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MPI@LHC – 2023

StringSpinner – adding spin to the PYTHIA string fragmentation

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University of Trieste and INFN Trieste

POLFRAG

Introduction

- StringSpinner is a package for the introduction of quark spin effects in PYTHIA 8 string fragmentation latest version in *AK, L. Lönnblad, CPC* ***292*** *(2023) 108886*
- Presently, can handle Deep Inelastic Scattering (DIS) @ LO

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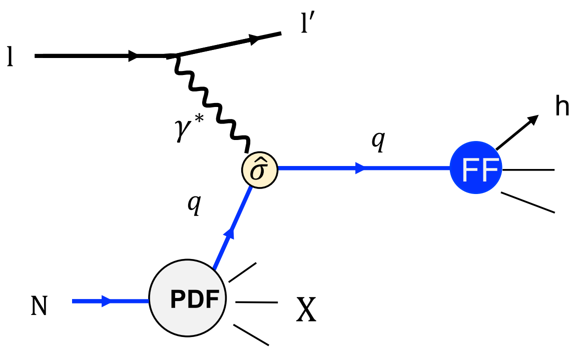
$$d\sigma^{lN\to l'hX} \propto 1 + D_{NN}\,|\vec{S}_T^{Nucl.}|\,A_{Coll}\sin\phi_{Coll} + \cdots$$

$$\phi_{Coll} = \phi_h + \phi_S - \pi$$

$$A_{Coll} \simeq \frac{\sum_q e_q^2\ h_1^q \times H_{1q}^{\perp h}}{\sum_q e_q^2\ f_1^q \times D_{1q}^h}$$



h_1^q transversity PDF: transverse polarization of quarks in a transversely polarized nucleon

$H_{1q}^{\perp h}$ Collins FF: fragmenttion of a transversely polarized quark in an unpolarized hadron

f_1^q and D_{1q}^h: spin-averaged PDF and FF

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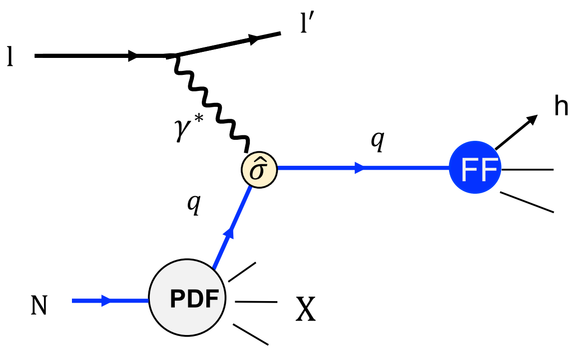

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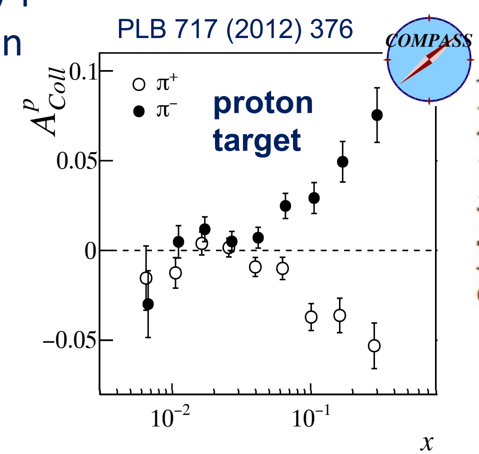
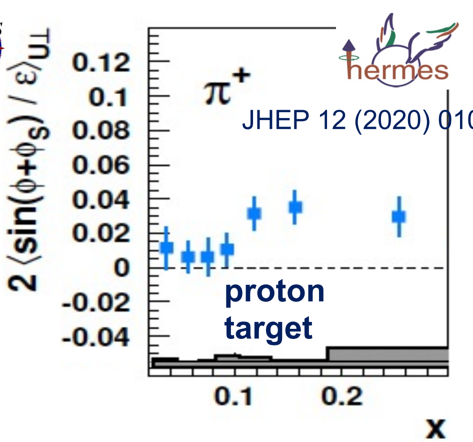
Found to be non-vanishing in a proton target by HERMES and COMPASS

PLB 717 (2012) 376





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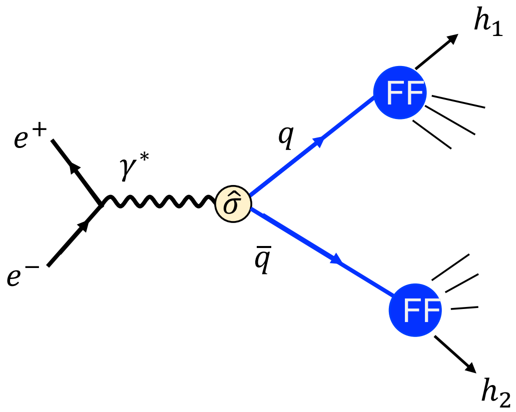

$$d\sigma^{e^+e^-\to h_1 h_h X} \propto 1 + a_{NN}\, A_{Coll}^{e^+e^-} \cos(\phi_1 + \phi_2)$$

$$A_{Coll}^{e^+e^-} \simeq \frac{\sum_q e_q^2 H_{1q}^{\perp h_1} \times H_{1\bar{q}}^{\perp h_2}}{\sum_q e_q^2\, D_{1q}^{h_1} \times D_{1\bar{q}}^{h_2}}$$

Many measurements by BELLE, BABAR, BESIII

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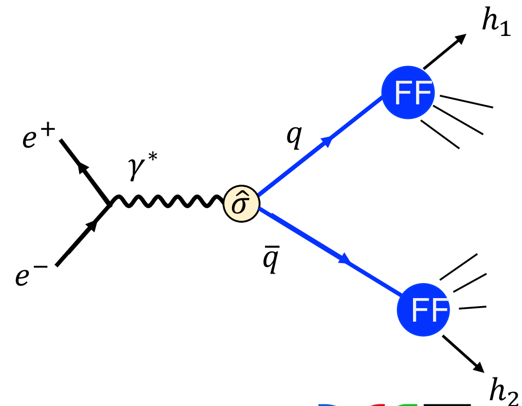

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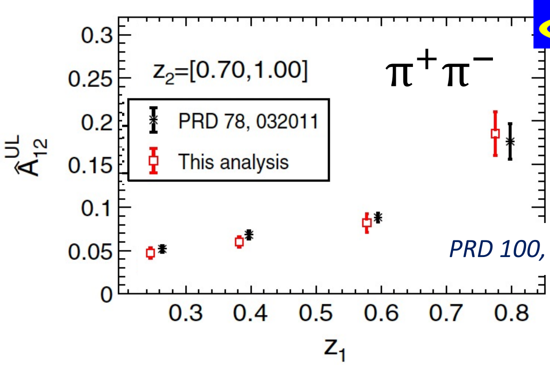
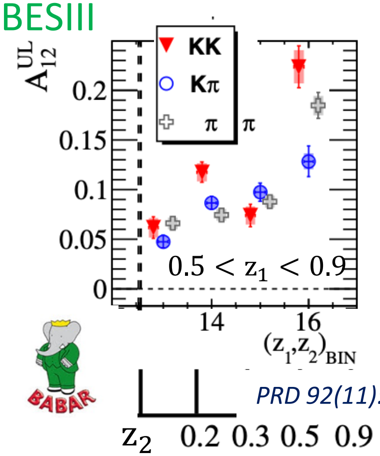
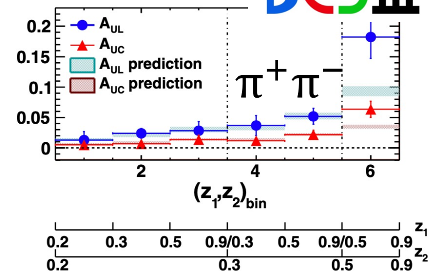


PRD 100, 092008 (2019)



PRD 92(11):111101, 2015



PRL 116(4):042001, 2016

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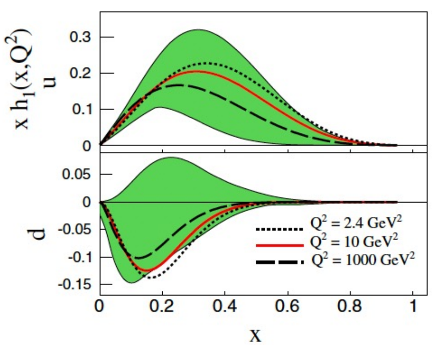
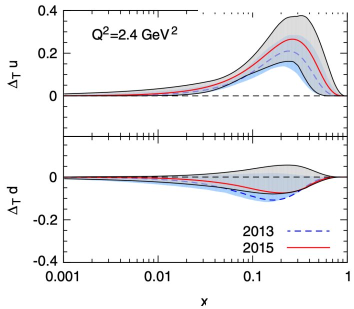


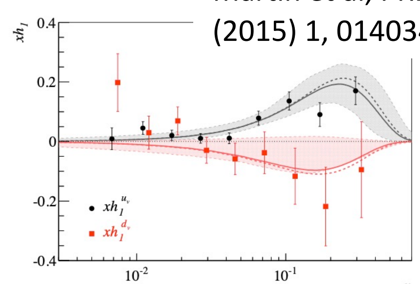
Kang et al., PRD 93 (1) (2016) 014009

Anselmino et al., PRD 92, 114023 (2015)



$h_1^q(x)$

Martin et al, PRD 91 (2015) 1, 014034



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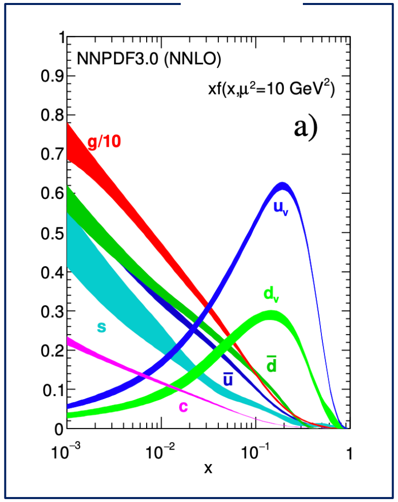

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- SIDIS and e^+e^- data used in global analyses aimed at extracting both transversity and the Collins FF

- A complete MCEG including these effects is lacking
  spin effects in perturbative processes already included

Richardson, JHEP 11, 029,
Richardson, Webster, EPJ, C (2020) 80:83,
K. Hamilton et al., JHEP 03 (2022) 193
..

- StringSpinner aims at filling this lack
  implements the string+$^3P_0$ model of polarized hadronization
  using the UserHooks of PYTHIA 8

Modeling spin-dependent hadronization: string+$^3P_0$

- Extension of the Lund string fragmentation model to include the quark spin

Artru, DSPIN-09, arXiv:1001.1061	2009	toy model
AK, Artru, Belghobsi, Bradamante, Martin, PRD 97, 074010 (2018)	2018	PS mesons
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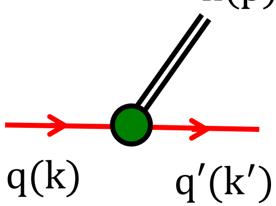
- Basic quantity – quark (and antiquark) splitting amplitude



$$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$$
$$Z_+ = p^+/k^+$$

$$T_{q',h,q} \propto \left[F^{\mathrm{Lund}}_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T)\right]^{1/2} \; [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \;\; \Gamma_{h,s_h}$$

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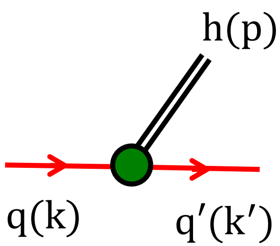

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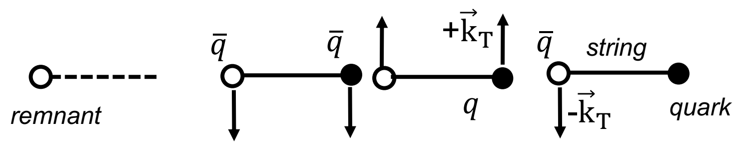
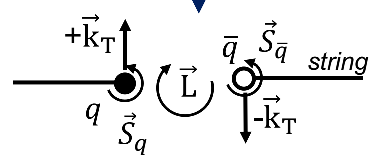
PS meson→ σ_z

Lund splitting fcnt.

$^3P_0$ wave function

coupling





VM with pol. $\vec{V}$

$$G_L V_L^* \mathbf{1} + G_T\, \vec{\sigma}_T \cdot \vec{V}_T^* \sigma_z$$

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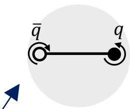

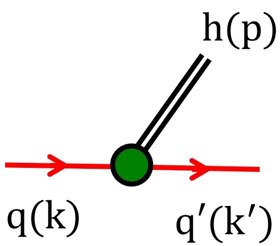

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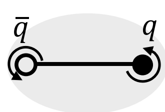
Lund splitting fcnt. — $^3P_0$ wave function — coupling

Free parameters: as in Lund Model

complex mass

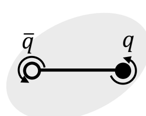
$\mathrm{Im}(\mu)$ responsible for transverse spin effects, e.g. Collins effect

$$f_L = \frac{|G_L/G_T|^2}{2+|G_L/G_T|^2}$$



fraction of long. pol. VMs

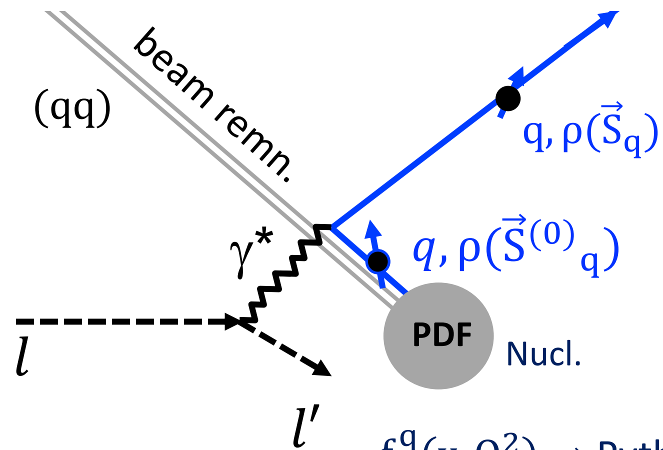
$$\theta_{LT} = \arg\left(\frac{G_L}{G_T}\right)$$



oblique pol.

Implementation of spin effects in PYTHIA hadronization for DIS

- Hard scattering event set up by Pythia
- Spin information is encoded in density matrices ρ



$f_1^q(x, Q^2) \rightarrow$ Pythia

$h_1^q(x) \rightarrow$parametriz.

$$\mathbf{S}_q^{(0)} = \frac{h_1^q}{f_1^q}\mathbf{S}_T^{\text{Nucl.}}$$

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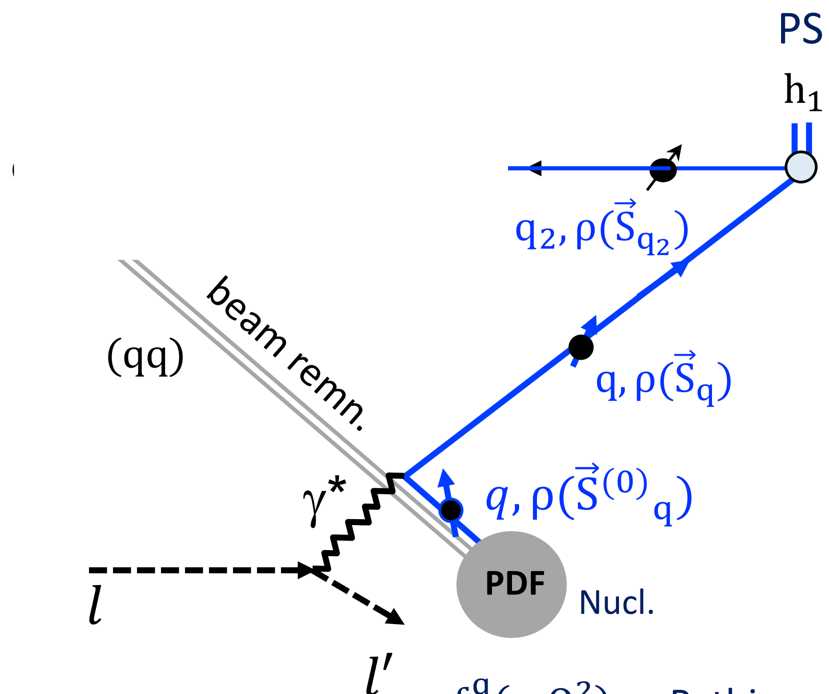
- Using the UserHooks class of PYTHIA
  - Each emitted hadron is accepted with probability according string+$^3P_0$

$$w_{q\to h+q'} = \frac{\mathrm{Tr}\, T_{q\to h+q'}\, \rho(q) T^\dagger_{q\to h+q'}}{\mathrm{Tr}\, T_{q\to h+q'}\, T^\dagger_{q\to h+q'}}$$

  - Propagate spin info to the next splitting using string+$^3P_0$



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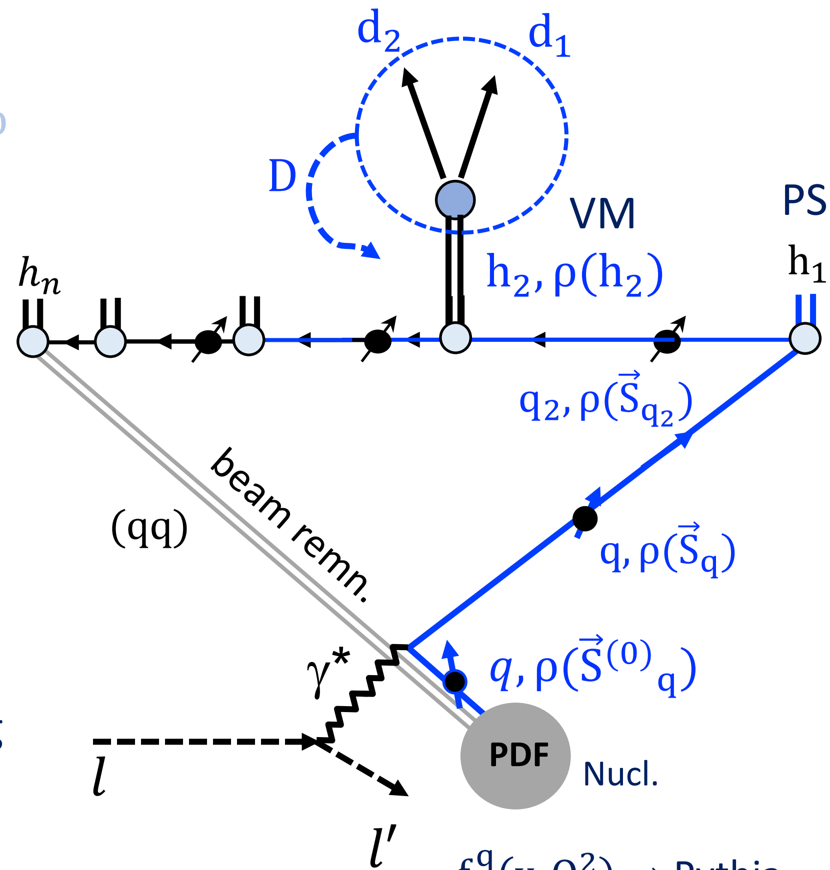


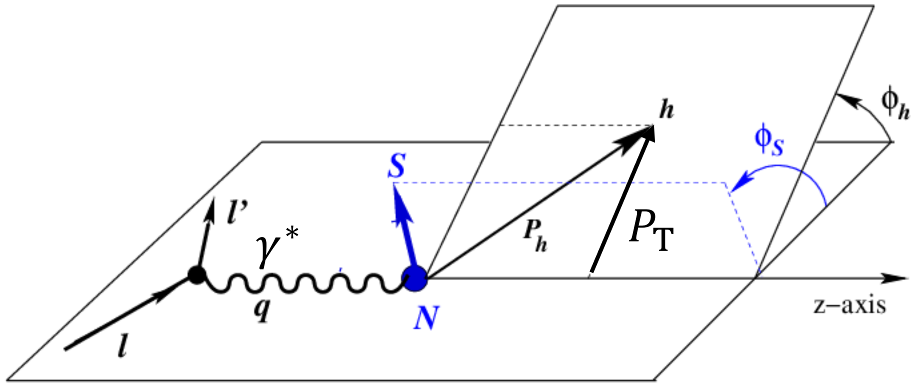
- Decays of PS mesons handled by Pythia
- Polarized decays of VMs handled externally using the UserHooks class and rules of string+$^3P_0$
  - i. Calculate ρ(VM)
  - ii. Perform polarized decay
  - iii. Return a decay matrix D [recipe of Collins '88, Knowles '88]
  - iv. Store decay products and pass them to Pythia later
  - v. Calculate ρ of next quark
- Iterate until exit condition of Pythia

$f_1^q(x, Q^2) \rightarrow$ Pythia

$h_1^q(x) \rightarrow$ parametriz.

$$\mathbf{S}_q^{(0)} = \frac{h_1^q}{f_1^q} \mathbf{S}_T^{\mathrm{Nucl.}}$$

Simulations of SIDIS with transversely polarized protons



Relevant hadronic variables

Bjorken x

fractional energy $z = P \cdot P_h / P \cdot q$

transverse momentum P_T

Results on

Collins asymmetries $A_{UT}^{\sin\phi_h+\phi_S-\pi}(x, z, P_T)$

Collins asymmetries for ρ^0 mesons $A_{UT}^{\sin\phi_{\rho^0}+\phi_S-\pi}(x, z, P_T)$

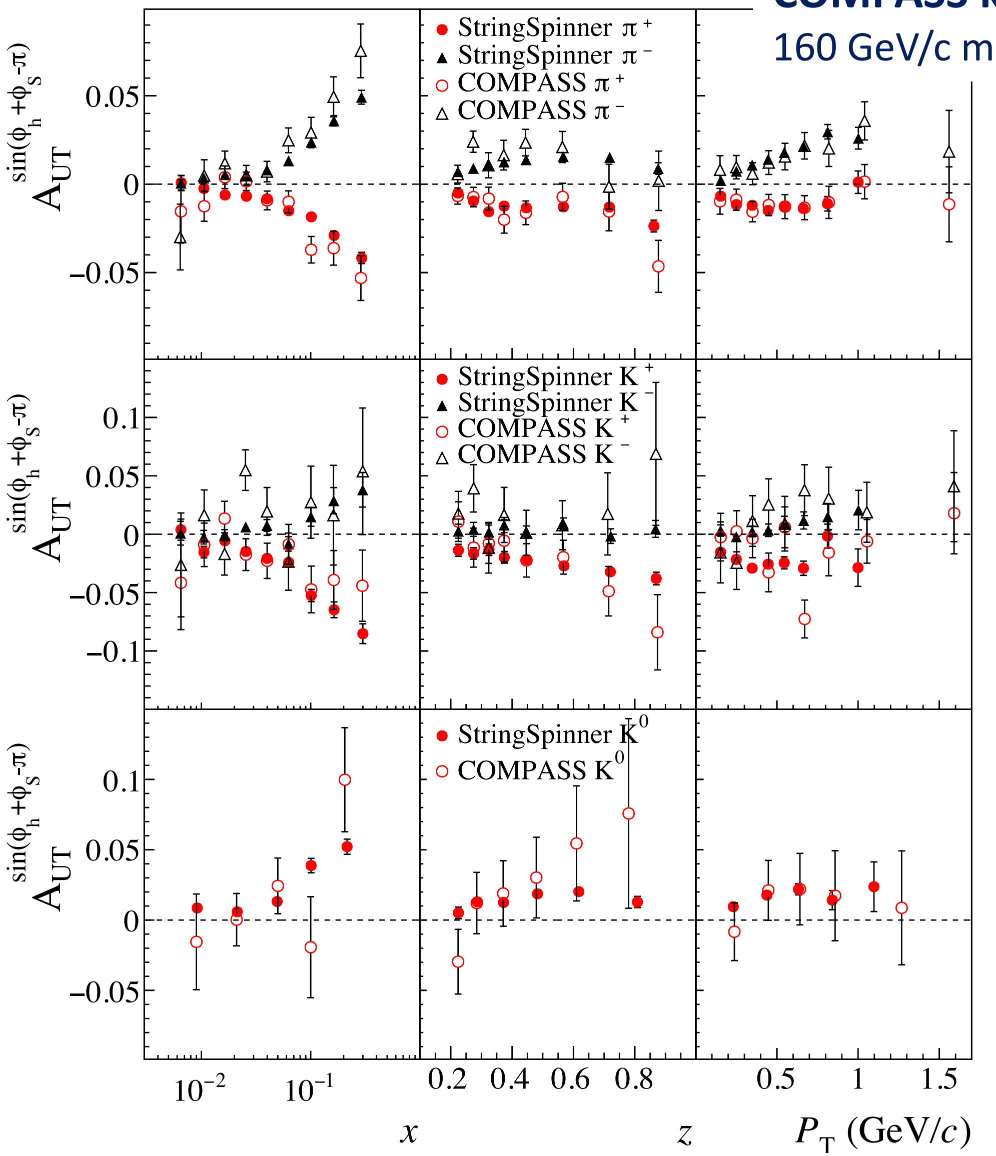
*more results in AK, L. Lönnblad, CPC **292** (2023) 108886*

Collins asymmetries for π and K

COMPASS kineamtics

160 GeV/c muons off protons

PLB 717 (2012) 376



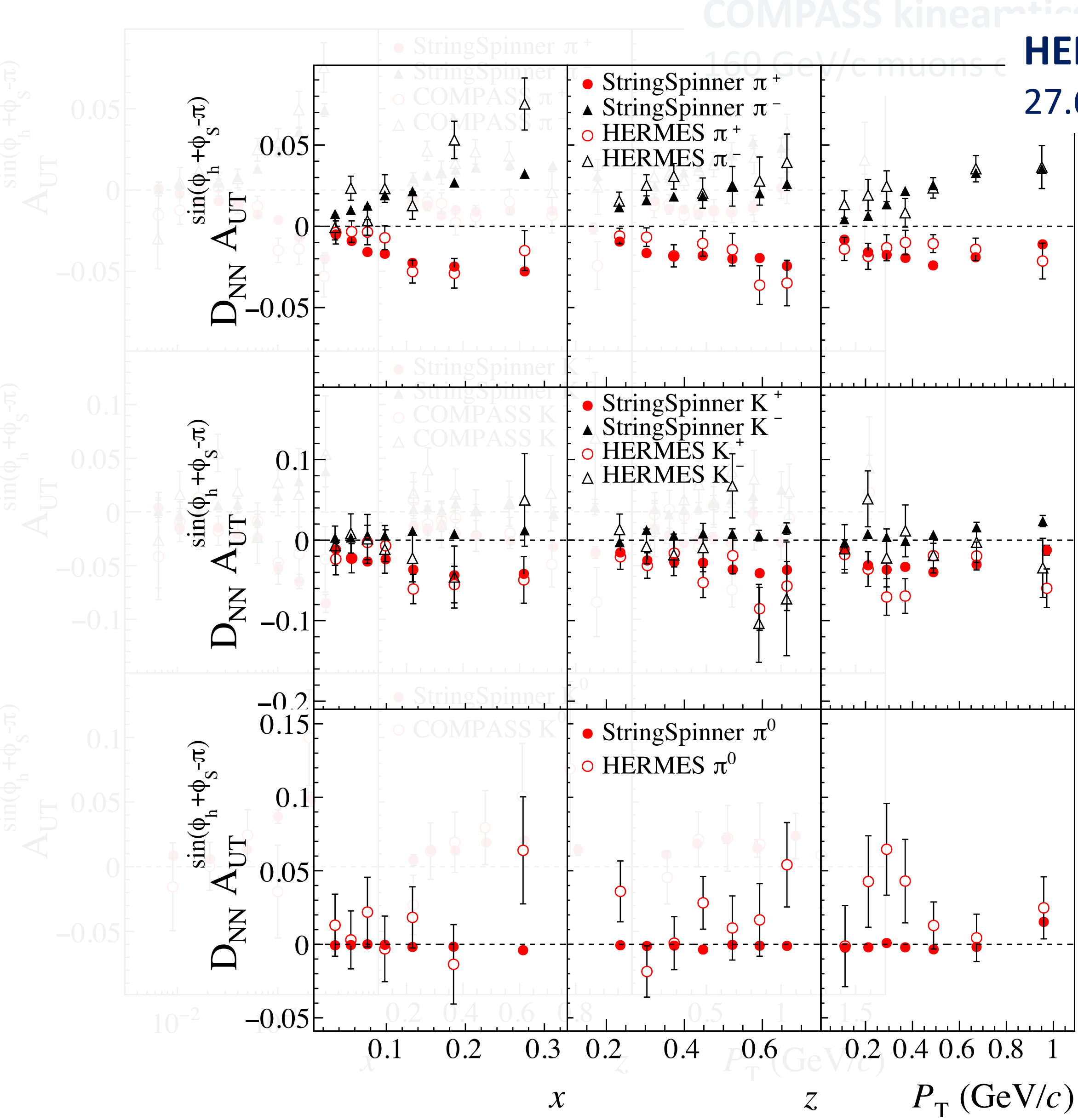
The model reproduces the main features of data!

Collins asymmetries for π and K

HERMES kineamtics

27.6 GeV/c e^+ off protons

JHEP 12 (2020) 010



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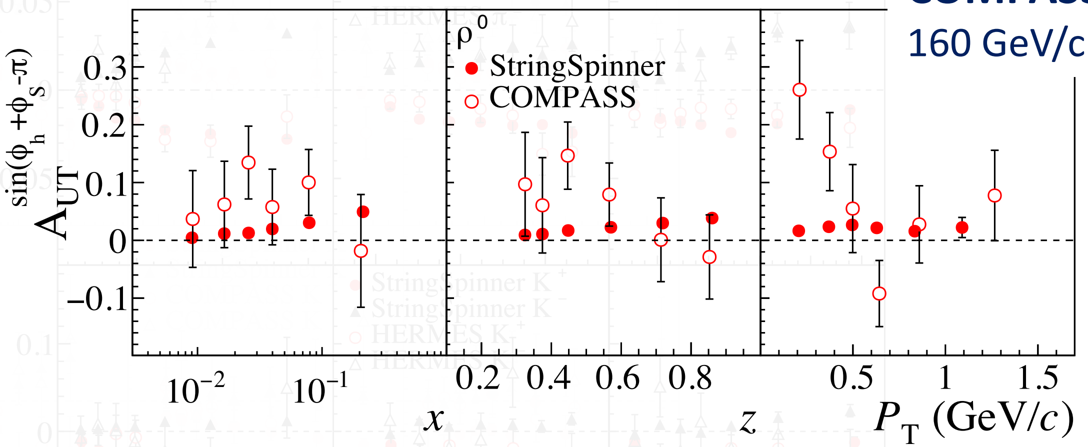
Collins asymmetries for ρ^0

COMPASS kineamtics

160 GeV/c muons off protons

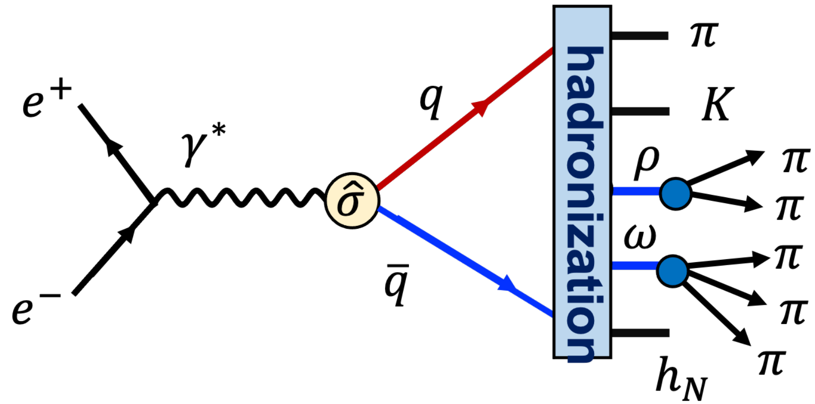
PLB 843 (2023) 137950



Implementation for DIS satisfactory

what about e^+e^- annihilation?

Recursive recipe for the simulation of $e^+e^- \to$ hadrons



Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

More complicated recipe than DIS, should account for

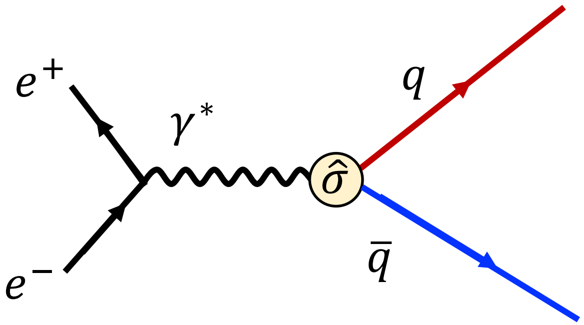
entanglement of q and $\bar{\text{q}}$

spin-correlations in string fragmentation

AK, X. Artru, [HADRON-2023] arXiv: 2311.03827

AK, X. Artru, in preparation

Recursive recipe for the simulation of $e^+e^- \to \text{hadrons}$



Steps:

1. Hard scattering

2. Joint spin density matrix

3. Hadron emission from q

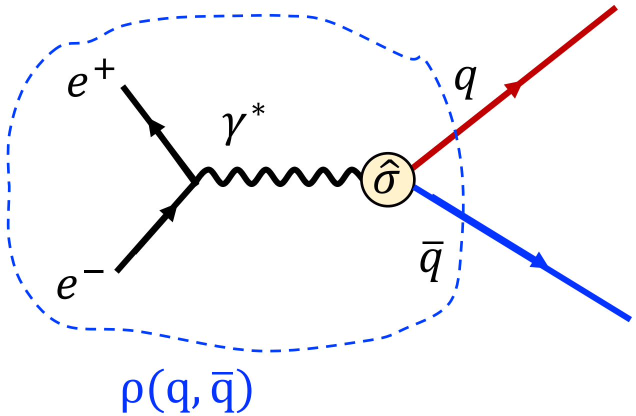
4. Update density matrix

5. Hadron emission from $\bar{q}$

6. Exit condition

- Set up the scattering $e^+e^- \to q\bar{q}$ in the c.m.s

Recursive recipe for the simulation of $e^+e^- \to \text{hadrons}$



Steps:

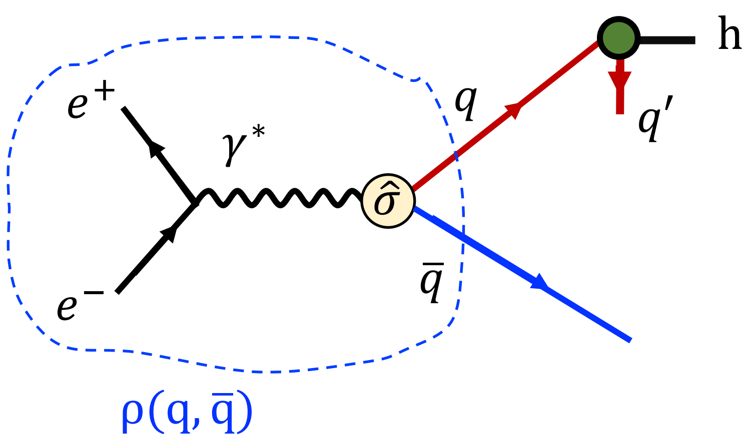
1. Hard scattering
2. **Joint spin density matrix**
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

- ❑ Set up the joint spin density matrix of the $q\bar{q}$ pair for γ^* exchange

$$\rho(q,\bar{q}) \propto 1_q \otimes 1_{\bar{q}} - \sigma_q^z \otimes \sigma_{\bar{q}}^z + \frac{\sin^2\theta}{1+\cos^2\theta}\left[\sigma_q^x \otimes \sigma_{\bar{q}}^x + \sigma_q^y \otimes \sigma_{\bar{q}}^y\right]$$

θ angle between e^- and q

Recursive recipe for the simulation of $e^+e^- \to$ hadrons



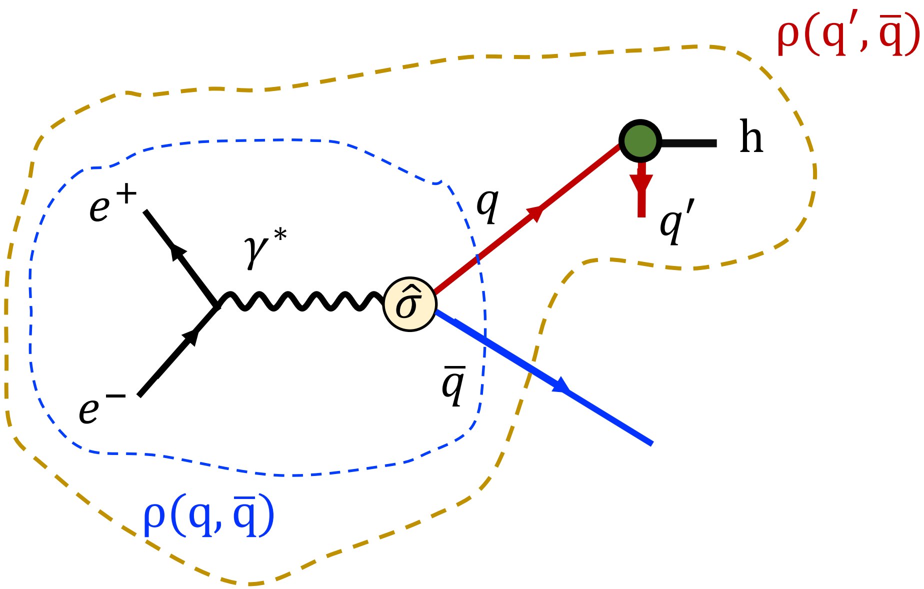
Steps:
1. Hard scattering
2. Joint spin density matrix
3. **Hadron emission from *q***
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

- Emit the first hadron with probability

$$dP(q \to h + q'; q\bar{q}) = \mathrm{Tr}_{q'\bar{q}} \mathbf{T}_{q',h,q}\, \rho(q,\bar{q})\, \mathbf{T}^{\dagger}_{q',h,q}$$
$$\mathbf{T}_{q',h,q} \equiv T_{q',h,q} \otimes 1_{\bar{q}}$$

(VM emission in the backup)

Recursive recipe for the simulation of $e^+e^- \to \text{hadrons}$



Steps:
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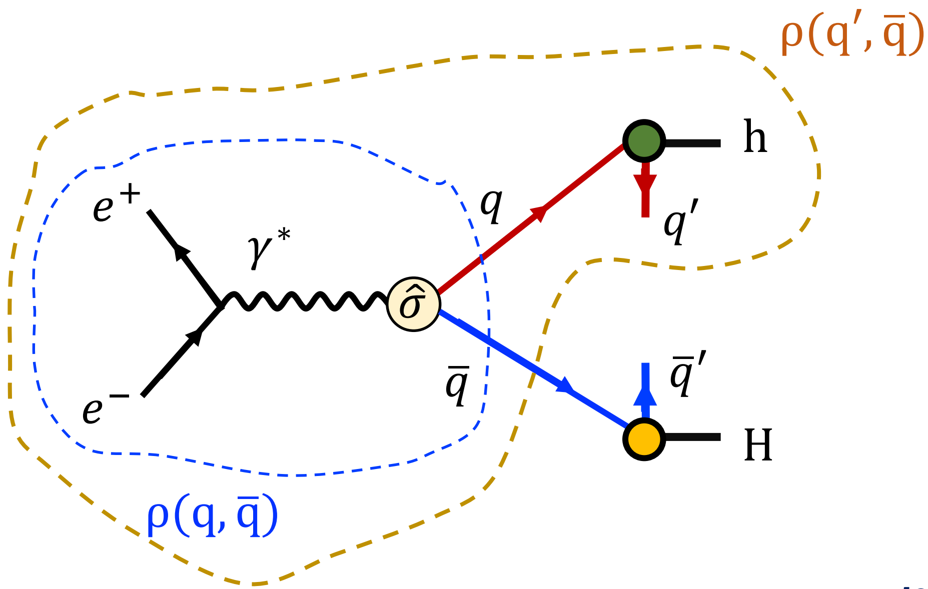
6. Exit condition

- ❑ Evaluate the spin density matrix $\rho(q'\bar{q})$

$$\rho(q', \bar{q}) = \mathbf{T}_{q',h,q} \;\; \rho(q, \bar{q}) \;\; \mathbf{T}^{\dagger}_{q',h,q}$$

includes the information on the emission of h

Recursive recipe for the simulation of $e^+e^- \to \text{hadrons}$



Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. **Hadron emission from $\bar{q}$**
6. Exit condition

- ❑ Emit a hadron from the $\bar{q}$ side with probability

[Collins NPB, 304:794–804, 1988, Knowles NPB, 310:571–588, 1988]

$$dP(\bar{q} \to H + \bar{q}'; q'\bar{q}) = \mathrm{Tr}_{q'\bar{q}'} \mathbf{T}_{\bar{q}',H,\bar{q}} \, \rho(q', \bar{q}) \, \mathbf{T}^{\dagger}_{\bar{q}',H,\bar{q}}$$

conditional probability of emitting H, having emitted h
→ correlations between the transverse momenta

- ❑ Iterate until exit condition

Simulations of e^+e^- with spin effects

- Now possible in Pythia 8.3 by using the StringSpinner package
  work still ongoing AK, L. Lönnblad, A. Martin

- Free parameters as in [AK and L. Lönnblad CPC 292 (2023) 108886],
  except $f_L = 0.33$
  and $\theta_{LT} = -\pi/6$
  by «eye» tuning, OK for both e^+e^- and SIDIS

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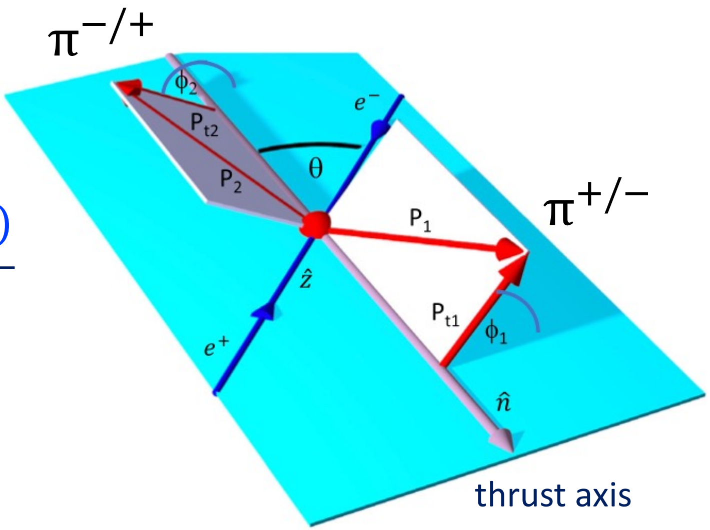
- Next slides → Collins asymmetries for back-to-back pions
  comparison with BELLE data (work ongoing for BaBar and BESIII)

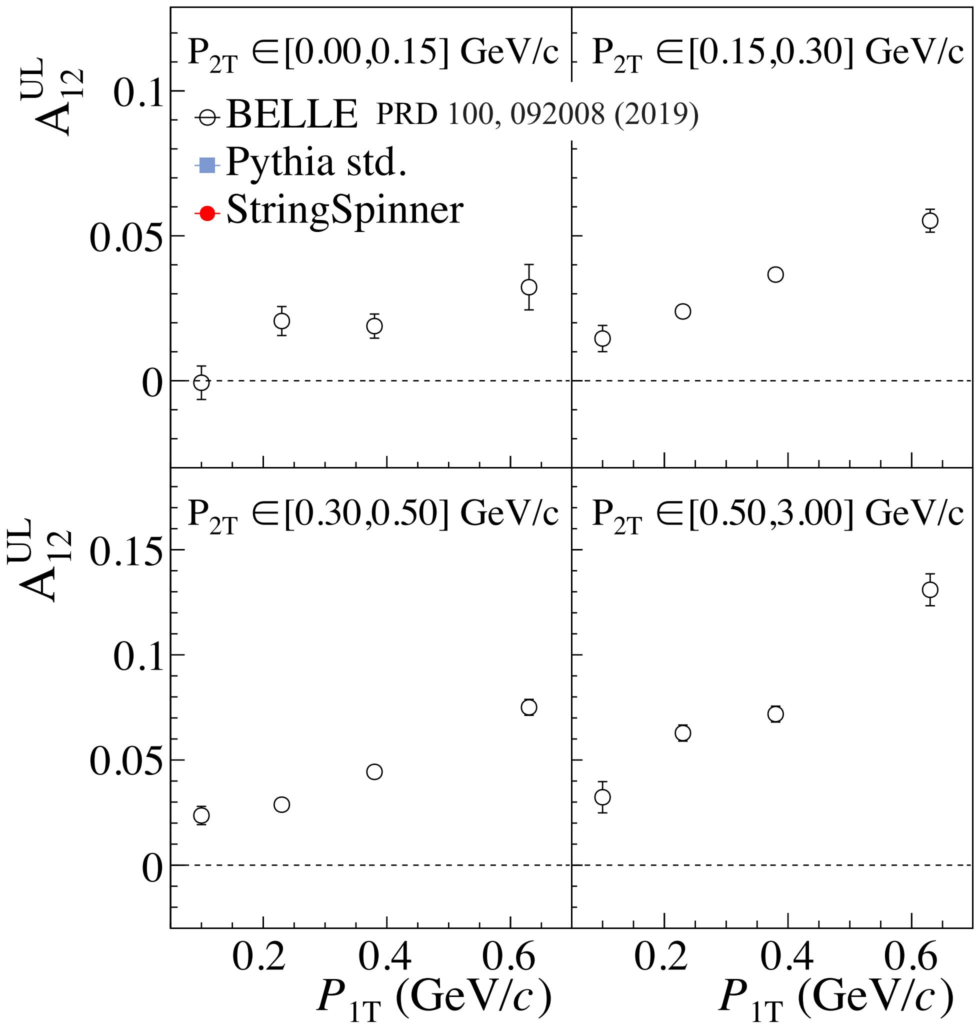
$$A_{12}^{UL} = \frac{\sum_q e_q^2 \; H_{1q}^{\perp h_1}(z_1, P_{T1}) H_{1\bar{q}}^{\perp h_2}(z_2, P_{T2})}{\sum_q e_q^2 \; D_{1q}^{h_1}(z_1, P_{T1}) D_{1\bar{q}}^{h_2}(z_2, P_{T2})}$$

$$z_i = \frac{2E_{h_i}}{\sqrt{s}}$$



A_{12}^{UL} asymmetry for back-to-back $\pi^{\pm} - \pi^{\mp}$

$P_{T1} \times P_{T2}$ - dependence



Asymmetries w.r.t thrust axis (not $q\bar{q}$ axis)

$T > 0.8$

$z > 0.2, P_T < 3.0$ GeV/c

$\alpha_O < 0.3$

A_{12}^{UL} asymmetry for back-to-back $\pi^{\pm} - \pi^{\mp}$

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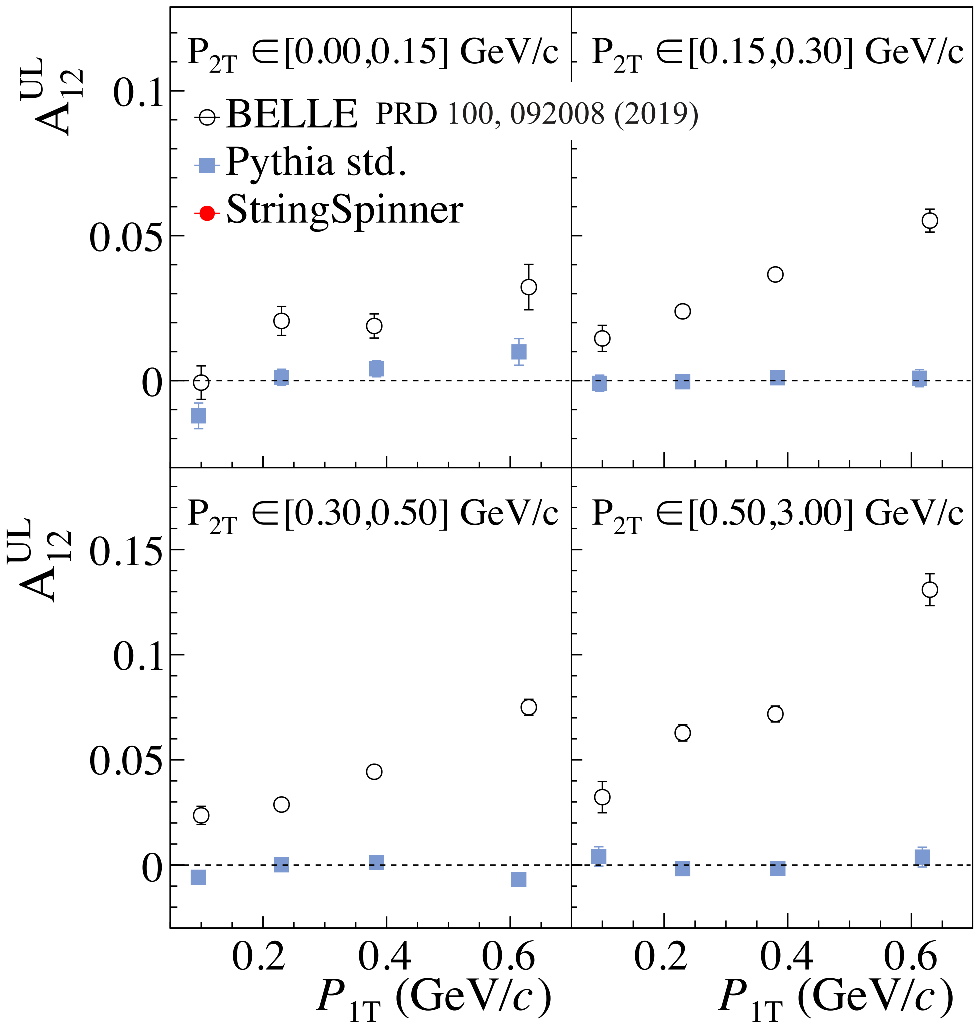

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Pythia results consistent with zero

A_{12}^{UL} asymmetry for back-to-back $\pi^{\pm} - \pi^{\mp}$

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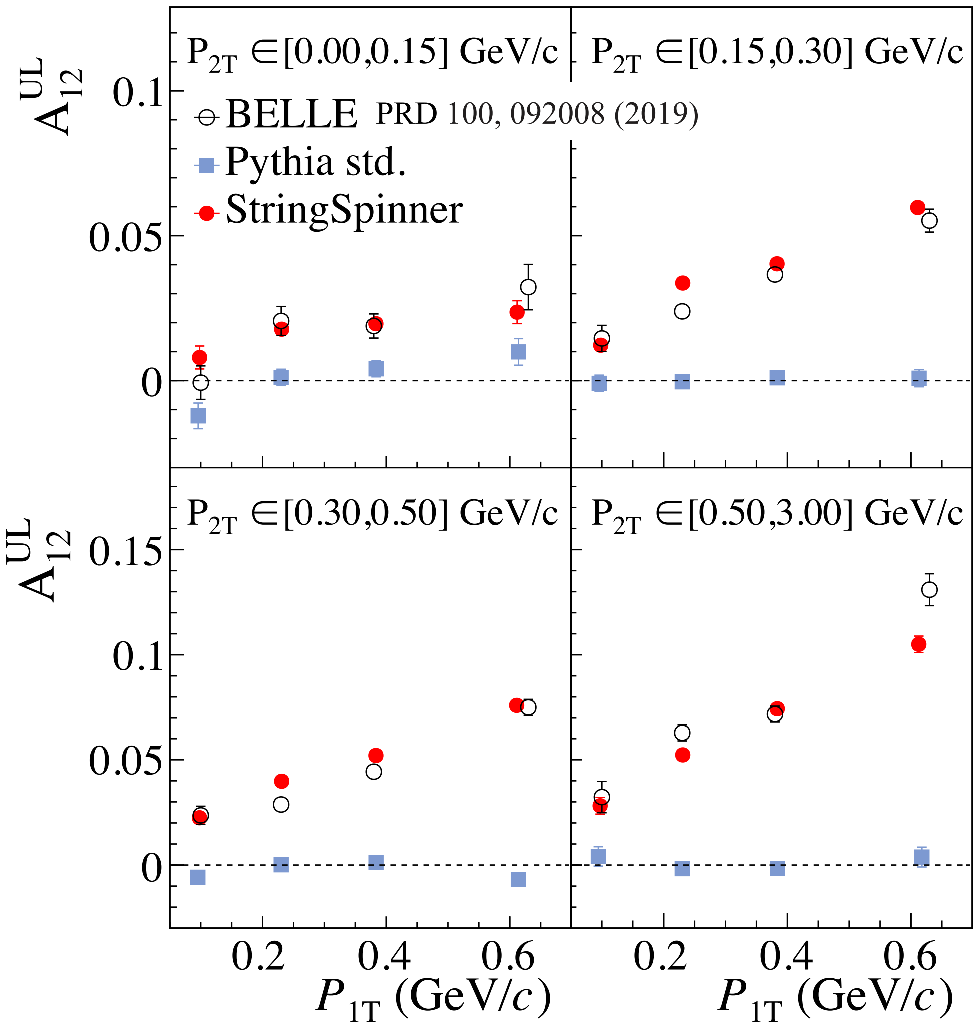


Asymmetries w.r.t thrust axis (not $q\bar{q}$ axis)

$T > 0.8$

$z > 0.2, P_T < 3.0$ GeV/c

$\alpha_O < 0.3$

Pythia results consistent with zero

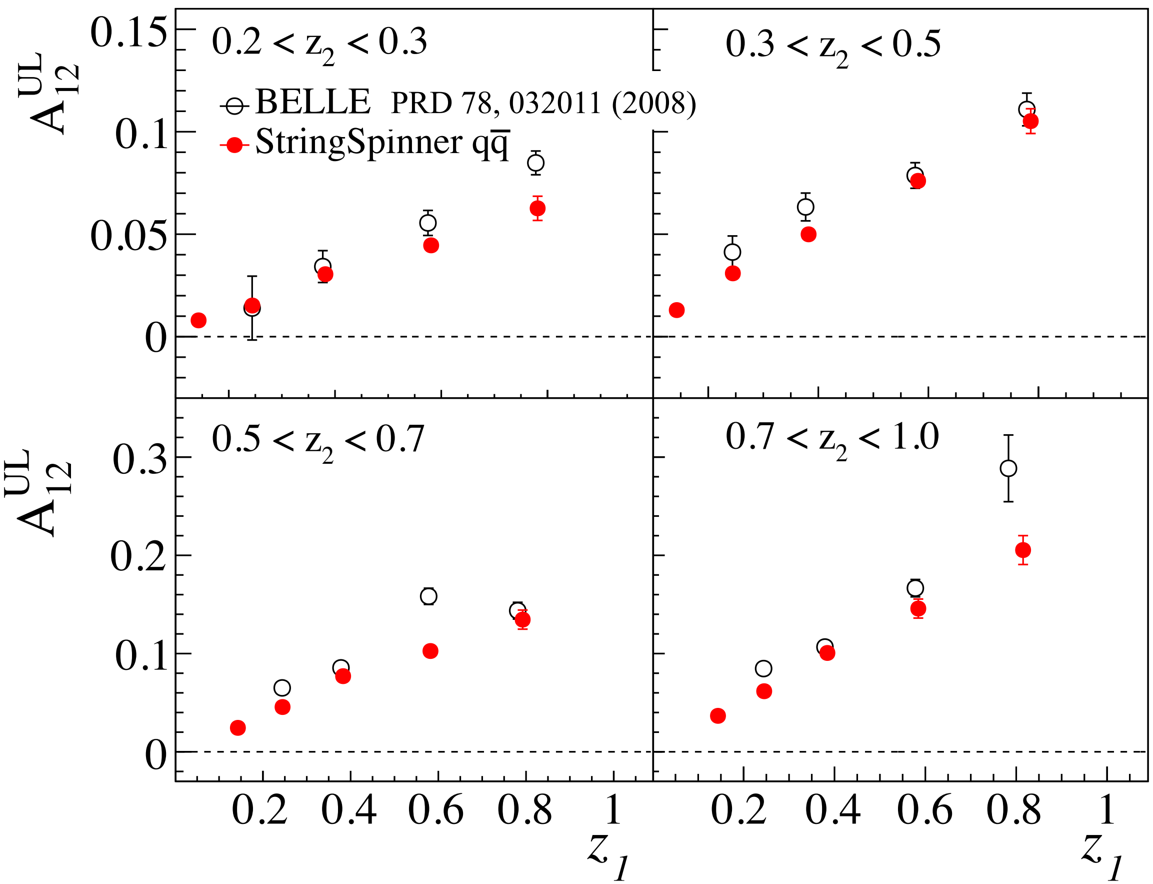
Satisfactory description from StringSpinner!

$z_1 \times z_2$ dependence OK, except few bins at large z → backup

Asymmetries for $\pi^0 - \pi^{\mp}$ and $\eta - \pi^{\mp}$ OK → backup

A_{12}^{UL} asymmetry for back-to-back $\pi^{\pm} - \pi^{\mp}$ w.r.t $q\bar{q}$ axis

$z_1 \times z_2$ - dependence



Belle asymmetries measured using the thrust axis, then rescaled to $q\bar{q}$ axis

Integrated over P_T

$T > 0.8, z > 0.2$

StringSpinner gives a satisfactory description!

Conclusions

- StringSpinner implements the quark spin effects in Pythia string fragmentation using the string+$^3P_0$ model
  - spin effects restricted to the productions of pseudoscalar and vector mesons

- Applied to DIS (public) and being developed for e^+e^- annihilation
  - @ LO, promising description of transverse-spin asymmetry data
    - more phenomenological studies ongoing: comparisons with BaBar and BESIII, calculation of the Artru-Collins asymmetries …

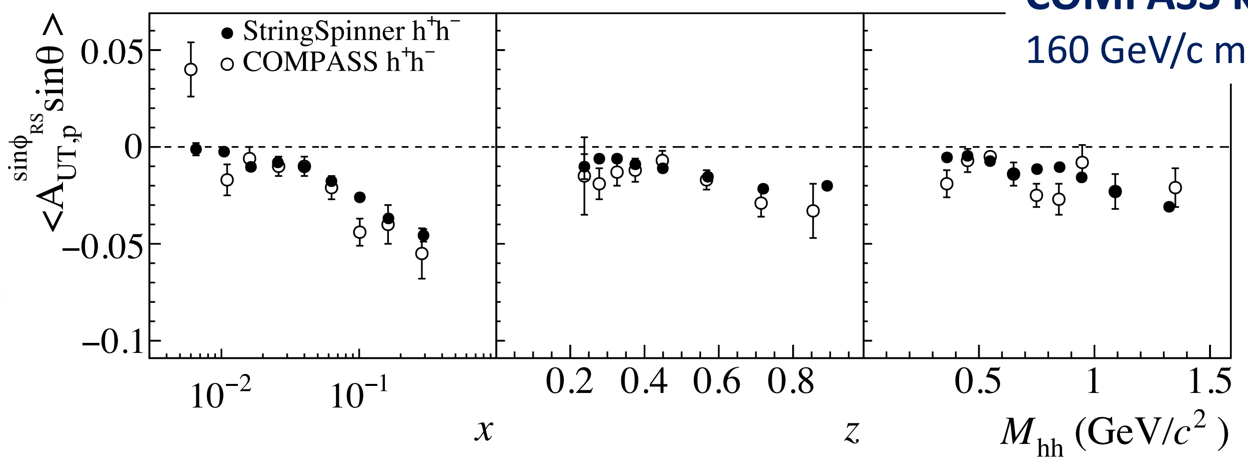
- More developments foreseen for the string+$^3P_0$ model
  - gluon emission (needed to connect with a parton shower)
  - baryon production
  - application to other processes (pp)

  and implementation in Pythia!

StringSpinner is still at an early stage, and many opportunities for exciting developments are possible!

Backup

Dihadron asymmetry for h^+h^-



COMPASS kineamtics

160 GeV/c muons off protons

Dihadron asymmetry ~ $h_1^q \times H_{1q}^{\sphericalangle hh}$

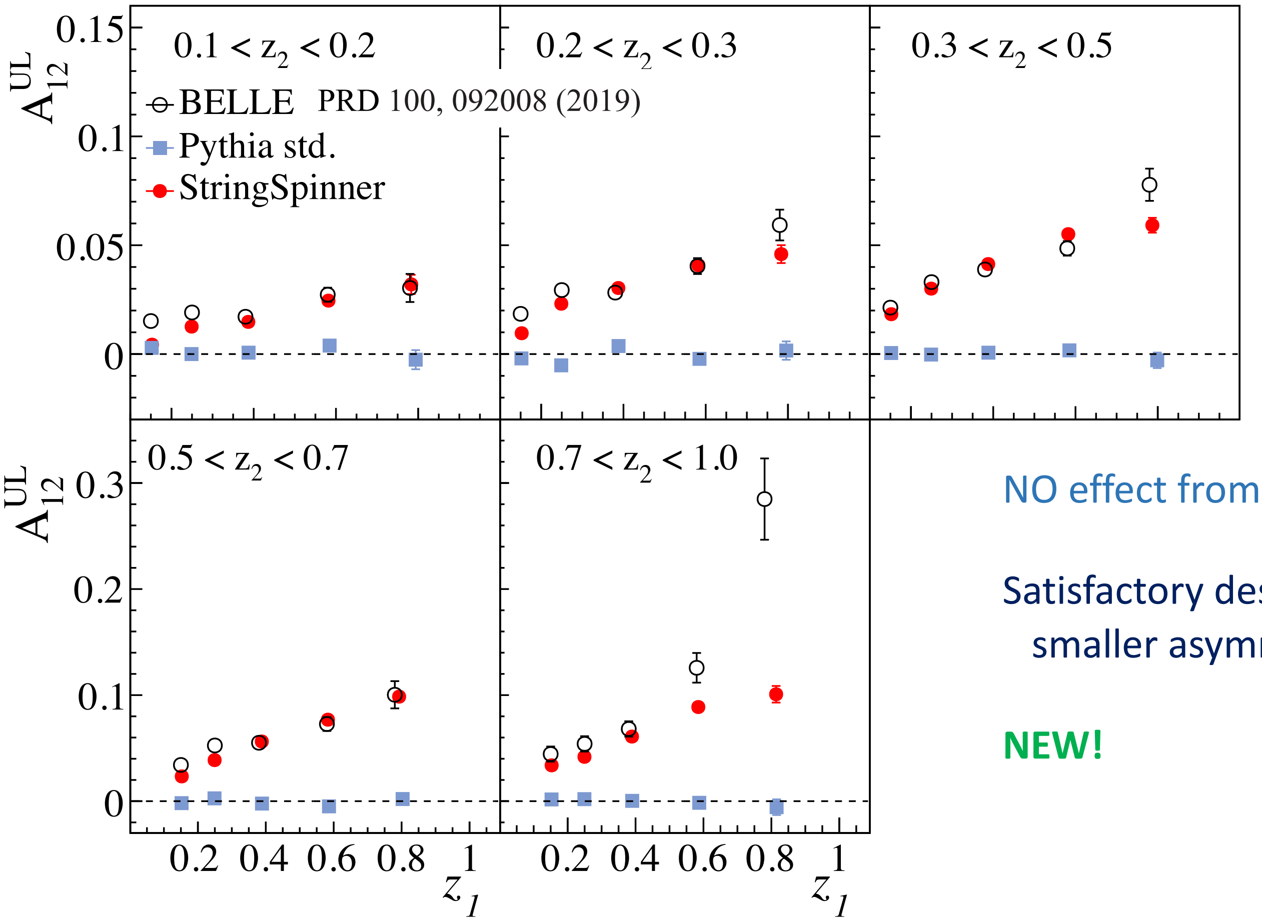
«Interference FF» (IFF)

another channel to access transversity..

The model reproduces the main features of data!

A_{12}^{UL} asymmetry for back-to-back $\pi^{\pm} - \pi^{\mp}$

$z_1 \times z_2$ - dependence



Asymmetries w.r.t thrust axis (not $q\bar{q}$ axis)

$T > 0.8$

$z > 0.1, P_T < 3.0$ GeV/c

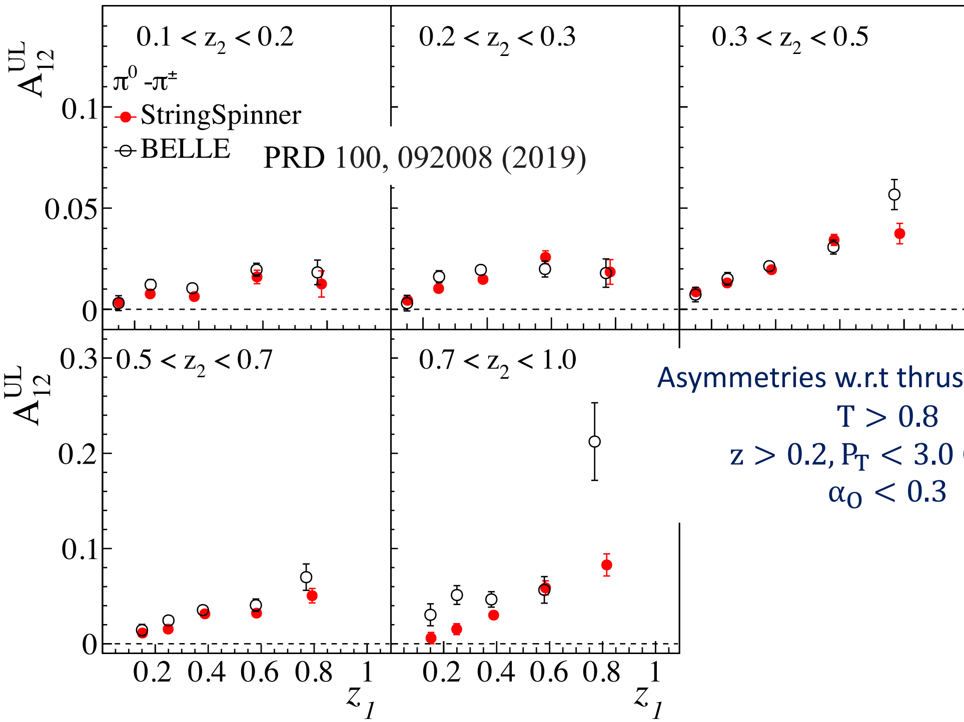
$\alpha_O < 0.3$

NO effect from Pythia

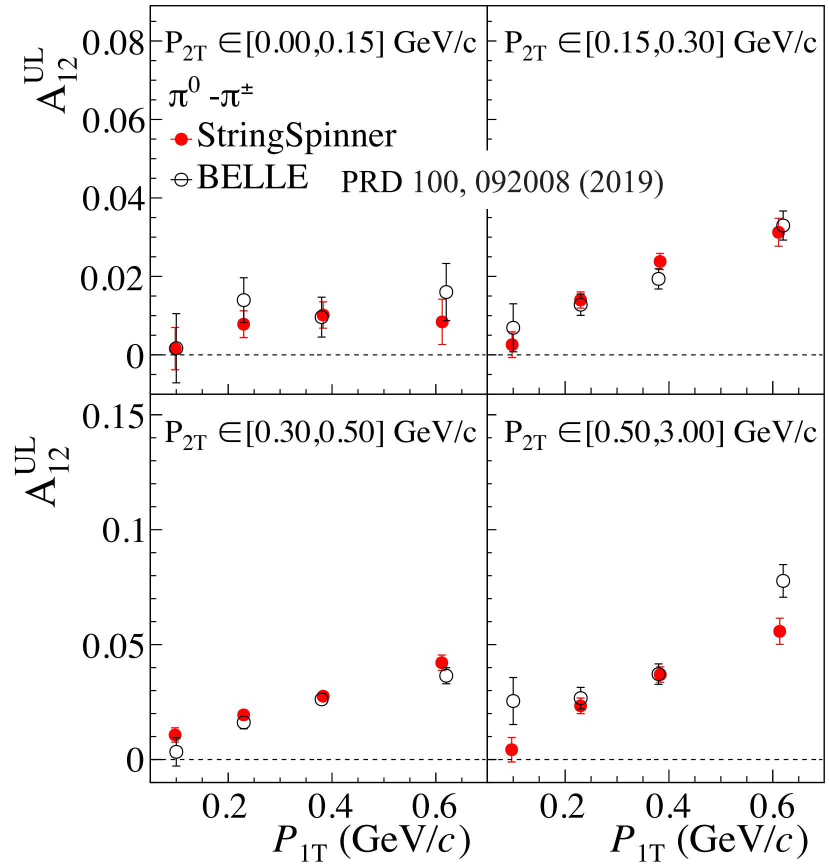
Satisfactory description from StringSpinner
smaller asymmetries at large z

NEW!

A_{12}^{UL} asymmetry for back-to-back $\pi^0 - \pi^{\mp}$



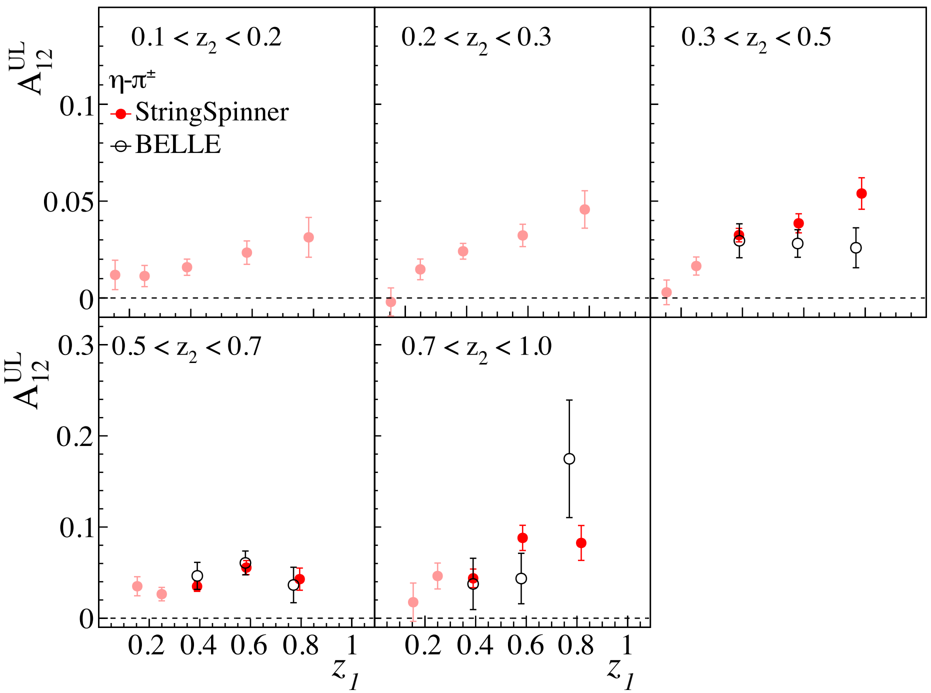
Asymmetries w.r.t thrust axis
$T > 0.8$
$z > 0.2, P_T < 3.0$ GeV/c
$\alpha_O < 0.3$

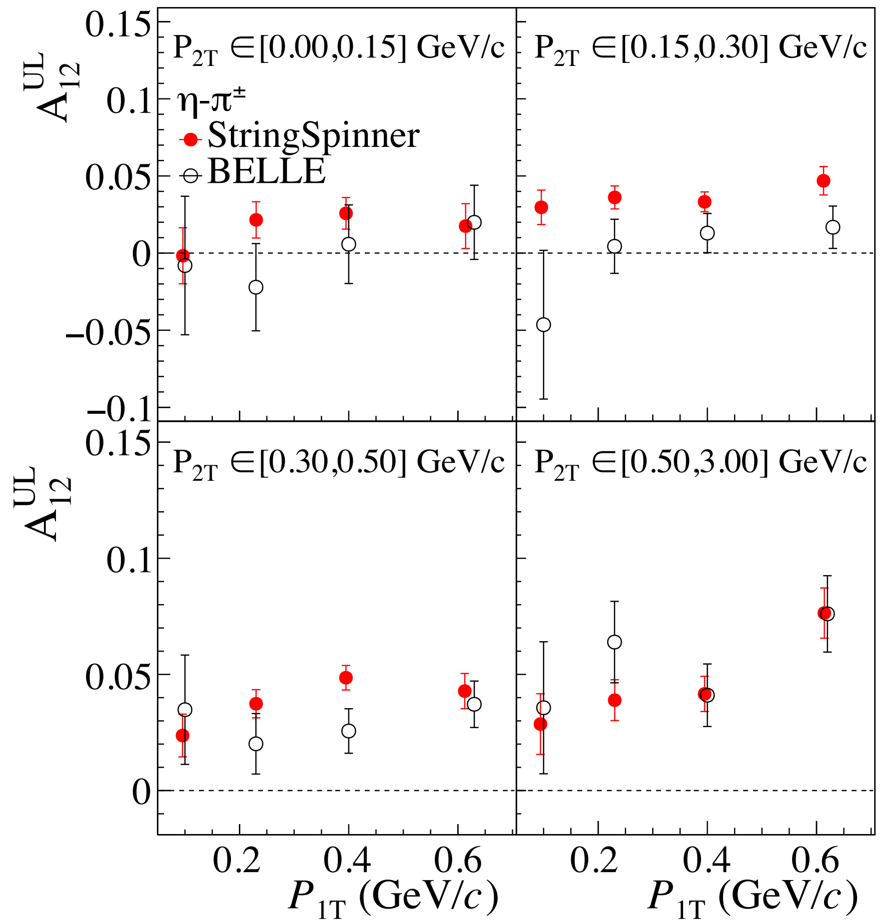


Satisfactory description
simulated asymmetries at large z lower

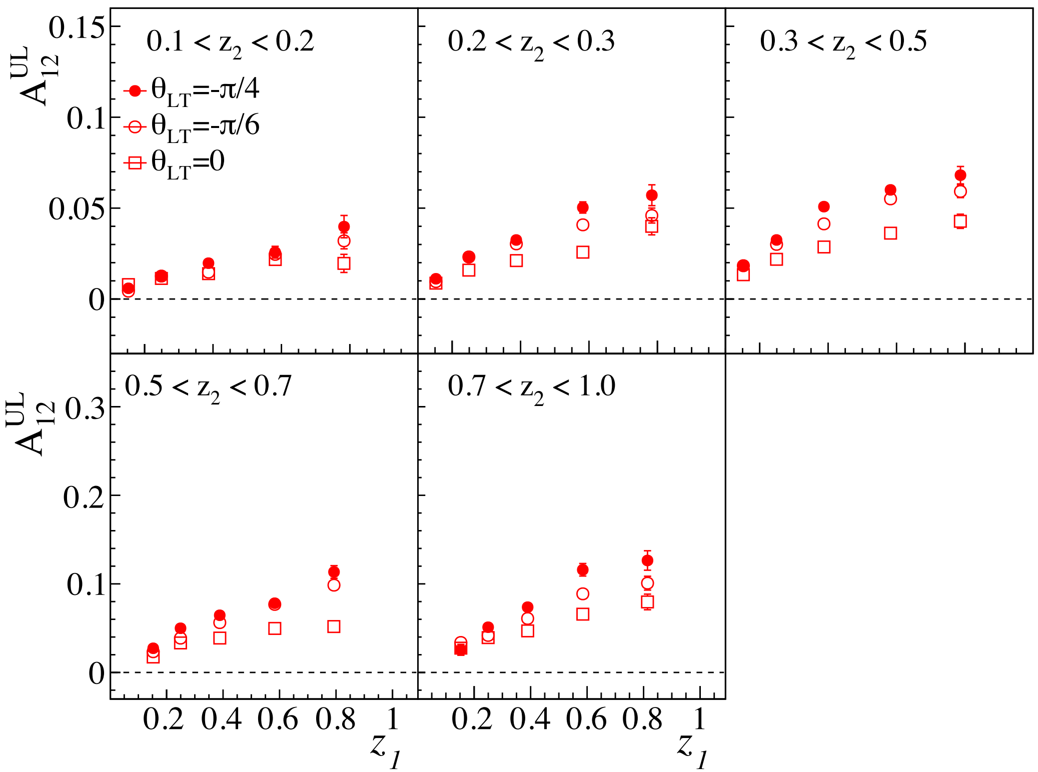
Asymmetries measured w.r.t the thrust axis
difficult to describe

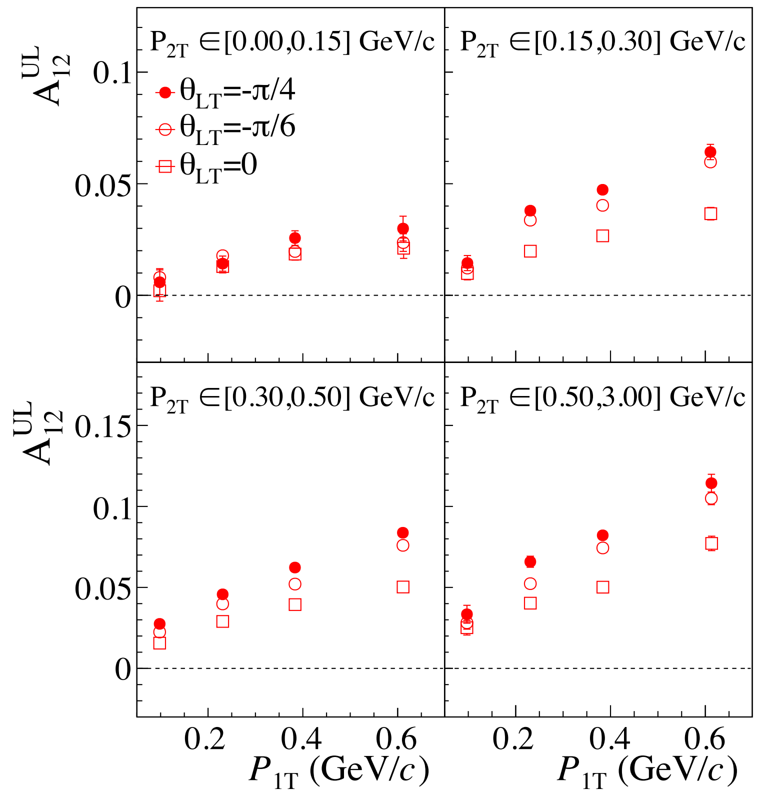
A_{12}^{UL} asymmetry for $\eta - \pi^{\pm}$





Sensitivity of asymmetries to free parameters





Asymmetries evaluated using the thrust axis

The oblique polarization θ_{LT} is varied, while all other parameters fixed

Relevant free parameters for string fragmentation used in simulations

(see AK, L. Lönnblad, arXiv: 2305.05058)

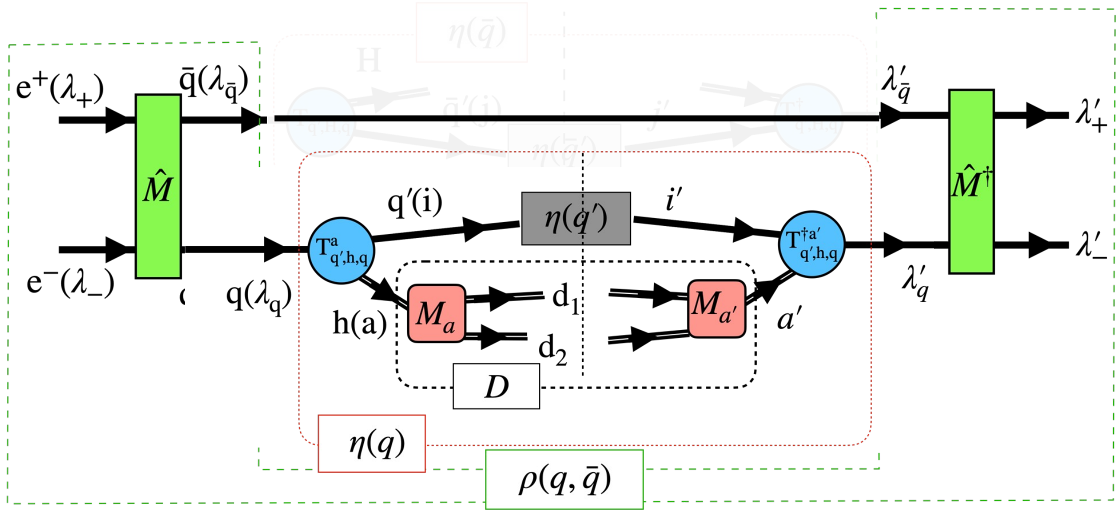
Pythia parameters

StringZ:aLund	default
StringZ:bLund	default
StringPT:sigma	default
StringPT:enhancedFraction	0.0
StringPT:enhancedWidth	0.0 GeV/c

String+$^3P_0$ parameters

$\mathrm{Re}(\mu)$	0.42 $\mathrm{GeV/c^2}$
$\mathrm{Im}(\mu)$	0.76 $\mathrm{GeV/c^2}$
f_L	0.33
θ_{LT}	$-\pi/6$

The recursive recipe for simulating e^+e^- annihilation: VM emission



For a vector meson h=VM

$$\rightarrow \eta(q) = \mathbf{T}^{a'\dagger}_{q',h=VM,q}\,\eta(q')\,\mathbf{T}^{a}_{q',h=VM,q} D_{a'a},\ \eta(q') = 1_{q'} \text{ and } \eta(\bar{q}) = 1_{\bar{q}}$$

Steps:

i) Emission probability density (summing over decay information, i.e. $D_{a'a} = \delta_{a'a}$)

$$\frac{dP(q \rightarrow h = VM + q'; q\bar{q})}{dM^2 dZ_+ Z_+^{-1} d^2p_T} = \mathrm{Tr}_{q'\bar{q}}\, \mathbf{T}^{a}_{q',h,q}\ \rho(q,\bar{q})\ \mathbf{T}^{a\dagger}_{q',h,q} = F_{q',h,q}(M^2, Z_+, p_T; k_T, C^{q\bar{q}})$$

ii) Calculate the spin density matrix of h=VM, and decay the meson

$$\rho_{aa'}(h) = \mathrm{Tr}_{q'\bar{q}}\, \mathbf{T}^{a}_{q',h,q}\, \rho(q,\bar{q})\, \mathbf{T}^{a'\dagger}_{q',h,q}$$

iii) Decay the meson $p \rightarrow p_1\, p_2..$

$$dN(p_1, p_2..)/d\Omega \propto M^{a}_{dec.}(p \rightarrow p_1 p_2..)\ \rho_{aa'}(h) M^{\dagger a'}_{dec.}(p \rightarrow p_1 p_2..)$$

iv) Build the decay matrix $D_{a'a}(p_1, p_2, ..) = M^{\dagger a'}_{dec.}(p \rightarrow p_1 p_2..)\ M^{a}_{dec.}(p \rightarrow p_1 p_2..)$