Double transverse momentum dependent parton distributions matched to DPDFs

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in collaboration with Markus Diehl

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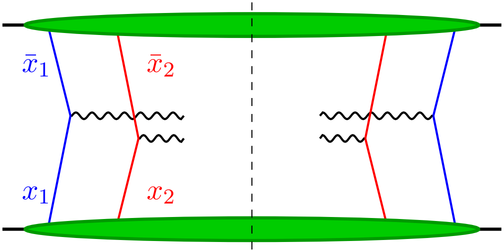
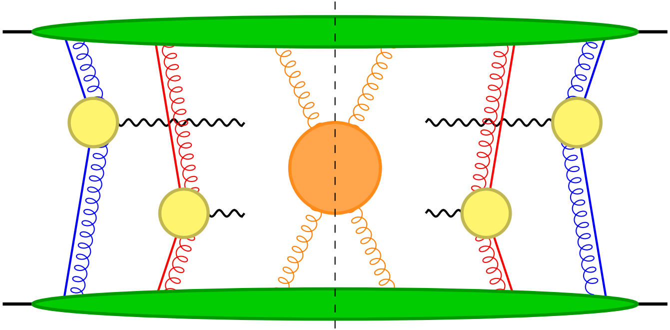
Motivation: transverse-momentum dependence in like-sign W production



- Single parton scattering (SPS) – α_S^2-suppressed compared to DPS, 2+ jets.
- Enhancement of double parton densities at small x.
- First observation – CMS, Phys.Rev.Lett. 131 (2023)

Double parton factorization: leading-power graphs

Leading-power Feynman graph for DPS:

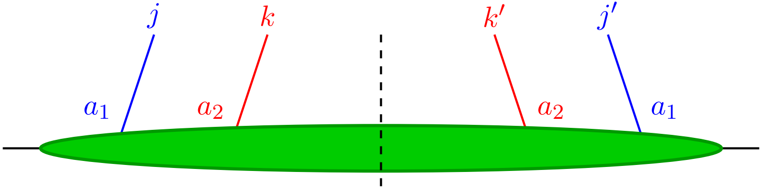


2 hard scales: $Q_{1,2}$.

Soft gluons exchange $\rightarrow$ Collins-Soper evolution (see M. Diehl's talk on Wednesday).

Diehl, Ostermeier, Schäfer, *Elements of a theory for multiparton interactions in QCD*

Colour structure of DPDs



Parton carry colour indices $\to$ project on SU(3) representations.

For quarks one can either project on $\delta_{jj'}\delta_{kk'}$ (colour singlet) or $t^c_{jj'}t^c_{kk'}$ (colour octet).

Gluons $\to$ 1, S, A, 10, $\overline{10}$, 27.

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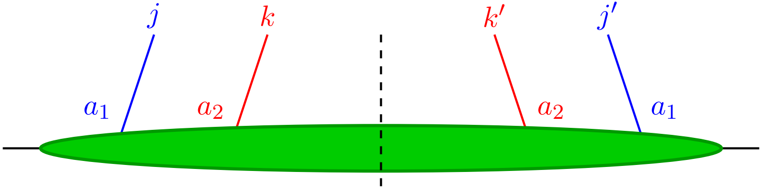

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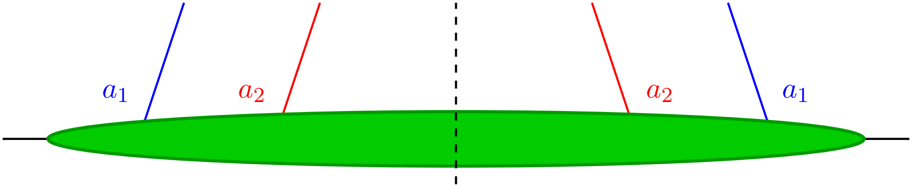
Notation

${}^{R_1R_2}F_{a_1a_2}$ for parton a_i in representation R_i.

DTMD in transverse position space

$$\frac{d\sigma}{\prod_{j=1,2} dx_i d\bar{x}_i d^2\mathbf{q}_i} = \frac{1}{C}\sum_{a,b}\hat{\sigma}_{a_1b_1}\hat{\sigma}_{a_2b_2}\prod_{i=1,2}\int d^2\mathbf{k}_i\, d^2\bar{\mathbf{k}}_i\, \delta^{(2)}\left(\mathbf{q}_i - \mathbf{k}_i - \bar{\mathbf{k}}_i\right) \times \int d^2\mathbf{y}\, F_{a_1a_2}\left(x_i, \mathbf{k}_i, \mathbf{y}\right) F_{b_1b_2}\left(\bar{x}_i, \bar{\mathbf{k}}_i, \mathbf{y}\right) . \tag{1}$$



$\mathbf{y}$ – transverse distance between partons 1 and 2.

DTMD in transverse position space

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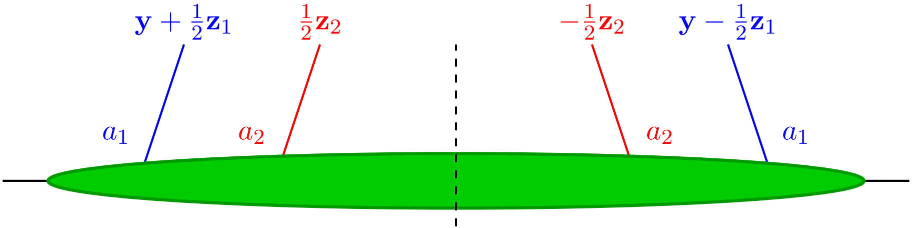
$$\times \int d^2\mathbf{y}\; F_{a_1 a_2}\big(x_i, \mathbf{z}_i, \mathbf{y}\big) F_{b_1 b_2}\big(\bar{x}_i, \mathbf{z}_i, \mathbf{y}\big)\,. \tag{2}$$



Relevant regions in the position space

$\mathbf{z}_i$ is Fourier-conjugate to $\mathbf{q}_i$ $\rightarrow$ assuming large transverse momenta:

$$\Lambda \ll |\mathbf{q}_i| \ll Q_i \ ,$$

the most important contribution stems from

$$|\mathbf{z}_i|^{-1} \gtrsim |\mathbf{q}_i| \gg \Lambda \ .$$

$\mathbf{y}$ $\rightarrow$ need to consider 2 regions:

- $|\mathbf{y}| \sim |\mathbf{z}| \ll \Lambda^{-1}$: DTMD can be expressed in terms of PDFs (splitting) and twist-four distributions.
- $|\mathbf{y}| \gg |\mathbf{z}|$: use operator product expansion to match DTMDs onto collinear double parton distributions (DPDF).

Large y approximation

Matching formula

$$ {}^{R_1 R_2}F_{a_1,a_2}(x, \bar{x}, \mathbf{y}, \mathbf{z}_i; \mu_{0i}, \zeta) = \sum_{b_1,b_2} \sum_{R'_1,R'_2} {}^{R_1\bar{R}'_1}C_{a_1 b_1} \underset{x_1}{\otimes} {}^{R_2\bar{R}'_2}C_{a_2 b_2} \underset{x_2}{\otimes} {}^{R'_1 R'_2}F^{coll.}_{b_1 b_2}(x'_i, \mathbf{y}; \mu_{0i}, \zeta) \,, \quad (3) $$

with the kernels

$$ {}^{R_1\bar{R}'_1}C_{a_1 b_1} = {}^{R_1\bar{R}'_1}C_{a_1 b_1}(x_1, \mathbf{z}_1; \mu_{01}, x_1^2\zeta), \quad (4) $$

known up to order α_S in both colour channels.

Scales in the problem

- The natural choice of scales, at which DPDFs is computed, is given by $|\mathbf{y}|$ (see also talks by P. Plößl and M. Diehl).
- Matching should be computed at scales $\mu_{0i} \propto 1/|\mathbf{z}_i|$
  $\to$ evolve DPDFs to the matching scales using DGLAP equations:

$$\frac{\partial}{\partial \log \mu} {}^{R_1 R_2}F_{a_1 a_2}(x_i, y; \mu_i, \zeta) = - {}^{R_1}\gamma_J(\mu_1) \log \frac{x_1 \sqrt{\zeta}}{\mu_1} {}^{R_1 R_2}F_{a_1 a_2}(x_i, y; \mu_i, \zeta) + 2 \sum_{b_1, R_1'} P_{a_1 b_1}(x_1; \mu_1) \underset{x_1}{\otimes} {}^{R_1' R_2}F_{b_1 a_2}(x_i', y; \mu_i, \zeta) \, . \tag{5}$$

More detailed discussion $\to$ see M. Diehl's talk.

Scales in the problem

- Rapidity dependence → use the Collins-Soper equation for the matching kernels:

$$\frac{\partial}{\partial \log \sqrt{\zeta}} {}^{R_1 R_1'}C_{a_1 b_1}(x_1, \mathbf{z}_1; \mu_1, \zeta) = {}^{R_1}K_{a_1}(\mathbf{z}_1; \mu_1) {}^{R_1 R_1'}C_{a_1 b_1}(x_1, \mathbf{z}_1; \mu_1, \zeta) \,. \quad (6)$$

${}^{R_1}K_{a_1}$ = colour factor × singlet TMD Collins-Soper kernel.

Solution

$${}^{R_1 R_1'}C_{a_1 b_1}(x_1, \mathbf{z}_1; \mu_1, \zeta') = \exp\left({}^{R_1}K_{a_1}(\mathbf{z}_1; \mu_1) \log \frac{\sqrt{\zeta'}}{\sqrt{\zeta}} \right) {}^{R_1 R_1'}C_{a_1 b_1}(x_1, \mathbf{z}_1; \mu_1, \zeta) \,. \quad (7)$$

Scales in the problem

Finally, matched DTMDs are evolved to the final scales $\sim Q_i$

Evolution equations for DTMDs

$$\frac{\partial}{\partial \log \sqrt{\zeta}} {}^{R_1R_2}F_{a_1a_2} = \left[{}^{R_1}J(\mathbf{y};\mu_i) + {}^{R_1}K(\mathbf{z}_1;\mu_1) + {}^{R_2}K(\mathbf{z}_2;\mu_2)\right] {}^{R_1R_2}F_{a_1a_2} \,,$$
$$\frac{\partial}{\partial \log \mu_1} {}^{R_1R_2}F_{a_1a_2} = \left[\gamma_{a_1} - \gamma_{K,a_1} \log \frac{x_1\sqrt{\zeta}}{\mu}\right] {}^{R_1R_2}F_{a_1a_2} \,. \tag{8}$$

${}^{1}J = 0$ **at all orders!**

Effects of the rapidity evolution

Solution of the Collins-Soper equation for DTMDs

$$
\begin{aligned}
&{}^{R_1R_2}F_{a_1a_2}(x_i, \mathbf{z}_i, \mathbf{y}; \mu_{0i}, \zeta) \\
&= \exp\left(\left[{}^{R_1}J(\mathbf{y}; \mu_{0i}) + {}^{R_1}K(\mathbf{z}_1; \mu_{01}) + {}^{R_2}K(\mathbf{z}_2; \mu_{02})\right] \log\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\right) {}^{R_1R_2}F_{a_1a_2}(x_i, \mathbf{z}_i, \mathbf{y}; \mu_{0i}, \zeta_0).
\end{aligned} \tag{9}
$$

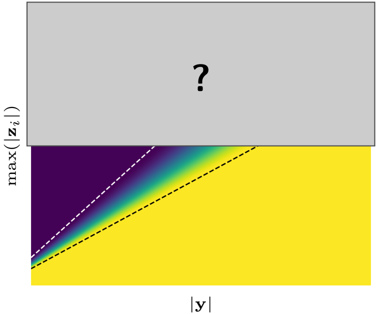
Large-y behavior of the Collins-Soper kernel J – see [arXiv:2310.16432]

$$
{}^{R}J(\mathbf{y}; \mu_i) = {}^{R}J^{pert.}(\mathbf{y}^*; \mu_i) + \Delta\, {}^{R}J(\mathbf{y}), \quad \mathbf{y}^* - \text{regularized distance}, \tag{10}
$$

$\Delta\, {}^{8}J$ is $\mathcal{O}(-1)$ at large $\mathbf{y}$.

For high final scales one has a strong suppression of the color non-singlet channel at large $\mathbf{y}$.

Extending the approximation to larger $\mathbf{z}$



- Small $\mathbf{z} \to$ interpolate between two approximations in the overlap region.

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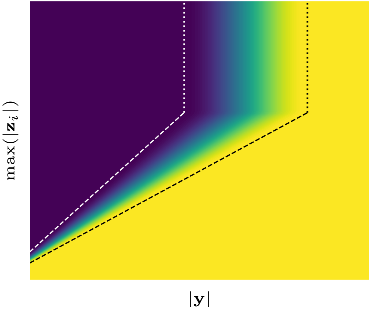
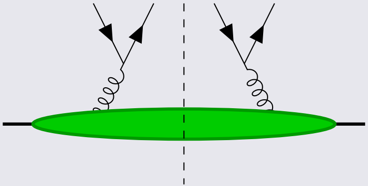


- Small $\mathbf{z}$ $\rightarrow$ interpolate between two approximations in the overlap region.
- $|\mathbf{y}| > y_{\max}$ $\rightarrow$ match DTMD at regularized distances $\mathbf{z}^*$, multiply by $e^{-\text{const.}\times \mathbf{z}_i^2}$.
- At small y extrapolate the short-distance result. Interpolate for intermediate $\mathbf{y}$.

Initial conditions for DPDFs

Splitting part



Perturbative splitting at scale $\propto |\mathbf{y}^*|^{-1}$
$\times$ Gaussian fall-off.

Intrinsic part

Product of PDFs $\times$ further factors, see Diehl, Gaunt, Lang, Plößl, Schäfer [2001.10428].

$$ {}^{R_1 R_2}F_{ab}^{intr.} = \text{colour factor}(R_1 R_2) \times {}^{11}F_{ab}^{intr.} \,. \tag{11} $$

This form saturates the positivity bounds in colour space.

Numerical results – matched DTMDs in z-space

- ChiliPDF – C++ library for evolution and interpolation of parton distributions
  Diehl, Nagar, Plößl, Tackmann [2305.0484]

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- Hard scales $\mu_i = \sqrt{\zeta} = M_W$, $\quad \sqrt{s} = 13$ TeV.
- $x_1 = x_2 = M_W/(13\,\mathrm{TeV}) \approx 6 \times 10^{-3}$.
- $|\mathbf{z}_1| = 0.1\,\mathrm{GeV}^{-1} =$ const.

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- $|\mathbf{z}_1| = 0.1\,\mathrm{GeV}^{-1} =$ const.
- Estimation of higher-order corrections
  → compare results obtained using 2 different choices of the matching scales:

$$\mu_0(\mathbf{z}) = \frac{b_0}{|\mathbf{z}^*|}, \frac{2b_0}{|\mathbf{z}^*|}, \qquad b_0 = 1.123... \tag{12}$$

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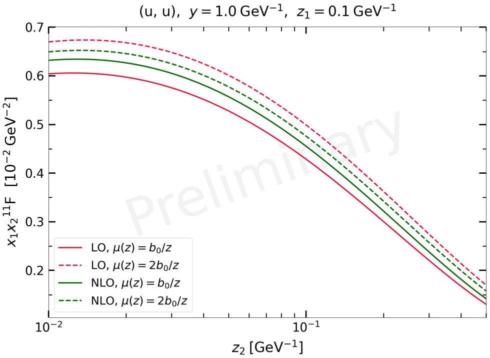

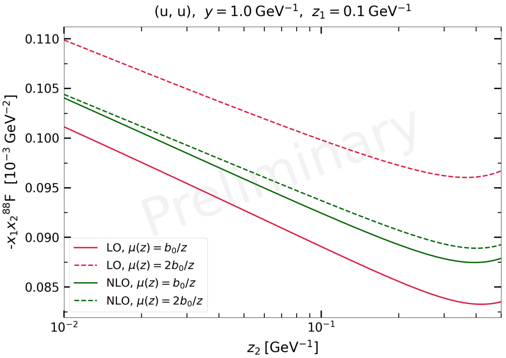

Left: singlet, right: colour octet.

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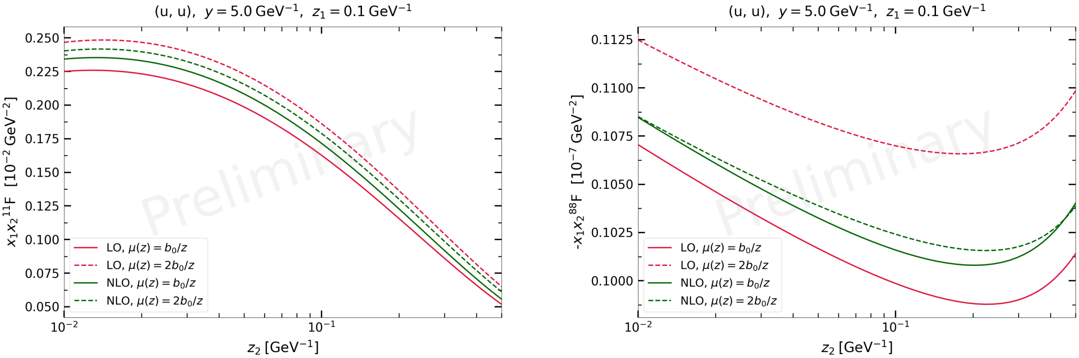

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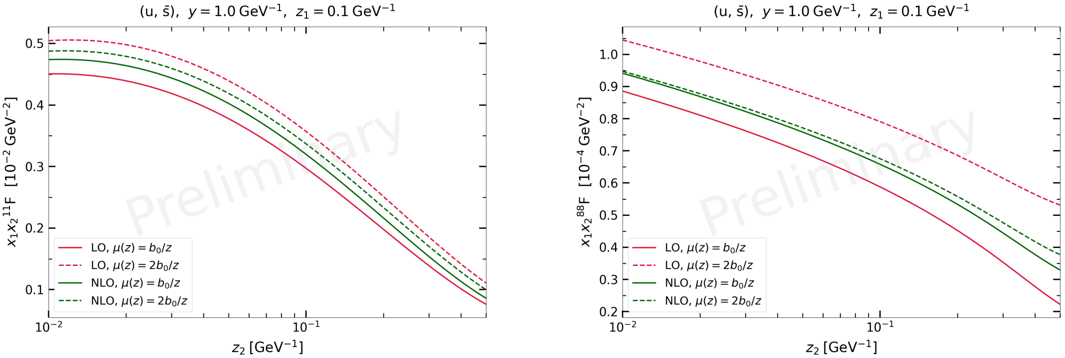

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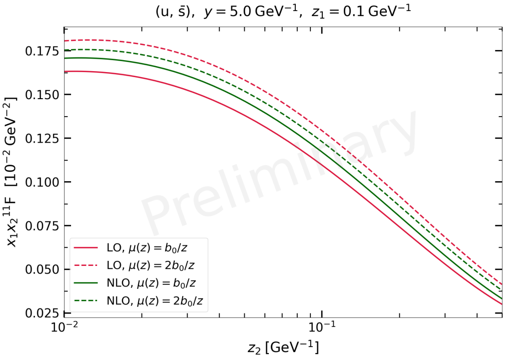

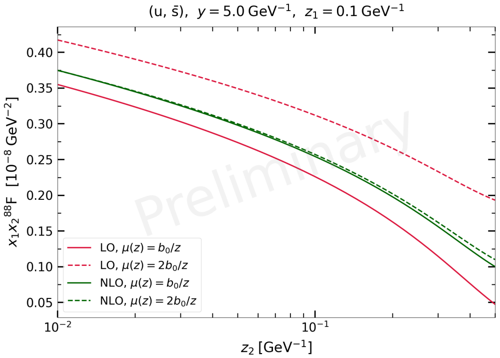

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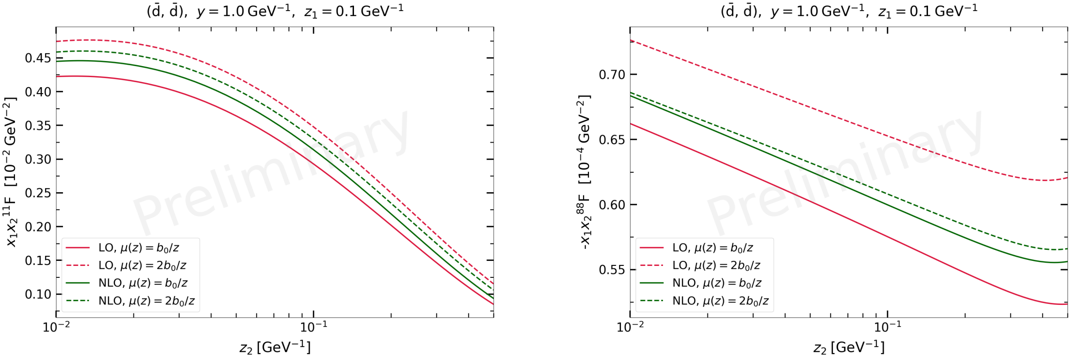

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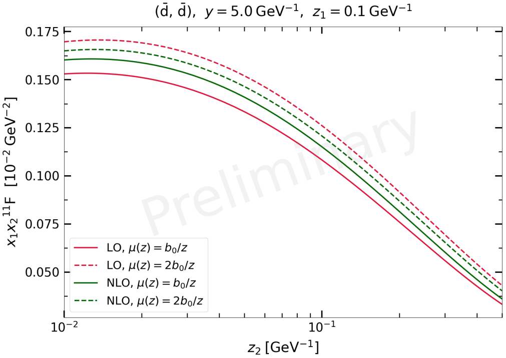

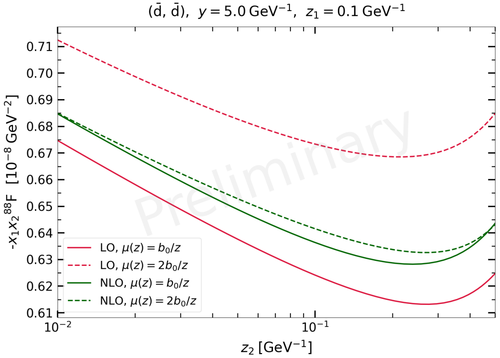

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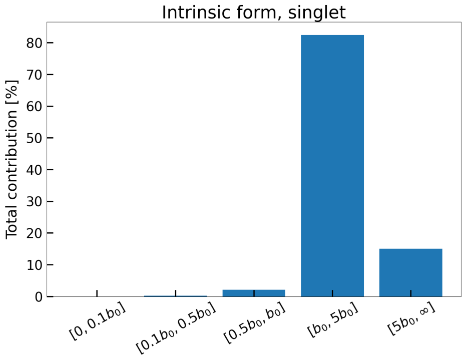
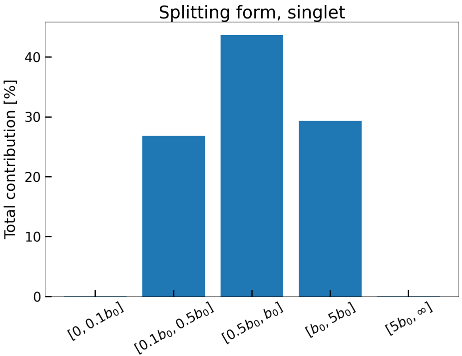
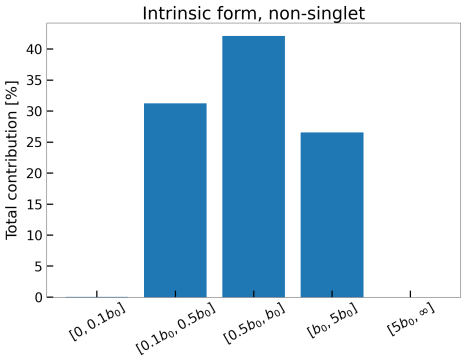
Numerical results – contributions from different y-regions

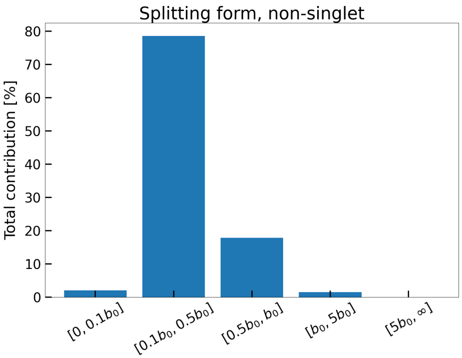




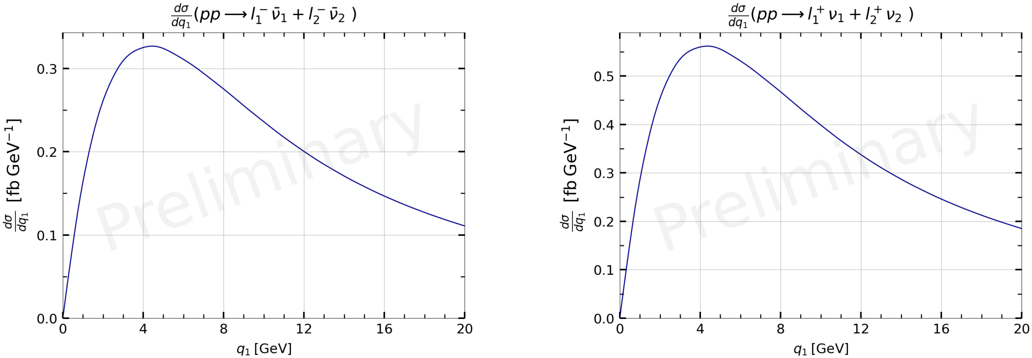
$$\frac{d\sigma^{S,spl.}_{W^+W^+}}{dY_1 dY_2}\Big|_{Y_j=0} \sim 10^{-2} \times \frac{d\sigma^{S,intr.}_{W^+W^+}}{dY_1 dY_2}\Big|_{Y_j=0} .$$

Numerical results – contributions from different y-regions





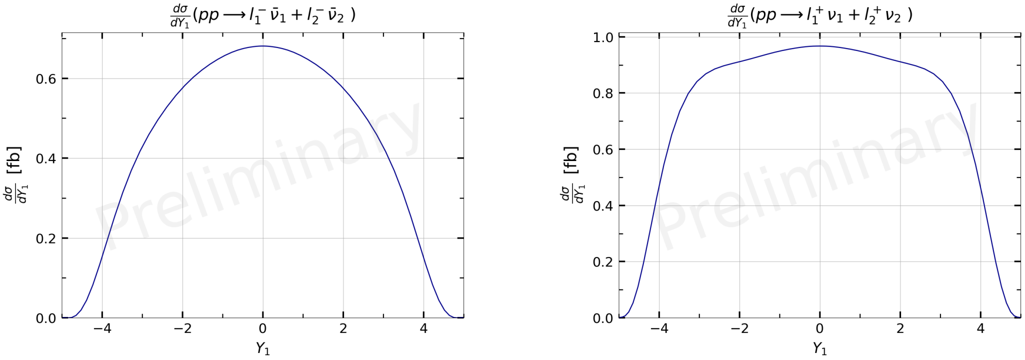
$$\frac{d\sigma^{NS,spl.}_{W^+W^+}}{dY_1 dY_2}\Big|_{Y_j=0} \sim 10^{-1} \times \frac{d\sigma^{NS,intr.}_{W^+W^+}}{dY_1 dY_2}\Big|_{Y_j=0} \sim 10^{-4} \times \frac{d\sigma^{S,intr.}_{W^+W^+}}{dY_1 dY_2}\Big|_{Y_j=0} .$$


$\frac{d\sigma}{dq_1}(pp \longrightarrow l_1^- \bar{\nu}_1 + l_2^- \bar{\nu}_2)$
$\frac{d\sigma}{dq_1}$ [fb GeV^{-1}]
q_1 [GeV]
Preliminary
$\frac{d\sigma}{dq_1}(pp \longrightarrow l_1^+ \nu_1 + l_2^+ \nu_2)$
$\frac{d\sigma}{dq_1}$ [fb GeV^{-1}]
q_1 [GeV]
Preliminary

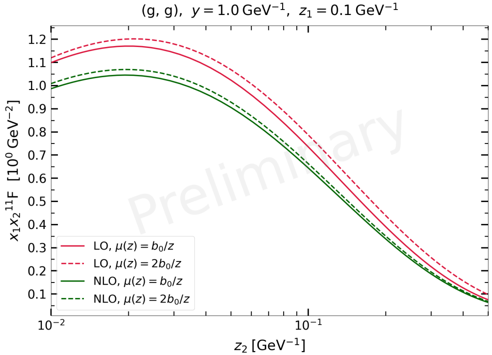
Summary

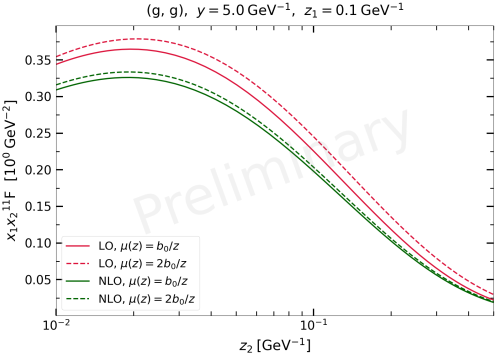
- Using the operator product expansion we obtain the description of DPS at large transverse momenta.
- Large-y approximation $\rightarrow$ dominant contribution from the singlet intrinsic part in the region of nonperturbative $\mathbf{y}$.
- Contribution from non-singlet DPDs strongly suppressed.
- Future work:
  - Impact of the non-perturbative input.
  - Study of the short-distance region.

Backup – rapidity dependence

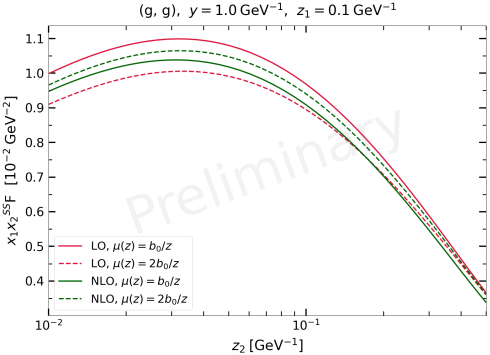


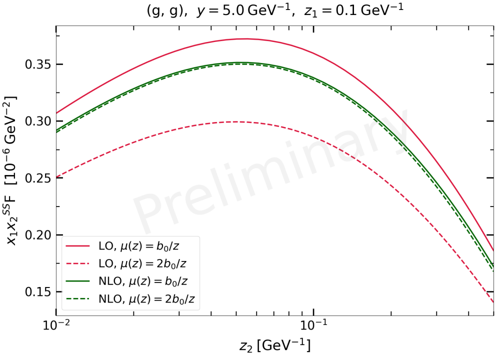
Backup – gluon DTMDs – $(R_1, R_2) = (1, 1)$



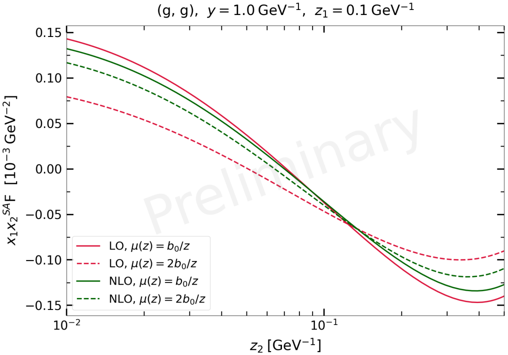


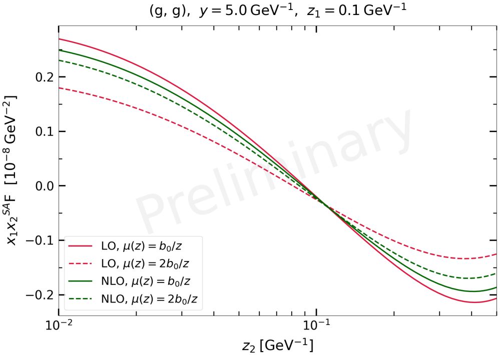
Backup – gluon DTMDs – $(R_1, R_2) = (S, S)$





Backup – gluon DTMDs – $(R_1, R_2) = (S, A)$





Backup – gluon DTMDs – $(R_1, R_2) = (27, 27)$