibration of magnetometer

- Mandatory
  - Hall sensor
    - Sensitivity, linearity, sensing angle, offset, ...
  - Fluxmeter
    - Geometrical factor (area), offset, ...
  - NMR
    - Aiming ultimate absolute accuracy
      - Chemical shift, probe intrinsic effect,....

Calibration of magnetometer

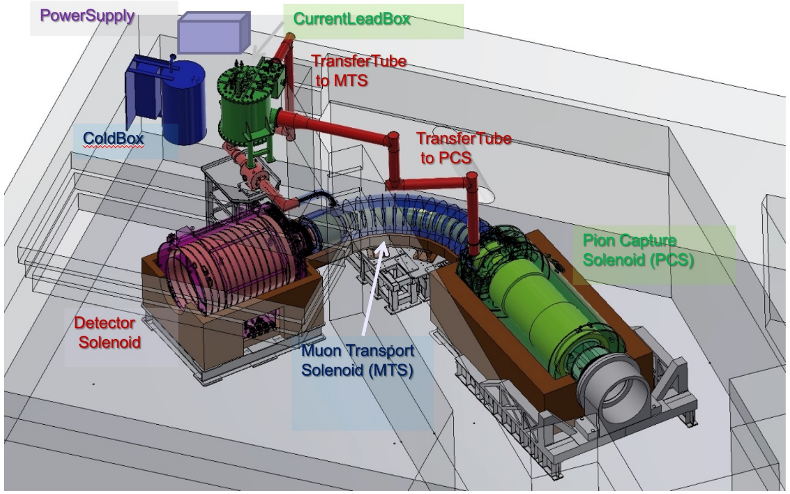
- How to calibrate magnetometer
  - place the magnetometer to be calibrated in a region with a precisely known magnetic field, and compare the output value with the field.
  - If sensor is sensitive to field direction
    - must place it in the right direction

  - Require
    - Reference electromagnet
      - ☐ Uniform field -> Helmholtz coil, dimensionally accurate dipole magnet, MRI magnet
      - ☐ Accurate current source

    - Magnetometer as a secondary standard
      - ☐ It must have higher accuracy than one to be calibrated
      - ☐ NMR magnetometer is generally used

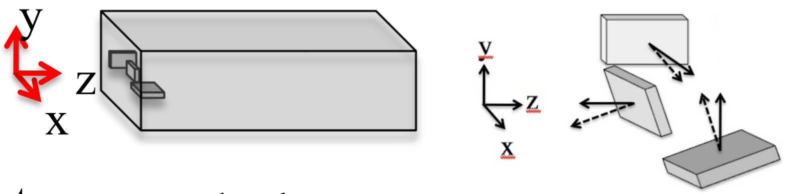
Calibration ~ ex) 3-axis Hall probe

- 3-axis Hall probe for COMET experiment
  - Superconducting solenoid
    - 5 T at maximum
    - Required accuracy for magnetic field measurement
      - < 0.1 %



- Purpose of calibration
  - Sensitivity up to 5 T
  - Orthogonality of 3 sensors

    1 ° error causes 0.015 % uncertainty



Ideal

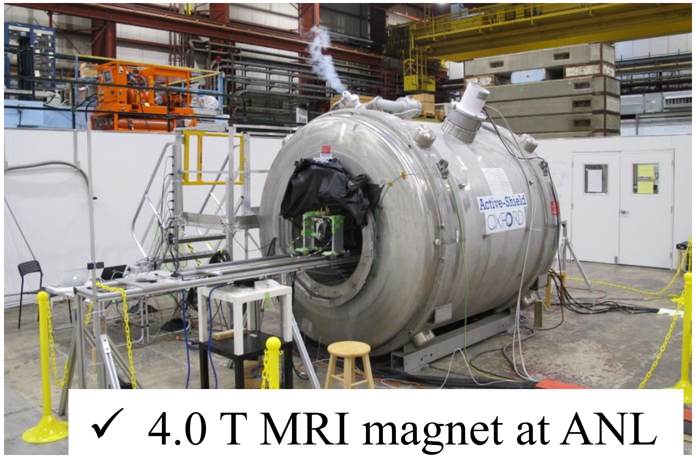
In practice

*Nominal value from vendor : ±0.5 °

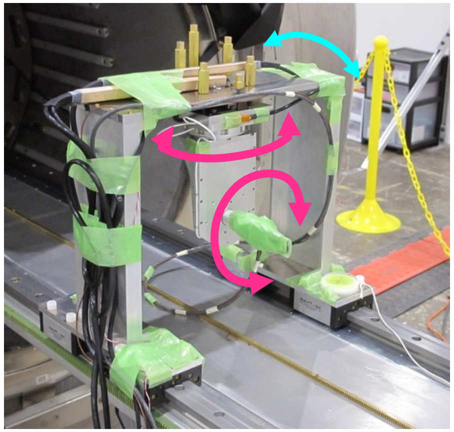
Calibration ~ ex) 3-axis Hall probe

- Use MRI magnet
  - need higher magnetic field

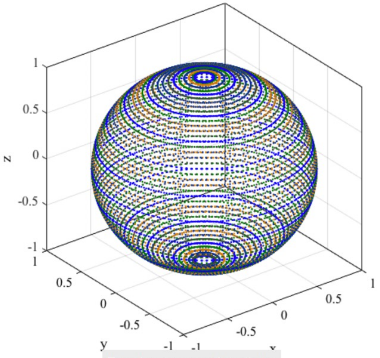
Collaboration with FNAL, ANL



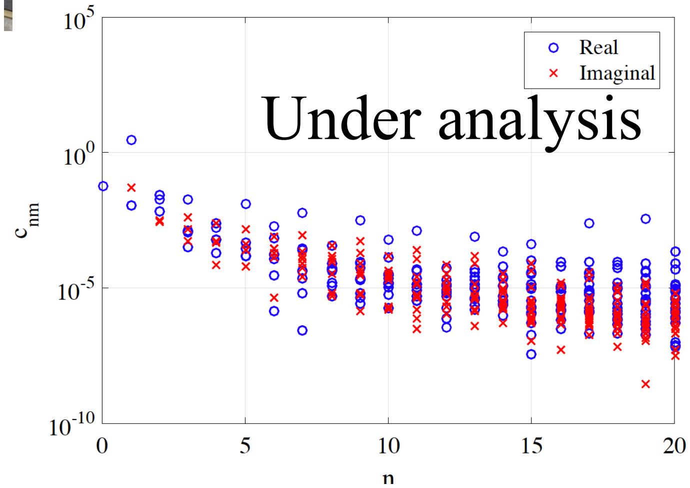
✓ 4.0 T MRI magnet at ANL



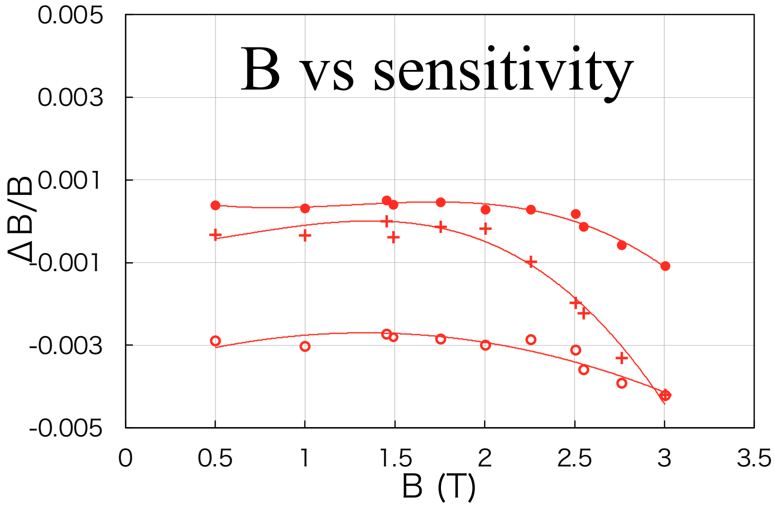
- Probe rotation stage
  - two-axis rotation by ultra sonic motors
  - one angle can be adjusted by spacer



Real scanned points



Under analysis



B vs sensitivity

$$B_{\mathrm{Hall}}(B,\theta,\phi)=\sum_{n=0}^{n_{\max}}\sum_{m=-n}^{n} c_{nm}(B)Y_{nm}(\theta,\phi)$$

$$c_{nm}(B)=\int_0^{2\pi}d\phi\int_0^{\pi}\sin\theta d\theta\cdot Y^*_{nm}(\theta,\phi)B_{\mathrm{Hall}}(B,\theta,\phi)$$

F. Bergsma., https://cds.cern.ch/record/1072471/files/cer-002727968.pdf

Calibration ~ ex) NMR magnetometer

- NMR magnetometer : the gold standard of magnetic field measurement
  - Resolution : 1 ppb ~ 1 ppm (depending on uniformity)
  - Absolute accuracy : ~ 10 ppm (depending on NMR sample, and structure)

Correction factor

$$B_p = (1 - \delta_t)B. \tag{2}$$

B_p : Magnetic field at the location of a proton
B : External magnetic field

$$\delta_t = \sigma(H_2O) + \delta_b + \delta_p + \delta_s. \tag{3}$$

1. $\sigma(H2O)$: Internal diamagnetic shielding in the water molecule
2. δ_b : Bulk diamagnetism of the water sample (shape effect)
3. δ_p : Paramagnetic impurities in the water sample
4. δ_S : Magnetization effect of all materials surrounding NMR sample (Material effect)

X. Fei et al., “Precision measurement of the magnetic field in terms of the free-proton NMR frequency”, NIM-A, 394, pp. 349-356, 1997. DOI: 10.1016/S0168-9002(97)84161-7

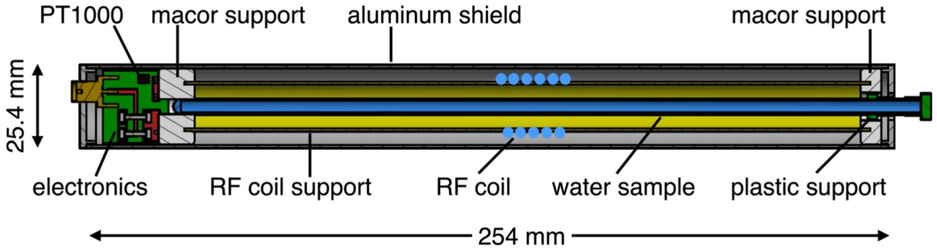
Calibration ~ ex) NMR magnetometer

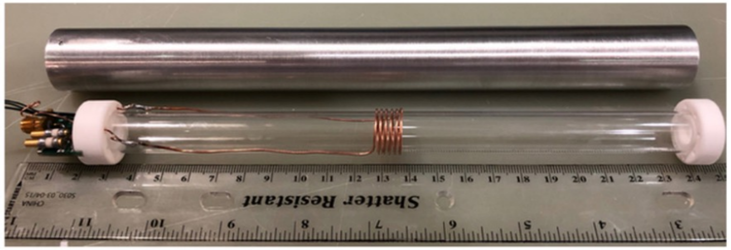
- How to calibrate the correction factors?
  - Only be realized by probe with careful construction
    - Diamagnetic shielding effect (chemical shift)
      - ☐ Studied precisely, it is well known at the level of 11 ppb
    - Bulk diamagnetism (shape effect)
      - ☐ Could cancel it by using spherical shape sample
    - Paramagnetic impurities
      - ☐ Practically negligible by using pure water
    - Material effect
      - ☐ Most annoying factor

<u>Standard probe</u>

✓ Example of standard probe

- Probe for Fermilab *g*-2 experiment





Accuracy : ~17 ppb (mat. effect : 12 ppb)

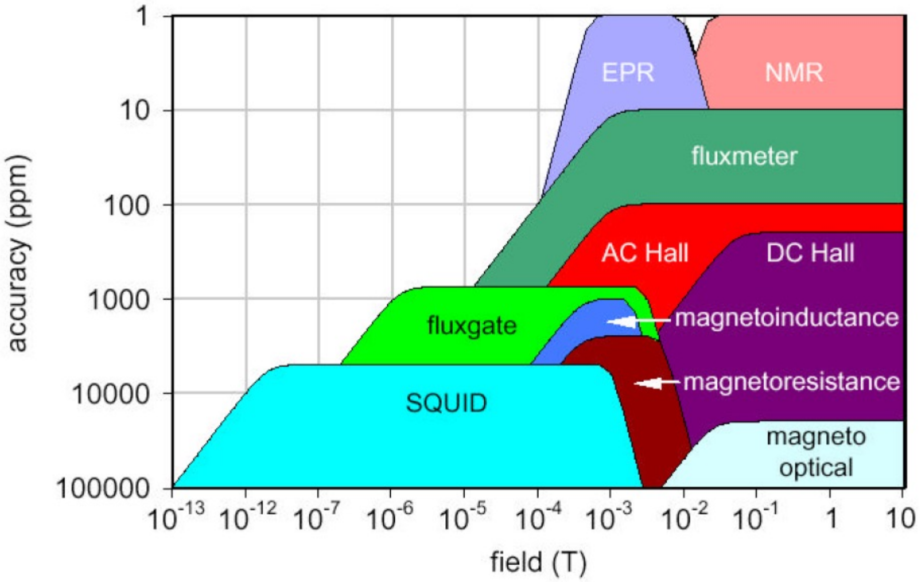
D. Flay et al, JINST, 16, P12041, 2021, DOI:10.1088/1748-0221/16/12/P12041

Outlines

1. Introduction
2. 2D and 3D expression of magnetic field
3. Typical magnetometers
4. Calibration of magnetometers
5. Summary

Summary

- Many types of magnetometers in the world
  - First job for magnetic field measurement
    - SELECT proper magnetometer considering measurement condition and target



- Ultimate performance
  - precise calibration can only realize

Thank you for your kind attention!