Small x resummation for parton distribution functions

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Small-x logarithms and collinear factorisation

Collinear factorisation:

$$\sigma\left(x, Q^2\right) = \int_x^1 \frac{\mathrm{d}z}{z} C_i\left(z, \alpha_s(Q^2)\right) f_i\left(\frac{x}{z}, Q^2\right)$$

DGLAP evolution:

$$\mu^2 \frac{\mathrm{d}f_i\left(x, \mu^2\right)}{\mathrm{d}\mu^2} = \int_x^1 \frac{\mathrm{d}z}{z} P_{ij}\left(z, \alpha_s(\mu^2)\right) f_j\left(\frac{x}{z}, \mu^2\right)$$

Perturbative expasion of C_i, $P_{ij} \rightarrow$ **logarithmic enhancements**

Small-x logarithms: $\alpha_s^n \frac{1}{x} \log^k\left(\frac{1}{x}\right) \quad 0 \leq k \leq n-1$ (single logs)

$\alpha_s \log\left(\frac{1}{x}\right) \sim 1$ (fixed order) perturbation theory fails $\rightarrow$ **all-order resummation**

Small-x resummation

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k_t-factorisation: [Catani, Hautmann hep-ph/9405388]

$$\sigma\left(x,Q^2\right)=\int_x^1\frac{\mathrm{d}z}{z}\int\mathrm{d}k_t^2\,\mathcal{C}_g\left(\frac{x}{z},\alpha_s,Q^2,k_t^2\right)\mathcal{F}_g\left(z,Q^2,k_t^2\right)+\ldots$$

BFKL equation singlet sector, $t=\log(1/x)$:

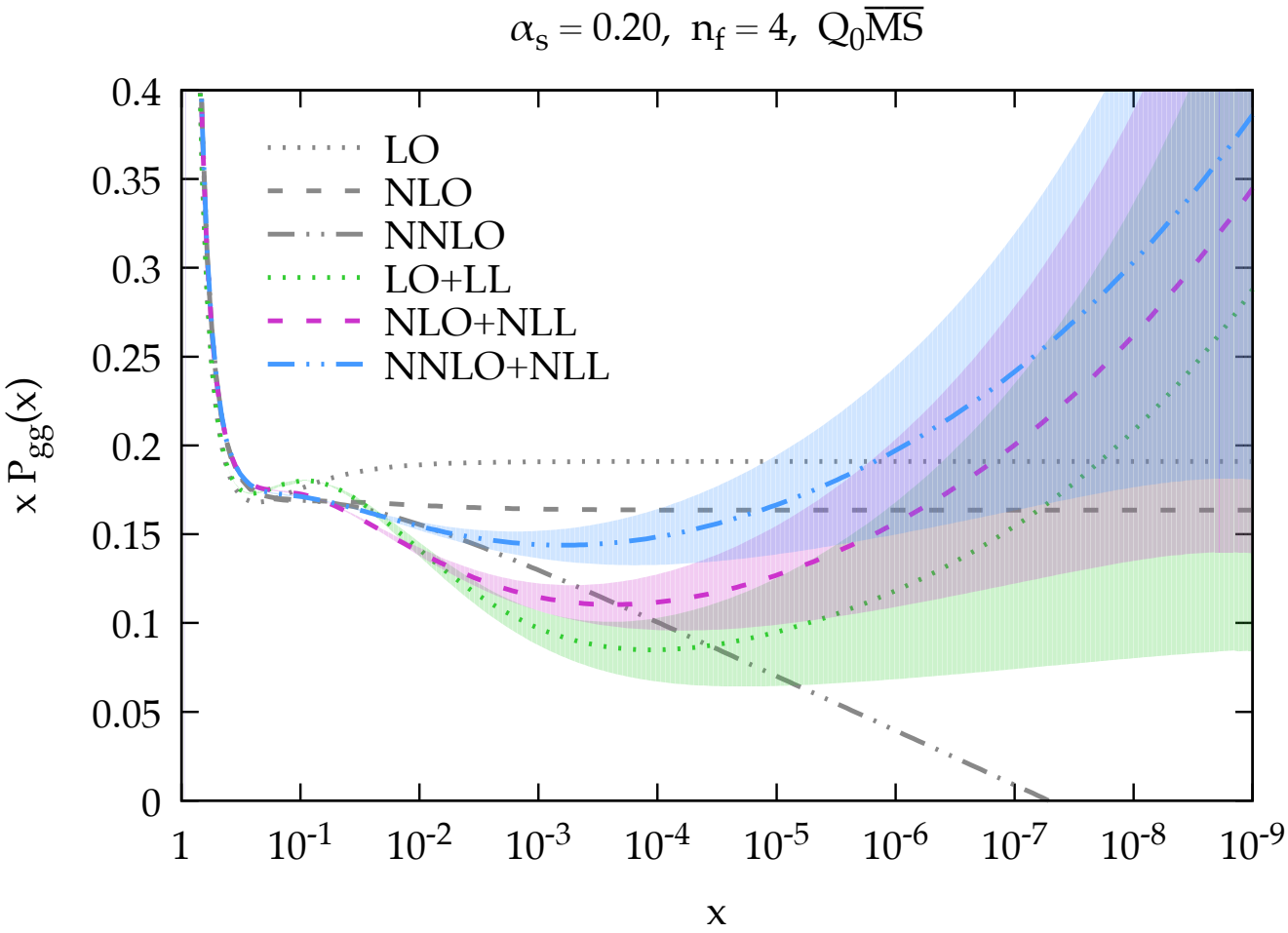
$$\frac{\mathrm{d}}{\mathrm{d}t}f\left(t,q^2\right)=\int_0^\infty\frac{\mathrm{d}k^2}{k^2}K\left(\frac{q^2}{k^2},\alpha_s\right)f\left(t,k^2\right)$$

for further reading: [Altarelli, Forte hep-ph/9703417], [Bonvini 1212.0480]

Small-x resummation in the gluon-gluon splitting function

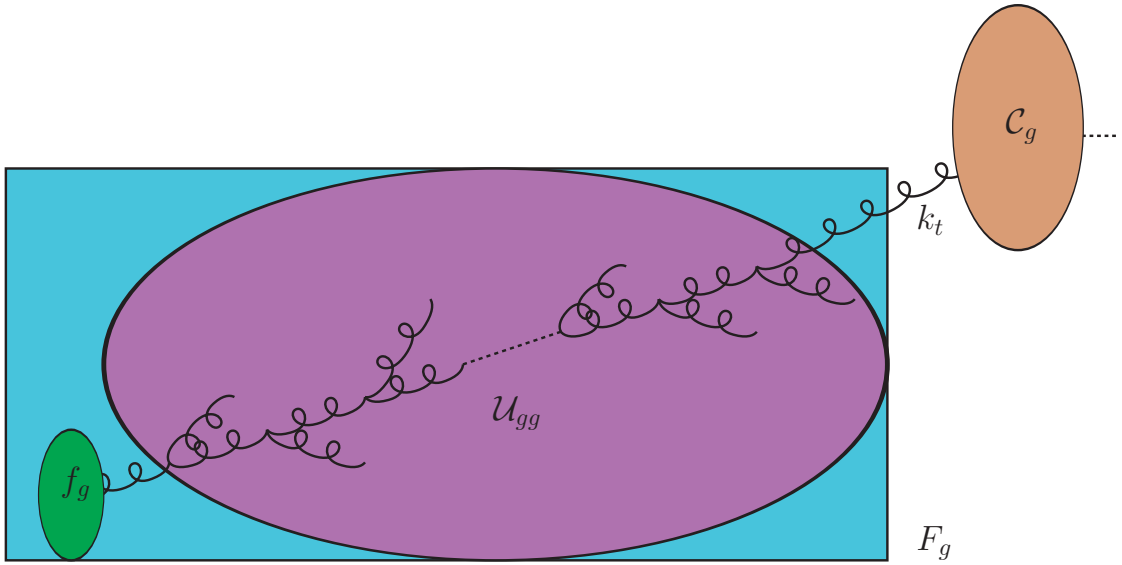
Resummation with HELL

[Bonvini, Marzani, Peraro 1607.02153]
[Bonvini, Marzani, Muselli 1708.07510] [Bonvini, Marzani 1805.06460]



For $Q \sim 5$ GeV $\rightarrow$ **perturbative instability** from $x \sim 10^{-3}$ and below.

k_t-factorisation and small-x resummation



Mellin transformation:

$$h(N) = \int_0^1 \mathrm{d}z\, z^N h(z)\,,$$

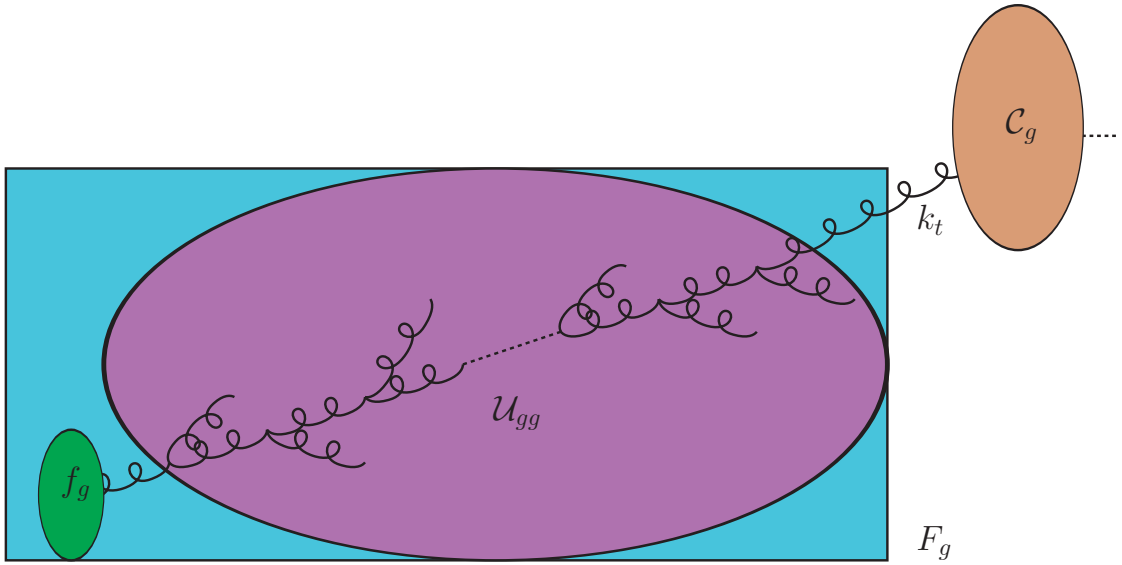
Collinear factorisation:

$$\sigma\left(N, Q^2\right) = C_i\left(N, \alpha_s, \mu_{\mathrm{F}}^2\right) f_i\left(N, \mu_{\mathrm{F}}^2\right)$$

k_t-factorisation: [Catani, Hautmann hep-ph/9405388]

$$\sigma\left(N, Q^2\right) = \int \mathrm{d}k_t^2\, \mathcal{C}_g\left(N, \alpha_s, Q^2, k_t^2\right) \mathcal{F}_g\left(N, Q^2, k_t^2\right)$$

k_t-factorisation and small-x resummation



Mellin-space formalism, fixed coupling:

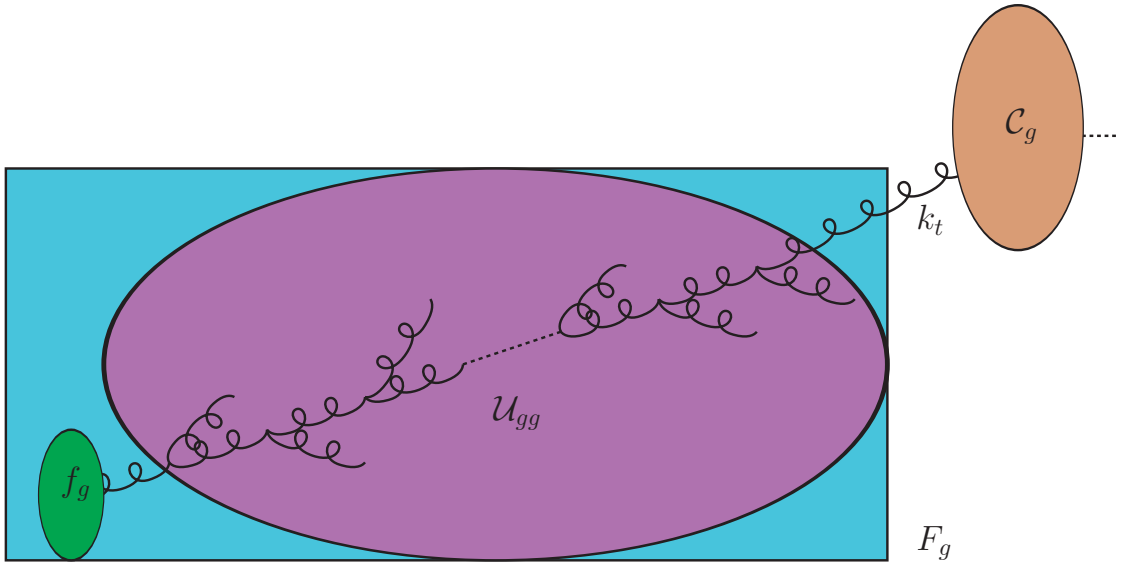
$$U\left(N, k_t^2, Q^2\right) = \gamma(\alpha_s, N)\left(\frac{k_t^2}{Q^2}\right)^{\gamma(\alpha_s, N)},$$

$$\mathcal{F}_g\left(N, k_t^2, Q^2\right) = U_{gg}\left(N, k_t^2, Q^2\right) f_g\left(N, Q^2\right)$$

Anomalous dimension resummation [Altarelli, Ball, Forte 0802.0032]

Direct space formalism [Bonvini, Marzani, Peraro 1607.02153]

k_t-factorisation and small-x resummation



Return to direct space $\rightarrow$ `HELL` method

$$C_g\left(z,\alpha_s,Q^2\right)=\int_z^1\frac{\mathrm{d}z'}{z'}\int\mathrm{d}k_t^2\,\mathcal{C}_g\left(\frac{z}{z'},\alpha_s,Q^2,k_t^2\right)U_{gg}\left(z',k_t^2,Q^2\right)$$

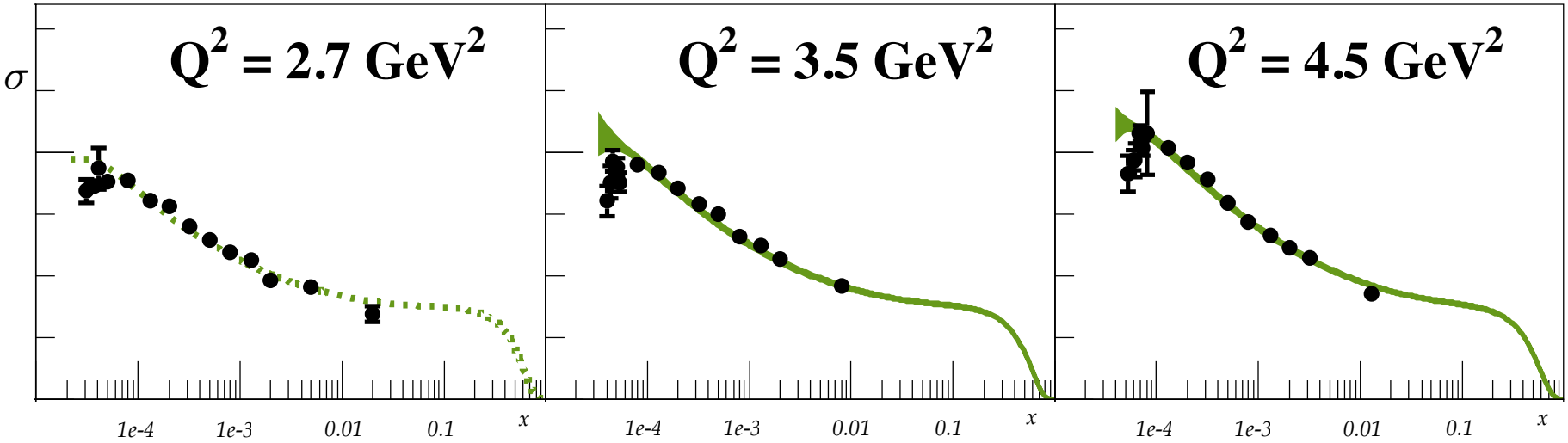
Resummation with `HELL` [Bonvini, Marzani, Peraro 1607.02153]

[Bonvini, Marzani, Muselli 1708.07510] [Bonvini, Marzani 1805.06460]

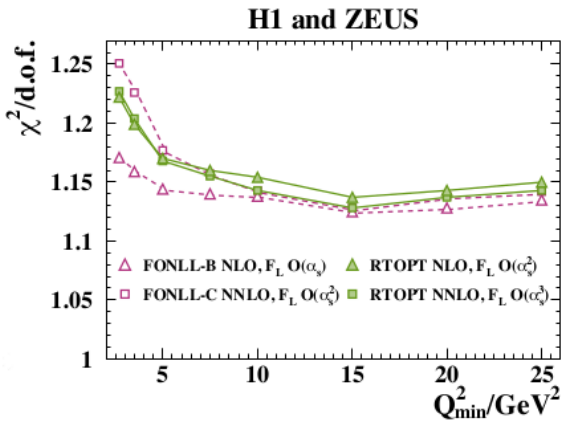
Low x at HERA

Deep-inelastic scattering (DIS) data from HERA extend down to $x \sim 3 \times 10^{-5}$

Tension between HERA data with theory at low Q^2 and low x



deterioration of the χ^2 when including low-Q^2 data



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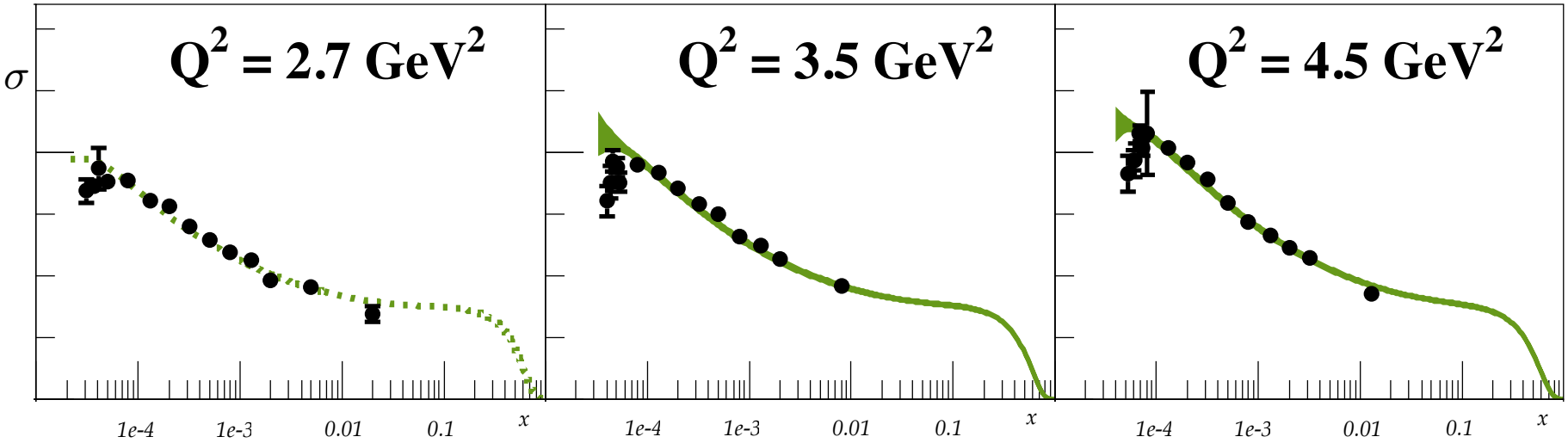

Attempts to explain this deviation with higher twists, phenomenological models, ...

$$\sigma_{r,\mathrm{NC}} = F_2(x_{\mathrm{Bj}}, Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x_{\mathrm{Bj}}, Q^2) \qquad y = \frac{Q^2}{x_{\mathrm{Bj}} s}$$

Larger F_L can drive the turnover in DIS

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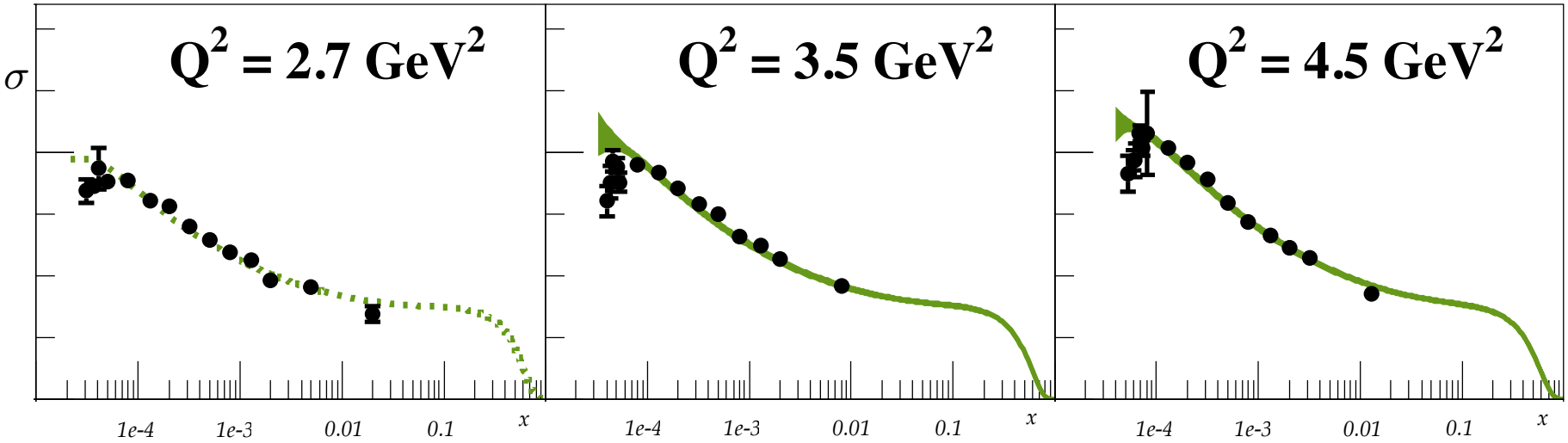
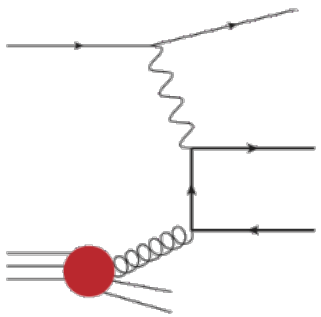


$F_L = \mathcal{O}(\alpha_s)$ and gluon dominated

$\rightarrow$sensitivity to small-x resummation

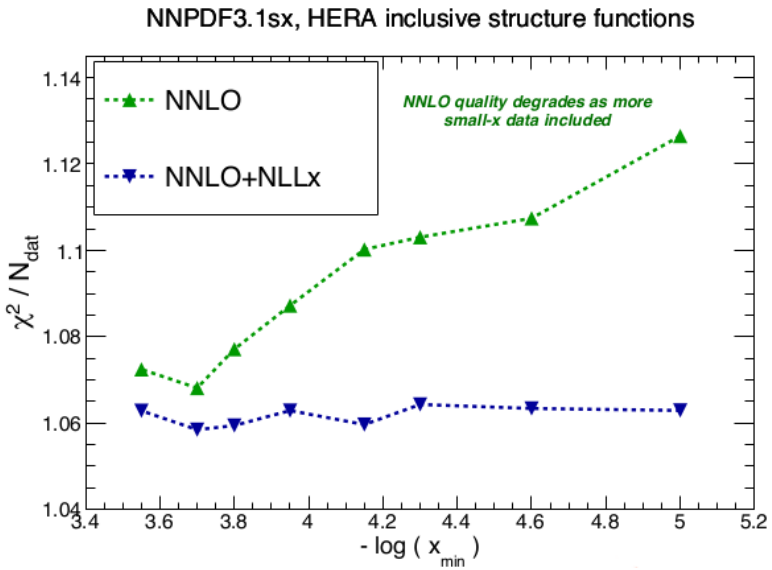


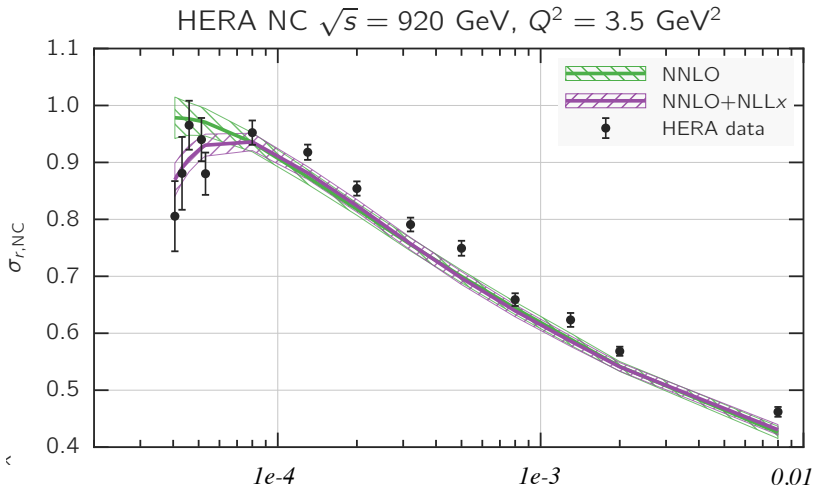
Fits with small-x resummed theory

Successful description of this region when including small-x resummation!

- NNPDF3.1 framework [Ball,Bertone,Bonvini,Marzani,Rojo,Rottoli 1710.05935]
- xFitter framework [xFitterCollaboration,Bonvini 1802.00064]

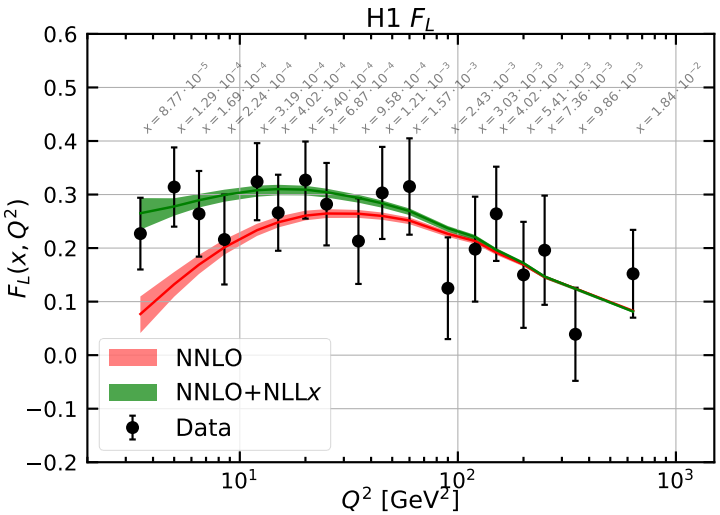
Turnover reproduced →



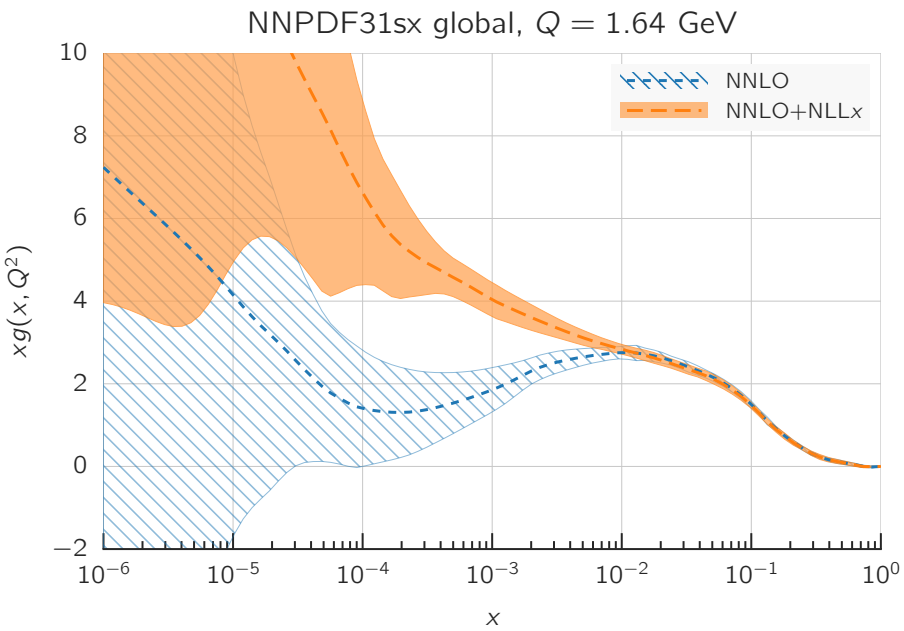


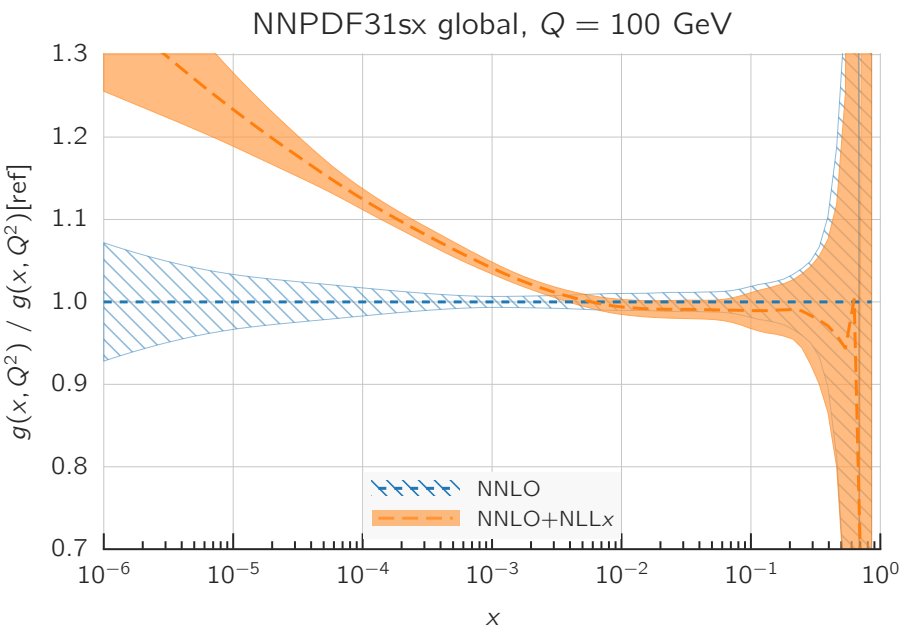
$\chi^2/N_{\rm dat}$	NNLO	NNLO+NLLx
xFitter	**1.23**	**1.17**
NNPDF3.1sx	**1.130**	**1.100**
		reduction!

No extra parameters →**refined theory only**



Impact on gluon PDF





Dramatic effect of resummation on the gluon PDF at $x \lesssim 10^{-3}$

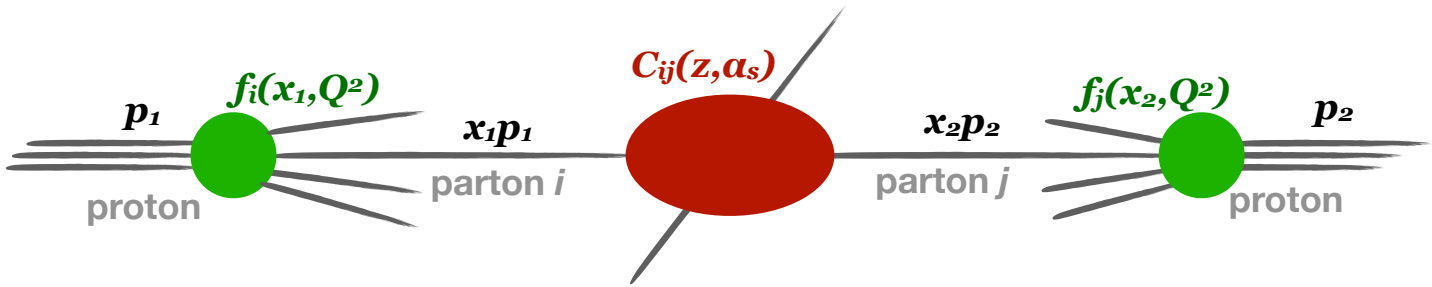
- Larger effect at higher energy scales $\rightarrow$impact for LHC and FCC-hh
- Onset of saturation at some point

Resummation of LHC observables

Differential cross section in collinear factorization

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\ldots} = \int_x^1 \frac{\mathrm{d}z}{z} \int \mathrm{d}\hat{y}\, \mathscr{L}_{ij}\left(\frac{x}{z}, \hat{y}, Q^2\right) \frac{\mathrm{d}C_{ij}}{\mathrm{d}y\ldots}(z, Y-\hat{y}, \ldots, \alpha_s)$$

$$\mathscr{L}_{ij}\left(x, \hat{y}, Q^2\right) = f_i(\sqrt{x}e^{\hat{y}}, Q^2)\, f_j(\sqrt{x}e^{-\hat{y}}, Q^2)\, \vartheta(e^{-2|\hat{y}|} - x)$$



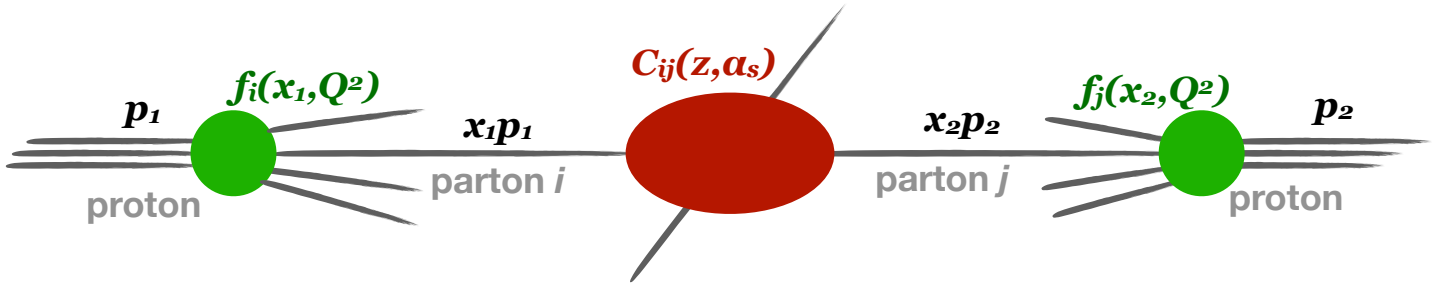
$\frac{Q^2}{s} = x < z$ **note:** typically $\sqrt{z}\mathrm{e}^{\pm\hat{y}} \sim \sqrt{x}$

Processes considered so far in HELL:

- $gg \to H$ (inclusive cross section) $\to$(pending fully differential) [Bonvini,Marzani 1802.07758] [Bonvini 1805.08785]
- $c\bar{c}$, $b\bar{b}$ pair production (fully differential) [Bonvini,FS 2211.10142]
- Drell-Yan (fully differential) *(in progress)*

Differential resummation at proton collider

$$\frac{\mathrm{d}C_{gg}}{\mathrm{d}y\dots}(z,y,\dots) = \int_0^\infty \mathrm{d}k_1^2 \int_0^\infty \mathrm{d}k_2^2 \int_z^1 \frac{\mathrm{d}x}{x} \int \mathrm{d}\hat{y}\, \frac{\mathrm{d}\mathcal{C}_{gg}}{\mathrm{d}y\dots}(x, y-\hat{y}, k_1^2, k_2^2, \dots, \alpha_s)$$
$$\times \frac{\mathrm{d}}{\mathrm{d}k_1^2} U\left(\sqrt{\frac{z}{x}}e^{\hat{y}}, k_1^2, Q^2\right) \frac{\mathrm{d}}{\mathrm{d}k_2^2} U\left(\sqrt{\frac{z}{x}}e^{-\hat{y}}, k_2^2, Q^2\right) \vartheta\left(e^{-2|\hat{y}|} - \frac{z}{x}\right)$$



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Impact of resummation in ggH

$gg \to H$ inclusive cross section

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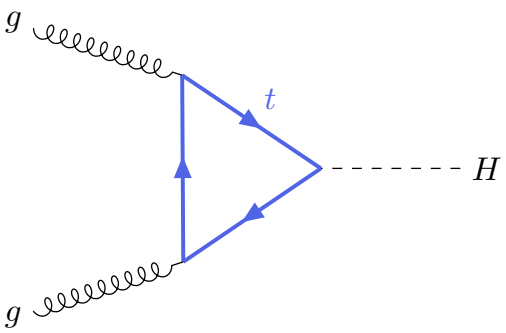
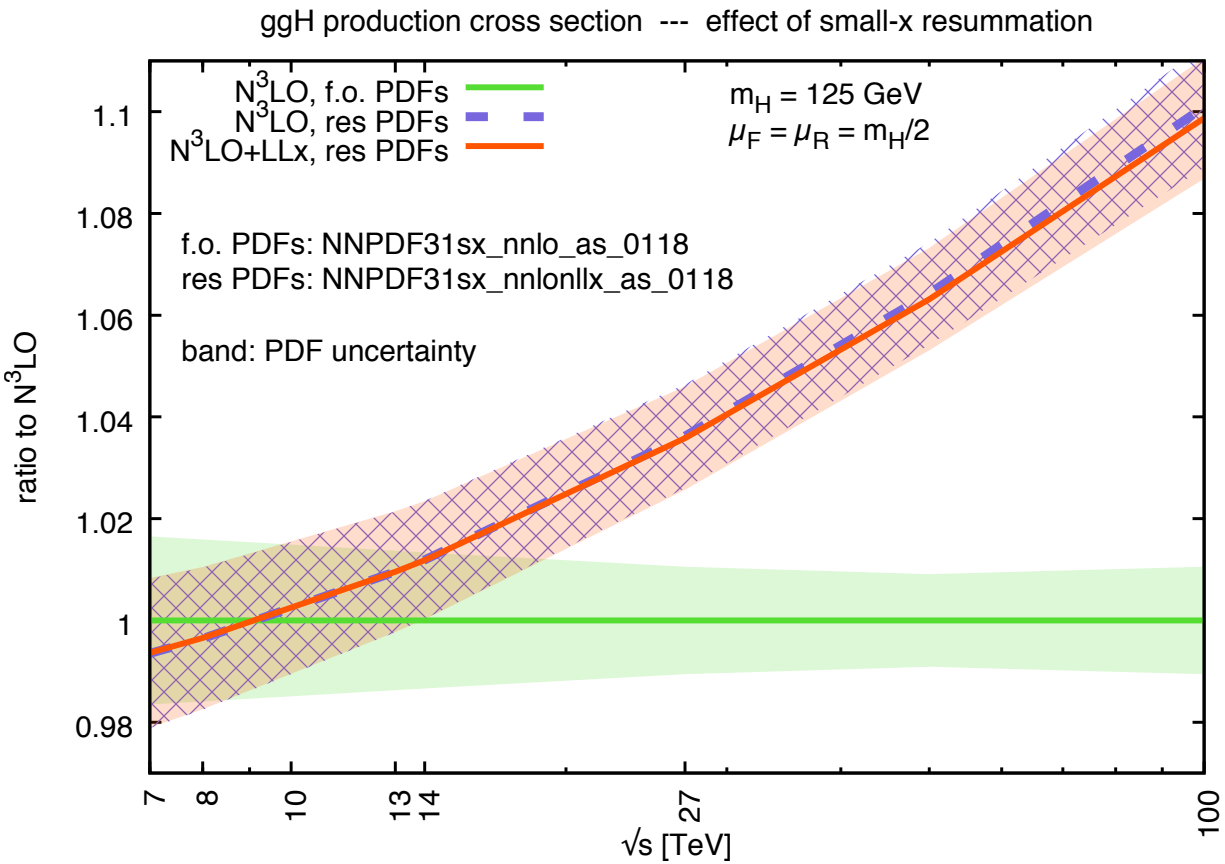




ggH cross section at FCC-hh can be $\sim 10\%$ larger than expected with NNLO PDFs!

At LHC $+1\%$ effect+ expected enhancement at differential level

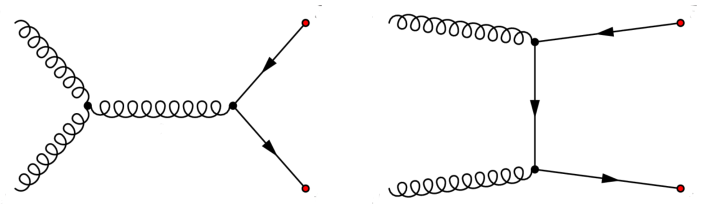
Progress toward full NLLx resummation

[Celiberto et al. 2008.00501]

[Celiberto et al. 2205.02681]

Charm (and bottom) pair production at LHC

Fully differential heavy-quark pair production [Bonvini,FS 2211.10142]

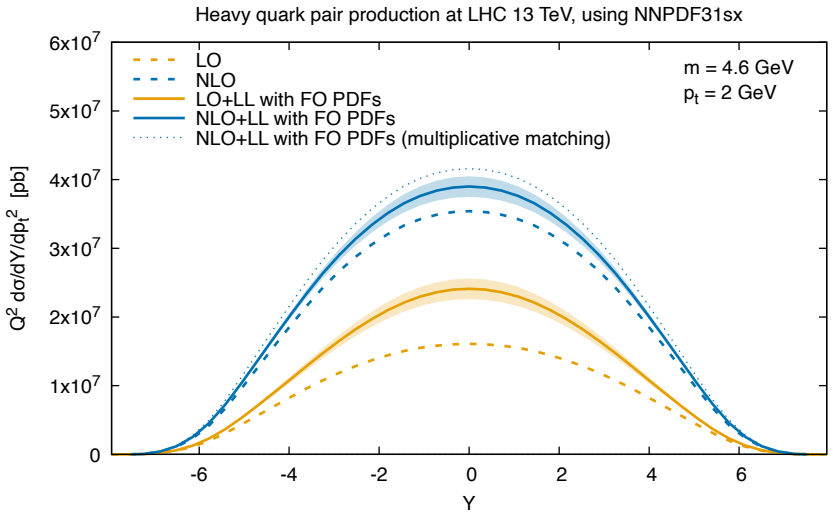


$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}q_t} \rightarrow$ pair kinematics

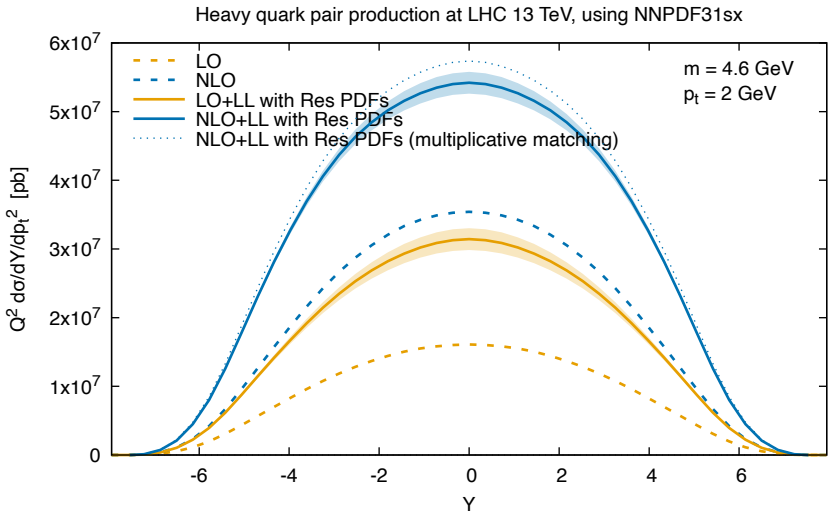
$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}p_t} \rightarrow$ single kinematics

Small-x resummation crucial for **charm and bottom production**

Key process at forward physics experiment e.g. FPF [Feng et al 2203.05090]



FO PDFs ↑ **↓ Resummed PDFs**



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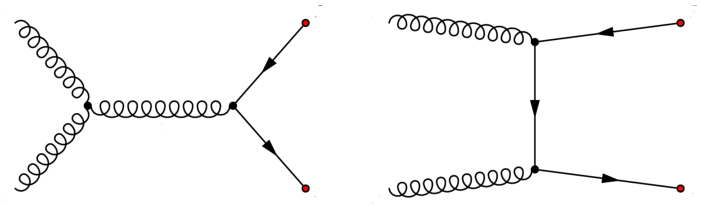

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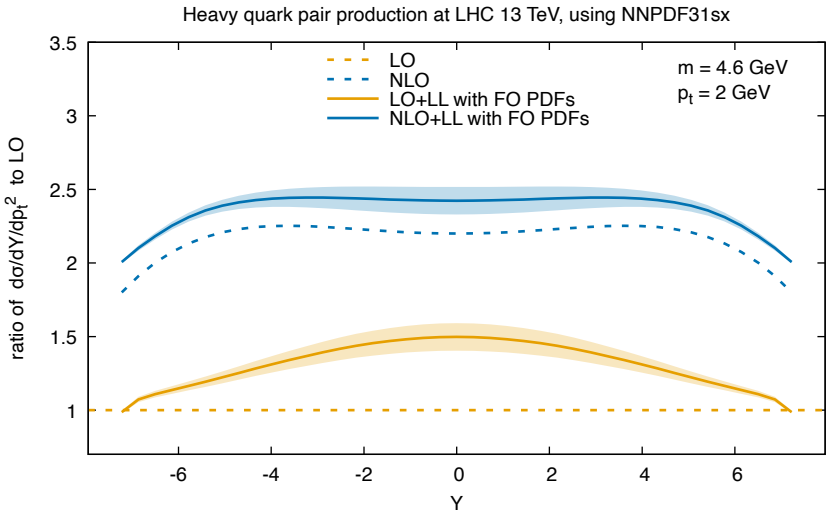

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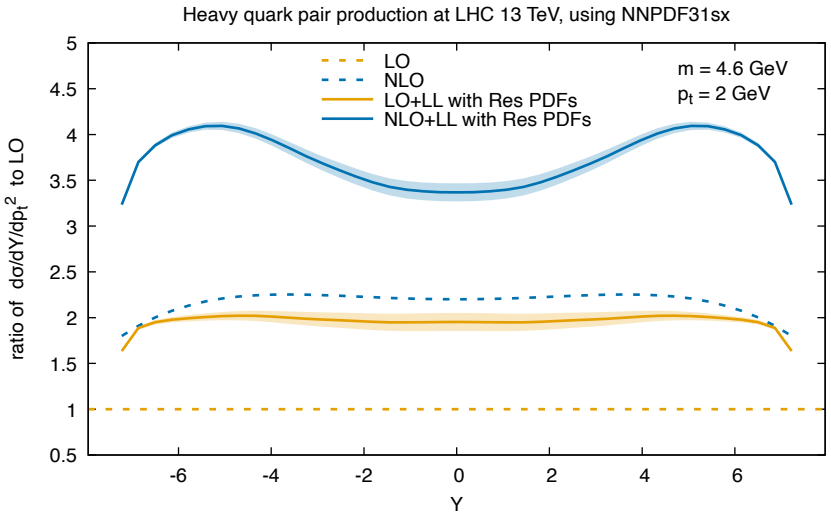

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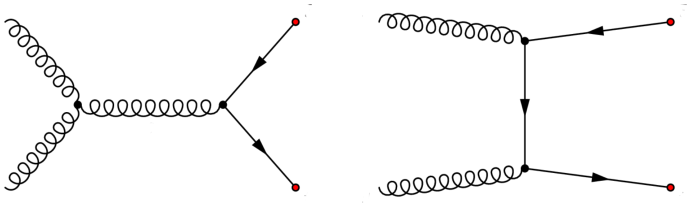

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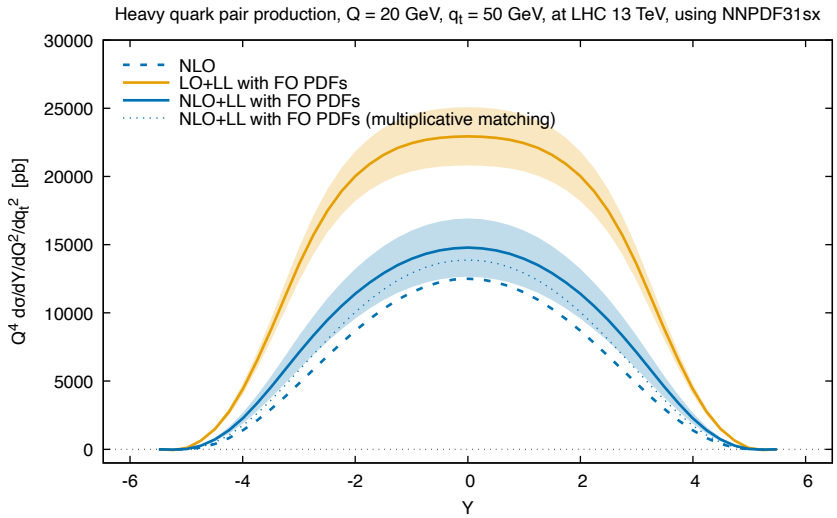

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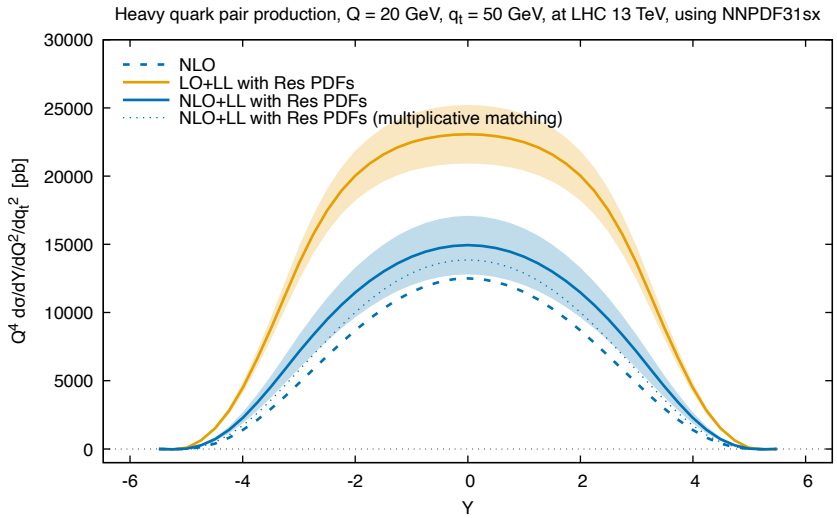
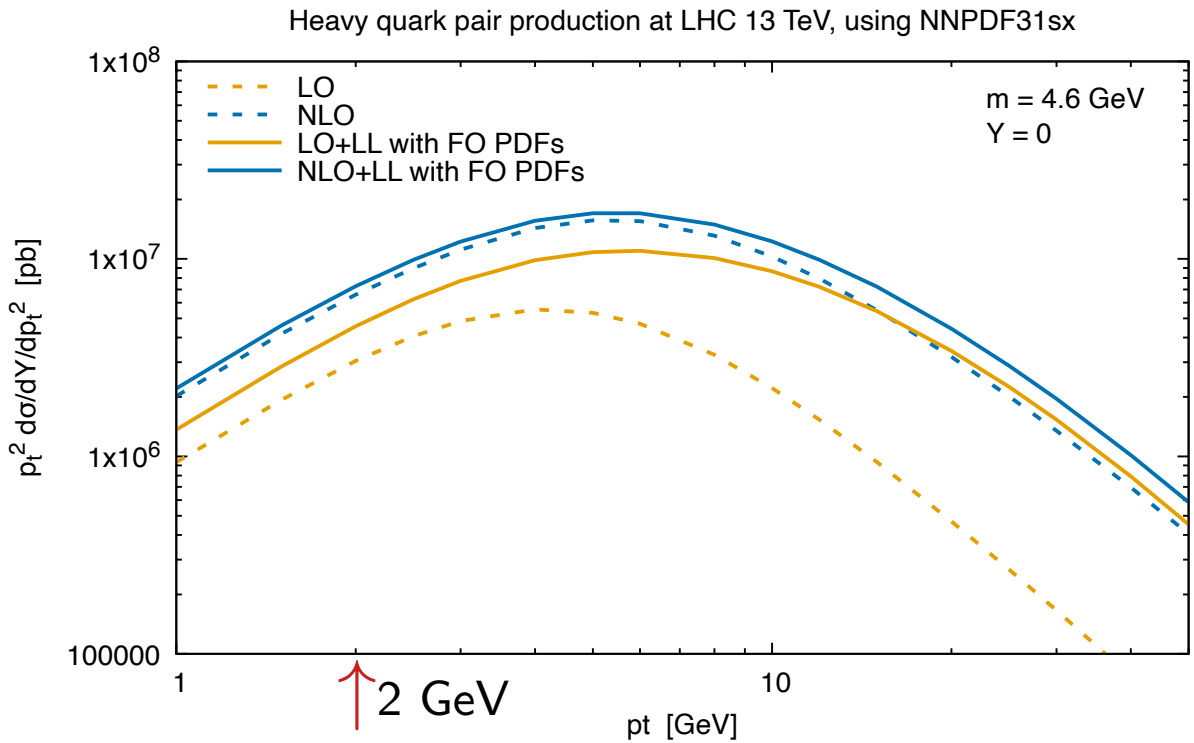
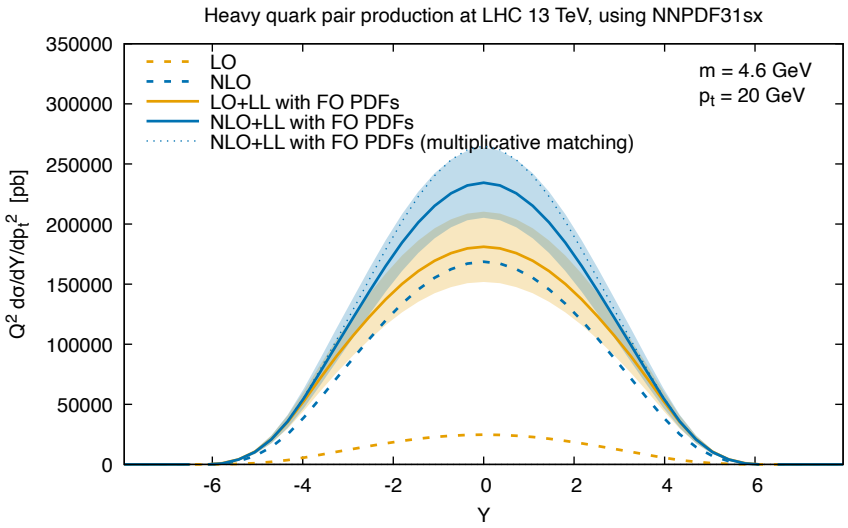


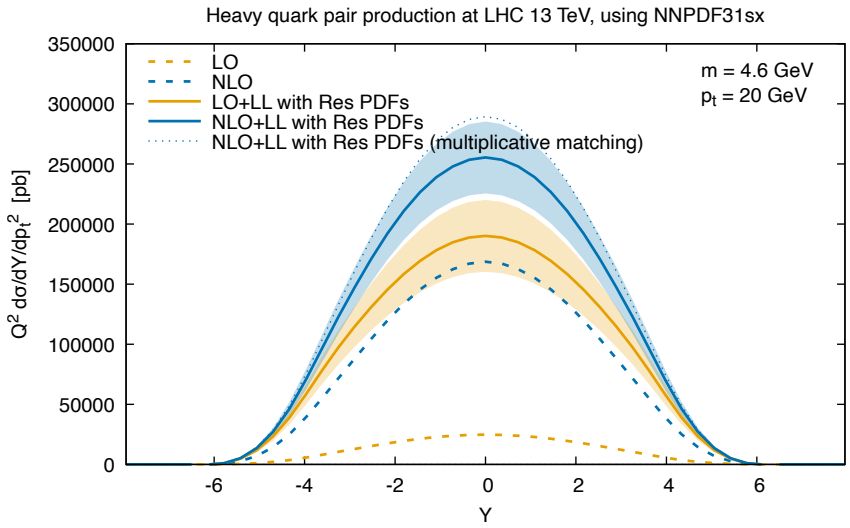
Results for $\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}p_t}$ in **single quark kinematics** [Bonvini,FS 2211.10142]



Heavy-quark pair production at LHC

Results for $\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}p_t}$ in **single quark kinematics** [Bonvini,FS 2211.10142]





Conclusion and outlook

Key messages:

- Resummation is needed at small-x, especially $x \lesssim 10^{-3}$
- Significant impact expected at LHC at low invariant mass and large rapidity
- Future colliders will be sensitive to this effect

Outlook:

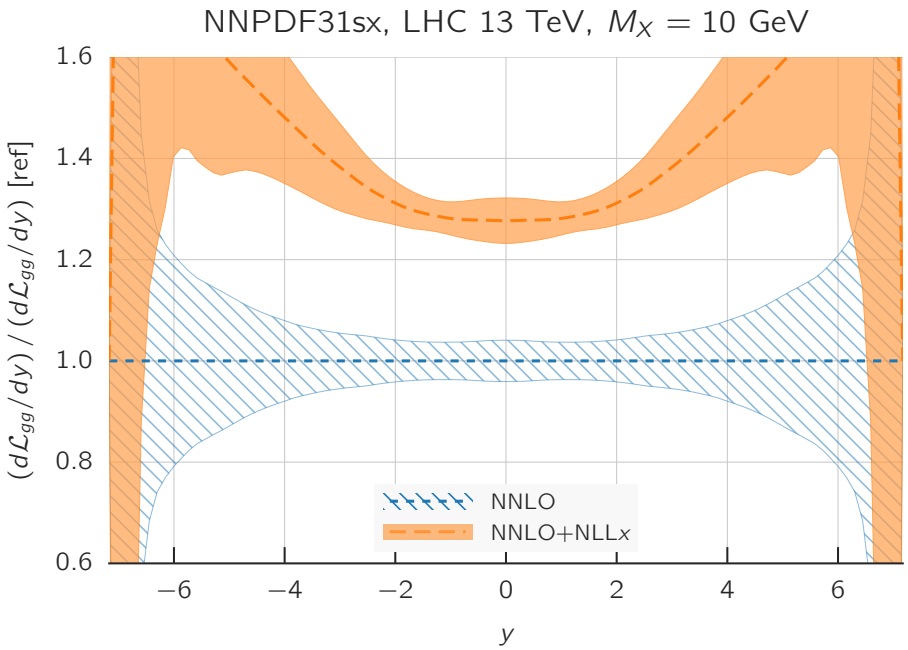
- For $c\bar{c}, b\bar{b}$ we considered only quark-level final states
  $\rightarrow$ add hadronization - update of resummed PDFs with LHCb data [Aaij et al. 1510.01707]
  $\rightarrow$Match to NNLO results - missing NLLx coefficient function [Mazzitelli, Ratti, Wiesemann, Zanderighi 2302.01645]
- Differential formalism can be applied to other LHC processes
  $\rightarrow$ Drell-Yan (urgent) or Higgs production (easier)

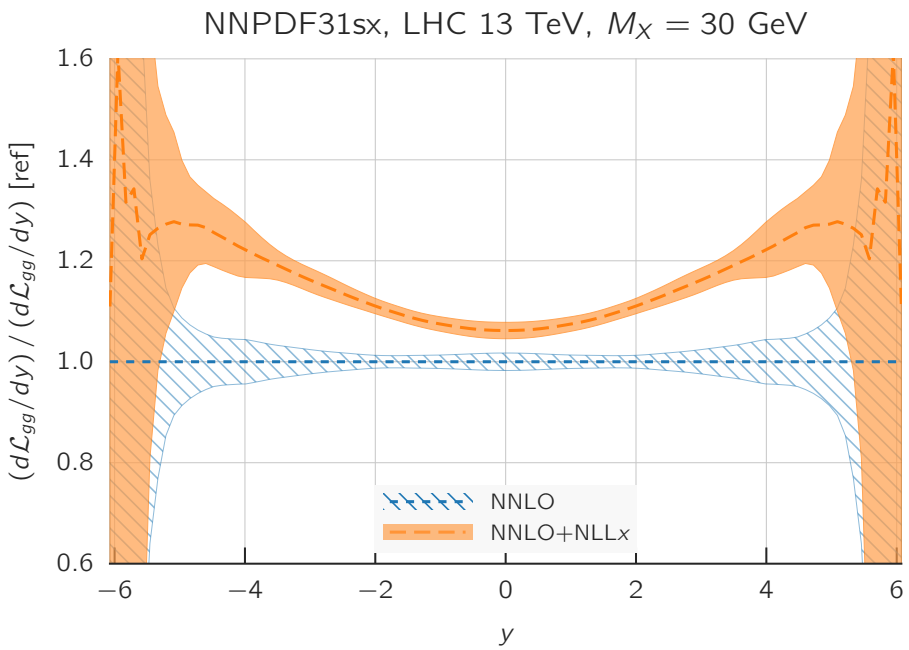
Thanks for you attention!

Backup

Impact on parton luminosities

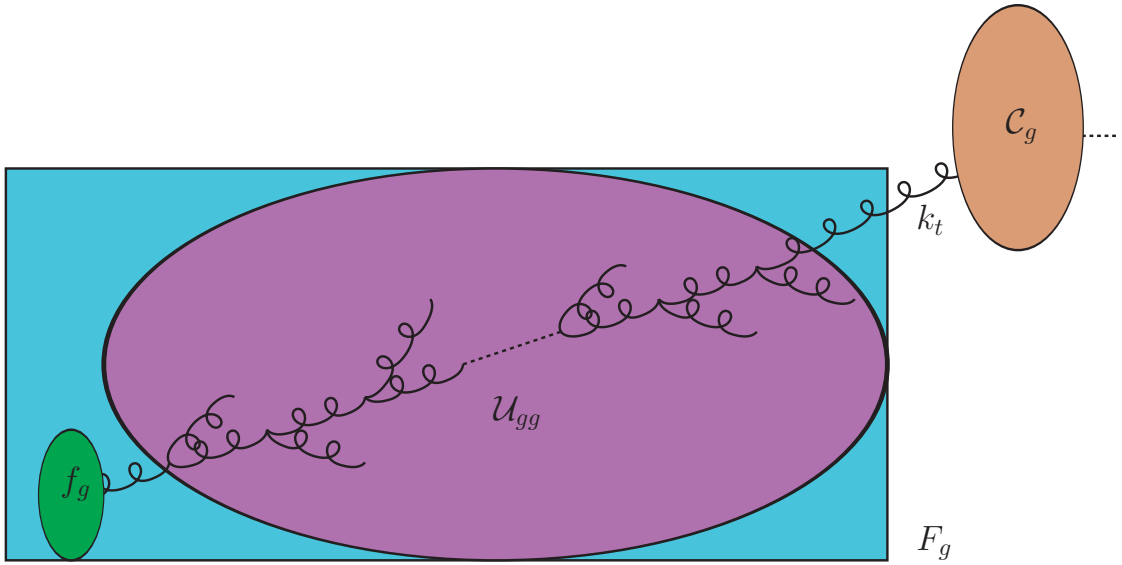
Gluon-gluon parton luminosities $\mathscr{L}_{gg}$





Significant impact for small invariant masses, especially at large rapidities

k_t-factorisation and small-x resummation



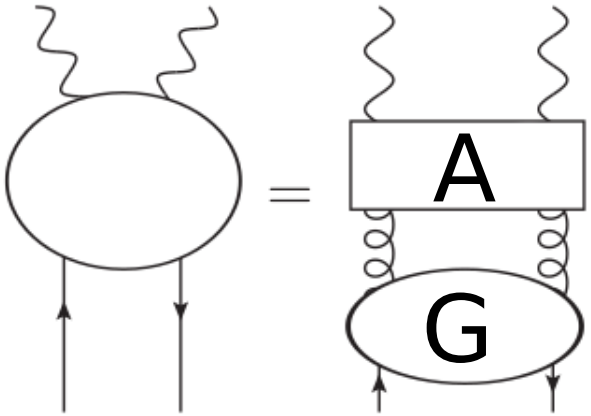
$\mathcal{C}(\cdots) \rightarrow$ off-shell continuation collinear Born counterpart.

- This is the only process dependent part of the computation.
- 2 Gluon Irreducible (2GI) in t channel $\leftarrow$ gauge-invariance at Born level.
- Off-shellness of the incoming gluon

$$k_{\text{collinear}} \rightarrow k_{\text{in}} = z p_1 + k_t, \qquad \frac{1}{D-2} \sum_{\lambda} \varepsilon_{\lambda}^{\mu}(k_{\text{in}}) \varepsilon_{\lambda}^{*\nu}(k_{\text{in}}) \rightarrow -\frac{k_t^{\mu} k_t^{\nu}}{k_t^2}$$

$$\left\langle -\frac{k_t^{\mu} k_t^{\nu}}{k_t^2} \right\rangle_{\hat{k}_t} \xrightarrow[|k_t| \rightarrow 0]{} \frac{1}{D-2} d^{\mu\nu}(z_1 p_1, n)$$

Factorisation of the off-shell coefficient function



2GI decomposition [Catani, Ciafaloni, Hautmann Nucl.Phys.B 366 (1991) 135-188]

$$Q^2\sigma = \frac{Q^2}{2S}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\left[A_{0,\mu\nu}(k,q) + \frac{\alpha_s}{4\pi}A_{1,\mu\nu}(k,q,n)\right]d^{\mu\mu'}(k,n)d^{\nu\nu'}(k,n)G_{\mu'\nu'}(p_1,k)\,.$$

at LO

n = Light cone gauge axis

$$\frac{Q^2}{2S}\int \frac{\mathrm{d}^4k}{(2\pi)^4}[A_{0,\mu\nu}(k,q)]d^{\mu\mu'}(k,n)d^{\nu\nu'}(k,n)G_{\mu'\nu'}(p_1,k) =$$
$$\int_x^1 \frac{\mathrm{d}z}{z}\int \mathrm{d}k_t^2\,\mathcal{C}_g\left(\frac{x}{z},\alpha_s,Q^2,k_t^2\right)\mathcal{F}_g\left(z,Q^2,k_t^2\right)$$

ansatz for G at NLL $\leftarrow$ from HELL evolutor + NLL resummation of an. dim

NNPDF and xFitter fits with small-x resummation

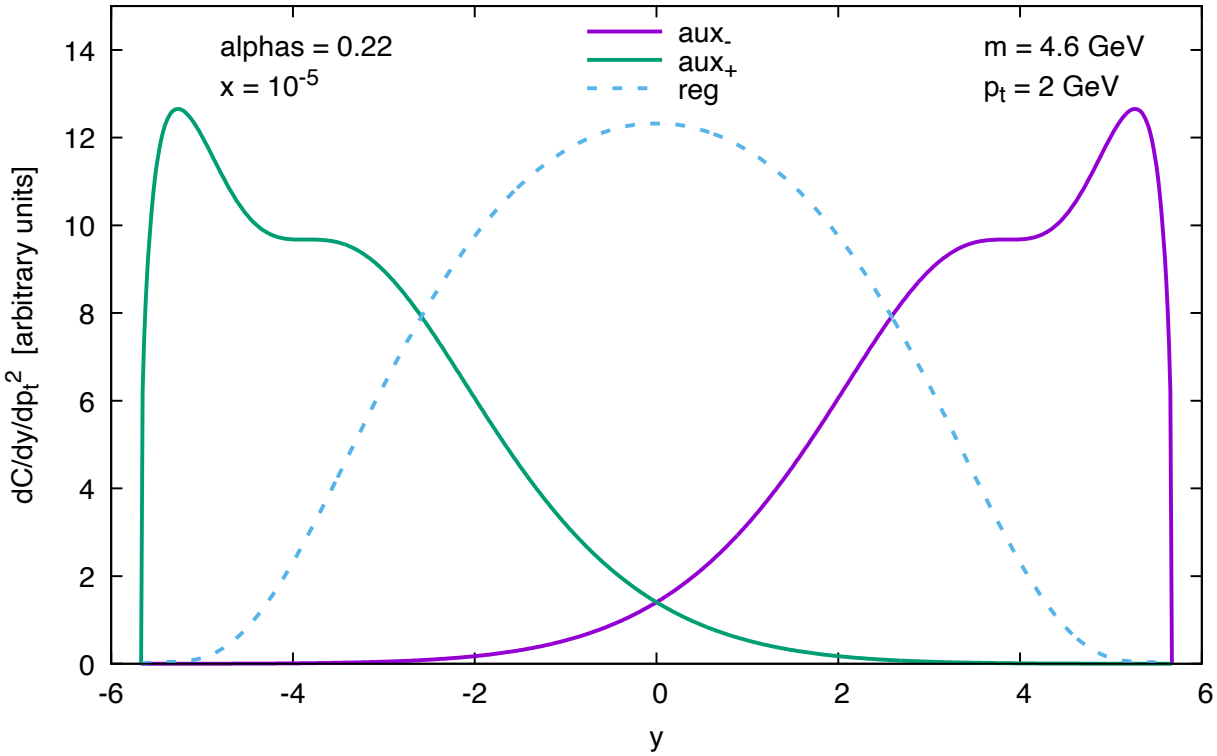
APFEL+HELL $\rightarrow$ make possible a PDF fit with small-x resummation

NNPDF3.1sx [1710.05935]	xFitter [1802.00064]
NeuralNet parametrization of PDFs	polynomial paramterization
MonteCarlo uncertainty	Hessian uncertainty
VFNS: FONLL	VFNS: FONLL
charm PDF is fitted	charm PDF perturbatively generated
DIS+tevatron+LHC (~ 4000 datapoints)	only HERA data (~ 1200 datapoints)
NLO, NLO+NLL, NNLO, NNLO+NLL	NNLO, NNLO+NLL

One interesting difference in the HERA data we include:

Lowest Q^2 HERA bins	NNPDF3.1/HERAPDF2.0	NNPDF3.1sx/xFitter
$Q^2 = 3.5$ GeV^2	included	included
$Q^2 = 2.7$ GeV^2	excluded	included
$Q^2 = 2.0$ GeV^2	excluded	excluded

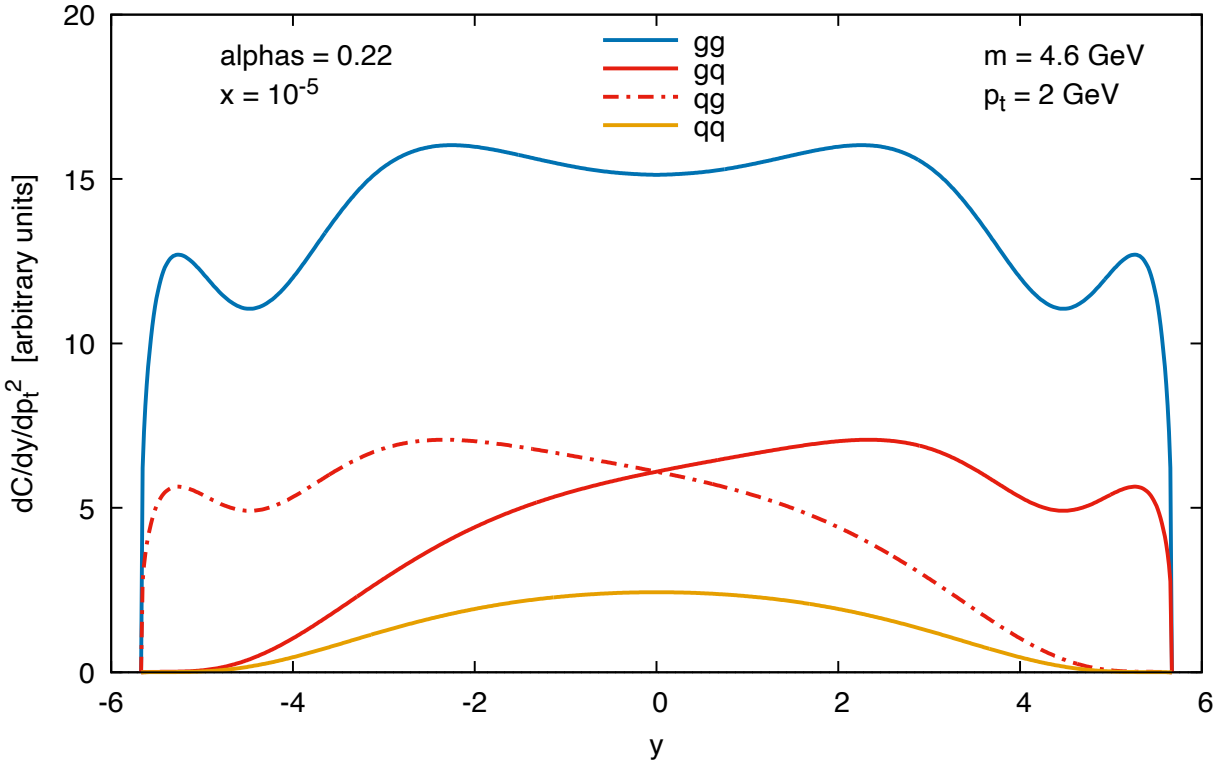
Results for $\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}p_t}$ in **single quark kinematics**



- Pure resummed
- Open heavy quark final state $\rightarrow$ add fragmentation to compare with exp. data
- Auxilliary channels provide dominant contribution $\rightarrow$ **hybrid factorisation**

Heavy-quark pair production at LHC

Results for $\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}p_t}$ in **single quark kinematics**

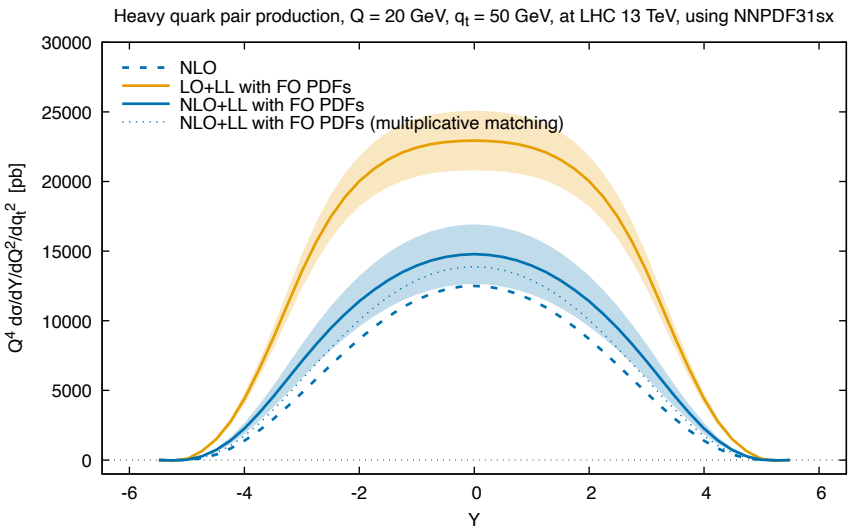


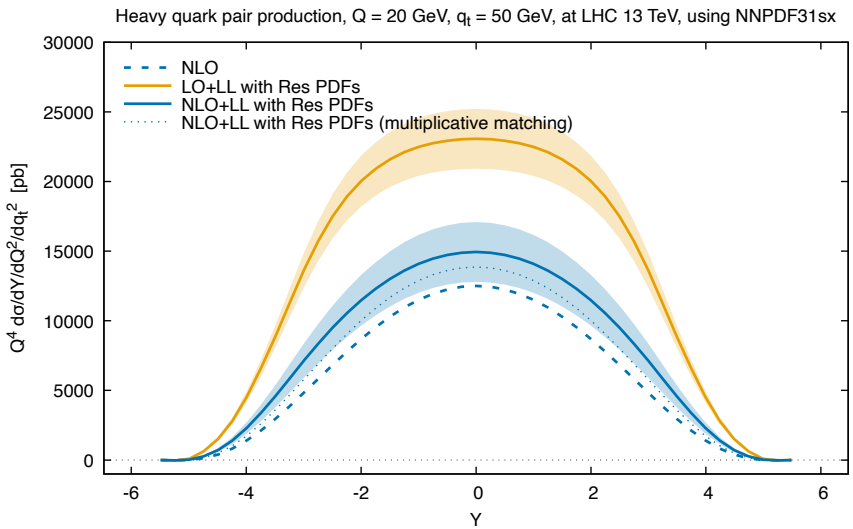
- Pure resummed
- Open heavy quark final state $\rightarrow$ add fragmentation to compare with exp. data

$$\mathcal{F}_g(N,\xi) = \left[U_{\mathrm{reg}}\left(N, Q^2\xi, \mu_{\mathrm{F}}^2\right) + \delta(\xi)\right] f_g(N,\mu_{\mathrm{F}}^2) + \frac{C_F}{C_A} U_{\mathrm{reg}}\left(N, Q^2\xi, \mu_{\mathrm{F}}^2\right) f_q(N,\mu_{\mathrm{F}}^2).$$

Heavy-quark pair production at LHC

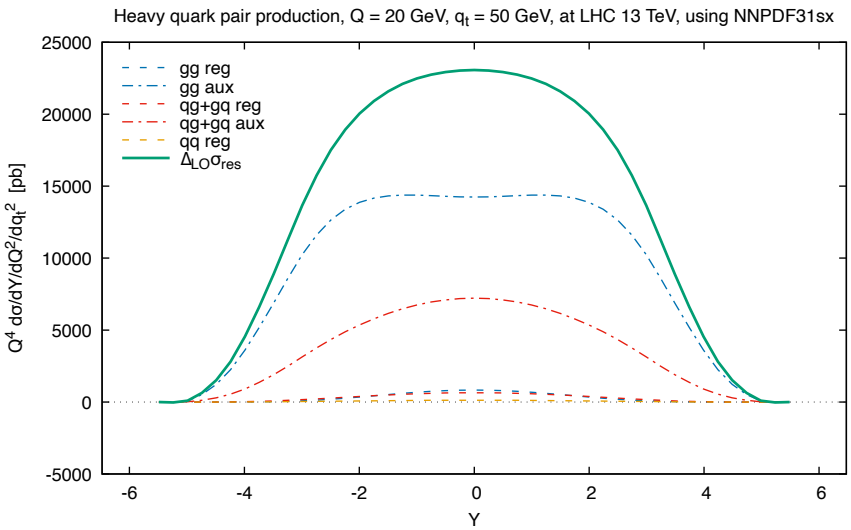
New results for $\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\, dY\, \mathrm{d}q_t}$ in **pair kinematics**

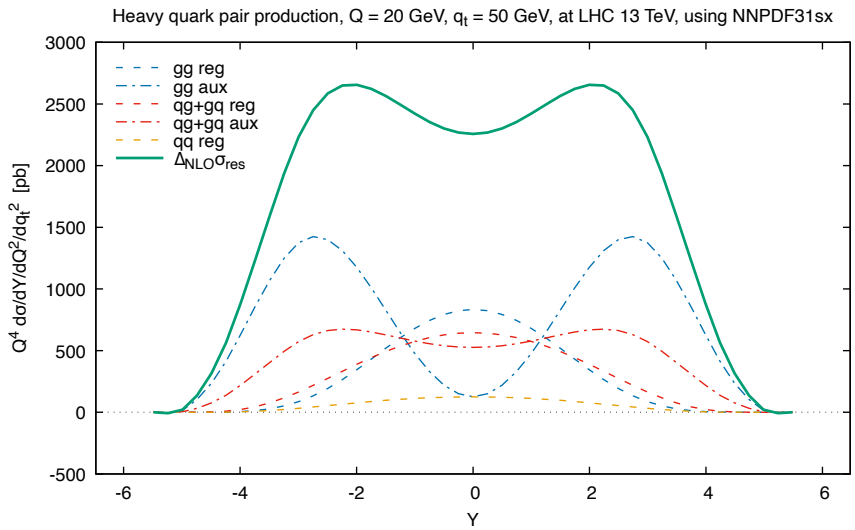




Heavy-quark pair production at LHC

New results for $\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\, dY\, \mathrm{d}q_t}$ in **pair kinematics**





Heavy-quark pair production at LHC

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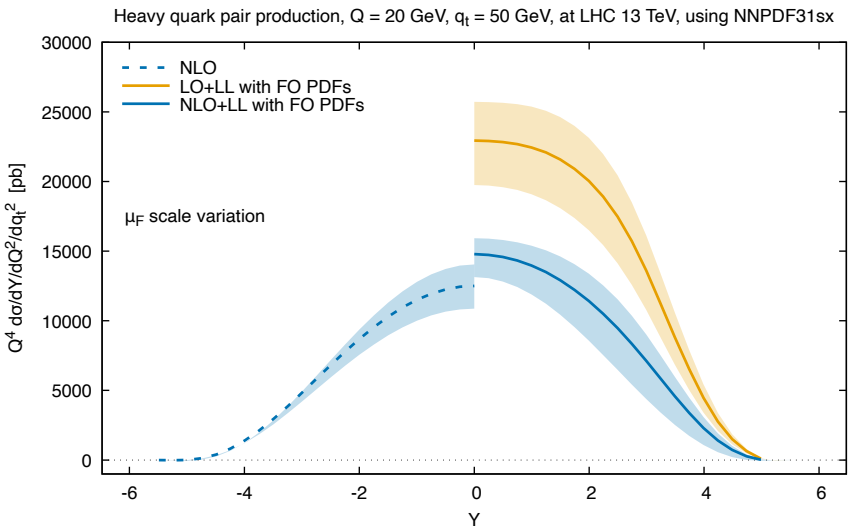
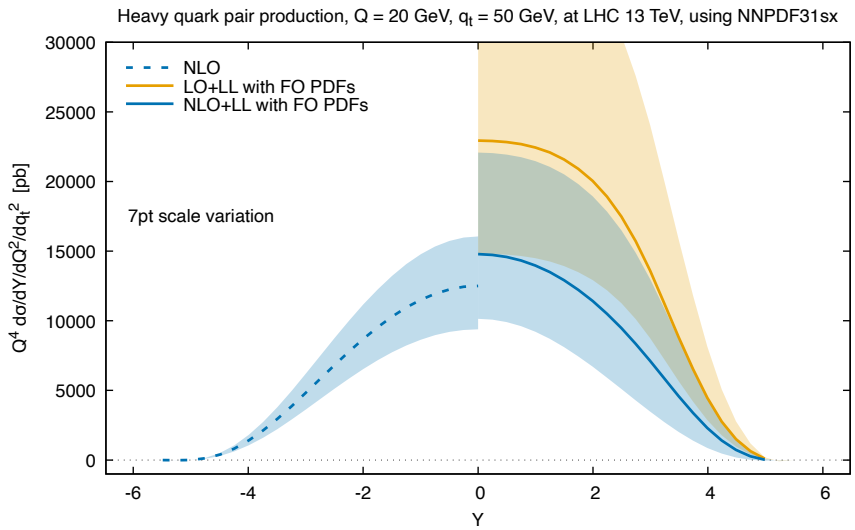
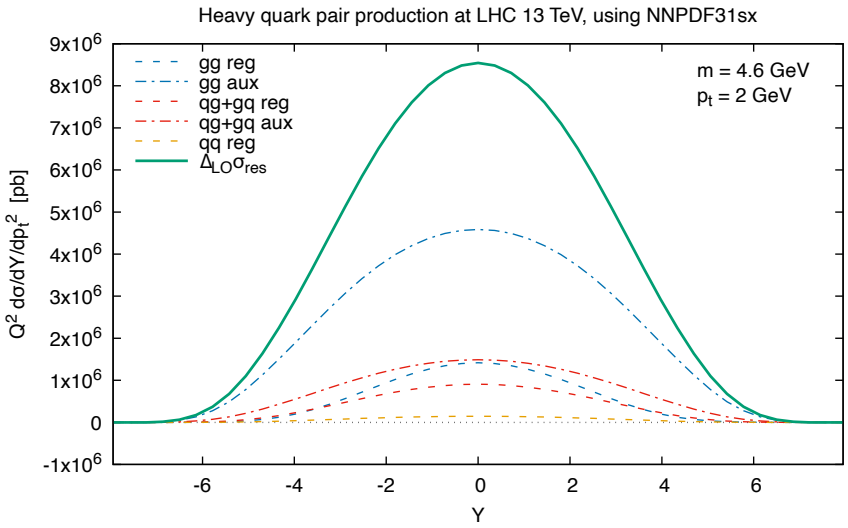
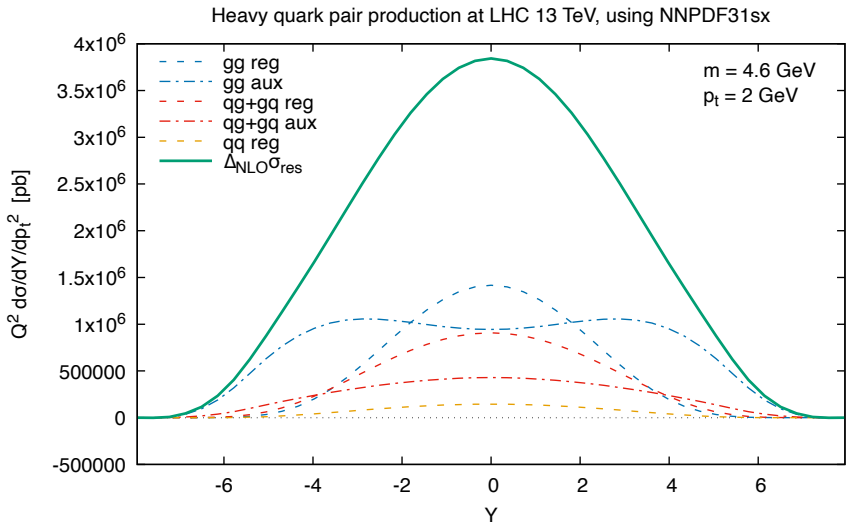


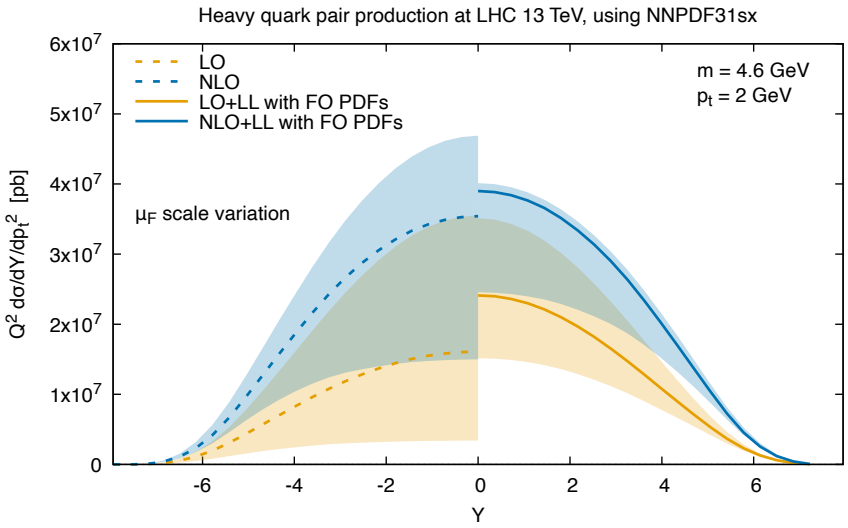


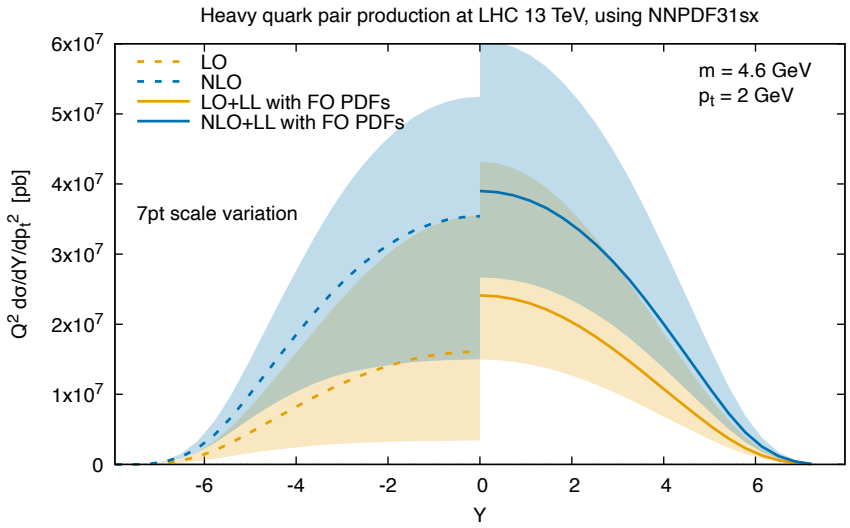
Results for $\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}p_t}$ in **single quark kinematics**



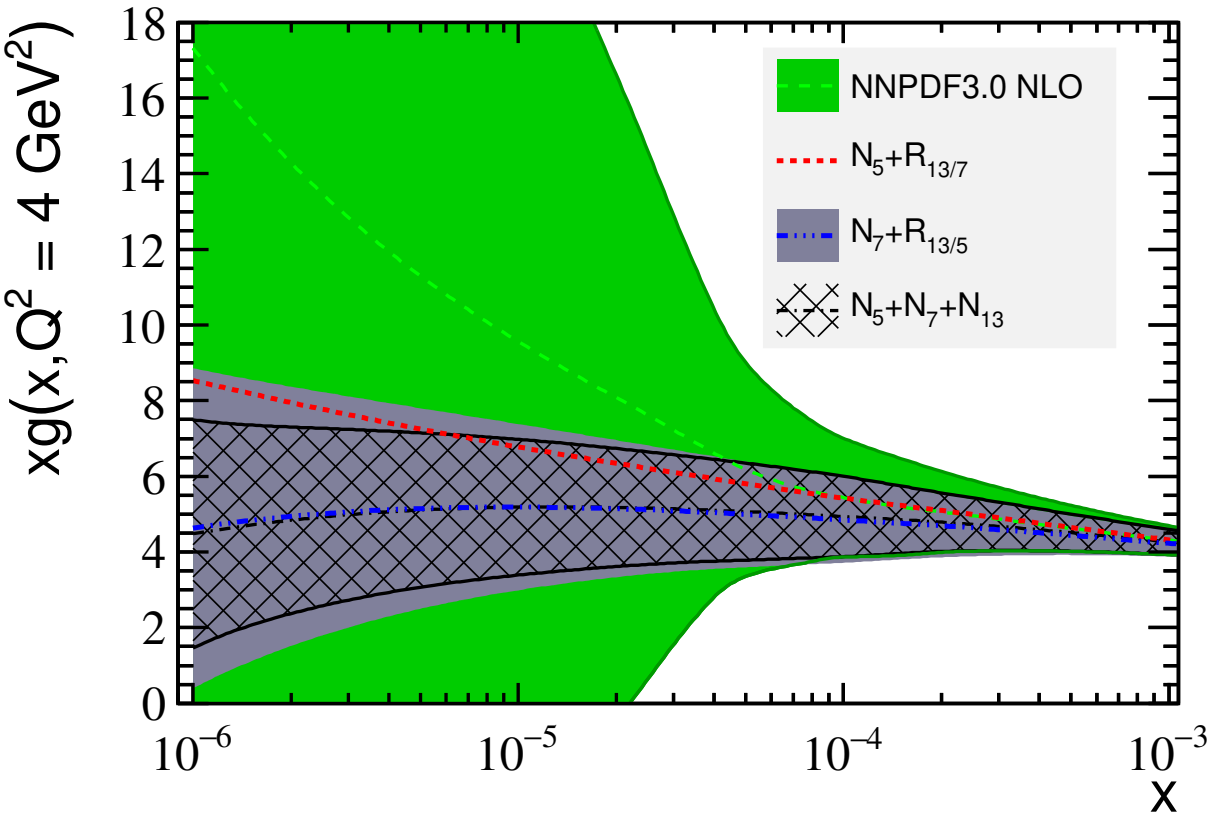


Results for $\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}p_t}$ in **single quark kinematics**





Impact of heavy meson data on gluon PDFGluon PDF uncertainty reduction from heavy quarks in LHC Run 2



[Gauld, Rojo 1610.09373]