Associated production of heavy quarkonia and *D* mesons in the high energy factorization

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Outline

1. Introduction
2. Factorization approaches
3. Hadronization models
4. Numerical methods
5. QQ production
6. Q D production
7. Conclusions

Description of production of hadrons in QCD is consist of *pQCD* and *hadronization* of final states.

i. Light and heavy–light hadrons hadroproduction:
   - fragmentation mechanism.

Studied in processes: $p+p \to D+X,\, D\,\bar{D}+X,\, D\,Z/W+X, \ldots$

ii. Quarkonium hadroproduction:
   - fragmentation mechanism (dominant at large $p_T^{\mathcal{Q}} \gtrsim 15$ GeV[Kniehl, Nefedov, and Saleev '16]);
   - fusion mechanisms (CSM, NRQCD, (I)CEM);

Studied in processes (LHC data): inclusive production $p+p \to \mathcal{Q}\,(nS)+X$ and associated productions:

$J/\psi\,\Upsilon\,X$	DØ: $\sqrt{s}=1.8$ TeV
	LHCb: $\sqrt{s}=13$ TeV, $p_T^{\psi,\Upsilon}<10,30$ GeV
$J/\psi\,J/\psi\,X$	LHCb: $\sqrt{s}=7,13$ TeV, $p_T^{\psi}<10$ GeV
	CMS: $\sqrt{s}=7$ *TeV,* $p_T^{\psi}>6.5$ *GeV*
	ATLAS: $\sqrt{s}=8$ *TeV,* $p_T^{\psi}>8.5$ *GeV*
$\Upsilon\,\Upsilon\,X$	CMS: $\sqrt{s}=13$ TeV
$J/\psi\,Z\,X$	*ATLAS:* $\sqrt{s}=8$ *TeV,* $p_T^{\psi}>8.5$ *GeV*
$J/\psi\,W\,X$	*ATLAS:* $\sqrt{s}=7,8$ *TeV,* $p_T^{\psi}>8.5$ *GeV*
$\mathcal{Q}\,D\,X$	LHCb: $\sqrt{s}=7$ TeV, $p_T^{\psi,\Upsilon}<13,15$ GeV

«Prompt» production $pp \to J/\psi\,J/\psi\,X$–production without B–hadron decay via $b \to J/\psi\,X$.

- In this talk;
- *Can be found in*[Chernyshev and Saleev '22,23].

Processes of inclusive and associated quarkonia hadroproduction are a good tools to study *hadronization*.

Factorization approaches

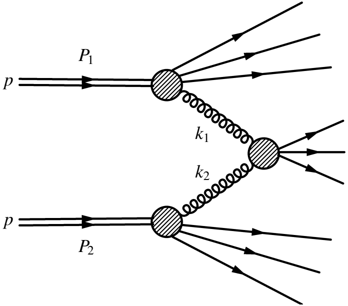
Collinear Parton Model (CPM)

- On–shell initial states: $k_i^\mu = x_i P_i^\mu, \quad k_i^2 = 0$;
- Collinear factorization: $a\,(k_1) + b\,(k_2) \to \ldots\,(k_f)$

$$P_{1,2} = \frac{\sqrt{s}}{2}(1,0,0,\pm 1)$$

$$d\sigma(k_f, \sqrt{s}) = \sum_{a,\,\bar{b}} \left[f_{a/p}(x_1,\mu^2)\, f_{b/p}(x_2,\mu^2) \right] \otimes d\hat{\sigma}_{ab}(t_1,t_2,k_f)$$

$$= \sum_{a,\,\bar{b}} \int dx_1\, f_{a/p}(x_1,\mu^2) \int dx_2\, f_{b/p}(x_2,\mu^2) \times d\hat{\sigma}_{ab}(t_1,t_2,k_f),$$



where $a,b \in \{g,q,\bar{q}\}$. Partonic cross–section:

$$d\hat{\sigma}_{ab}(t_1,t_2,k_f) =$$

$$(2\pi)^4\, \delta^{(4)}(k_1 + k_2 - \sum k_f)\, \frac{\overline{|\mathcal{A}(a\,b \to \ldots)|^2}}{I}\, d\Phi(k_f)$$

with flux–factor $I \simeq 2x_1x_2s$;

- Framework: $\mu_{\mathrm{F}} \sim p_T > m_c \gg \Lambda_{\mathrm{QCD}}$.

Parton Reggeization Approach (PRA)[Nefedov, Saleev, and Shipilova '13]

- *Off–shell* initial states (Reggeized gluons R and quarks Q_q): $k_i^\mu = x_i P_i^\mu + k_{Ti}^\mu, \quad k^2 = -\mathbf{k}_{Ti}^2$;
- *Multi–Regge* kinematics: $i\,(k_1) + j\,(k_2) \to \ldots\,(k_f)$

$$d\sigma\left(k_f,\sqrt{s}\right) = \sum_{i,\,\bar{j}} \left[\Phi_{i/p}(x_1,t_1,\mu^2) \times \Phi_{j/p}(x_2,t_2,\mu^2)\right] \otimes d\hat{\sigma}_{ij}^{\rm PRA}\left(t_1,t_2,k_f\right)$$

$$= \sum_{i,\,\bar{j}} \int \frac{dx_1}{x_1} \int dt_1 \int \frac{d\phi_1}{2\pi}\, \Phi_{i/p}(x_1,t_1,\mu^2) \int \frac{dx_2}{x_2} \int dt_2 \int \frac{d\phi_2}{2\pi}\, \Phi_{j/p}(x_2,t_2,\mu^2) \times d\hat{\sigma}_{ij}^{\rm PRA}\left(t_1,t_2,k_f\right),$$

where $t_i = -k_{Ti}^2$ and $i,j \in \{R, Q_q, \bar{Q}_q\}$. Partonic cross section in PRA:

$$d\hat{\sigma}_{ij}^{\rm PRA}\left(t_1,t_2,k_f\right) = (2\pi)^4\,\delta^{(4)}\left(k_1+k_2-\sum k_f\right) \frac{\overline{|\mathcal{A}^{\rm PRA}(i\;j \to \ldots)|^2}}{I}\, d\Phi(k_f)$$

with flux–factor $I \simeq 2x_1x_2s$, $\overline{|\mathcal{A}^{\rm PRA}|^2}$ is calculated in the *Lipatov's EFT*[Lipatov '95];

- Exact normalization condition for the *modified unPDF*[Nefedov and Saleev '20]:

$$\int_0^{\mu^2} dt\; \Phi_{i/p}(x,t,\mu^2) = F_{i/p}(x,\mu^2),$$

here $F_{i/p}(x,\mu^2) = x\, f_{i/p}(x,\mu^2)$;

- Framework: $\mu_{\rm F} \ll \sqrt{s}$–«*small*» *x physics*.

unPDFs in the *modified KMRW model*:

$$\Phi_i(x,t,\mu) = \frac{\alpha_S(\mu)}{2\pi}\,\frac{T_i(t,\mu^2,x)}{t} \sum_{j=g,q,\bar{q}} \int_x^1 dz\, P_{ij}(z)\, F_j\left(\frac{x}{z},t\right)\, \theta\left(\Delta(t,\mu)-z\right),$$

where $T_i(t,\mu^2,x)$ is *Sudakov formfactor* with boundary conditions $T_i(t=0,\mu^2,x)=0$ and $T_i(t=\mu^2,\mu^2,x)=1$.
The solution for Sudakov formfactor[Nefedov and Saleev '20]:

$$T_i(t,\mu^2,x) = \exp\left[-\int_t^{\mu^2} \frac{dt'}{t'}\,\frac{\alpha_S(t')}{2\pi}\left(\tau_i(t',\mu^2)+\Delta\tau_i(t',\mu^2,x)\right)\right],$$

with

$$\begin{aligned}
\tau_i(t,\mu^2) &= \sum_j \int_0^1 dz\, zP_{ji}(z)\theta(\Delta(t,\mu^2)-z),\\
\Delta\tau_i(t,\mu^2,x) &= \sum_j \int_0^1 dz\, \theta(z-\Delta(t,\mu^2))\left[zP_{ji}(z) - \frac{F_j\left(\frac{x}{z},t\right)}{F_i(x,t)}P_{ij}(z)\theta(z-x)\right].
\end{aligned}$$

PRA smoothly interpolates QCD predictions between high–energy and low–energy regions as well as between small–p_T and large–p_T of final particles.

Main LO PRA publications:

- B.A. Kniehl, V.A. Saleev and D.V. Vasin, «Bottomonium production in the Regge limit of QCD»,
  Phys. Rev. D **74** (2006), 014024;
- M.A. Nefedov, V.A. Saleev and A.V. Shipilova, «Dijet azimuthal decorrelations at the LHC in the parton Reggeization approach»,
  Phys. Rev. D **87** (2013), 094030;
- A.V. Karpishkov, M.A. Nefedov and V.A. Saleev, «$B\bar{B}$ angular correlations at the LHC in parton Reggeization approach merged with higher-order matrix elements»,
  Phys. Rev. D **96** (2017), 096019;
- M.A. Nefedov and V.A. Saleev, «High-Energy Factorization for Drell–Yan process in pp and $p\bar{p}$ collisions with new Unintegrated PDFs»,
  Phys. Rev. D **102** (2020), 114018.

Quarkonium production

$M_{J/\psi} \simeq 3.097$ GeV and $M_{\Upsilon} \simeq 9.460$ GeV - *non–relativistic*. Quarkonium in the potential model:

$$\mathcal{V}_{\text{Cornell}}(r) = -C_F \frac{\alpha_S(1/r)}{r} + \sigma r \quad \Longrightarrow \quad \boxed{\alpha_S(M_Q v) \sim v \sim 0.2 - 0.3}$$

Color Singlet Model (CSM)[Baier, Ruckl, Berger, and Jones '83]

$$d\sigma\left(\mathcal{Q}[{}^3S_1^1]\right) = d\sigma\left(Q\bar{Q}[{}^3S_1^1]\right) \times \langle \mathcal{O}^{\mathcal{Q}}[{}^3S_1^1]\rangle,$$

where $\langle \mathcal{O}^{\mathcal{Q}}[{}^3S_1^1]\rangle \sim \mid \Psi_{\mathcal{Q}}(0) \mid^2$.

Non–relativistic QCD (NRQCD)[Bodwin, Braaten, and Lepage '95]

$$d\sigma(\mathcal{Q}) = \sum_n d\sigma\left(Q\bar{Q}[n]\right) \times \langle \mathcal{O}^{\mathcal{Q}}[n]\rangle,$$

here $n = {}^{2S+1}L_J^a$.

Main problems:

- octet contribution excess of predictions over experimental data in η_c production;
- polarization puzzle.

Color Evaporation Model (CEM)[Fritzsch and Halzen '77] ↪ **Improved CEM (ICEM)**[Ma and Vogt '16]

$i + j \to Q\,(p_Q) + \bar{Q}\,(p_{\bar{Q}}) \to \mathcal{Q}\,(p)$

Master formula for the differential cross section:

$$\frac{d\sigma_{\mathcal{Q}}}{d^3p} = \mathcal{F}^{\mathcal{Q}} \times \int_{M_{\mathcal{Q}}}^{2M_H} dM\, d^3p'\, \delta^{(3)}\left(\mathbf{p} - \frac{M_{\mathcal{Q}}}{M}\mathbf{p}'\right) \frac{d\sigma_{Q\bar{Q}}}{dM\, d^3p'} \quad + \quad \mathcal{O}\left(\lambda^2/M_{\mathcal{Q}}^2\right)$$

$$\simeq \mathcal{F}^{\mathcal{Q}} \times \int_{M_{\mathcal{Q}}}^{2M_H} dM\left[\theta(M-M_{\mathcal{Q}}) - \theta(M-2M_H)\right] \left.\frac{d\sigma_{Q\bar{Q}}}{d^3p'}\right|_{\left(\mathbf{p}'=\frac{M}{M_{\mathcal{Q}}}\mathbf{p}\right)}$$

- One free parameter $\mathcal{F}^{\mathcal{Q}}$ for each $\mathcal{Q}$;
- All quarkonium $\mathcal{Q}$ states are treated like $Q\bar{Q}$ pairs below $H\bar{H}$ threshold;
- All $Q\bar{Q}$ production diagrams included independent of color;
- The $Q\bar{Q}$ pairs produce quarkonium $\mathcal{Q}$ through proto–quarkonium state with 4–momentum $p' = p_Q + p_{\bar{Q}}$ with matching condition: $p = (M_{\mathcal{Q}}/M)\,p'$;
- ICEM predicts polarized quarkonium production[Cheung and Vogt '21].

ICEM can be considered as NRQCD without velocity scaling.

Recent ICEM publications:

- V. Cheung and R. Vogt, «Production and polarization of direct J/ψ up to $\mathcal{O}(\alpha_S^3)$ in the improved color evaporation model in collinear factorization», Phys. Rev. D **104** (2021), 094026;
- V. Cheung and R. Vogt, «Production and polarization of prompt J/ψ in the improved color evaporation model using the k_T–factorization approach», Phys. Rev. D **98** (2018), 114029;
- V. Cheung and R. Vogt, «Polarized Heavy Quarkonium Production in the Color Evaporation Model», Phys. Rev. D **95** (2017), 074021;
- J.–P. Lansberg et.al., «Complete NLO QCD study of single– and double–quarkonium hadroproduction in the colour–evaporation model at the Tevatron and the LHC», Phys. Lett. B **807** (2020), 135559;
- A.A. Chernyshev and V.A. Saleev, «Single and pair J/ψ production in the improved color evaporation model using the parton Reggeization approach», Phys. Rev. D. **106** (2022), 114006;
- V.A. Saleev and A.A. Chernyshev «Pair Production of J/ψ in the Color Evaporation Model and the Parton Reggeization Approach», Phys.Part.Nucl.Lett. **20** (2023), 389–394;
- A.A. Chernyshev and V.A. Saleev, «Pair Production of Heavy Quarkonia in the Color Evaporation Model», Physics of Atomic Nuclei **86** (2023), 1093–1098;
- A.A. Chernyshev and V.A. Saleev, «Associated production of J/ψ plus Z/W in the improved color evaporation model using the parton Reggeization approach», arXiv:2304.07481.

Open charm production

Fragmentation mechanism[D′Alesio and Murgia ′04]

$i+j \to c\,(q)\;(\to D\,(p)) + \bar{c}$

Master formula for the differential cross section:

$$\frac{d\sigma_D}{dp_{T\,D}^2\,dy_D} = \int_{z_{\rm cut}}^{1} dz\,\mathcal{D}_{c\to D}(z,\mu_0^2)\,\frac{d\sigma_{c\bar{c}}}{dp_{T\,c}^2\,dy_c}\bigg|_{q=q(z)}$$

- Parameter $z = \left(p^0 + |\mathbf{p}|\right) / \left(q^0 + |\mathbf{q}|\right)$;
- Parameter cut: $z_{\rm cut} = m_D / \left(q^0 + |\mathbf{q}|\right)$;
- Peterson's fragmentation function (FF)[Peterson et al. ′83]:

$$\mathcal{D}_{c\to D}(z,\mu_0^2) = \mathcal{N}\,\frac{z\,(1-z)^2}{[(1-z)^2+\varepsilon\,z]^2}, \qquad \int dz\,\mathcal{D}_{c\to D}(z,\mu_0^2) = f(c\to D),$$

  with $\varepsilon = 0.06$;

- Fragmentation fractions: $f(c\to D^0) = 0.542$ and $f(c\to D^+) = 0.225$[Gladilin ′15].

Numerical methods

There are 2 ways to perform calculations in the PRA:

- ReggeQCD$_{\text{[Nefedov]}}$–FeynCalc model file
  - Tree–level matrix elements for up to 2 → 4 partonic subprocesses with Reggeized partons;
- KaTie$_{\text{[Hameren '16]}}$–MC generator
  - Calculations up to 2 → 4 parton subprocesses with off–shell amplitudes;
  - Tree–level matrix elements from AVHLIB$_{\text{[Hameren '13]}}$;
  - Collinear PDFs from LHAPDF and TMD PDFs from TMDlib;
  - unPDF with exact normalization in the modified KMRW model$_{\text{[Nefedov and Saleev '20]}}$;
  - Multiparton interactions;
  - Event files in LHE format.

These two methods are equivalent at the stage of numerical calculations.

Pair J/ψ production (using ICEM and PRA)

Cross section for process $p+p \to J/\psi + J/\psi + X$:

$$d\sigma_{\psi\psi} = d\sigma_{\psi\psi}^{\text{SPS}} + d\sigma_{\psi\psi}^{\text{DPS}}$$

with SPS and DPS contributions:

- SPS master formula:

$$d\sigma_{\psi\psi}^{\text{SPS}} = \mathcal{F}^{\psi\psi} \times \sum_{i,\bar{j}} \prod_{k=1,2} \int_{M_\psi}^{2M_D} dM_k \left[\Phi_{i/p}(x_1,t_1,\mu^2) \times \Phi_{j/p}(x_2,t_2,\mu^2)\right] \otimes \frac{d\hat{\sigma}_{ij\to c\bar{c}c\bar{c}}}{dM_k}$$

- DPS pocket formula:

$$d\sigma_{\psi\psi}^{\text{DPS}} = (\mathcal{F}^{\psi})^2 \times \frac{1}{(1+\delta_{Q_1Q_2})\,\sigma_{\text{eff}}} \times \sum_{i,\bar{j}} \prod_{k=1,2} \int_{M_\psi}^{2M_D} dM_k \left[\Phi_{i/p}(x_1,t_1,\mu^2) \times \Phi_{j/p}(x_2,t_2,\mu^2)\right] \otimes \frac{d\hat{\sigma}_{ij\to c\bar{c}}}{dM_k},$$

where $\delta_{Q_1Q_2} = 1$ for $Q_1 = Q_2 = J/\psi$, $\mathcal{F}^{\psi}$ is fixed in inclusive J/ψ production, σ_{eff} considered as free parameter.

In general:

$$\mathcal{F}^{\psi\psi} \neq \mathcal{F}^{\psi} \times \mathcal{F}^{\psi} \quad \Longrightarrow \quad (\mathcal{F}^{\psi},\, \mathcal{F}^{\psi\psi}) \ \textit{are independent parameters}$$

$\mathcal{F}^{\psi}$ and $\mathcal{F}^{\Upsilon}$ extraction[1]

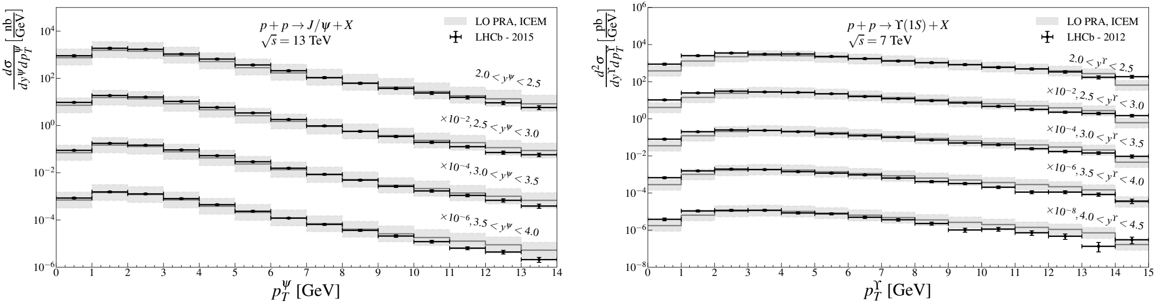


Figure 1: Spectra of inclusive J/ψ and Υ production on transverse momentum p_T^{ψ} of J/ψ and p_T^{Υ} of $\Upsilon(1S)$ mesons respectively. We found $\mathcal{F}^{\psi} = 0.020$ and $\mathcal{F}^{\Upsilon} = 0.021$ at LHC energies, plots are from[Chernyshev and Saleev '22,23].

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[1] The data are from LHCb Collaboration[Aaij et.al '15,12].

$\mathcal{F}^{\psi\psi}$ and σ_{eff} extraction

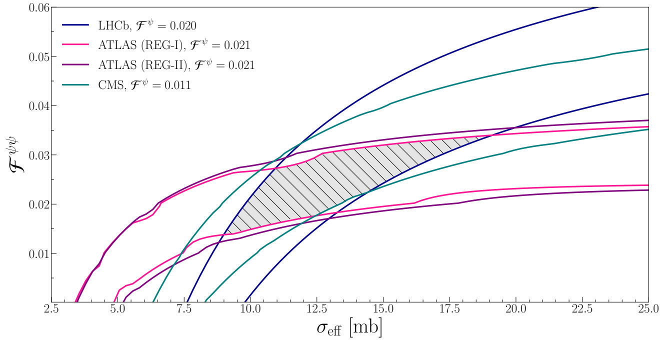

Figure 2: Plot is taken from [Chernyshev and Saleev '22].

Two lines for each experiment (k = ATLAS, CMS, LHCb) corresponds condition:

$$x_k = \frac{|\,\sigma^{\text{exp}} - \sigma^{\text{theor}}\,|}{\Delta\sigma^{\text{exp}}} \leq 1$$

Isolines corresponds more strong condition:

$$x = \sum_k x_k = 2.0, 1.5, 1.0$$

We obtained: $\boxed{\mathcal{F}^{\psi\psi} \simeq \mathcal{F}^{\psi} = 0.02 \text{ and } \sigma_{\mathrm{eff}} \simeq 11 \text{ mb}}$.

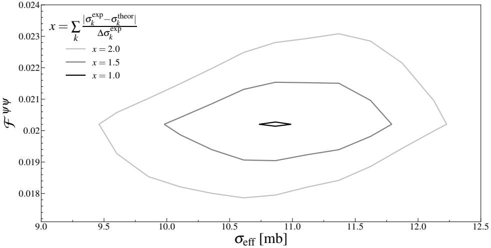

Figure 3: Regions of the parameters $\mathcal{F}^{\psi\psi}$ and σ_{eff} in the ICEM for pair J/ψ production, obtained as a result of data fitting[Chernyshev and Saleev '22]: $\sigma_{\mathrm{eff}} = 11.0 \pm 0.2$ (stat.).

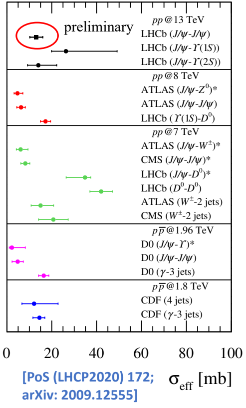

Figure 4: From Liupan An, talk at EPS-HEP2023: $\sigma_{\mathrm{eff}} = 13.1 \pm 1.8$ (stat.) ± 2.3 (syst.) mb (from new pair J/ψ data).

Our extracted value of $\sigma_{\mathrm{eff}} = 11$ mb is in good agreement with others.

Pair J/ψ production in the NRQCD + PRA

- Z.G. He, B.A. Kniehl, M.A. Nefedov and V.A. Saleev, «Double Prompt J/ψ Hadroproduction in the Parton Reggeization Approach with High–Energy Resummation», Phys. Rev. Lett. **123** (2019), 162002;

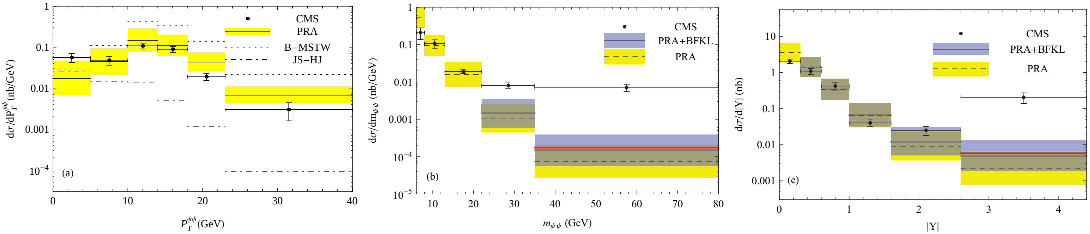

Figure 5: The plots are taken from [Zhi-Guo He et.al '19].

Results[2]

Ratio SPS / DPS:

$$R_{\psi\psi} = \frac{\sigma^{\text{SPS}}_{\psi\psi}}{\sigma^{\text{DPS}}_{\psi\psi}} \simeq 0.2$$

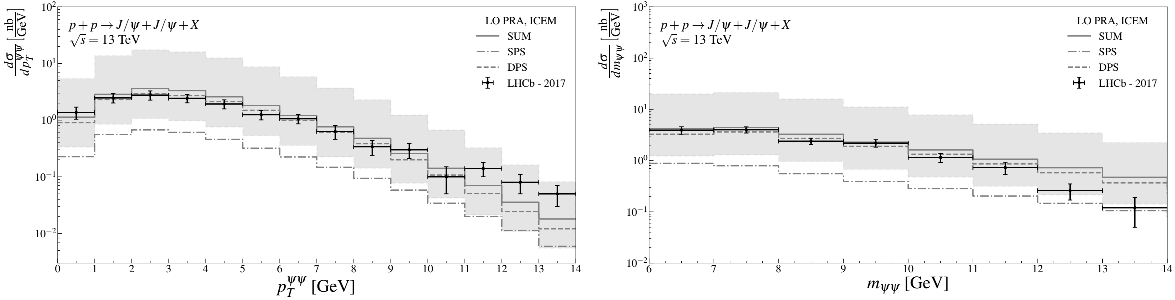

Figure 6: Spectra of pair J/ψ production on transverse momentum $p_T^{\psi\psi}$ and invariant mass $m_{\psi\psi}$ of J/ψ pair.

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[2] The data are from LHCb Collaboration[Aaij et.al '17].

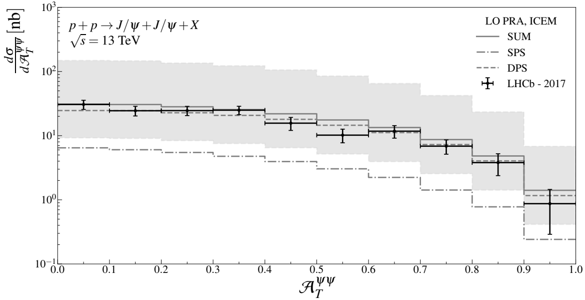

Figure 7: Spectra of pair J/ψ production on transverse asymmetry $\mathcal{A}_T^{\psi\psi}$ between two J/ψ.

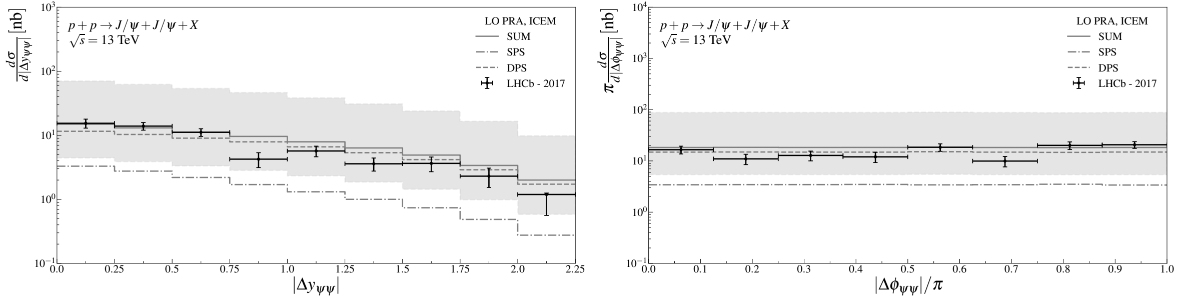

Figure 8: Correlation spectra of pair J/ψ production on rapidity difference $|\Delta y_{\psi\psi}|$ and azimuthal angle difference between two J/ψ.

Pair Υ production[3]

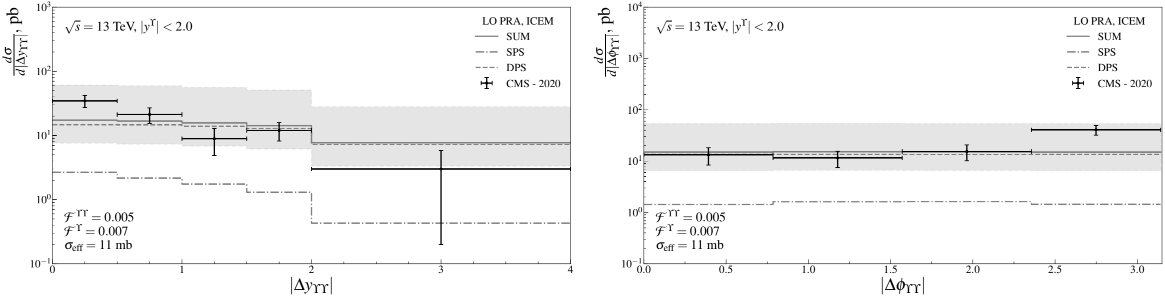

Figure 9: Correlation spectra of pair Υ production on rapidity difference $|\Delta y_{\Upsilon\Upsilon}|$ and azimuthal angle difference between two Υ.

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[3] The data are from CMS Collaboration[Sirunyan et.al '20].

Associated J/ψ Υ production[4]

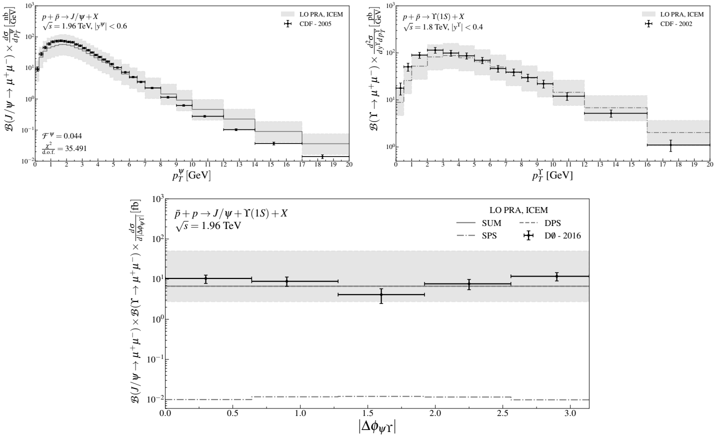

Figure 10: Correlation spectra of associated J/ψ Υ production on azimuthal angles difference $|\Delta\phi_{\psi\Upsilon}|$.

$$p+\bar{p}\to J/\psi+\Upsilon+X$$

Via SPS and DPS:

$$\mathcal{F}^{\psi\Upsilon}=\mathcal{F}^{\psi}\times\mathcal{F}^{\Upsilon}$$

and

$$\sigma_{\text{eff}}=11\text{ mb}$$

$\mathcal{F}^{\psi(\Upsilon)}$ is fixed in the inclusive $J/\psi(\Upsilon)$ production at the same energy.

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[4] The data are from D0 Collaboration$_{\text{[Abazov et.al. '16]}}$.

Associated $\mathcal{Q}$ D production (using ICEM, FM, and PRA)

Cross section for process $p+p\to\mathcal{Q}+D+X$ ($\mathcal{Q}=J/\psi,\Upsilon(1S)$):

$$d\sigma_{\mathcal{Q}D}=d\sigma^{\rm SPS}_{\mathcal{Q}D}+d\sigma^{\rm DPS}_{\mathcal{Q}D}$$

with SPS and DPS contributions:

- SPS master formula ($\mathcal{Q}[q\bar{q}]$):

$$d\sigma^{\rm SPS}_{\mathcal{Q}D}=\mathcal{F}^{\mathcal{Q}}\times\int_{M_{\mathcal{Q}}}^{2M_H}dM\;f(c\to D)\times\int dz\;\mathcal{D}_{c\to D}(z,\mu_0^2)$$
$$\times\sum_{i,\bar{j}}\left[\Phi_{i/p}(x_1,t_1,\mu^2)\times\Phi_{j/p}(x_2,t_2,\mu^2)\right]\otimes\frac{d\hat{\sigma}_{ij\to q\bar{q}c\bar{c}}}{dM}$$

- DPS pocket formula:

$$d\sigma^{\rm DPS}_{\mathcal{Q}D}=\frac{d\sigma^{\rm SPS}_{\mathcal{Q}}\times d\sigma^{\rm SPS}_{D}}{(1+\delta_{\mathcal{Q}D})\;\sigma_{\rm eff}},$$

  where $\delta_{\mathcal{Q}D}=0$.

All free parameters, $\mathcal{F}^{\mathcal{Q}}$ and $\sigma_{\rm eff}$, are fixed in other processes.

Inclusive D mesons production (using FM and PRA[5])

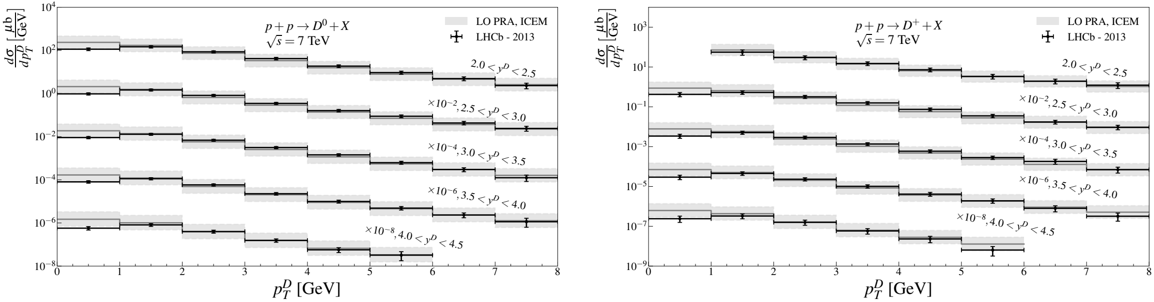

Figure 11: Spectra of inclusive D mesons production on transverse momentum p_T^D of $D^{0,+}$ mesons.

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[5] The data are from LHCb Collaboration[Aaij et.al ′13].

Results[6]

Comparison of the theoretical and experimental total cross sections:

Final state	Energy	Cross section	Exp.$\pm$(stat.)$\pm$(syst.)	LO PRA$\pm(\Delta_{\rm SPS})\pm(\Delta_{\rm DPS})$
$J/\psi+D^0$	$\sqrt{s}=7$ TeV	$\mathcal{B}(J/\psi\to\mu\bar{\mu})\times\sigma$	$9.7\pm0.2\pm0.7$ [nb]	$9.6\ ^{+0.4}_{-0.1}\ ^{+26.1}_{-5.9}$ [nb]
$J/\psi+D^+$	$\sqrt{s}=7$ TeV	$\mathcal{B}(J/\psi\to\mu\bar{\mu})\times\sigma$	$3.4\pm0.1\pm0.4$ [nb]	$3.9\ ^{+0.2}_{-0.02}\ ^{+10.8}_{-2.4}$ [nb]
$\Upsilon+D^0$	$\sqrt{s}=7$ TeV	$\mathcal{B}(\Upsilon\to\mu\bar{\mu})\times\sigma$	$155\pm21\pm7$ [pb]	$145\ ^{+16}_{-6}\ ^{+124}_{-65}$ [pb]
$\Upsilon+D^+$	$\sqrt{s}=7$ TeV	$\mathcal{B}(\Upsilon\to\mu\bar{\mu})\times\sigma$	$82\pm19\pm5$ [pb]	$78\ ^{+14}_{-2}\ ^{+140}_{-38}$ [pb]
$\Upsilon+D^0$	$\sqrt{s}=8$ TeV	$\mathcal{B}(\Upsilon\to\mu\bar{\mu})\times\sigma$	$250\pm28\pm11$ [pb]	$255\ ^{+25}_{-9}\ ^{+189}_{-113}$ [pb]
$\Upsilon+D^+$	$\sqrt{s}=8$ TeV	$\mathcal{B}(\Upsilon\to\mu\bar{\mu})\times\sigma$	$80\pm16\pm5$ [pb]	$85\ ^{+8}_{-3}\ ^{+63}_{-37}$ [pb]

Ratios SPS/DPS:

$$R_{\psi D}=\frac{\sigma^{\rm SPS}_{\psi D}}{\sigma^{\rm DPS}_{\psi D}}\simeq\frac{1}{13}\quad\text{and}\quad R_{\Upsilon D}=\frac{\sigma^{\rm SPS}_{\Upsilon D}}{\sigma^{\rm DPS}_{\Upsilon D}}\simeq\frac{1}{10}$$

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[6] The data are from LHCb Collaboration[Aaij et.al ′16].

Associated $J/\psi + D$ production

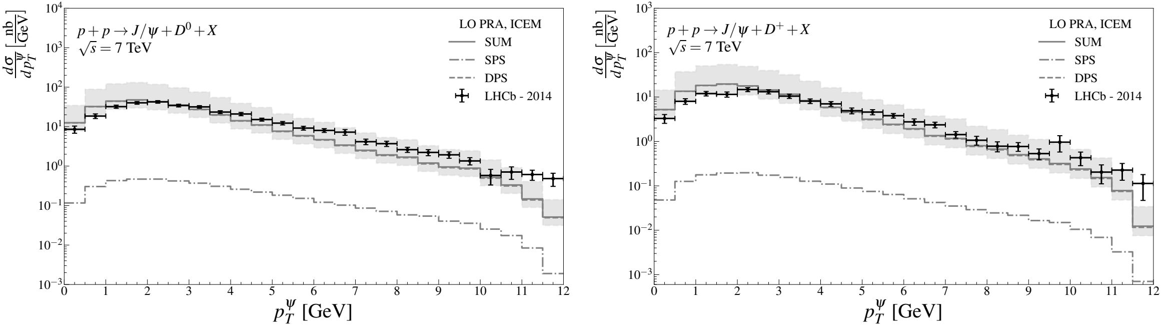

Figure 12: Spectra of associated $J/\psi + D$ production on transverse momentum p_T^ψ of J/ψ meson.

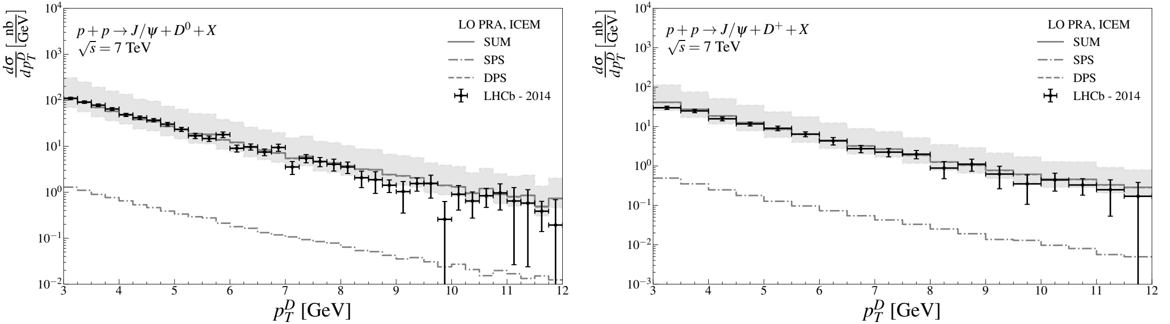

Figure 13: Spectra of associated $J/\psi + D$ production on transverse momentum p_T^D of D meson.

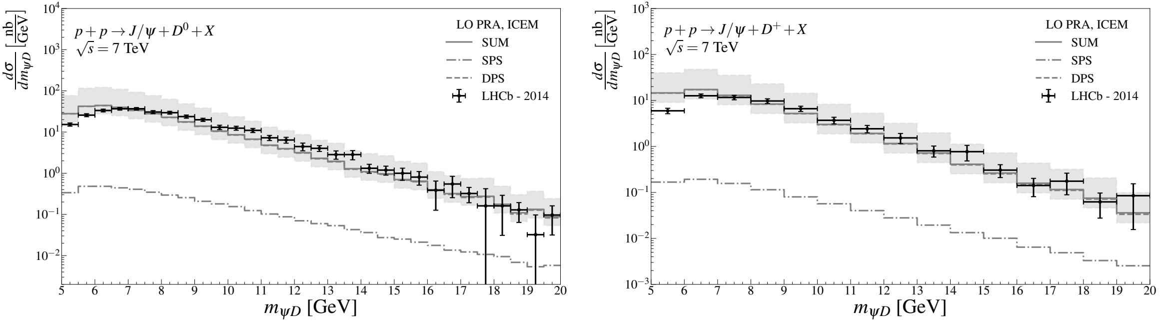

Figure 14: Spectra of associated $J/\psi + D$ production on invariant mass $m_{\psi D}$ of $J/\psi + D$ pair.

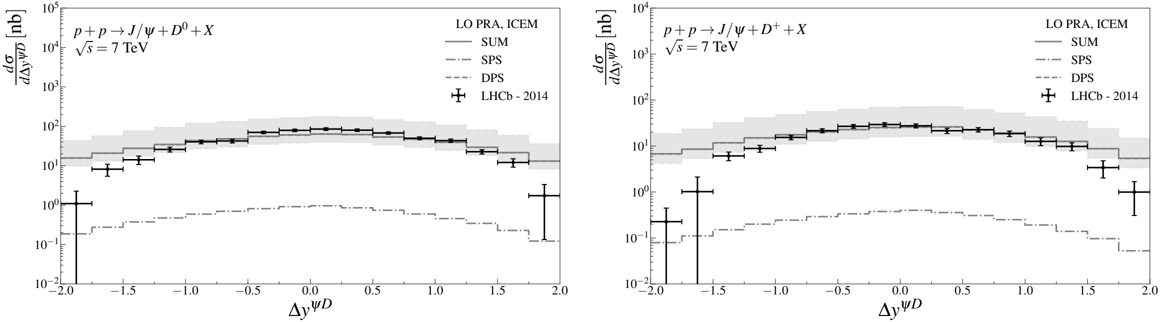

Figure 15: Correlation spectra of associated $J/\psi + D$ production on rapidity difference $\Delta y^{\psi D}$ between J/ψ and D mesons.

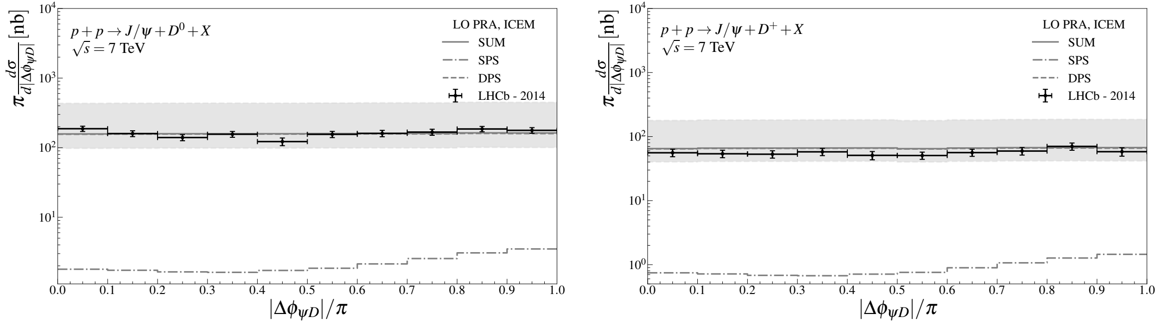

Figure 16: Correlation spectra of associated $J/\psi + D$ production on azimuthal angles difference $\Delta\phi_{\psi D}$ between J/ψ and D mesons.

Associated $\Upsilon + D$ production

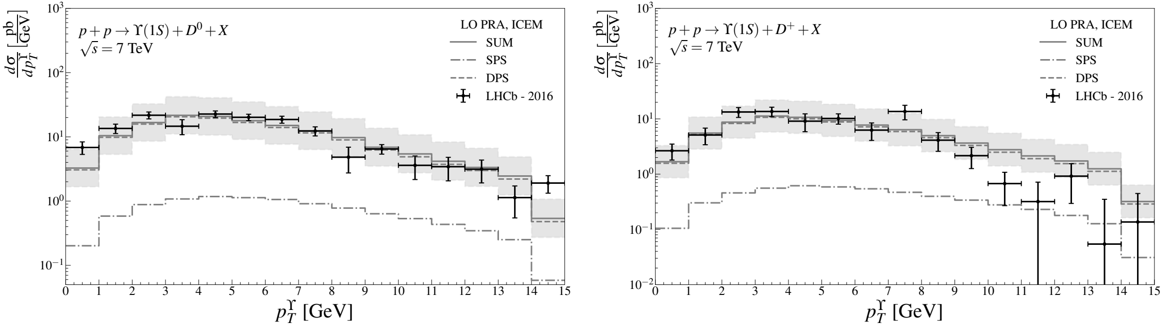

Figure 17: Spectra of associated $\Upsilon + D$ production on transverse momentum p_T^Υ of Υ meson.

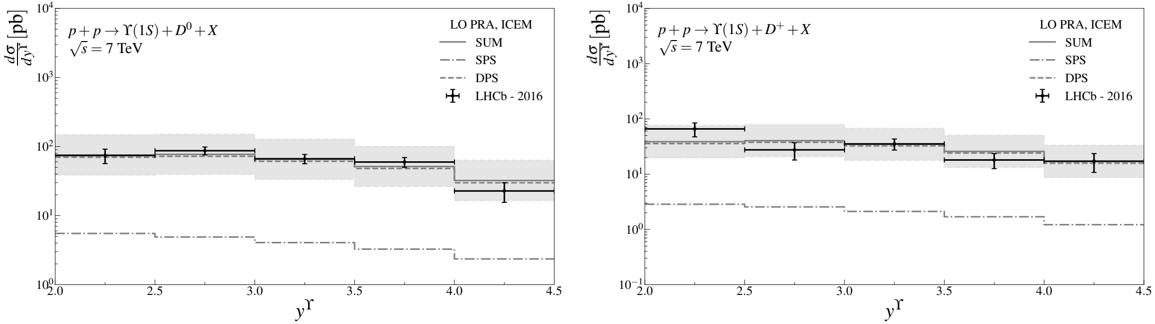

Figure 18: Spectra of associated $\Upsilon + D$ production on rapidity y^{Υ} of Υ meson.

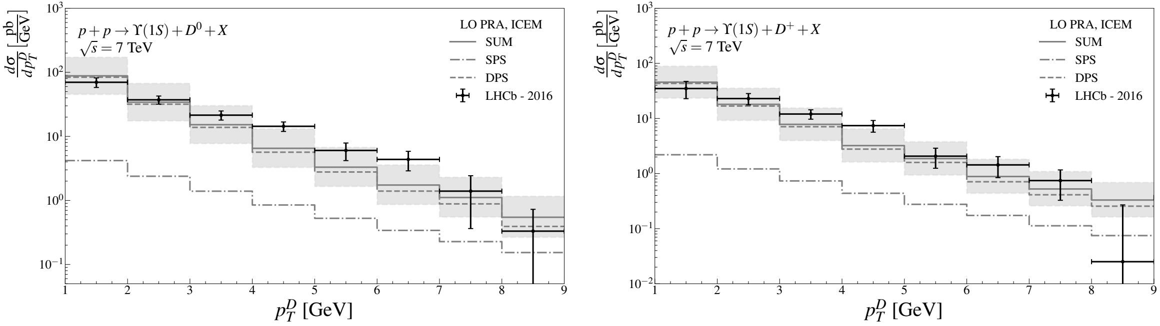

Figure 19: Spectra of associated $\Upsilon + D$ production on transverse momentum p_T^D of D meson.

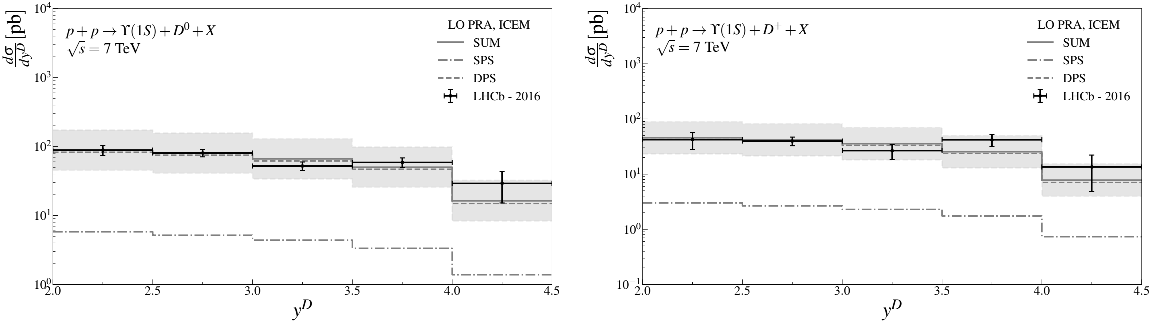

Figure 20: Spectra of associated $\Upsilon + D$ production on rapidity y^D of D meson.

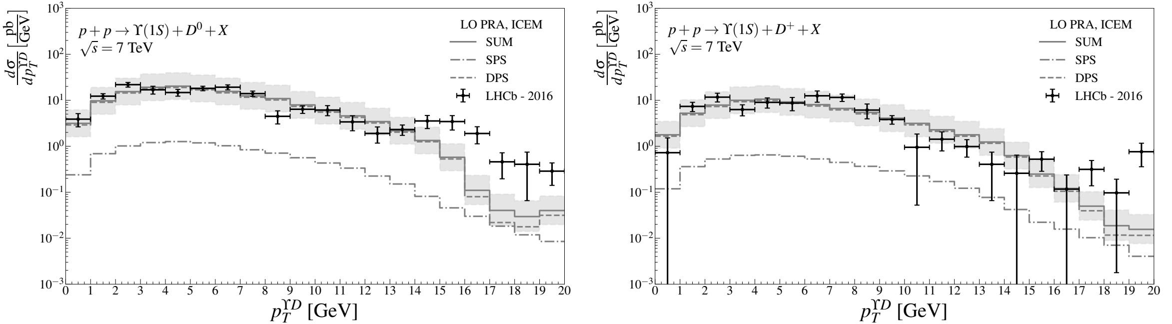

Figure 21: Spectra of associated $\Upsilon + D$ production on transverse momentum $p_T^{\Upsilon D}$ of $\Upsilon + D$ pair.

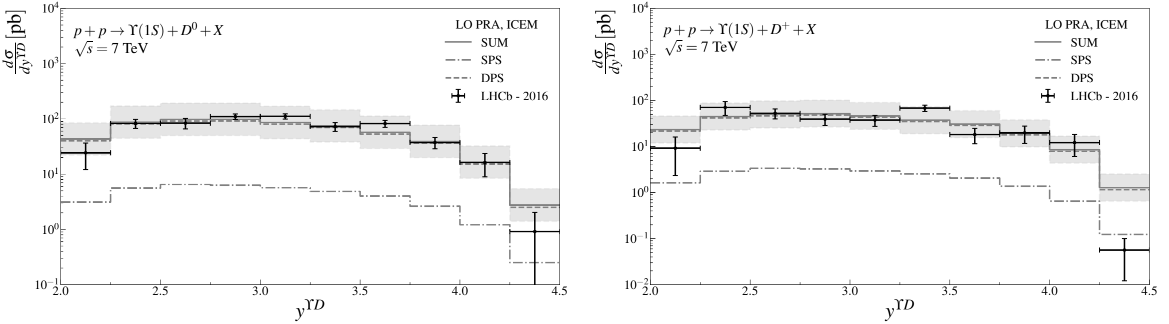

Figure 22: Spectra of associated $\Upsilon + D$ production on rapidity $y^{\Upsilon D}$ of $\Upsilon + D$ pair.

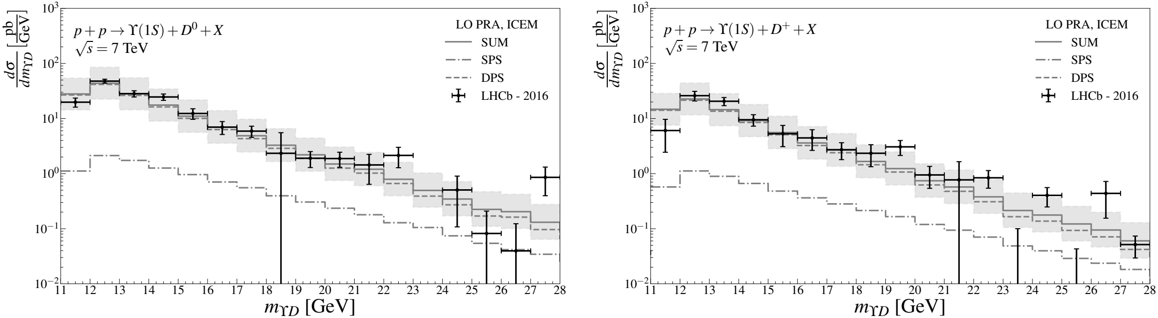

Figure 23: Spectra of associated $\Upsilon + D$ production on invariant mass $m_{\Upsilon D}$ of $\Upsilon + D$ pair.

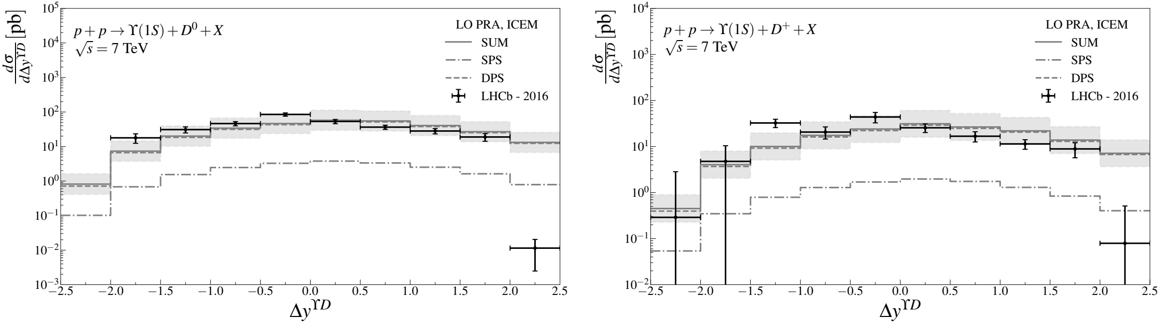

Figure 24: Correlation spectra of associated $\Upsilon + D$ production on rapidity difference $\Delta y^{\Upsilon D}$ between Υ and D mesons.

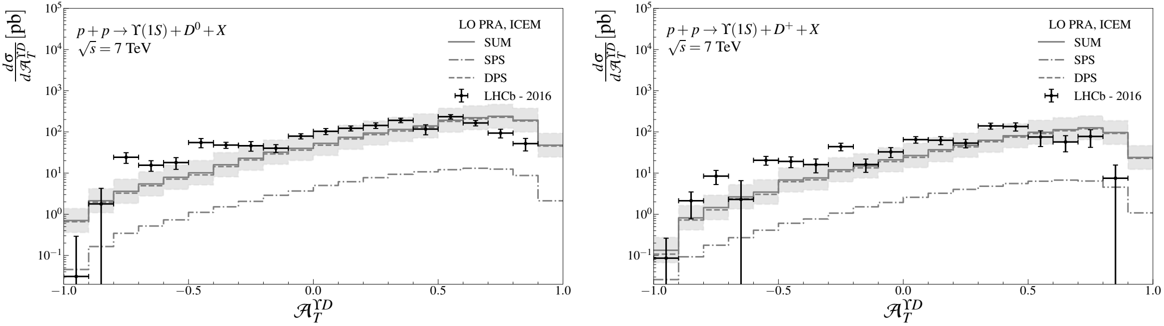

Figure 25: Correlation spectra of associated $\Upsilon + D$ production on transverse assymetry $\mathcal{A}_T^{\Upsilon D}$ between Υ and D mesons.

$$\mathcal{A}_T^{\Upsilon D} = \frac{|\ \mathbf{p}_T^{\Upsilon} - \mathbf{p}_T^{D}\ |}{|\ \mathbf{p}_T^{\Upsilon} + \mathbf{p}_T^{D}\ |}$$

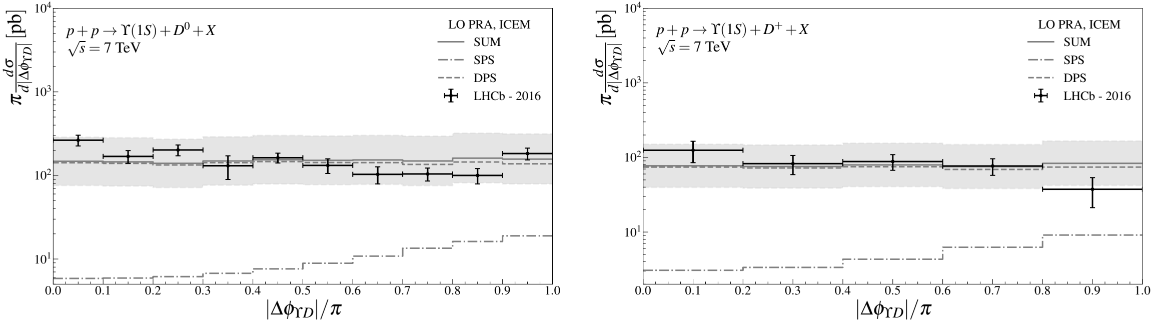

Figure 26: Correlation spectra of associated $\Upsilon+D$ production on azimuthal angles difference $\Delta\phi_{\Upsilon D}$ between Υ and D mesons.

Conclusions

i. Considering parameters $(\mathcal{F}^{\psi\psi}, \sigma_{\text{eff}})$ as independent, we obtained the following values of the parameters by data fitting on pair J/ψ production total cross sections: $\mathcal{F}^{\psi\psi} \simeq \mathcal{F}^{\psi} = 0.02$, $\sigma_{\text{eff}} = 11.0 \pm 0.2\,(\text{stat.})$ mb;

ii. At high energies, we obtained the following relations: $\mathcal{F}^{\psi\psi} \simeq \mathcal{F}^{\psi}, \quad \mathcal{F}^{\Upsilon\Upsilon} \simeq \mathcal{F}^{\Upsilon}, \quad \mathcal{F}^{\psi\Upsilon} \simeq \mathcal{F}^{\psi} \times \mathcal{F}^{\Upsilon}$;

iii. We obtained a quite satisfactory description for the $\mathcal{Q} + D$ associated production cross sections in the ICEM using the PRA without fitting any free parameters;

iv. Data on inclusive productions cross sections of $\mathcal{Q}$ and D mesons can be described self–consistent with the data on the associated production of $\mathcal{Q} + D$;

v. We find dominant role of the DPS production mechanism in the processes of pair $\mathcal{Q}$ and associated $\mathcal{Q} + D$ quarkonia hadroproduction in the forward LHCb kinematic region, such as:

$$R_{\psi\psi} \simeq 1/5, \qquad R_{\psi D} \simeq 1/13, \qquad R_{\Upsilon D} \simeq 1/10;$$

vi. The azimuthal angle difference spectra of $\Delta\phi_{\mathcal{Q}D}$ is flat due the DPS production mechanism;

vii. We obtained a self–consistent description for the following processes:

$$p + p \to \mathcal{Q} + \mathcal{Q} + X, \qquad p + \bar{p} \to J/\psi + \Upsilon + X, \qquad p + p \to J/\psi + Z/W + X, \qquad p + p \to \mathcal{Q} + D + X$$

with the same $\sigma_{\text{eff}} = 11$ mb parameter value (for details see [Chernyshev and Saleev '22,23]).

Thank you for your attention!

Extra slides

Kinematical cuts

Collaboration	Energy	Rapidity	Transverse momentum
		$J/\psi\, J/\psi$	
LHCb:	$\sqrt{s} = 13$ TeV	$y^{\psi} \in [2.0, 4.5]$	$p_T^{\psi} < 10$ GeV
		$J/\psi\, D$	
LHCb:	$\sqrt{s} = 7$ TeV	$y^{\psi} \in [2.0, 4.0]$	$p_T^{\psi} < 12$ GeV
		$y^{D} \in [2.0, 4.0]$	$p_T^{D} \in [3, 12]$ GeV
		$\Upsilon\, D$	
LHCb:	$\sqrt{s} = 7$ TeV	$y^{\Upsilon} \in [2.0, 4.5]$	$p_T^{\Upsilon} < 15$ GeV
		$y^{D} \in [2.0, 4.5]$	$p_T^{D} \in [1, 20]$ GeV

Table 1: Kinematical cuts of measurements. The data are from LHCb Collaboration[Aaij et.al '14, 16, 17].