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DOUBLE PARTON DISTRIBUTIONS ON THE LATTICE USING LARGE MOMENTUM EFFECTIVE THEORY

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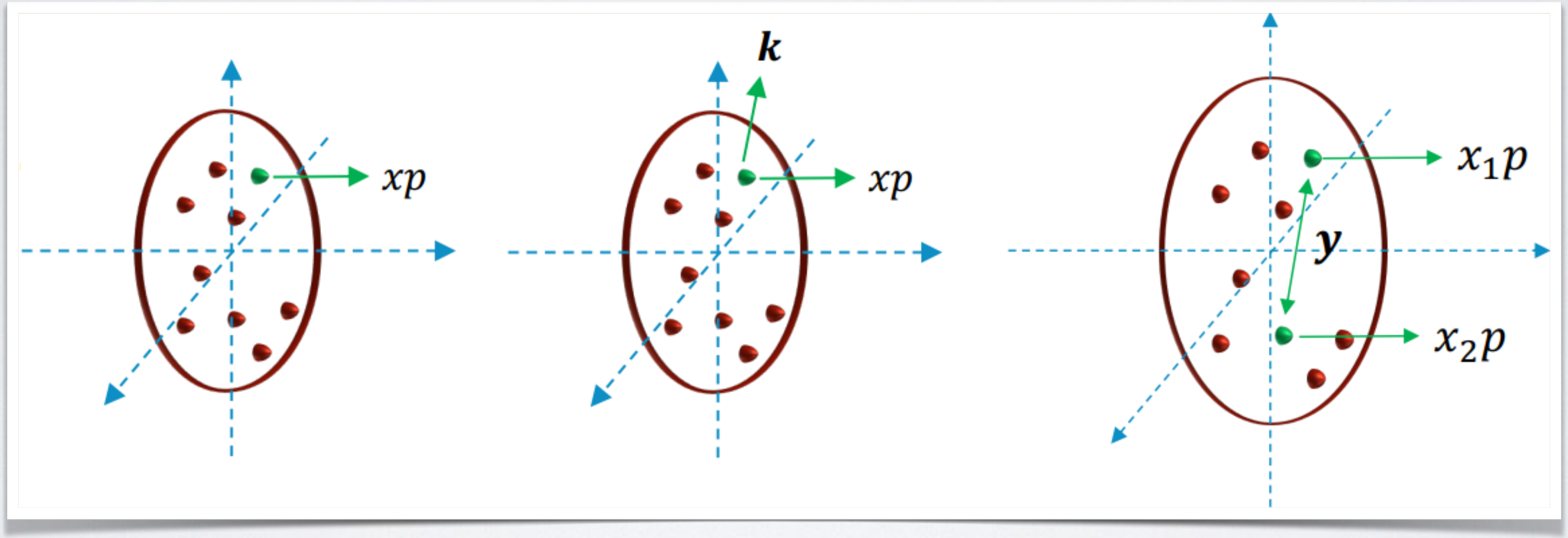
Based on work with Max Jaarsma
& Wouter Waalewijn

OUTLINE

- Important features of PDFs and DPDs
- Time and the lattice
- Quasi-PDFs and LaMET
- Quasi-DPDs
- Results

PDFS, TMDS, AND DPDS



(J. Gaunt, via M. Jaarsma)

- What's in a proton?
- Both TMDs and DPDs introduce a transverse component

PARTON DISTRIBUTIONS

- Quantum mechanics: Probability amplitude:

$$f(x) = \sum\!\!\!\!\!\!\int_X \mathrm{d}\Pi_X \int \mathrm{d}^4 p\, \delta(xP^+ - p^+)\delta^4(P - p - p_X)|\langle X|\psi(0)|P\rangle|^2$$

$$= \int_{-\infty}^{\infty} \frac{\mathrm{d}y^-}{2\pi} e^{-\mathrm{i}xP^+y^-} \langle P|\bar{\psi}(y^- n^\mu)\gamma^0 \psi(0)|P\rangle$$

- Light cone separation tied to x-dependence

- Wilson line for gauge invariance:

$$f(x)_q = \int_{-\infty}^{\infty} \frac{\mathrm{d}y^-}{2\pi} e^{-\mathrm{i}xP^+y^-} \langle P|\bar{\psi}(y^- n^\mu)\gamma^0 W_n(y^- \leftarrow 0)\psi(0)|P\rangle$$

[textbook: Schwartz]

DPDS

- Double the fun!

[Diehl, Ostermeier, Schaefer, '11]
[Manohar, Waalewijn, '12]

$$ {}^{R}F_{a_1 a_2} = -\pi P^+ \int \frac{db_1^-}{2\pi}\frac{db_2^-}{2\pi}\frac{db_3^-}{2\pi}\; e^{-ix_1 P^+ b_1^-}\, e^{-ix_2 P^+ b_2^-}\, e^{ix_1 P^+ b_3^-} $$
$$ \times \langle P|\, T^\dagger \Big[[\bar\psi W]_n(0^+, b_1^-, \mathbf{b}_\perp)\Gamma_{a_1} R_1\Big]_i \Big[[\bar\psi W]_n(b_2^-)\Gamma_{a_2} R_2\Big]_j $$
$$ \times T\Big[[W^\dagger\psi]_n(0^+, b_3^-, \mathbf{b}_\perp)\Big]_i \Big[[W^\dagger\psi]_n(0)\Big]_j\, |P\rangle $$

- Colour and spin correlations between the extracted partons possible

- Like PDF assumes infinite momentum proton

THE LATTICE

- Idea: Monte Carlo integration for the path integral

$$\langle \phi(x_1)\phi(x_2)\rangle = \int \mathcal{D}\phi \; e^{iS[\phi]} \; \phi(x_1)\phi(x_2)$$

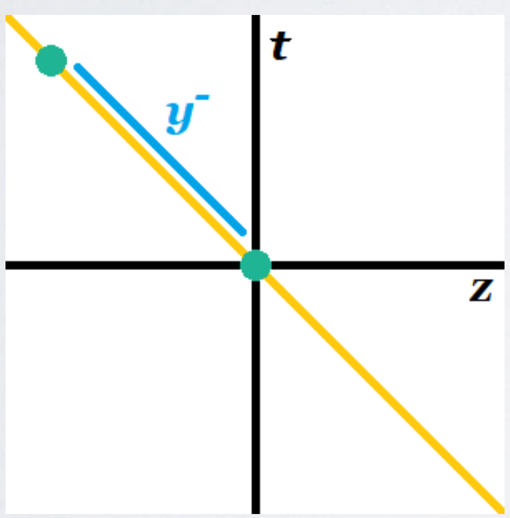
- Problem: Weight factor doesn't weigh

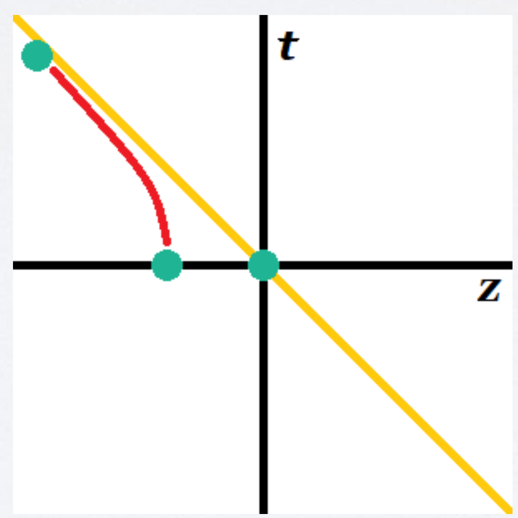
$\Rightarrow$ Wick rotate to imaginary time

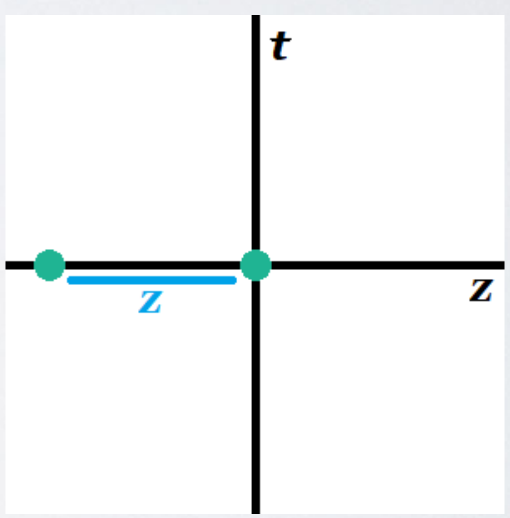
- But: $\bar{\psi}(y^- n^\mu)\gamma^0\psi(0)$ $\quad n^\mu = e_0^\mu + e_z^\mu$

TOWARDS LAMET

- Special relativity exists
- Boosted spacelike separations look lightlike







We want $\Rightarrow$ We MacGyver $\Leftarrow$ We have

THE QUASI PDF

[Ji, '13]

$$\tilde{f}(x, P^z) = \int \frac{dz}{4\pi}\, e^{-ixP^z z} \, \langle P| \bar{\psi}(z)\gamma^z W(z \leftarrow 0)\psi(0) |P\rangle$$

- Under a boost:

$$\frac{z}{\sqrt{2}}(-1^+, 1^-, \mathbf{0}_\perp) \to \frac{z}{\sqrt{2}}(-e^{-y_B}, e^{y_B}, \mathbf{0}_\perp)$$

- Proton momentum parametrises boost

- Expectation: $\lim_{P^z \to \infty} \tilde{f}(x, P^z) = f(x)$

THE QUASI PDF

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- Proton momentum parametrises boost

- Expecta

$$\lim_{P^z \to \infty} \tilde{f}(x, P^z) \neq f(x)$$

A TALE OF TWO LIMITS

Light cone PDF

$$\langle P | \bar{\psi}(y^-)\gamma^+ W(y^- \leftarrow 0)\psi(0) | P \rangle$$

- Lightlike separation
- Infinite momentum P^+

$\Rightarrow$ Lorentz invariant

$$\text{“} \lim_{\epsilon \to 0} \lim_{P^+ \to \infty} \text{”}$$

Quasi PDF

$$\langle P | \bar{\psi}(z)\gamma^z W(z \leftarrow 0)\psi(0) | P \rangle$$

- Spacelike separation
- Finite momentum P^z

$\Rightarrow$ Not Lorentz invariant

$$\text{“} \lim_{P^z \gg 0} \lim_{\frac{1}{a} \to \infty} \text{”}$$

LARGE MOMENTUM EFFECTIVE THEORY

- HQET: $m_Q \to \infty$ $\left\{ \begin{array}{ll} \text{After renormalising:} & \text{No EFT} \\ \text{Before renormalising:} & \text{EFT} \end{array} \right.$

- $P^+ \to \infty$ defines LaMET
  - P^+ isn't part of the Lagrangian $\Rightarrow$ effective states

[Ji, Liu, Liu, Zhang, Zhao, '20]

LARGE MOMENTUM EFFECTIVE THEORY

- HQET: $m_Q \to \infty$ { After renormalising: No EFT
  Before renormalising: EFT

- $P^+ \to \infty$ defines LaMET
  - P^+ isn't part of the Lagrangian $\Rightarrow$ effective states
  - The Lightcone PDF is the effective theory object

[Ji, Liu, Liu, Zhang, Zhao, '20]

MATCHING

- Matching is UV physics, and QCD is perturbative

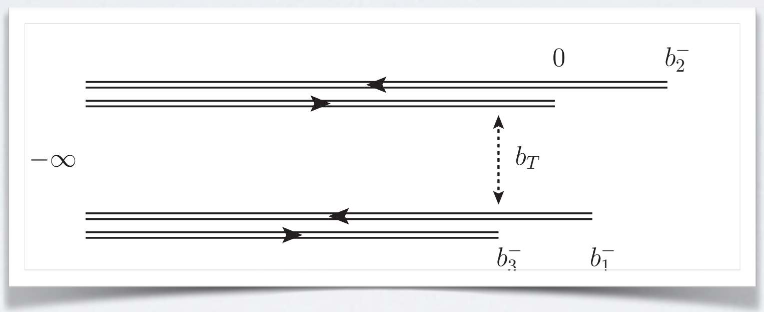
$$\tilde{f}(x, P^z, \mu) = \int \frac{dy}{y}\, \mathcal{C}\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) f(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(xP^z)^2}\right)$$

[Ji, '13]
[Ma, Qiu '14]
[Izubuchi, Ji, Jin, Stewart, Zhao '18]

- So what about DPDs?
  - Assumption: The same, up to difficulties
  - See Jianhui Zhang's talk

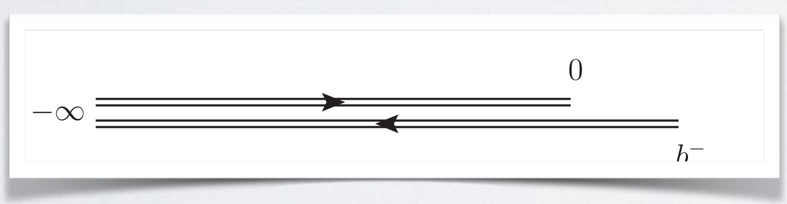
WILSON LINE TROUBLE

- There are four (Wilson) lines!

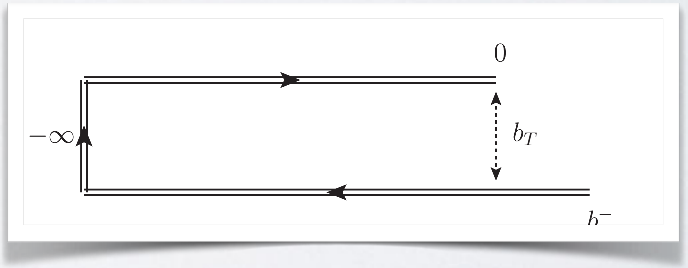


Colour-summed

Colour-correlated



Like PDF (x2)



Like TMD (x2)

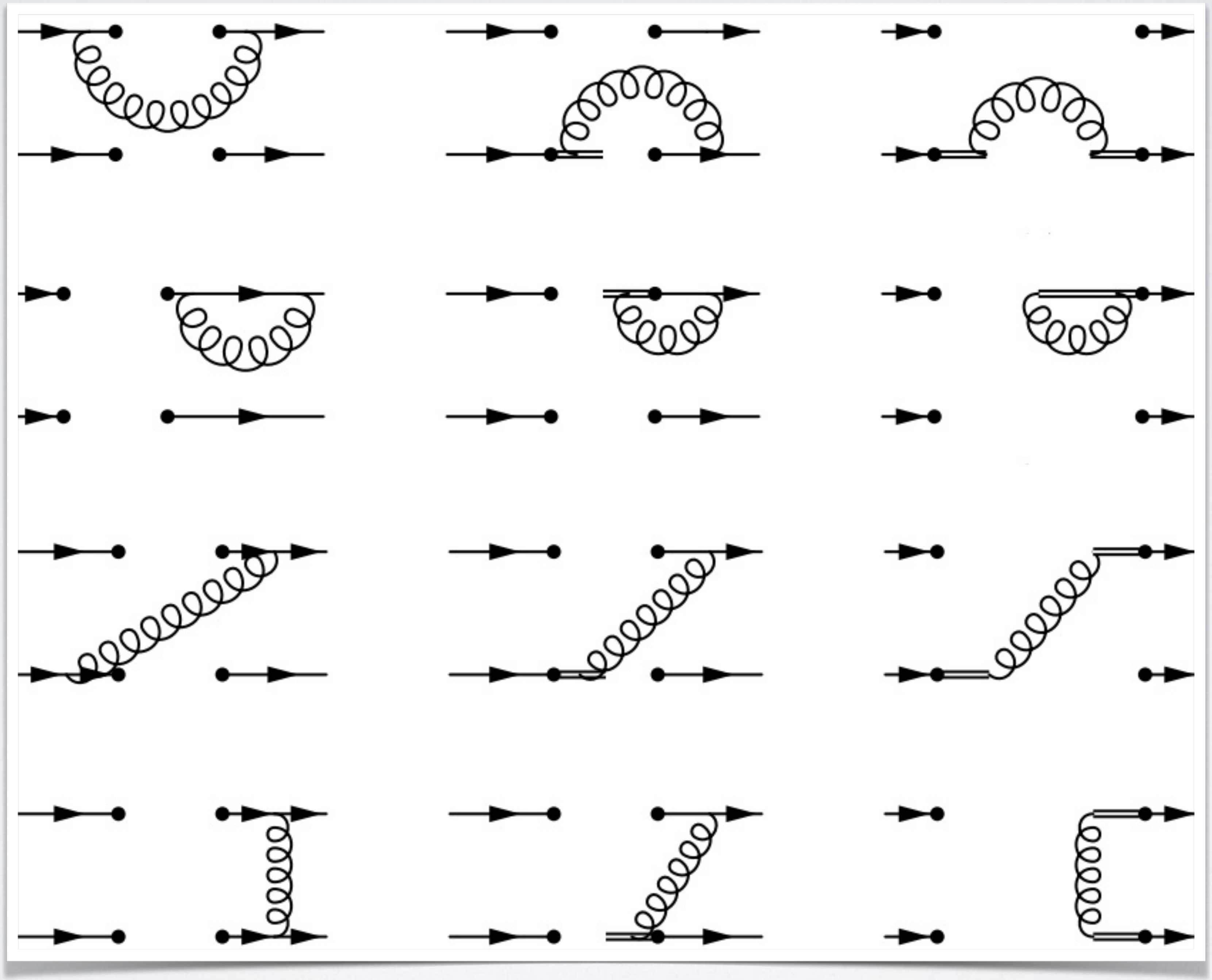
[Ebert, Stewart, Schindler, Zhao '22]

MATCHING CONJECTURE

$$
\begin{aligned}
&{}^{R}\tilde{F}^{\rm sub}_{a_1 a_2}(x_1, x_2, b_\perp, \mu, \tilde{\zeta}, P^z) \\
&= \int \frac{\mathrm{d}y_1}{y_1}\frac{\mathrm{d}y_2}{y_2}\, {}^{RR'}C_{a_1 a_2, a_1' a_2'}\left(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{(x_1 P^z)^2}{\mu^2}, \frac{(x_2 P^z)^2}{\mu^2}, \frac{\tilde{\zeta}}{\mu^2}\right) \\
&\qquad \times \exp\left[\frac{1}{2}\, {}^{R'}\gamma_\zeta(b_\perp, \mu)\log\left(\frac{\tilde{\zeta}}{\zeta}\right)\right] {}^{R'}F^{\rm sub}_{a_1' a_2'}(x_1, x_2, b_\perp, \mu, \zeta) \\
&\quad + \text{mixing with single PDFs} \\
&\quad + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{1}{x_i b_\perp P^z}, \frac{\Lambda^2_{\rm QCD}}{(x_i P^z)^2}\right)
\end{aligned}
$$

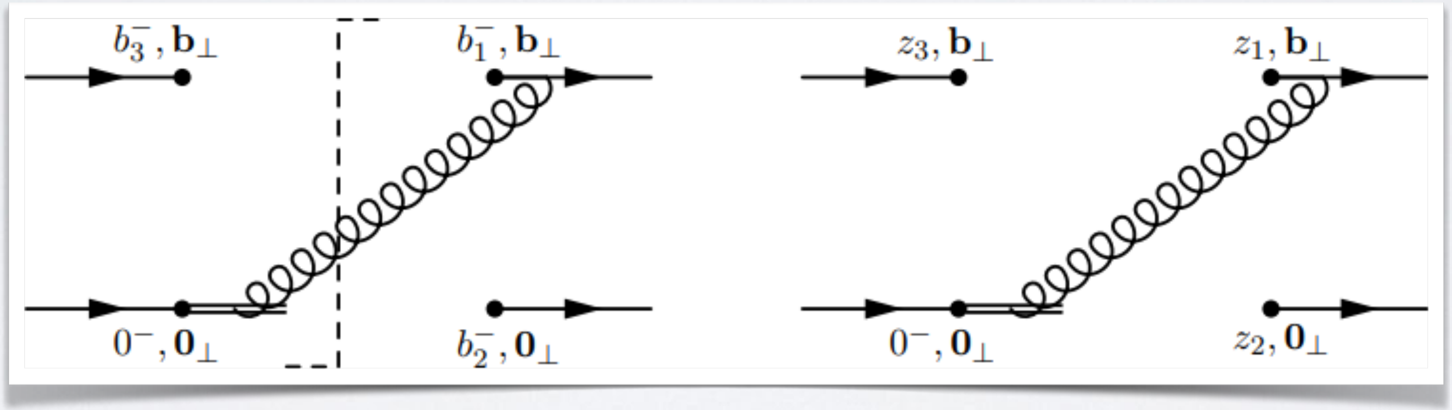
- We don't prove it, but we can check it

THE DIAGRAMS



AN EXAMPLE

- Calculate the diagrams and check if the matching is pure UV



$$\Delta^{\text{example}} = -4\pi\delta(1-x_1)\delta(1-x_2)\left[\frac{1}{\epsilon_{\text{ir}}} + \log\left(\frac{\mu^2\mathbf{b}_\perp^2}{b_0^2}\right)\right]\left[2 + \log\left(\frac{\delta^2}{p_1^+ p_2^+}\right)\right]$$
$$+ \text{ other terms}$$

- Individual diagrams have infrared dependence…

THE KERNELS

- …but the sum of all is pure UV:

$$ {}^{1}C_{a_1a_2}\left(\frac{x_1}{y_1},\frac{x_2}{y_2},\frac{\mu}{|y_1|P^z},\frac{\mu}{|y_2|P^z}\right) = \mathcal{C}_{a_1}\left(\frac{x_1}{y_1},\frac{\mu}{|y_1|P^z}\right)\mathcal{C}_{a_2}\left(\frac{x_2}{y_2},\frac{\mu}{|y_2|P^z}\right) $$

$$ {}^{8}C^{(1)}_{a_1a_2} = \left(1-\frac{N_c}{2C_F}\right){}^{1}C^{(1)}_{a_1a_2} + \delta\left(1-\tfrac{x_1}{y_1}\right)\delta\left(1-\tfrac{x_2}{y_2}\right) \times N_c\left[2\log\left(\frac{\tilde{\zeta}}{\mu^2}\right) - \tfrac{1}{2}\log^2\left(\frac{(2y_1P^z)^2}{\mu^2}\right) - \tfrac{1}{2}\log^2\left(\frac{(2y_2P^z)^2}{\mu^2}\right) - \frac{5}{2} + \frac{\pi^2}{6}\right] $$

- Note also: No mixing of colour and spin structures at 1-loop

CONCLUSIONS AND OUTLOOK

- We verified the conjectured matching of Quasi and light cone DPDs at 1-loop order
- We find relations to the single-parton PDFs, and absence of colour and spin mixing
- Open questions: Proof, soft functions, lattice,…

THANK YOU!