Event-by-event hadron correlations in ALICE experiment

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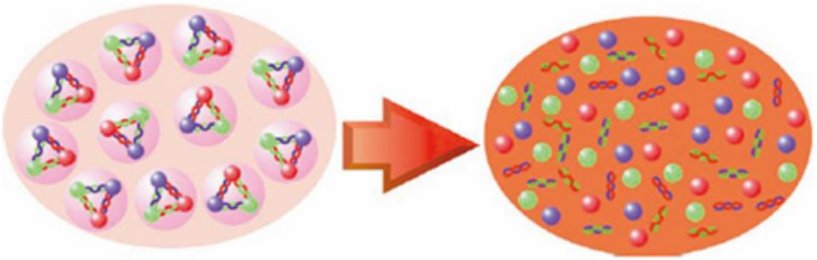
Outline

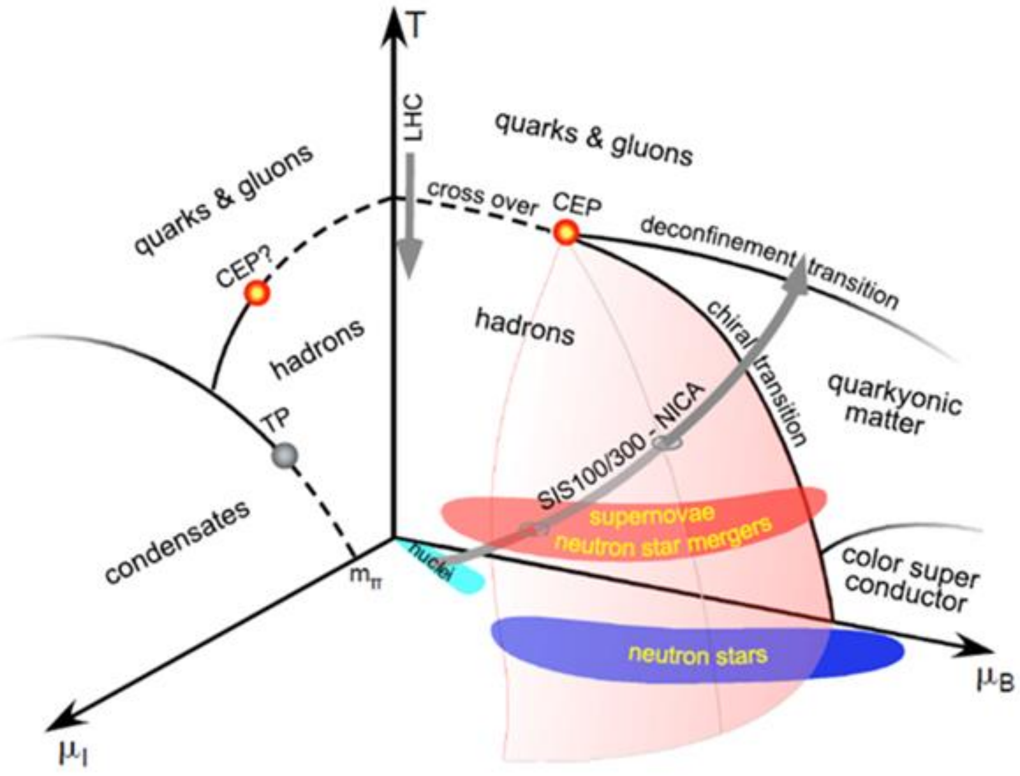
- Introduction
  - QCD phase diagram and criticality
- Fluctuations and correlations
  - EbyE physics
- Recent EbyE publications in ALICE

Quark-Gluon Plasma and QCD

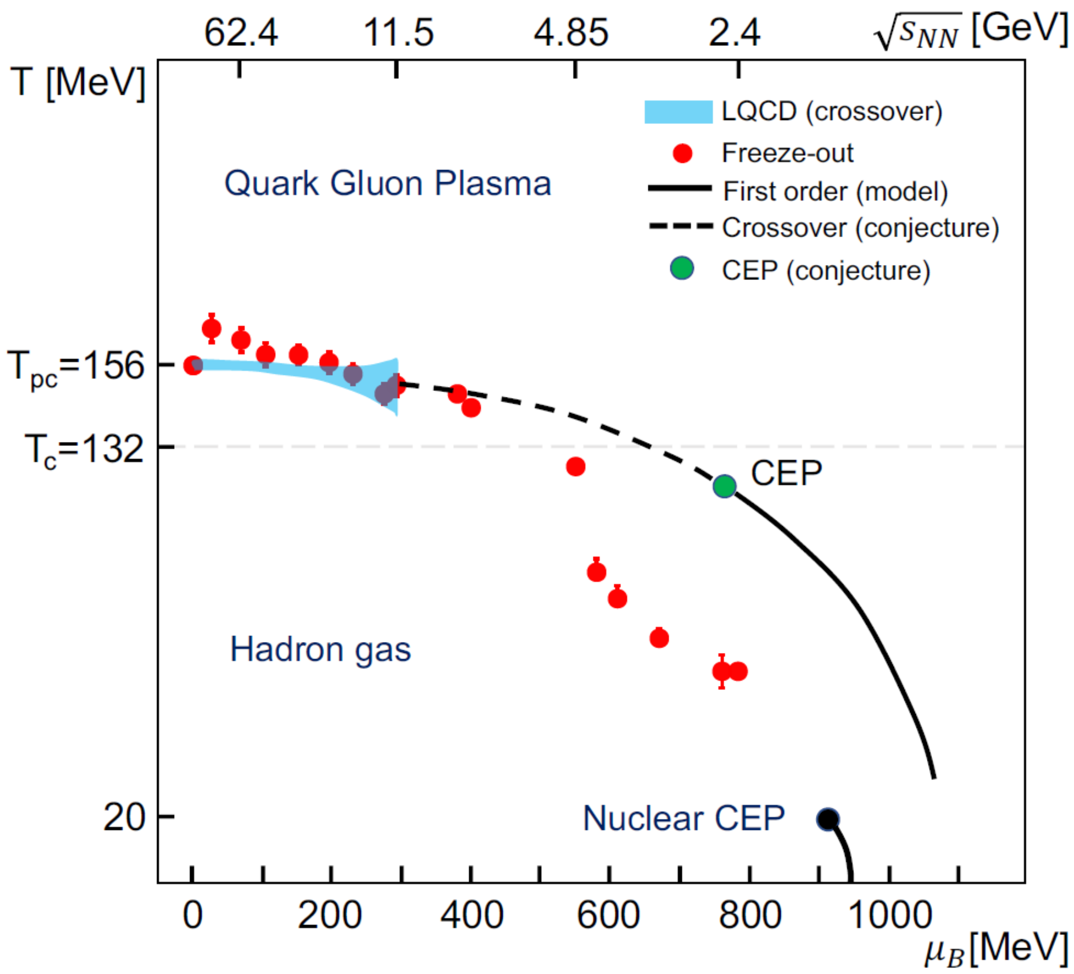
- Extreme state of matter in which quarks and gluons can move freely over distances comparable to the size of hadrons



- Phase diagram of Quantum Chromodynamics



Nature of QCD phase transitions



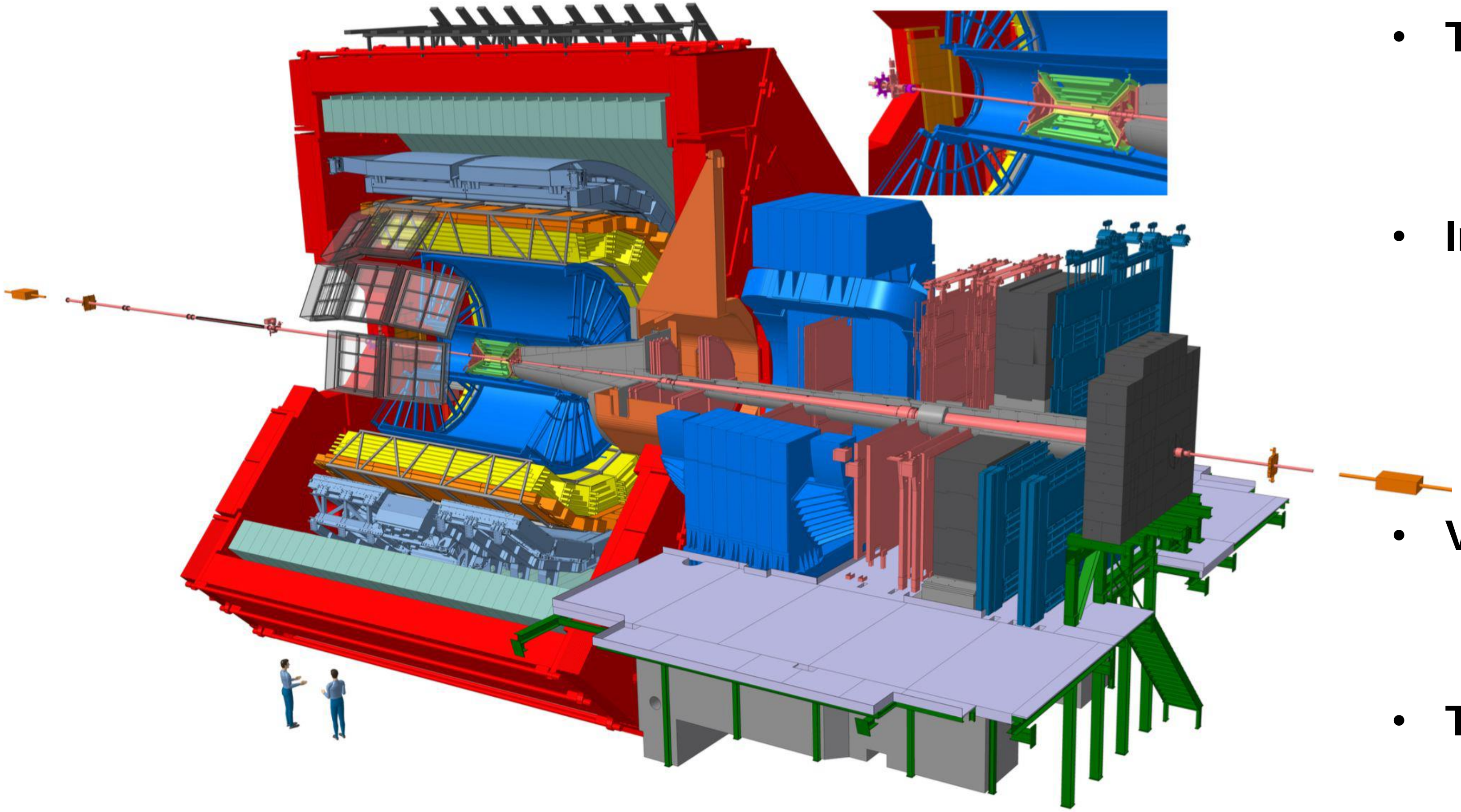
- QCD phase diagram of strongly interacting nuclear matter can be explored in ultrarelativistic heavy-ion collisions.
- The state of quark–gluon plasma (QGP) is probed as a function of temperature and baryon chemical potential.
- What is the nature of phase transitions in QCD phase diagram (smooth crossover, 1st or 2nd order phase transition, etc.)?
- Existence of critical point?

F. Gross et al., *"50 Years of Quantum Chromodynamics"*, 2212.11107

Event-by-event (EbyE) physics

- Physical quantities are expected to display a qualitatively different behaviour in case of a phase transition, and can be signalled by anomalous fluctuations and correlations in a number of observables. H. Heiselberg *Phys.Rept.* 351 (2001) 161-194
- EbyE fluctuations of multiplicity, net-charge, mean transverse momentum, etc., can be used to probe dynamical fluctuations due to production QGP.

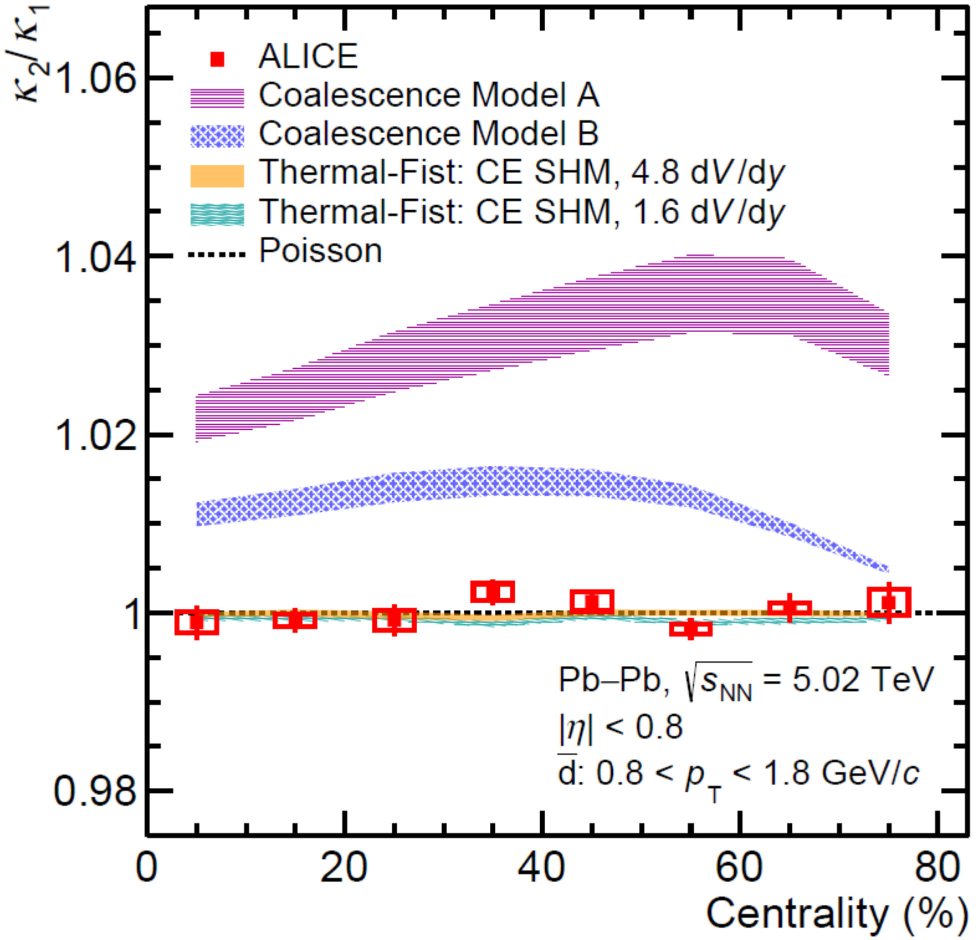
ALICE detector in Run 2



- **Time Projection Chamber (TPC)**
  - Main tracking detector
  - Particle identification (PID) via dE/dx
- **Inner Tracking System (ITS)**
  - 6 layers of silicon detectors (SPD, SDD, SSD)
  - Primary vertex reconstruction
  - Tracking and PID via dE/dx
- **V0 detectors**
  - Centrality estimator
  - Trigger
- **Time-Of-Flight**
  - PID via particle velocity

Recent EbyE publications in ALICE

Antideuteron number fluctuations



Higher order cumulants and Pearson correlation coefficient

$$\kappa_1 = \langle n \rangle,$$

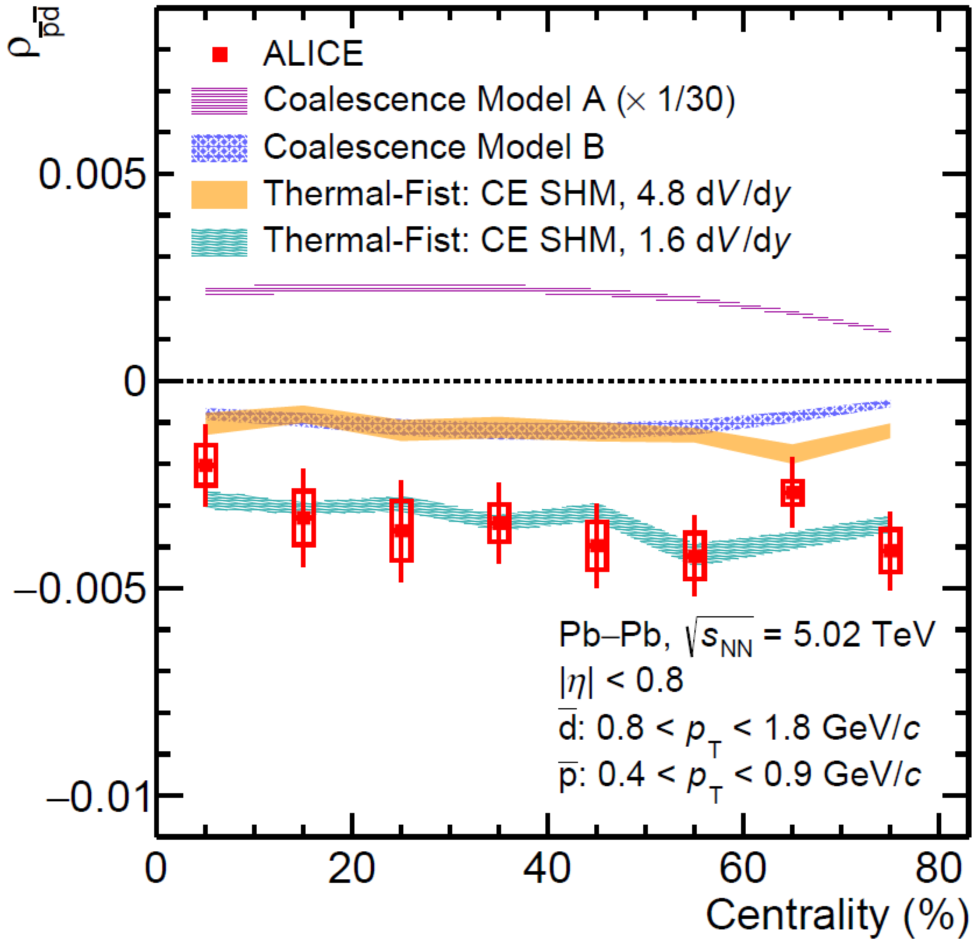
$$\kappa_m = \langle (n - \langle n \rangle)^m \rangle,$$

$$\rho_{ab} = \langle (n_a - \langle n_a \rangle)(n_b - \langle n_b \rangle) \rangle / \sqrt{\kappa_{2a}\kappa_{2b}},$$

- Ratio of the second to the first order cumulant for antideuterons is found to be consistent with unity within uncertainties as expected from a Poisson distribution.
- Measurements consistent with statistical hadronisation models (SHM).
- Deviations from coalescence models.

ALICE Collaboration, *"First measurement of antideuteron number fluctuations at energies available at the Large Hadron Collider",* Phys. Rev. Lett. 131 (2023) 041901, 2204.10166

Antideuteron number fluctuations



Higher order cumulants and Pearson correlation coefficient

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$$\rho_{ab} = \langle (n_a - \langle n_a \rangle)(n_b - \langle n_b \rangle) \rangle / \sqrt{\kappa_{2a} \kappa_{2b}},$$

- A significant negative correlation between antiprotons and antideuterons is observed in all collision centralities.
- CE version of SHM with correlation volume for baryon number conservation of $V_c = 1.6\ dV/dy$ captures data.

ALICE Collaboration, *"First measurement of antideuteron number fluctuations at energies available at the Large Hadron Collider"*, Phys. Rev. Lett. 131 (2023) 041901, 2204.10166

Net-baryon fluctuations

- Fluctuations of conserved charges are sensitive probes for the equation of state and are related to the thermodynamic susceptibilities – calculable in the framework of LQCD:

$$\chi_{klmn}^{\mathrm{B,S,Q,C}} = \frac{\partial^{(k+l+m+n)}(P(\hat{\mu}_{\mathrm{B}}, \hat{\mu}_{\mathrm{S}}, \hat{\mu}_{\mathrm{Q}}, \hat{\mu}_{\mathrm{C}})/T^4)}{\partial\hat{\mu}_{\mathrm{B}}^{k}\partial\hat{\mu}_{\mathrm{S}}^{l}\partial\hat{\mu}_{\mathrm{Q}}^{m}\partial\hat{\mu}_{\mathrm{C}}^{n}}\Big|_{\vec{\mu}=0}$$

- Transition from chiral crossover to a second-order transition – signs of criticality expected to show up starting only with the 6th order cumulants of net-charge distributions.

- Currently available in terms of statistics: 2nd and 3rd order cumulants of net-proton distributions:

$\Delta N_B = X = N_B - N_{\bar{B}}$ $\kappa_n \rightarrow$ cumulants (i.e. $\kappa_2 \equiv \langle X^2\rangle - \langle X\rangle^2$)

$$\hat{\chi}_2^B = \frac{\kappa_2(\Delta N_B)}{VT^3} \quad \Rightarrow \quad \frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} = \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B}$$

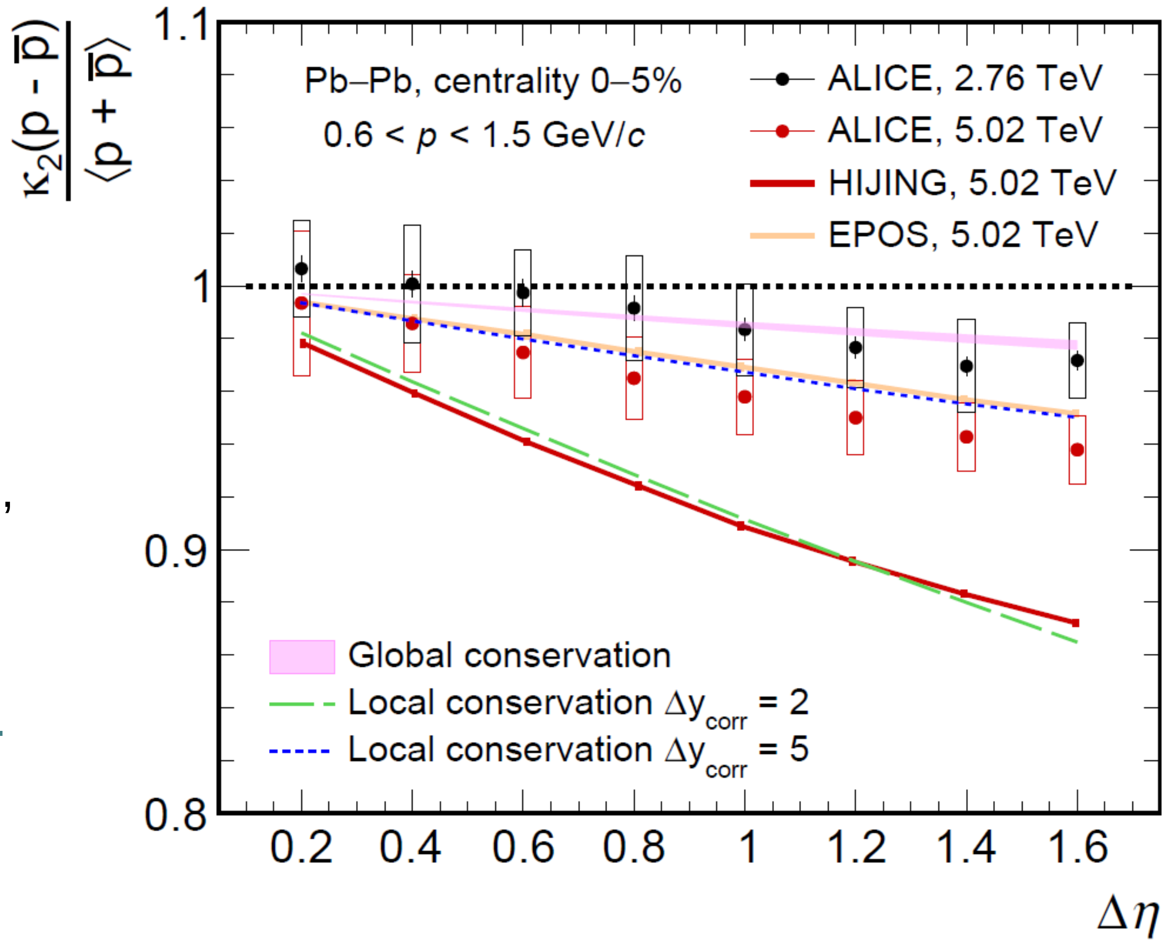
ALICE Collaboration, *"Closing in on critical net-baryon fluctuations at LHC energies: cumulants up to third order in Pb−Pb collisions"*, Phys. Lett. B 844 (2023) 137545, 2206.03343

Net-baryon fluctuations

- Deviation from Skellam baseline (i.e. statistically independent Poisson limit) is consistent with baryon number conservation:

$$\kappa_n^{\mathrm{Skellam}}(\mathrm{p}-\overline{\mathrm{p}}) = \langle \mathrm{p} \rangle + (-1)^n \langle \overline{\mathrm{p}} \rangle$$

- As a function of the width of the pseudorapidity interval, the fluctuations are increasingly reduced
  - Larger interval – increasing relevance of baryon number conservation;
  - Narrowest interval – statistically independent Poisson limit.



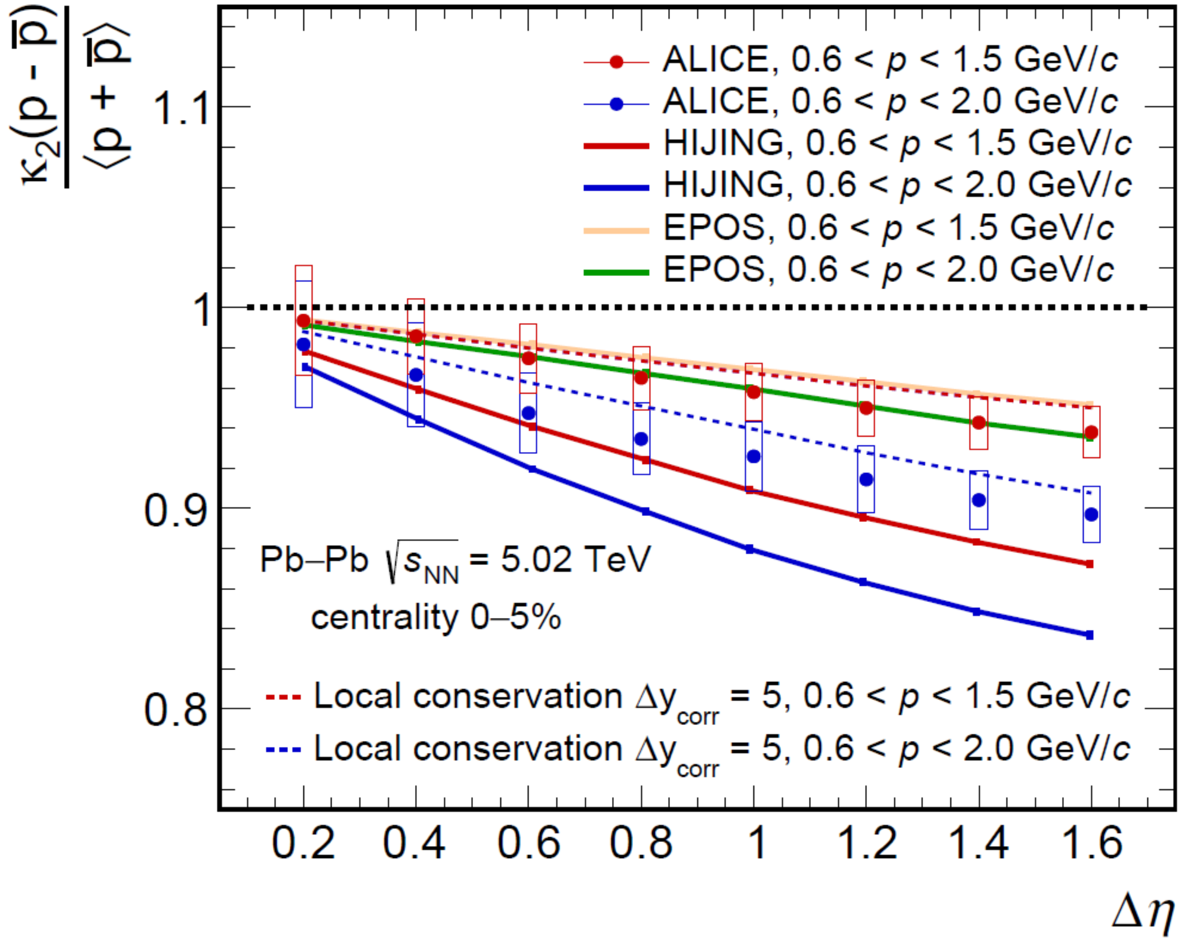
ALICE Collaboration, *"Closing in on critical net-baryon fluctuations at LHC energies: cumulants up to third order in Pb−Pb collisions"*, Phys. Lett. B 844 (2023) 137545, 2206.03343

Net-baryon fluctuations

- Comparison to models: EPOS and HIJING.
- ALICE data suggest long-range correlations, Δy = ±2.5 unit or longer → earlier in time.

  A. Dumitru et al., Nucl. Phys. A 810 (2008) 91

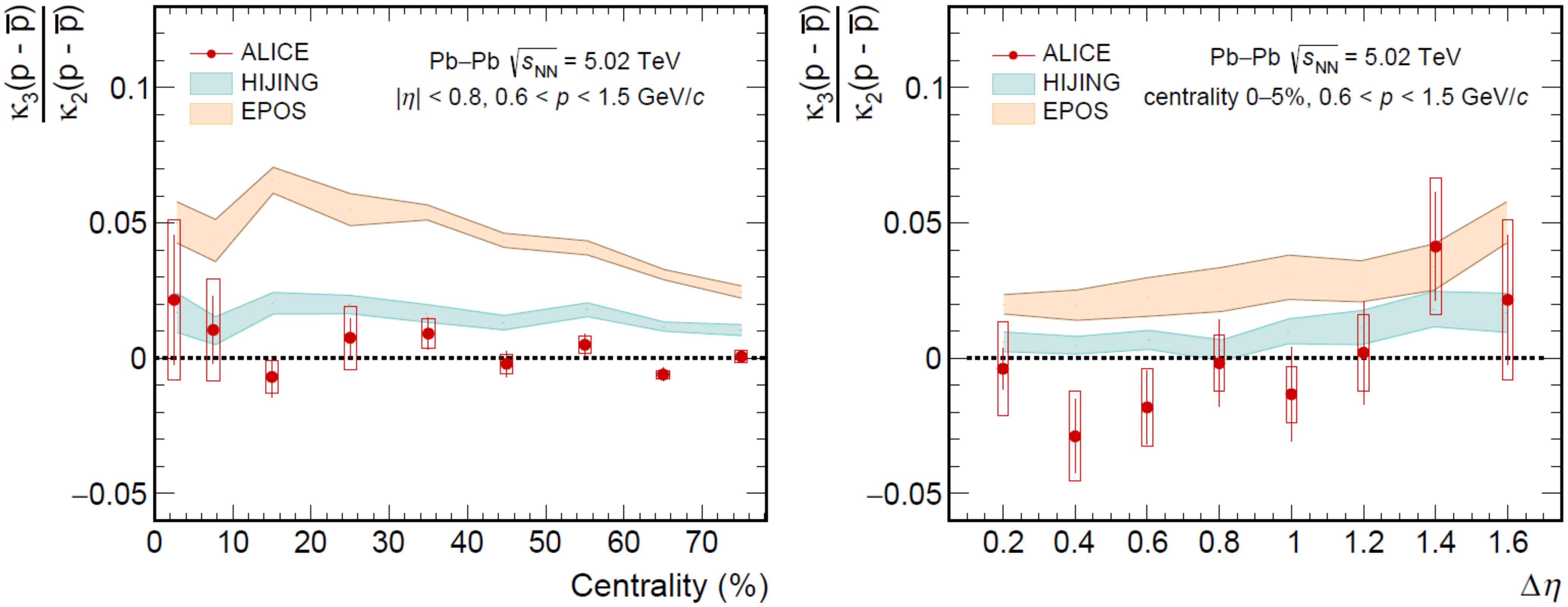
- EPOS agrees with ALICE data but HIJING deviates significantly.
- Event generators based on string fragmentation (HIJING) conserve baryon number over Δy = ±1 unit.



ALICE Collaboration, *"Closing in on critical net-baryon fluctuations at LHC energies: cumulants up to third order in Pb−Pb collisions"*, Phys. Lett. B 844 (2023) 137545, 2206.03343

Net-baryon fluctuations

- First results for 3rd order cumulants of net protons:



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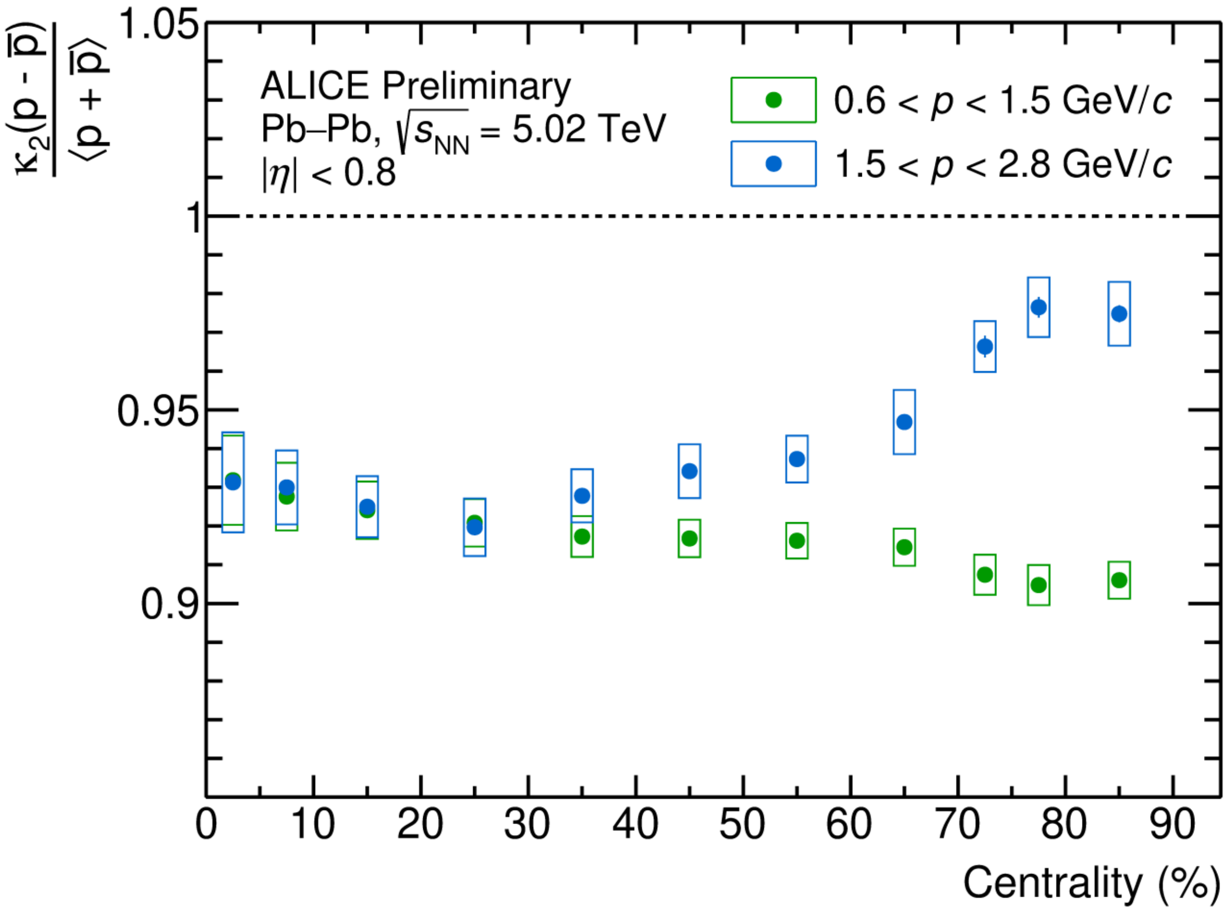
Net-proton fluctuations: larger momenta and more peripheral

- Can we measure the magnetic field produced in peripheral collisions?
- First measurement of net-proton cumulants above p = 2 GeV/*c.*
- Low momenta: weak centrality dependence (due to radial flow?)
- High momenta: significant increase towards peripheral collisions
  - Magnetic field effect as expected by the LQCD?
  - Proton clusters?



Net-Ξ net-kaon correlation

- Event-by-event strangeness fluctuations – is thermalisation reached in all systems at LHC?
- Charged kaons and Ξ – negligible effects of heavy resonance decays.
- Net-particle fluctuations:

$$\kappa_2(\overline{\Xi}^+ - \Xi^-) = \kappa_2(\overline{\Xi}^+) + \kappa_2(\Xi^-) - 2\kappa_{11}(\overline{\Xi}^+, \Xi^-)$$

$$\kappa_{11}(\Delta\Xi, \Delta \mathrm{K}) = \boxed{\kappa_{11}(\overline{\Xi}^+, \mathrm{K}^+) + \kappa_{11}(\Xi^-, \mathrm{K}^-)} - \boxed{\kappa_{11}(\overline{\Xi}^+, \mathrm{K}^-) - \kappa_{11}(\Xi^-, \mathrm{K}^+)}$$

same-sign   opposite-sign

- Net-particle correlation:

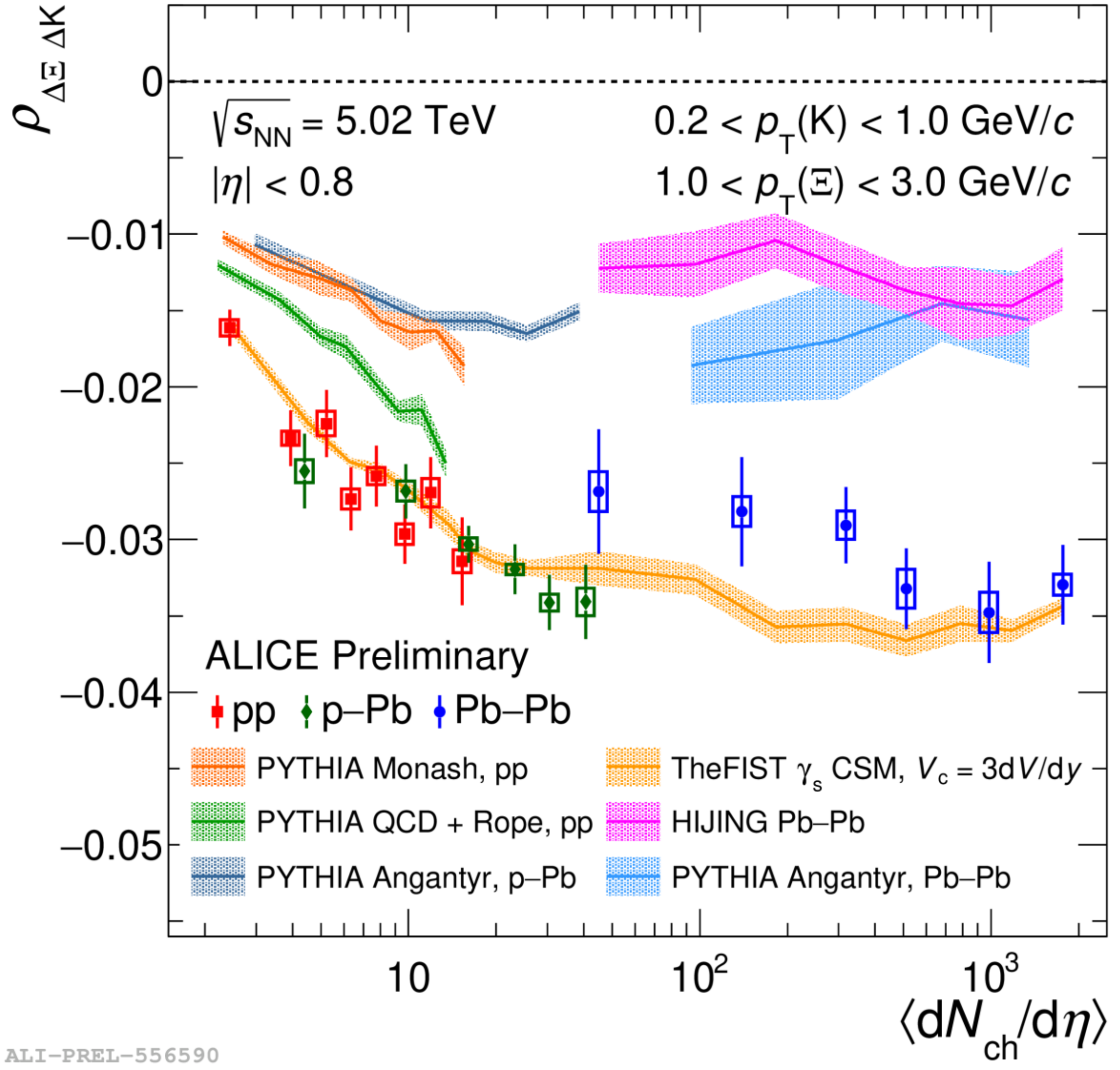
$$\rho_{\Delta\Xi\Delta\mathrm{K}} = \frac{\kappa_{11}(\Delta\Xi, \Delta\mathrm{K})}{\sqrt{\kappa_{2,\Delta\Xi}\kappa_{2,\Delta\mathrm{K}}}}$$

$$\Delta\Xi = n_{\overline{\Xi}^+} - n_{\Xi^-}$$
$$\Delta\mathrm{K} = n_{\mathrm{K}^+} - n_{\mathrm{K}^-}$$

M. Caccio, QM23 talk

Net-Ξ net-kaon correlation

- Continuity of correlation from small to large systems.
- Predictions from the Thermal-FIST canonical statistical model (CSM) describe the data well, across different colliding systems, while PYTHIA and HIJING fail.
- Large correlation length for strangeness (~3dV/dy) is observed.



M. Caccio, QM23 talk

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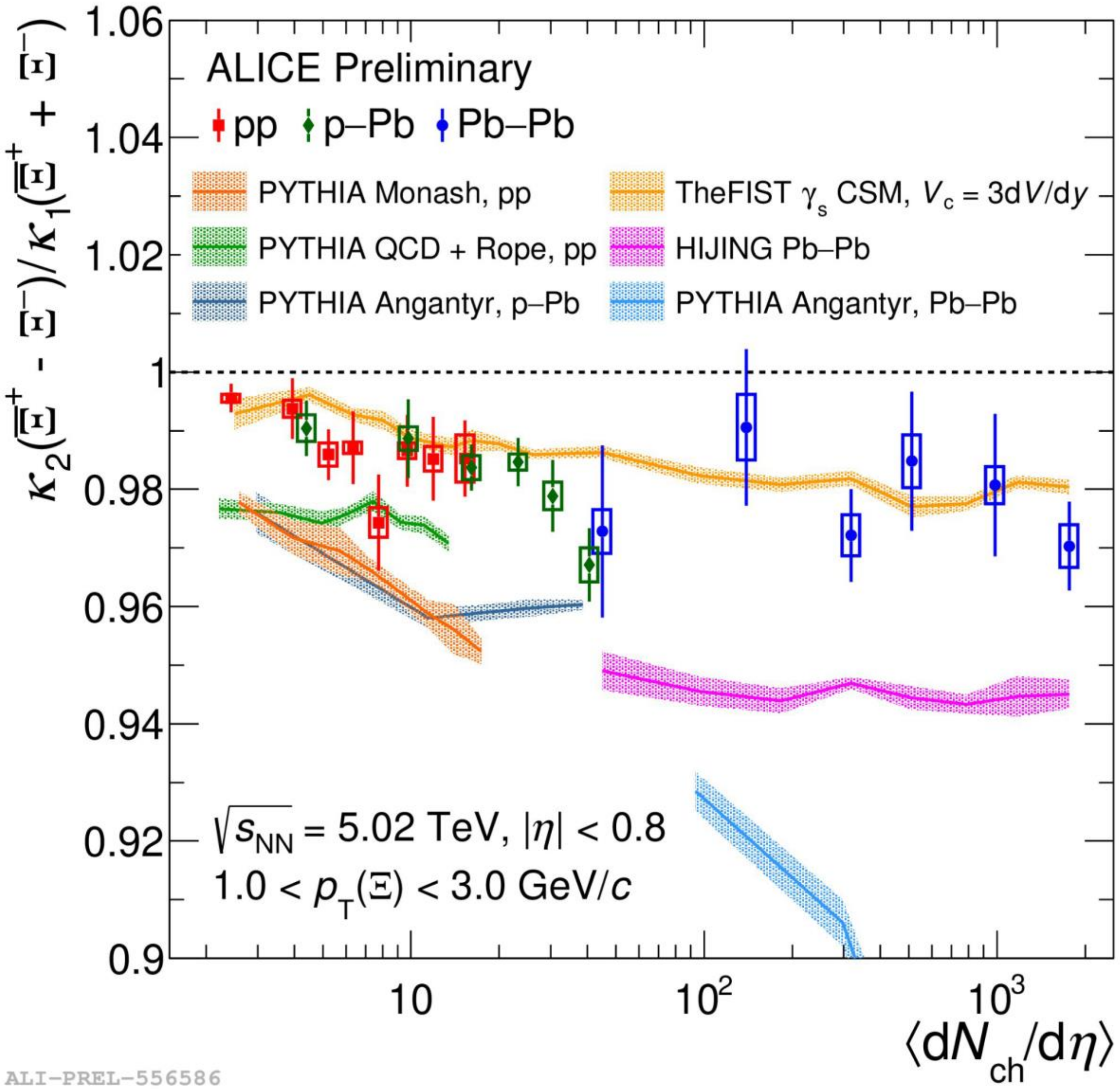


M. Caccio, QM23 talk

Net-charge fluctuations

- EbyE fluctuations of conserved quantities in a finite phase space window, like net charge, baryon number and strangeness, are considered to be sensitive indicators for de-confined phase transition.
- Dynamical net-charge fluctuations observable is defined as:

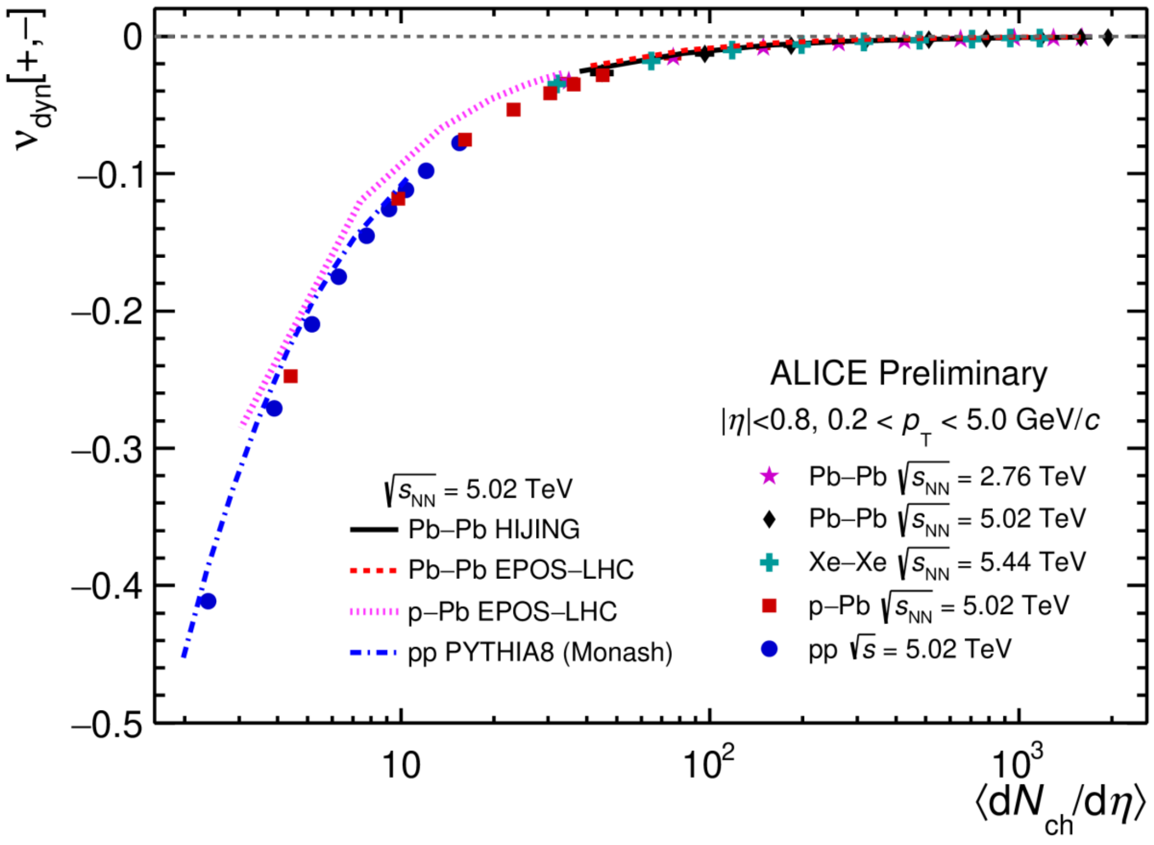
$$\nu_{[+-,dyn]} = \frac{\langle N_+(N_+ - 1)\rangle}{\langle N_+\rangle^2} + \frac{\langle N_-(N_- - 1)\rangle}{\langle N_-\rangle^2} - 2\,\frac{\langle N_+ N_-\rangle}{\langle N_+\rangle\langle N_-\rangle}$$

C. Pruneau et al., Phys. Rev. C 66, 044904 (2002)

- N_+ and N_- – number of charged particles in the phase space of interest.
- This observable measures deviation from Poissonian behaviour.
- Robust against detection efficiency losses.

Net-charge fluctuations

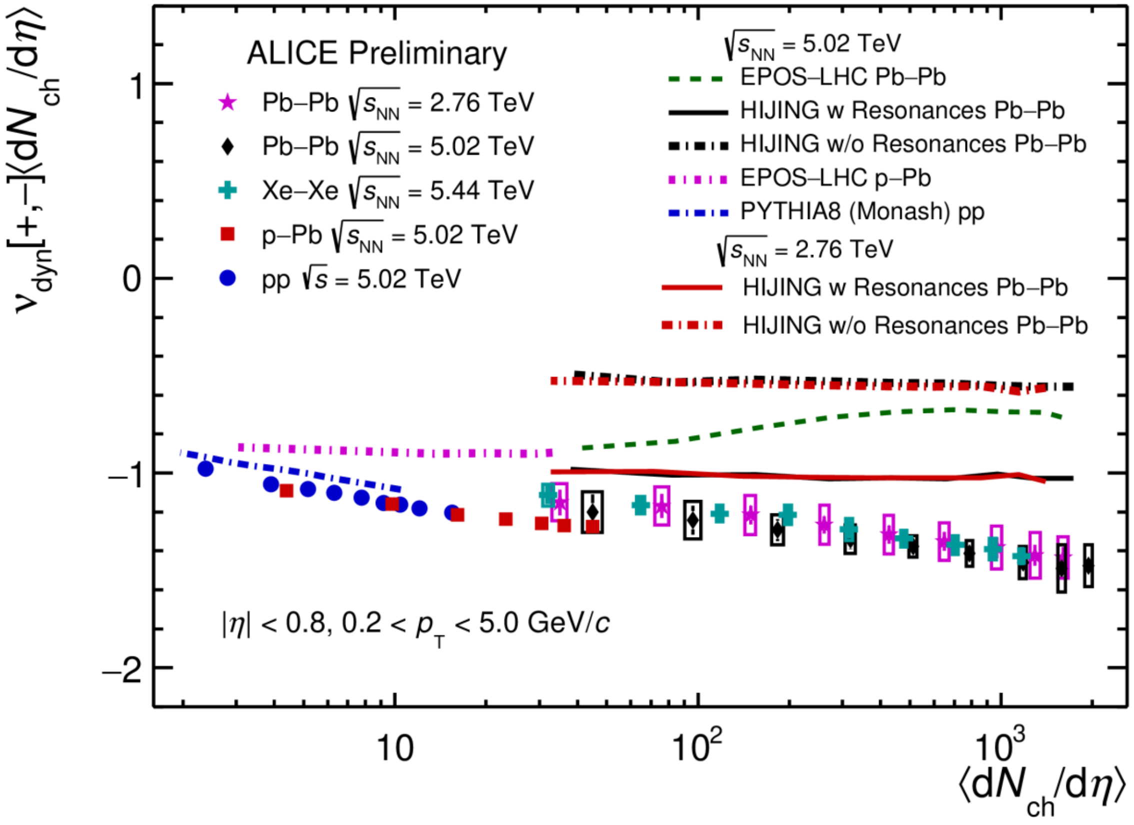
- **Negative** $\nu_{dyn}[+,-]$ indicates the dominance of correlation between positive and negative charged particles.
- Smooth evolution with multiplicity across various collision systems.
- MC event generators show similar centrality dependence as data.



S. Khan, PoS EPS-HEP2021 (2022) 319, proceedings

Net-charge fluctuations

- Scaling of $\nu_{dyn}[+,-]$ with respect to charged-particle density at midrapidity.
- HIJING predicts no centrality dependence – heavy-ion collisions are treated as superpositions of independent nucleon-nucleon collisions.
- Significant contribution of net-charge fluctuations can arise due to the resonance decays.



S. Khan, PoS EPS-HEP2021 (2022) 319, proceedings

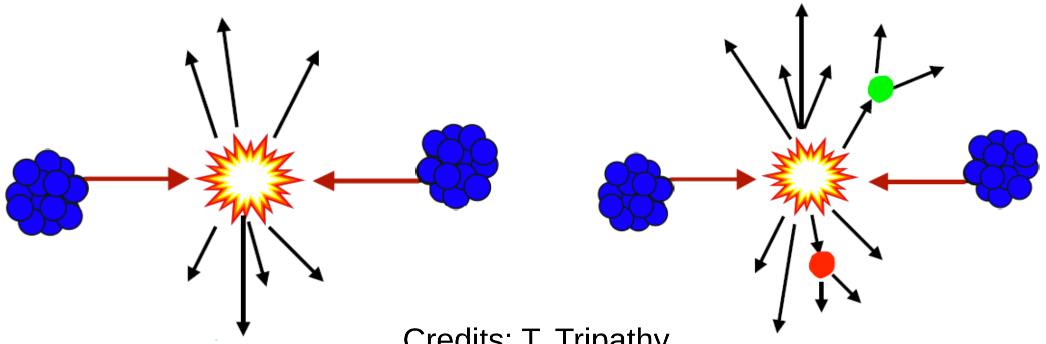
Event-by-event fluctuations of $\langle p_{\mathrm{T}} \rangle$

- $\langle p_{\mathrm{T}} \rangle$ is a proxy for the local temperature of the produced system in high-energy nuclear collisions.
- Analyses are performed in 2 different collision systems: Xe–Xe and Pb–Pb.
- Separation of statistical and dynamical fluctuations.
- Observable: two-particle correlator $\langle \Delta p_{\mathrm{T}} \Delta p_{\mathrm{T}} \rangle$

$$\langle \Delta p_{\mathrm{Ti}} \Delta p_{\mathrm{Tj}} \rangle = \left\langle \frac{\sum_{i,j \neq i} (p_{\mathrm{Ti}} - \langle\langle p_{\mathrm{T}} \rangle\rangle)(p_{\mathrm{Tj}} - \langle\langle p_{\mathrm{T}} \rangle\rangle)}{N_{\mathrm{ch}}(N_{\mathrm{ch}} - 1)} \right\rangle$$

$$\langle \Delta p_{\mathrm{Ti}} \Delta p_{\mathrm{Tj}} \rangle = \left\langle \frac{(Q_1)^2 - Q_2}{N_{\mathrm{ch}}(N_{\mathrm{ch}} - 1)} \right\rangle - \left\langle \frac{Q_1}{N_{\mathrm{ch}}} \right\rangle^2, \text{ where } Q_{\mathrm{n}} = \sum_{i=1}^{N} (p_{\mathrm{T,i}})^n$$

G. Giacalone, F. G. Gardim, J. Noronha-Hostler, and J-Y. Ollitrault
Phys. Rev. C **103**, 024910 (2021)

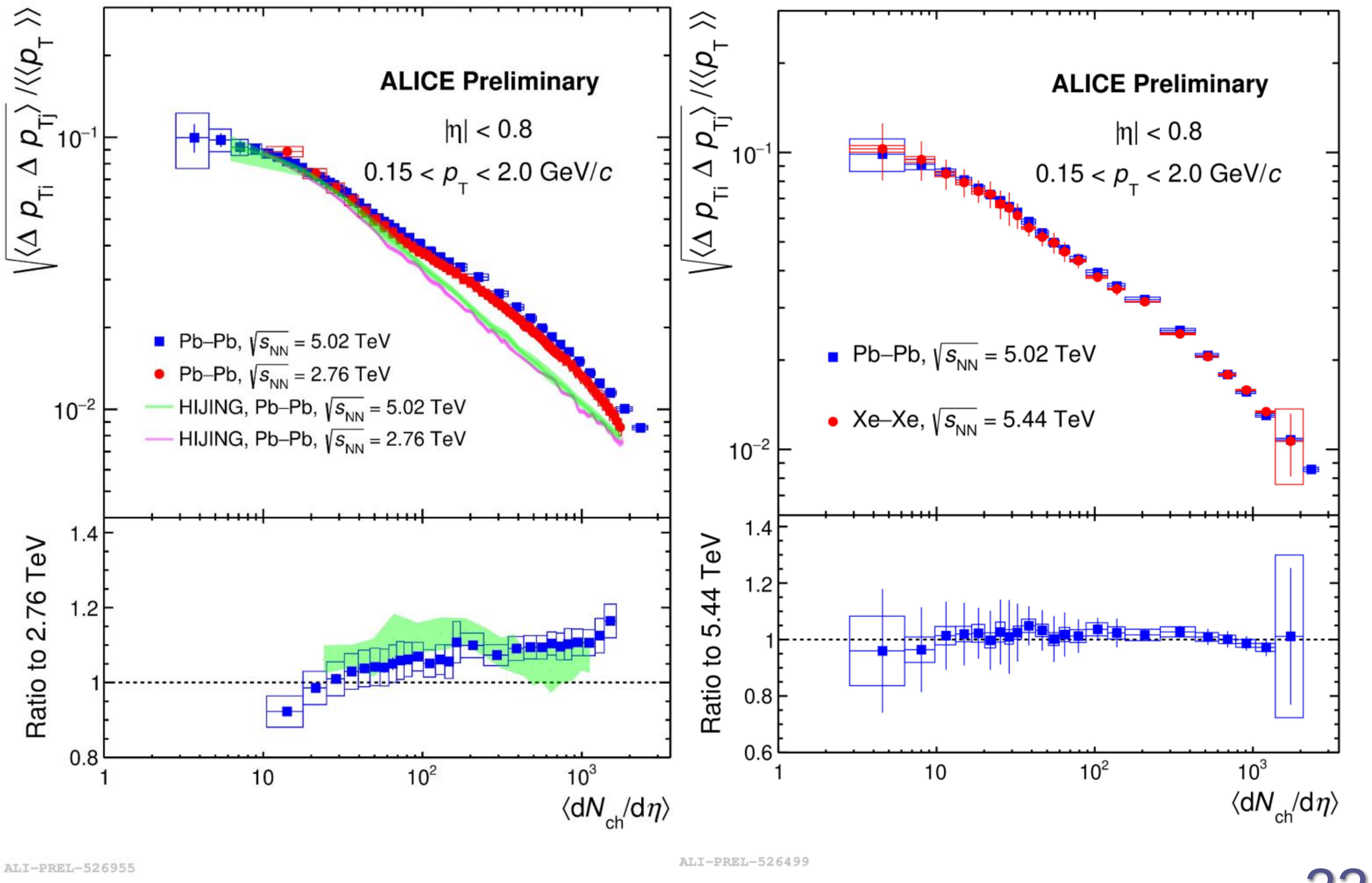


Credits: T. Tripathy

LHS – only trivial statistical fluctuations
RHS – dynamical fluctuations (mini jets, resonances, etc,)

Event-by-event fluctuations of $\langle p_T \rangle$

- Evolution of the correlator strength with charged particle pseudorapidity density as a function of
  - beam energy;
  - collision system size.
- Progressive dilution with multiplicity in all three systems ⇔ increase of number of correlated particle sources vs $\langle dN_{ch}/d\eta \rangle$.
- Deviation from HIJING suggests the presence of radial flow.



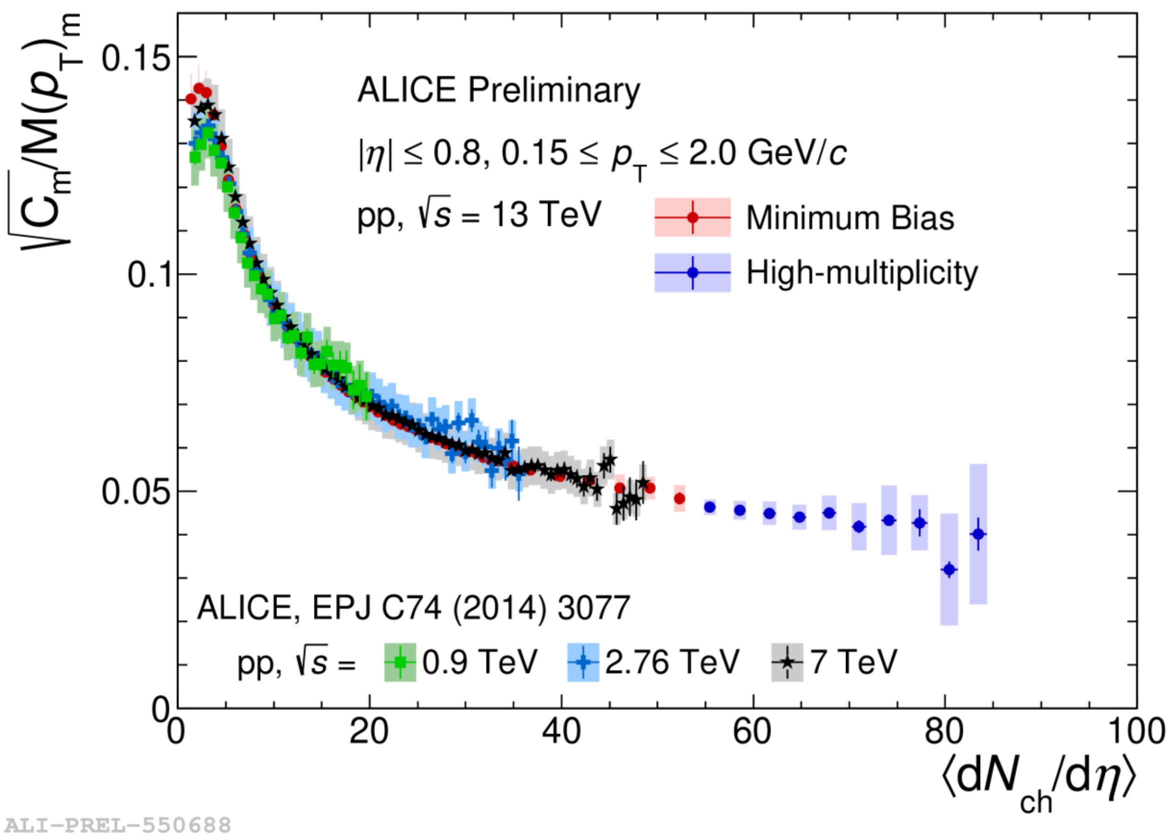
B. Ali, T. Tripathy QM23 poster

Mean p_{T} fluctuations

- Mean p_{T} fluctuations in 13 TeV pp data:
  - High-multiplicity pp data;
  - Differential fluctuations of identified particles.
- Observable: Normalized two-particle transverse momentum correlator $\sqrt{C_m}/M(p_{\mathrm{T}})$

$$C_m = \frac{1}{\sum_{k=1}^{n_{evt,m}} N_k^{pairs}} \sum_{k=1}^{n_{evt,m}} \sum_{i=1}^{N_{acc,k}} \sum_{j=i+1}^{N_{acc,k}} \left(p_{T,i} - M(p_T)_m\right) * \left(p_{T,j} - M(p_T)_m\right)$$

$$M(p_T)_m = \frac{1}{\sum_{k=1}^{n_{evt,m}} N_{acc,k}} \sum_{k=1}^{n_{evt,m}} \sum_{i=1}^{N_{acc,k}} p_{T,i}$$



B. Ali, T. Tripathy QM23 poster

Skewness and kurtosis of $\langle p_{\mathrm{T}} \rangle$ fluctuations

- Main idea: Fluctuations in temperature between different phases in QCD phase diagram are inscribed in event-by-event $\langle p_{\mathrm{T}} \rangle$ fluctuations of final-state particles.
- Two categories of fluctuations:
  - Statistical – trivial, due to finite multiplicity;
  - Dynamical – encode nontrivial physics.
- Main challenge: How to disentangle dynamical fluctuations from the ones which are non-thermodynamic in nature (fluctuations of initial positions of participating nucleons, etc.)?
- Higher moments of $\langle p_{\mathrm{T}} \rangle$ fluctuations: **skewness** and **kurtosis**

$$\gamma_{\langle p_{\mathrm{T}} \rangle} = \frac{\langle \Delta p_{\mathrm{T},i} \Delta p_{\mathrm{T},j} \Delta p_{\mathrm{T},k} \rangle}{\langle \Delta p_{\mathrm{T},i} \Delta p_{\mathrm{T},j} \rangle^{3/2}}$$

Standardized skewness

$$\Gamma_{\langle p_{\mathrm{T}} \rangle} = \frac{\langle \Delta p_{\mathrm{T},i} \Delta p_{\mathrm{T},j} \Delta p_{\mathrm{T},k} \rangle \langle \langle p_{\mathrm{T}} \rangle \rangle}{\langle \Delta p_{\mathrm{T},i} \Delta p_{\mathrm{T},j} \rangle^{2}}$$

Intensive skewness

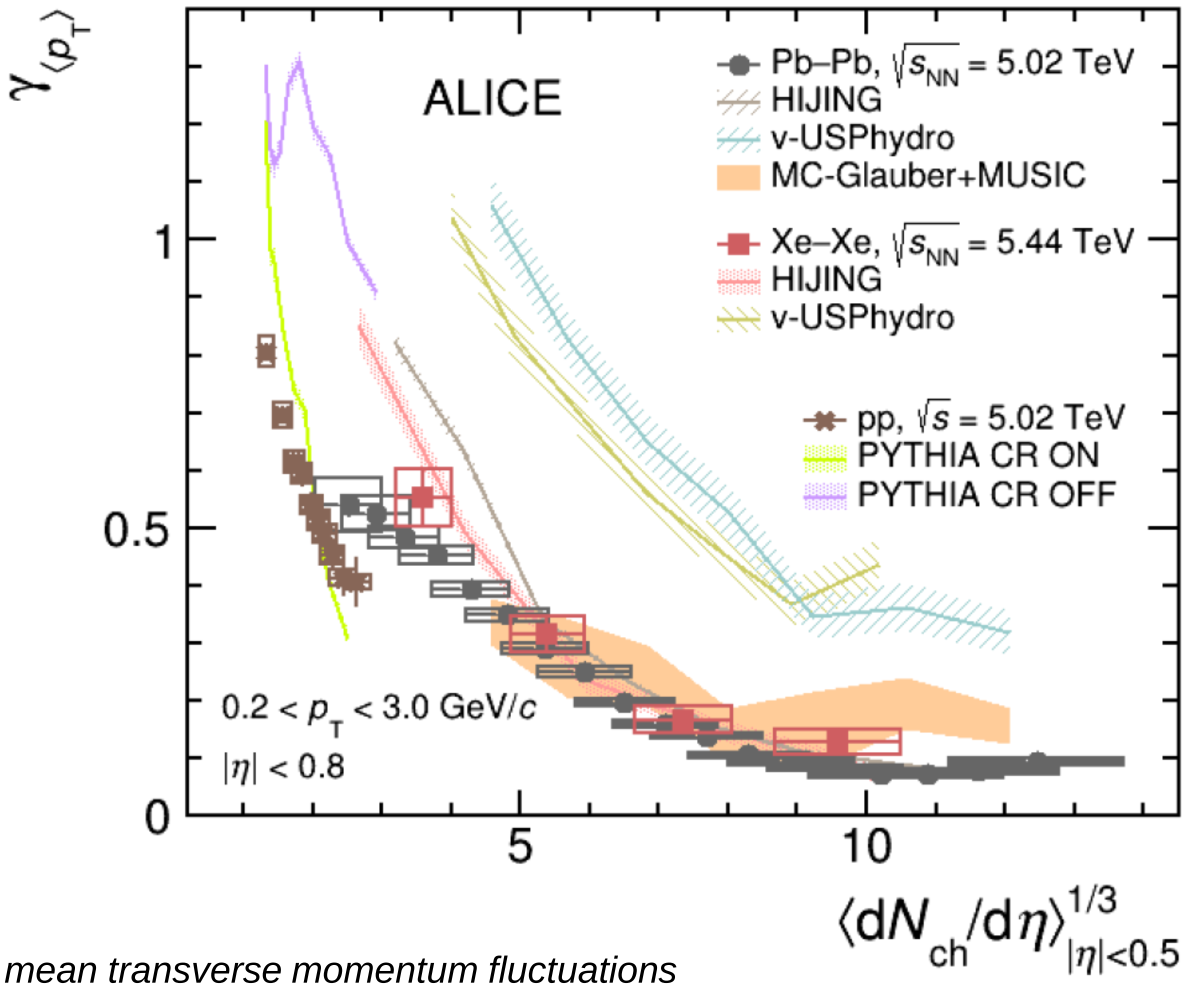
$$\kappa_{\langle p_{\mathrm{T}} \rangle} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \Delta p_l \rangle}{\langle \Delta p_i \Delta p_j \rangle^{2}}$$

Kurtosis

G. Giacalone et al, Phys. Rev. C **103** (2021) 2, 024910, 2004.09799

Skewness and kurtosis of $\langle p_T \rangle$ fluctuations

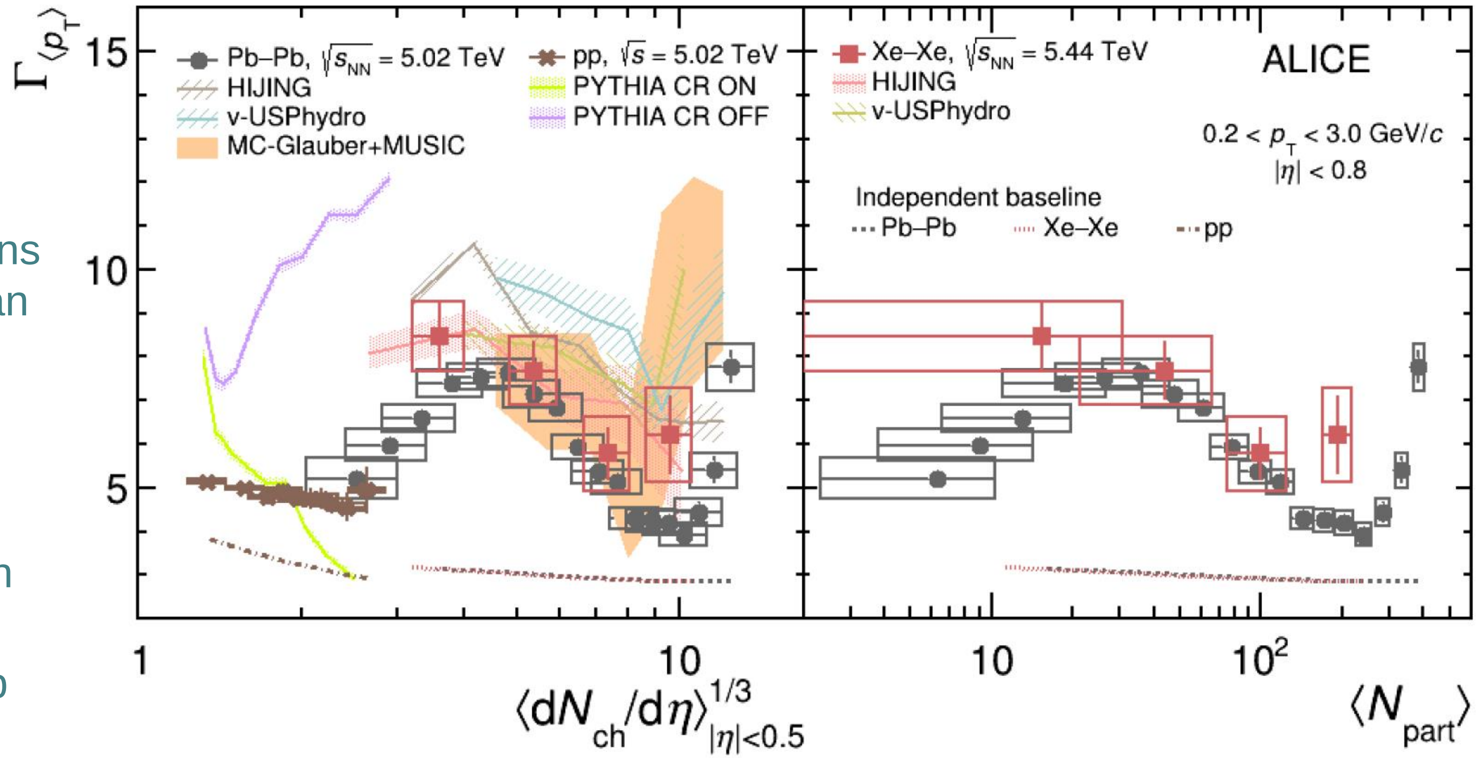
- Measurements performed in three different collision systems: Pb–Pb, Xe–Xe and pp.
- Common proxy for system size: $\langle dN_{ch}/d\eta \rangle^{1/3}_{|\eta|<0.5}$
- Main results for standardized skewness:
  - Positive standardized skewness of $\langle p_T \rangle$ fluctuations in Pb–Pb, Xe–Xe and pp collisions – an essential consequence of hydrodynamic evolution;
  - However, positive skewness of $\langle p_T \rangle$ fluctuations also for small system size – difficult to reconcile with hydro;
  - Hydro model MUSIC with Monte Carlo Glauber initial conditions qualitatively describes skewness;
  - PYTHIA captures qualitatively the same measurements in pp collisions (colour reconnection (CR) mechanism plays a pivotal role).



ALICE Collaboration, *"Skewness and kurtosis of mean transverse momentum fluctuations at the LHC energies"*, Submitted to PLB, 2308.16217

Skewness and kurtosis of $\langle p_T \rangle$ fluctuations

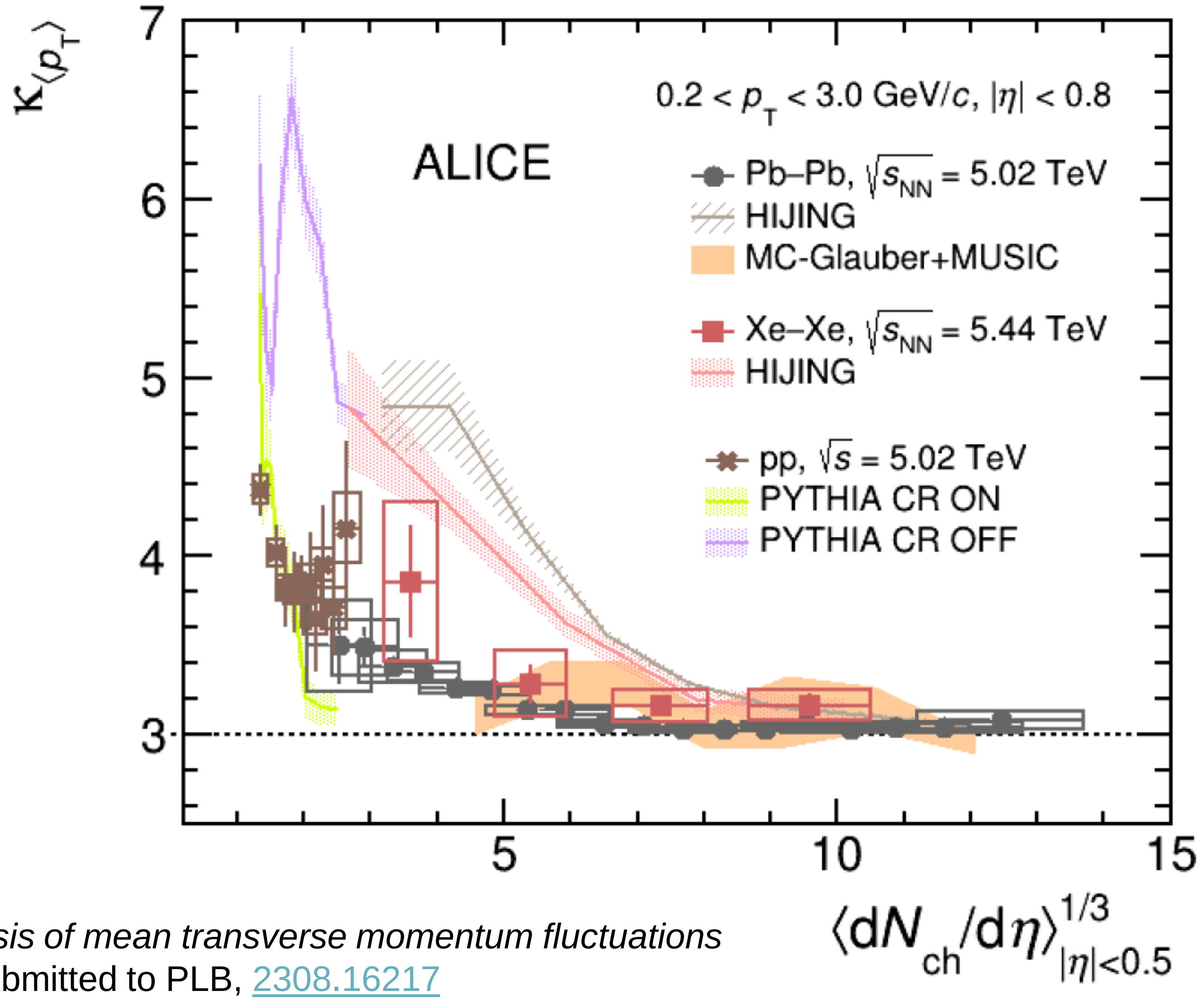
- Intensive skewness, as a function of system size and N_{part}.
- Main results:
  - Positive intensive skewness of $\langle p_T \rangle$ fluctuations in Pb–Pb, Xe–Xe and pp collisions, larger than the independent baseline;
  - Neither version of PYTHIA (with and without color reconnection mechanism) can describe pp data;
  - Only hydro-based models capture the sudden rise in most central collisions;
  - Non-trivial system size dependence in Pb–Pb and Xe–Xe, monotonic decrease in pp.



ALICE Collaboration, *"Skewness and kurtosis of mean transverse momentum fluctuations at the LHC energies"*, Submitted to PLB, 2308.16217

Skewness and kurtosis of $\langle p_T \rangle$ fluctuations

- Kurtosis as a function of system size
- Main results:
  - Kurtosis of $\langle p_T \rangle$ fluctuations in Pb–Pb, Xe–Xe and pp collisions decreases as system size increases;
  - Kurtosis approaches independent Gaussian baseline only in most central collisions;
  - Only PYTHIA with color reconnection can qualitatively describe pp data;
  - MC-Glauber+MUSIC captures Pb–Pb data in most central collisions;
  - HIJING overestimates the data.



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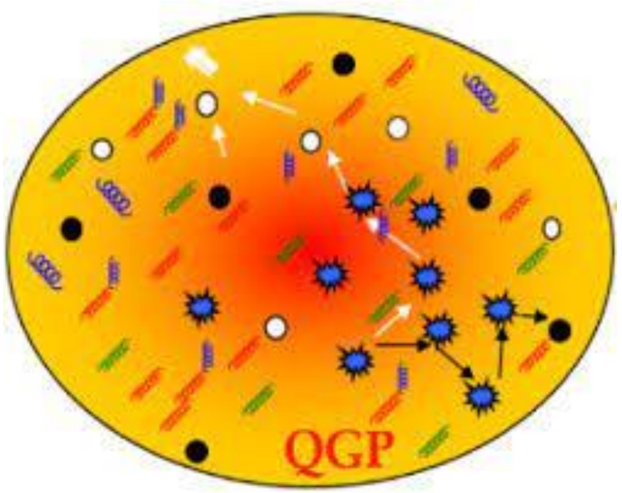
Intermittency analysis of charged-particle production

- The final distribution of particles in phase–space cells depends upon the dynamics of initial processes involving partons.
- Universal property of critical phenomena – existence of clusters of all sizes without characteristic scale.
- Scaled factorial moments – by definition, filter out statistical Poisson noise and isolate only dynamical fluctuations:



$$F_q(\delta^d) = \frac{\langle n!/(n-q)!\rangle}{\langle n\rangle^q}$$

q – order of moment
δ^d – bin size in d-dimensional space
n – bin multiplicity

- If multiplicity distribution is Poissonian, for any bin size δ it follows:

$$F_q(\delta) = 1$$

R.C. Hwa, J. Pan, Phys. Lett. B 297, 35 (1992); R.C. Hwa, C.B. Yang, Phys. Rev. C **85**, 044914 (2012)

Intermittency analysis of charged-particle production

- Studying scaling behaviour of the spatial distributions of the produced particles – **intermittency**
  - Power-law scaling behaviour:

$$F_q(\delta) \propto \delta^{-\varphi_q}$$

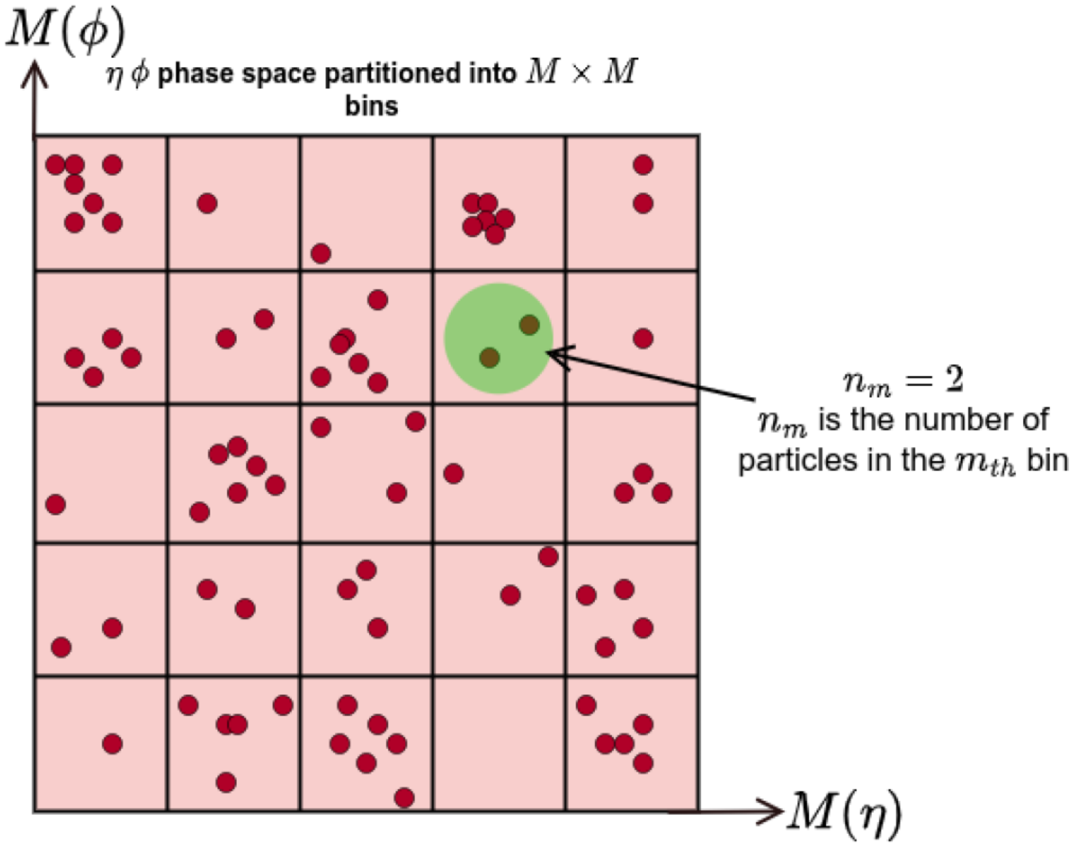
φ_q – intermittency index (constant at any q)
M – number of bins
$\delta = 1/\mathrm{M}$ – bin resolution

- 2nd order-phase transition is characterized by:

$$F_q \propto F_2^{(q-1)^\nu}, \quad \nu = 1.304$$

ν – scaling exponent, universal quantity

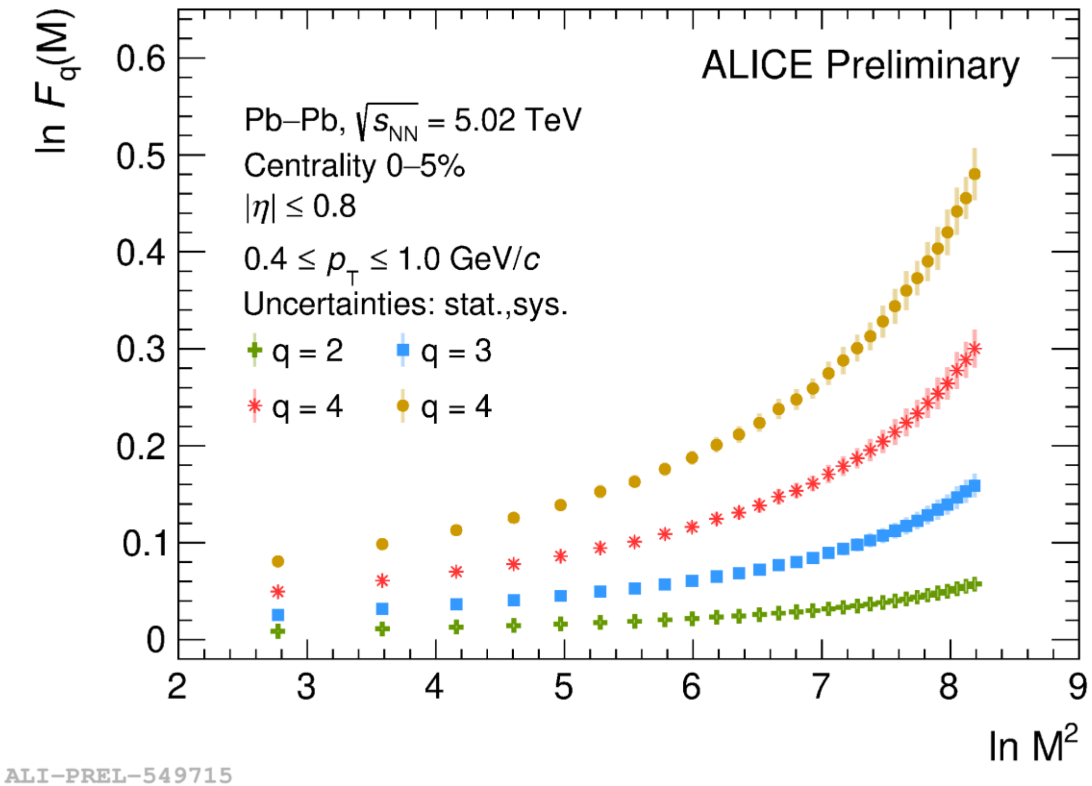
- First preliminary results on intermittency studies at LHC energies, ongoing discussion on their interpretation.



R. Gupta, S. K Malik, S. Sharma, QM23 poster

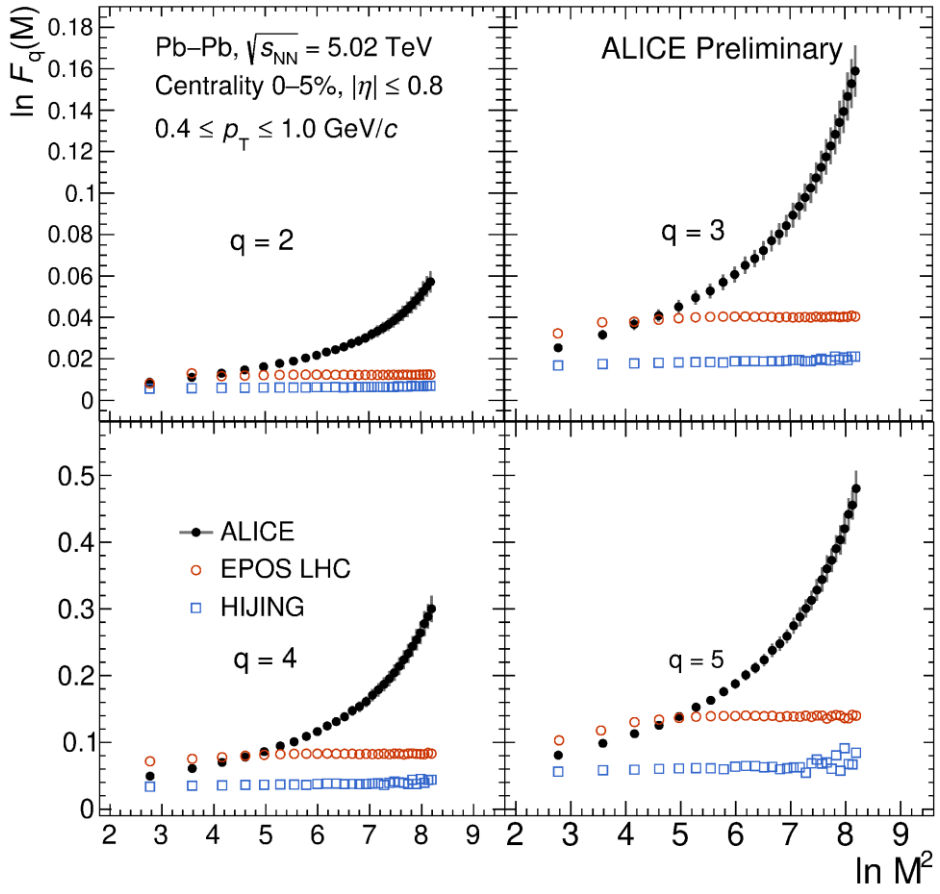
Intermittency analysis of charged-particle production

- Power-law growth of F_q with the increase in the number of bins (M) – scale-invariant pattern:



ALI-PREL-549715

- Qualitative and quantitative differences are observed between data and models:



ALI-PREL-559667

R. Gupta, S. K Malik, S. Sharma, QM23 poster

Antideuteron number fluctuations

Net-baryon fluctuations

Net-charge fluctuations

Event-by-event fluctuations of $\langle p_{\mathrm{T}} \rangle$

Intermittency analysis

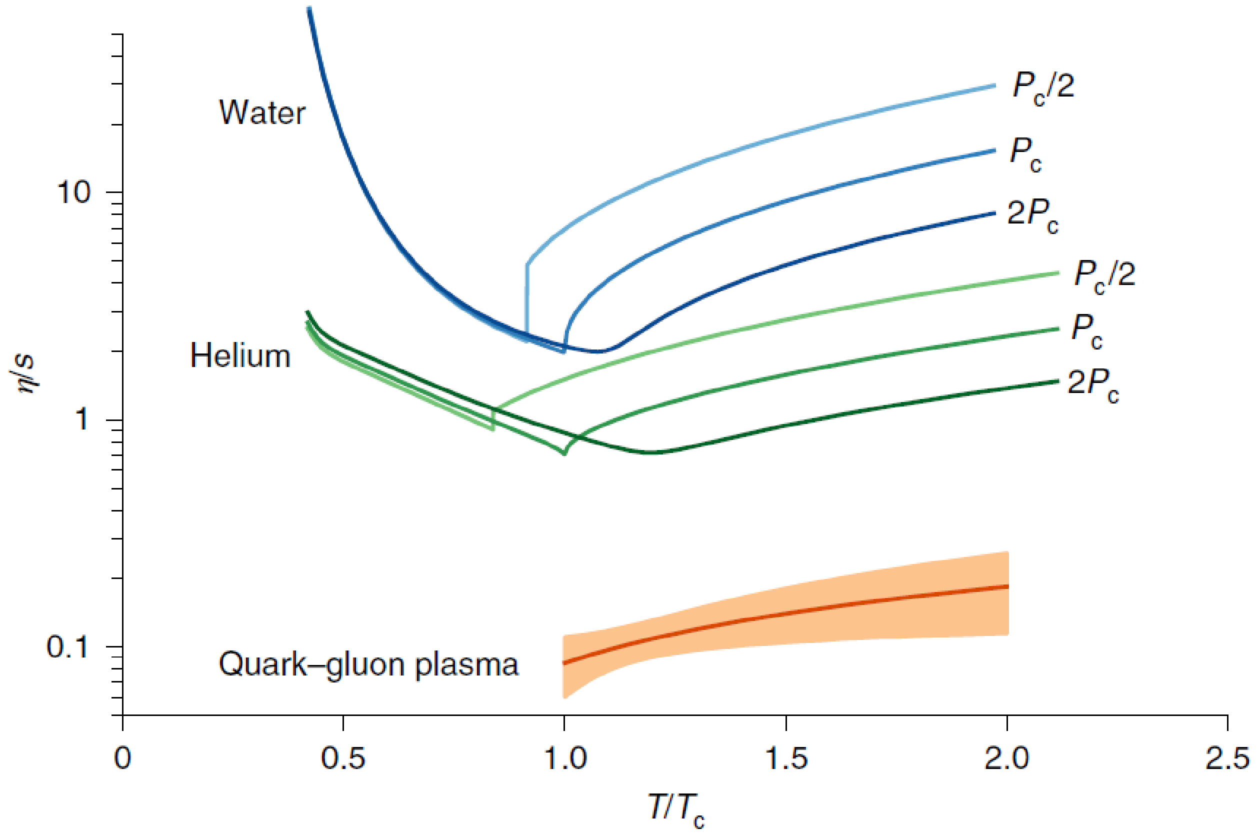
Event-by-event hadron correlations

Thanks!

Backup slides

Example #1: Transport properties of QGP

- Temperature dependence of QGP's specific shear viscosity (*η/s)* is smallest of all known substances



Bernhard, J.E., Moreland, J.S. & Bass, S.A. *Nat. Phys.* **15**, 1113–1117 (2019),

A bit of math

- The mathematical foundation of cumulants is well established!

Theorem: A cumulant $\langle X_i X_j \ldots \rangle_c$ is zero if the elements $X_i, X_j, \ldots$ are divided in two or more groups which are statistically independent.

Collorary: A cumulant is zero if one of the variables in it is independent of the others. Conversely, a cumulant is not zero if and only if the variables in it are statistically connected.

Kubo, Journal of the Physical Society of Japan, Vol. 17, No. 7, (1962)

- Careful reading is mandatory, one statement is not covered:

$$\kappa = 0 \Leftarrow \text{variables are independent}\,,$$
$$\kappa = 0 \not\Rightarrow \text{variables are independent}\,,$$
$$\kappa \neq 0 \Leftrightarrow \text{variables are not independent}\,.$$

Cumulant can be trivially zero due to underlying symmetries!

Fluctuations, p.d.f., moments, cumulants

- Properties of random (stochastic) observable v of interest are specified by functional form of probability density function (p.d.f.) $f(v)$
- Different moments carry by definition independent information about the underlying p.d.f. $f(v_n)$

$$\left\langle v_n^k \right\rangle \equiv \int v_n^k f(v_n)\, dv_n$$

- Two completely different p.d.f.'s $f(v_n)$ can have first moment $\langle v_n \rangle$ to be the same, and all higher-order moments different
- Is it mathematically equivalent to specify functional form $f(v_n)$ and all its moments $\langle v_n^k \rangle$?
- A priori it is not guaranteed that a p.d.f. $f(v_n)$ is uniquely determined by its moments $\langle v_n^k \rangle$
  - Necessary and sufficient conditions have been worked out only recently

$$K[f] \equiv \int_0^\infty \frac{-\ln f(x^2)}{1+x^2}\, dx \quad \Rightarrow \quad K[f] = \infty$$

Krein-Lin conditions (1997)

$$L(x) \equiv -\frac{x f'(x)}{f(x)} \quad \Rightarrow \quad \lim_{x\to\infty} L(x) = \infty$$

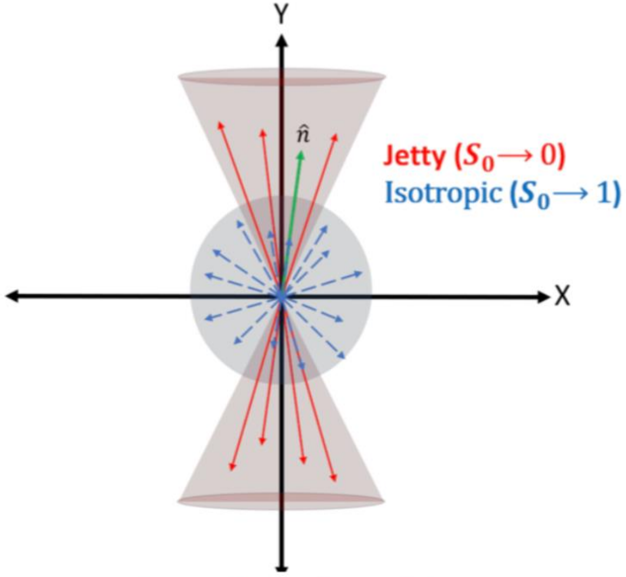
J. Stoyanov, Section 3 in *'Determinacy of distributions by their moments'*, Proceedings 2006

Spherocity

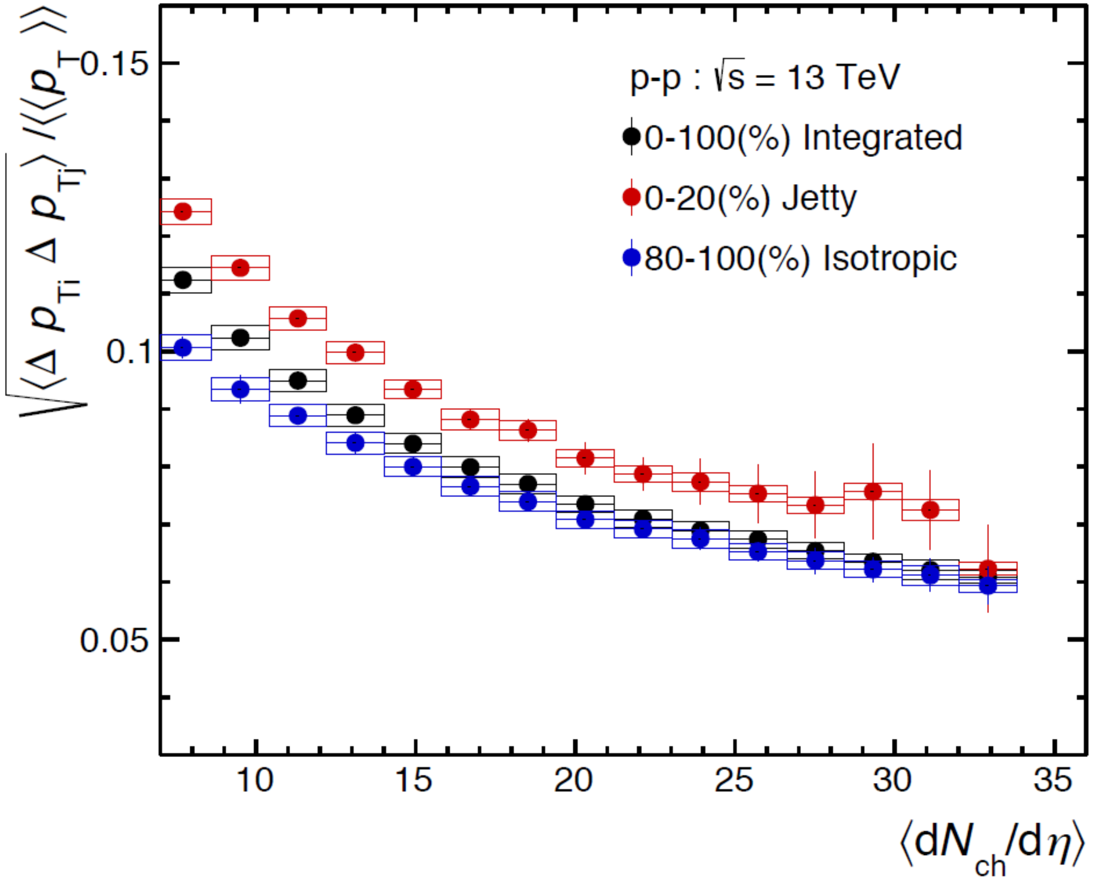
- How to differentiate hard and soft processes?
  - Hard processes are “jetty” => back-to-back particle emission
  - Absence of jets => soft QCD, isotropic particle emission

- Transverse spherocity S_0

$$S_o = \frac{\pi^2}{4} \min_{\hat{n}=(n_x,n_y,0)} \left(\frac{\sum_i |\vec{p}_{Ti} \times \hat{n}|}{\sum_i p_{Ti}} \right)^2$$



A. Khuntia, S. Tripathy, A. Bisht and R. Sahoo, J. Phys. G48, 035102 (2021)



2-particle cumulants in general

- Cumulants are alternative to moments to describe stochastic properties of variable
- If 2 p.d.f.'s have the same moments, they will also have the same cumulants, and vice versa
  - True both for univariate and multivariate case
- X_i denotes the general i-th stochastic variable
- The most general decomposition of 2-particle correlation is:

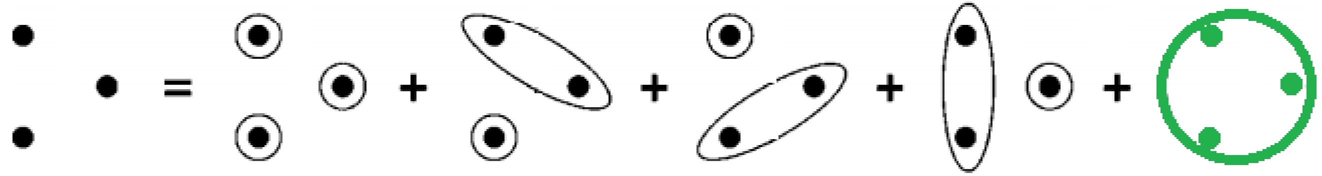
$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c$$

- By definition, the 2nd term on RHS is 2-particle cumulant
- Cumulants cannot be measured directly, however:

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

3-particle cumulants in general

- The most general decomposition of 3-particle correlation is:



- Or written mathematically:

$$\begin{aligned}\langle X_1X_2X_3\rangle &= \langle X_1\rangle\langle X_2\rangle\langle X_3\rangle\\ &+ \langle X_1X_2\rangle_c\langle X_3\rangle+\langle X_1X_3\rangle_c\langle X_2\rangle+\langle X_2X_3\rangle_c\langle X_1\rangle\\ &+ \langle X_1X_2X_3\rangle_c\end{aligned}$$

- The key point: 2-particle cumulants were expressed independently in terms of measured correlations in the previous step!

$$\langle X_1X_2\rangle_c = \langle X_1X_2\rangle - \langle X_1\rangle\langle X_2\rangle$$

3-particle cumulants in general

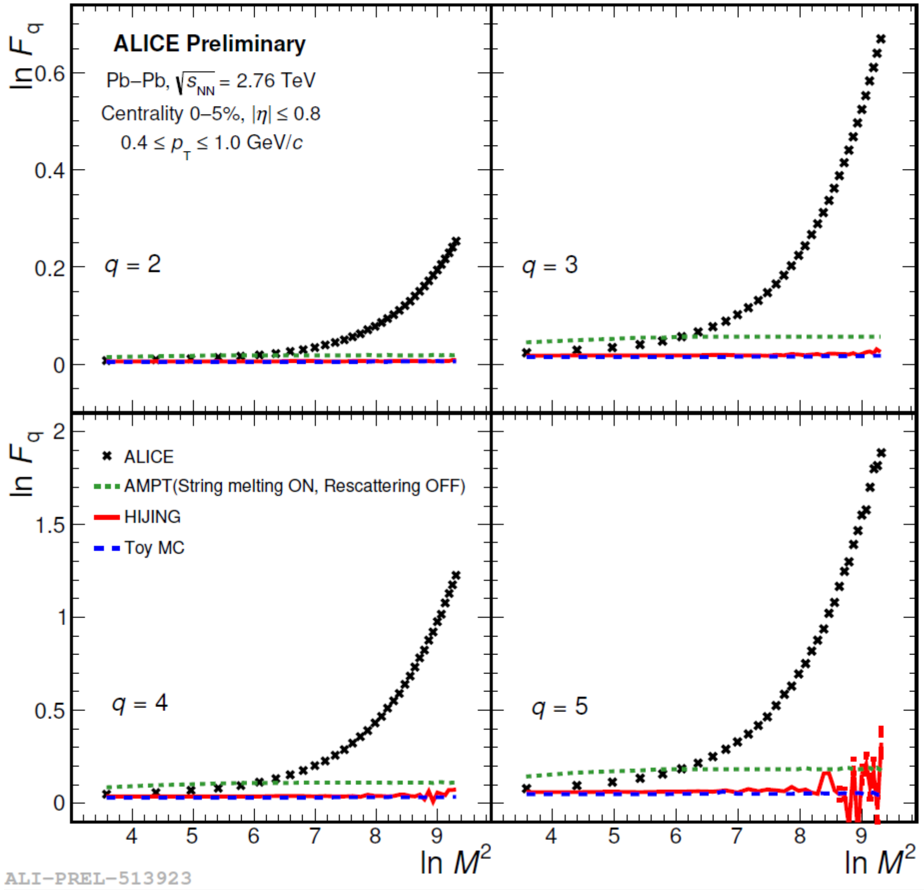
- Working recursively from higher to lower orders, we eventually have 3-particle cumulant expressed in terms of measured 3-, 2-, and 1-particle averages

$$
\begin{aligned}
\langle X_1 X_2 X_3 \rangle_c &= \langle X_1 X_2 X_3 \rangle \\
&- \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle \\
&+ 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle
\end{aligned}
$$

- In the same way, cumulants can be expressed in terms of measurable averages for any number of particles
  - The number of terms grows rapidly

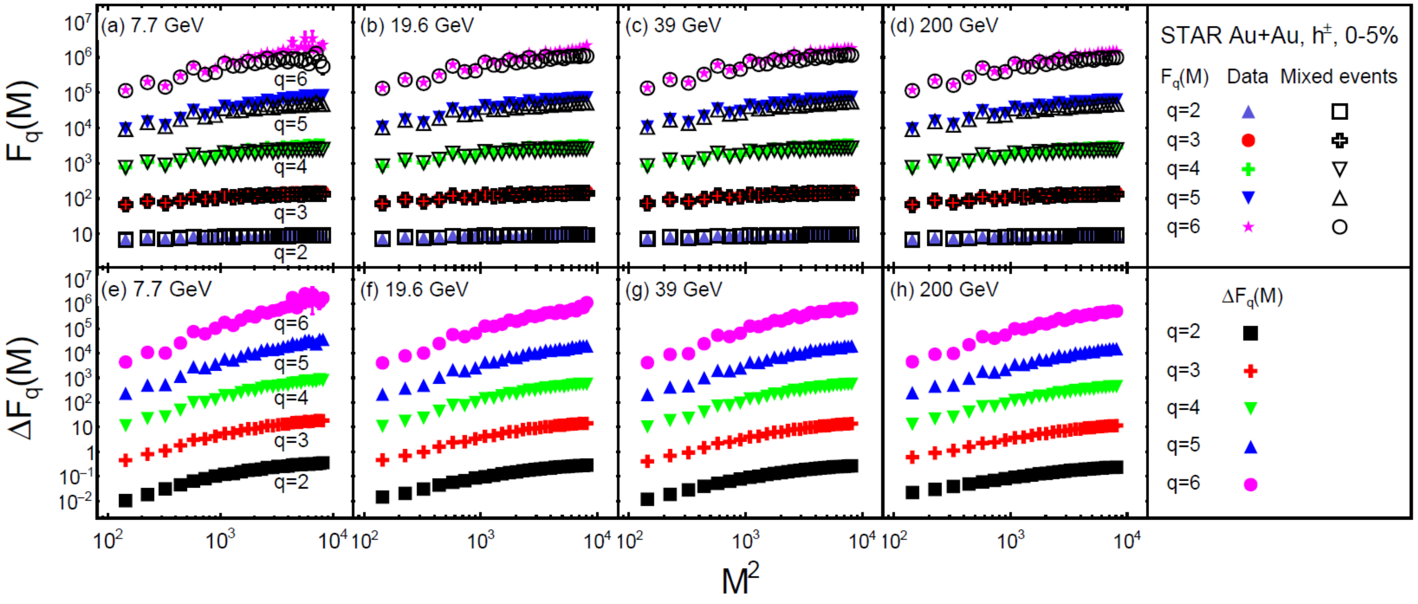
Intermittency analysis of charged-particle production

- ALICE preliminary:



- Very challenging results and interpretation of results

- Results from STAR Collaboration



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