Evolution of colour dependent double parton distributions: a quantitative study

M. Diehl

Deutsches Elektronen-Synchrotron DESY

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What this talk is about

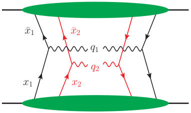
the importance of colour correlations between the initial partons in double parton scattering

- with two hard scales
  e.g. like-sign W pairs, W plus dijets, two dijets, ...
- with one scale hard and another one less hard (but still perturbative)
  underlying event kinematics

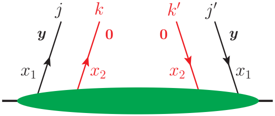
Colour correlations in DPDs and DPS

DPS cross section formula:

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{\hat{\sigma}_{a_1 b_1 \to A_1}\, \hat{\sigma}_{a_2 b_2 \to A_2}}{1 + \delta_{A_1 A_2}} \int d^2\boldsymbol{y}\; F_{a_1 a_2}(x_1, x_2, \boldsymbol{y})\, F_{b_1 b_2}(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$



- describe colour state of the partons in each DPD:



- project colour of each parton pair on SU(N_c) representations
- colour singlet: $\delta_{jj'}\, \delta_{kk'}$
  summed over each parton's colour as in usual PDFs
- colour octet: $t^a_{jj'}\, t^a_{kk'}$
  describes colour correlations
- for gluons have $8_A, 8_S, 10, \overline{10}, 27$

- notation: ${}^{R_1 R_2}F_{a_1 a_2}$ for parton a_i in colour representation R_i
  need $\dim(R_1) = \dim(R_2)$ for overall colour singlet of the 4 parton lines

Colour correlations in DPDs and DPS

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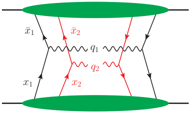

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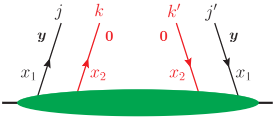

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- colour non-singlet DPDs introduced by M Mekhfi, 1985
  but neglected in almost all DPS studies — why?

Colour correlations in DPDs and DPS

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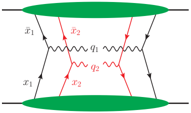
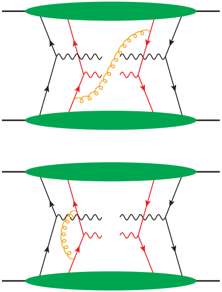


- soft gluon corrections



- infrared region cancels between real and virtual graphs for colour singlet DPDs
- but not for other colour channels
  $\rightsquigarrow$ suppressed by Sudakov logarithms
  M Mekhfi, X Artru 1988

Colour correlations in DPDs and DPS

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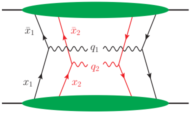

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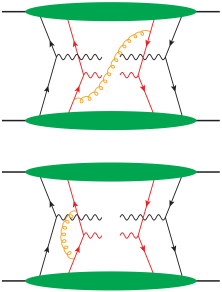
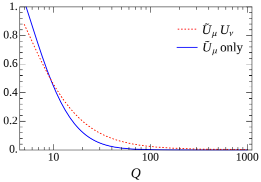


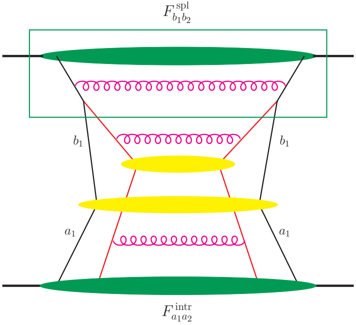
- first quantitative estimate:
  A Manohar, W Waalewijn arXiv:1202:3794



here: logarithms built up between
IR scale $\Lambda = 1.4\,\text{GeV}$ and hard scale Q

DPD splitting: enter a third scale

- at small y can distinguish between splitting and intrinsic part of DPD
  MD, J Gaunt, K Schönwald 2017
- $\rightsquigarrow$ three scales $\Lambda < 1/y < Q$



- from Λ to $1/y$:
  DGLAP evolution of PDF inside F_{spl}
- from $1/y$ to Q:
  evolution of DPDs, including Sudakov logarithms in colour non-singlet case
  $\rightsquigarrow$ Sudakov suppression reduced if $\Lambda \ll 1/y$

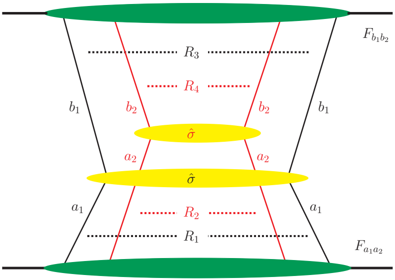
observed by B Blok, J Mehl, 2022
working in momentum representation (using Δ = Fourier conjugate of y)

Cross section formula including colour dependence:

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{{}^{\overline{R}_1 \overline{R}_3}\hat{\sigma}_{a_1 b_1 \to A_1}\; {}^{\overline{R}_2 \overline{R}_4}\hat{\sigma}_{a_2 b_2 \to A_2}}{1 + \delta_{A_1 A_2}}\; {}^{R_1 R_2, R_3 R_4}\mathcal{L}_{a_1 a_2, b_1 b_2}$$

with double parton luminosities

$${}^{R_1 R_2, R_3 R_4}\mathcal{L}_{a_1 a_2, b_1 b_2} = \int d^2\boldsymbol{y}\; {}^{R_1 R_2}F_{a_1 a_2}(x_1, x_2, y; \mu_1, \mu_2, \zeta)\; {}^{R_3 R_4}F_{b_1 b_2}(\bar{x}_1, \bar{x}_2, y; \mu_1, \mu_2, \bar{\zeta})$$

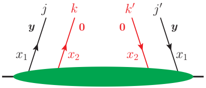


- $\hat{\sigma}$ becomes colour dependent
- for $ab \to$ colour singlet:

  $${}^{R\overline{R}'}\hat{\sigma}_{ab} = \pm\delta_{RR'}\; {}^{11}\hat{\sigma}_{ab}$$

- colour labels of DPDs and $\hat{\sigma}$s must match
  no "colour interference term"
  as proposed in B Blok, J Mehl, 2022

Evolution of colour dependent DPDs



variables:

μ_1, μ_2 $\leftrightarrow$ cutoff on virtuality of first/second parton

ζ $\leftrightarrow$ cutoff on soft gluon rapidities

- roughly: DPD includes rapidities between Y_p and $\ln \sqrt{s/\zeta}$
  in collision c.m., $\sqrt{s}$ = c.m. energy, Y_p = proton rapidity
- colour singlet DPDs ^{11}F :
  soft gluon effects cancel between real and virtual graphs
  $\rightsquigarrow$ no ζ dependence

more technical detail → backup slides

Evolution of colour dependent DPDs

- Collins-Soper equation:

$$\frac{d}{d \ln \sqrt{\zeta}} \, {}^{R_1 R_2} F_{a_1 a_2}(x_1, x_2, y; \mu_1, \mu_2, \zeta) = {}^{R_1} J(y; \mu_1, \mu_2) \, {}^{R_1 R_2} F_{a_1 a_2}(x_1, x_2, y; \mu_1, \mu_2, \zeta) \, .$$

with RGE $$\frac{d}{d \ln \mu_1} \, {}^{R} J(y; \mu_1, \mu_2) = - \, {}^{R} \gamma_J(\mu_1)$$

- Collins-Soper kernel ${}^{R}J$ depends on R only via $\dim(R)$
  for colour singlet: ${}^{1}J = 0$
- same form for TMDs and double TMDs (with different kernels)
- solution

$$ {}^{R_1 R_2} F_{a_1 a_2}(x_1, x_2, y; \mu_1, \mu_2, \zeta) = \exp\left[{}^{R_1} J(y; \mu_1, \mu_2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right] {}^{R_1 R_2} F_{a_1 a_2}(x_1, x_2, y; \mu_1, \mu_2, \zeta_0)$$

- simple in y space but **not** if use Fourier conjugate momentum Δ

more on Δ representation $\rightarrow$ backup slides

Evolution of colour dependent DPDs

- DGLAP equations:

$$\frac{d}{d\ln\mu_1} \, {}^{R_1R_2}F_{a_1a_2}(x_1,x_2,y;\mu_1,\mu_2,\zeta)$$

$$= - \, {}^{R_1}\gamma_J(\mu_1) \; \ln\left(\frac{x_1\sqrt{\zeta}}{\mu_1}\right) \, {}^{R_1R_2}F_{a_1a_2}(x_1,x_2,y;\mu_1,\mu_2,\zeta)$$

$$+ \, 2\sum_{b_1,R_1'} \int_{x_1}^{1} \frac{dz}{z} \, {}^{R_1\overline{R}_1'}P_{a_1b_1}\left(\frac{x_1}{z};\mu_1\right) \, {}^{R_1'R_2}F_{b_1a_2}(z,x_2,y;\mu_1,\mu_2,\zeta)$$

likewise for μ_2 dependence

- kernels ${}^{RR'}P$ known at NLO — F Fabry, MD, A Vladimirov 2022
- term with $\gamma_J \rightsquigarrow$ exponentiated Sudakov logarithms
- quark-gluon mixing:

$$ {}^{SR_2}F_{ga_2} \longleftrightarrow {}^{8R_2}F_{qa_2} + {}^{8R_2}F_{\bar{q}a_2}$$

$$ {}^{AR_2}F_{ga_2} \longleftrightarrow {}^{8R_2}F_{qa_2} - {}^{8R_2}F_{\bar{q}a_2}$$

- solved numerically in ChiliPDF

MD, R Nagar, P Plößl, F Tackmann 2023; F Fabry, PhD thesis 2023

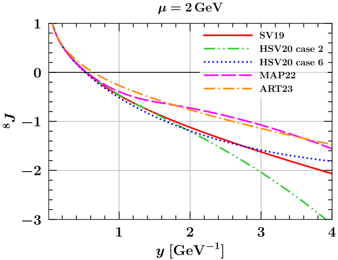
A quantitative study

MD, Florian Fabry, Peter Plößl

arXiv:2310.16432

Collins-Soper kernel

- exact relation: ${}^{8}J(y;\mu,\mu) = K_g(y;\mu)$
  $K_g =$ Collins-Soper kernel for single-gluon TMDs A Vladimirov, 2016
- we assume Casimir scaling $K_g(b)/C_A = K_q(b)/C_F$ for all distances b
  holds at small b up to $\mathcal{O}(\alpha_s^3)$
  and use a selection of recent fits of $K_q(b)$ to data
  which agree reasonably well with determinations in lattice QCD



$\rightsquigarrow$ strong DPD suppression at large y from Collin-Soper evolution factor

$$\exp\left[{}^{R}J(y;\mu_1,\mu_2)\ln\frac{\sqrt{\zeta}}{\sqrt{\zeta_0}}\right]$$

for $\zeta \gg \zeta_0$

at large y have $\ln\sqrt{\frac{\zeta}{\zeta_0}} \sim \ln\frac{Q}{\Lambda}$

- for ${}^{R}J$ with $R = 10, 27$ assume Casimir scaling relative to ${}^{8}J$

DPDs: initial conditions

- make ansatz

$$ {}^{R_1 R_2}F_{a_1 a_2} = {}^{R_1 R_2}F^{\text{spl}}_{a_1 a_2} + {}^{R_1 R_2}F^{\text{intr}}_{a_1 a_2} $$

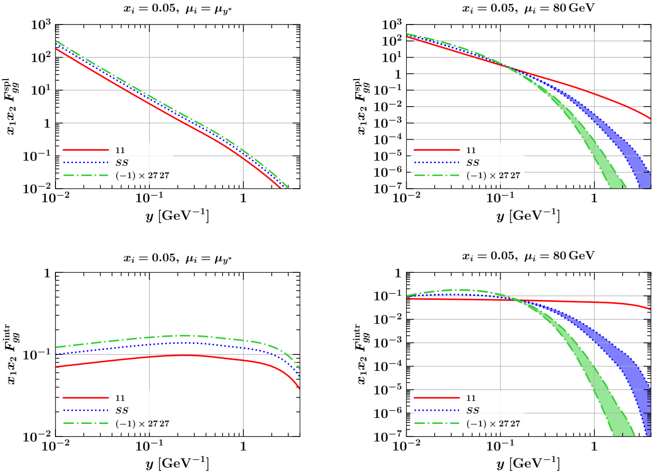
at initial scales $\mu_1 = \mu_2 = \mu_{y^*}$ and $\zeta_0 = \mu_{y^*}^2/(x_1 x_2)$

- μ_{y^*} is $\sim 1/y$ at small y and saturates at large y
- ${}^{R_1 R_2}F^{\text{spl}}_{a_1 a_2}$ = perturbative form (LO) $\times$ Gaussian to suppress large y
- ${}^{11}F^{\text{intr}}_{a_1 a_2}$ = product of two PDFs with further factors
  developed in MD, Gaunt, Lang, Plößl, Schäfer 2020 to approximately fulfil DPD sum rules
- for non-singlet channels assume

$$ {}^{R_1 R_2}F^{\text{intr}}_{a_1 a_2} = \text{colour factor}\,(R_1, R_2) \times {}^{11}F^{\text{intr}}_{a_1 a_2} $$

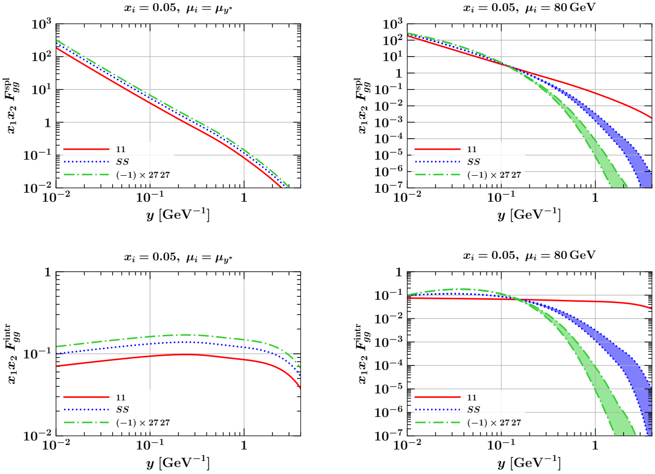
  such that positivity bounds in colour space are saturated
  T Kasemets, P Mulders 2014; MD, J Gaunt, P Pichini, P Plößl 2021
  in a loose sense maximises colour correlation effects at initial scale

Evolved DPDs



- plot DPDs for $\mu_1 = \mu_2$ and $\zeta = \mu_1 \mu_2 / (x_1 x_2)$
- bands: range of models for Collins-Soper kernel

Evolved DPDs

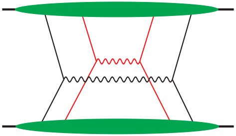


- large y: increasingly strong suppression of colour non-singlets
- small y: little evolution from μ_{y^*} to final μ_i $\rightsquigarrow$ no suppression

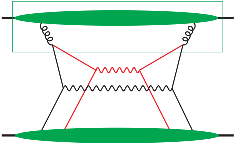
Double parton luminosities

- following plots show double parton luminosities separately for

2v2:

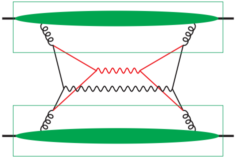


1v2 + 2v1:



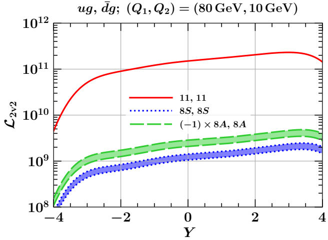
+ mirror graph

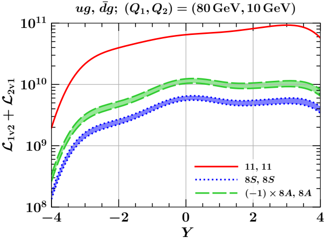
1v1:

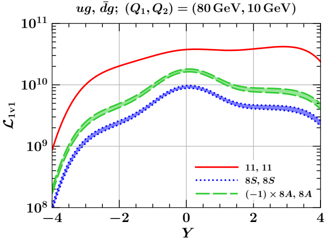


- luminosities are integrated over $y \geq b_0 / \min(Q_1, Q_2)$ $b_0 \approx 1.12$
- plotted vs. Y, where subsystems have rapidities $Y_1 = Y$ and $Y_2 = -Y$

Double parton luminosities: W^+ plus dijet in underlying-event kinematics





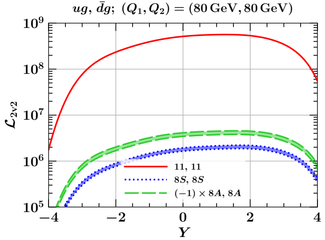


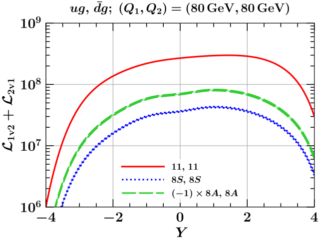
suppression of colour non-singlet DPDs:

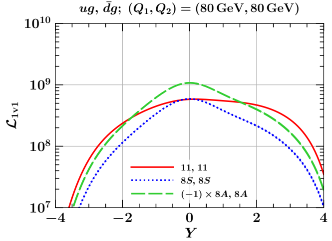
- strong for 2v2
- moderate
  for 1v2 + 2v2 and 1v1
  (dominated by smaller y)

four-gluon luminosities → backup slides

Double parton luminosities: W^+ plus dijet





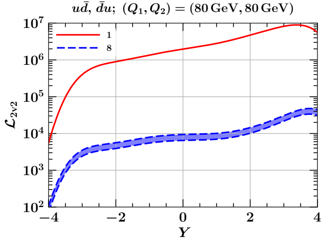


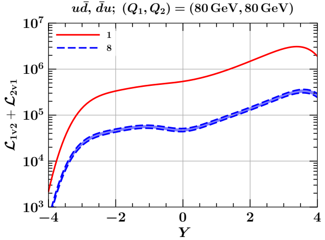
suppression of colour non-singlet DPDs:

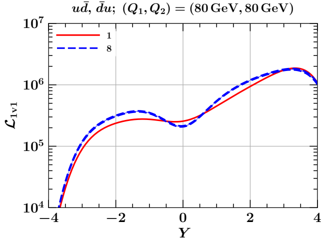
- strong for 2v2
- moderate or absent for 1v2 + 2v2 and 1v1 (dominated by smaller y)

four-gluon luminosities → backup slides

Double parton luminosities: W^+W^+







suppression of colour non-singlet DPDs:

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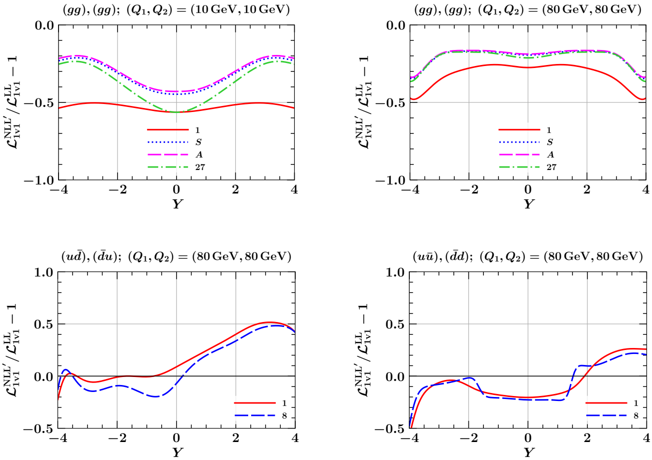
four-gluon luminosities → backup slides

Double parton luminosities: higher order corrections



- noticeable impact when including NLO in evolution (and PDFs)
  see effects up to 50% across all colour channels

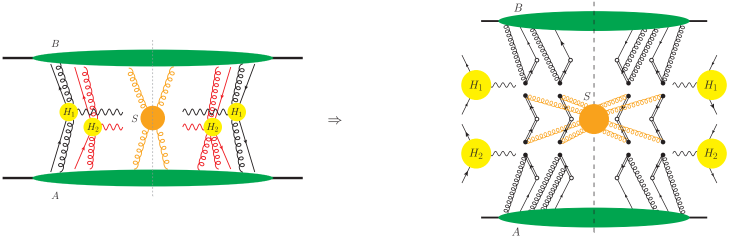
Summary

- have theory description for DPS including colour correlations
  with colour dependent DPDs in y space
- can systematically include higher-order corrections
  evolution kernels known up to NLO; NLO corrections seen to be important
- colour correlations
  - strongly suppressed for 2v2
    large y cut away by Collins-Soper kernel
  - can remain important with $1 \to 2$ splitting in one or both protons
    contributions of $\mathcal{O}(10\%)$ or more, even at the electroweak scale
- open questions/perspectives
  - size and shape of colour dependent DPDs $\to$ next talk by R Rahn
  - compute colour dependent hard cross sections $\hat{\sigma}$
    quite easy at LO, more work at NLO
  - include in DPS parton showers (e.g. dShower by J Gaunt et al)
    and event generators
    $\rightsquigarrow$ improved underlying event description?

Backup slides

Soft gluons and factorisation

- can generalise factorisation proof from single to double Drell-Yan

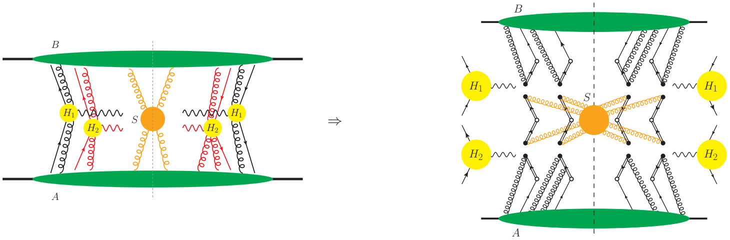


- works for
  - collinear factorisation (k_T integrated)
    → this talk and talk by P Plößl (Tuesday)
  - TMD factorisation (identified k_T, for colour-singlet final states only)
    → talk by O Grocholski (Monday)

MD, J Gaunt, D Ostermeier, P Plößl, A Schäfer 2015; M Buffing, T Kasemets, MD 2017; MD, R Nagar 2018; see also A Manohar, W Waalewijn 2012

Soft gluons and factorisation

- can generalise factorisation proof from single to double Drell-Yan

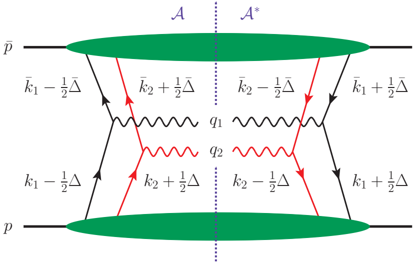


- steps include:
  - decouple soft gluons from partons in protons/proton fragments
    ⇝ DPS soft factor (vev of 4 Wilson line pairs)
  - split soft factor into two parts, absorb one part into each DPD
    ⇝ DPDs depend on rapidity parameter ζ

$$\log \sqrt{\zeta} \;\leftrightarrow\; \text{rapidity at which soft factor is split}$$

  - Collins-Soper kernel = derivative of soft factor w.r.t. rapidity

Transverse momentum vs. distance in DPDs

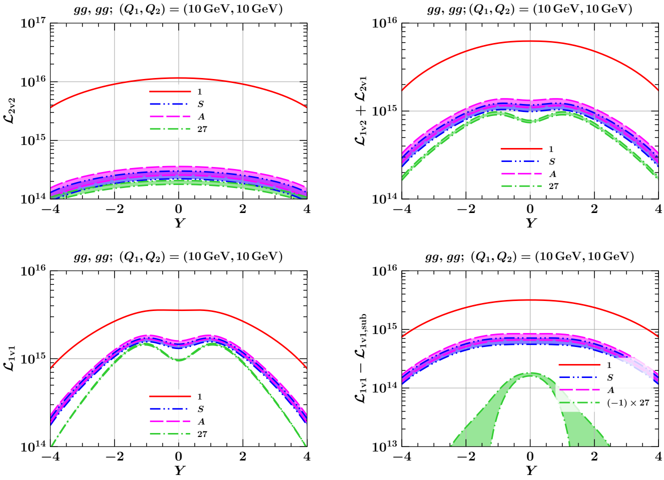


$\bar{\Delta} = -\Delta$

- large (plus or minus) parton momentum components fixed by final state
  $\rightsquigarrow$ equal in amplitude $\mathcal{A}$ and conjugate amplitude $\mathcal{A}^*$
- transverse parton momenta not equal in $\mathcal{A}$ and in $\mathcal{A}^*$
  cross section $\propto \int d^2\boldsymbol{\Delta}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{\Delta}) F(\bar{x}_i, \bar{\boldsymbol{k}}_i, -\boldsymbol{\Delta})$
- Fourier trf. to impact parameter: $F(x_i, \boldsymbol{k}_i, \boldsymbol{\Delta}) \to F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$
  cross section $\propto \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$
- interpretation: $\boldsymbol{y}$ = transv. dist. between two scattering partons
  = equal in both colliding protons

Double parton luminosities: four gluons

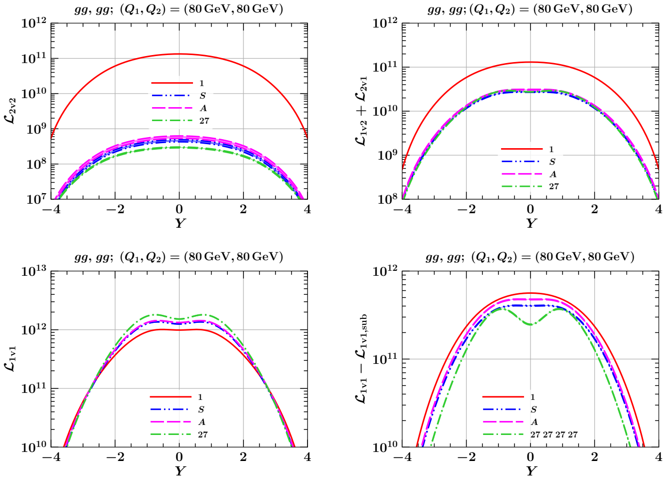
four gluons, $Q_1 = Q_2 = 10\,\text{GeV}$



subtraction term in lower right panel → talk by P Plößl (Tuesday)

Double parton luminosities: four gluons

four gluons, $Q_1 = Q_2 = 80\,\mathrm{GeV}$



subtraction term in lower right panel → talk by P Plößl (Tuesday)

Double parton luminosities: four gluons

four gluons, $Q_1 = 80\,\text{GeV},\ Q_2 = 10\,\text{GeV}$