Double parton distributions in the nucleon from lattice simulations

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Universität Regensburg

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Work done in collaboration with

C. Zimmermann *(U. Regensburg)*, **M. Diehl** *(DESY Hamburg)*,
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Content

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Introduction

- Double parton scattering processes (DPS), e.g. Double Drell-Yan (DDY), are important standard model contributions in LHC experiments, especially after high-luminosity upgrade, e.g. for the description of $W^{++/--}$ production
- Fundamental description by double parton distributions (DPDs) :

$$\frac{\mathrm{d}\sigma_{\mathrm{DPS},i_1 i_2,j_1 j_2}}{\mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_1' \mathrm{d}x_2'} \propto \int \mathrm{d}^2\boldsymbol{y}\; F_{i_1 i_2}(x_1, x_2, \boldsymbol{y}) F_{j_1 j_2}(x_1', x_2', \boldsymbol{y})$$

- DPDs are non-perturbative objects, unknown from experiments so far, but can be accessed via lattice simulations
- Results for the pion *[arXiv:1807.03073]*, *[arXiv:2006.14826]*
- Results for **flavor diagonal** case published in *[arXiv:2106.03451]*
- Possible interferences w.r.t. flavor, color and fermion number which are in general considered to be supressed but **lattice simulations allow model testing**

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- **Ensemble dependency** of the results, based on lattice data obtained from the **H102** and **S400** ensemble
- **Invariant functions** and their ratios compared to the $SU(6)$ **model**
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Double Parton Distributions

- Light cone coordinates: x^μ: $x^\pm = (x^0 \pm x^3)/\sqrt{2}$, $\boldsymbol{x} = (x^1, x^2)$
- Proton rapidly moving in 3-direction, i.e. $p^+ \sim Q \gg \Lambda \sim m$, $\boldsymbol{p} = \mathbf{0}$, $p^- \sim \Lambda^2/Q$

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Definition of proton DPDs for quarks *[arXiv:1111.0910]*

$$F_{ab}(x_1, x_2, \boldsymbol{y}) := 2p^+ \int \mathrm{d}y^- \left[\prod_{j=1,2} \int \frac{\mathrm{d}z_j^-}{2\pi} e^{ix_j p^+ z_j^-} \right] \times$$
$$\times \tfrac{1}{2} \sum\nolimits_\lambda \langle p, \lambda | \, \mathcal{O}_a(y, z_1^-) \mathcal{O}_b(0, z_2^-) \, | p, \lambda \rangle \Big|_{y^+=0}$$

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Light cone operators

$$\mathcal{O}_a(y, z^-) = \bar{q}(y - \tfrac{z}{2}) \Gamma_a q(y + \tfrac{z}{2}) \Big|_{\boldsymbol{z}=\boldsymbol{0}, z^+=0}$$

- $\bar{q}$, q quark operators for certain flavor (light-like distance z^-)
- Γ_a quark polarization

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Twist-2 components: Quark polarizations

operators	twist-2 comp.	polarization
$V_q^\mu = \bar{q}\gamma^\mu q$	$V_q^+ = \mathcal{O}_q$	$q : q^\uparrow + q^\downarrow$ (unpolarized)
$A_q^\mu = \bar{q}\gamma^\mu \gamma_5 q$	$A_q^+ = \mathcal{O}_{\Delta q}$	$\Delta q : q^\uparrow - q^\downarrow$ (longitudinal)
$T_q^{\mu\nu} = \bar{q} i \sigma^{\mu\nu} \gamma_5 q$	$T_q^{+j} = \mathcal{O}_{\delta q}^j$	$\delta q^j : q^{\uparrow,j} - q^{\downarrow,j}$ (transverse)

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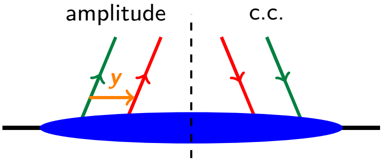

Joint probability to find quark *a* with momentum $x_1 p^+$ and quark *b* with momentum $x_2 p^+$ at transverse distance $\boldsymbol{y}$ ($|x_1| + |x_2| \leq 1$)

Double Parton Distributions

Interference distributions

$$F_{duud}(x_1, x_2, y) := 2p^+ \int \mathrm{d}y^- \left[\prod_{j=1,2} \int \frac{\mathrm{d}z_j^-}{2\pi} e^{ix_j p^+ z_j^-} \right] \times$$
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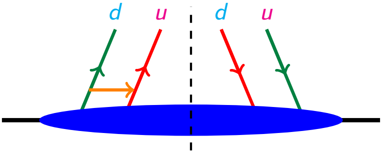

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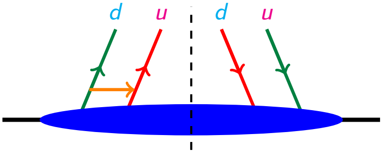

Flavor changing operators

$$\mathcal{O}_{ud}(y, z^-) = \bar{u}(y - \tfrac{z}{2}) \Gamma_a d(y + \tfrac{z}{2}) \Big|_{\mathbf{z}=\mathbf{0}, z^+=0}$$
$$\mathcal{O}_{du}(y, z^-) = \bar{d}(y - \tfrac{z}{2}) \Gamma_a u(y + \tfrac{z}{2}) \Big|_{\mathbf{z}=\mathbf{0}, z^+=0}$$

Double Parton Distributions: Factorization

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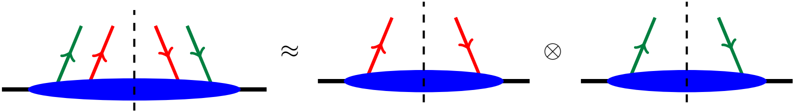

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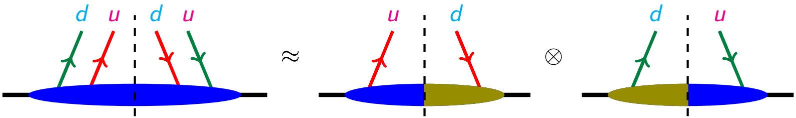

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Lattice QCD

$$\langle \mathcal{O}[q,\bar{q},U]\rangle = \frac{1}{Z}\int \left[\prod_{x\in\Lambda} \mathrm{d}\bar{q}(x)\mathrm{d}q(x)\mathrm{d}U(x)\right] \mathcal{O}[q,\bar{q},U] e^{-S_F[\bar{q},q,U]-S_G[U]}$$

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- Reduce spacetime to a lattice
- Finite volume $\Rightarrow$ IR regularization
- Finite lattice spacing $\Rightarrow$ UV regularization
- Evaluate the fermionic part (Grassmann variables) using Wick's theorem $\Rightarrow$ **Wick contractions (graphs)**
- Euclidean spacetime: $e^{iS} \rightarrow e^{-S}$ suitable weight for Monte Carlo integration
- Evaluate gauge integral by Monte Carlo integration $\Rightarrow$ gauge ensembles of N configuration, statistical error $\propto N^{-\frac{1}{2}}$:

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$$\int \left[\prod_x \mathrm{d}U(x)\right] \det\{\mathcal{D}[U]\} e^{-S[U]}\, \mathcal{O}[q,\bar{q},U] \to \sum_{U\sim P(U)}^{\text{ensemble}} \mathcal{O}[q,\bar{q},U],$$

Lattice QCD

$$\langle \mathcal{O}[q,\bar{q},U]\rangle = \frac{1}{Z}\int \left[\prod_{x\in\Lambda} \mathrm{d}\bar{q}(x)\mathrm{d}q(x)\mathrm{d}U(x)\right] \mathcal{O}[q,\bar{q},U] e^{-S_F[\bar{q},q,U]-S_G[U]}$$

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- Reduce spacetime to a lattice
- Finite volume $\Rightarrow$ IR regularization
- Finite lattice spacing $\Rightarrow$ UV regularization
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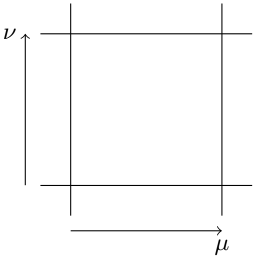
Lattice QCD

Reduce spacetime $\mathbb{R}^4$ to finite lattice with spacing a, extensions $L^3 \times T$:

- put fermions on the grid points
- replace derivatives by symmetric differential quotient and integrals by sums
- restore gauge invariance (gauge links $U_\mu(x) \sim e^{iaA_\mu(x)}$)
- add pure gauge part, the plaquette, $\beta = 3g^{-2}$



$$S[q, \bar{q}, U] = \int \mathrm{d}^4 x \bar{q}(x) \mathcal{D} q(x)$$

$$\mathcal{D} = i\gamma_\mu \partial^\mu - m\mathbb{1}$$

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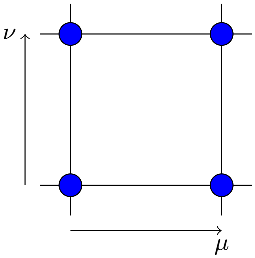

$$S[q, \bar{q}, U] = a^4 \sum_{x,y \in \Lambda_4} \bar{q}(x) \mathcal{D}(x|y) q(y)$$

$$\mathcal{D}(x|y) = \gamma_\mu \frac{\delta_{x+\hat{\mu},y} - \delta_{x-\hat{\mu},y}}{2a} - m\delta_{x,y}$$

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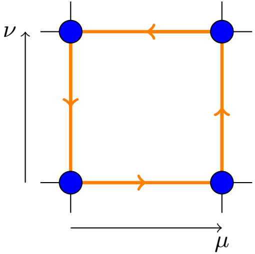

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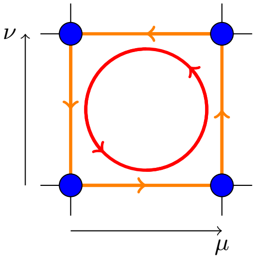

$$S[q,\bar{q},U] = a^4 \sum_{x,y\in\Lambda_4} \bar{q}(x)\mathcal{D}(x|y)q(y) + \frac{\beta}{3}\sum_{x\in\Lambda}\sum_{\mu<\nu} \mathrm{Re\,tr}\{\mathbb{1} - U_{\mu\nu}(x)\}$$

$$\mathcal{D}(x|y) = \gamma_\mu \frac{U_\mu(x)\delta_{x+\hat{\mu},y} - U_\mu^\dagger(x-\hat{\mu})\delta_{x-\hat{\mu},y}}{2a} - m\delta_{x,y}$$

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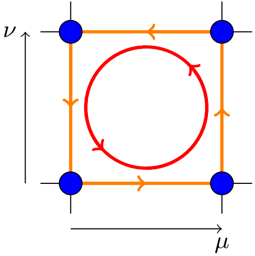

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Double parton distributions on the lattice

Accessible quantities

$$\langle p|\, \mathcal{O}_a(y, z_1^-)\mathcal{O}_b(0, z_2^-)\, |p\rangle \xrightarrow[y^+=0,\ \text{twist-2}]{p^+ \int \mathrm{d}y^- \,\mathrm{d}z_i^- \, e^{-iz_i x_i p^+}} F_{ab}(x_i, \boldsymbol{y})$$

Double parton distributions on the lattice

Accessible quantities

$F_{ab}(x_i, \boldsymbol{y})$

$p^+ \int \mathrm{d}y^- \, \mathrm{d}z_i^- \, e^{-i z_i x_i p^+}$

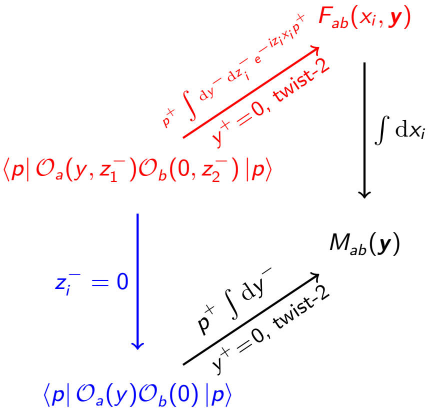
$y^+ = 0$, twist-2

$\langle p | \, \mathcal{O}_a(y, z_1^-) \mathcal{O}_b(0, z_2^-) \, | p \rangle$

not accessible on the lattice
if $z_i^- > 0$

Double parton distributions on the lattice

Accessible quantities



Double parton distributions on the lattice

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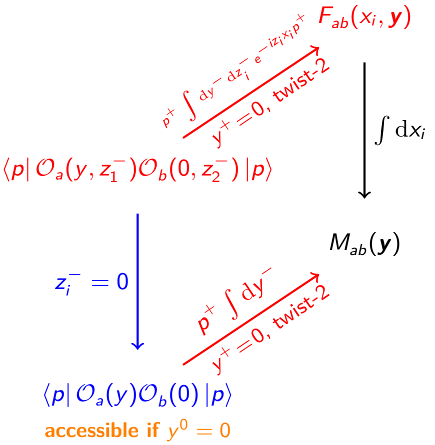

Double parton distributions on the lattice

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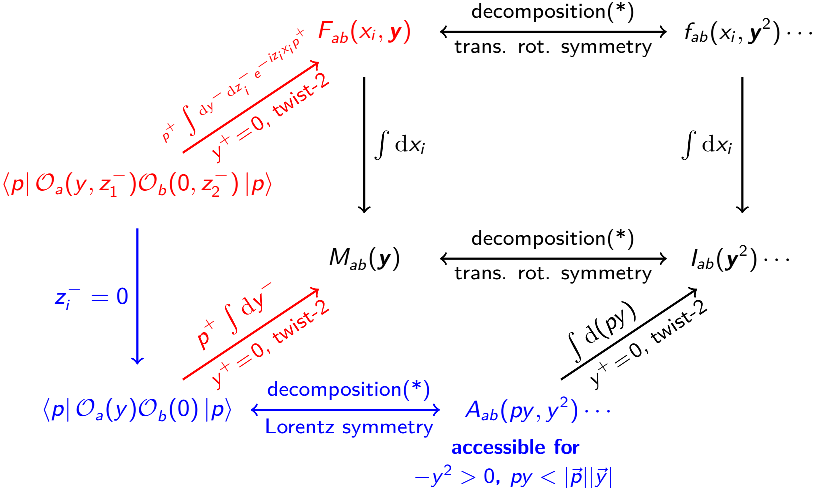
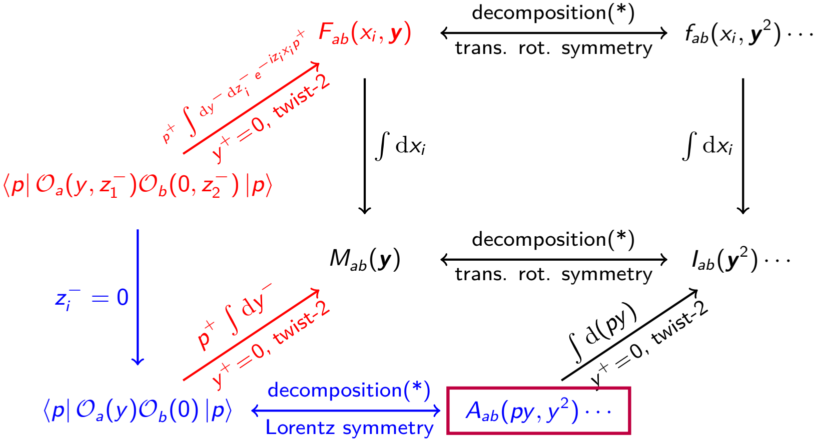


(*) into basis tensors and scalar functions

Double parton distributions on the lattice

Accessible quantities



Results for these quantities

(*) into basis tensors and scalar functions

Content

Two-current matrix elements on the lattice

Access via 4- and 2-point functions

$$C_{\rm 4pt}^{\vec{p},ij}(t,\tau,\vec{y}) = \sum_{\vec{x}} e^{-i\vec{p}(\vec{z}'-\vec{z})} \left\langle \mathrm{tr}\left\{ P_+ \mathcal{P}^{\vec{p}}(\vec{z}',t)\; \mathcal{O}_i^{q_1 q_2}(\vec{0},\tau)\; \mathcal{O}_j^{q_3 q_4}(\vec{y},\tau)\; \overline{\mathcal{P}}^{\vec{p}}(\vec{z},0) \right\} \right\rangle$$

$$C_{\rm 2pt}^{\vec{p}}(t) = \sum_{\vec{x}} e^{-i\vec{p}(\vec{z}'-\vec{z})} \left\langle \mathrm{tr}\left\{ P_+ \mathcal{P}^{\vec{p}}(\vec{z}',t)\; \overline{\mathcal{P}}^{\vec{p}}(\vec{z},0) \right\} \right\rangle$$

Two-current matrix elements on the lattice

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with $P_+ = \frac{1}{2}(\mathbb{1} + \gamma_4)$ and Proton interpolators:

$$\mathcal{P}^{\vec{p}}(\vec{x},t) = \epsilon_{abc} u_a(x) \left[u_b^T(x) C\gamma_5 d_c(x) \right] \Big|_{x^4=t}$$

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yielding the spin averaged matrix element:

$$\mathcal{M}_{ij}(p,y)|_{y^0=0} = \frac{1}{2}\sum_{\lambda} \langle p,\lambda|\, \mathcal{O}_i(y)\mathcal{O}_j(0) |p,\lambda\rangle\ |_{y^0=0} = 2V\sqrt{m^2+\vec{p}^2}\ \frac{C_{4\text{pt}}^{\vec{p},ij}(t,\tau,\vec{y})}{C_{2\text{pt}}^{\vec{p}}(t)}\Bigg|_{0\ll\tau\ll t}$$

Two-current matrix elements on the lattice

Connection between Mellin Moments and matrix elements:

$$M_{ab}^{(1,1)}(\zeta, \boldsymbol{y}) = 2(p^+)^{-1} \int \mathrm{d}y^- \, e^{-i\zeta p^+ y^-} \; \mathcal{M}_{ba}(p, y)|_{\,y^+=0}$$

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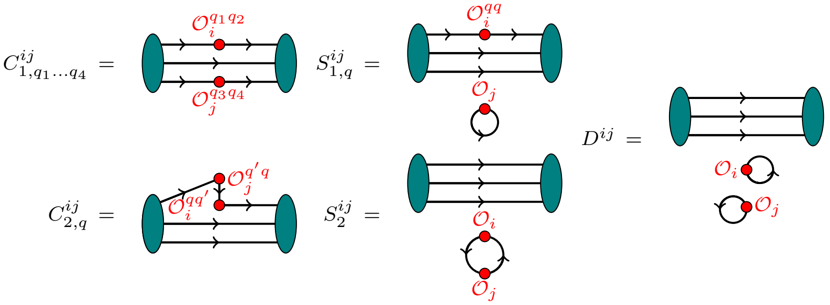
Connection between matrix elements and invariant functions for leading twist:

$$\begin{aligned}
\mathcal{M}_{VV,qq'}^{++}(p, y) &= 4\mathcal{M}_{qq'} = 2(p^+)^2 A_{q'q}(py, y^2) \\
\mathcal{M}_{AA,qq'}^{++}(p, y) &= 4\mathcal{M}_{\Delta q \Delta q'} = 2(p^+)^2 A_{\Delta q' \Delta q}(py, y^2) \\
\mathcal{M}_{TV,qq'}^{j++}(p, y) &= 4\mathcal{M}_{\delta q q'}^{j} = 2(p^+)^2 y^j m A_{q' \delta q}(py, y^2) \\
\mathcal{M}_{VT,qq'}^{+j+}(p, y) &= 4\mathcal{M}_{q \delta q'}^{j} = 2(p^+)^2 y^j m A_{\delta q' q}(py, y^2) \\
\mathcal{M}_{TT,qq'}^{j+l+}(p, y) &= 4\mathcal{M}_{\delta q \delta q'}^{jl} = 2(p^+)^2 \left[\delta^{jl} A_{\delta q' \delta q}(py, y^2) - (2y^j y^l + \delta^{jl} y^2) m^2 B_{\delta q' \delta q}(py, y^2)\right]
\end{aligned}$$

Two-current matrix elements on the lattice

Wick contractions



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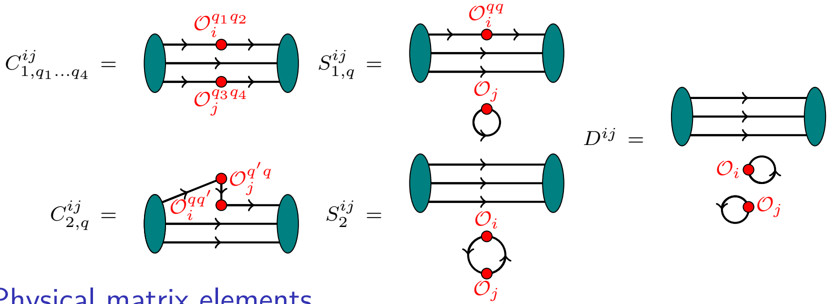

Physical matrix elements

$$\langle p|\, \mathcal{O}_i^{uu}(\vec{0})\mathcal{O}_j^{dd}(\vec{y})\,|p\rangle = C^{ij,\vec{p}}_{1,uudd}(\vec{y}) + S^{ij,\vec{p}}_{1,u}(\vec{y}) + S^{ji,\vec{p}}_{1,d}(-\vec{y}) + D^{ij,\vec{p}}(\vec{y})$$

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$$\langle p|\, \mathcal{O}_i^{du}(\vec{0})\mathcal{O}_j^{ud}(\vec{y})\,|p\rangle = C^{ij,\vec{p}}_{1,duud}(\vec{y}) + C^{ij,\vec{p}}_{2,u}(\vec{y}) + C^{ji,\vec{p}}_{2,d}(-\vec{y}) + S^{ij,\vec{p}}_{2}(\vec{y})$$

Two-current matrix elements on the lattice

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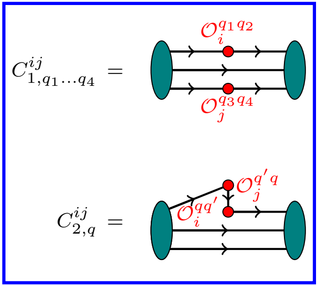

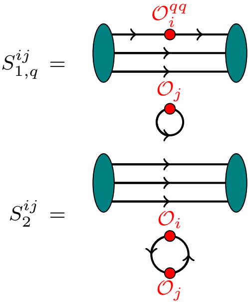

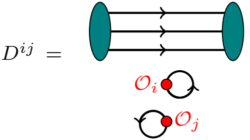

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Content

Lattice Setup

CLS ensembles ($n_f = 2 + 1$, Wilson fermions, order-a improved *[arXiv:1411.3982]*):

id	β	a[fm]	$L^3 \times T$	$\kappa_{l/s}$	$m_{\pi/K}$[MeV]	$m_\pi L$	conf.	used
H102	3.4	0.0856	$32^3 \times 96$	0.136865	355	4.9	2037	990
				0.136549339	441			
S400	3.46	0.076	$32^3 \times 128$	0.136984	354	4.4	2873	1000
				0.136702387	442			

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Renormalization for $\beta = 3.4$, including conversion to $\overline{\mathrm{MS}}$ at $\mu = 2\mathrm{GeV}$ *[arXiv:2012.06284]*:

	β	V	A	T
Z	3.4	0.7128	0.7525	0.8335
Z	3.46	0.7220	0.7594	0.8470

- Calculations done for Momenta up to $|\vec{p}| = \sqrt{12}\frac{2\pi}{La} \approx \begin{cases} 1.57\ \mathrm{GeV}(H102) \\ 1.76\ \mathrm{GeV}(S400) \end{cases}$
- Point source at $t_{\rm src} = \begin{cases} 48a(H102) \\ 64a(S400) \end{cases}$ (spatial positions randomly selected)
- Source sink separation

$$t = t_{\rm snk} - t_{\rm src} = \begin{cases} S400 \begin{cases} 13a & \vec{p} = \vec{0} \\ 11a & \vec{p} \neq \vec{0} \end{cases} \\ H102 \begin{cases} 12a & \vec{p} = \vec{0} \\ 10a & \vec{p} \neq \vec{0} \end{cases} \end{cases}$$

- Insertion time $\tau \in [t_{\rm src} + 3a, t_{\rm snk} - 3a]$ fitted for C_1
- Other contractions: $\tau = t_{\rm src} + t/2$ fixed

- Calculations done for Momenta up to $|\vec{p}| = \sqrt{12}\frac{2\pi}{La} \approx \begin{cases} 1.57\ \mathrm{GeV}(H102) \\ 1.76\ \mathrm{GeV}(S400) \end{cases}$
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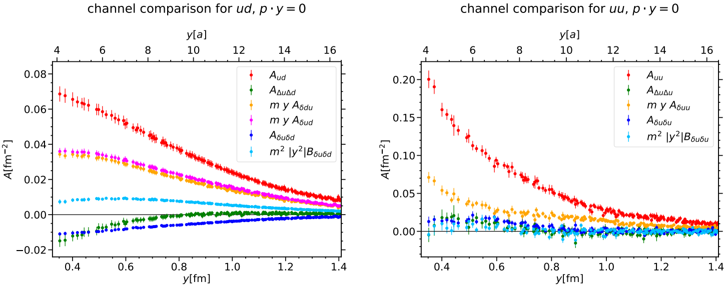
Results for $A(py = 0, y^2)$: Polarization dependence

Invariant functions $A(py = 0, y^2)$, connected graphs only (notation $y = \sqrt{-y^2}$, $y^2 = y^\mu y_\mu$):



- Signal of good quality for most channels
- ud: Clear contributions from all polarized channels (large for δud, δdu)
- uu: Polarization effects suppressed, but visible for δuu

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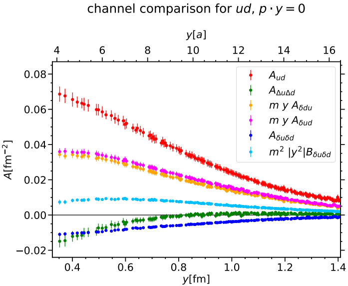

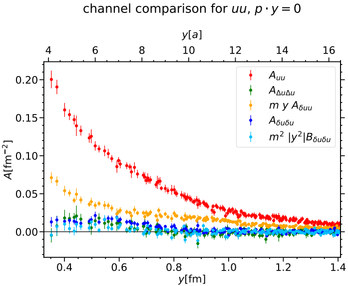

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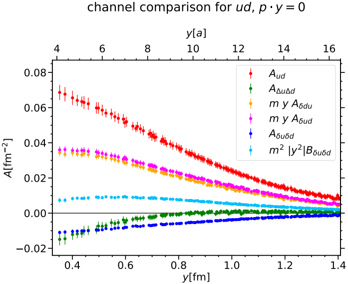

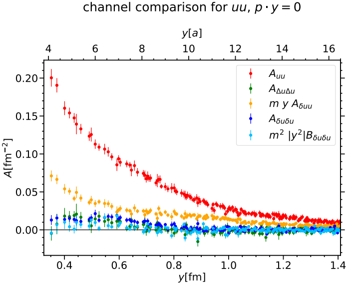

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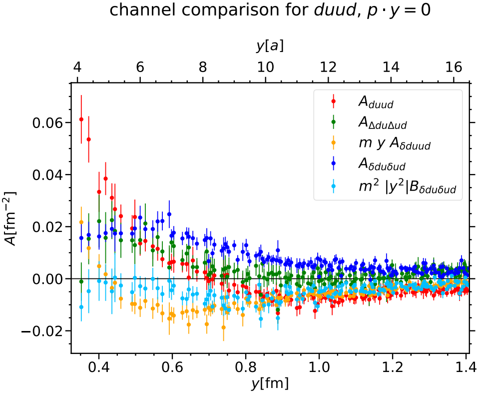

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- Clear signal for interference case despite this is not resolved in simple models

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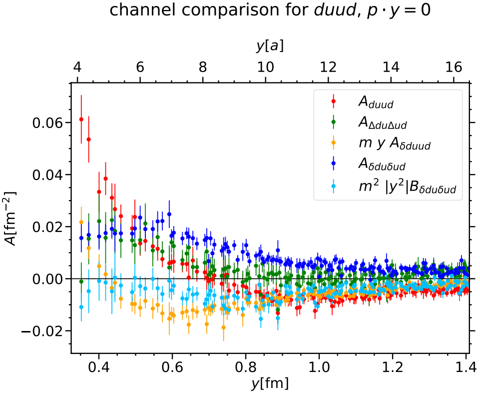

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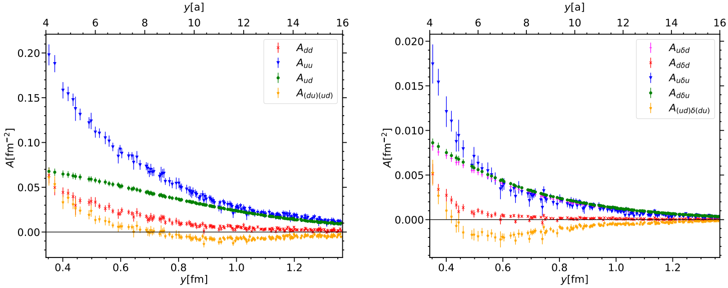

- Clear flavor dependence visible, behavior of *uu* and *dd* different from *du*
- Size of interference effects comparable to *dd*, sign change possible, **signal as clear as in the falvor diagonal case**
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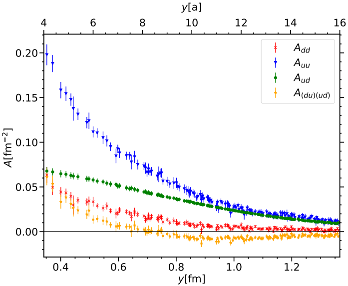

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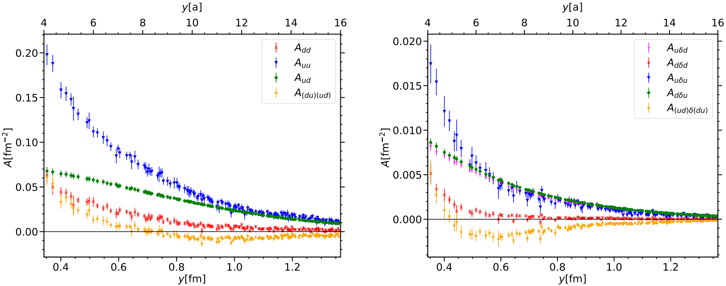

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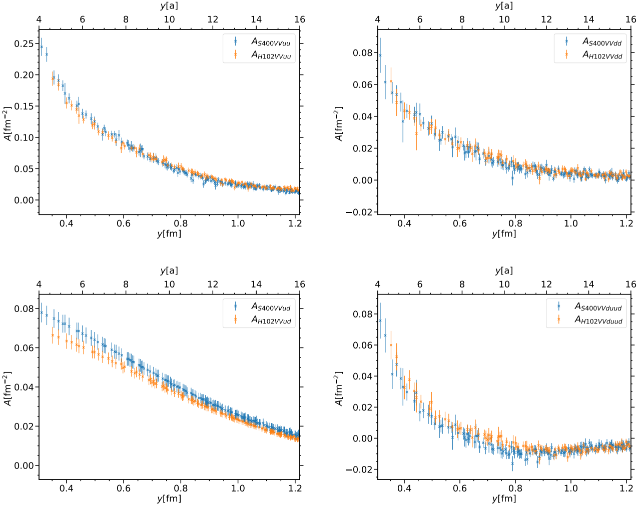
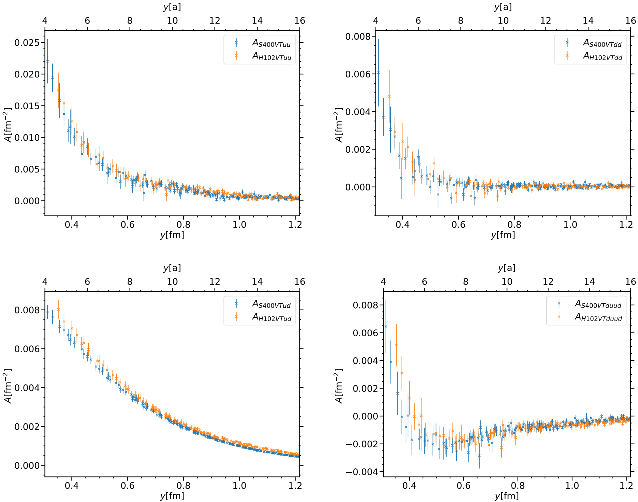
Results for $A(py = 0, y^2)$: a-dependence (unpolarized)



- Differences between the quantities obtained from $H102$ and $S400$ are within one sigma, no strong a - dependence visible

Results for $A(py = 0, y^2)$: a-dependence (one channel transverse polarized)



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Results for $A(py = 0, y^2)$: Comparison with $SU(6)$-model

Using the following operators

$$
\begin{aligned}
O_{uu} \widehat{=} & (\bar{u}^{\uparrow} u^{\uparrow})(\bar{u}^{\uparrow} u^{\uparrow}) \pm (\bar{u}^{\uparrow} u^{\uparrow})(\bar{u}^{\downarrow} u^{\downarrow}) \pm (\bar{u}^{\downarrow} u^{\downarrow})(\bar{u}^{\uparrow} u^{\uparrow}) + (\bar{u}^{\downarrow} u^{\downarrow})(\bar{u}^{\downarrow} u^{\downarrow}) \\
O_{ud} \widehat{=} & (\bar{u}^{\uparrow} u^{\uparrow})(\bar{d}^{\uparrow} d^{\uparrow}) \pm (\bar{u}^{\uparrow} u^{\uparrow})(\bar{d}^{\downarrow} d^{\downarrow}) \pm (\bar{u}^{\downarrow} u^{\downarrow})(\bar{d}^{\uparrow} d^{\uparrow}) + (\bar{u}^{\downarrow} u^{\downarrow})(\bar{d}^{\downarrow} d^{\downarrow}) \\
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O_{duud} \widehat{=} & (\bar{d}^{\uparrow} u^{\uparrow})(\bar{u}^{\uparrow} d^{\uparrow}) \pm (\bar{d}^{\uparrow} u^{\uparrow})(\bar{u}^{\downarrow} d^{\downarrow}) \pm (\bar{d}^{\downarrow} u^{\downarrow})(\bar{u}^{\uparrow} d^{\uparrow}) + (\bar{d}^{\downarrow} u^{\downarrow})(\bar{u}^{\downarrow} d^{\downarrow})
\end{aligned} \tag{1}
$$

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together with the $SU(6)$-symmetric proton wave-function:

$$|p^{\uparrow}\rangle = \frac{1}{3\sqrt{2}} \Big[|u^{\uparrow} u^{\downarrow} d^{\uparrow}\rangle + |u^{\downarrow} u^{\uparrow} d^{\uparrow}\rangle - 2\,|u^{\uparrow} u^{\uparrow} d^{\downarrow}\rangle + |u^{\uparrow} d^{\uparrow} u^{\downarrow}\rangle + |u^{\downarrow} d^{\uparrow} u^{\uparrow}\rangle - $$
$$-2\,|u^{\uparrow} d^{\downarrow} u^{\uparrow}\rangle + |d^{\uparrow} u^{\uparrow} u^{\downarrow}\rangle + |d^{\uparrow} u^{\downarrow} u^{\uparrow}\rangle - 2\,|d^{\downarrow} u^{\uparrow} u^{\uparrow}\rangle \Big] \ .$$

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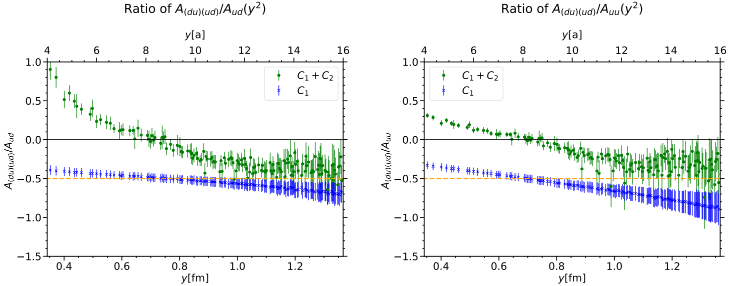
yields the following ratios:

$$\frac{f_{duud}}{f_{ud}} = -\frac{1}{2}\,, \qquad \frac{f_{duud}}{f_{uu}} = -\frac{1}{2}\,, \qquad \frac{f_{\Delta du \Delta ud}}{f_{ud}} = +\frac{5}{6}\,, \qquad \frac{f_{\Delta u \Delta d}}{f_{ud}} = -\frac{2}{3}$$

Results for $A(py = 0, y^2)$: Comparison with $SU(6)$-model



- C_1 data for unpolarized quarks roughly coincides with $SU(6)$ prediction (orange line)
- Large deviations in particular for small y when considering all leading contractions
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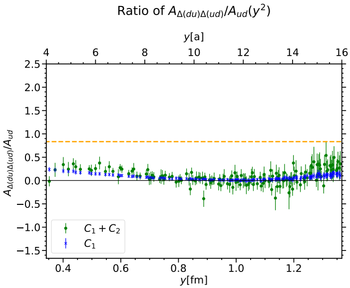

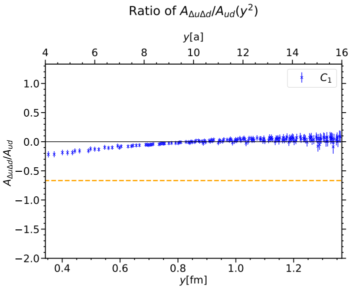

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Factorization tests

Factorization in terms of impact parameter distributions $f_q(x, \boldsymbol{b})$:

$$F_{qq'}(x_1, x_2, \boldsymbol{y}) \approx \int \mathrm{d}^2\boldsymbol{b}\; f_q(x_1, \boldsymbol{b} + \boldsymbol{y})\; f_{q'}(x_2, \boldsymbol{b})$$

Factorization tests

The invariant functions $A(py, y^2)$ depend on the nucleon form factors $F_1(t)$ and $F_2(t)$:

$$A_{qq'}(py = 0, y^2) \approx \frac{1}{2\pi} \int_{-1}^{1} \mathrm{d}\zeta \frac{(1-\zeta/2)^2}{1-\zeta} \int \mathrm{d}r \; r \; J_0(yr) \times \\ \times \left[\left(1 - \frac{\zeta^2}{(2-\zeta)^2} \right) F_1^q(t) \; F_1^{q'}(t) + \dots \right]$$

$\Rightarrow$ These can be obtained from the lattice

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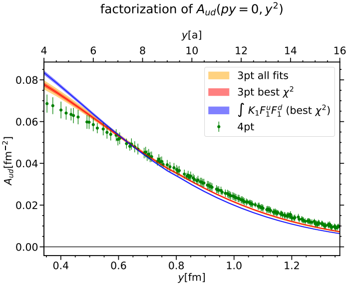

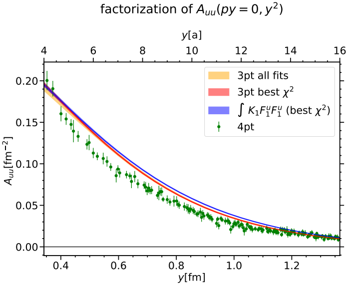

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The "transition distributions" f_{ud},f_{du} can be decomposed into impact parameter distributions f_u, f_d via isospin symmetry:

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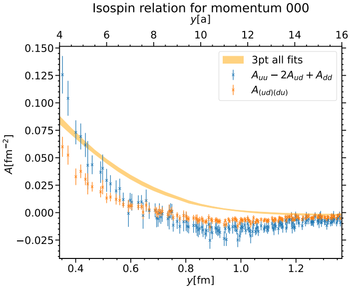

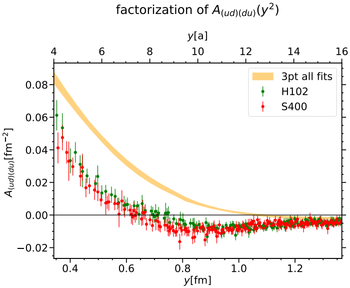

Content

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Questions?