Hot spot model of nucleon and double parton scattering

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European J. of Physics C83 (2023) 5, 415

1. Introduction

The hot spot model-one of the popular approaches to the nucleon structure. Basic idea—the gluon density is not homogeneous, but has fluctuations, and is concentrated in several points –black spots. In particular in these points the density can reach black limit values, while being low in other parts of a nucleon. The special interest are the the case of three black spots, since they can be associated with constituent quarks.

The aim of the present talk is to calculate the DPS rate in the hot spots model. We shall see that the hot spots model with parameters determined from the virtual Compton scattering in HERA is in tension with the experimental data on DPS in pp collisions as determined by CMS and ATLAS experiments.

2. Hot spots model. Nonperturbative hot spots first Levin and Frankfurt(1968), pQCD hot spots-A.H. Mueller (1991)
Recently a new model based on hot spots idea was proposed
H. Mäntysaari, B. Schenke, (2017), H. Mäntysaari, B. Schenke, C. Shen, W. Zhao (2022), H.Mäntysaari,F.Salazar,B.Schenke, (2022)
The parameters of the model are fixed by studing the reaction $\gamma p \to J/\psi + gap + Y$
Basic ideas of hot spot model:
1. the hot spot distribution density is given by
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$$\rho\left(\left\{\vec{b}_i\right\}_{i=1}^{N_q} |\vec{c}\right) = \frac{2\pi B_p}{N_q}\delta^{(2)}\left(\frac{\sum_{i=1}^{N_q}\vec{b}_i}{N_q} - \vec{c}\right) \times \left(\prod_{i=1}^{N_q}\frac{1}{2\pi B_p}e^{-\frac{(\vec{b}_i-\vec{c})^2}{2B_p}}\right),$$

Here the distribution is gaussian, c is the hadron center of mass coordiante, b_i are the hot spot positions, B_p is the distribution width and N_q us the number of hot spots

The gluon density is given by

$$\rho\left(\vec{r},\left\{\vec{b}_i,p_i\right\}_{i=1}^{N_q}|\vec{c}\right)=\frac{2\pi B_p}{N_q}\delta^{(2)}\left(\frac{\sum_{i=1}^{N_q}\vec{b}_i}{N_q}-\vec{c}\right)$$
$$\times\left(\prod_{i=1}^{N_q}\frac{1}{2\pi B_p}e^{-\frac{(b_i-\vec{c})^2}{2B_p}}\right)$$
$$\times\left(\frac{1}{N_q}\sum_{i=1}^{N_q}p_i\frac{1}{2\pi B_q}e^{-\frac{(\vec{r}-\vec{b}_i)^2}{2B_q}}\right).$$

Which is normalized to one in other words this is a probability to find a parton in a point r.
The strenght of the hot spots is assumed to have random distribution

$$P(\log(p_i))=\frac{1}{\sqrt{2\pi}\sigma}\exp(-\log(p_i)^2/2\sigma^2),$$

For average value we have E is given by: $E\left[p^n\right]=e^{\frac{n^2\sigma^2}{2}}.$

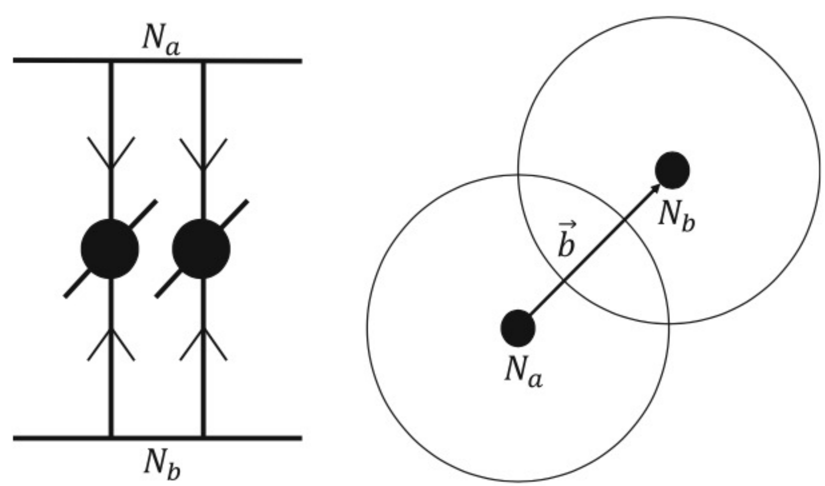
3.Some comments on DPS. Recall some basic ideas of DPS that were discussed a lot in these conferences. The DPS is characterised by effective cross section so that the total DPS cross section is given by

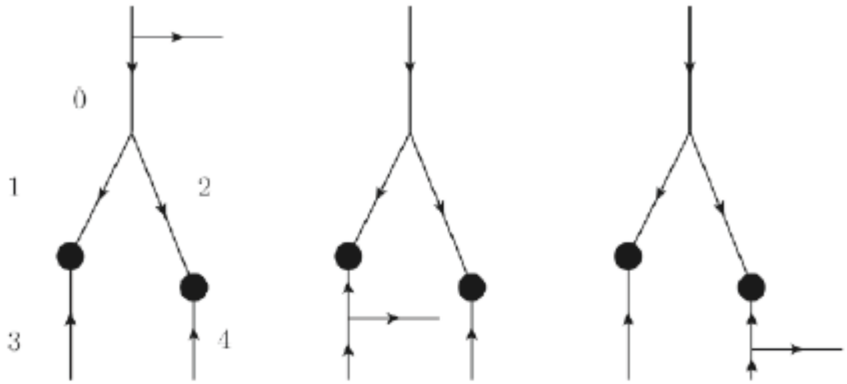
$$\sigma_{DPS} = \frac{\sigma_1 \sigma_2}{\sigma_{\mathrm{eff}}},$$

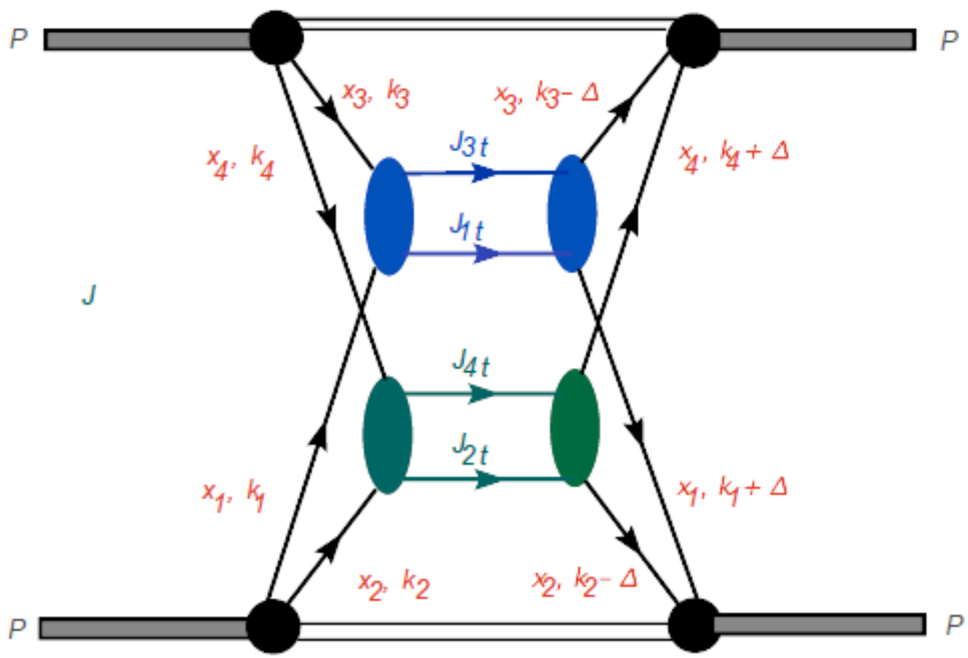
Where sigma1,2 are the cross sections of two hard processes.

- In the mean field approximation the effective cross section is given by

$$\frac{1}{\sigma_{\mathrm{eff}}} = \frac{\int \frac{d^2\Delta}{(2\pi)^2} {}_2G(x_1, x_2, Q_1^2, Q_2^2, \Delta)\, {}_2G(x_3, x_4, Q_1^2, Q_2^2, \Delta)}{f(x_1, Q_1^2) f(x_2, Q_2^2) f(x_3, Q_3^2) f(x_4, Q_4^2)}.$$







G. 1: Kinematics of double hard collision - momenta of t

In the mean field approximation the two-GPD is a product of two one-GPDs,

$$ {}_2G(x_1, x_2, Q_1^2, Q_2^2, \Delta) = {}_1G(x_1, Q_1^2, \Delta)\, {}_1G(x_1, Q_1^2, \Delta), $$

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$$ {}_1G(x_1, Q_1^2, \Delta) = f(x_1, Q_1^2) F_{2g}(\Delta, x_1), $$

The latter can be represented as a product of conventional PDF and two-gluon formfactor, that can be determined in a model independent way by parametrisation of HERA data (Frankfurt, Strikman, 2003)

In coordinate space we can rewrite this equation in the form:

$$ {}_1G(x, Q^2, \vec{r}) = f(x, Q^2)\rho(\vec{r});\ \rho(\vec{r}) = \int \frac{d^2\Delta}{(2\pi)^2} F_{2g}(\Delta, x_1) \exp(i\vec{\Delta}\vec{r}) $$

And thus obtain the effective cross section in terms of normalized to one gluon density.

$$ \frac{1}{\sigma_{\rm eff}} = \int d^2b \left(\int d^2r \rho(\vec{r})\rho(\vec{r} - \vec{b})\right)^2, $$

4. Calculation of DPS effective cross section in hot spot model

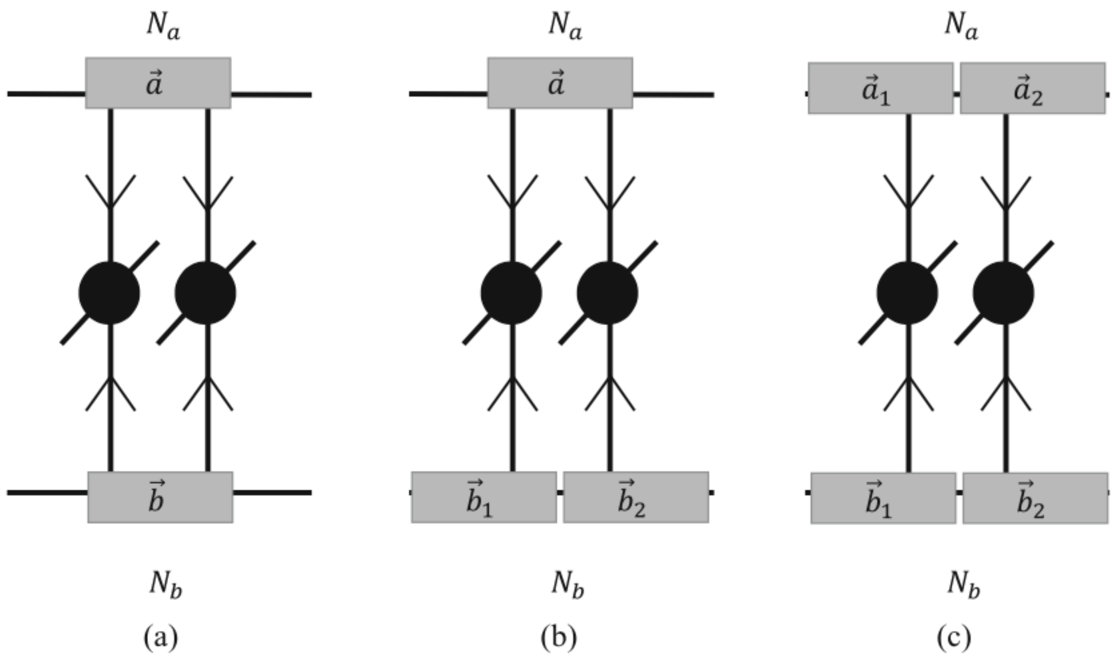

Fig. 2 Three distinct classes of diagrams for DPS scattering in the hot spots model

- Class I: The two partons come from one hot spot for both protons, or $i_1 = i_2$ and $j_1 = j_2$.
- Class II: One proton emits the two partons from a single hot spot while the other emits them from two different hot-spots, or $i_1 = i_2$ and $j_1 \neq j_2$ or $i_1 \neq i_2$ and $j_1 = j_2$.

• *Class III: Each proton emits the two partons from different hot spots, or* $i_1 \neq i_2$ *and* $j_1 \neq j_2$.

$$\int d^2 r \rho\left(\vec{r}, \{\vec{a}_i\}_{i=1}^{N_q}, \vec{c}\right) \rho\left(\vec{r}, \left\{\vec{b}_j\right\}_{j=1}^{N_q}, \vec{c}+\vec{b}\right)$$

$$\propto \sum_{i,j} \int d^2 r e^{-\frac{(\vec{r}-\vec{a}_i)^2+\left(\vec{r}-\vec{b}_j\right)^2}{2B_q}}$$

$$= \pi B_q \sum_{i,j} e^{-\frac{\left(\vec{a}_i-\vec{b}_j\right)^2}{4B_q}}.$$

We now take the square of this expression and integrate over the hot spot positions. We shall also need a representation of delta function:

$$\delta^{(2)}\left(\frac{\sum_{i=1}^{N_q} \vec{a}_i}{N_q} - \vec{c}\right) \delta^{(2)}\left(\frac{\sum_{j=1}^{N_q} \vec{b}_j}{N_q} - \left(\vec{c}+\vec{b}\right)\right)$$

$$= \int \frac{d^2 s_1 d^2 s_2}{(2\pi)^4} e^{i\vec{s}_1 \cdot \left(\frac{\sum_{i=1}^{N_q} \vec{a}_i}{N_q} - \vec{c}\right)} e^{i\vec{s}_2 \cdot \left(\frac{\sum_{j=1}^{N_q} \vec{b}_j}{N_q} - \left(\vec{c}+\vec{b}\right)\right)}$$

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Then for the case A we obtain:

$$A = \frac{p^2 \tilde{p}^2}{4N_q^4 (2\pi)^6 B_q^2} \left(\int d^5x e^{-\vec{x}^T M_A \vec{x}} \right)^2,$$

$$M_A = \begin{pmatrix} \frac{B_p+B_q}{2B_pB_q} & -\frac{1}{2B_q} & 0 & -\frac{i}{2N_q} & 0 \\ -\frac{1}{2B_q} & \frac{B_p+B_q}{2B_pB_q} & -\frac{1}{2B_p} & 0 & -\frac{i}{2N_q} \\ 0 & -\frac{1}{2B_p} & \frac{1}{2B_p} & 0 & \frac{i}{2N_q} \\ -\frac{i}{2N_q} & 0 & 0 & \frac{N_q-1}{2N_q^2}B_p & 0 \\ 0 & -\frac{i}{2N_q} & \frac{i}{2N_q} & 0 & \frac{N_q-1}{2N_q^2}B_p \end{pmatrix},$$

After taking this gaussian integral we obtain

$$A = \frac{p^2 \tilde{p}^2}{8\pi B_q {N_q}^2}.$$

Or averaging over hot spot strengths in both hadrons we obtain

$$A = \frac{E\left[p^2\right]^2}{8\pi B_q {N_q}^2} = \frac{e^{4\sigma^2}}{8\pi B_q {N_q}^2}.$$

For the case B we obtain in the same way

$$B = \frac{\left(N_q - 1\right) E\left[p^2\right] E\left[p\right]^2}{2\pi\left(B_p + 2B_q\right) {N_q}^2} = \frac{\left(N_q - 1\right) e^{3\sigma^2}}{2\pi\left(B_p + 2B_q\right) {N_q}^2},$$

For the case C in the same way we get

$$C = \frac{p_1 \tilde{p}_1 p_2 \tilde{p}_2 \left(N_q - 1\right)^2}{8\pi\left(B_p + B_q\right) N_q^2},$$

Or after averaging we get

$$C = \frac{E\left[p\right]^4 \left(N_q - 1\right)^2}{8\pi\left(B_p + B_q\right) N_q^2} = \frac{\left(N_q - 1\right)^2 e^{2\sigma^2}}{8\pi\left(B_p + B_q\right) N_q^2},$$

For the total effective cross section expressed through the parameters of the hot spot model we obtain

$$\sigma_{\text{eff}} = (A + B + C)^{-1} = \frac{8\pi N_q^2}{\frac{e^{2\sigma^2}}{B_q} + \frac{4(N_q-1)e^{\sigma^2}}{B_p+2B_q} + \frac{(N_q-1)^2}{B_p+B_q}}.$$

According to the fit of the experimental data there are two suitables values of parameters:

Parameter	Description	Variable N_q	$N_q \equiv 3$
N_q	Number of hot spots	$6.79^{+2.93}_{-4.83}$	3
σ	Magnitude of hot spots strength fluctuations	$0.833^{+0.194}_{-0.441}$	$0.563^{+0.143}_{-0.141}$
$B_q\,[GeV^{-2}]$	Hot spot size	$0.474^{+0.434}_{-0.286}$	$0.346^{+0.282}_{-0.202}$
$B_p\,[GeV^{-2}]$	Proton size	$4.02^{+1.73}_{0.728}$	$4.45^{+0.801}_{-0.803}$

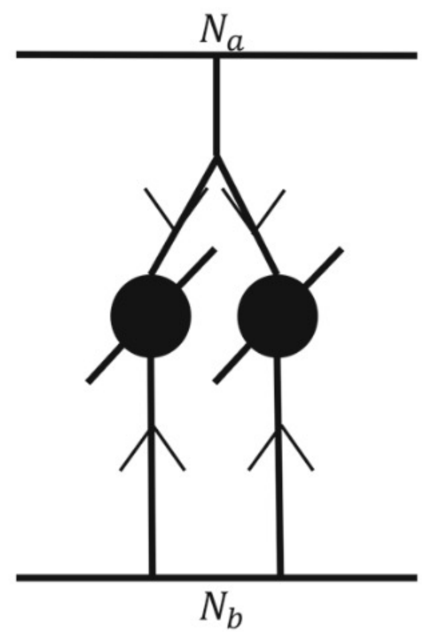
leading to:

$$\sigma_{\text{eff}} \approx 17 \text{ mb for variable } N_q = 6.79,$$

$$\sigma_{\text{eff}} \approx 10.5 \text{ mb for } N_q = 3.$$

5. Comparison with the experimental data. Such data exists for both ATLAS and CMS measurements for jet energy $p_T > 20$ GeV.

For CMS 2021 and ATLAS 2016 (latest) measurements we have for Effective cross section 13 ± 3 mb *For earliar CMS meauserements we have* 20 ± 5 mb *These results seem to be compatible with the results for the DPS effective cross section for the hot spot model. However this does not take into account pQCD evolution. The hot spot model parameters were obtained for scales of order 1 GeV. So we need to go from the scales of 20 GeV to scales 1 GeV for DPS (or vice versa). The main source of such pQCD evolution are the so called* $1 \to 2$ *Processes (Gaunt and Stirling 2010,2011, Blok, Dokshitzer Frankfurt and Strikman 2011,2012,2014 Diehl, Ostermeier and Schafer 2012, Manohar and Waalewijn 2012*



$$\sigma_{\rm eff}(20\ GeV)/\sigma_{\rm eff}(1\ GeV) \sim 0.6.$$

Blok and Gunnellini 2015

Measurement	σ_{eff} at scale 20 GeV	σ_{eff} at scale ~1 GeV
CMS 2021 [27] and ATLAS 2016 [28]	13 ± 3 mb	20 ± 5 mb
CMS2016 [29]	20 ± 5 mb	32 ± 8 mb
Hot spots fit $N_q = 3$		~ 10 mb
Hot spots fit variable N_q		~ 17 mb

We see that with pQCD taken into account the results of DPS measurements and hot spot model for DPS are incompatible. However we do not know the accuracy of the parmeters, thus we shall just say that there is a tension between the effective cross section of DPS in hot spot model and experimental DPS data.