Energy-energy correlations in heavy-ion collisions

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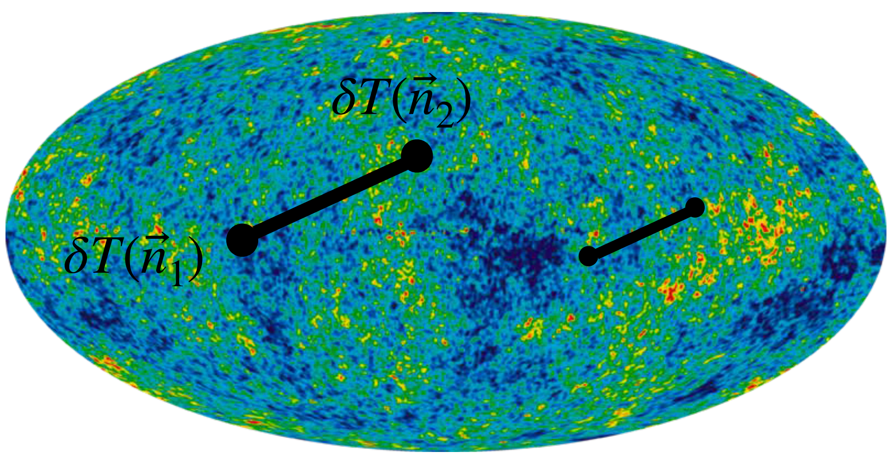
CA, Dominguez, Elayavalli, Holguin, Marquet, Moult, 2209.11236

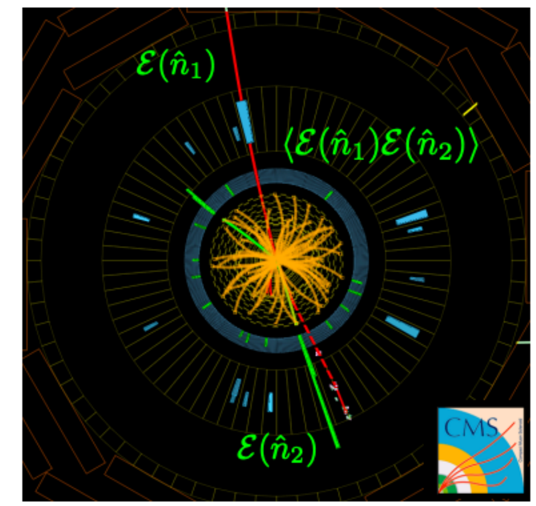
CA, Dominguez, Holguin, Marquet, Moult, 2303.03413

CA, Dominguez, Holguin, Marquet, I. Moult, 2307.15110

Energy Correlators

- Fundamental objects that encode the dynamics of the underlying theory



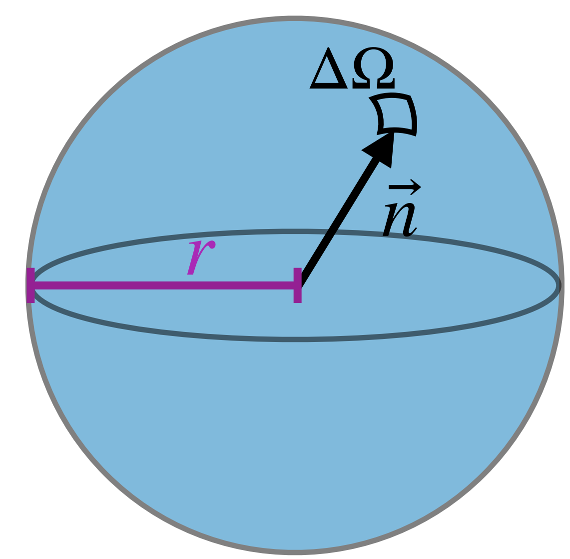


- Correlators $\langle \mathscr{E}(\vec{n}_1)\mathscr{E}(\vec{n}_2)\cdots\mathscr{E}(\vec{n}_k)\rangle$ of the **energy flux**:

Sterman, Korchemsky, Nucl. Phys. B 555 (1999) 335

$$\mathscr{E}(\vec{n}) = \lim_{r\to\infty}\int \mathrm{d}t\, r^2 n^i\, T_{0i}(t, r\vec{n})$$

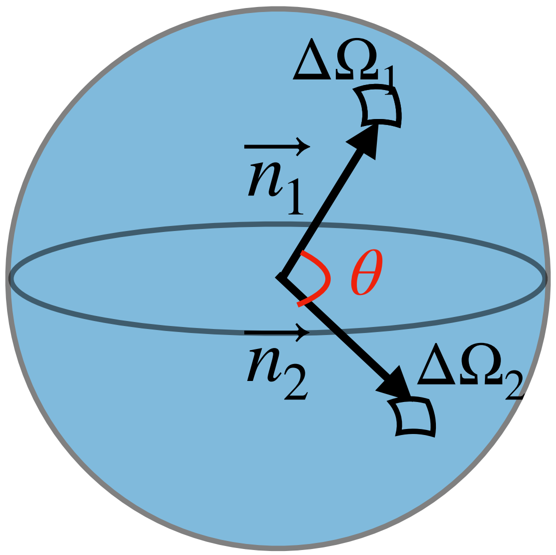
$$\mathscr{E}(\vec{n})\,|X\rangle = \sum_i E_i\,\delta^{(2)}(\vec{n}-\vec{n}_i)\,|X\rangle$$



- 1-point correlator: $\langle X|\mathscr{E}(\vec{n})|X\rangle \propto \sum_i E_i$

Total energy flux through an area element

Two-point correlator



- **2-point correlator (EEC):**

$$\frac{\langle \mathcal{E}^n(\vec{n}_1)\mathcal{E}^n(\vec{n}_2)\rangle}{Q^{2n}} = \frac{1}{\sigma}\sum_{ij}\int \frac{d\sigma_{ij}}{d\vec{n}_i d\vec{n}_j}\frac{E_i^n E_j^n}{Q^{2n}}\delta^{(2)}(\vec{n}_i-\vec{n}_1)\delta^{(2)}(\vec{n}_j-\vec{n}_2)$$

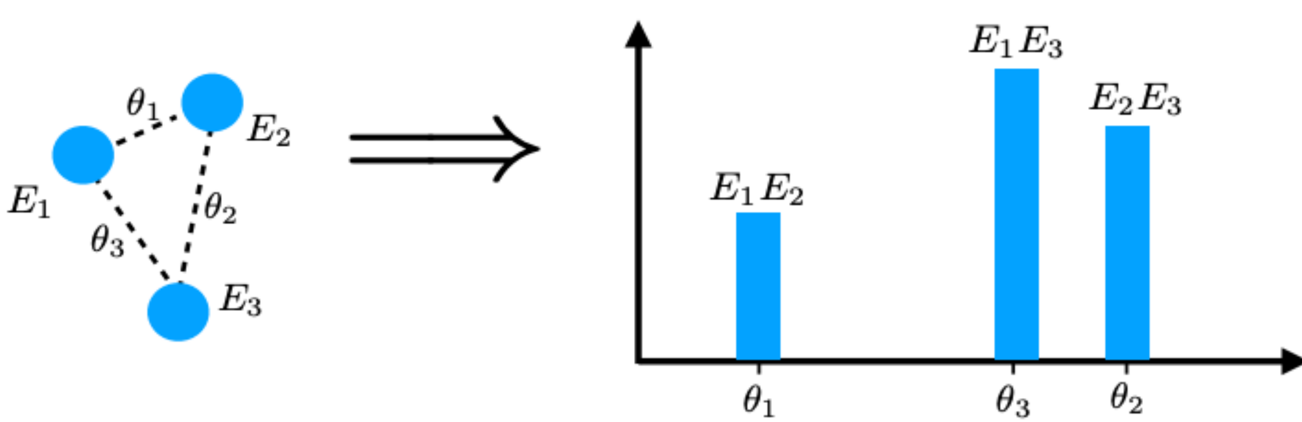
Inclusive cross section to produce two particles *i* and *j*

Hard scale of the process

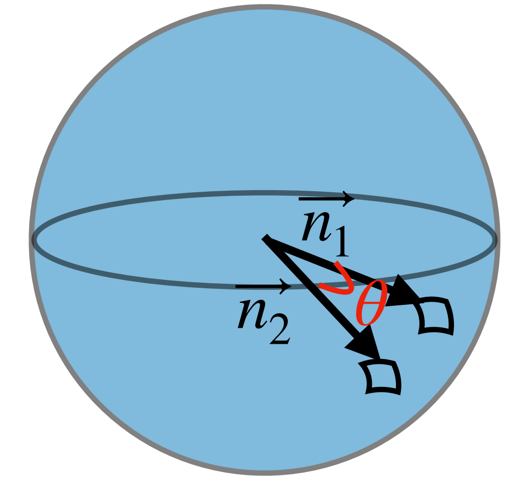
- As function of the relative angle only:

$$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma}\sum_{i,j}\int dE_{i,j}\,\frac{d\sigma}{d\theta dE_i dE_j}\,\frac{E_i^n E_j^n}{Q^{2n}}$$

See also:
Barata, Milano, Sadofyev
2308.01294

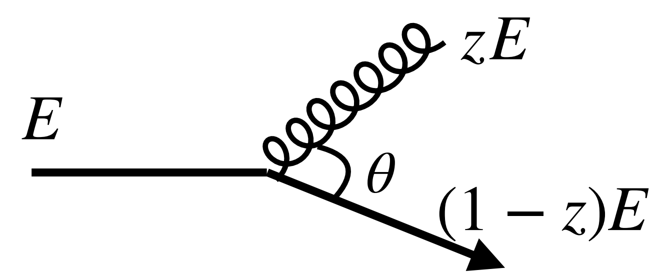


EEC within p-p jets



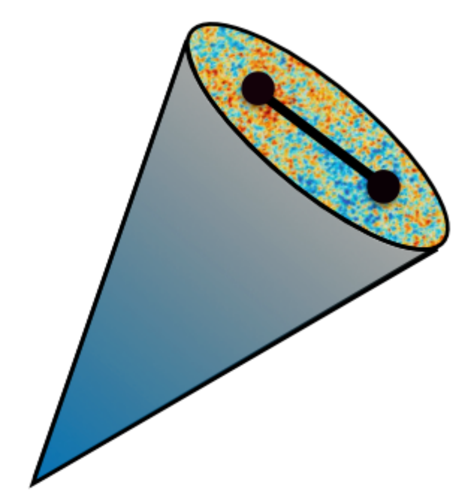
- **2-point correlator:** $$\frac{d\Sigma^{(n)}}{d\theta} = \frac{1}{\sigma}\sum_{i,j}\int dE_{i,j}\,\frac{d\sigma}{d\theta\, dE_i\, dE_j}\,\frac{E_i^n E_j^n}{Q^{2n}}$$

- EEC for a massless **quark** jet in **vacuum** at LO:



$$\frac{\mathrm{d}\sigma_{qg}^{\mathrm{vac}}}{\mathrm{d}z\mathrm{d}\theta} = \frac{\alpha_s C_F\,\sigma}{\pi}\,\frac{1+(1-z)^2}{z\,\theta} + \mathcal{O}(\alpha_s^2,\theta) \quad \Longrightarrow \quad \frac{\mathrm{d}\Sigma^{(1)}}{\mathrm{d}\theta} \propto \frac{1}{\theta}$$

- Within jets: **collinear** (or OPE) limit of EECs



$$\langle X|\,\mathscr{E}(\vec{n}_1)\mathscr{E}\vec{n}_2)\,|X\rangle \xrightarrow{\theta\to 0} \sum_i \theta^{(\tau_i-4)/2}\,\mathcal{O}_i(\vec{n}_1)$$

Power-law scaling according to CFT!

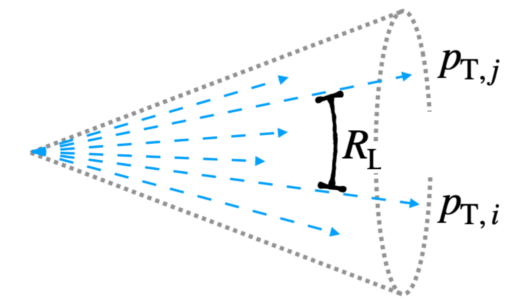
Hoffman, Maldacena, 0803.1467

$$\frac{d\Sigma^{(1)}}{d\theta} \propto \frac{1}{\theta^{1-\gamma(3)}}$$

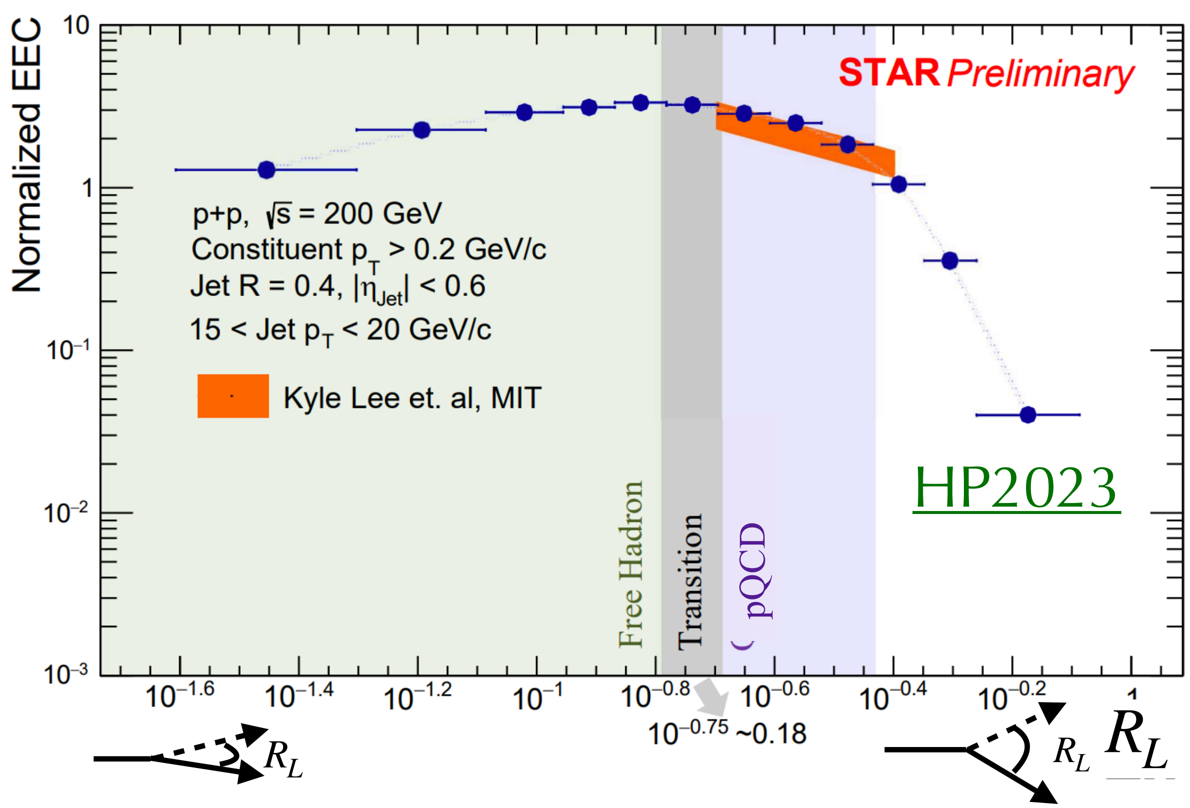
$\gamma(3)$: twist-2 spin-3 QCD anomalous dimension

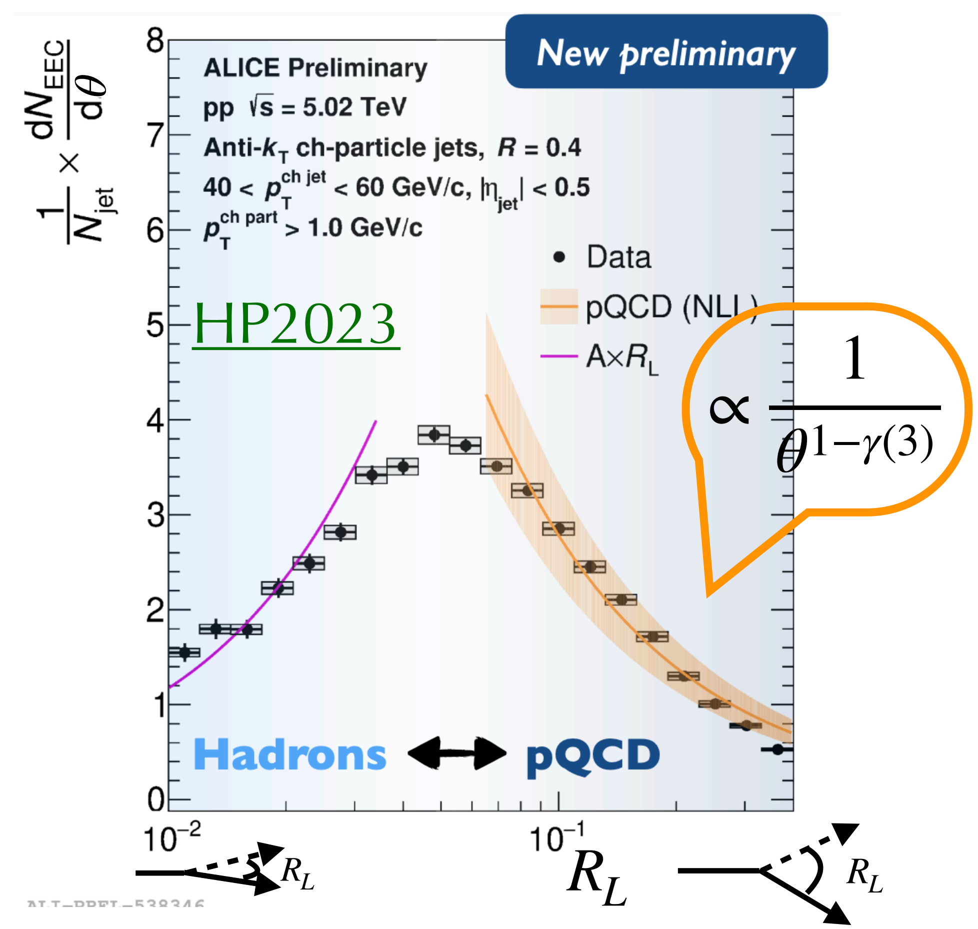
EEC in vacuum

$R_L = \sqrt{\Delta\phi^2 + \Delta\eta^2}$



- **First measurements** of the EEC in **p-p collisions** announced in HP2023

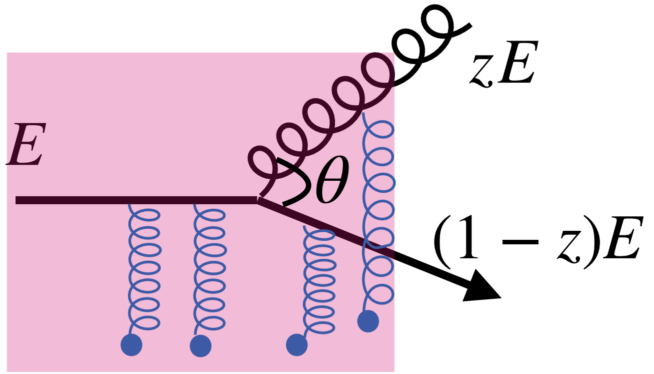




- ✅ Clear **separation** between **perturbative** and **non-perturbative**
- ✅ p-p baseline under control (good agreement with pQCD predictions)
- ✅ Reduced sensitivity to soft

Komiske, Moult, Thaler, Zhu 2201.07800

EEC in HICs



- EEC for a **heavy-ion** jet initiated by a **massless quark**:

$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \frac{1}{\sigma_{qg}} \int \mathrm{d}z \, \frac{\mathrm{d}\sigma_{qg}}{\mathrm{d}z\mathrm{d}\theta} z^n (1-z)^n + \mathcal{O}\left(\frac{\mu_s}{E}\right)$$

- We can always define F_{med} such as

$$\frac{\mathrm{d}\sigma_{qg}}{\mathrm{d}\theta dz} = \left(1 + F_{\text{med}}(z,\theta)\right) \frac{\mathrm{d}\sigma_{qg}^{\text{vac}}}{\mathrm{d}\theta dz} \qquad F_{\text{med}}(z,\theta) \xrightarrow{\theta<\theta_L} 0$$

Additional **energy loss** ($E_q + E_g \neq E$) is **subleading!**

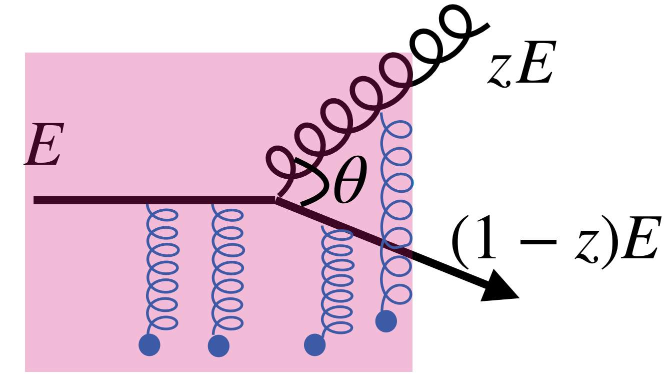
For energy loss effects see:
Barata, Mehtar-Tani, 2307.08943

- We do not expect medium modification at small angles, thus vacuum collinear resummation should still be valid

$$\frac{\mathrm{d}\Sigma^{(n)}}{\mathrm{d}\theta} = \left(\frac{1}{\sigma_{qg}} \int \mathrm{d}z \, \left(g^{(n)}(\theta,\alpha_s) + F_{\text{med}}(z,\theta)\right) \frac{\mathrm{d}\sigma^{\text{vac}}}{\mathrm{d}\theta\mathrm{d}z} z^n (1-z)^n \right) \left(1 + \mathcal{O}\left(\frac{\bar{\mu}_s}{Q}\right)\right) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{\theta Q}\right)$$

$$g^{(1)}(\theta,\alpha) = \theta^{\gamma(3)} + \mathcal{O}(\theta) \quad \Longrightarrow \quad \frac{d\Sigma^{(1)\,\text{vac}}}{d\theta} \sim \frac{1}{\theta^{1-\gamma(3)}}$$

EEC in HICs



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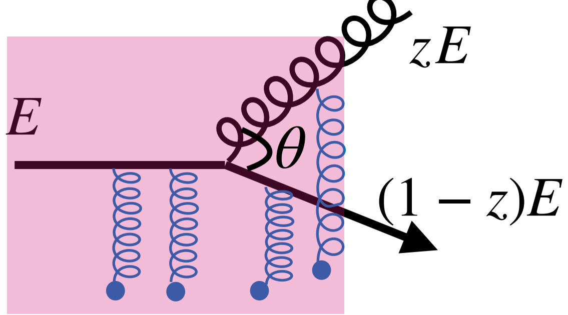
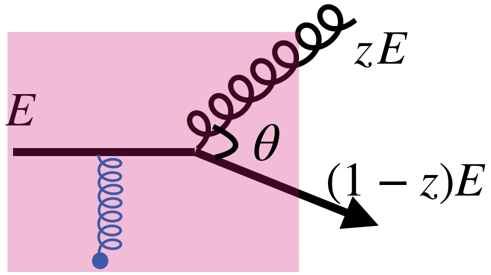
Our idealized models



- Two available approaches to compute the in-medium splitting:
  - **1 single scattering (GLV)**
    Ovanesyan, Vitev 1103.1074, 1109.5619



  - ***Tilted* Wilson lines (multiple scatterings resummed):**
    - Assumes *semi-hard* splittings (z not too small)
    - All partons propagate along straight line trajectories
    - Neglects broadening

Dominguez, Milhano, Salgado, Tywoniuk, Vila, 1907.03653

Isaksen, Tywoniuk 2107.02542

- **Static brick** with length L
- **Harmonic oscillator** (HO) approximation employed $n\sigma(r) \approx \hat{q}r^2/2$
- The strength of the interactions is encoded in the **jet quenching parameter** $\hat{q}$, which measures the average transverse momentum transferred per unit length

Time and angular scales (HO)

- For a static medium of length L within the HO one can read off the relevant scales directly from the formulas:

  2 competing angular scales: θ_L and θ_c

  - (Vacuum) formation time:

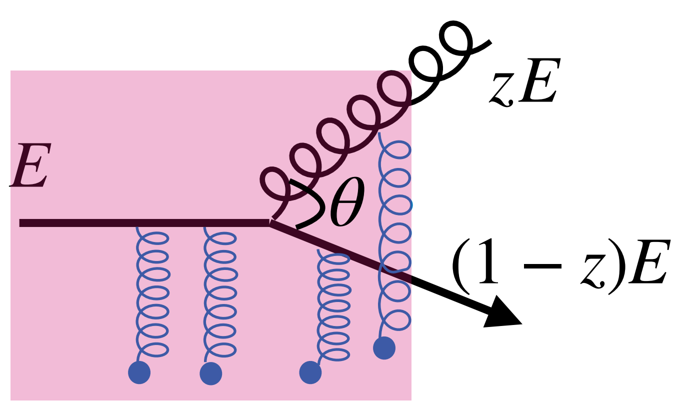
$$t_f = \frac{2}{z(1-z)E\theta^2} \xrightarrow{t_f \leq L} \theta_L \sim (EL)^{-1/2}$$

Below θ_L all emissions have a formation time larger than L

  - Decoherence time:

$$S_{12}(\tau) = e^{-\frac{1}{12}\hat{q}(1+z^2)\theta^2\tau^3} \qquad t_d \sim (\hat{q}\theta^2)^{-1/3} \xrightarrow{t_d \leq L} \theta_c \sim (\hat{q}L^3)^{-1/2}$$

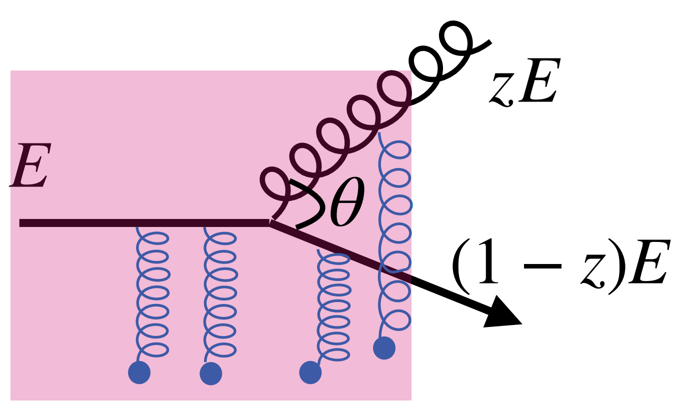
Below θ_c splittings do not color decohere and the medium does not resolve them



If $\theta_L > \theta_c$: θ_c becomes irrelevant

Time and angular scales (HO)

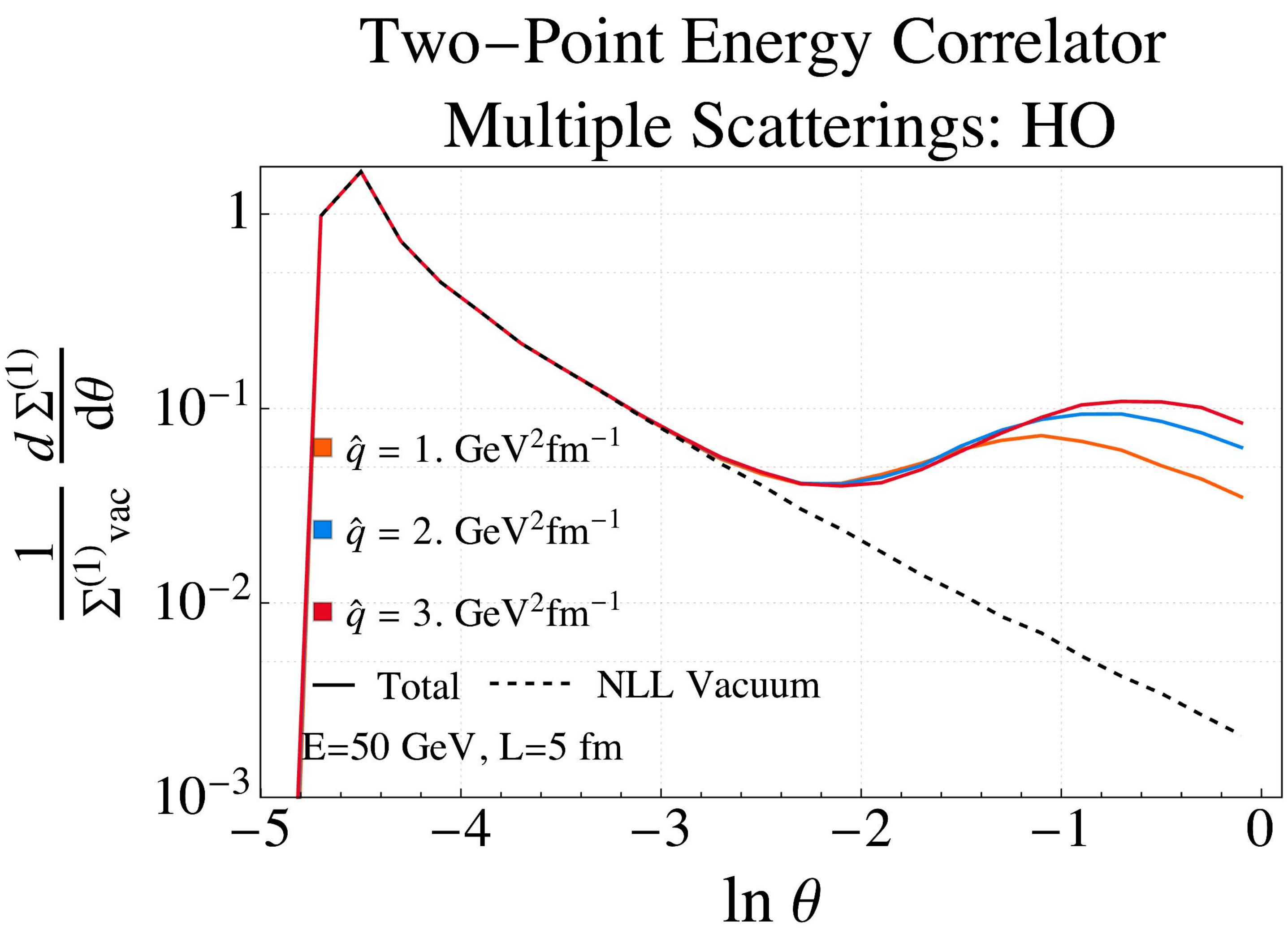
Can be extended to include a more **realistic interactions or expanding media**, but then we would not know the scales directly from the equations



If $\theta_L > \theta_c$: θ_c becomes irrelevant

Results HO

$\theta_L \gg \theta_c (E \ll \hat{q}L^2)$



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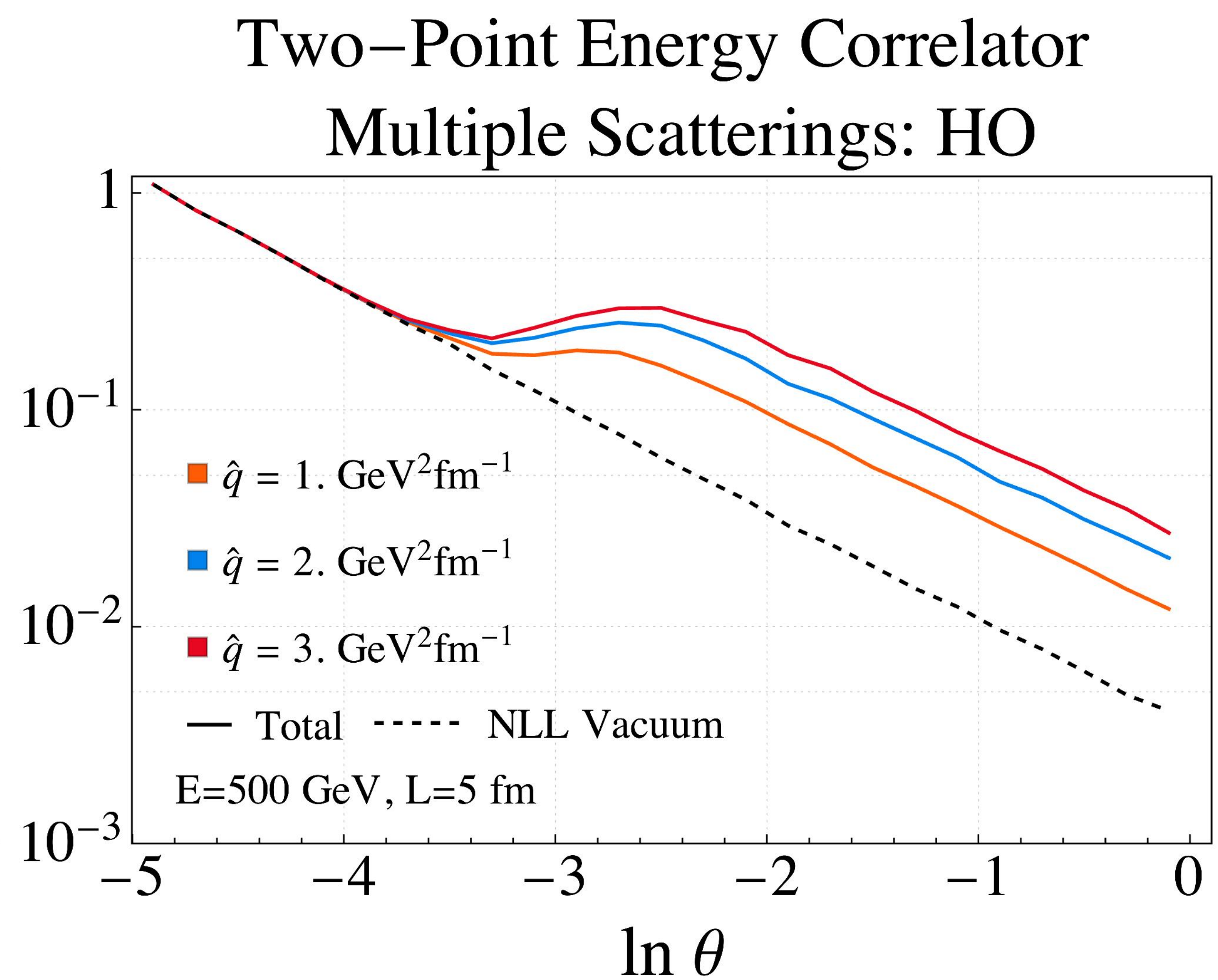

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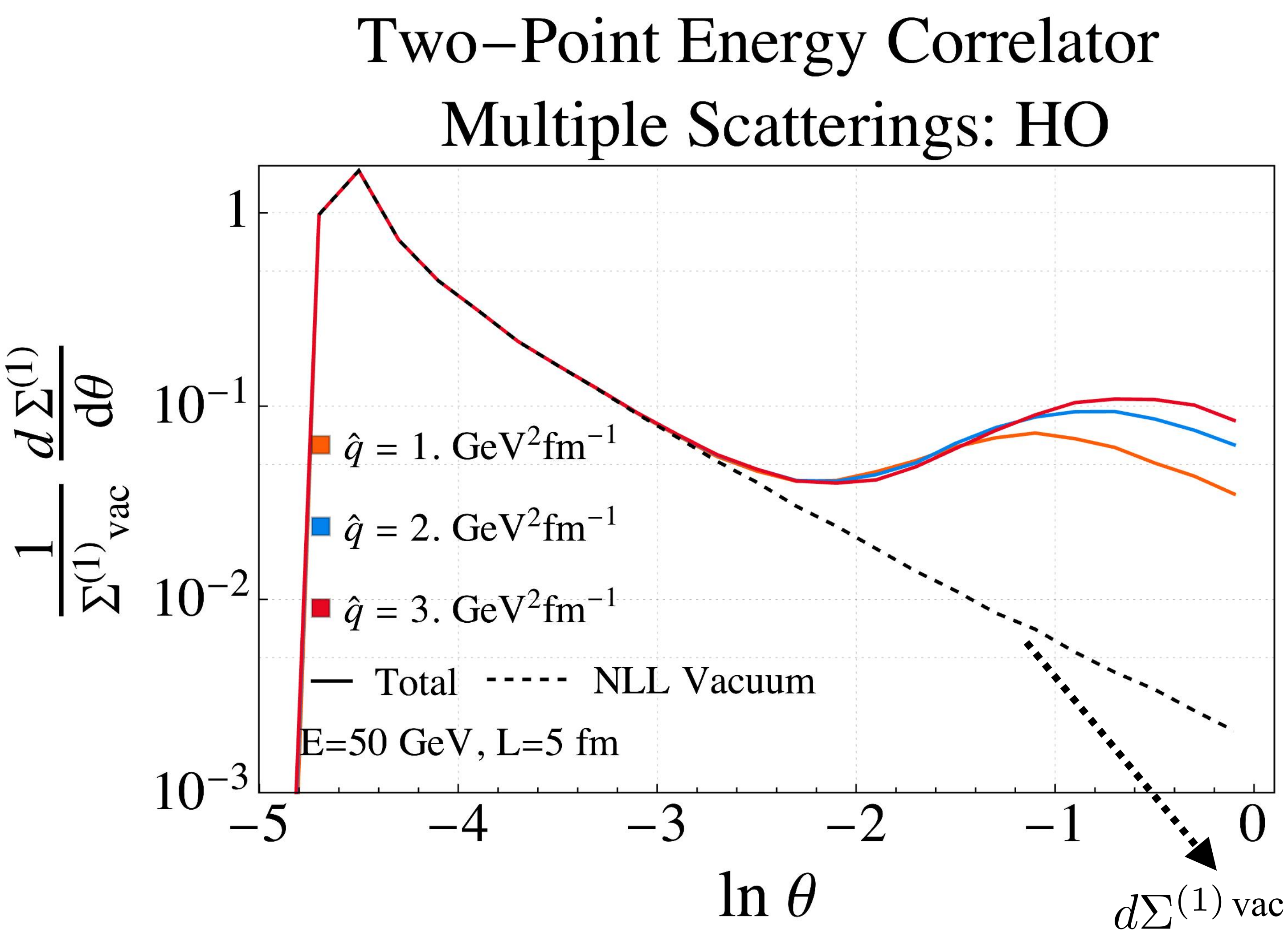
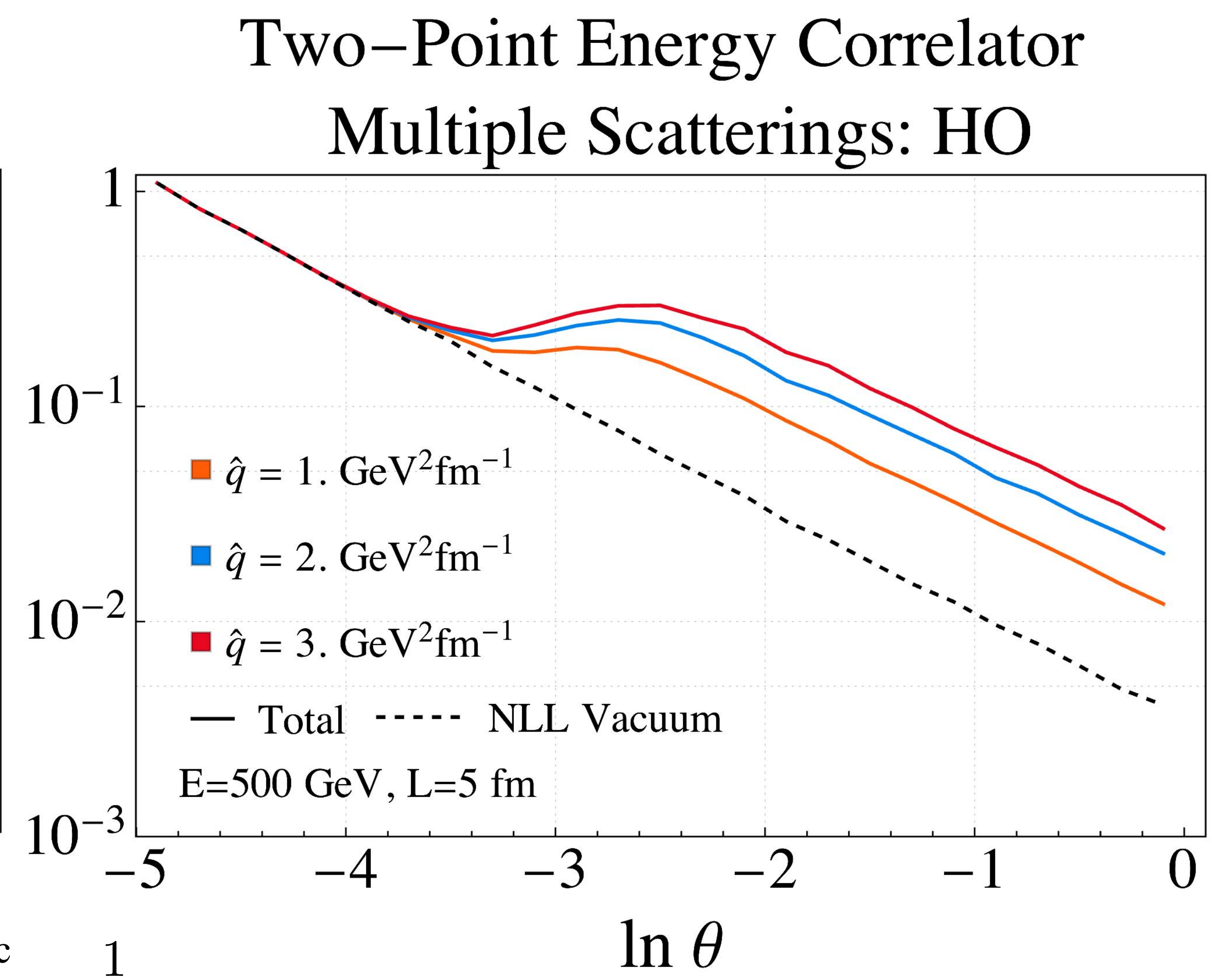


$$\frac{d\Sigma^{(1)\,\text{vac}}}{d\theta} \sim \frac{1}{\theta^{1-\gamma(3)}}$$

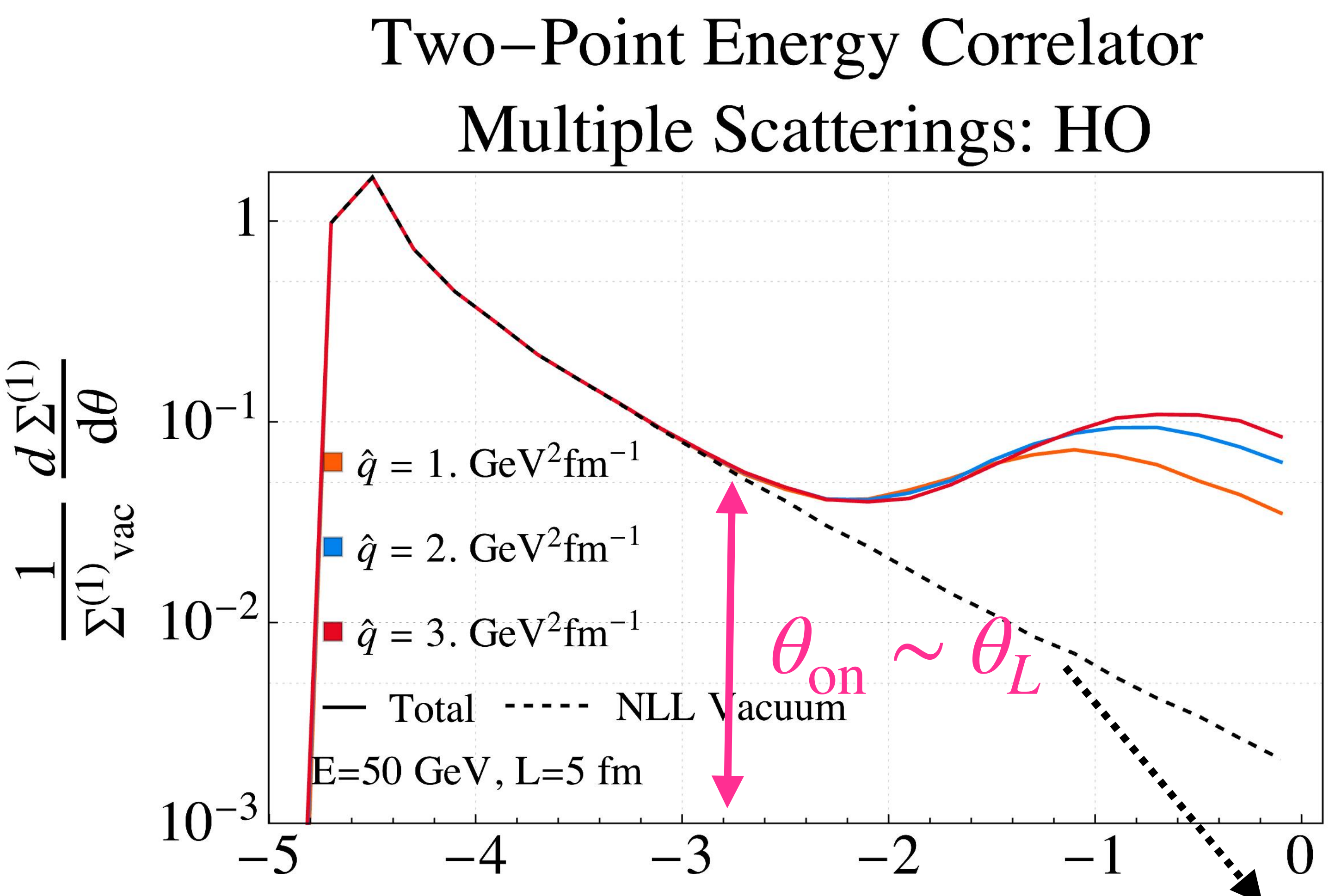
$\theta_L \ll \theta_c \ (E \gg \hat{q}L^2)$



- **No** medium-induced enhancement at **small angles**

Results HO

$\theta_L \gg \theta_c \left(E \ll \hat{q}L^2\right)$



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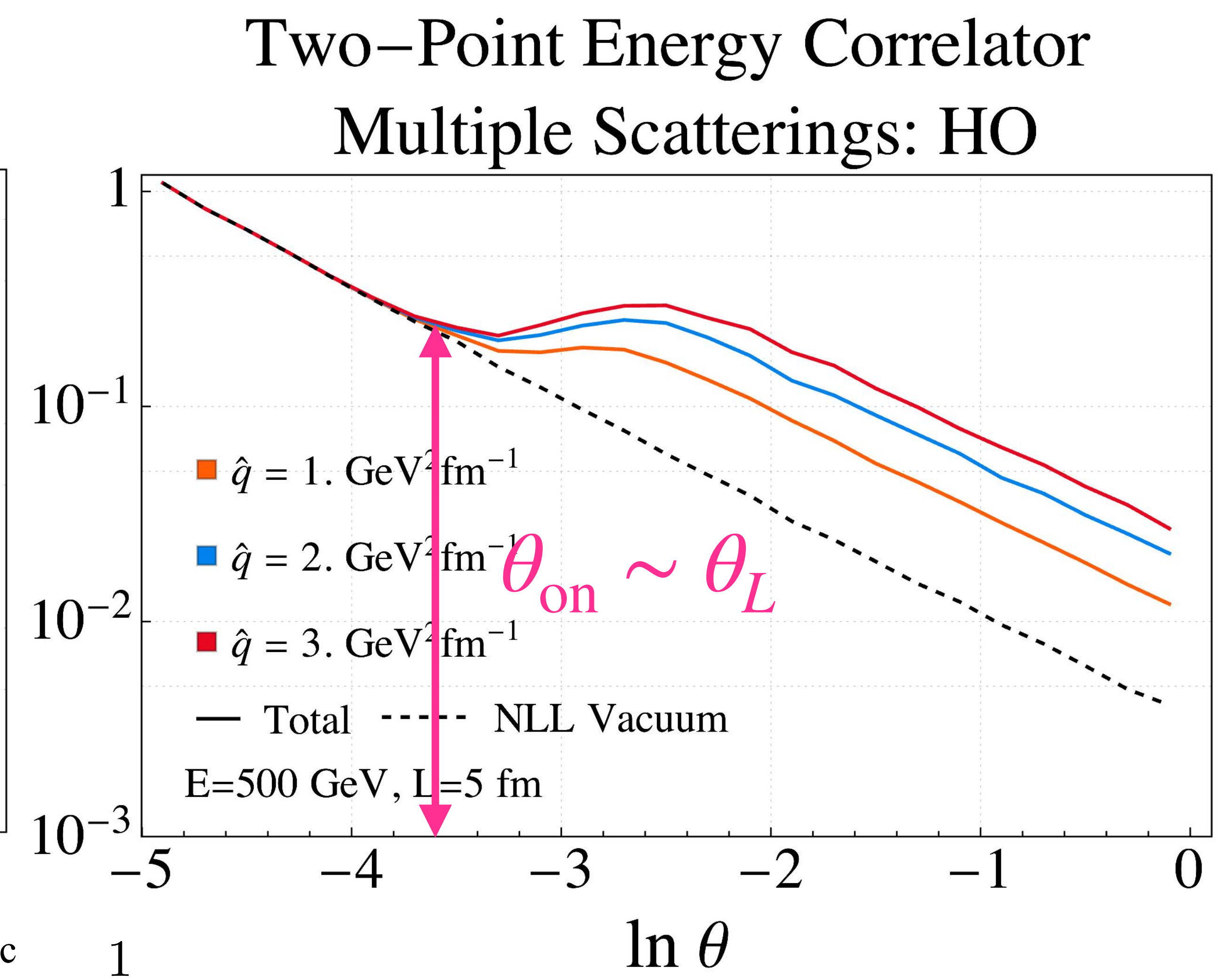
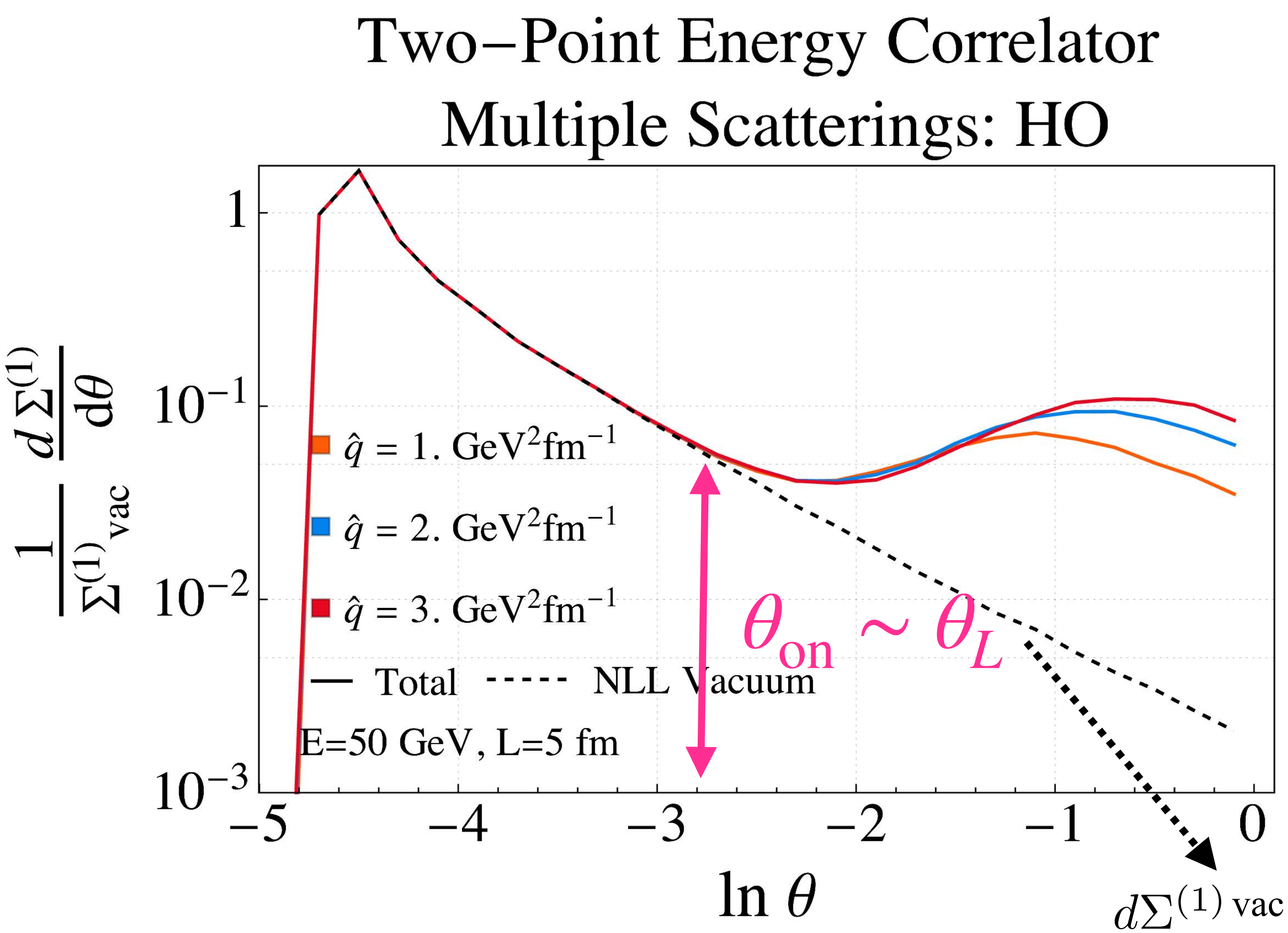


- **No** medium-induced enhancement at **small angles**
- Onset angle seems to be independent of $\hat{q}$

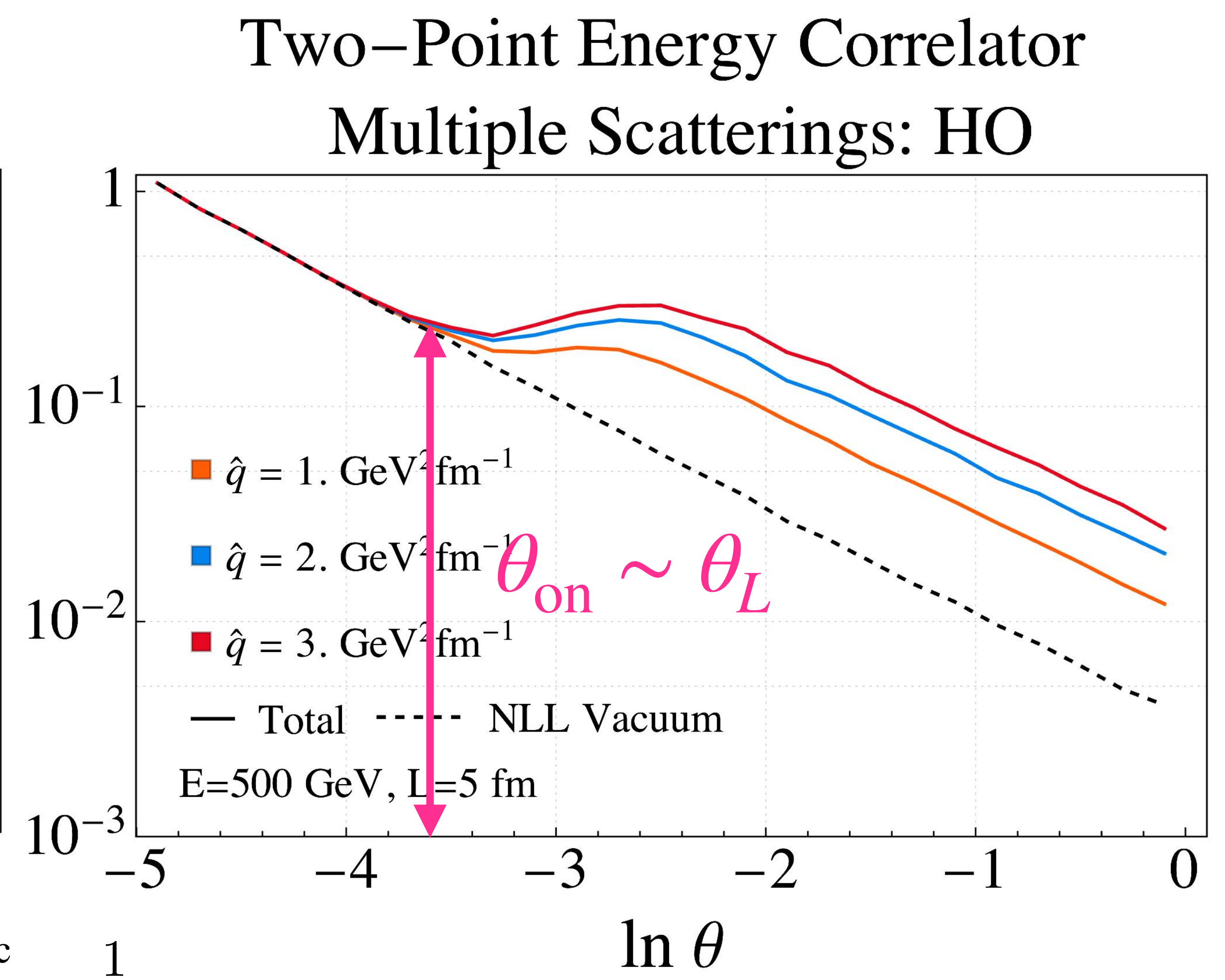
Results HO

$\theta_L \gg \theta_c (E \ll \hat{q}L^2)$



$$\frac{d\Sigma^{(1)\,\text{vac}}}{d\theta} \sim \frac{1}{\theta^{1-\gamma(3)}}$$

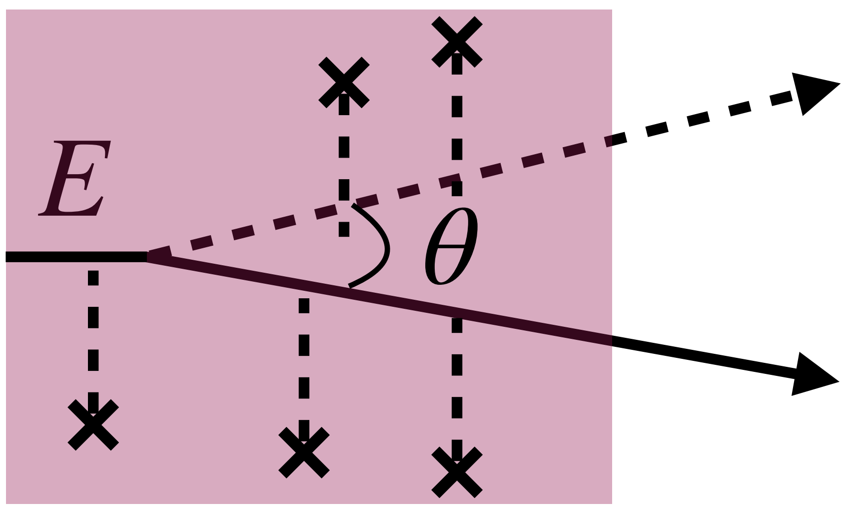
$\theta_L \ll \theta_c \ (E \gg \hat{q}L^2)$



- **No** medium-induced enhancement at **small angles**
- Onset angle seems to be independent of $\hat{q}$
- Varying $\hat{q}$ has different effects in the two regimes

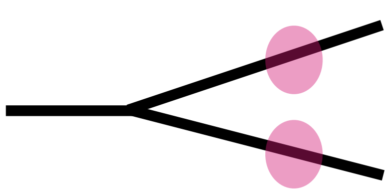
Interpretation

$\theta_L \gg \theta_c \quad (E \ll \hat{q}L^2)$

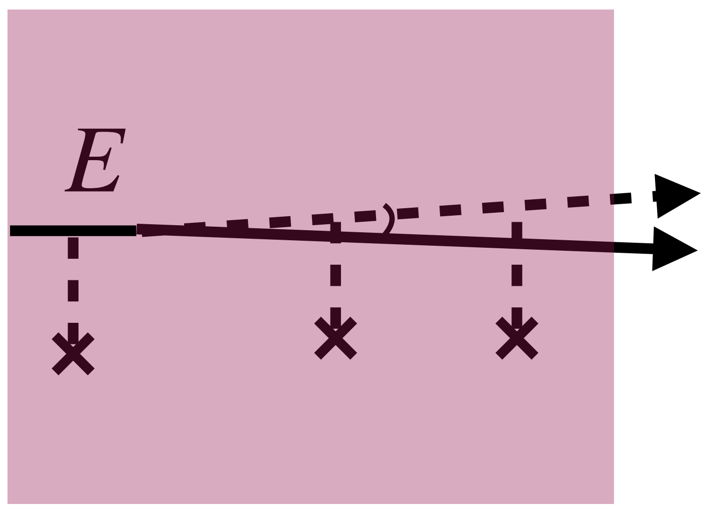


For $\theta \gg \theta_L \Rightarrow \theta \gg \theta_c$

The medium resolves the emission

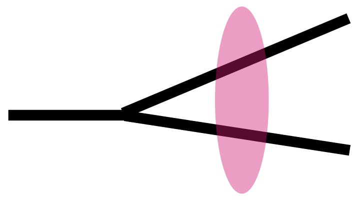


$\theta_L \ll \theta_c \quad (E \gg \hat{q}L^2)$



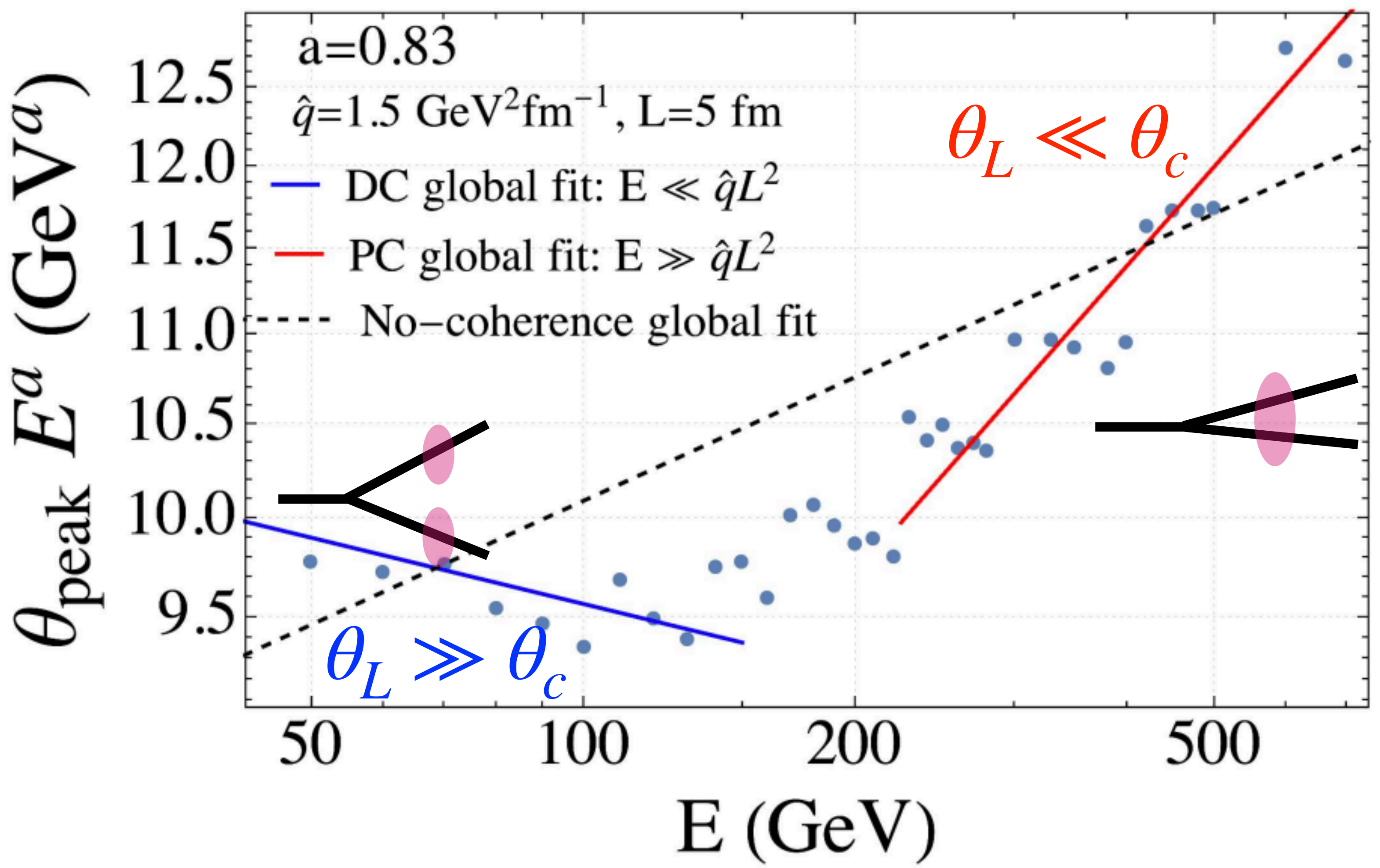
For $\theta_c \gg \theta \gg \theta_L$:

The medium does NOT resolve the emission



Coherence transition

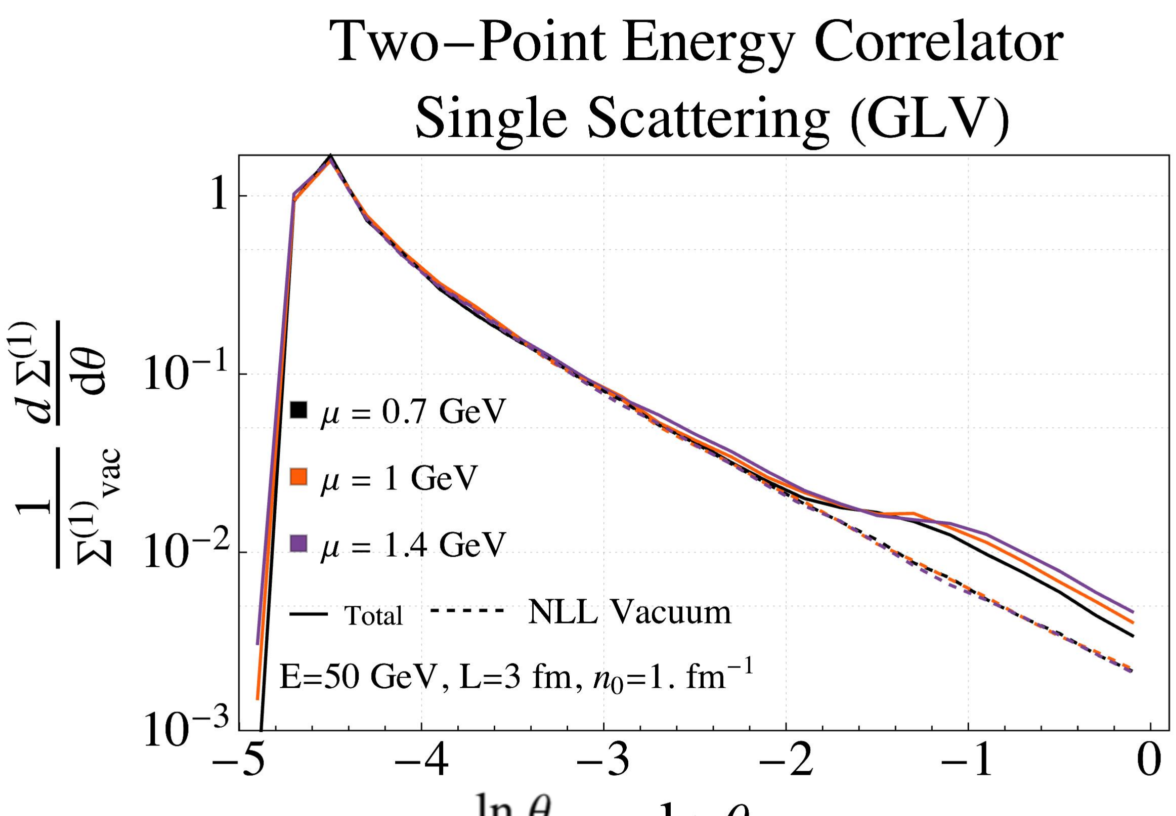
CA, Dominguez, Holguin, Marquet, Moult, 2303.03413



- Extracted the peak angle $\theta_{\rm peak}$ for 332 sets of parameters with $E \in [50,700]$ GeV, $L \in [0.2,10]$ fm, $\hat{q} \in [1,3]\,{\rm GeV}^2/{\rm fm}$
- Performed separate fits in the two different regions for the scaling behavior of the peak angle with respect to the 3 parameters

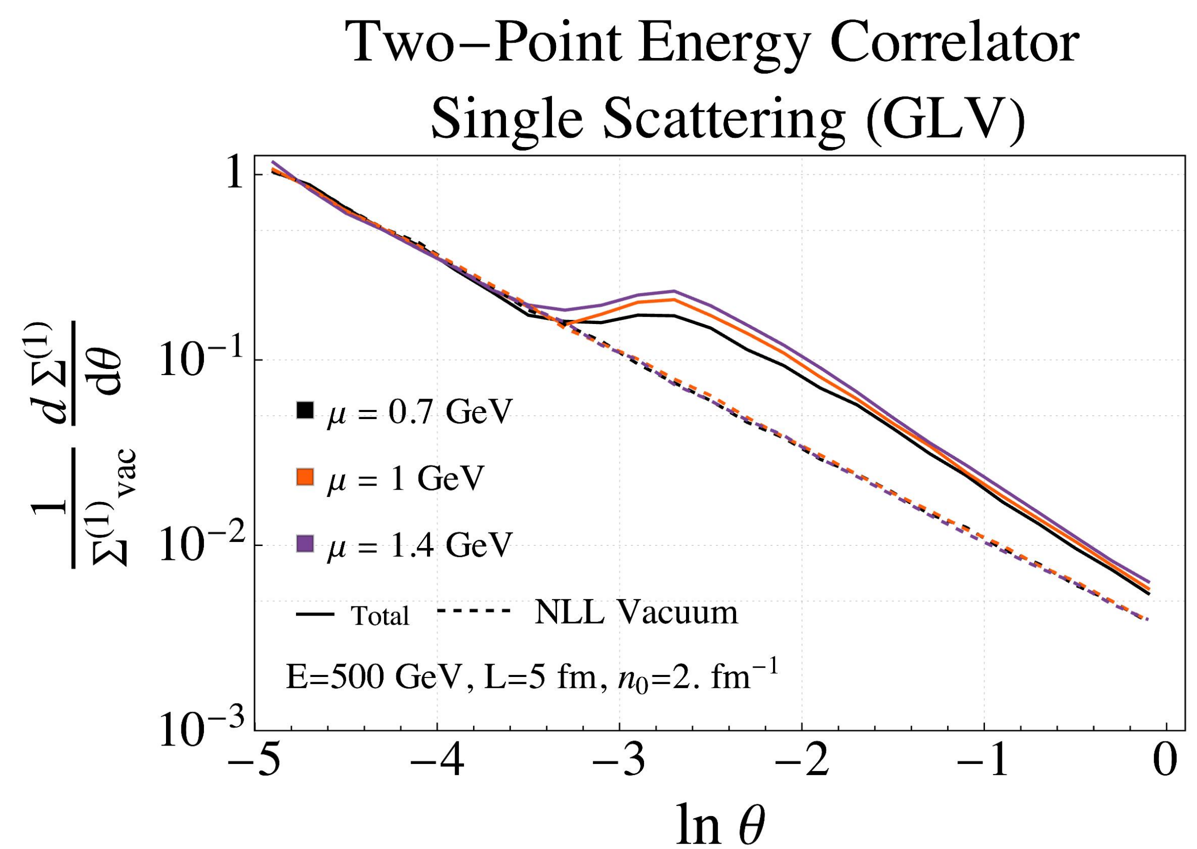
Results GLV

$\theta_L \gg \theta_c$



ln θ

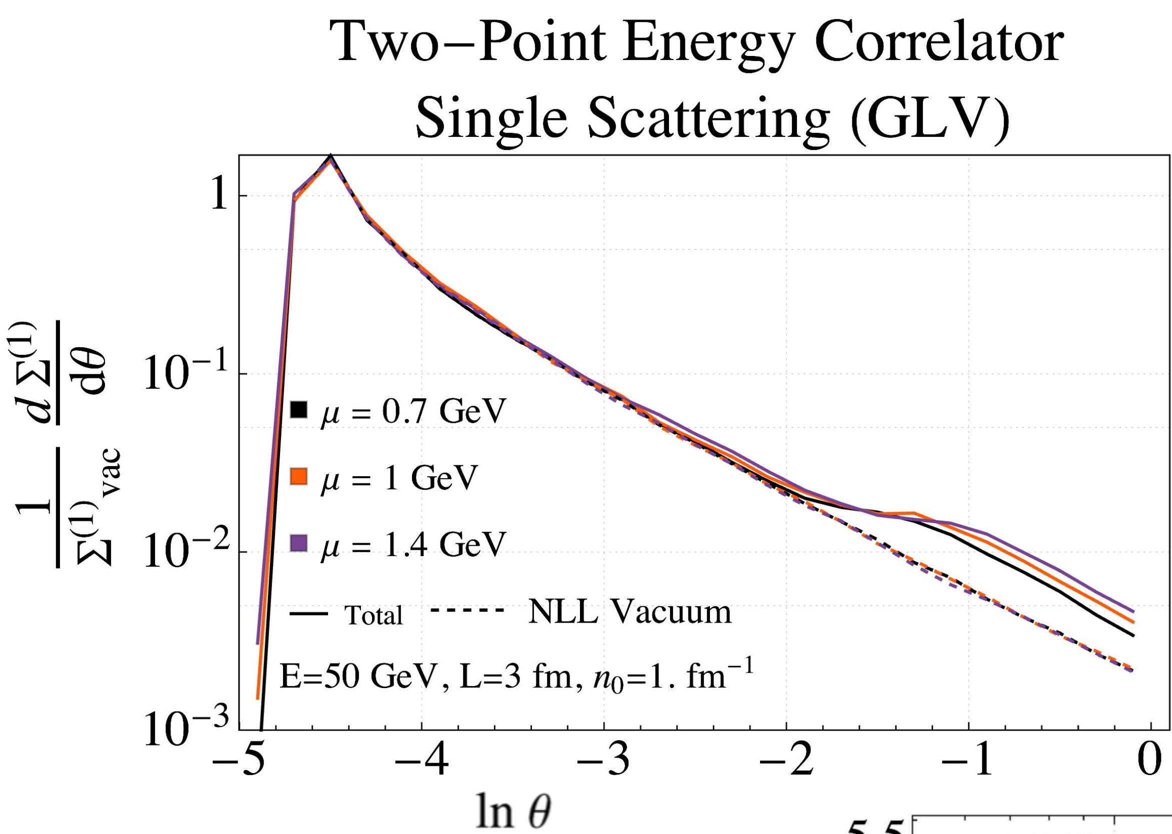
$\theta_L \ll \theta_c$



GLV calculation from:
Ovanesyan, Vitev,
1109.5619

Results GLV

$\theta_L \gg \theta_c$



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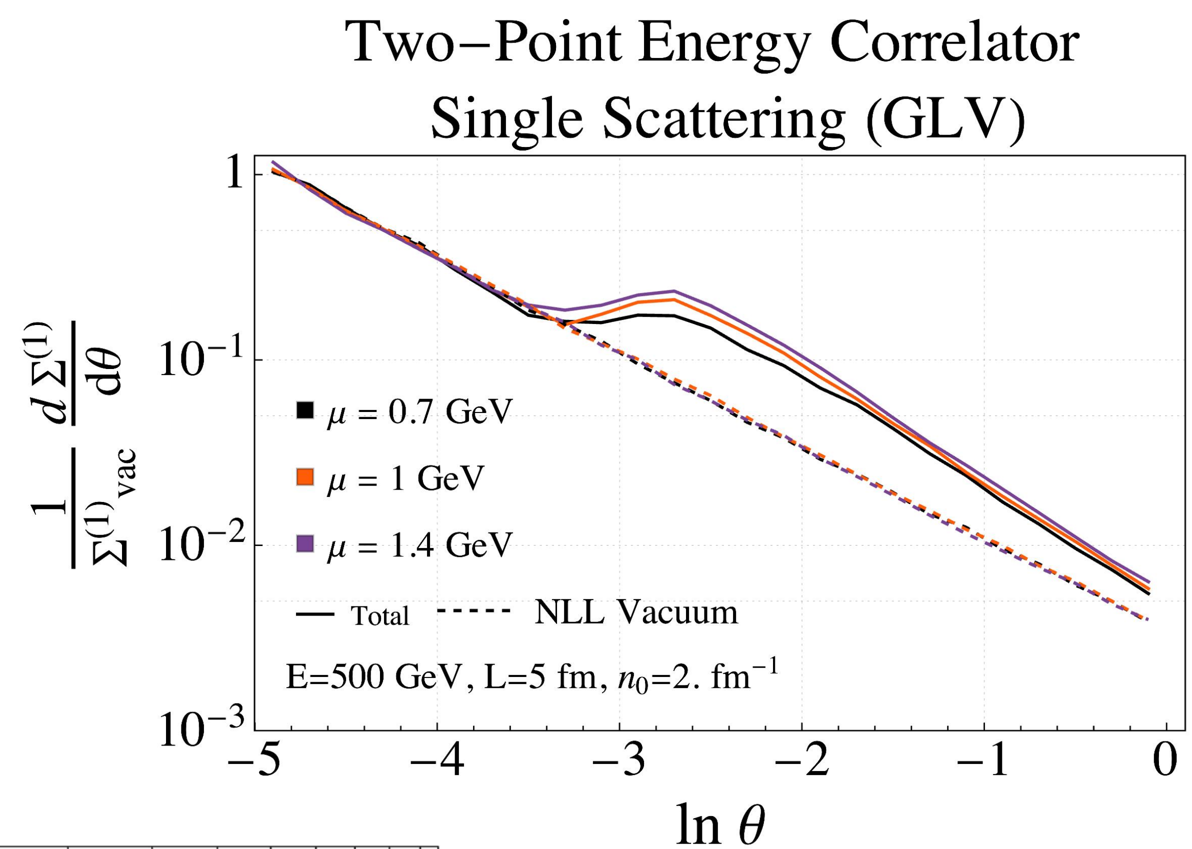
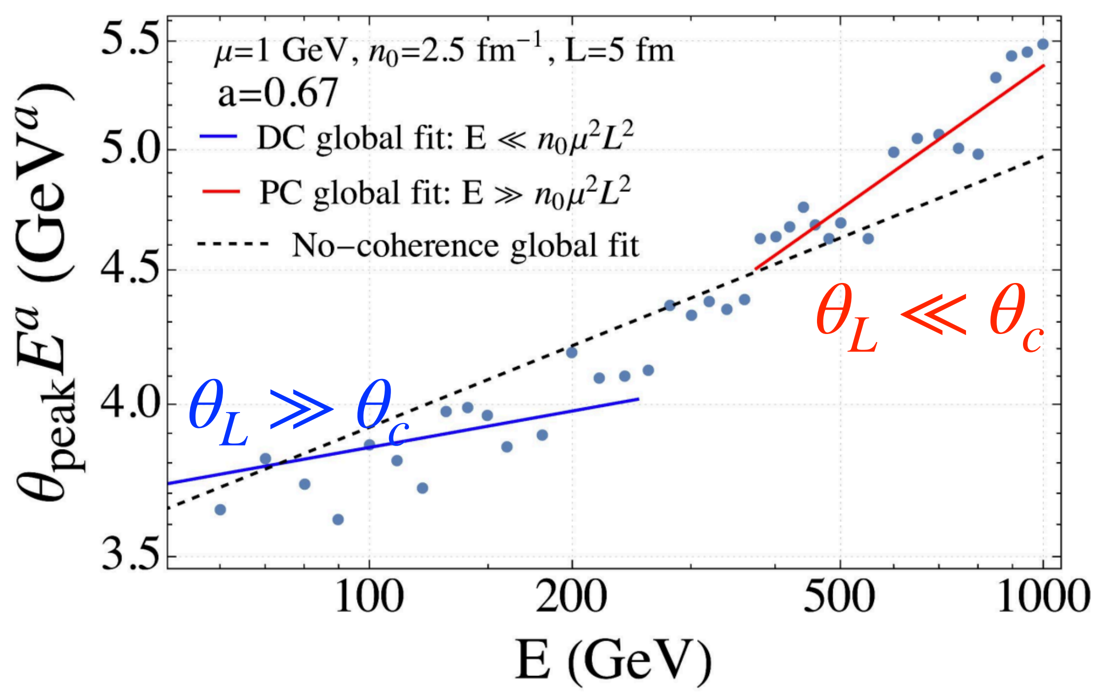




GLV calculation from:
Ovanesyan, Vitev,
1109.5619

Coherence transition not as clearly observed **as** in the **multiple scattering** case

Heavy-flavor jets

CA, Dominguez, Holguin, Marquet, I. Moult, 2307.15110

HF jets

- Same formalism:

$$\frac{d\Sigma^{(n)}}{d\theta} = \left(\frac{1}{\sigma_{qg}}\int dz\,\left(g^{(n)}(\theta,\alpha_s) + F_{med}(z,\theta)\right)\frac{d\sigma^{vac}}{d\theta dz} z^n(1-z)^n\right)\left(1+\mathcal{O}\left(\frac{\bar{\mu}_s}{Q}\right)\right)+\mathcal{O}\left(\frac{\Lambda_{QCD}}{\theta Q}\right)$$

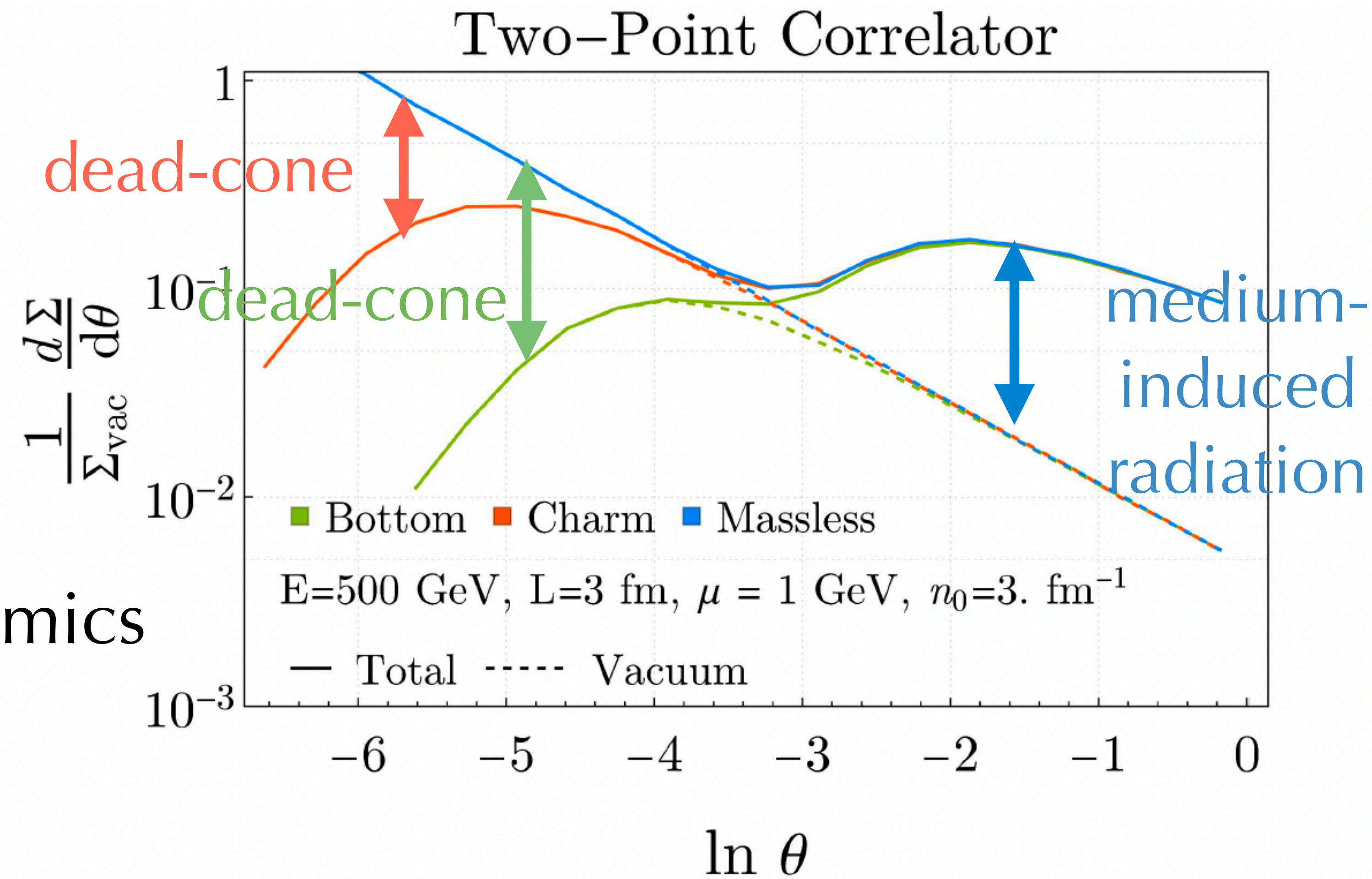
$\mathcal{O}(\alpha_s)$ $Q \to Qg$ collinear splitting function

$$t_f = \frac{2}{z(1-z)E\theta^2} \Rightarrow t_f = \frac{2}{z(1-z)E\left(\theta + \frac{\Theta_0}{1-z}\right)^2}$$

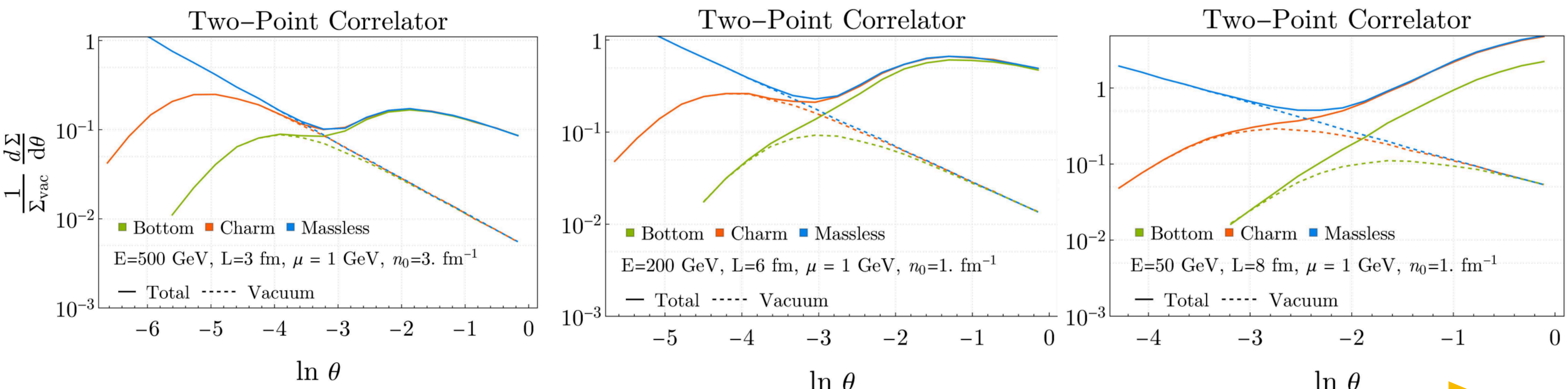
- Two competing scales:

$$\Theta_0 \propto \frac{m_Q}{E} \qquad \theta_L \propto \frac{1}{\sqrt{EL}}$$

If $\theta_L \gg \Theta_0$: two separate dynamics



HF jets: filling the dead-cone



$\frac{\theta_L}{\Theta_0} \to 1$: Filling the dead-cone

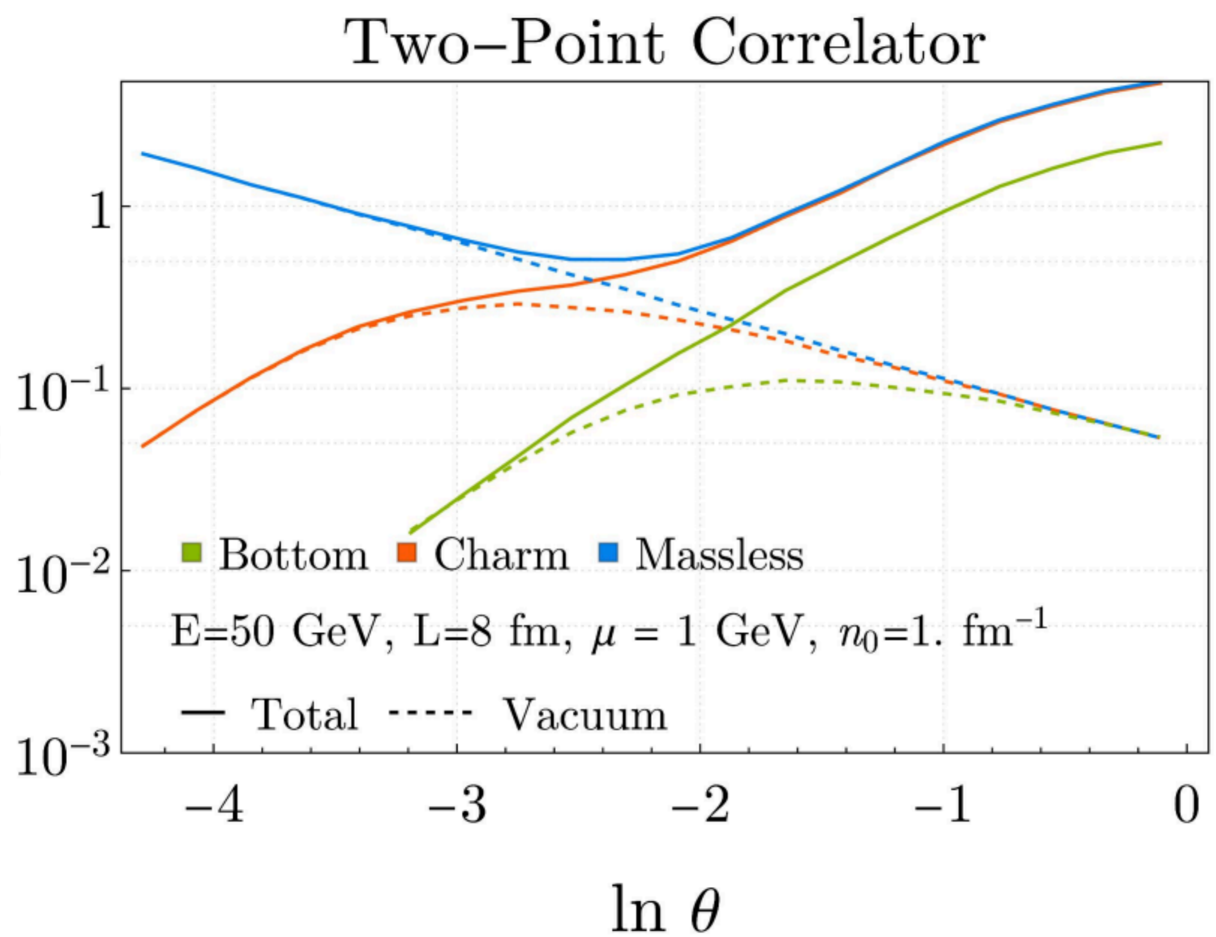
Armesto, Salgado, Wiedemann, arXiv: hep-ph/0312106

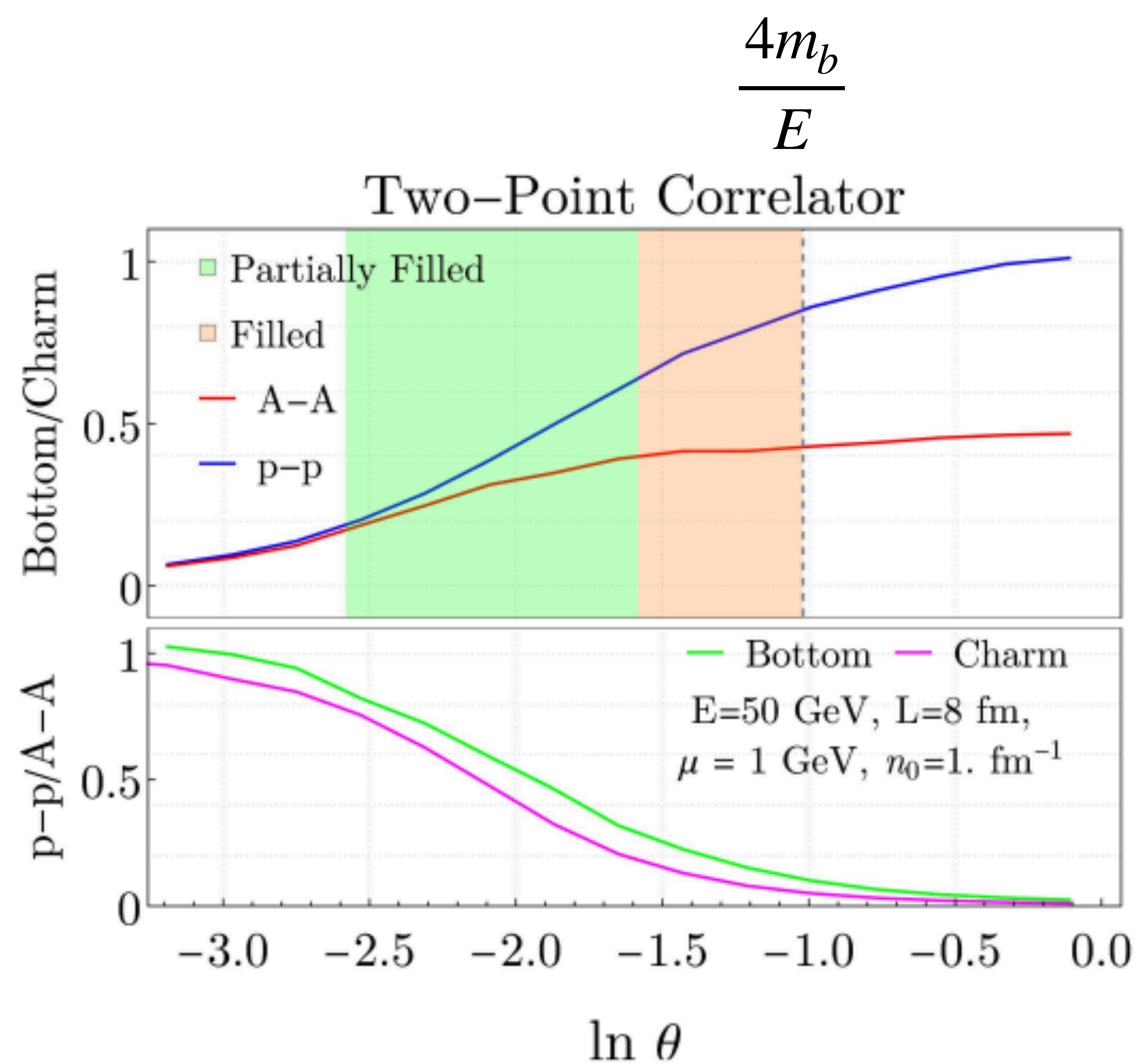
EEC sensitive to the **dead-cone and its medium modifications**

CA, Dominguez, Holguin, Marquet, I. Moult, 2307.15110

HF jets: filling the dead-cone

- Look at the b/c ratio:





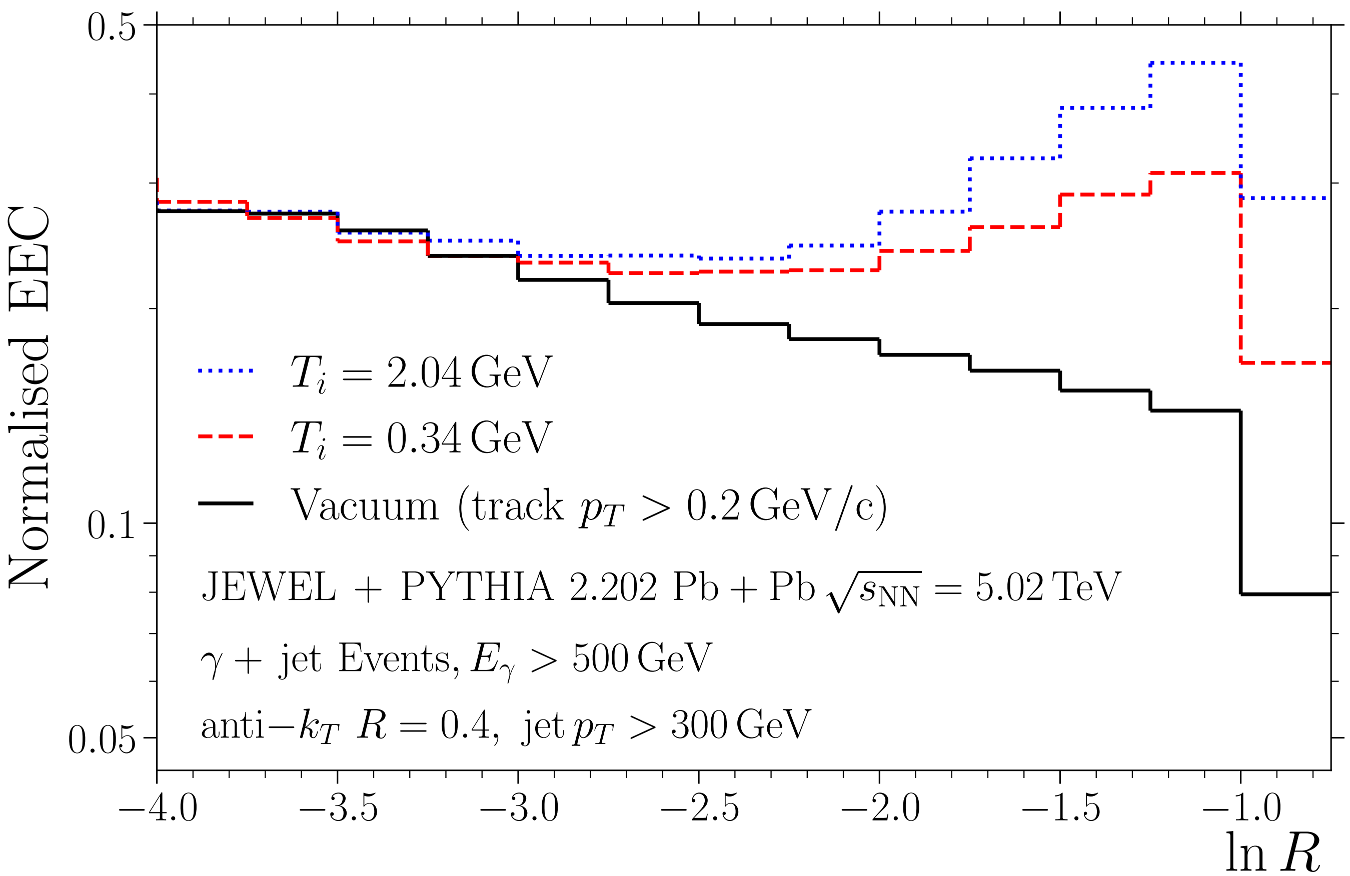
Conclusions

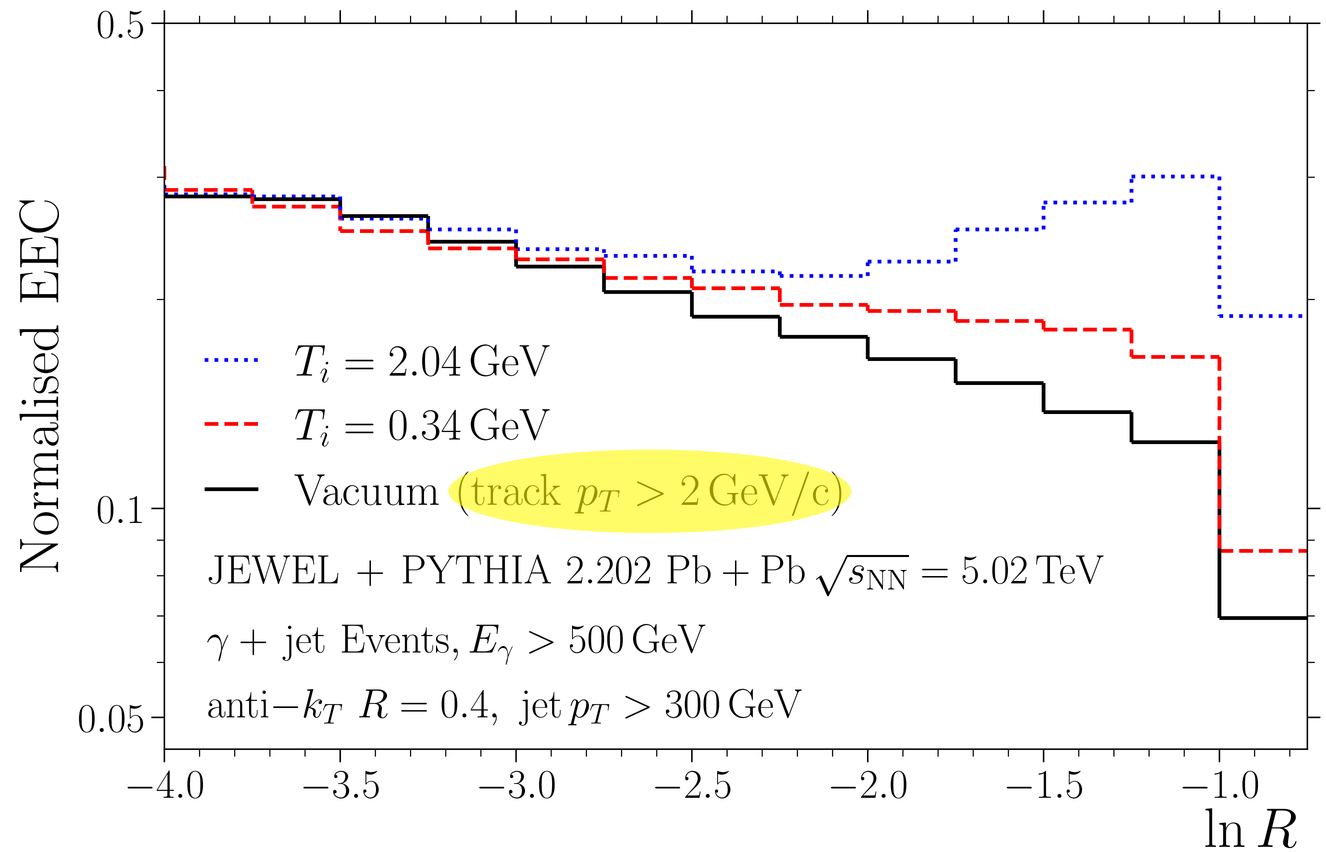
- **Energy Correlators** have **great potential for jet substructure** studies in HICs
    - Experimentally accessible
    - **No need of de-clustering**
    - Expected to be **less sensitive to soft physics** than traditional jet substructure observables: hadronization, and background are usually subleading
    - In **vacuum**: they can be computed perturbatively at very **high accuracy**
- Characteristic features of the calculation of the in-medium splittings are clearly imprinted in these observables
- **Many new developments to come!**

Thank you!

Results from JEWEL

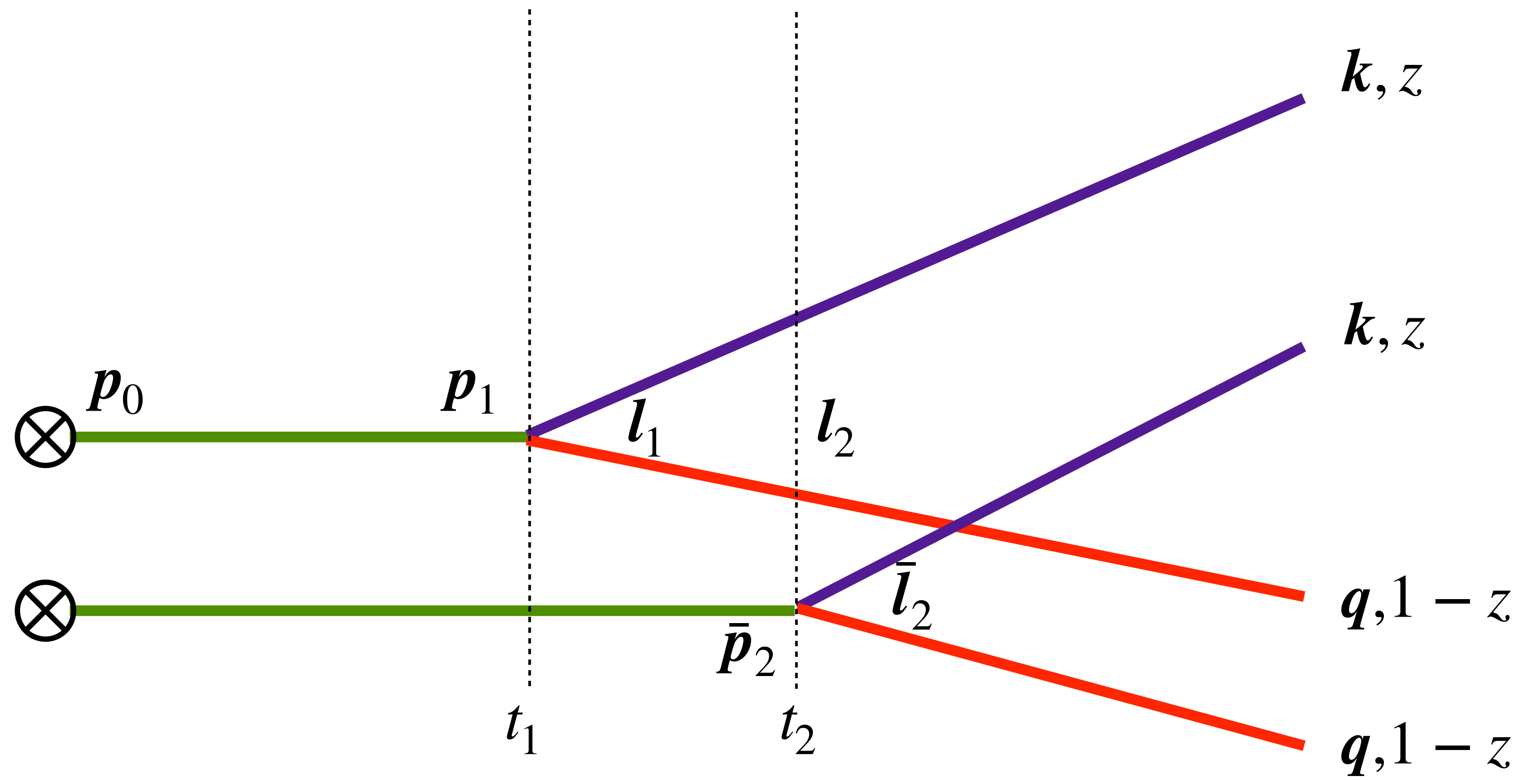
- An analysis on JEWEL is on the way





Features in the curves seem resilient against a hadron cut $p_T \gtrsim 2\,\mathrm{GeV}$

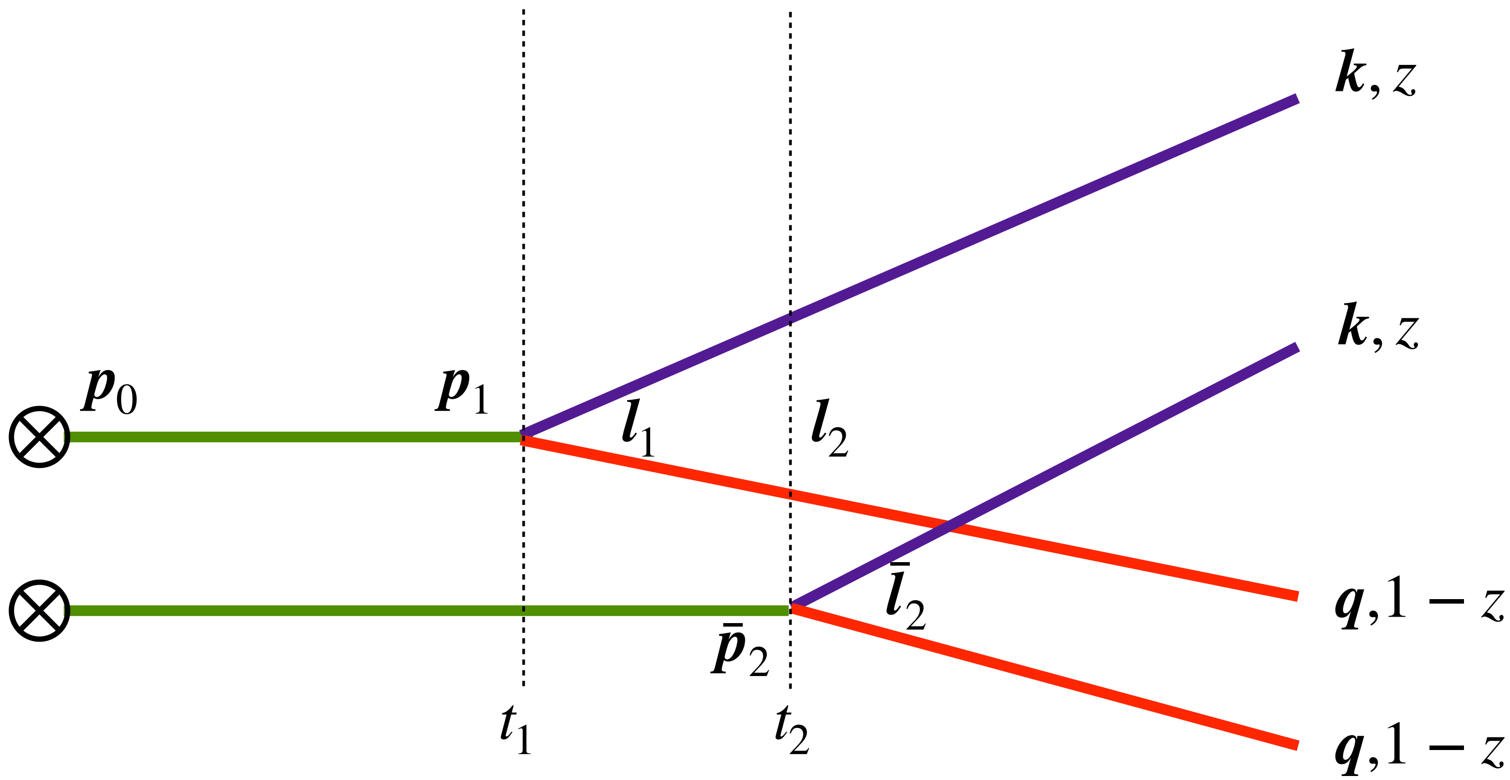
Double differential cross section



$$\frac{d\sigma}{d\Omega_k d\Omega_q} = \frac{g^2}{z(1-z)E^2} P_{a\to bc}(z)\, 2\mathrm{Re} \int_{\boldsymbol{p}_0 \boldsymbol{p}_1 \bar{\boldsymbol{p}}_2 \boldsymbol{l}_1 \boldsymbol{l}_2 \bar{\boldsymbol{l}}_2} \int_{t_0}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 (\boldsymbol{l}_1 \cdot \bar{\boldsymbol{l}}_2)$$
$$\times\, \mathcal{S}^{(4)}((1-z)\boldsymbol{k} - z\boldsymbol{q}, L; \boldsymbol{l}_2, \bar{\boldsymbol{l}}_2, t_2; \boldsymbol{k}+\boldsymbol{q}-\bar{\boldsymbol{p}}_2, z)$$
$$\times\, \mathcal{K}^{(3)}(\boldsymbol{l}_2, t_2; \boldsymbol{l}_1, t_1; \bar{\boldsymbol{p}}_2 - \boldsymbol{p}_1, z)\, \mathcal{P}_{R_a}(\boldsymbol{p}_1 - \boldsymbol{p}_0; t_1, t_0) \frac{d\sigma_{hard}}{d\Omega_{p_0}}$$

Blaizot, Iancu, FD, Mehtar-Tani 1209.4585
Apolinario, Armesto, Milhano, Salgado 1407.0599

Double differential cross section



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$$\left\langle \mathcal{G}\mathcal{G}^{\dagger} \right\rangle$$

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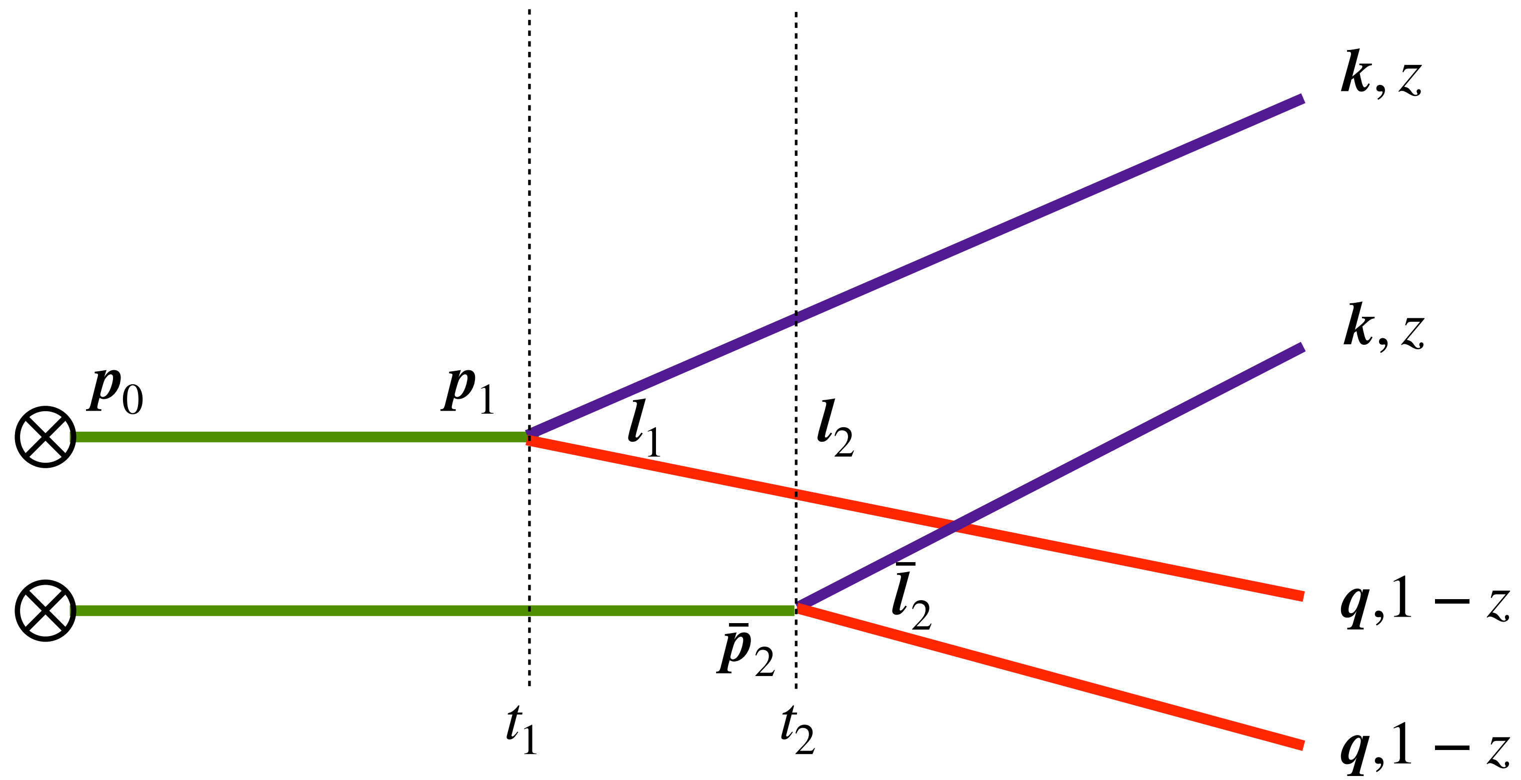

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$\langle \mathcal{G}\mathcal{G}\mathcal{G}^\dagger \rangle$ $\langle \mathcal{G}\mathcal{G}^\dagger \rangle$

Blaizot, Iancu, FD, Mehtar-Tani 1209.4585
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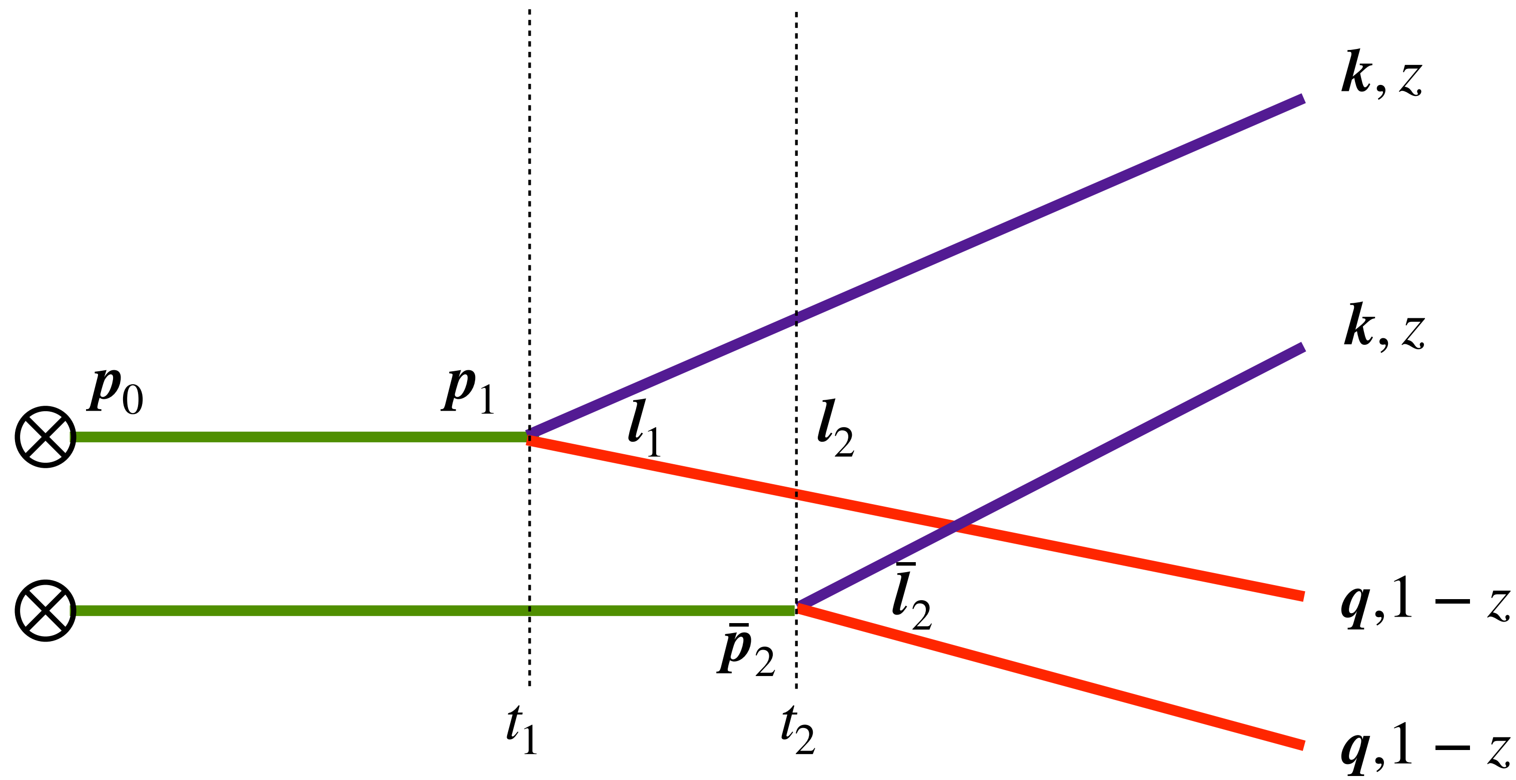

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$\mathcal{S}^{(4)}$: $\langle \mathcal{G}\mathcal{G}\mathcal{G}^\dagger\mathcal{G}^\dagger \rangle$

$\mathcal{K}^{(3)}$: $\langle \mathcal{G}\mathcal{G}\mathcal{G}^\dagger \rangle$

$\mathcal{P}_{R_a}$: $\langle \mathcal{G}\mathcal{G}^\dagger \rangle$

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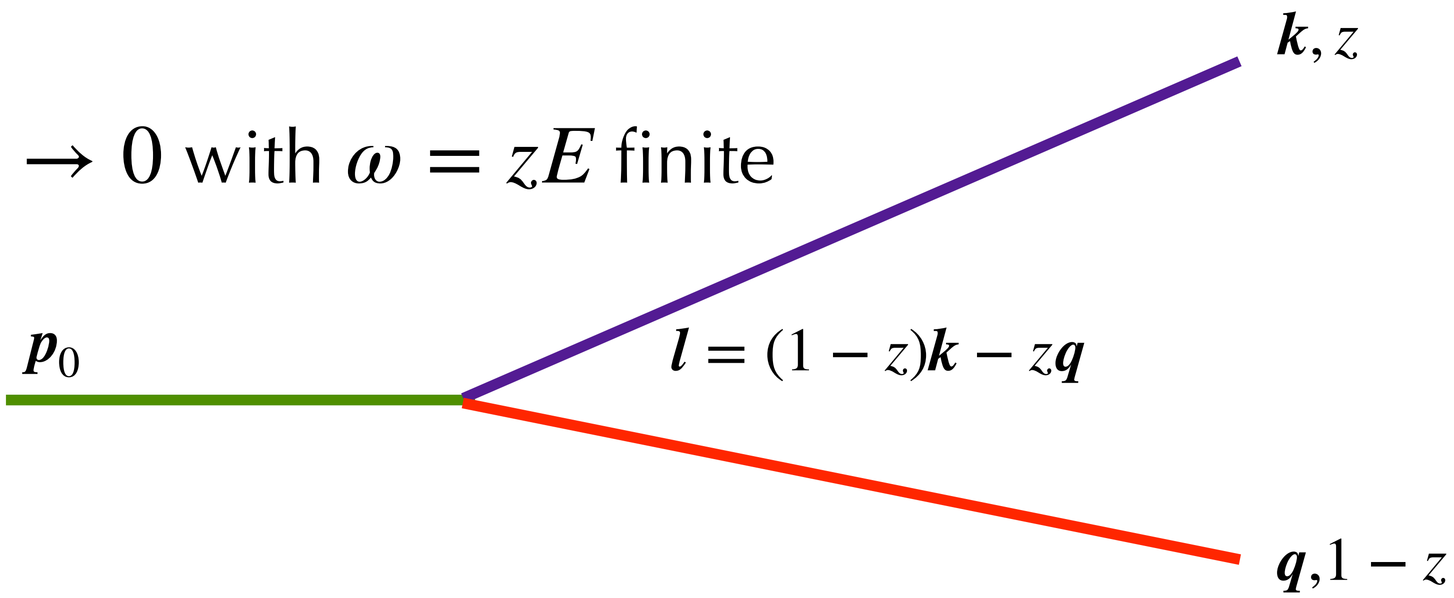
Soft limit

$z \to 0$ with $\omega = zE$ finite



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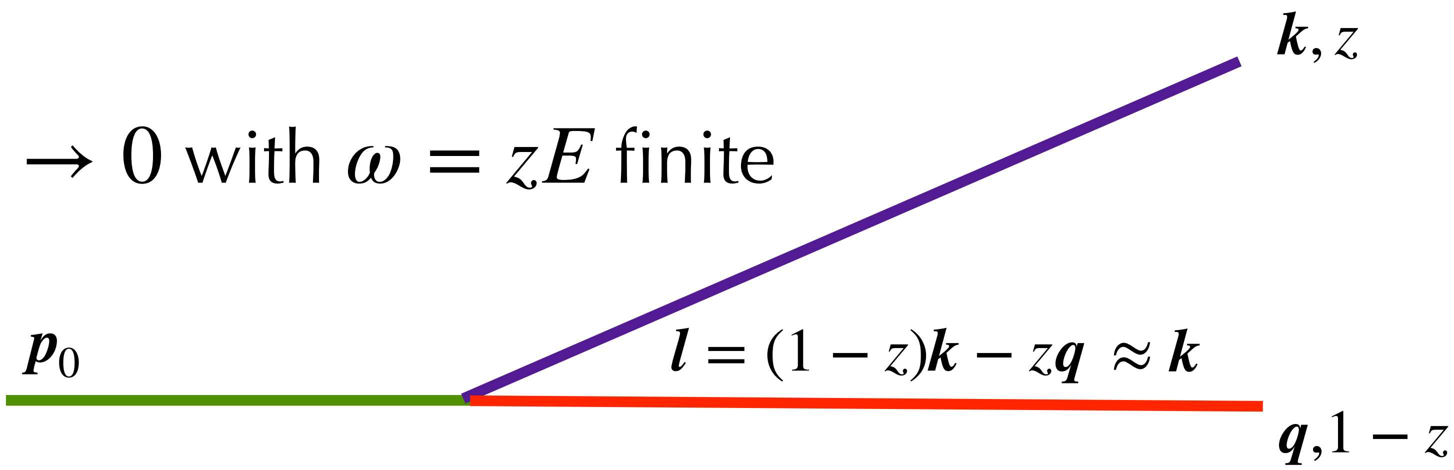
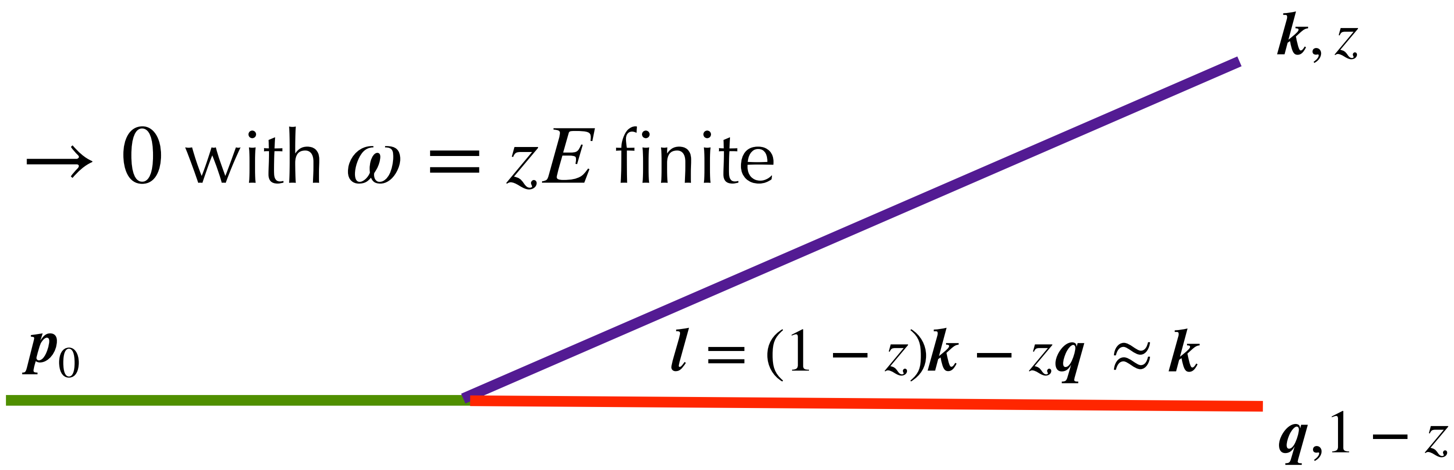


Angle of emission depends only on transverse momentum of the soft particle

Soft limit

$z \to 0$ with $\omega = zE$ finite



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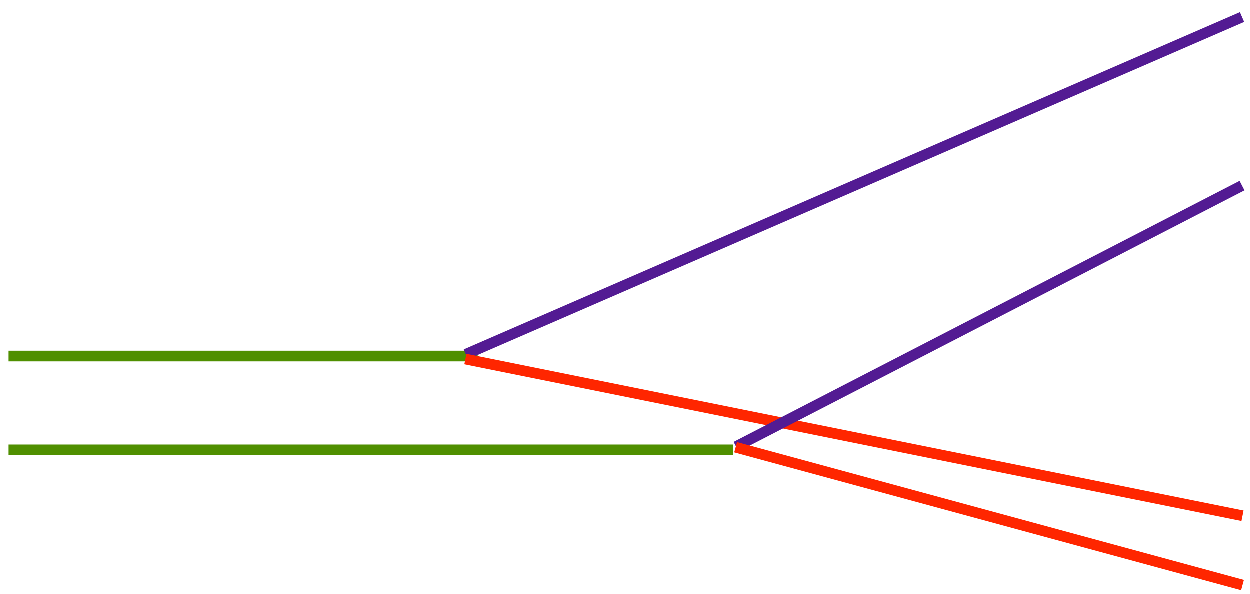

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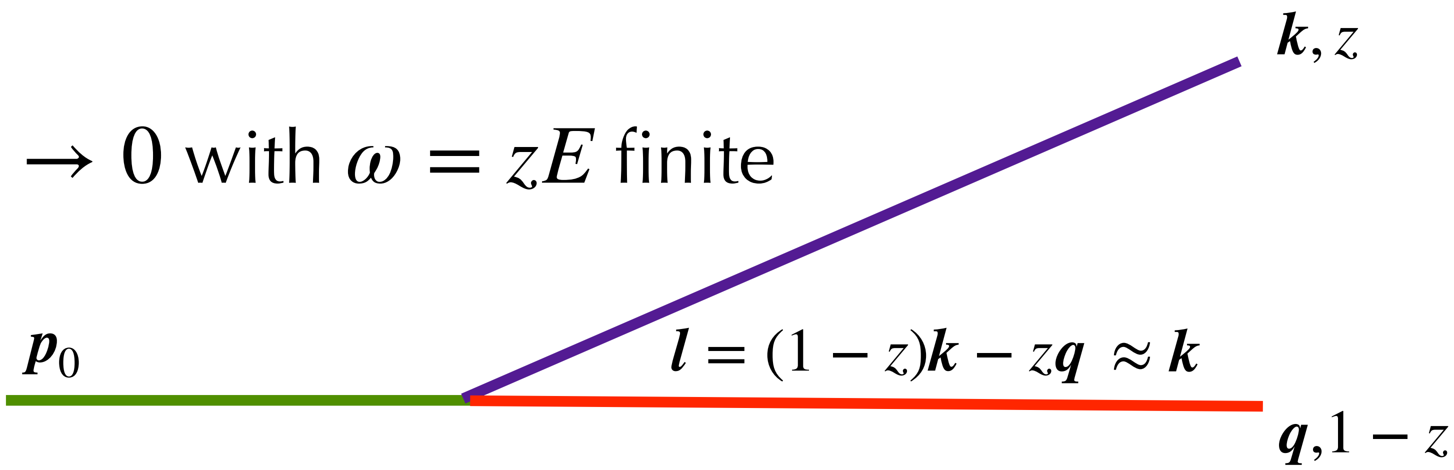

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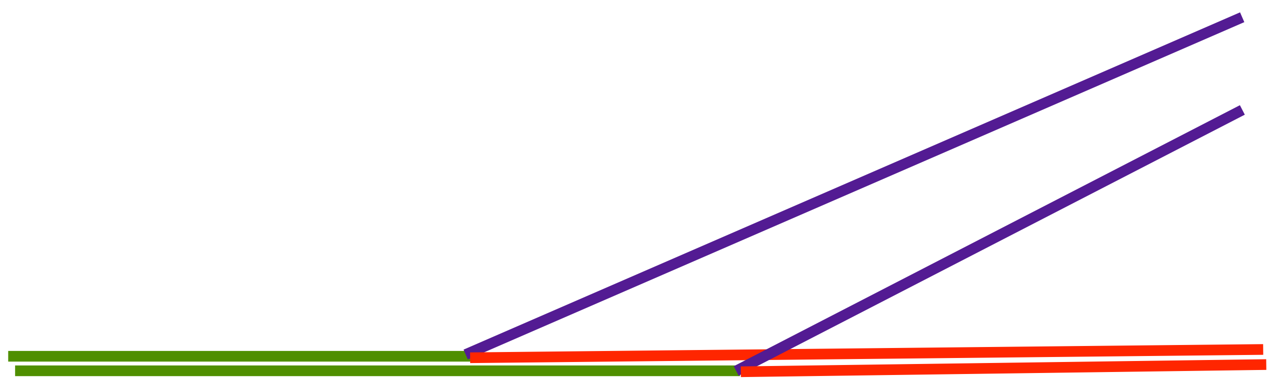

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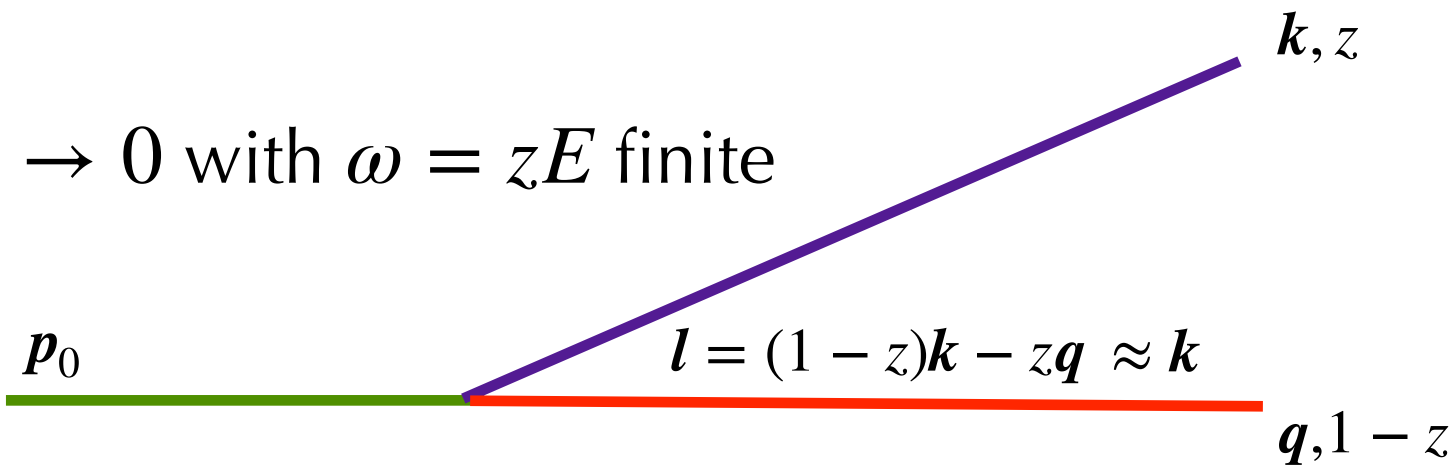

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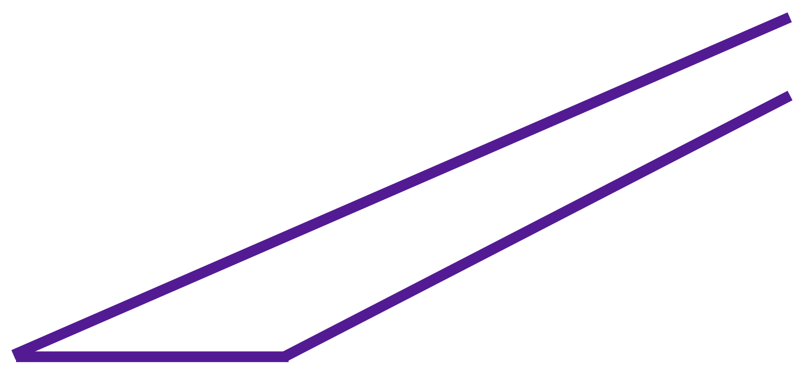
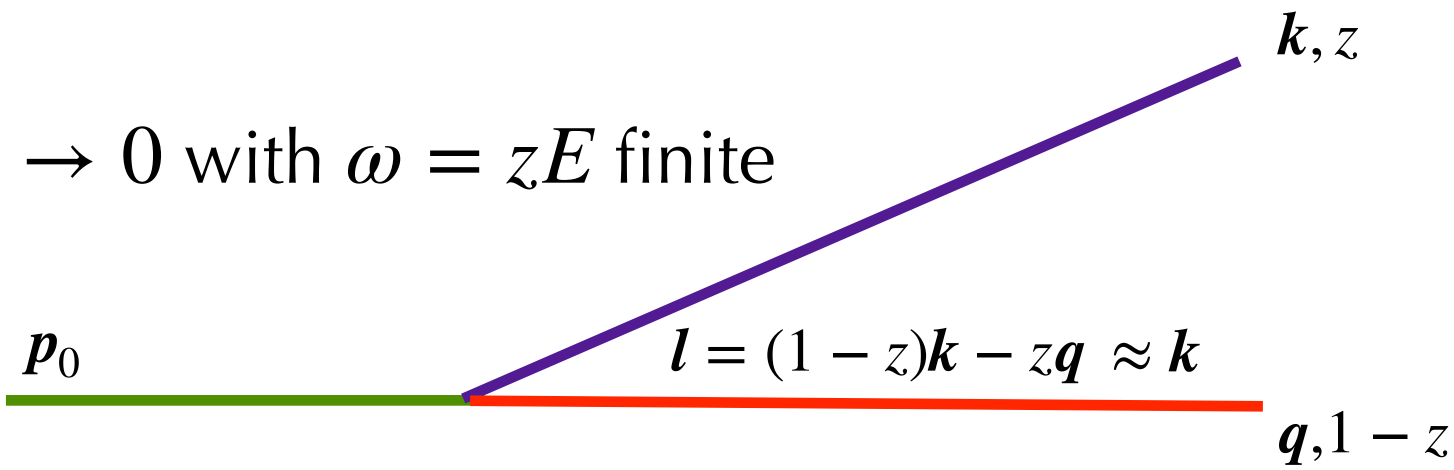


Initial and final broadening of the hard particle cancels out

Soft limit

$z \to 0$ with $\omega = zE$ finite



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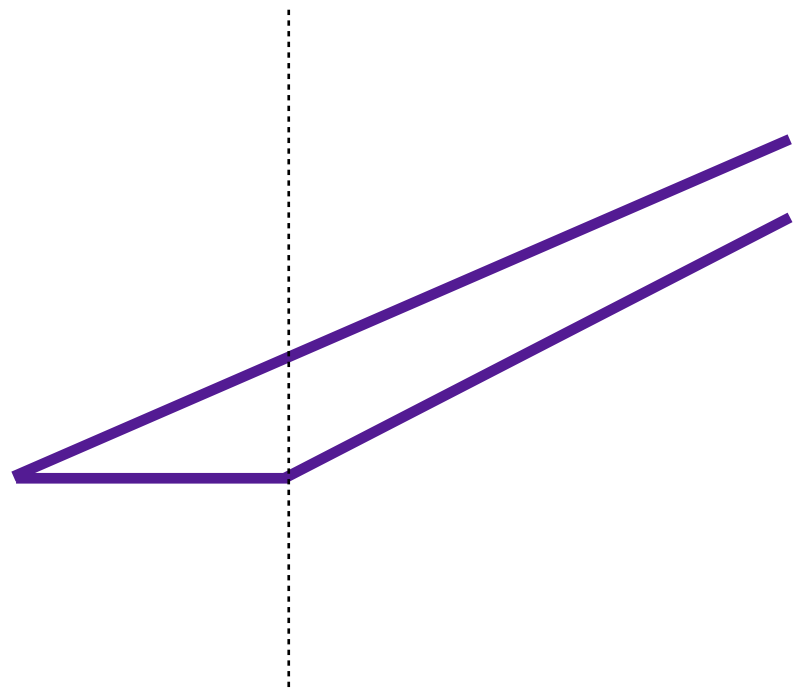

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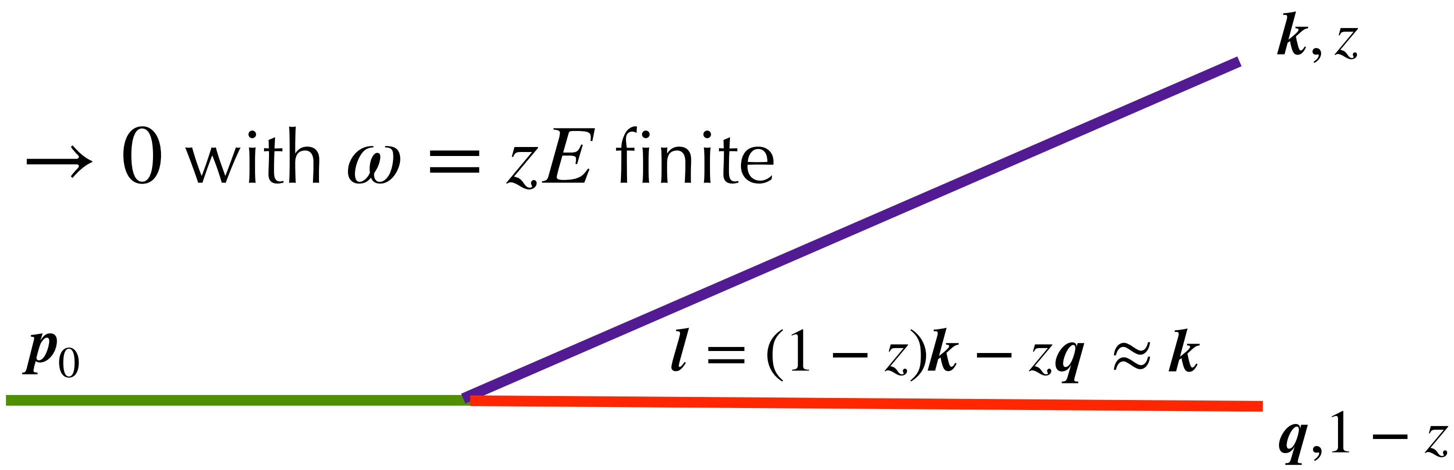

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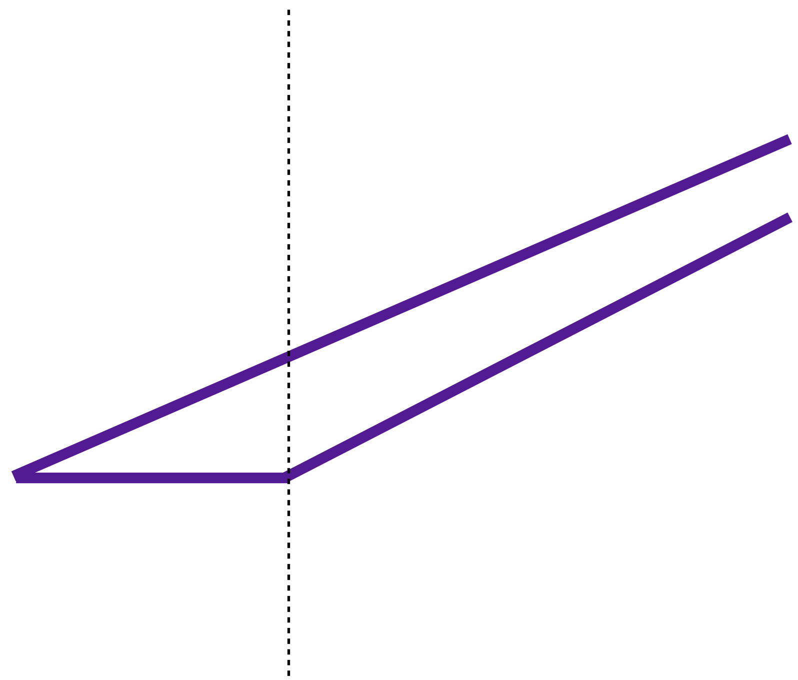

Initial and final broadening of the hard particle cancels out

$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega^2} \mathrm{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \widetilde{\mathcal{K}}(t',\boldsymbol{q};t,\boldsymbol{p})\mathcal{P}(\infty,\boldsymbol{k};t',\boldsymbol{q})$$

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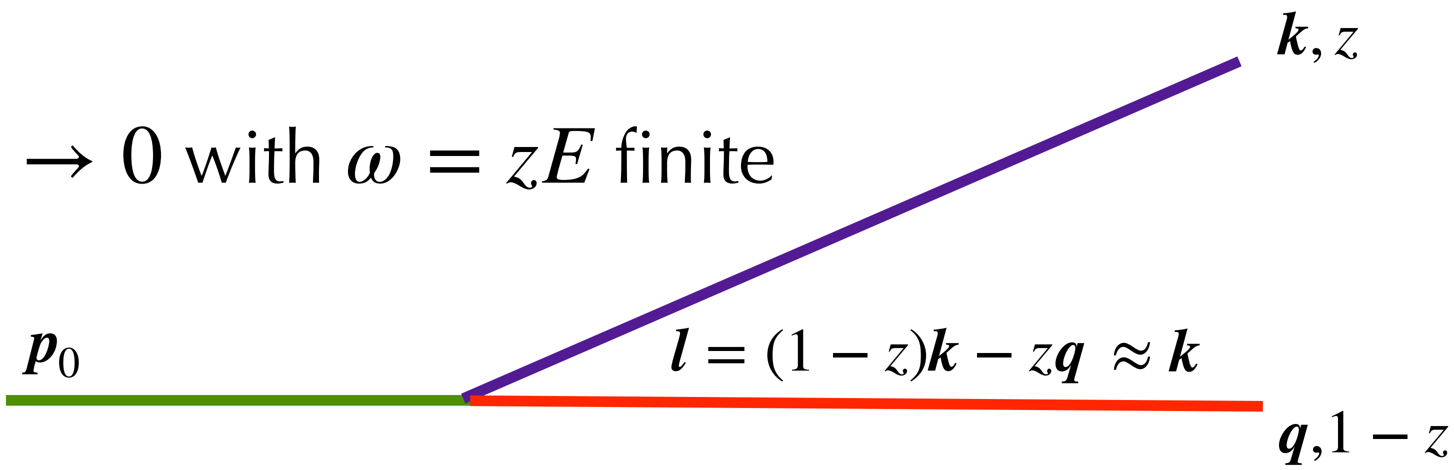

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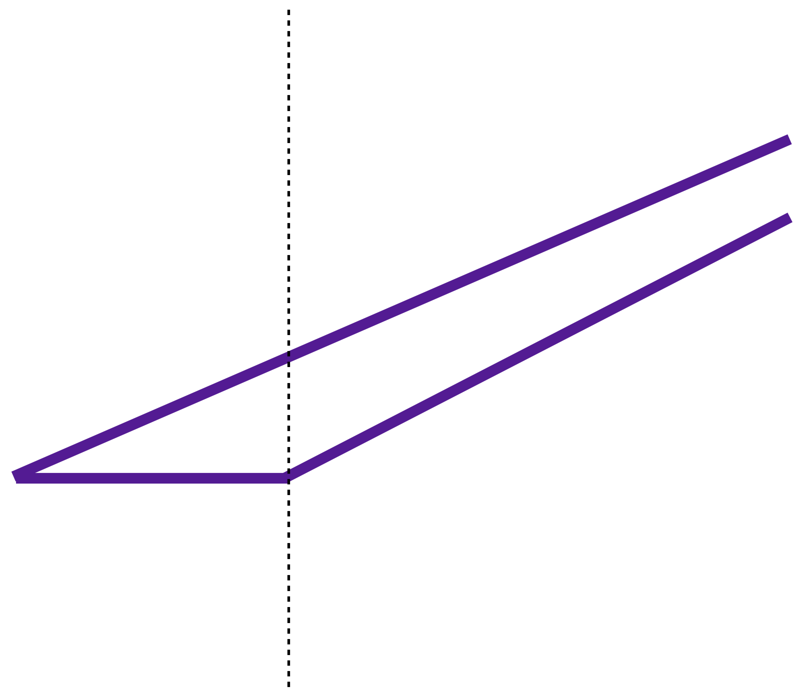

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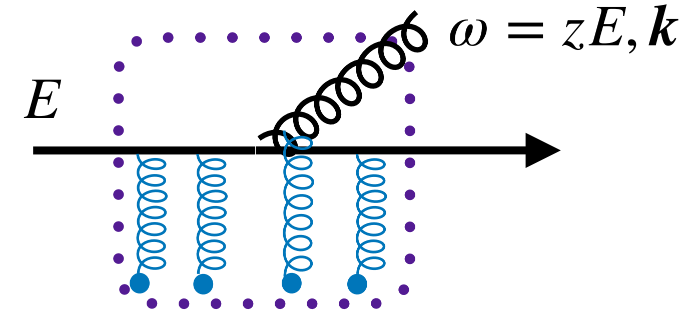
Recently evaluated numerically with multiple scatterings and realistic interactions

Andres, Apolinario, Dominguez 2002.01517

Andres, Dominguez, Gonzalez Martinez 2011.0652

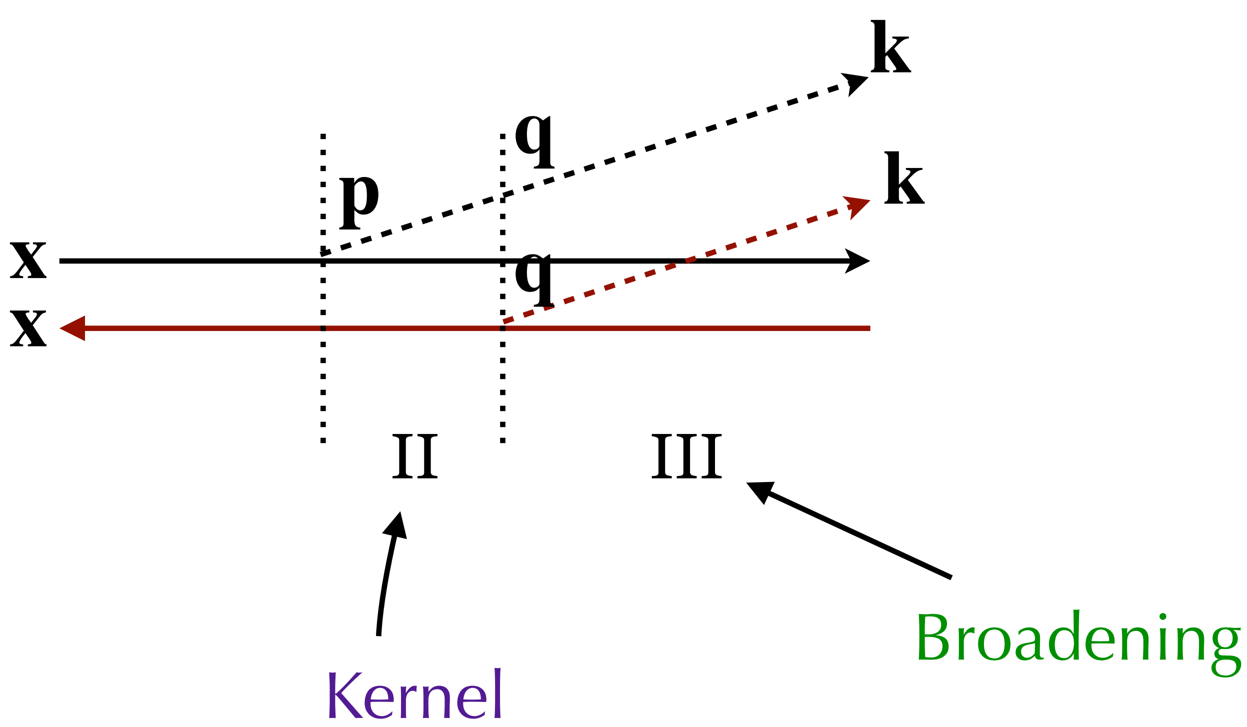
Medium-induced gluon spectrum

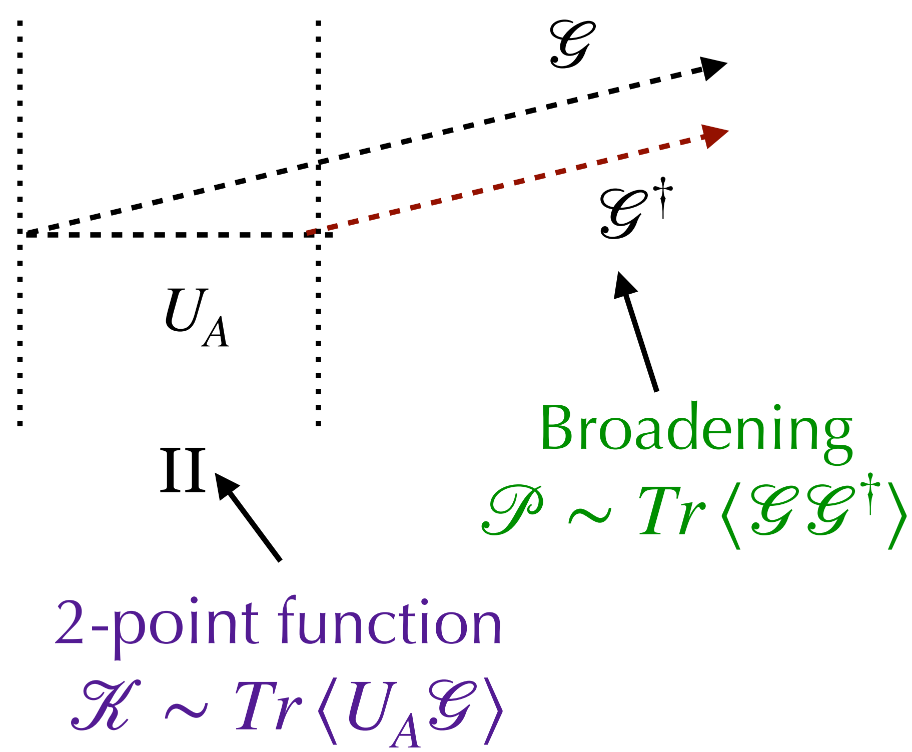


- For a soft emitted gluon ($z \ll 1$)

$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega^2} \mathrm{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \widetilde{\mathcal{K}}(t', \boldsymbol{q}; t, \boldsymbol{p}) \mathcal{P}(\infty, \boldsymbol{k}; t', \boldsymbol{q})$$

BDMPS-Z





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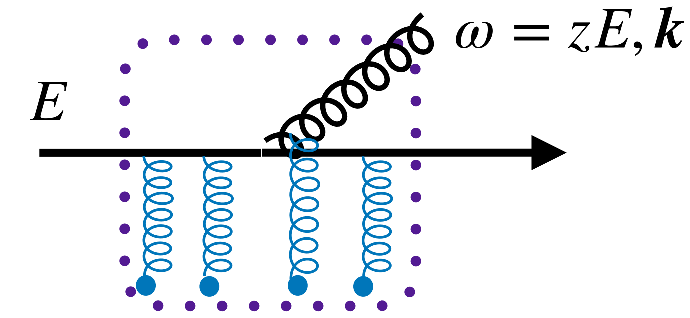

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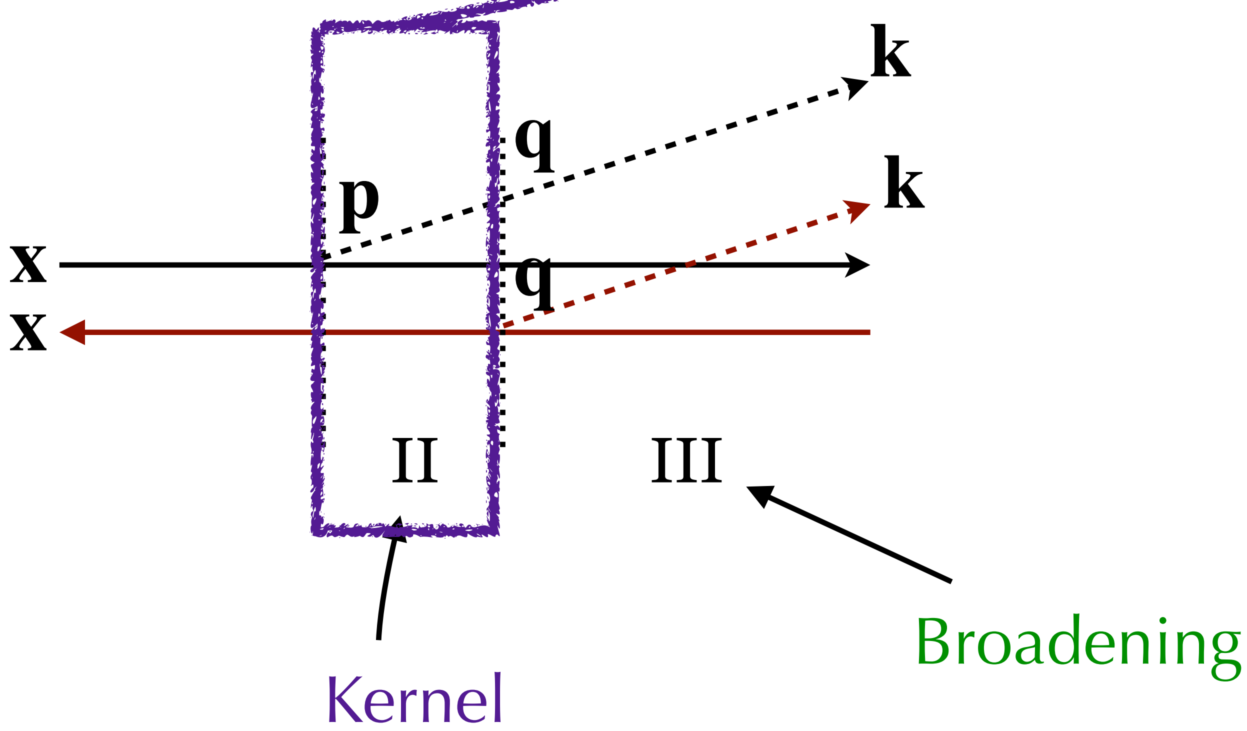

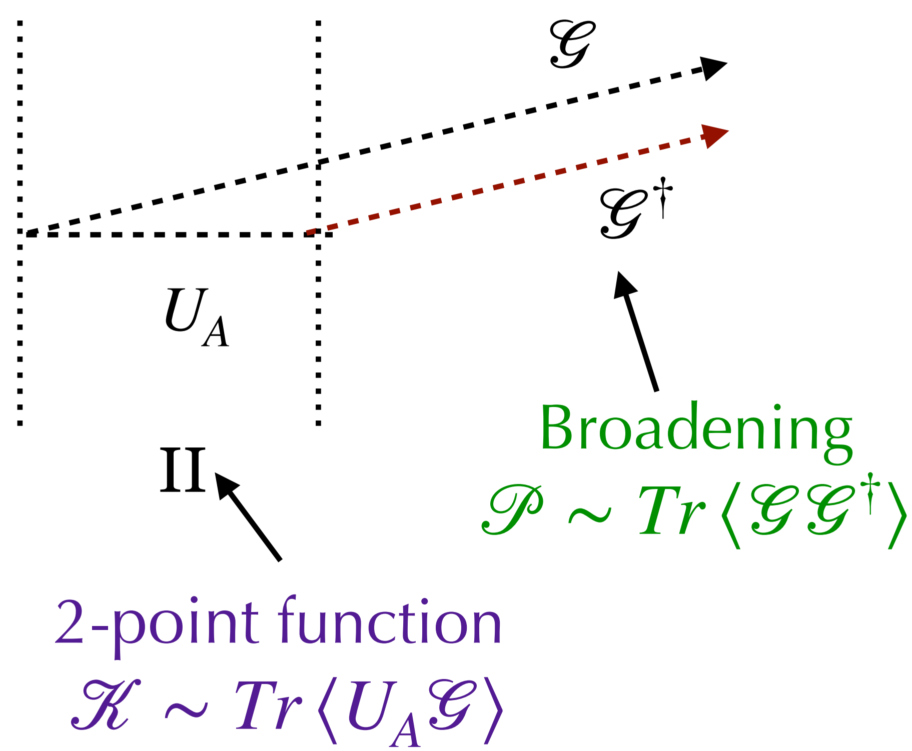

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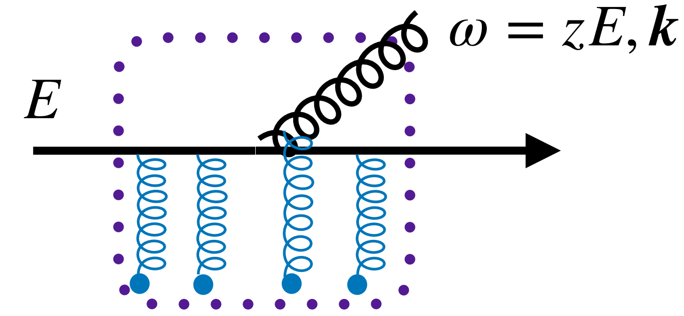

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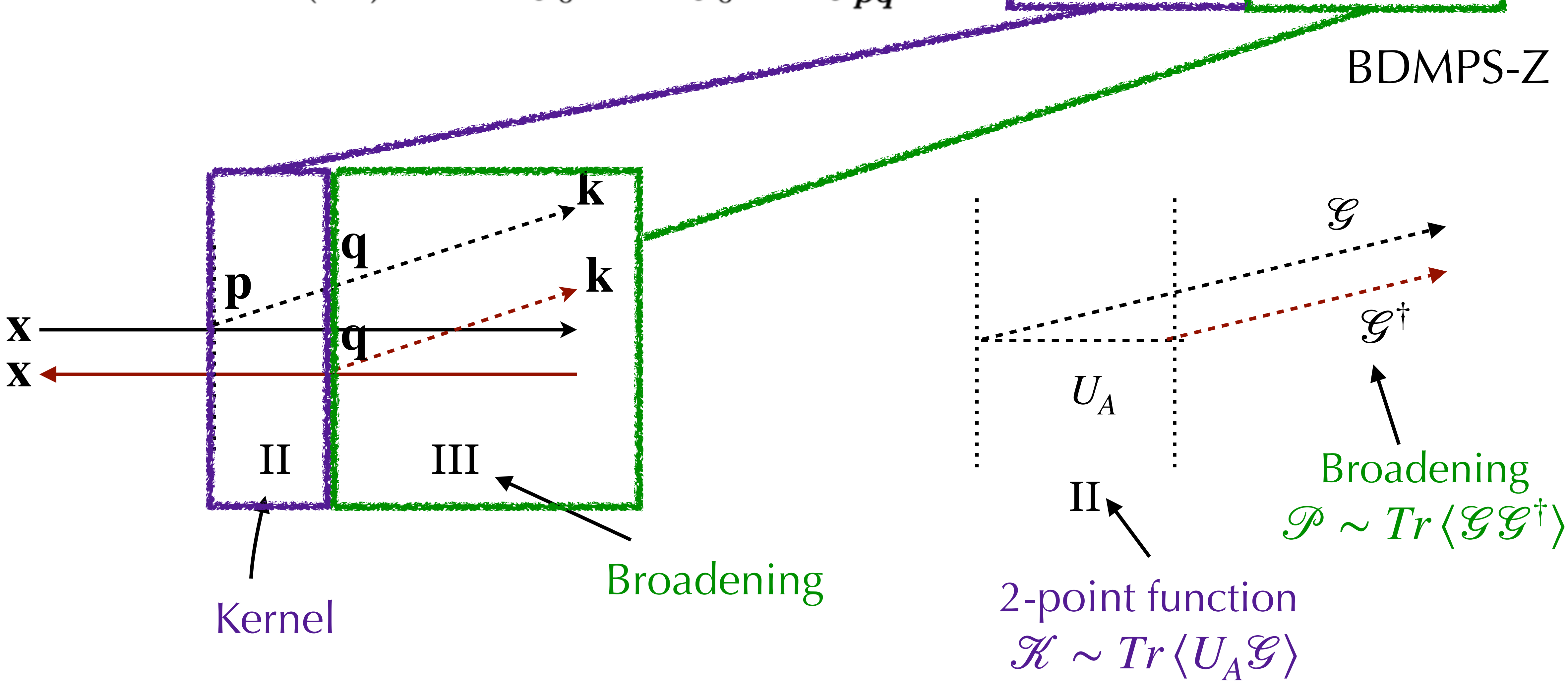

Semi-hard approximation

Dominguez, Milhano, Salgado, Tywoniuk, Vila 1907.03653
Isaksen, Tywoniuk 2107.02542

- Use high-energy limit of propagators: vacuum propagator times a Wilson line in the classical trajectory

$$\mathcal{G}_R(t_2,\boldsymbol{p}_2;t_1,\boldsymbol{p}_1;\omega) \to (2\pi)^2\delta^{(2)}(\boldsymbol{p}_2-\boldsymbol{p}_1)\, e^{-i\frac{p_2^2}{2\omega}(t_2-t_1)}V_R(t_2,t_1;[\boldsymbol{n}t])$$

- Calculate averages of Wilson lines in the large-N_c limit (calculations also available for finite N_c). All averages can be expressed in terms of fundamental dipoles and quadrupoles

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\, V_1 V_2^\dagger \right\rangle = S_{12} \qquad \frac{1}{N_c}\left\langle \mathrm{Tr}\, V_1 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger \right\rangle = Q$$