

# Fitting of a Deep Generative Hadronization Model

## Andrzej Siódmok

### Towards a Deep Learning Model for Hadronization

---

Aishik Ghosh,<sup>a,b</sup> Xiangyang Ju,<sup>b</sup> Benjamin Nachman,<sup>b,c</sup> and Andrzej Siódmok<sup>d</sup>

<sup>a</sup>Department of Physics and Astronomy, University of California, Irvine, CA 92697, USA

<sup>b</sup>Physics Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

<sup>c</sup>Berkeley Institute for Data Science, University of California, Berkeley, CA 94720, USA

<sup>d</sup>Jagiellonian University, Krakow, Poland

2203.12660

### Fitting a Deep Generative Hadronization Model

---

Jay Chan,<sup>a,b</sup> Xiangyang Ju,<sup>b</sup> Adam Kania,<sup>e</sup> Benjamin Nachman,<sup>b,c</sup> Vishnu Sangli,<sup>d,b</sup> and Andrzej Siódmok<sup>d</sup>

<sup>a</sup>Department of Physics, University of Wisconsin-Madison, Madison, WI 53706, USA

<sup>b</sup>Physics Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

<sup>c</sup>Berkeley Institute for Data Science, University of California, Berkeley, CA 94720, USA

<sup>d</sup>Department of Physics, University of California, Berkeley, CA 94720, USA

<sup>e</sup>Jagiellonian University, Krakow, Poland

2305.17169

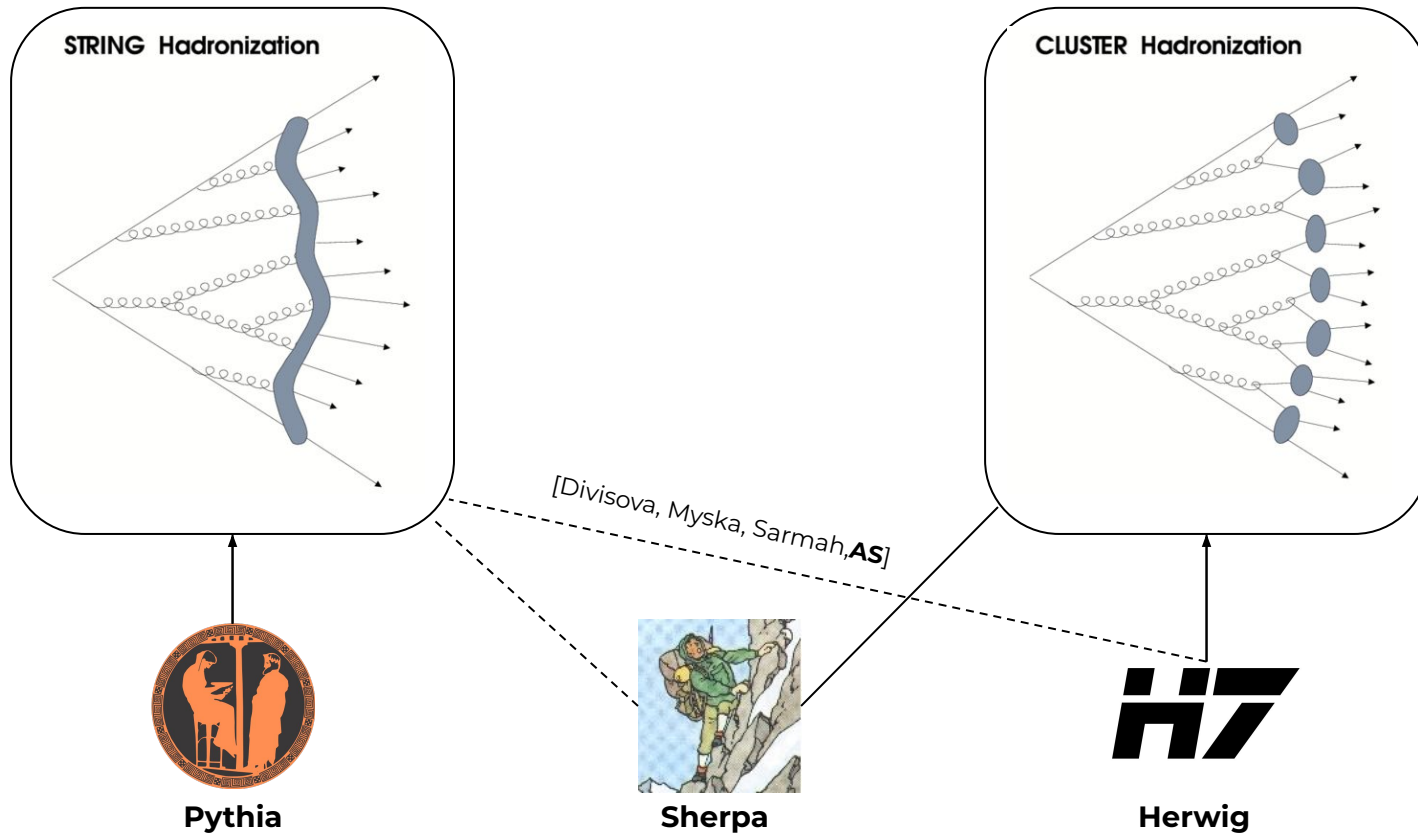


# Motivation - Hadronization

## Hadronization:

→ Increased control of perturbative corrections ⇒ more often LHC measurements are limited by non-perturbative components, such as hadronization.

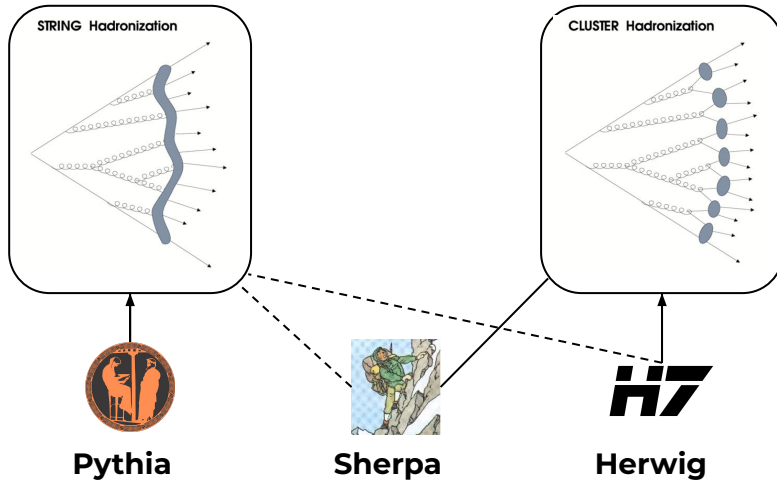
- $W$  mass measurement using a new method [Freytsis et al. JHEP 1902 (2019) 003]
- Extraction of the strong coupling in [M. Johnson, D. Maître, Phys.Rev. D97 (2018) no.5]
- Top mass [S. Argyropoulos, T. Sjöstrand, JHEP 1411 (2014) 043]
- ...



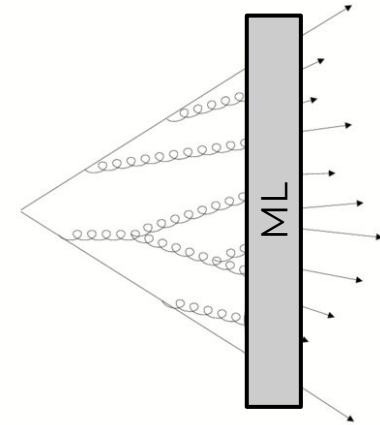
# Hadronization models

## Hadronization:

Early 1980's



Early 2020's  
(lot of progress in ML)

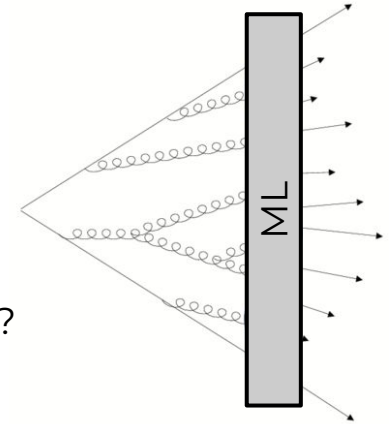


Idea of using Machine Learning (ML) for hadronization.

# Motivation for Machine learning hadronization

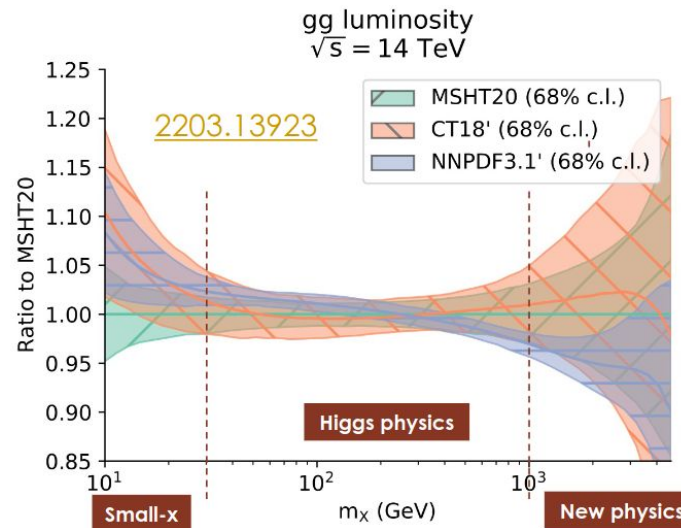
## Idea of using Machine Learning (ML) for hadronization.

- Existing hadronization models are highly parameterized functions.
- Hadronization is a fitting problem
  - Can ML hadronization be more flexible to fit the data?
  - Can ML hadronization extract more information from the data?  
[can accommodate unbinned and high-dimensional inputs]



## NNPDF

NNPDF used successfully ML to nonperturbative Parton Density Functions (PDF). Hadronization is closely related to fragmentation functions (FF) which were considered the counterpart of PDFs.



# Recent progress: Machine learning hadronization

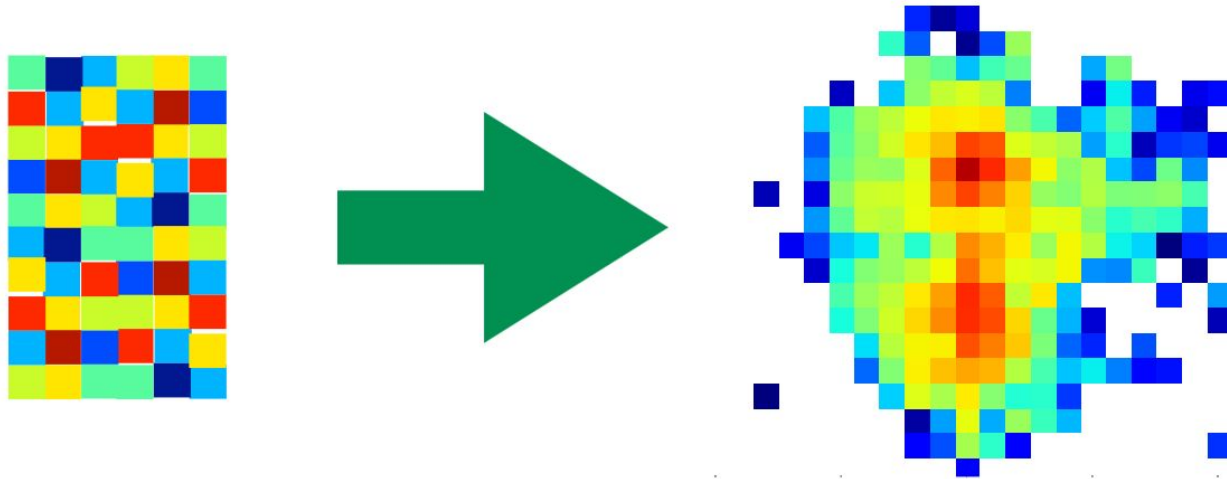
First steps for ML hadronization:

- HADML - [A. Ghosh, Xi. Ju, B. Nachman **AS**, *Phys.Rev.D* 106 (2022) 9]
- MLhad - [P. Ilten, T. Menzo, A. Youssef and J. Zupan, *SciPost Phys.* 14, 027 (2023)]

	MLhad	HADML
Deep generative model:	Variational Autoencoder	Generative Adversarial Networks
Trained on:	String model	Cluster model
Recent progress:	<p><i>“Reweighting Monte Carlo Predictions and Automated Fragmentation Variations in Pythia 8”</i></p> <p>[Bierlich, Ilten, Menzo, Mrenna, Szewc, Wilkinson, Youssef, Zupan, 2308.13459]</p> <p>(see Christian’s talk)</p>	<p><i>“Fitting a Deep Generative Hadronization Model”</i></p> <p>[J. Chan, X. Ju, A. Kania, B. Nachman, V. Sangli and <b>AS</b>, <i>JHEP</i> 09 (2023) 084]</p>

# What is a deep generative model?

A **generator** is nothing other than a function that maps random numbers to structure.

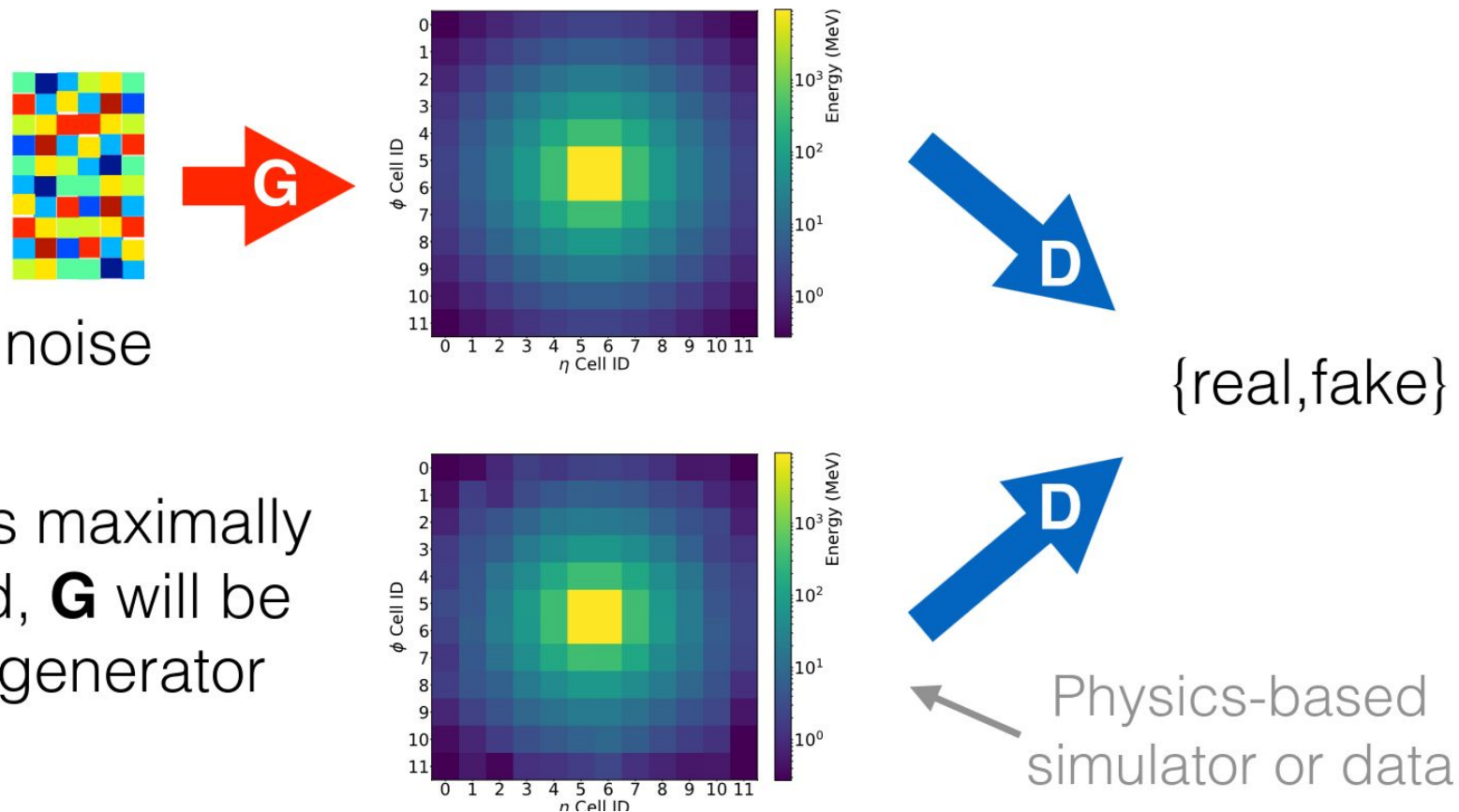


Deep generative models: the map is a deep neural network.

# Our tool of choice: GANs

[Goodfellow et al. "Generative adversarial nets". arxiv:1406.2661]

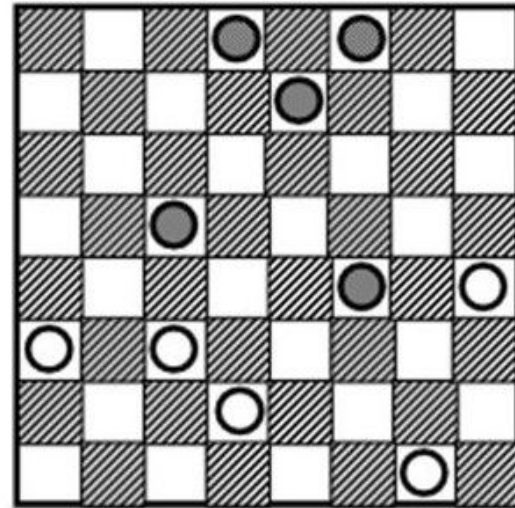
Generative Adversarial Networks (GANs):  
A two-network game where one **maps noise to structure**  
and one **classifies images as fake or real**.



When **D** is maximally confused, **G** will be a good generator

# Adversarial Networks

**Arthur Lee Samuel** (1959) wrote a program that learnt to play checkers well enough to beat him.



- He popularized the term "**machine learning**" in 1959.
- The program chose its move based on a **minimax** strategy, meaning it made the move assuming that the opponent was trying to optimize the value of the same function from its point of view.
- He also had it play thousands of **games against itself** as another way of learning.

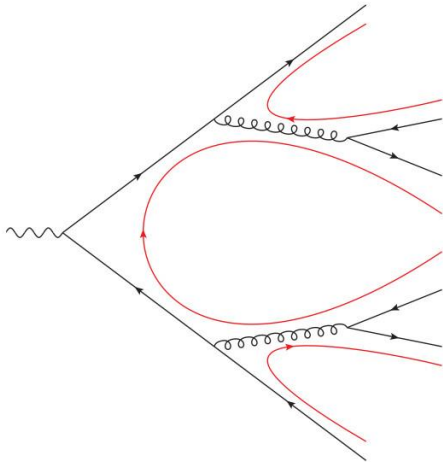


# Cluster hadronization model

**The philosophy of the model:** use information from perturbative QCD as an input for hadronization.

QCD **pre-confinement** discovered by Amati & Veneziano:

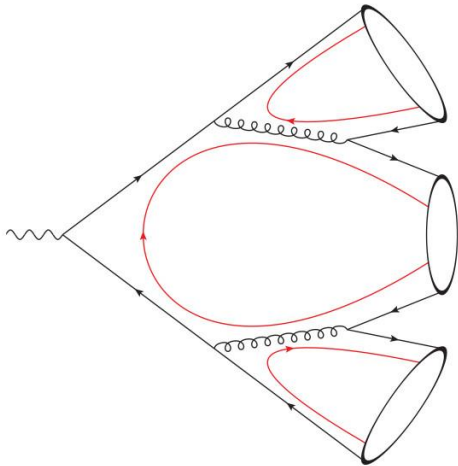
- QCD provide pre-confinement of colour



# Cluster hadronization model

**The philosophy of the model:** use information from perturbative QCD as an input for hadronization.

QCD **pre-confinement** discovered by Amati & Veneziano:

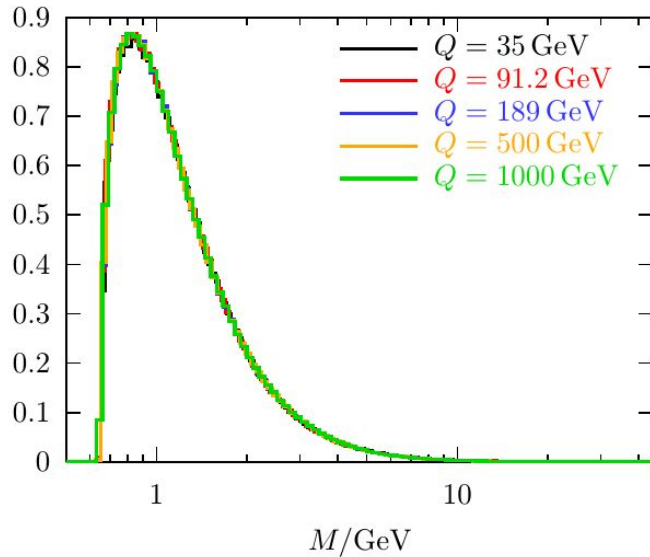


- QCD provide pre-confinement of colour
- Colour-singlet pair end up close in phase space and form highly excited hadronic states, the clusters

# Cluster hadronization model

**The philosophy of the model:** use information from perturbative QCD as an input for hadronization.

QCD **pre-confinement** discovered by Amati & Veneziano:



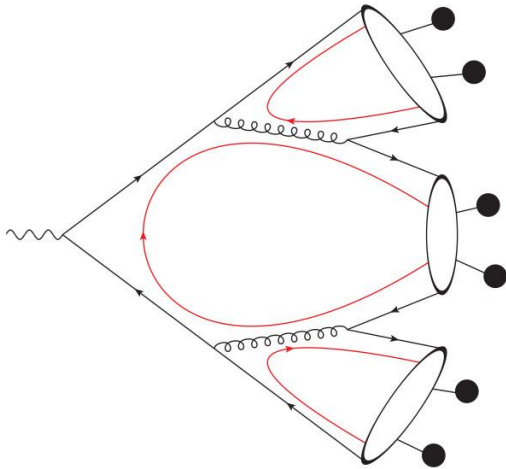
- QCD provide pre-confinement of colour
- Colour-singlet pair end up close in phase space and form highly excited hadronic states, the clusters
- Pre-confinement states that the spectra of clusters are independent of the hard process and energy of the collision

[S. Gieseke, A. Ribon, MH Seymour,  
P Stephens, B Webber JHEP 0402 (2004) 005]

# Cluster hadronization model

**The philosophy of the model:** use information from perturbative QCD as an input for hadronization.

QCD **pre-confinement** discovered by Amati & Veneziano:

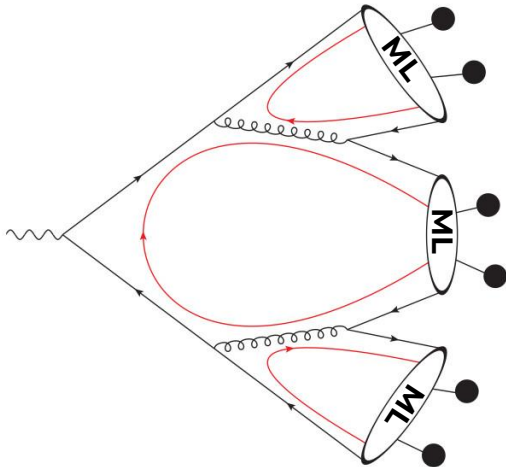


- QCD provide pre-confinement of colour
- Colour-singlet pair end up close in phase space and form highly excited hadronic states, the clusters
- Pre-confinement states that the spectra of clusters are independent of the hard process and energy of the collision
- Peaked at low mass (1-10 GeV) typically decay into 2 hadrons

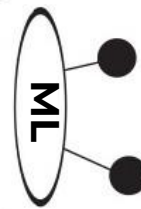
# Cluster hadronization model

**The philosophy of the model:** use information from perturbative QCD as an input for hadronization.

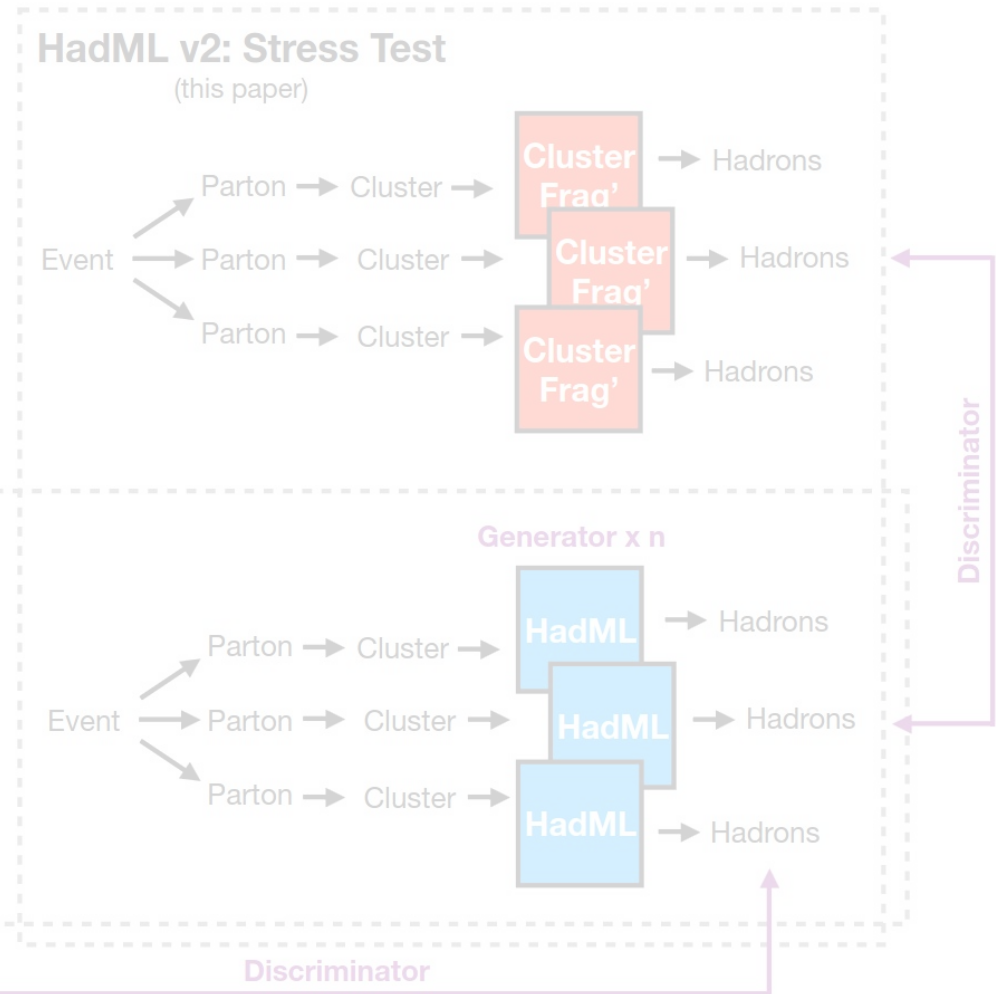
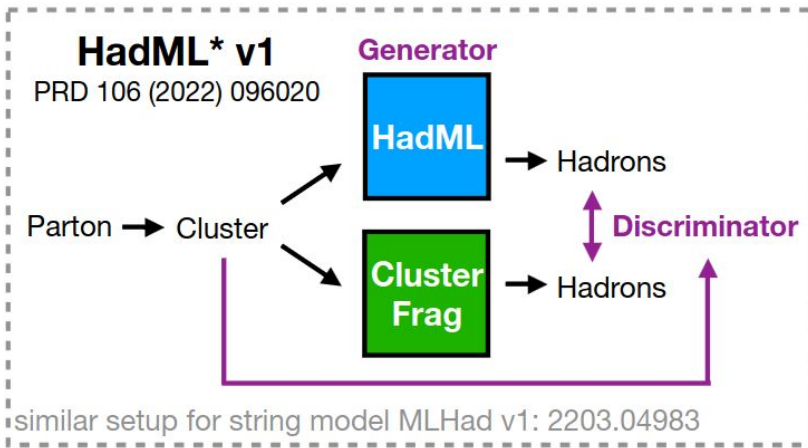
QCD **pre-confinement** discovered by Amati & Veneziano:



- QCD provide pre-confinement of colour
- Colour-singlet pair end up close in phase space and form highly excited hadronic states, the clusters
- Pre-confinement states that the spectra of clusters are independent of the hard process and energy of the collision
- Peaked at low mass (1-10 GeV) typically decay into 2 hadrons
- **ML hadronization**  
1st step: generate kinematics of a cluster decay:



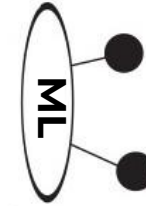
# Road map for today



# Towards a Deep Learning Model for Hadronization

## ML hadronization

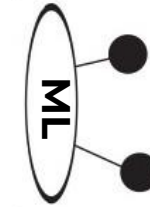
1st step: generate kinematics of a cluster decay to 2 hadrons



# Towards a Deep Learning Model for Hadronization

## ML hadronization

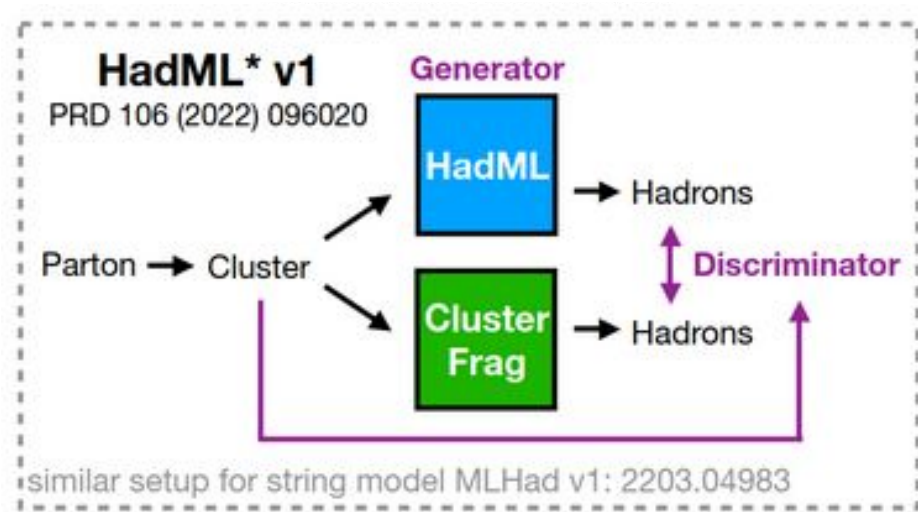
1st step: generate kinematics of a cluster decay to 2 hadrons



## How?

We have a conditional GAN, with cluster 4-vector input and two hadron 4-vector outputs.

## Generative Adversarial Net

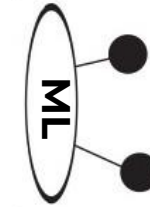




# Towards a Deep Learning Model for Hadronization

## ML hadronization

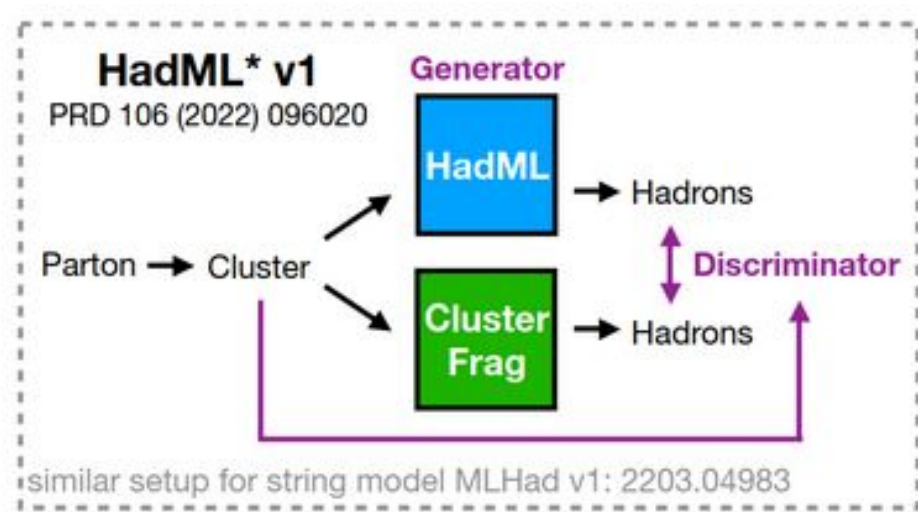
1st step: generate kinematics of a cluster decay to 2 hadrons



## How?

We have a conditional GAN, with cluster 4-vector input and two hadron 4-vector outputs.

## Generative Adversarial Net

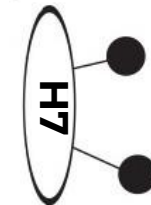


## Training data:



$e^+e^-$  collisions at  
 $\sqrt{s} = 91.2$  GeV

Cluster  $(E, p_x, p_y, p_z)$



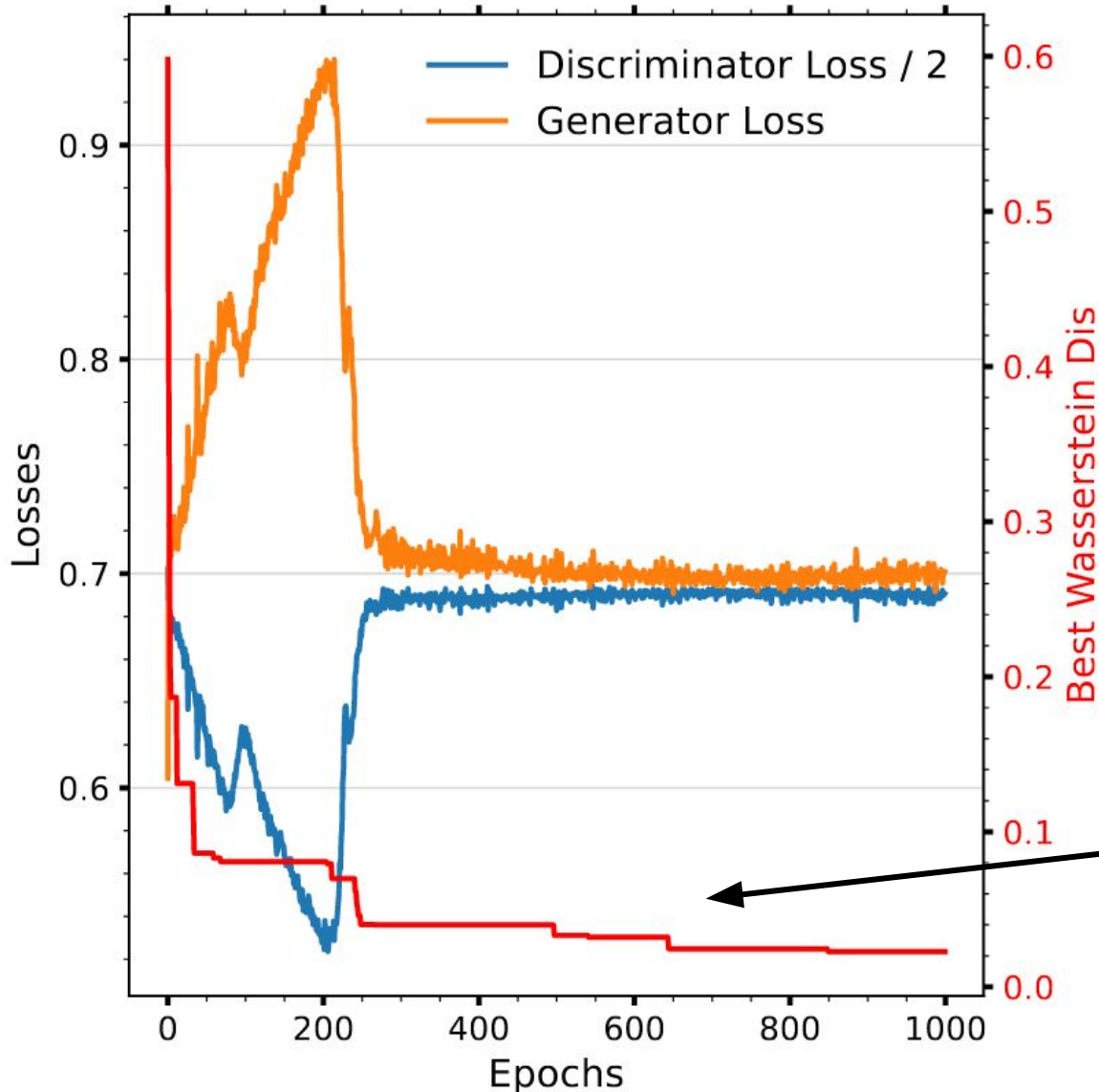
$\pi^0(E, p_x, p_y, p_z)$

$\pi^0(E, p_x, p_y, p_z)$

## Simplification:

considering only pions and generating two angles in the cluster rest frame.

# Training HADML v1

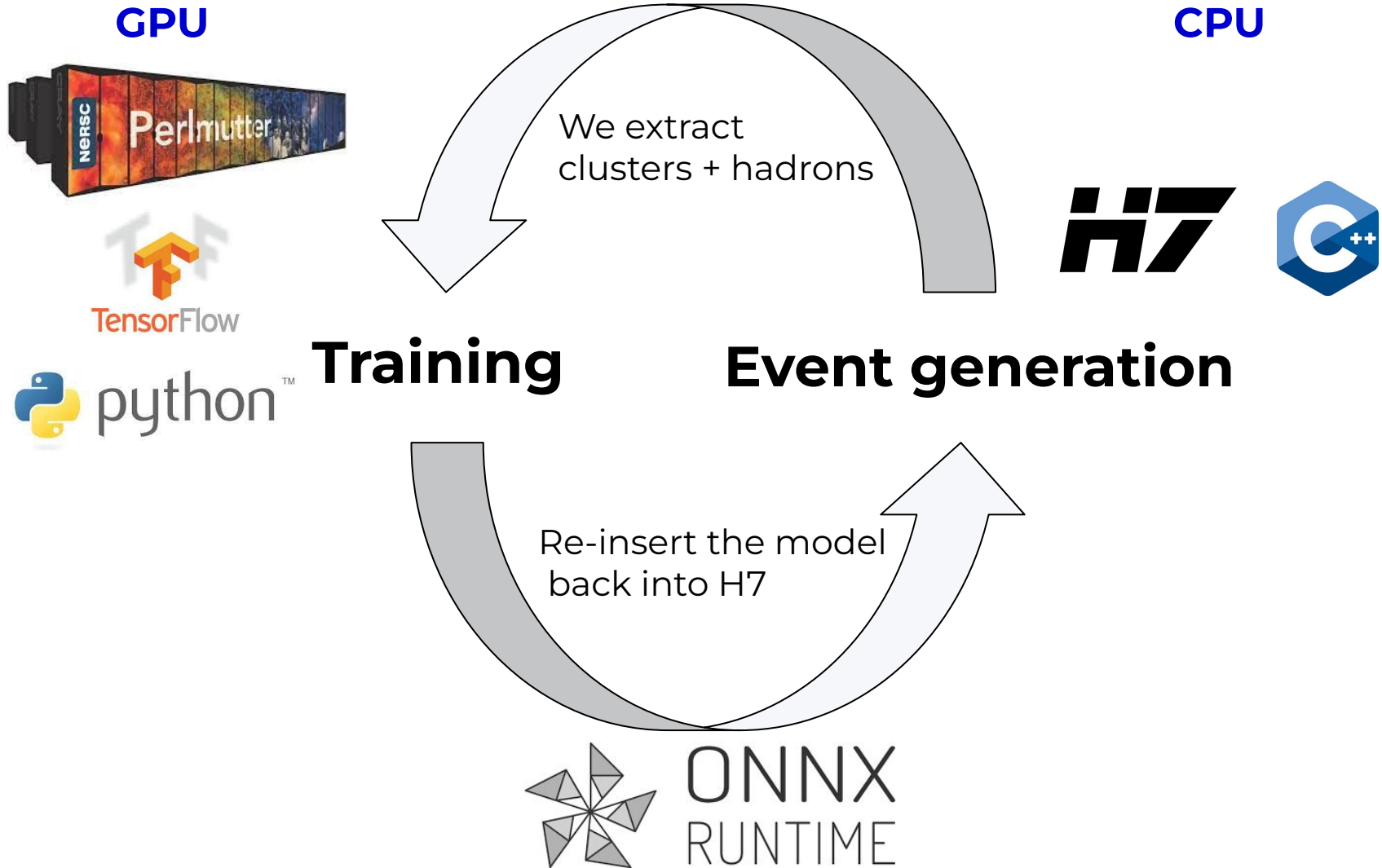


We have a conditional GAN, with cluster 4-vector input and two hadron 4-vector outputs.

Simplification:  
considering only pions and generating two angles in the cluster rest frame.

This is a typical learning curve for GAN training

# Integration into Herwig



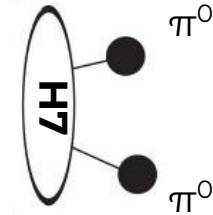
This then allows us to run a full event generator and produce plots

# Performance: Pions

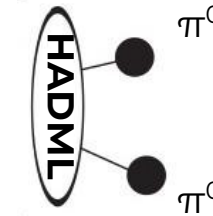
## Low-level Validation

(similar to training data)

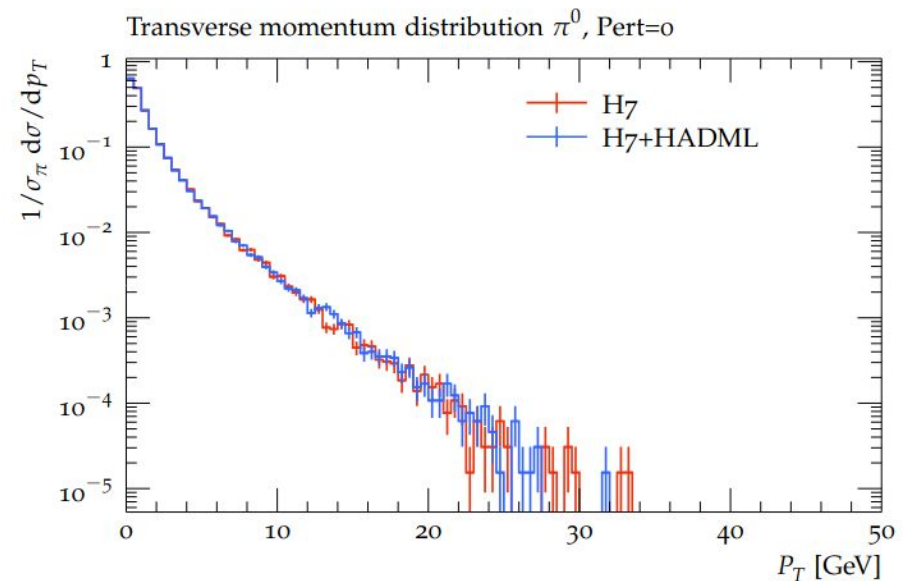
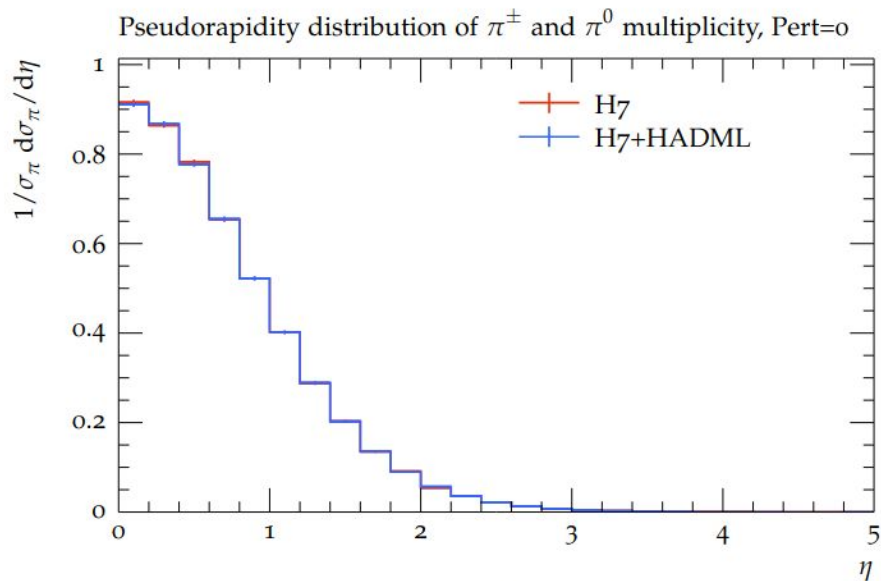
$e^+e^-$  collisions at  
 $\sqrt{s} = 91.2$  GeV



VS



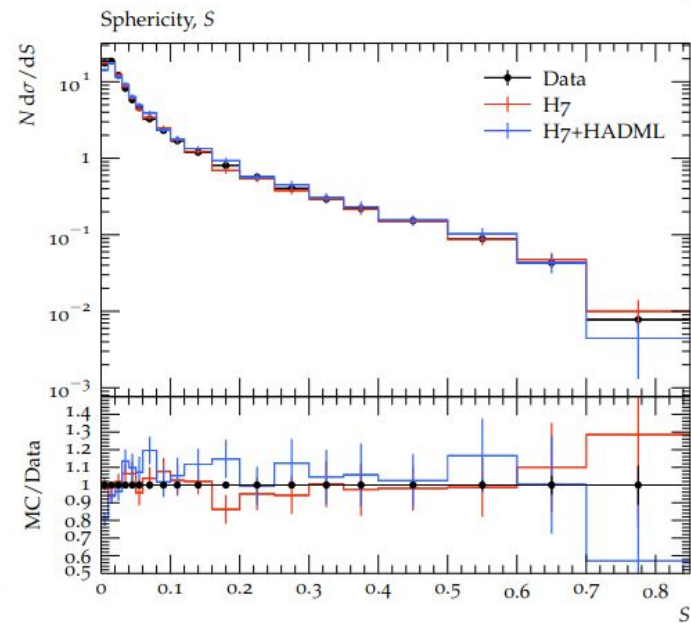
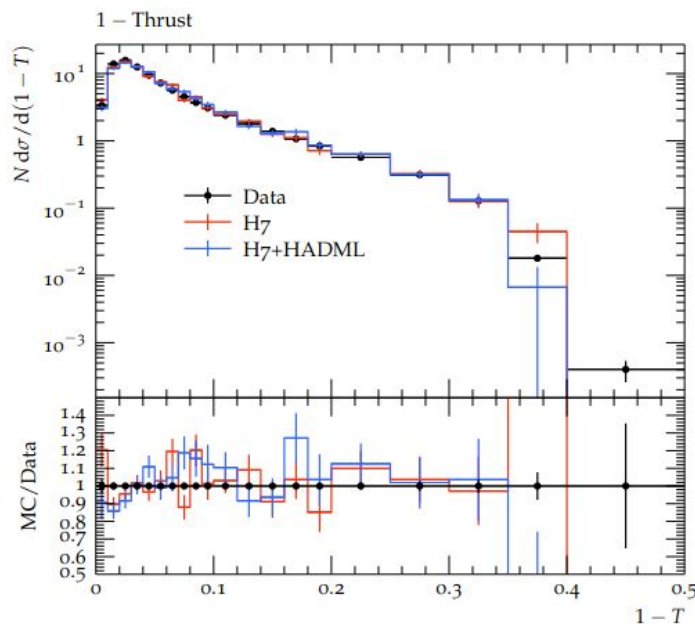
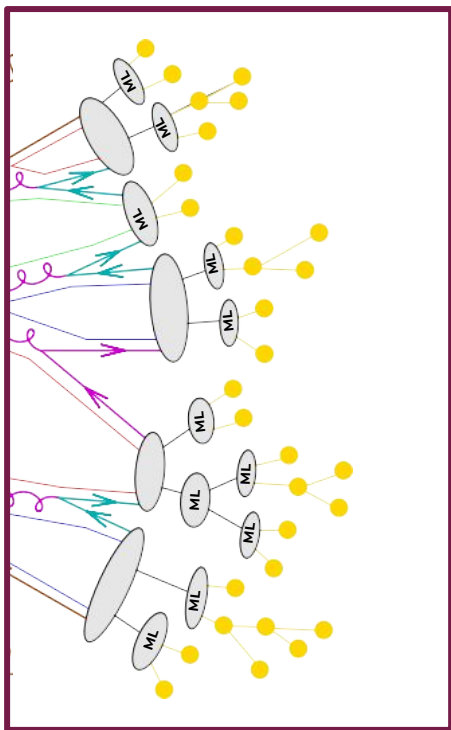
$\pi^0$  kinematic variables



# Performance: Data!

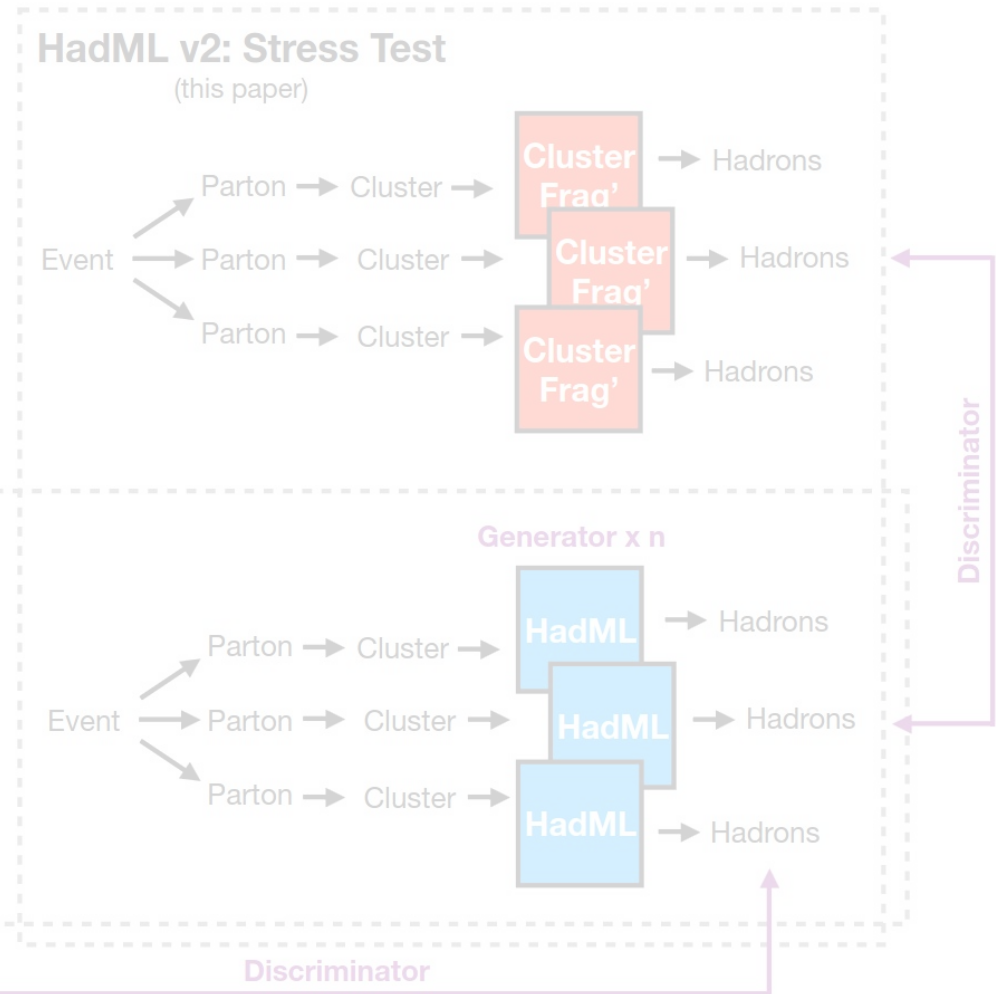
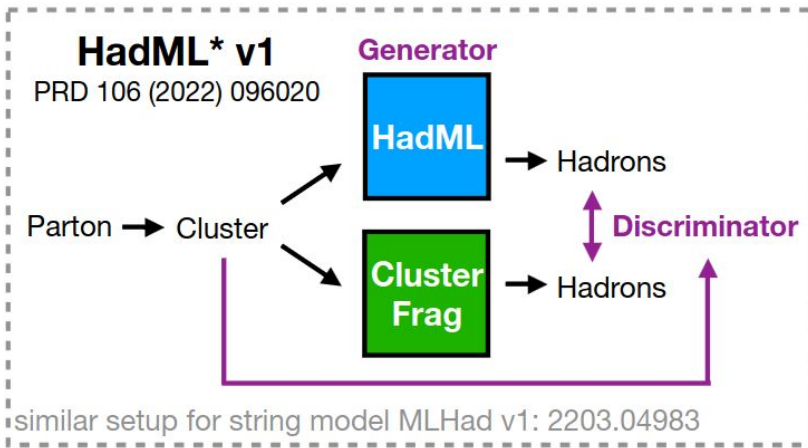
With a “full” model, we can compare directly to data!

## LEP DELPHI Data



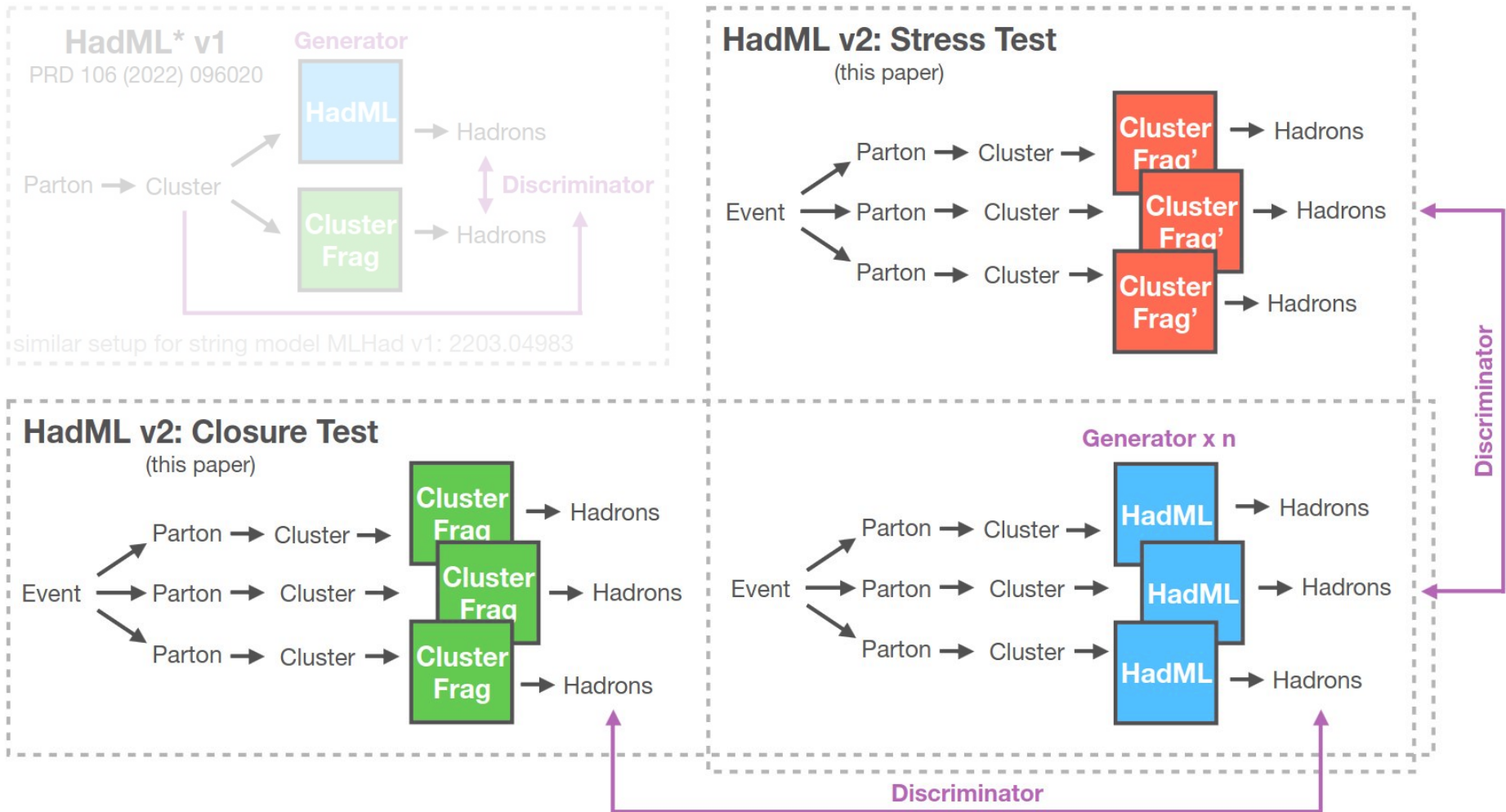
N.B. we have trained on H7, so we don't expect to be any better than it at modeling the data.

# Road map for today



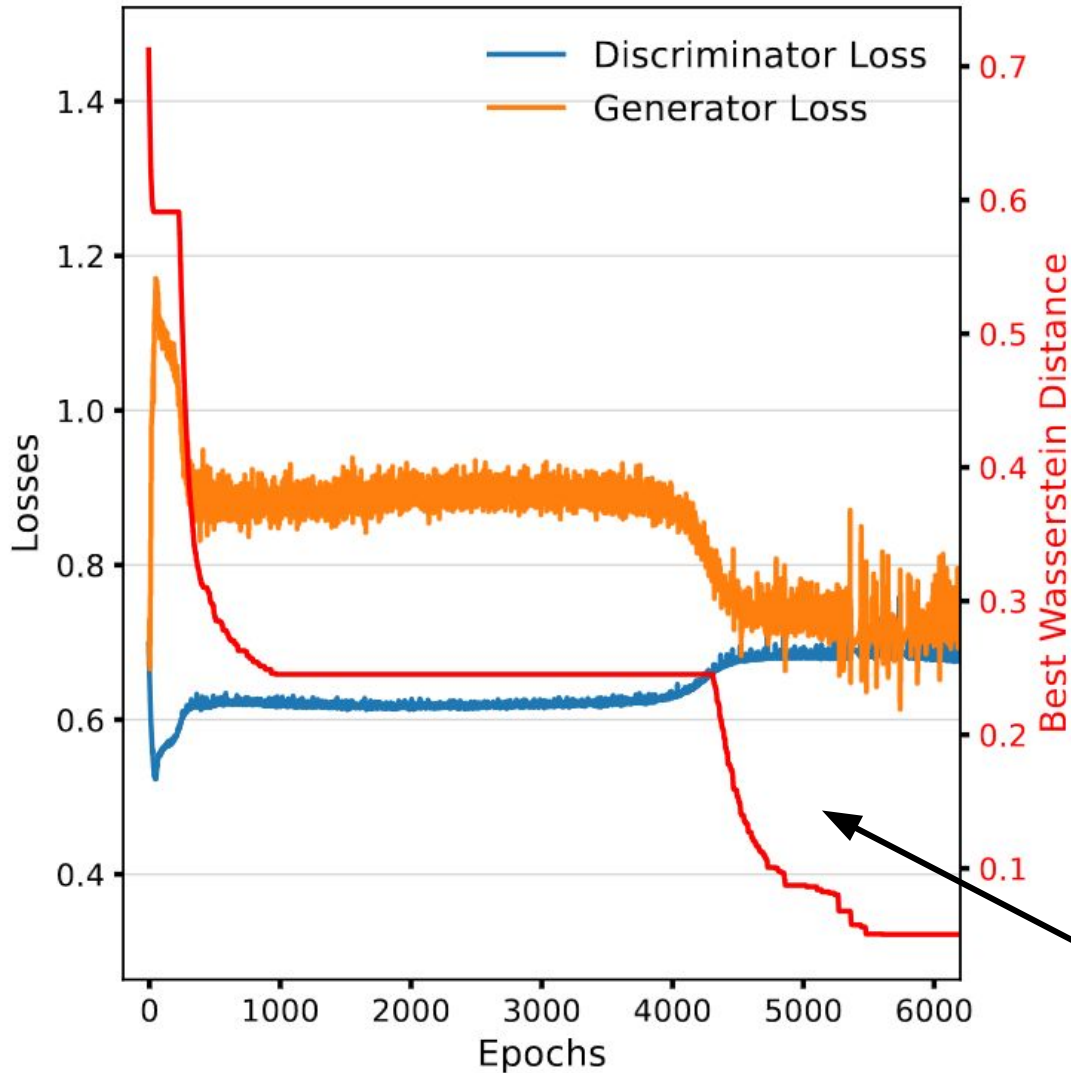


# Road map for today



Protocol for fitting a deep generative hadronization model in a realistic data setting, where we only have access to a set of hadrons in data.

# Training HADML v2



Now, the generator is local (per cluster), but the discriminator is global (whole event).

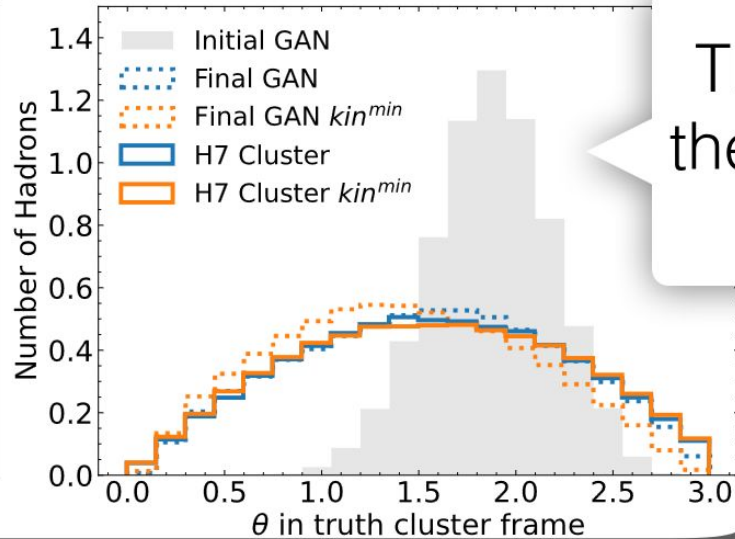
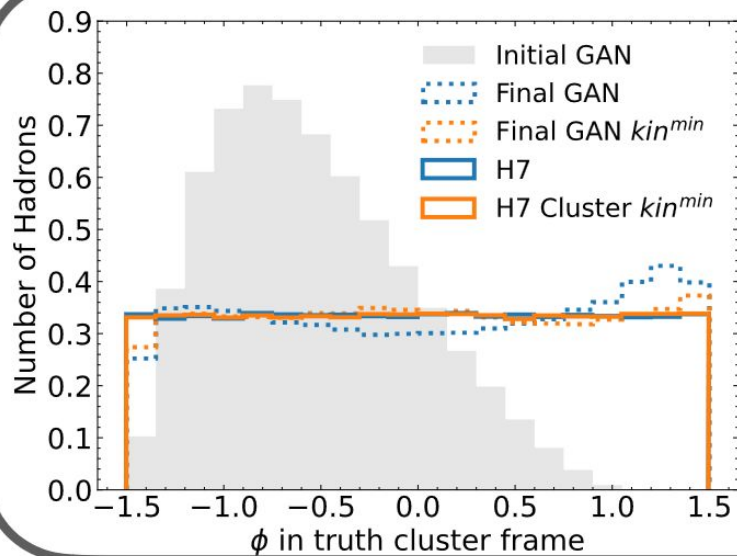
Discriminator is a permutation-invariant architecture called Deep Sets.

Simplification only  
Pions

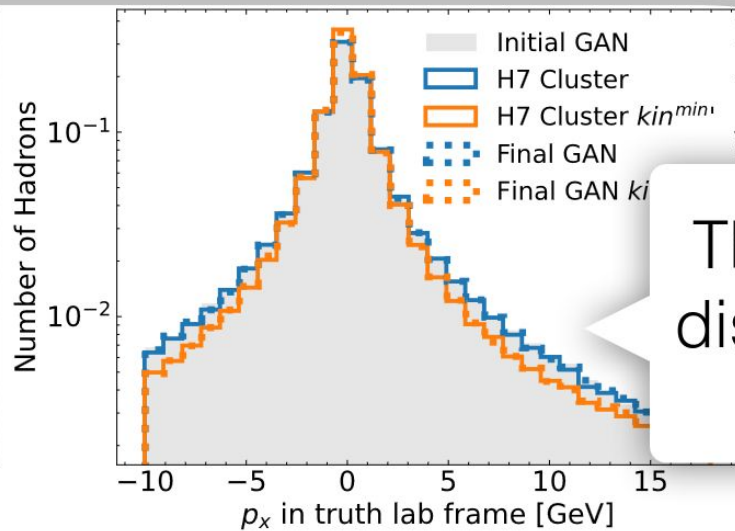
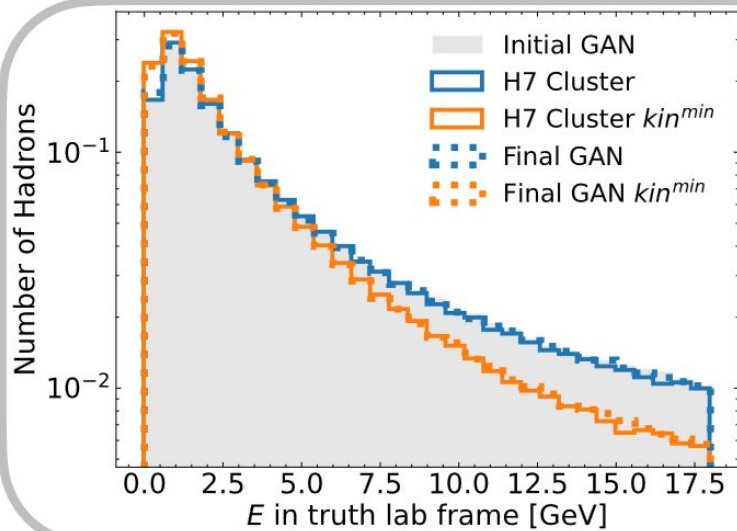
Still works !



# Performance

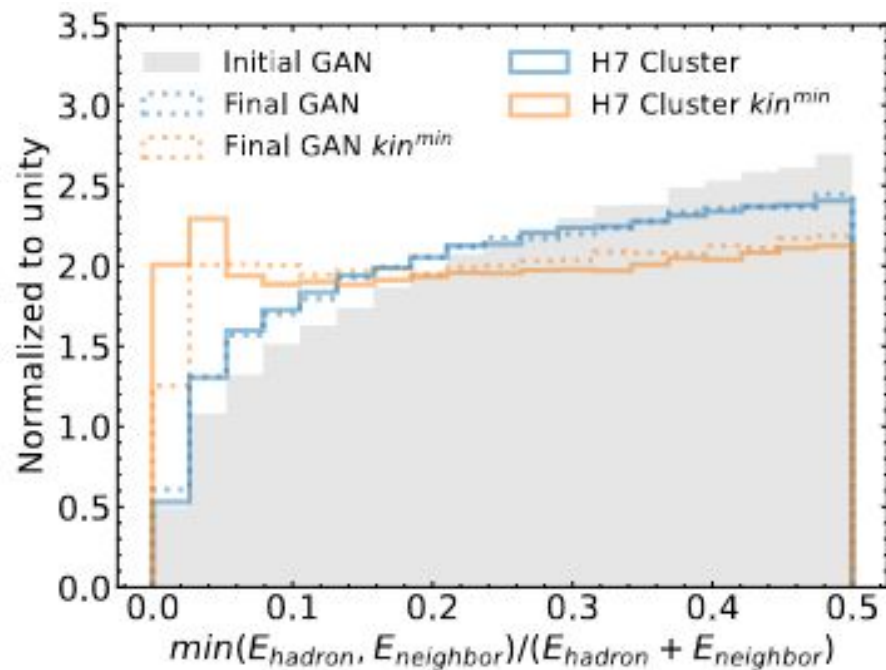
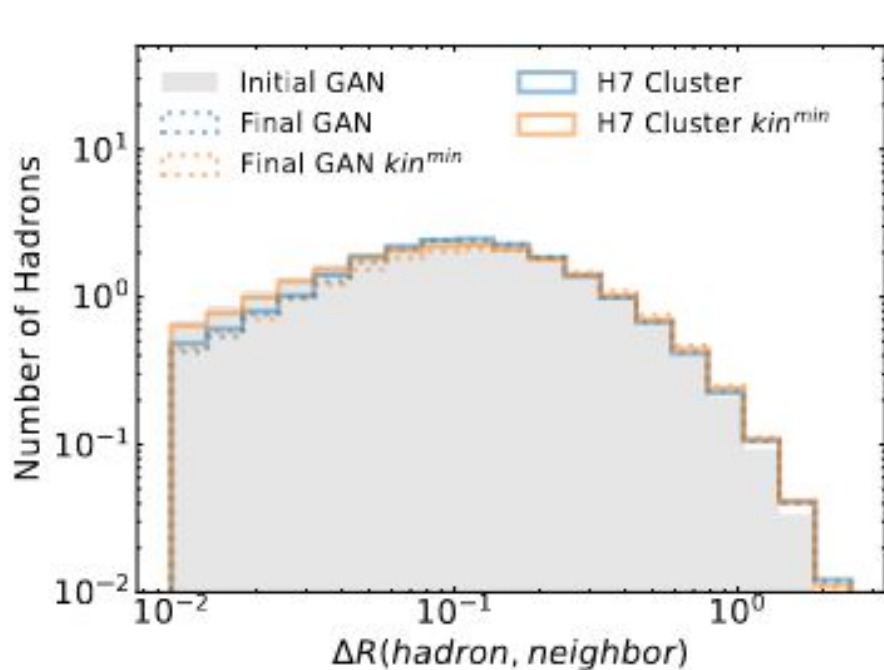


This is what the generator "sees"



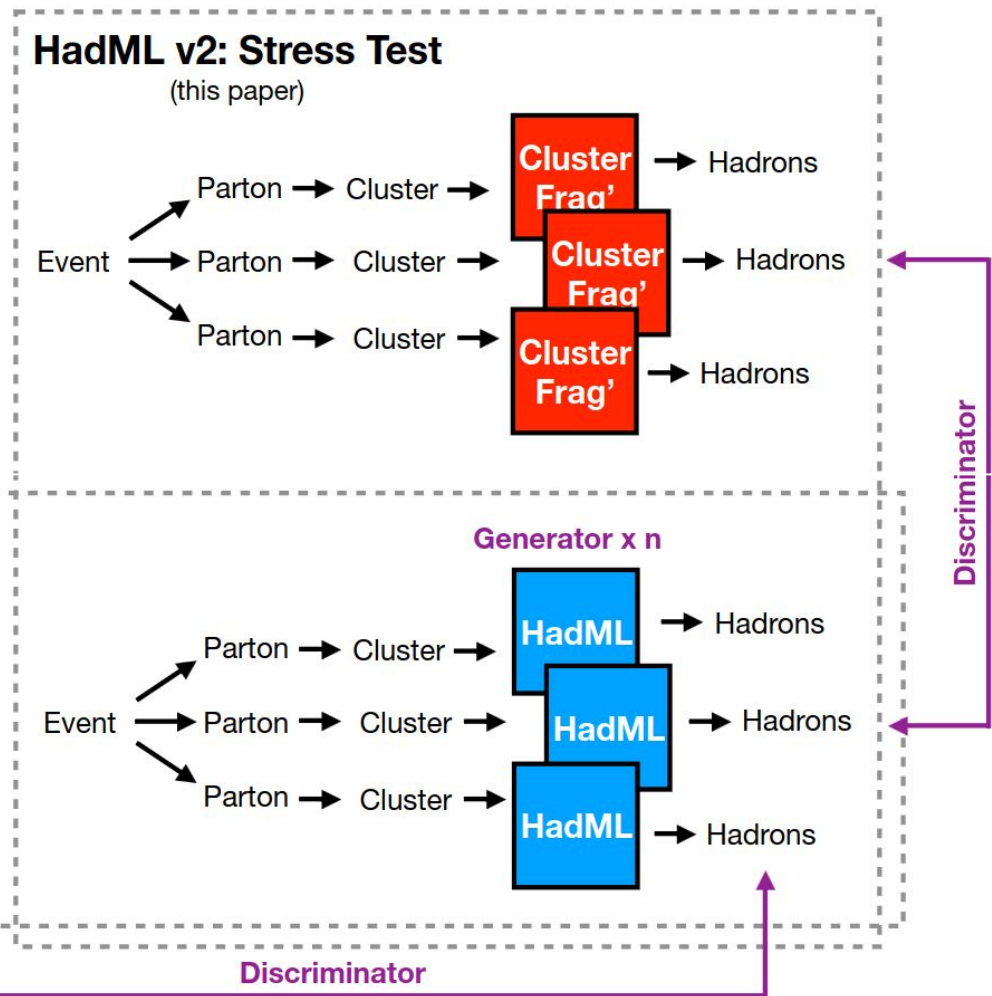
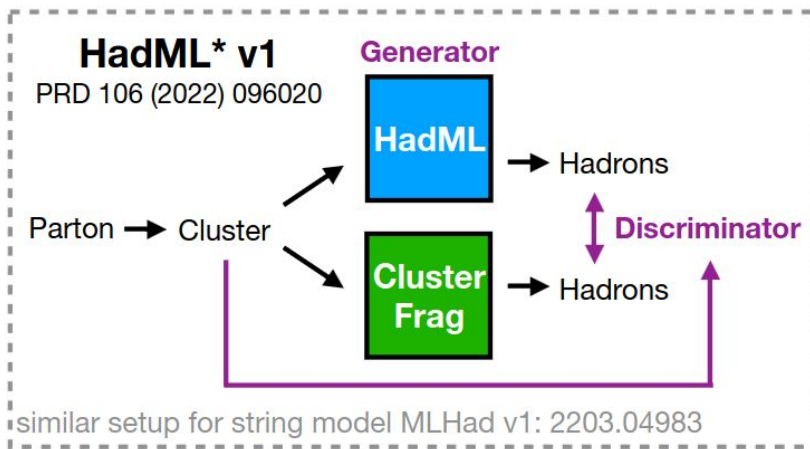
This is what discriminator "sees"

# Performance: going beyond inputs and outputs



$$\text{MINIMAL } \Delta R^2 = \Delta\phi^2 + \Delta\eta^2$$

# Summary

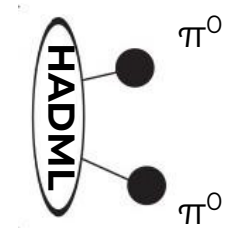


A key advantage of this fitting protocol over other methods is that it can accommodate unbinned and high-dimensional inputs.

**The approach could also be used to tune (without binning) data to a parametric physics model (for example cluster) as well. However, this would require making the cluster model differentiable.**

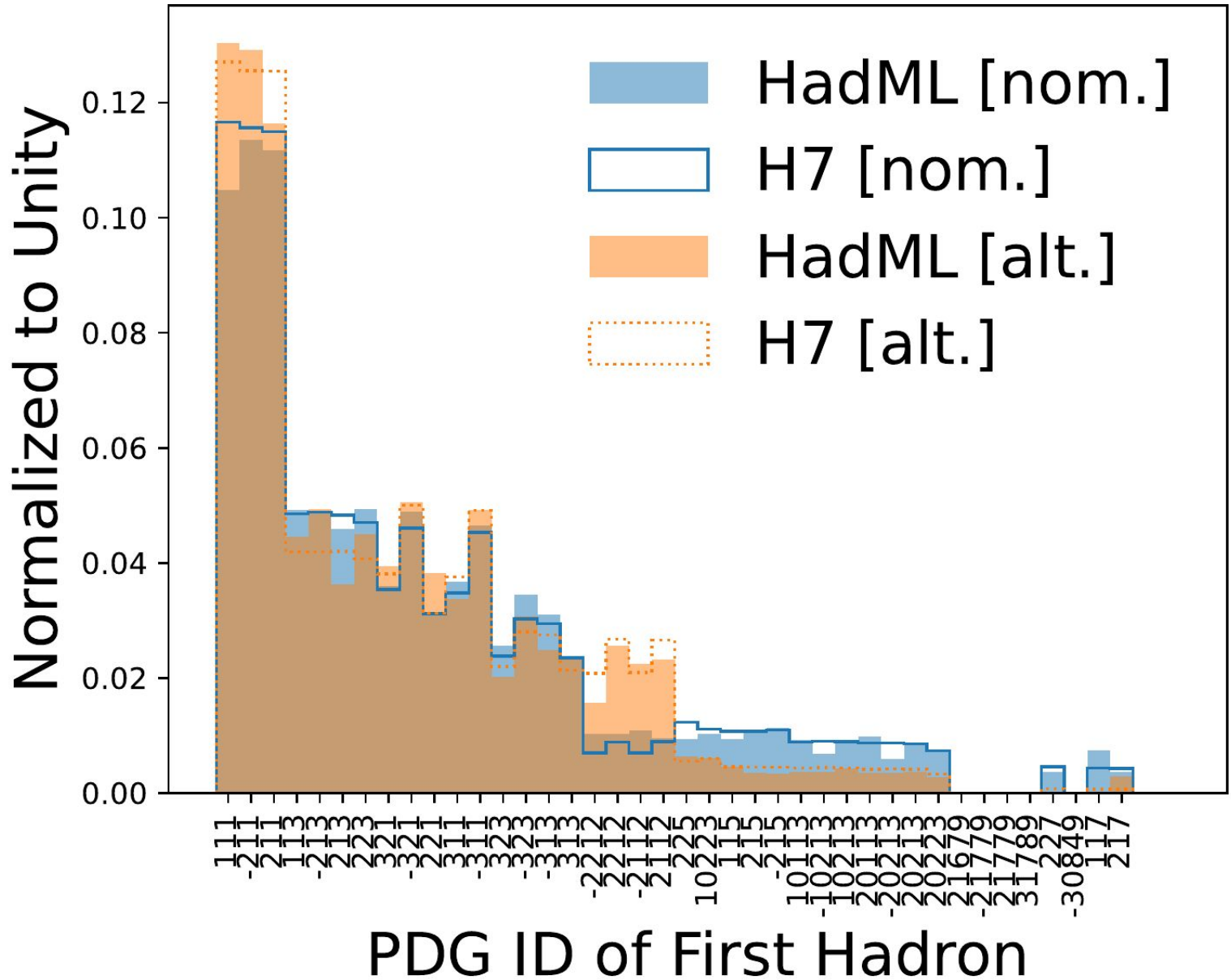
# Outlook

- For HADML, we have made significant progress, but there are still multiple steps to build and tune a full-fledged hadronization model.



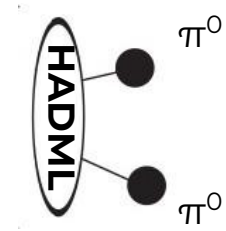
## What is next?

- Number of technical and methodological step needed:
  - Directly accommodate multiple hadron species with their relative probabilities



# Outlook

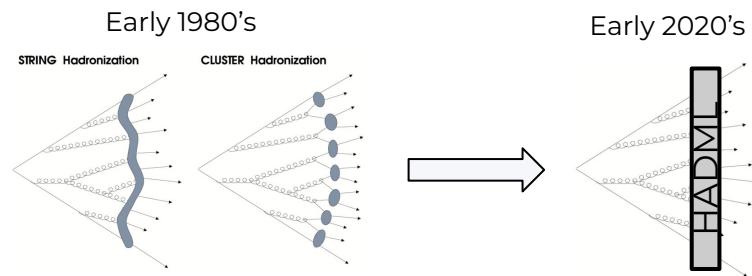
- For HADML, we have made significant progress, but there are still multiple steps to build and tune a full-fledged hadronization model.
- HADML is naturally suited for GPUs



## What is next?

- Number of technical and methodological step needed:
  - Directly accommodate multiple hadron species with their relative probabilities
  - Include heavy clusters (so far done by Herwig)
  - Hyperparameter optimization, including the investigation of alternative generative models
  - More flexible model with a capacity to mimic the cluster or string models and beyond.
  - Tune to the LEP data

There is still a multi-year program ahead of us, but it will be worth it!



**So Stay tuned!**



# Advertisement

A postdoc in ML/HEP position



JAGIELLONIAN UNIVERSITY  
IN KRAKÓW



If you are interested please contact me:  
[andrzej.siodmok@cern.ch](mailto:andrzej.siodmok@cern.ch)

# Discriminator HadML v1 vs v2

## HadML v1

The loss function:

$$L = - \sum_{\lambda \sim \text{HERWIG}, z \sim p(z)} (\log(D(\tau(\lambda))) + \log(1 - D(G(z, \lambda))))$$

## HadML v2

The discriminator function is modified, we parameterize it as a Deep Sets model

$$D_E(x) = F \left( \frac{1}{n} \sum_{i=1}^n \Phi(h_i, \omega_{D_\Phi}), \omega_F \right) \longleftarrow \text{invariant under permutations of hadrons}$$

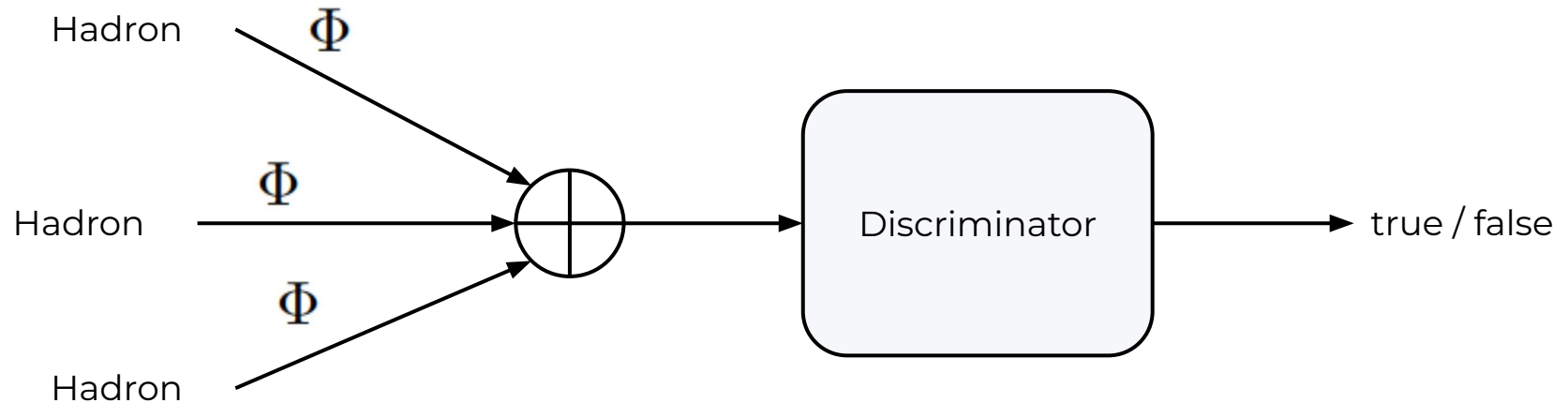
$\Phi$  embeds a set of hadrons into a fixed-length latent space and  $F$  acts on the average

$$L = - \sum_{x \sim \text{data}} \log(D_E(x)) - \sum_{\{G\} \sim \text{HERWIG}, z \sim p(z)} \log(1 - D_E(\{G(z, \lambda)\}))$$

The approach could also be used to fit (without binning) data to a parametric physics model (for example cluster) as well. However, this would require making the cluster model differentiable.



# Discriminator HadML v2

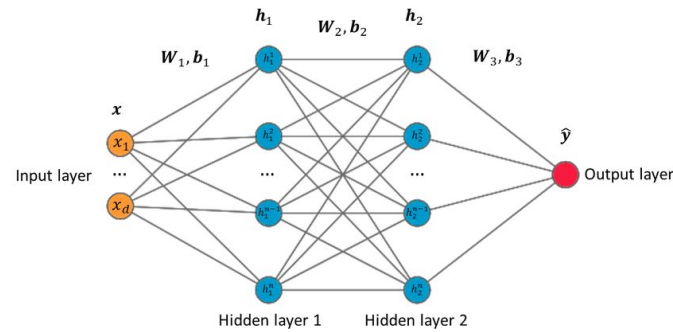


The discriminator function is modified, we parameterize it as a Deep Sets model

$$D_E(x) = F\left(\frac{1}{n} \sum_{i=1}^n \Phi(h_i, \omega_{D_\Phi}), \omega_F\right) \leftarrow \text{invariant under permutations of hadrons}$$

# Architecture: conditional GAN

**Generator and the Discriminator are composed of two-layer perceptron**  
(each a fully connected, hidden size 256, a batch normalization layer, LeakyReLU activation function)



## Generator

### Input

Cluster  $(E, p_x, p_y, p_z)$  and 10 noise features sampled from a Gaussian distribution

### Output (in the cluster frame)

$\phi$  - polar angle  
 $\theta$  - azimuthal angle

} we reconstruct the four vectors of the two outgoing hadrons

## Discriminator

### Input

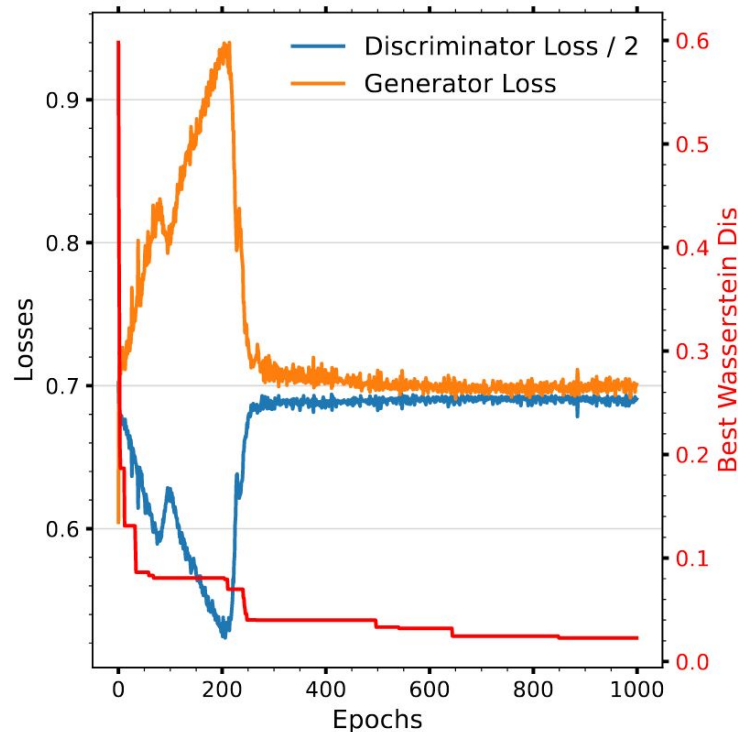
$\phi$  and  $\theta$  labeled as signal (generated by Herwig) or background (generated by Generator)

### Output

Score that is higher for events from Herwig and lower for events from the Generator

# Training

- **Data normalization:** cluster's four vector and angular variables are scaled to be between -1 and 1 (tanh activation function as the last layer of the Generator)
- **Discriminator** and the **Generator** are trained separately and alternately by two independent Adam optimizers with a learning rate of  $10^{-4}$ , for 1000 epochs



- **The best model** for events with partons of  $P_{\text{ert}} = 0$ , is found at the epoch 849 with a total Wasserstein distance of 0.0228.

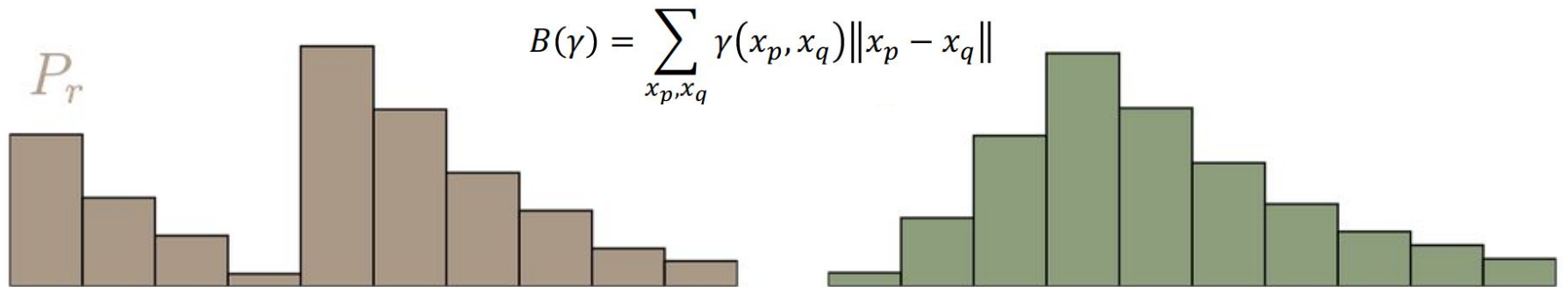
# Wasserstein distance

## The Wasserstein distance

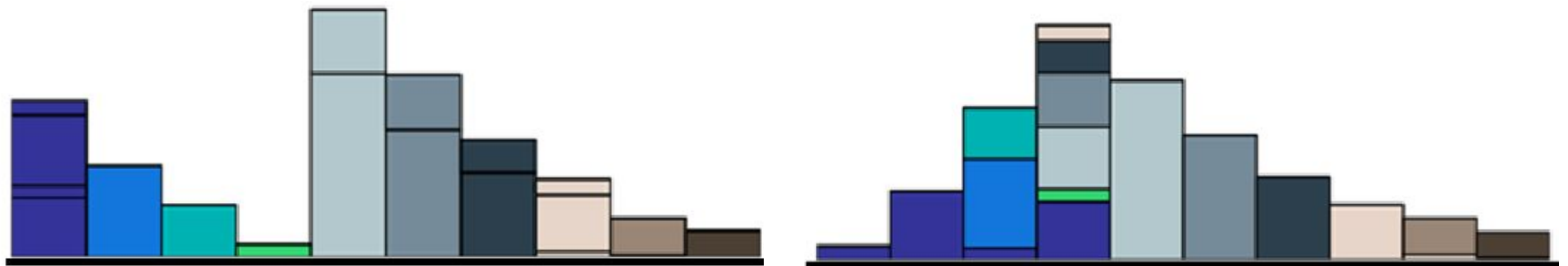
- For discrete probability distributions, the Wasserstein distance is called the earth mover's distance (EMD):
- EMD is the minimal total amount of work it takes to transform one heap into the other.

$$W(P, Q) = \min_{\gamma \in \Pi} B(\gamma)$$

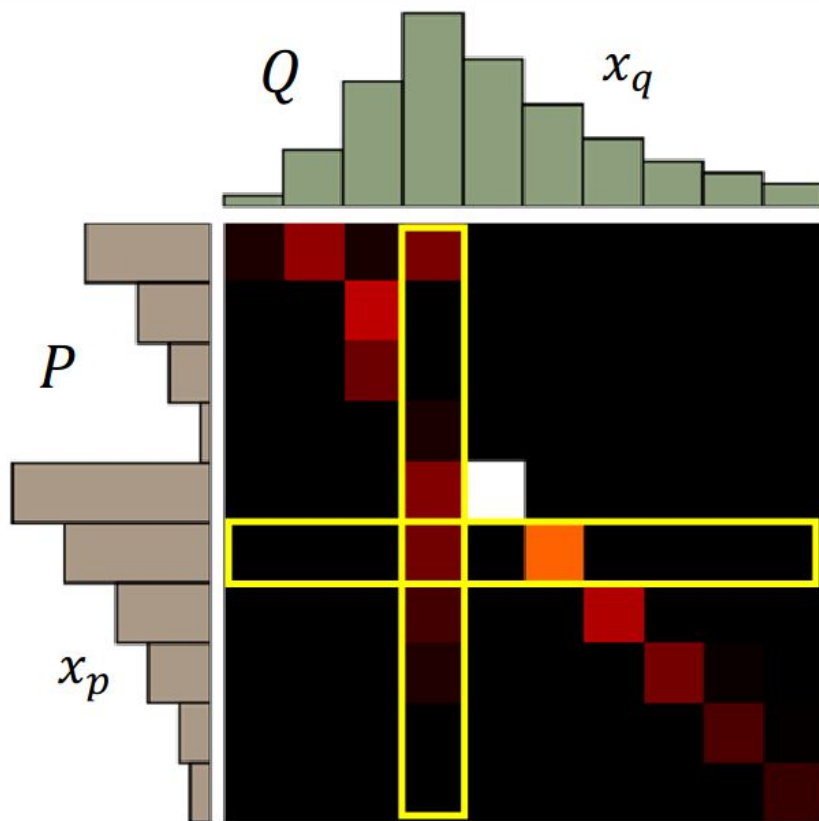
- Work is defined as the amount of earth in a chunk times the distance it was moved.



Best “moving plans” of this example



# Wasserstein distance



moving plan  $\gamma$   
All possible plan  $\Pi$

A “moving plan” is a matrix  
The value of the element is the amount of earth from one position to another.

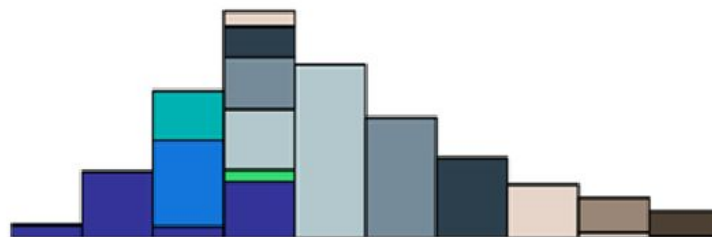
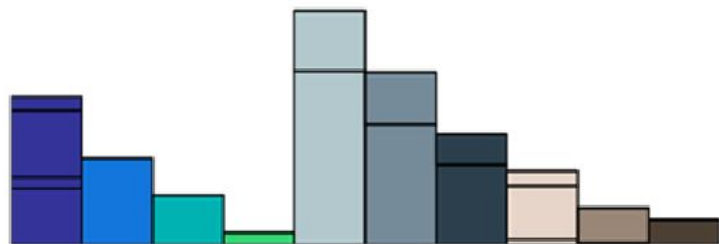
Average distance of a plan  $\gamma$ :

$$B(\gamma) = \sum_{x_p, x_q} \gamma(x_p, x_q) \|x_p - x_q\|$$

Earth Mover’s Distance:

$$W(P, Q) = \min_{\gamma \in \Pi} B(\gamma)$$

The best plan



# Minimax Loss

In the paper that introduced GANs, the generator tries to minimize the following function while the discriminator tries to maximize it:

$$E_x[\log(D(x))] + E_z[\log(1 - D(G(z)))]$$

In this function:

- $D(x)$  is the discriminator's estimate of the probability that real data instance  $x$  is real.
- $E_x$  is the expected value over all real data instances.
- $G(z)$  is the generator's output when given noise  $z$ .
- $D(G(z))$  is the discriminator's estimate of the probability that a fake instance is real.
- $E_z$  is the expected value over all random inputs to the generator (in effect, the expected value over all generated fake instances  $G(z)$ ).
- The formula derives from the [cross-entropy](#) between the real and generated distributions.

The generator can't directly affect the  $\log(D(x))$  term in the function, so, for the generator, minimizing the loss is equivalent to minimizing  $\log(1 - D(G(z)))$ .

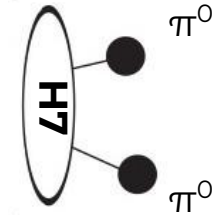
# Performance: Energy of the collisions

## Low-level Validation

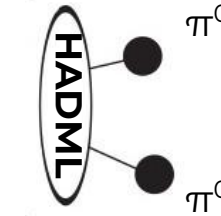
(beyond training data different energy)

$e^+e^-$  collisions at

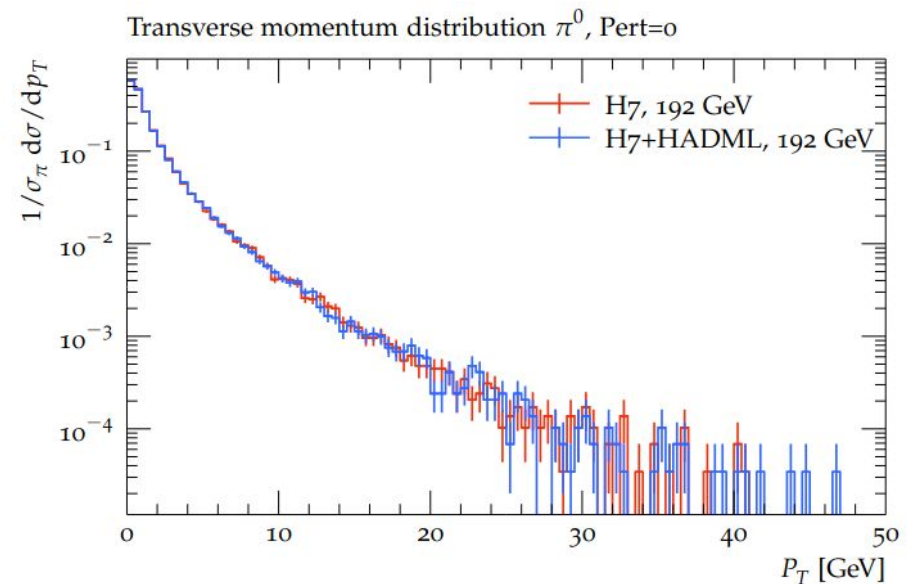
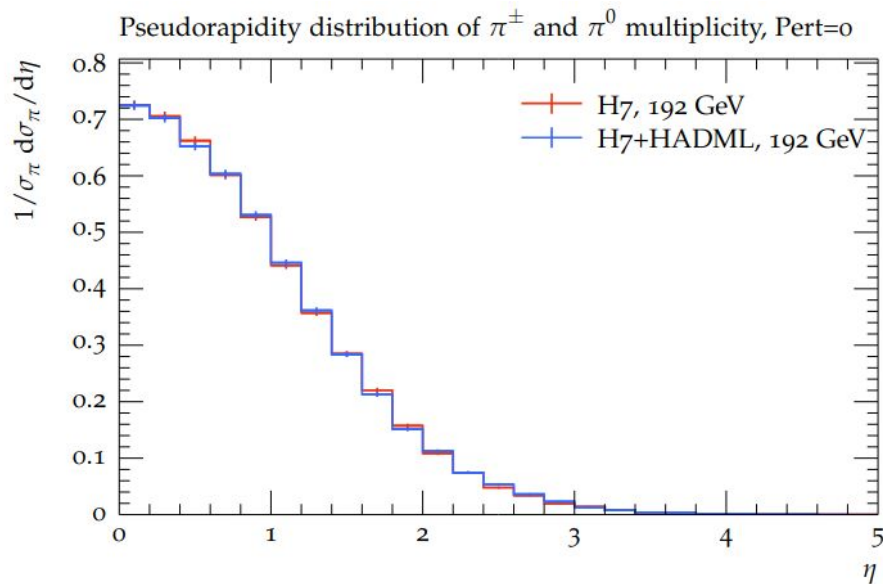
$$\sqrt{s} = 192 \text{ GeV}$$



VS



$\pi^0$  kinematic variables

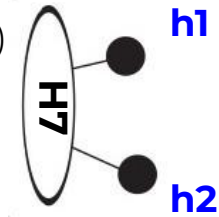


# Performance: All Hadrons

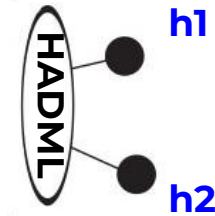
## Low-level Validation

(beyond training data different hadrons)

$e^+e^-$  collisions at  
 $\sqrt{s} = 91.2$  GeV



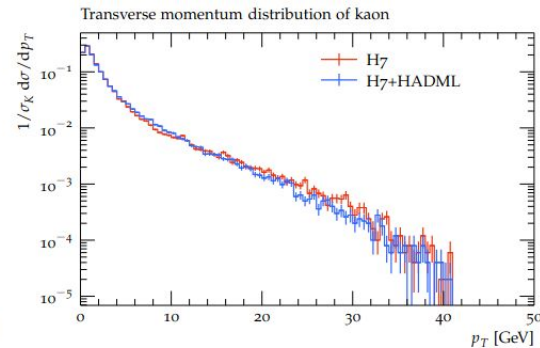
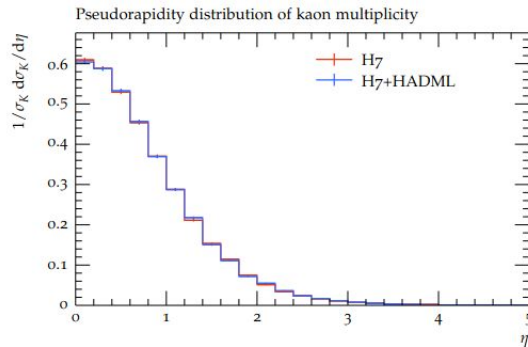
VS



**h** kinematic variables

As a crude “full” model, we simply take the PIDs from Herwig and the kinematics from the GAN.

**Kaons**



**Lambda**

