# Colour evolution and infrared physics

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At the
MPI@LHC Workshop
Manchester | 23 November 2023

[mainly advocating Plätzer — JHEP 07 (2023) 126]

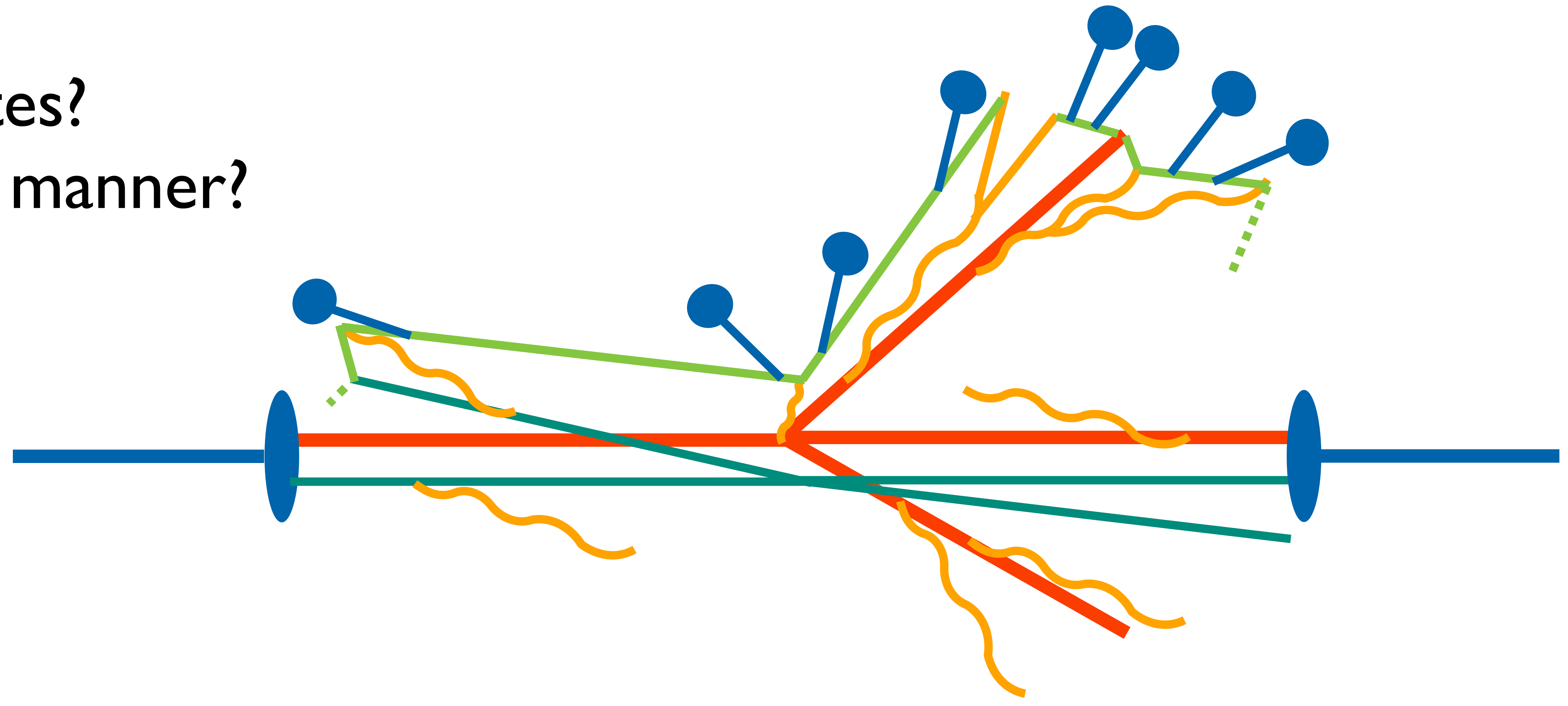
# Current challenges for event generators

How do we accurately describe details of final states?
How do we quantify precision in a comprehensive manner?

Matching beyond NLO QCD?
Solve shower bottlenecks first?

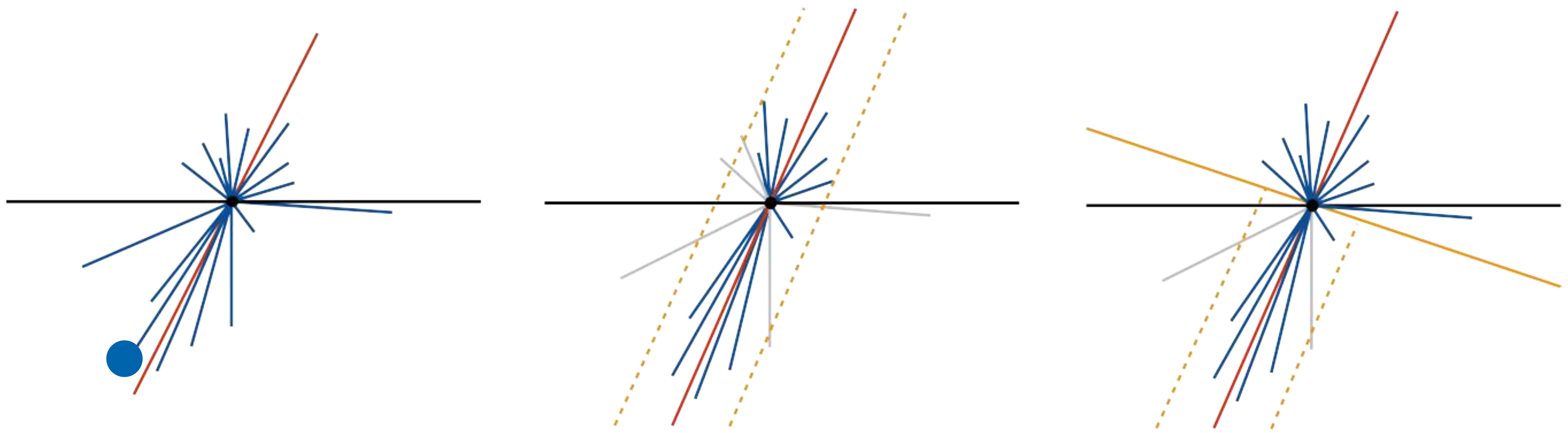
How to benchmark precision of QCD algorithms?
How to accurately include EW and QED?

How to constrain hadronization models?
What is their response to perturbative variations?



$$d\sigma \sim \mathrm{L} \times d\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

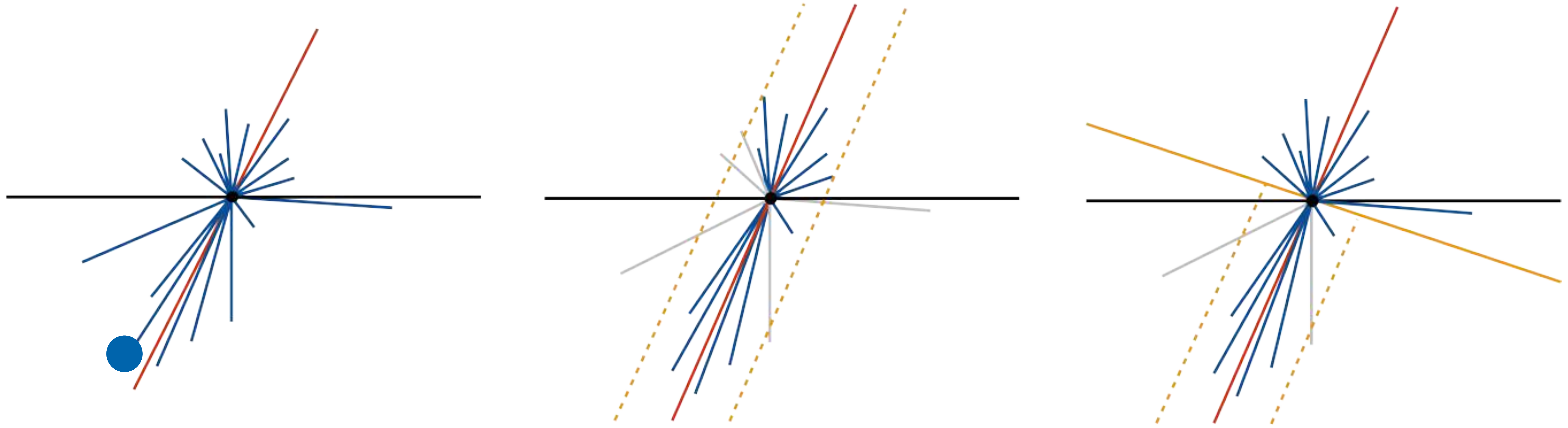
# Accuracy of Parton Showers



Fragmentation is fine if we get
collinear physics right.

# Accuracy of Parton Showers

[Catani, Trentadue, Webber, Marchesini ….]



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Global event shapes from coherent branching — for two jets.

$$H(\alpha_s) \times \exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

LL — qualitative

NLL — quantitative

NNLL — precision

$$\alpha_s L \sim 1$$

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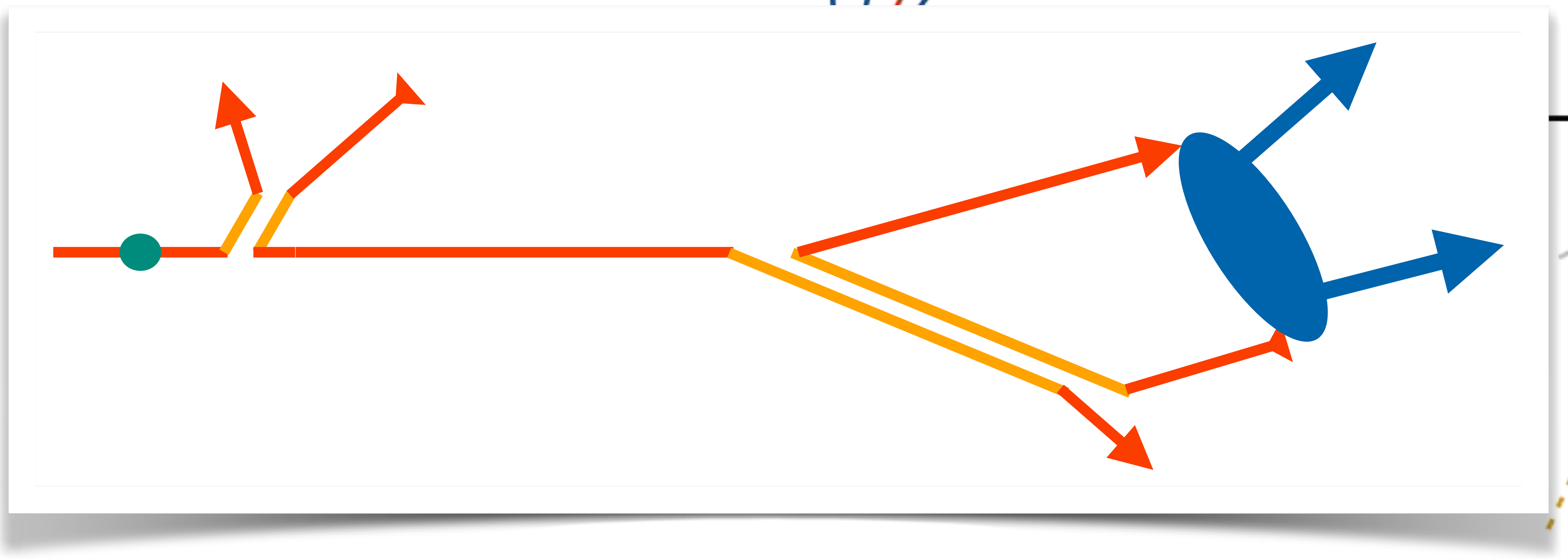

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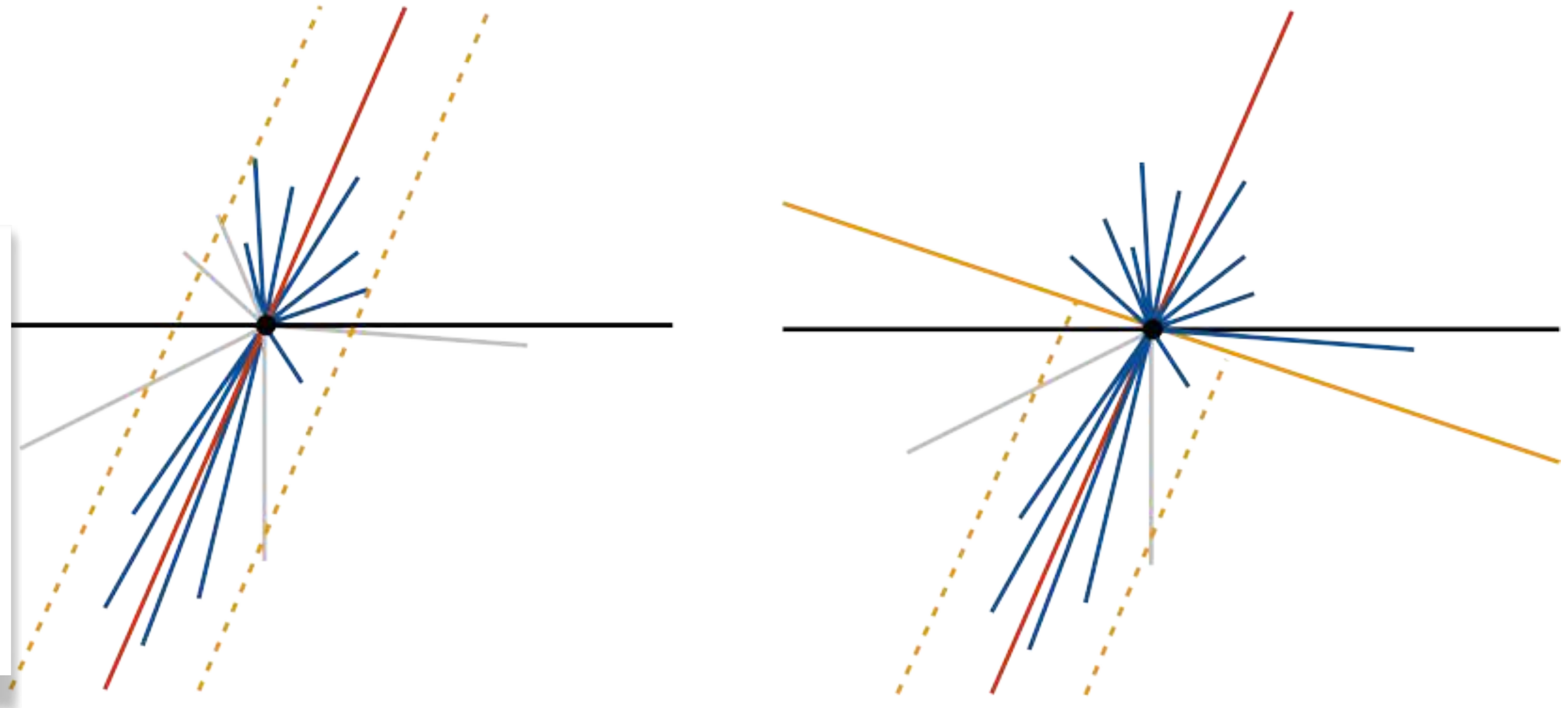

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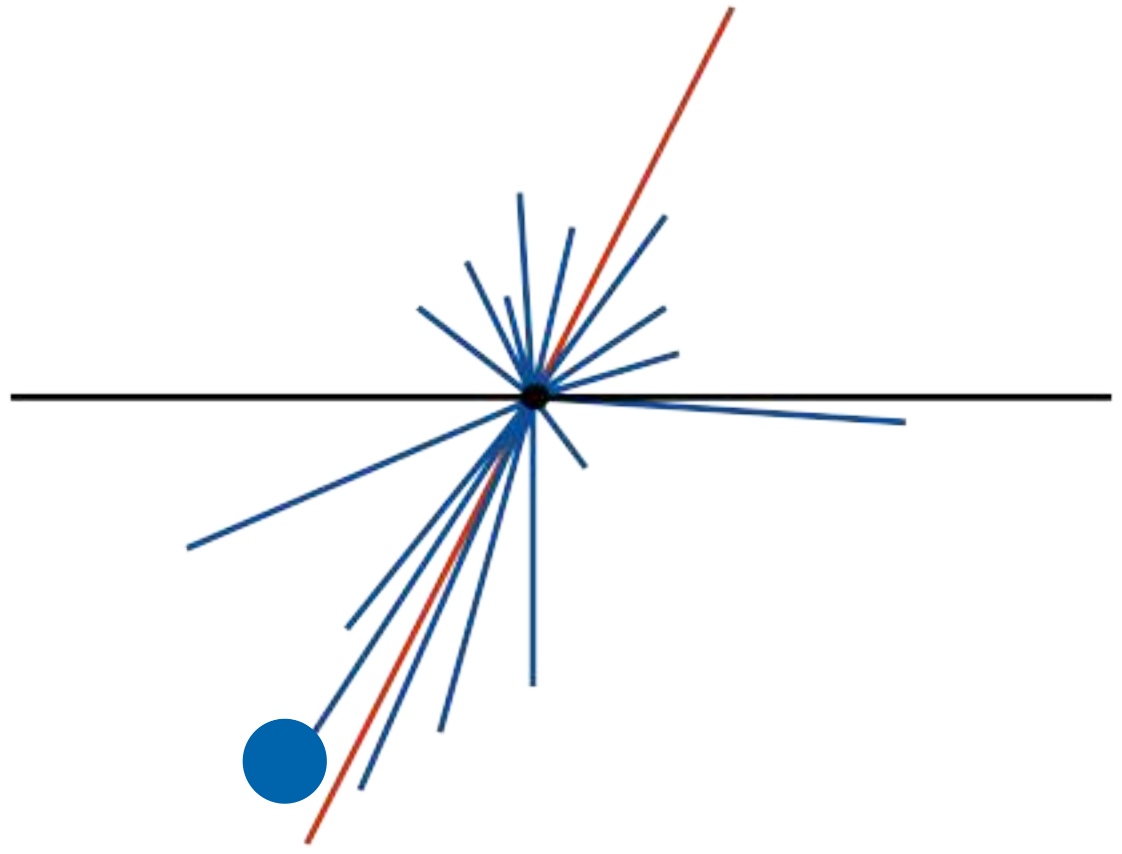

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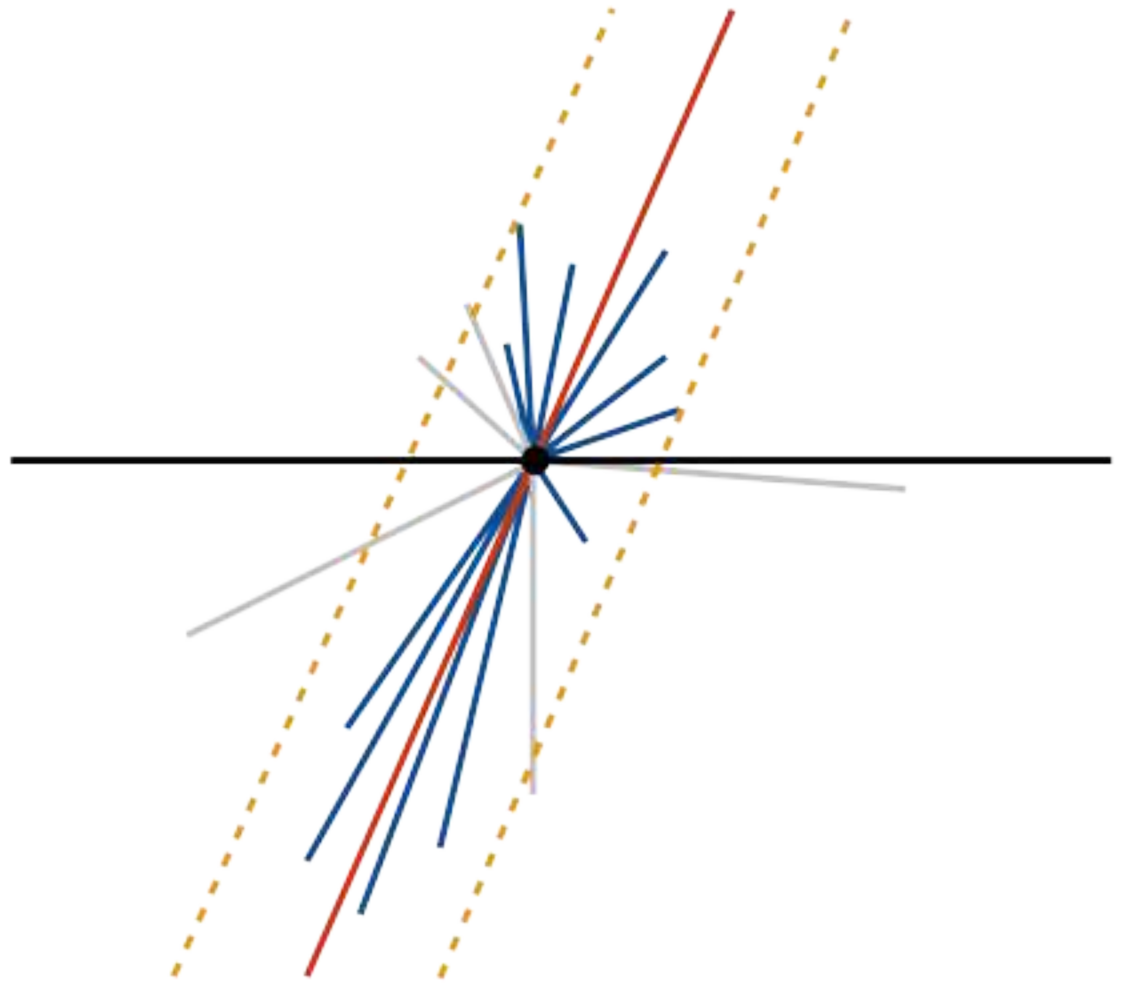

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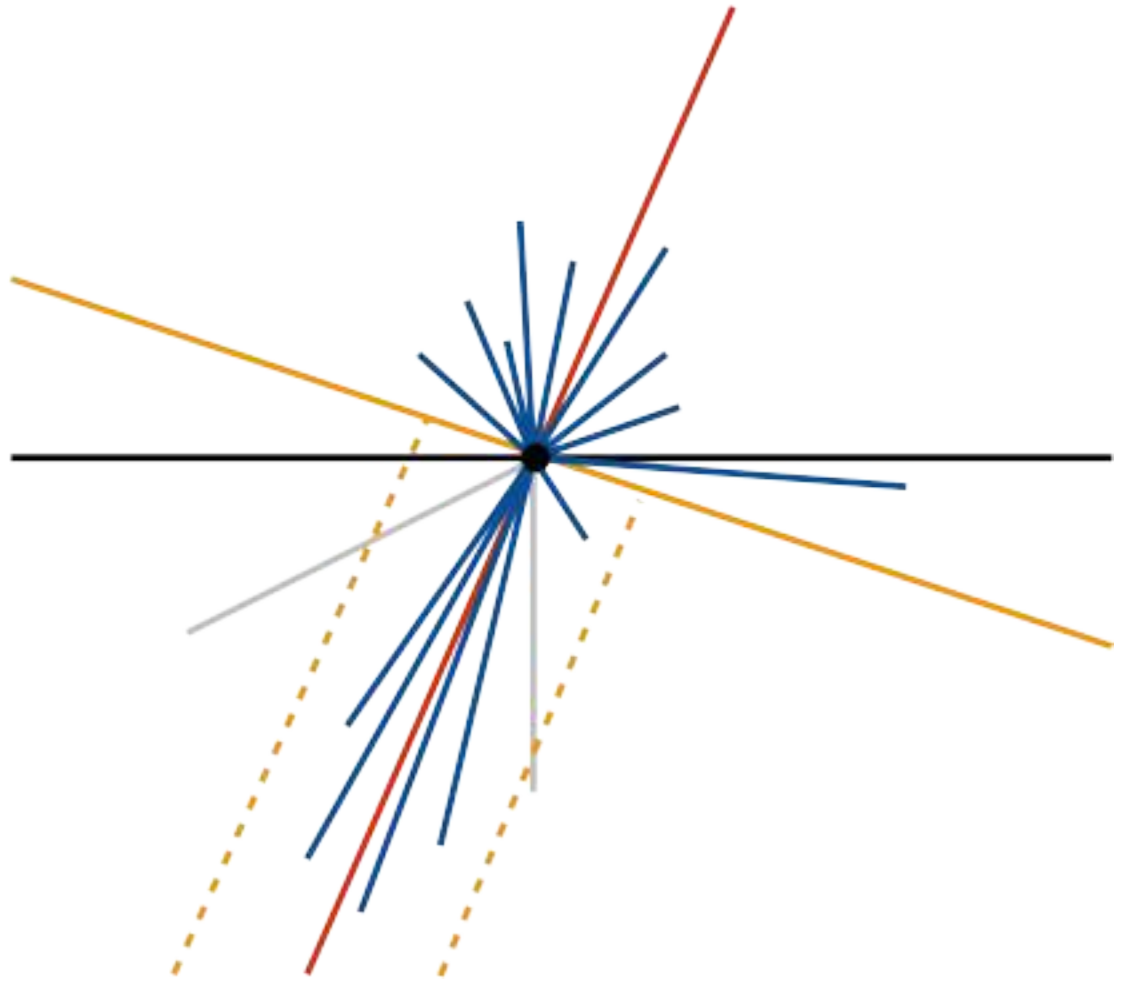


Coherence breaks down for non-global observables.

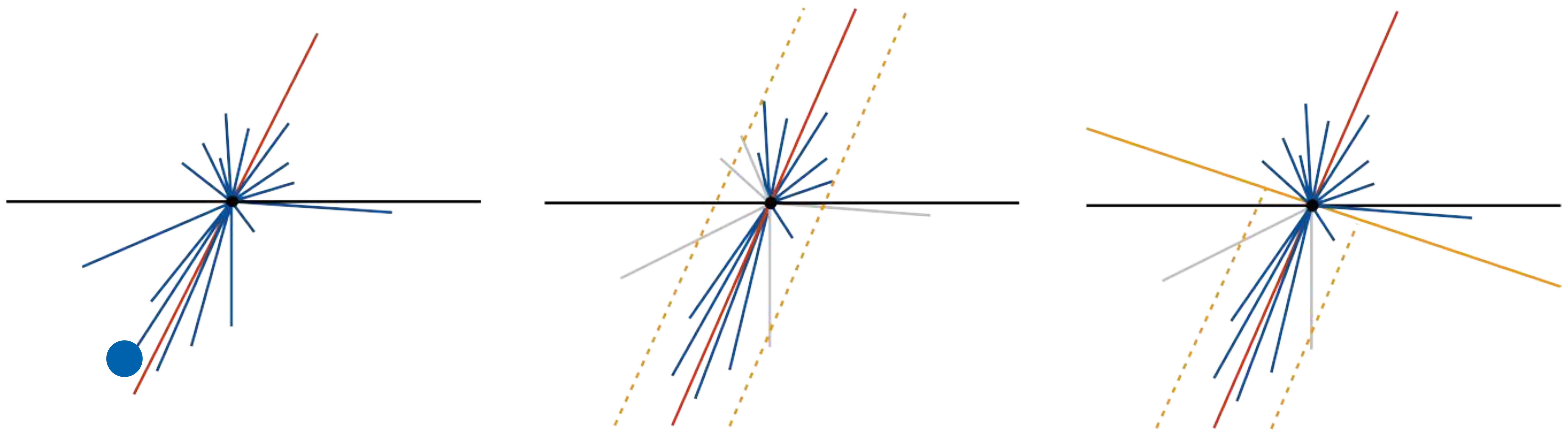
$T_k T_l T_i \cdot T_j T_m T_n$

large-N limit ↓

$$\frac{\partial G_{ab}(t)}{\partial t} = -\int_{\text{in}} \frac{\mathrm{d}\Omega_k}{4\pi}\omega_{ab}(k) G_{ab}(t) + \int_{\text{out}} \frac{\mathrm{d}\Omega_k}{4\pi}\omega_{ab}(k)\Big[G_{ak}(t)G_{kb}(t) - G_{ab}(t)\Big]$$

[Banfi, Marchesini, Smye '02]

# Event generator accuracy



(N)NLO with matching

NLL with coherent branching
Issues in dipole showers

Issues in coherent branching
LL with dipole showers

Can we push this to $\mathrm{NLL}_{\mathrm{global}}$ / $\mathrm{LL}_{\text{non-global}}$ in one (dipole) algorithm?

$\alpha_s L \sim 1 \qquad \alpha_s N^2 \sim 1$

# Event generator accuracy

## Progress in improving the PS accuracy

- **Assessing the logarithmic accuracy of a shower**
  Herwig [1904.11866, 2107.04051], Deductor [2011.04777], Forshaw, Holguin, Plätzer [2003.06400]
  PanScales [1805.09327, 2002.11114], Alaric [2110.05964], …

- **Triple collinear / double soft splittings**
  Dulat, Höche, Krauss, Gellersen, Prestel [1705.00982, 1705.00742, 1805.03757, 2110.05964]
  Li & Skands [1611.00013], Löschner, Plätzer, Simpson Dore [2112.14454], …

- Matching to fixed-order **see Alexander's talk**
  NLO; i.e. Frixione & Webber [0204244], Nason [0409146], …
  NNLO; i.e. UNNLOPS [1407.3773], MiNNLOps [1908.06987], Vincia [2108.07133], …
  NNNLO; Prestel [2106.03206], Bertone, Prestel [2202.01082]

- Colour (and spin) correlations **see Simon's talk**
  Forshaw, Holguin, Plätzer, Sjödahl [1201.0260, 1808.00332, 1905.08686, 2007.09648, 2011.15087]
  Deductor [0706.0017, 1401.6364, 1501.00778, 1902.02105], Herwig [1807.01955], Plätzer & Ruffa [2012.15215]
  PanScales [2011.10054, 2103.16526, 2111.01161], …

- Electroweak corrections
  Vincia [2002.09248, 2108.10786], Pythia [1401.5238], Herwig [2108.10817], …

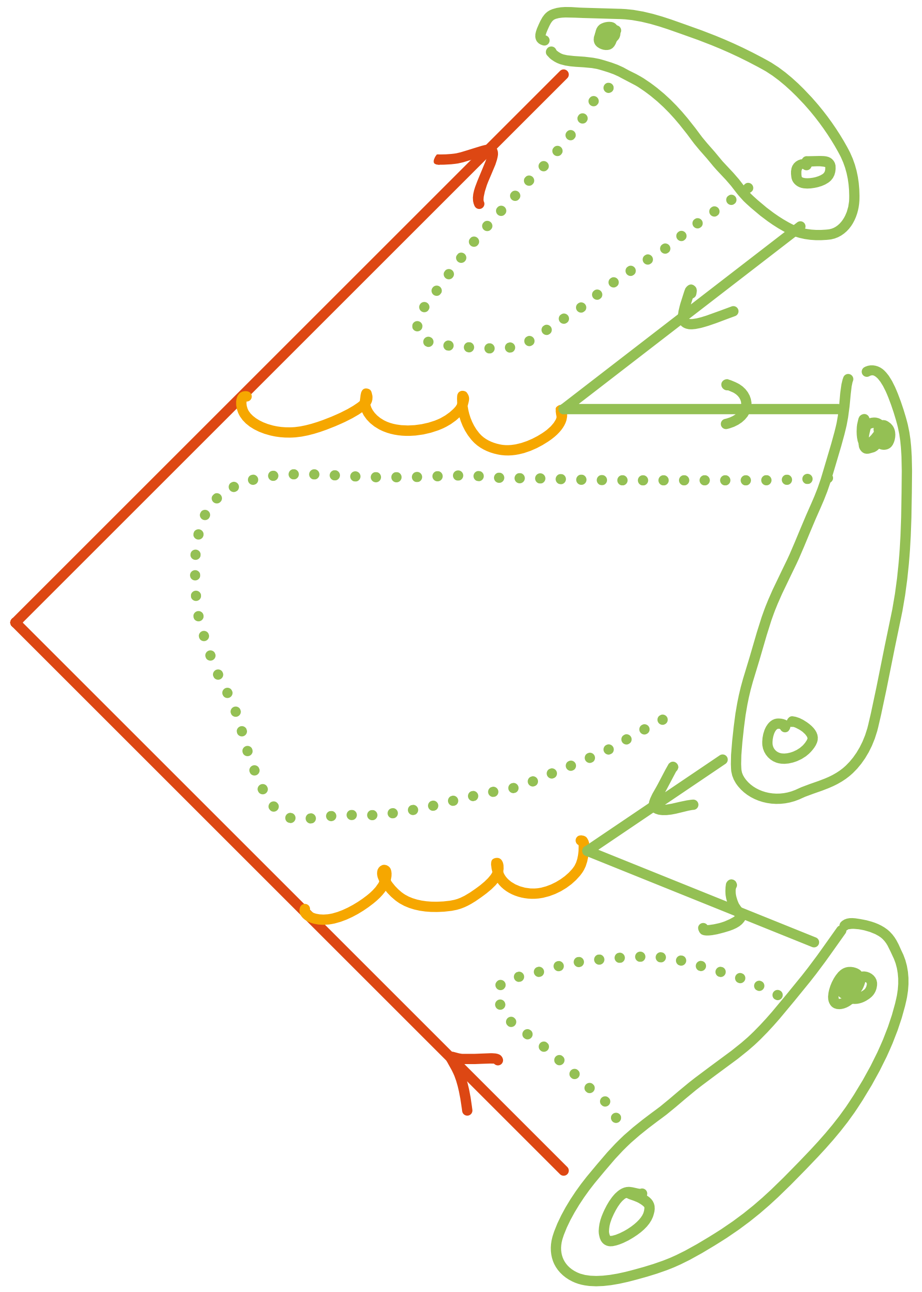
Super-active field of research:
taken from Melissa van Bleekveld's talk at the CERN workshop on parton showers for future colliders.

nching
vers

C … $L \sim 1$ $\alpha_s N^2 \sim 1$

# Hadronization

$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$



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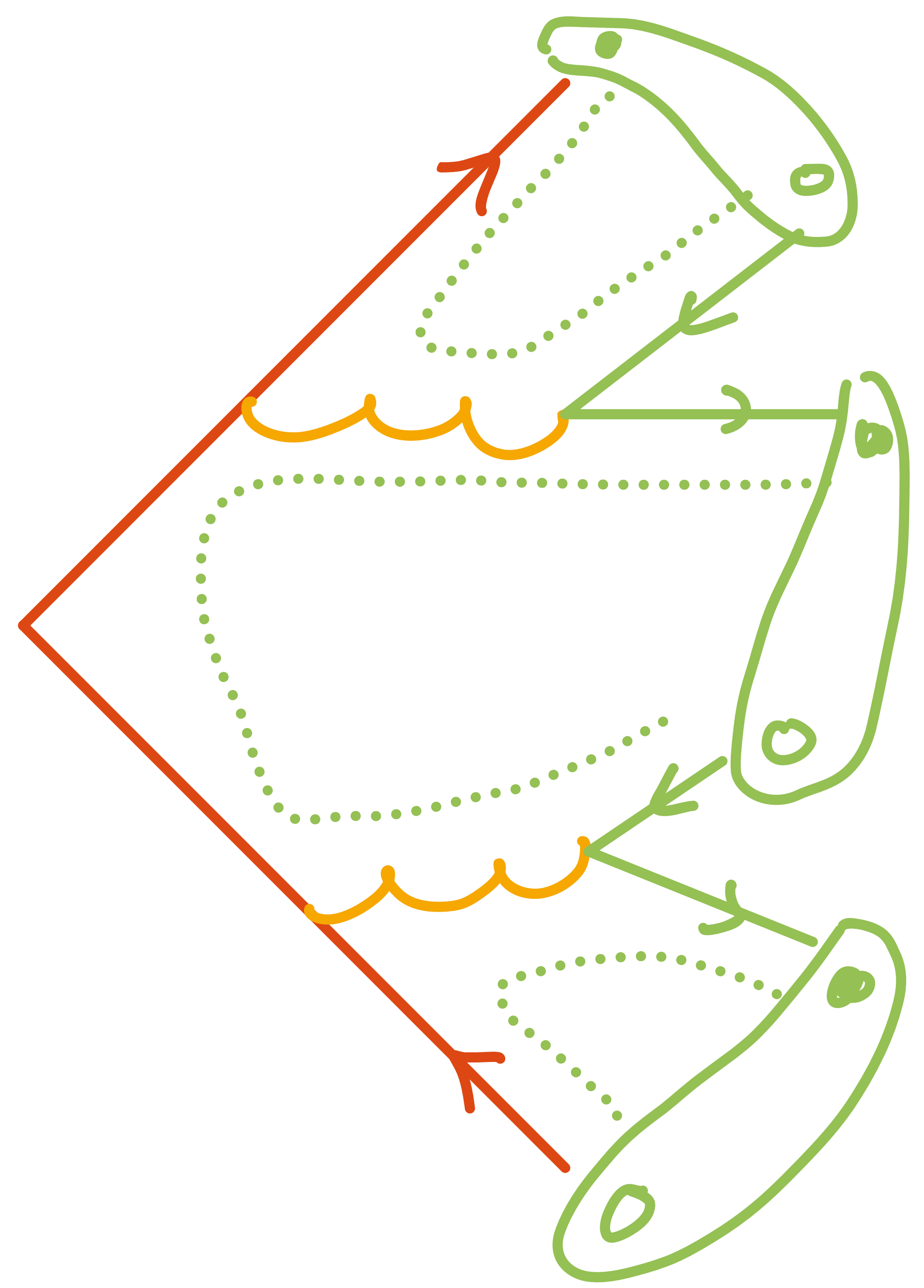
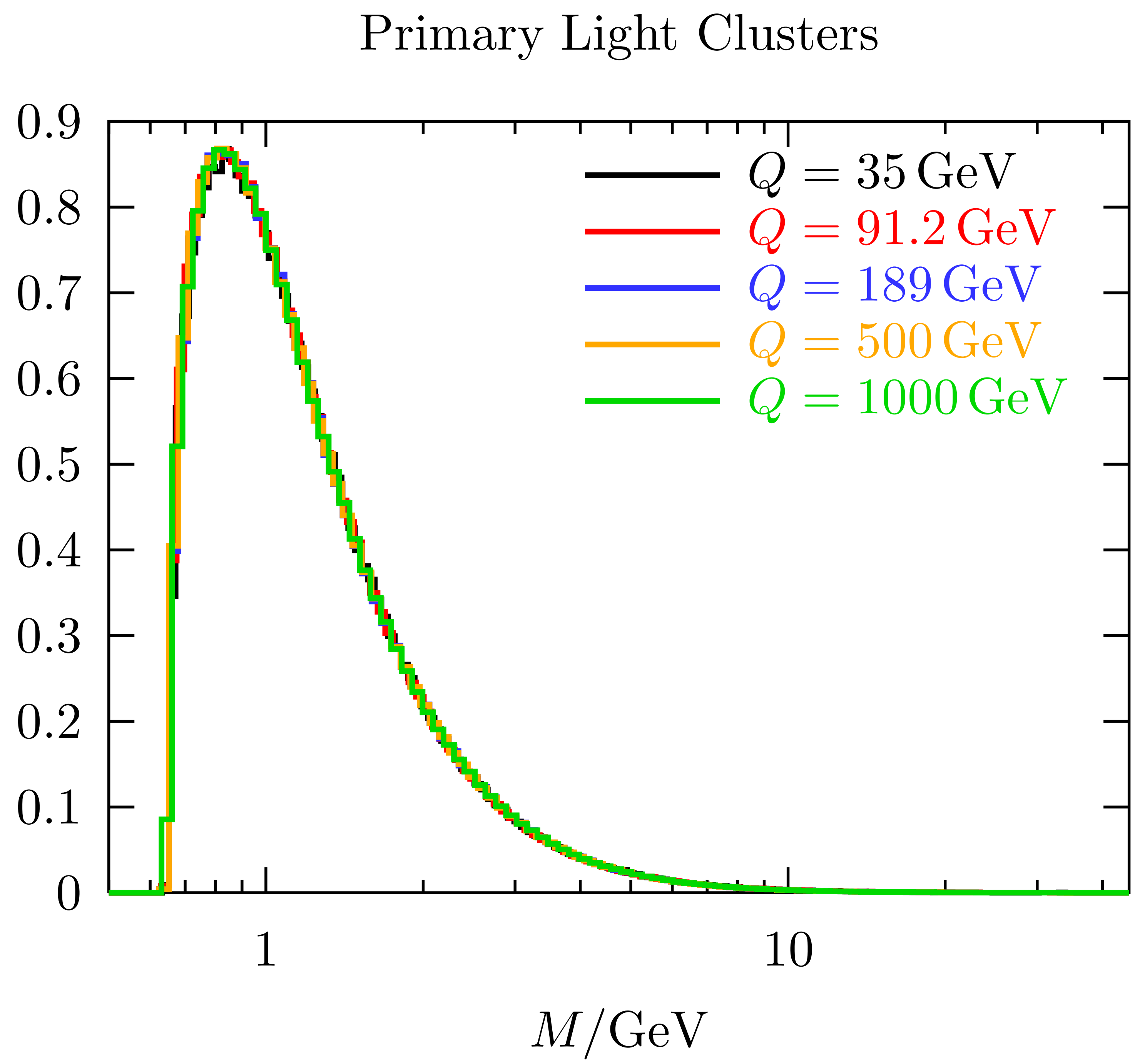


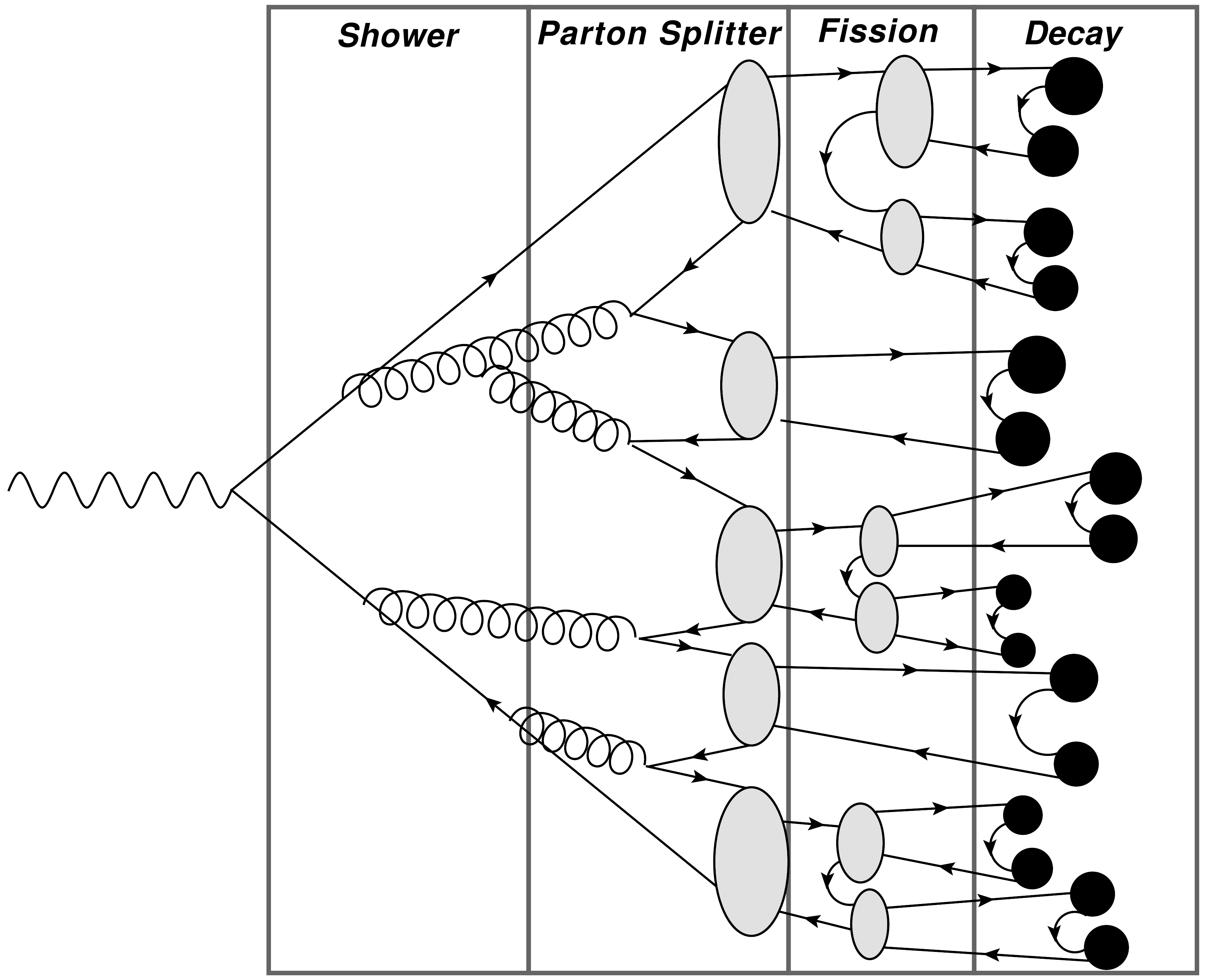
**Universal cluster spectrum: pre-confinement.**




[Herwig++ 1.0 release — Gieseke et al. JHEP 02 (2004) 005]

# Hadronization

$$d\sigma \sim L \times d\sigma_H(Q) \times PS(Q \to \mu) \times MPI \times Had(\mu \to \Lambda) \times \dots$$

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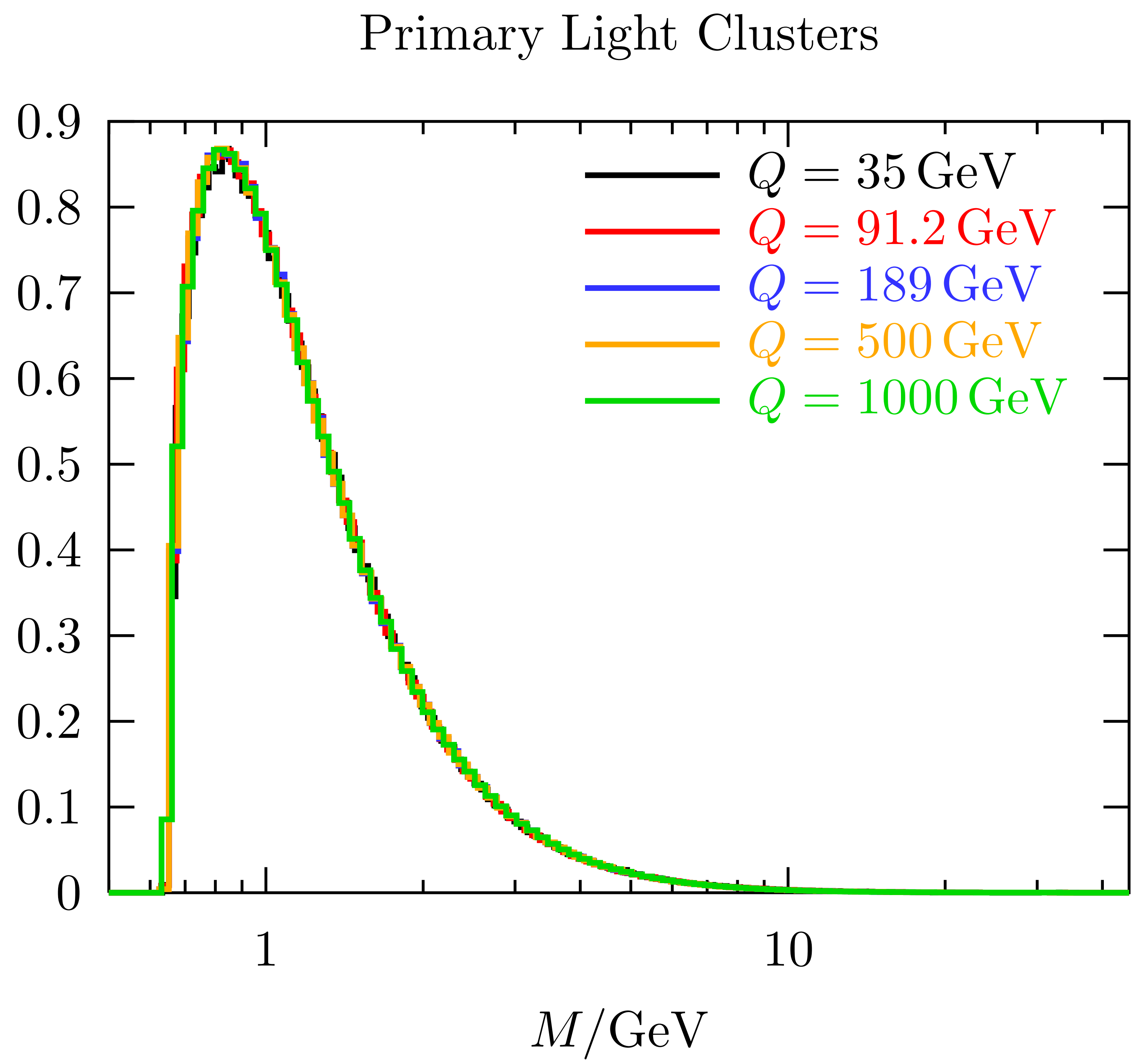


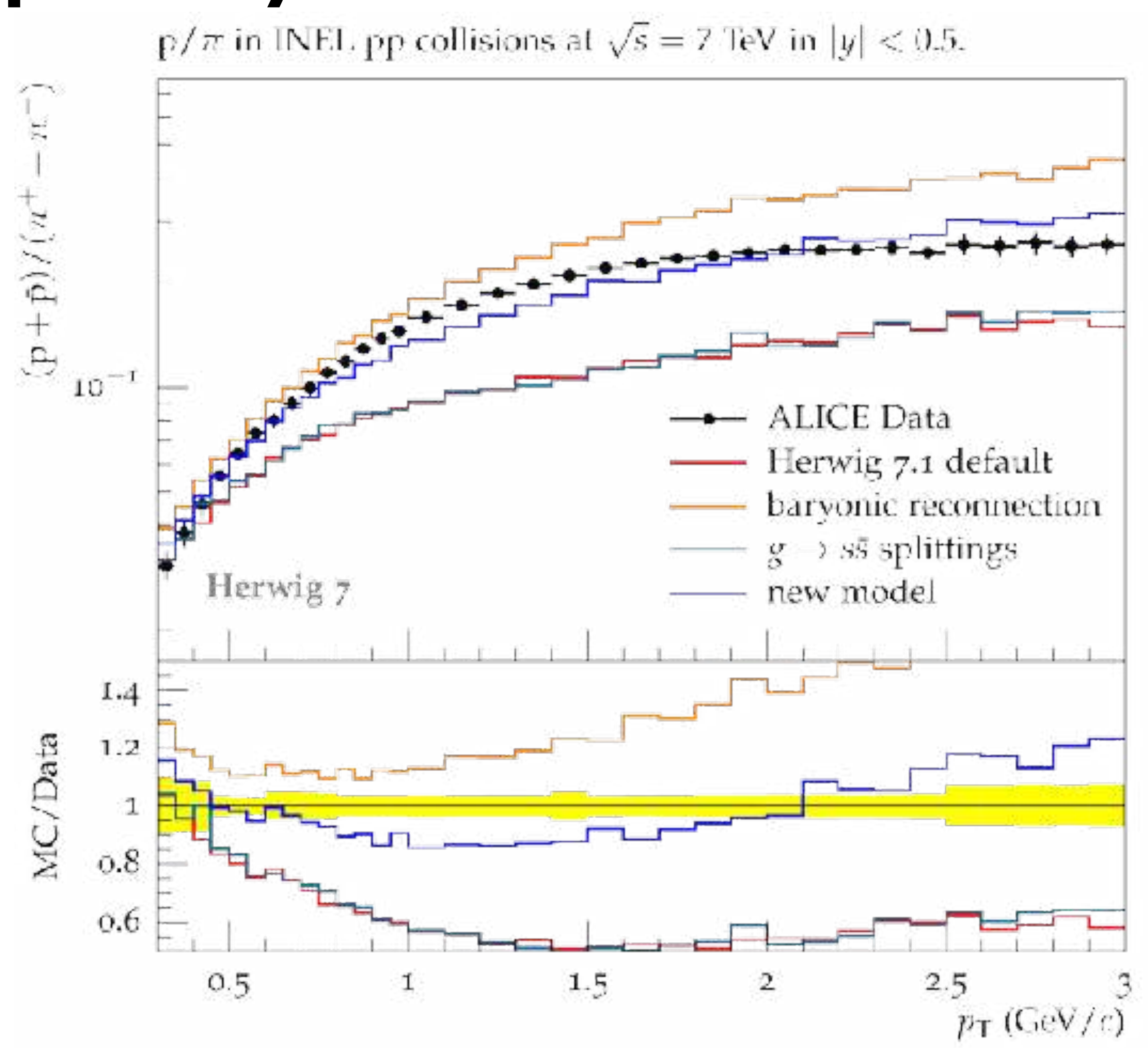

[Figure by Patrick Kirchgaesser]

[Herwig++ 1.0 release — Gieseke et al. JHEP 02 (2004) 005]

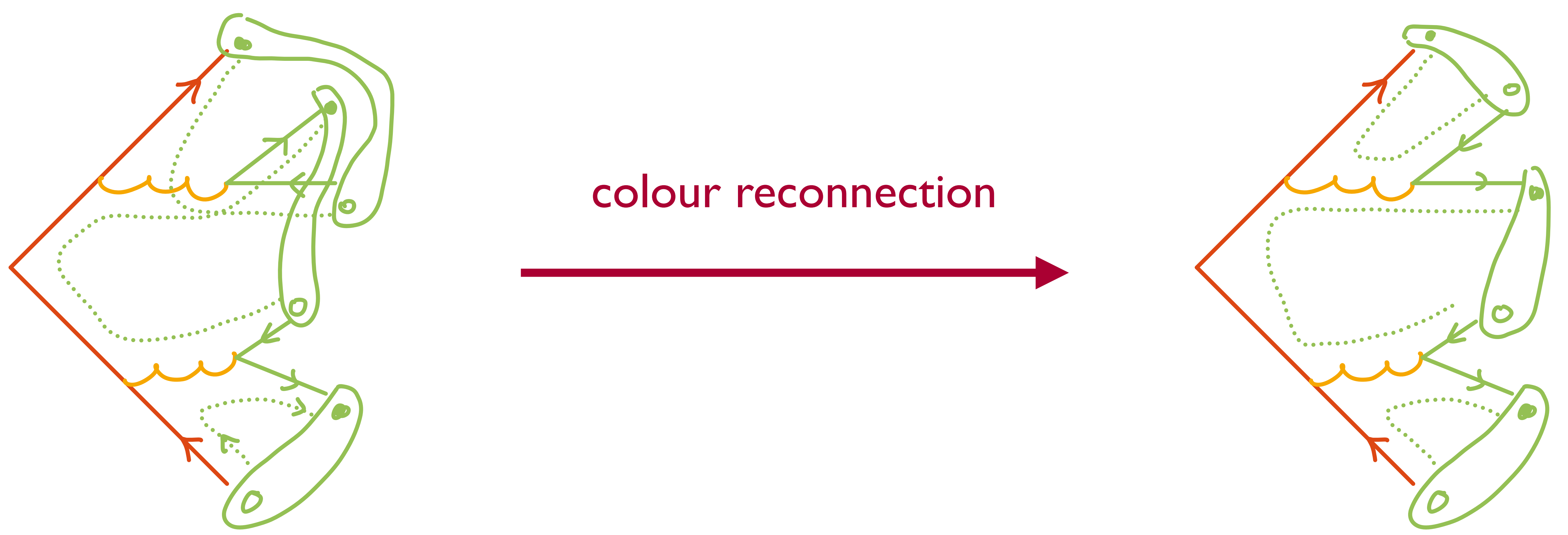
# Colour Reconnection

Ignorance about colour correlations results in clusters which are too heavy.

**proton yield**




**pt**




Most sophisticated algorithms in the cluster model now include baryons and non-trivial momentum information.


[Gieseke, Kirchgaesser, Plätzer – '18]
[Gieseke, Kirchgaesser, Plätzer, Siodmok – '18]

# Colour

Colour reconnection and hadronization is about subleading-N.
So are shower accuracy and interference terms.

| Colour factor algorithms | Colour ME corrections | Full amplitude evolution |
| --- | --- | --- |
| Coherent, NLL-accurate dipole showers | Colour-exact real emissions as far as possible | Colour-exact real and virtual corrections |
| [Gustafson] [PanScales '21] [Forshaw, Holguin, Plätzer '21] | [Plätzer, Sjödahl '12, '18] [Höche, Reichelt '20] | [Forshaw, Plätzer + … '13 …] [Nagy, Soper '12 …] |

$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

# Amplitude evolution

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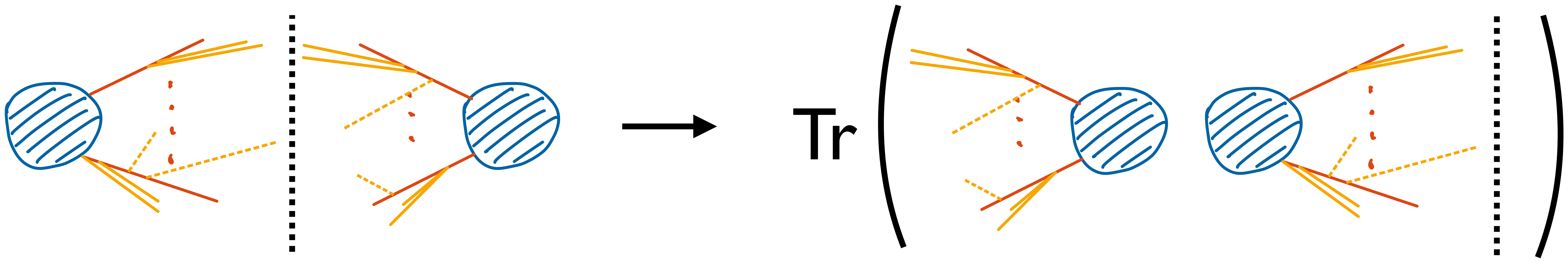
$$\mathrm{d}\sigma \sim \mathrm{Tr}\left[\mathbf{PS}(Q \to \mu)\mathrm{d}\mathbf{H}(Q)\mathbf{PS}^{\dagger}(Q \to \mu)\mathbf{Had}(\mu \to \Lambda)\right]$$

# Amplitude evolution: CVolver

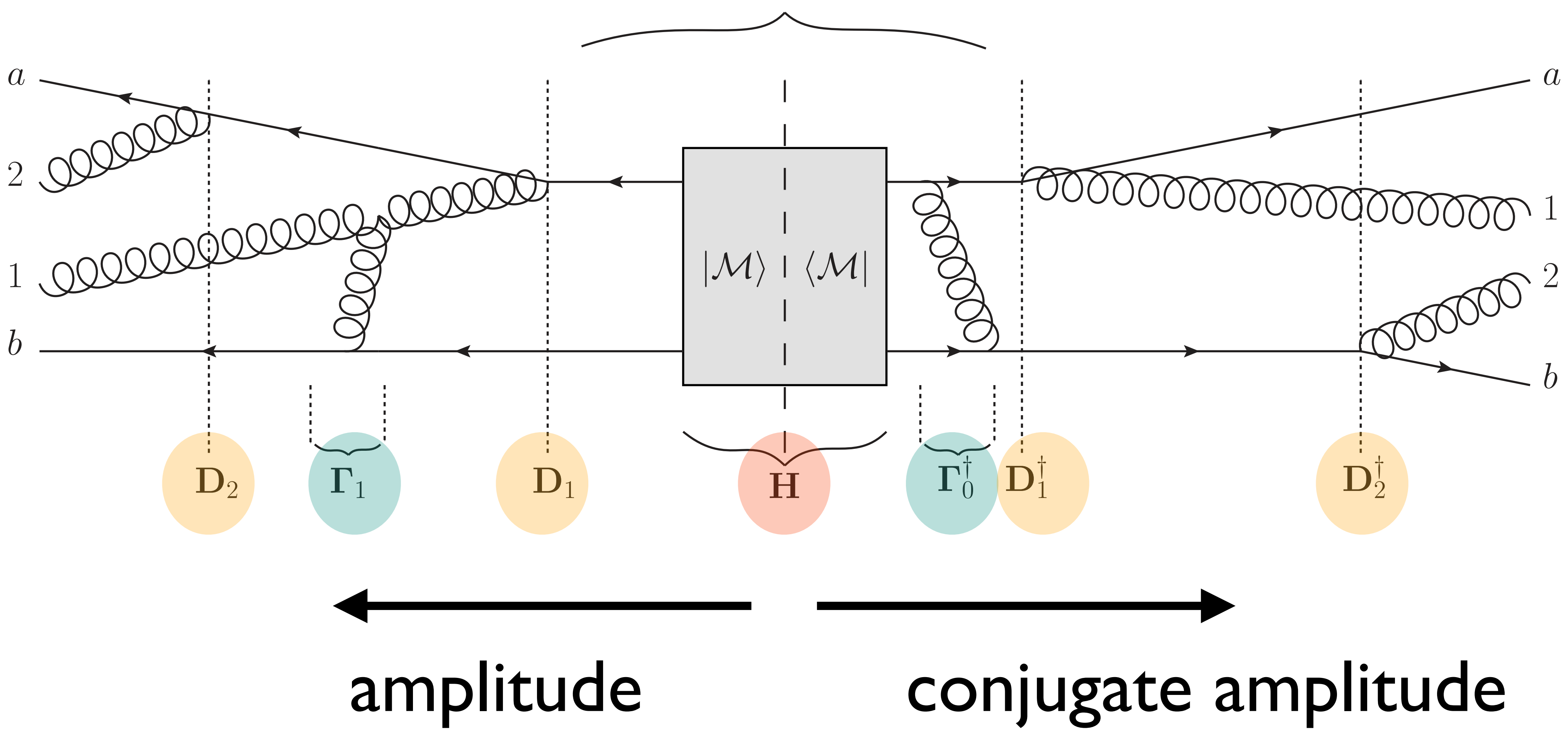



$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\, \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}\, \mathbf{D}_n(k)\, \mathbf{A}_{n-1}(k)\, \mathbf{D}_n^\dagger(k)\, \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}^\dagger(k')}$$

Markovian algorithm at the amplitude level:
Iterate gluon exchanges and emission.

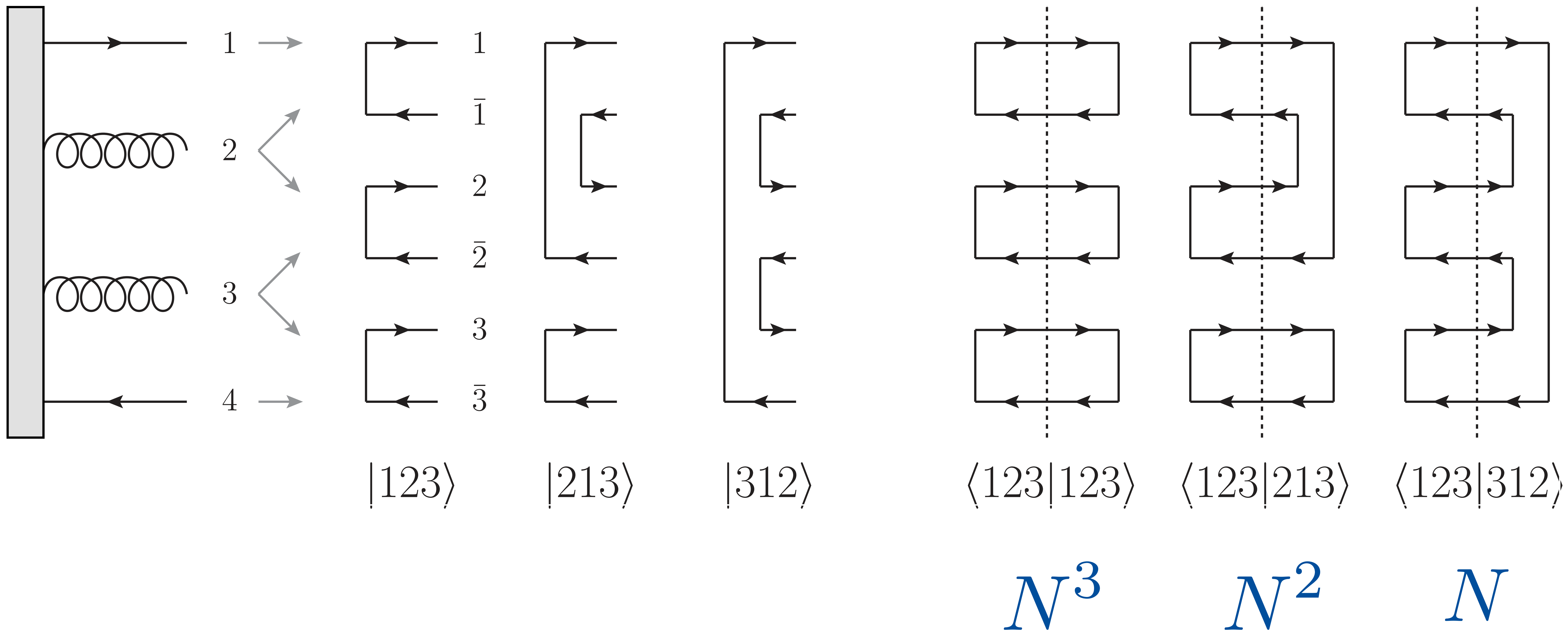
Different histories in amplitude and conjugate amplitude needed to include interference.

[Angeles, De Angelis, Forshaw, Plätzer, Seymour – '18]
[Forshaw, Holguin, Plätzer – '19]



# Tracking colour

Decompose amplitudes in flow of colour charge. $(t^a)^i{}_k (t^a)^j{}_l = T_R \left( \delta^i_l \delta^j_k - \frac{1}{N} \delta^i_k \delta^j_l \right)$



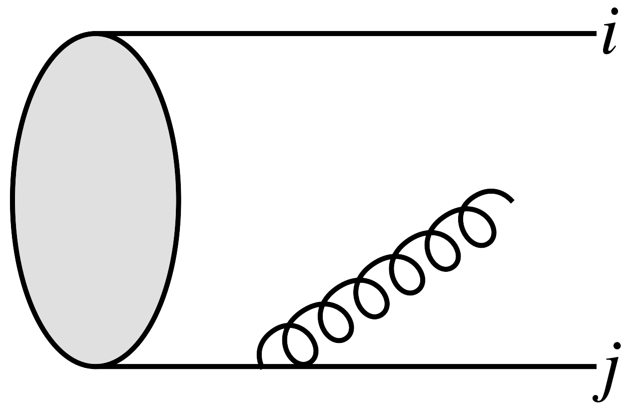

Suppression of interferences outside of colour connected dipoles.

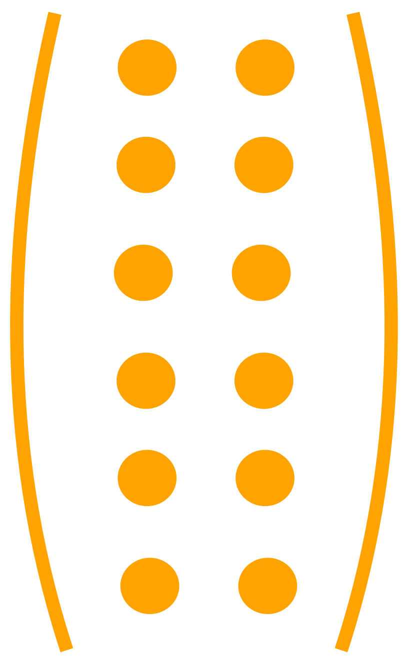
[Plätzer '13]
[Angeles, De Angelis, Forshaw, Plätzer, Seymour '18]

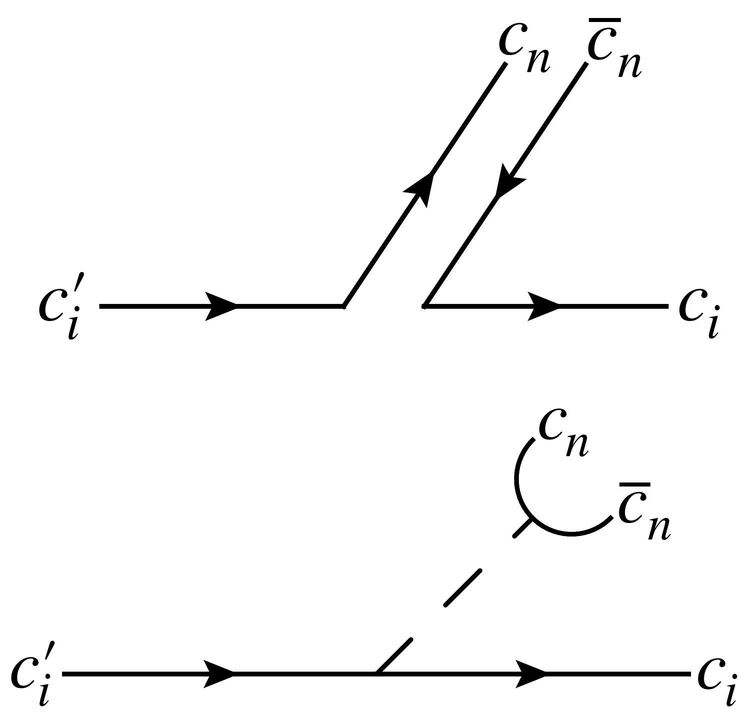
# Tracking colour

## Gluon emission

$$\mathbf{D}_n(k)$$



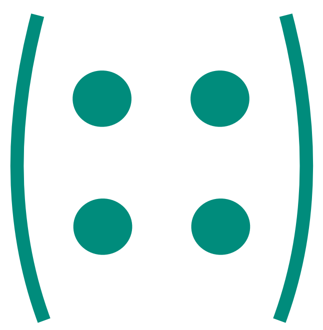



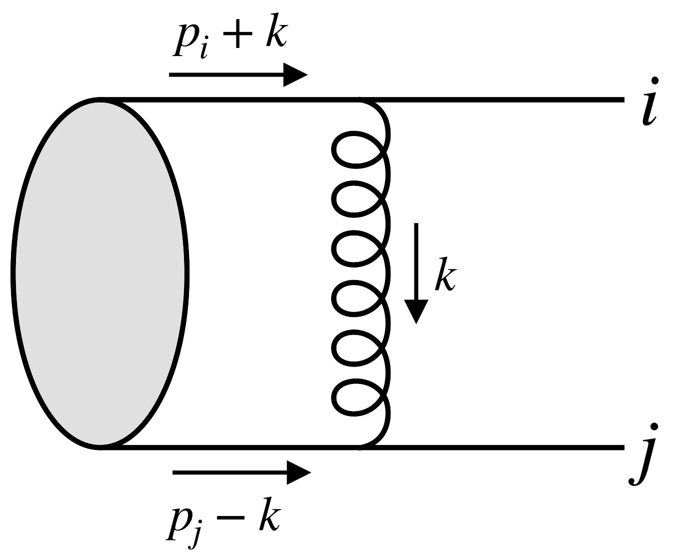



Explicit suppression in 1/N

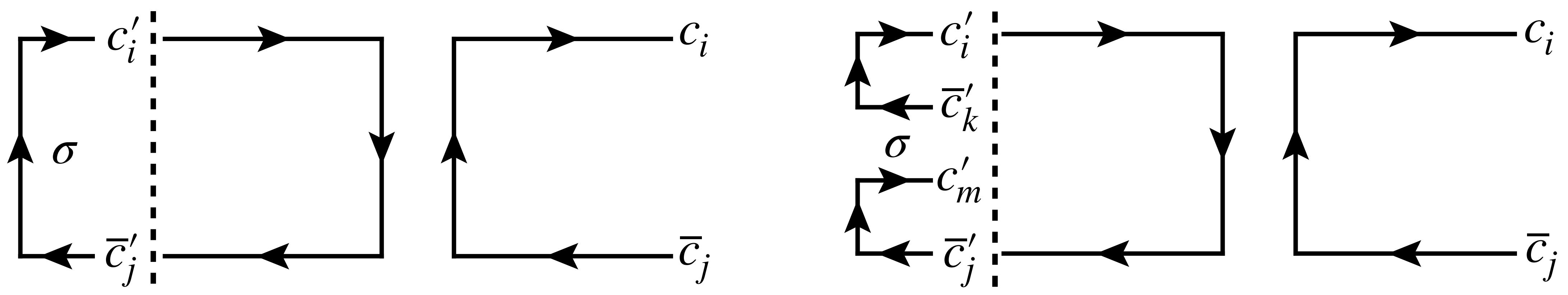
## Gluon exchange

$$\mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}$$






$$[\tau|\mathbf{\Gamma}|\sigma\rangle = (\alpha_s N)[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle + (\alpha_s N)^2[\tau|\mathbf{\Gamma}^{(2)}|\sigma\rangle + \ldots$$




$$[\tau|\mathbf{\Gamma}^{(1)}|\sigma\rangle = \left(\Gamma_\sigma^{(1)} + \frac{1}{N^2}\rho^{(1)}\right)\delta_{\sigma\tau} + \frac{1}{N}\Sigma_{\sigma\tau}^{(1)}$$

dipole flips — implicit suppression in 1/N

Systematically expand around large-N limit summing towers of terms enhanced by $\alpha_S N$

[Plätzer – '13] — diagrams from [Ruffa, MSc thesis 2020]

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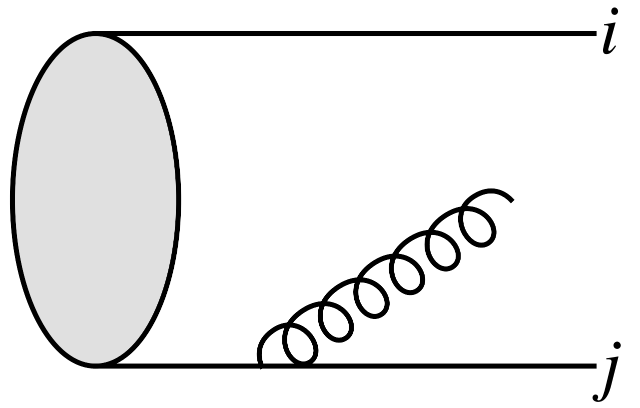
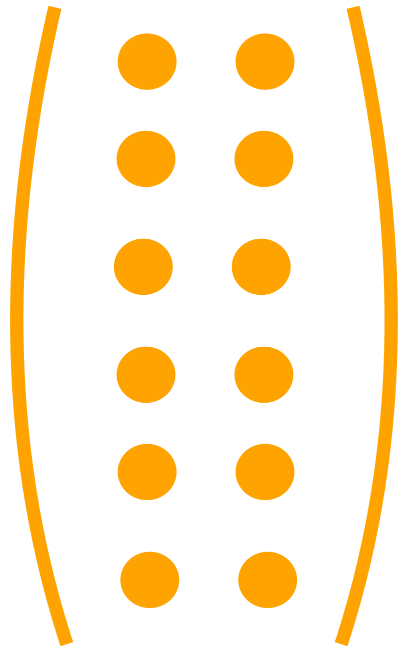
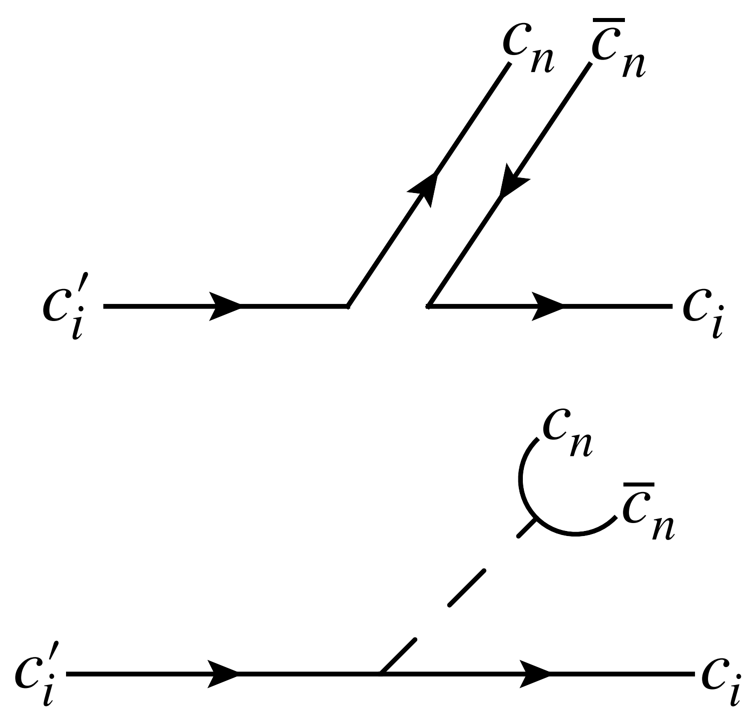
## Gluon emission

$$\mathbf{D}_n(k)$$



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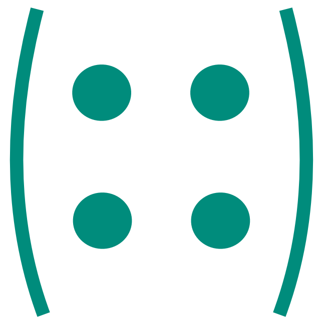

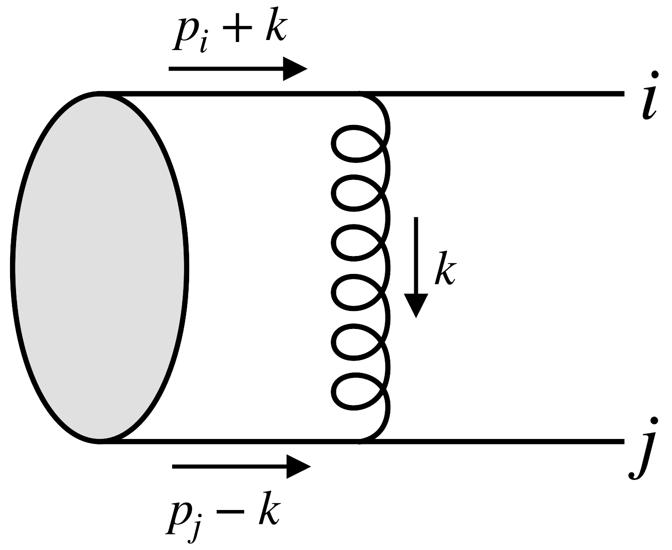


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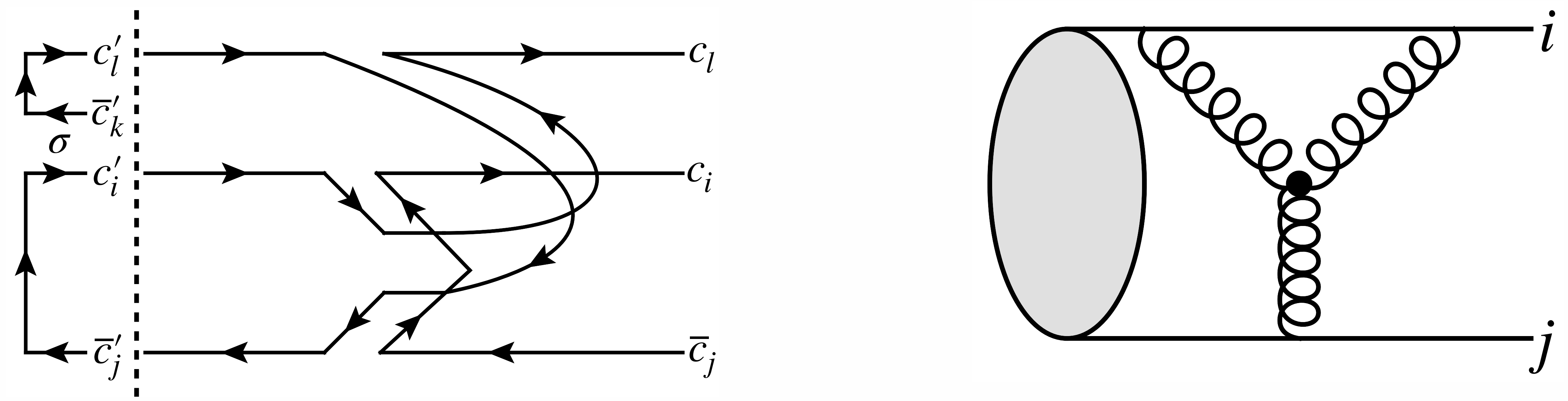


[Plätzer, Ruffa — '21]

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# Amplitude evolution — CVolver

**CVolver** solves evolution equations in colour flow space

[De Angelis, Forshaw, Plätzer '21]
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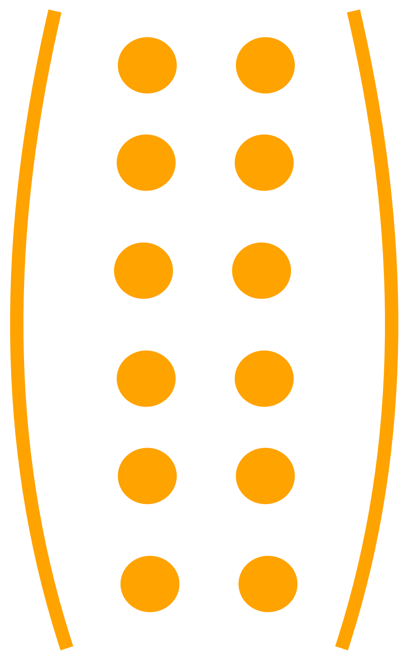

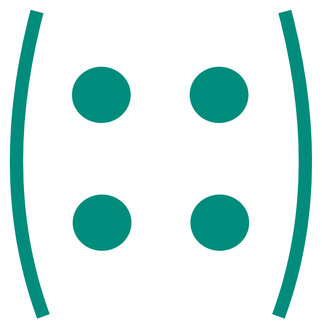

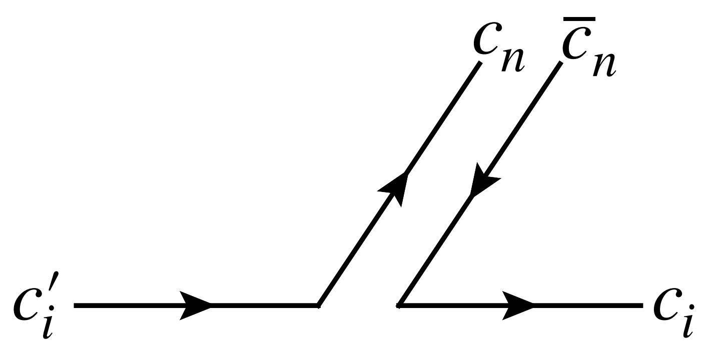


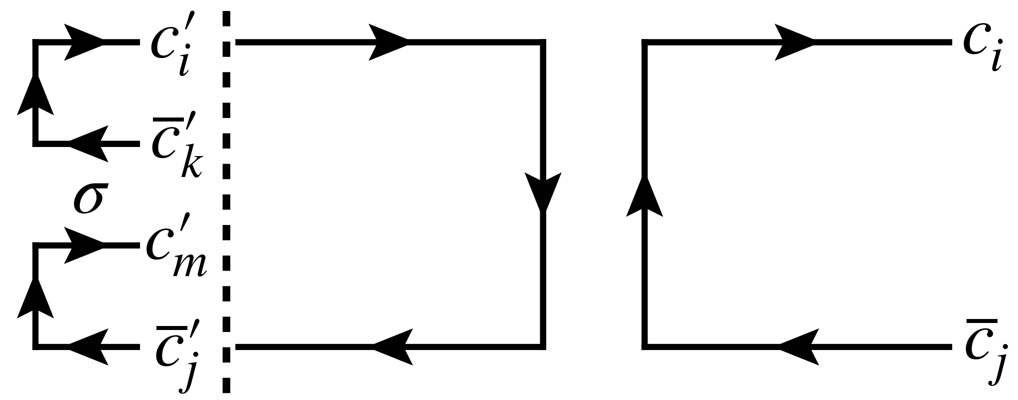


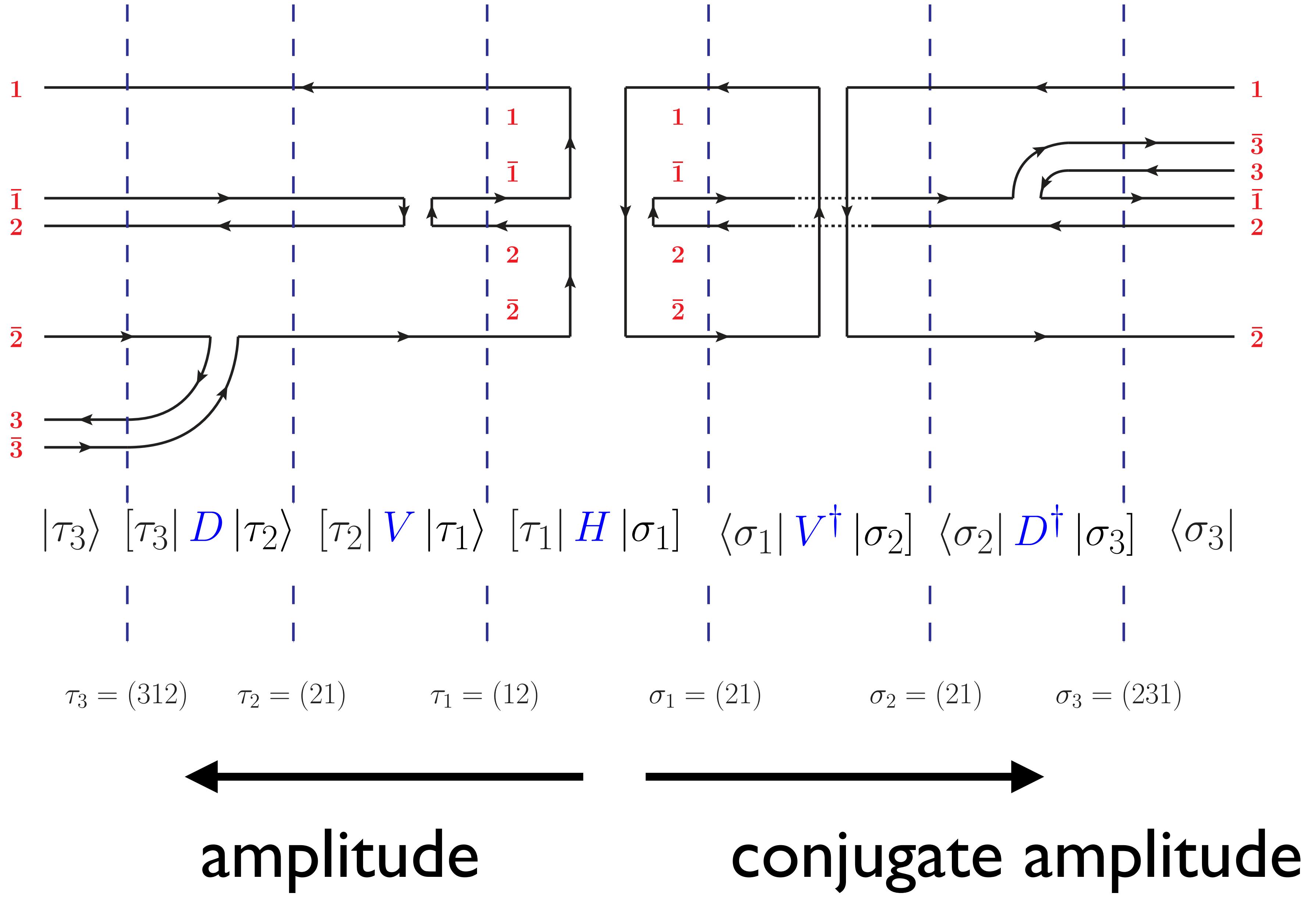

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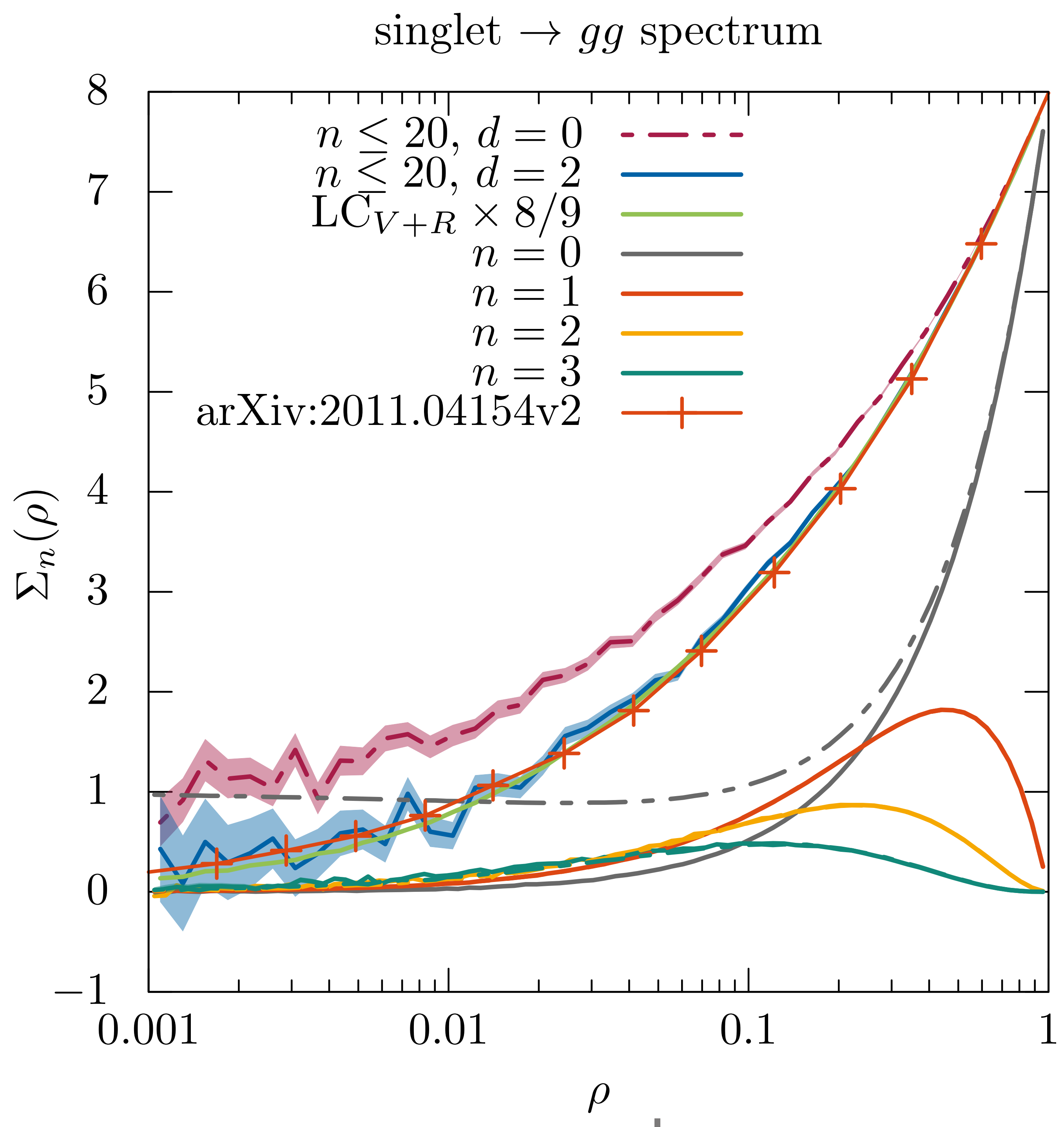
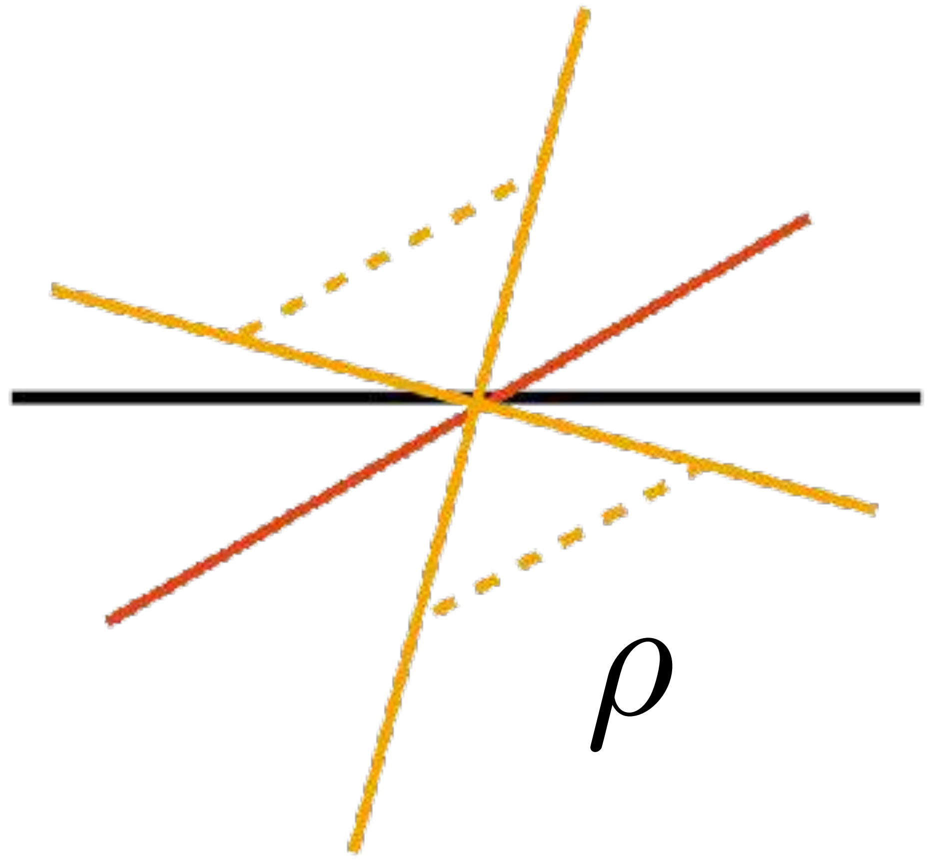
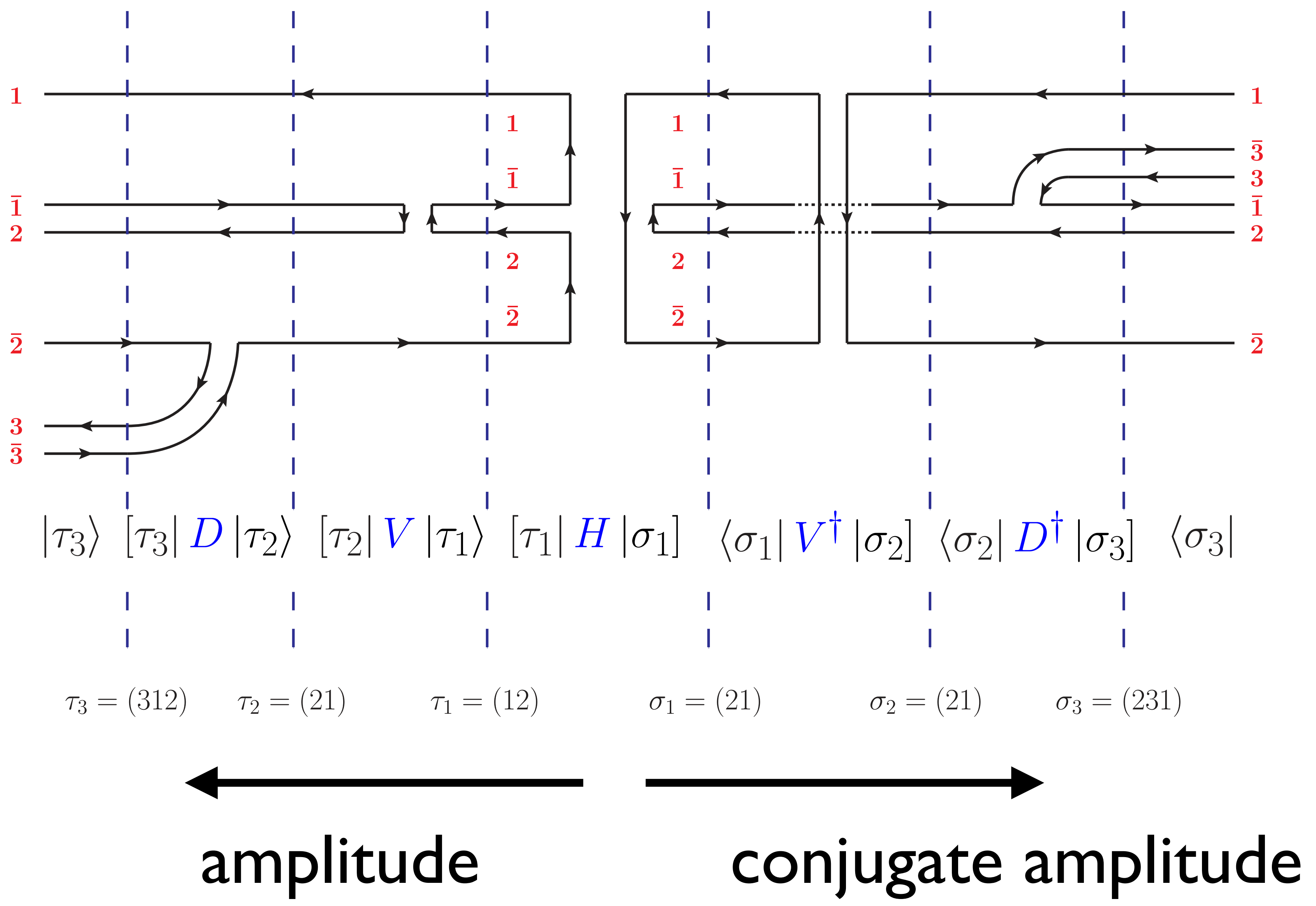


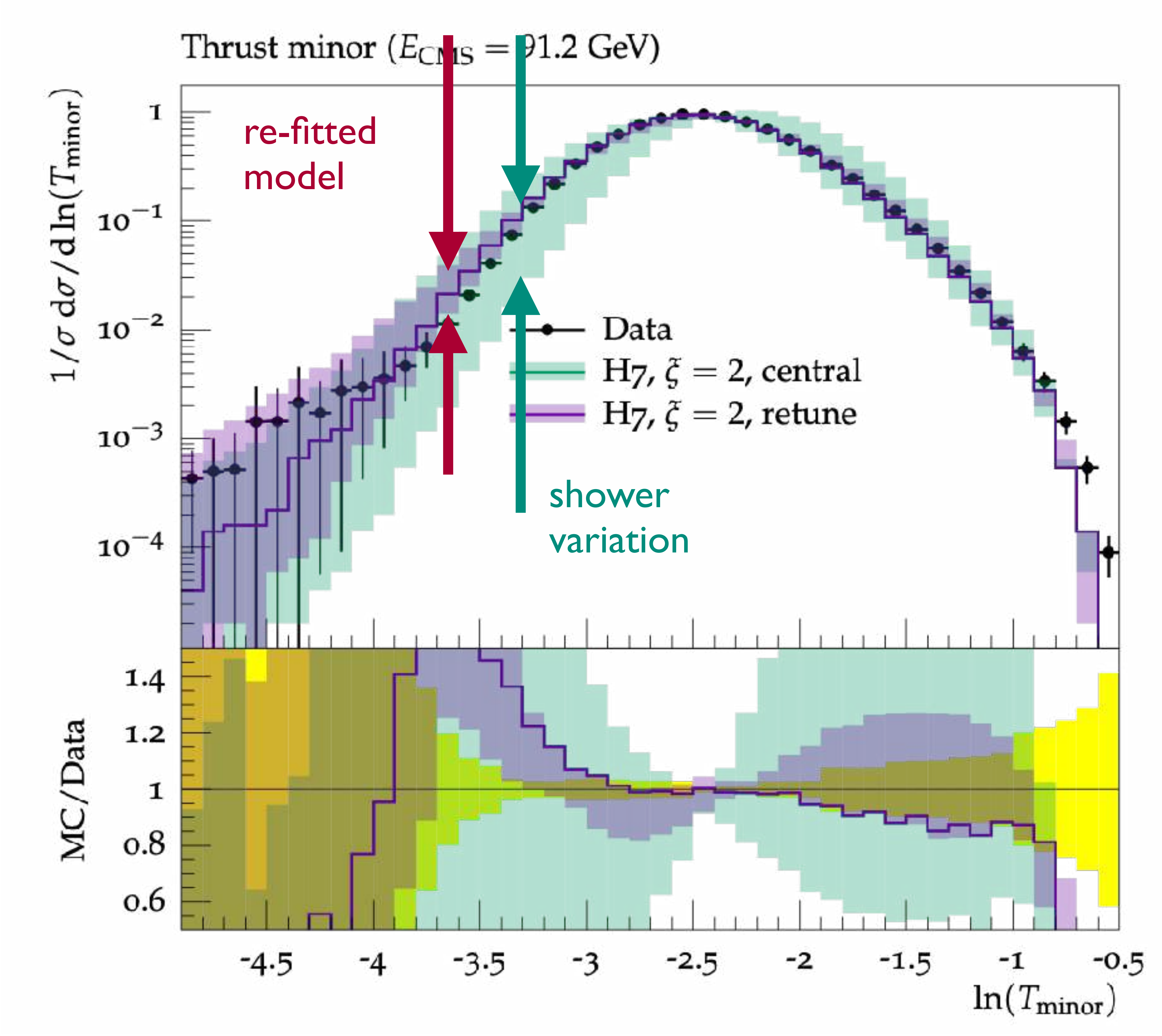

$$\Sigma(\rho) = \sum_n \int \mathrm{d}\sigma(\{p_i\}) \prod_i \theta_{\text{in}}(\rho - E_i)$$




complete agreement with Hatta et al. using equivalent Langevin formulation

# Hadronization?

[Bellm, Lönnblad, Plätzer, Prestel, Samitz, Siodmok, Hoang — Les Houches 2017]



# Factorisation and evolution

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[ \mathbf{M}_n \mathbf{U}_{nm} \right] \mathrm{d}\phi_m u(\phi_m)$$

[Plätzer – '22]

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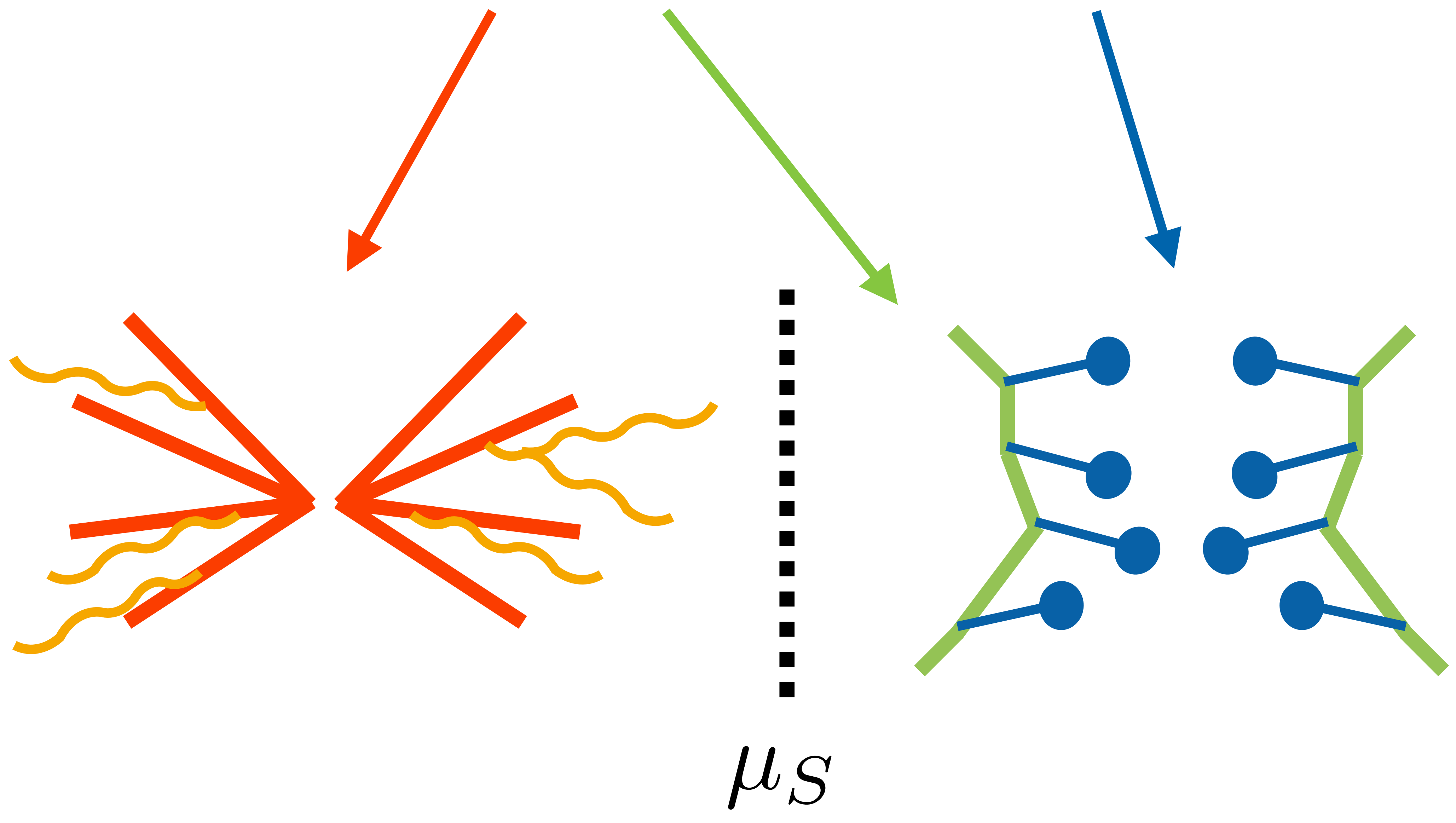


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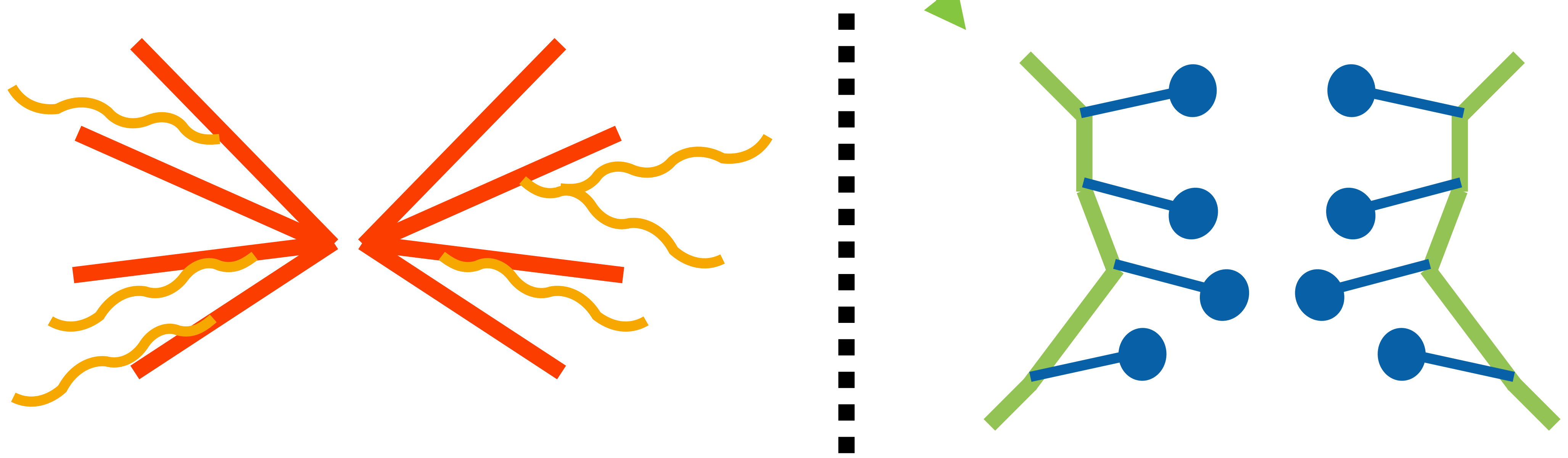


$\mu_S$

calculate building blocks

derive evolution

construct model response

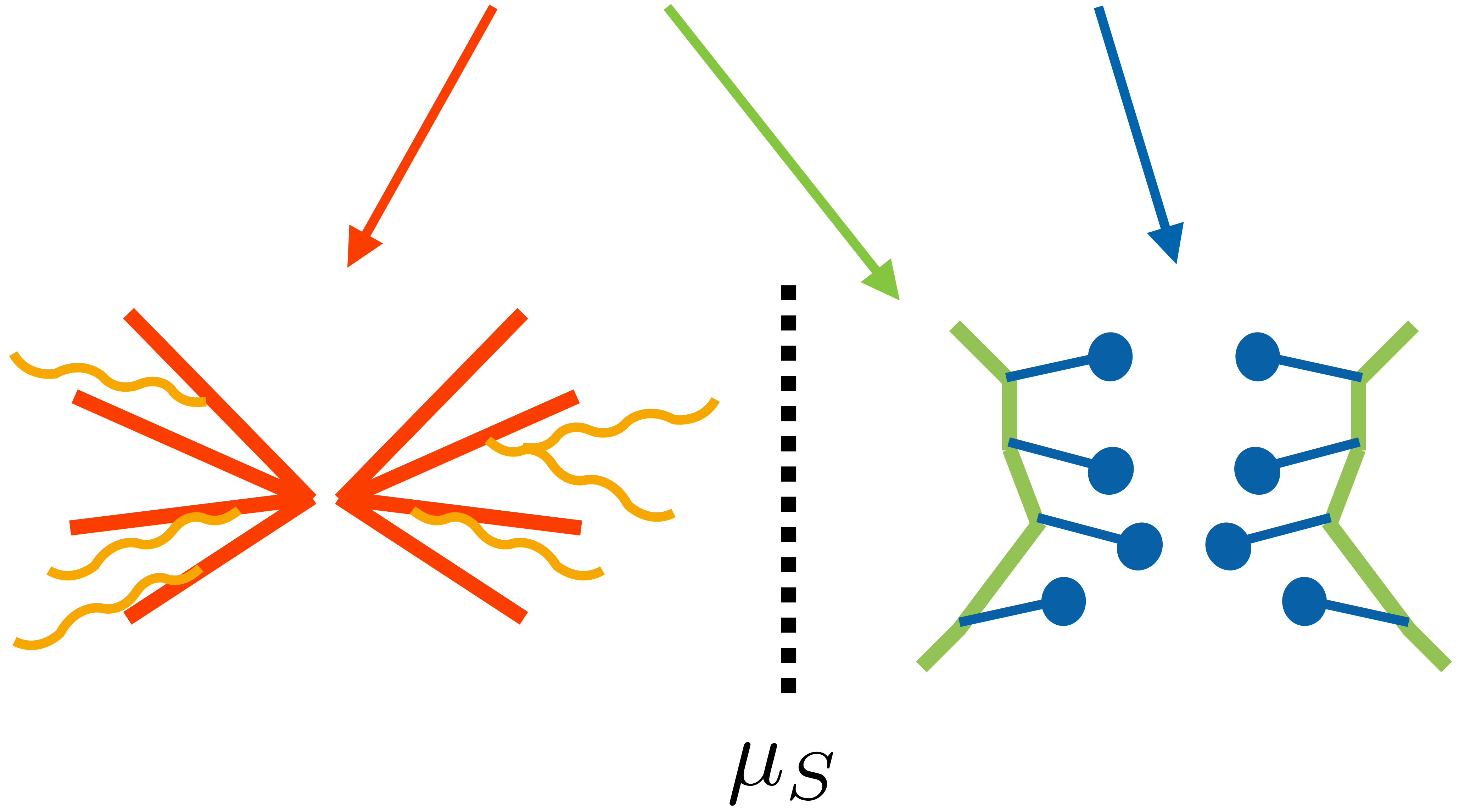
constrain by data

[Plätzer – '22]

# Factorisation and evolution

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$




Not limited to a hadronization model — can also re-arrange partonic observables in this way.

[e.g. resummation of NGL in SCET — Becher, Neubert et al.]

[Plätzer – '22]

# Redefinitions of “bare” operators

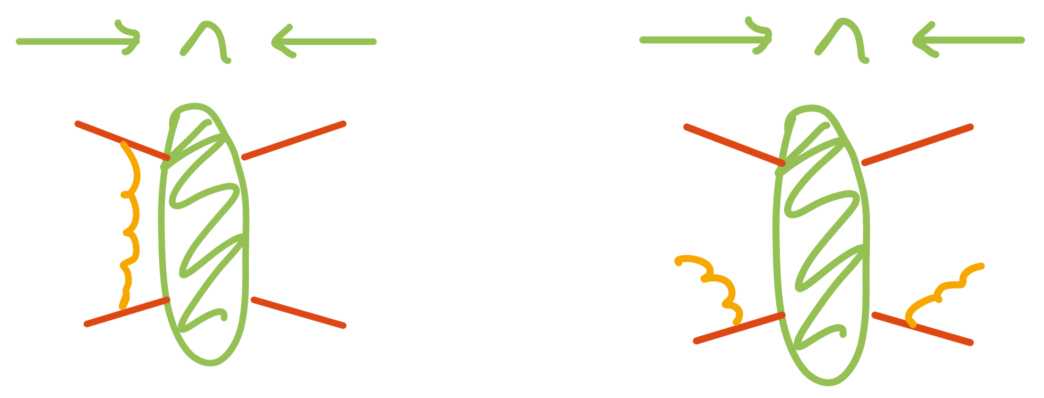
How do we consistently hadronize in light of (improved) shower algorithms?

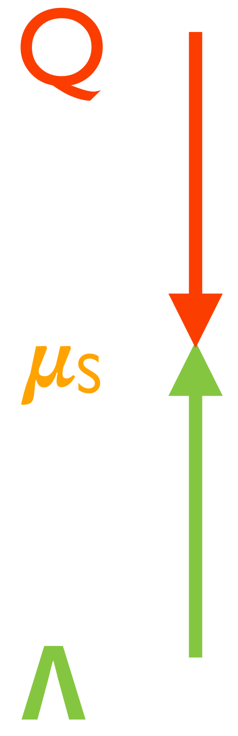
How to do this at subleading N and higher order shower evolution?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Subtract IR divergencies in unresolved regions

$$\mathbf{U}_n = \mathcal{X}_n \left[\mathbf{S}(\mu_S), \mu_S\right]$$
$$= \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$






Re-arrange to resum IR enhancements

$$\mathbf{M}_n Z_g^n = \mathcal{Z}_n \left[\mathbf{A}(\mu_S), \mu_S\right]$$
$$= \mathbf{Z}_n \mathbf{A}_n \mathbf{Z}_n^\dagger + \sum_{s=1}^{n} \alpha_S^s \mathbf{E}_n^{(s)} \mathbf{A}_{n-s} \mathbf{E}_n^{(s)\dagger}$$

$$\mathbf{M}_n = \sum_{l=0}^{\infty} \alpha_0^l \, \mathbf{M}_n^{(l)}$$

Cross section is RG invariant

$$\sigma = \sum_n \alpha_S^n \int \mathrm{Tr}\left[\mathbf{A}_n(\mu_S) \mathbf{S}_n(\mu_S)\right] \mathrm{d}\phi_n$$

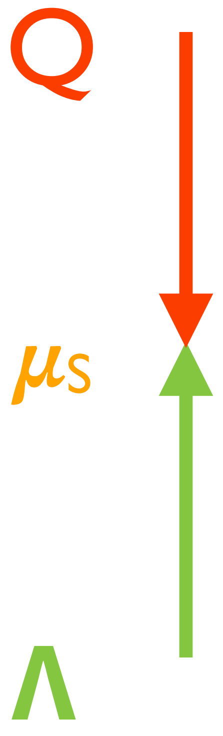
# Redefinitions of “bare” operators

How do we consistently hadronize in light of (improved) shower algorithms?

How to do this at subleading N and higher order shower evolution?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Redefinitions of hard and soft factor **inverse** to each other:




$$\mathbf{Z}_n = \mathbf{X}_n^{-1}$$

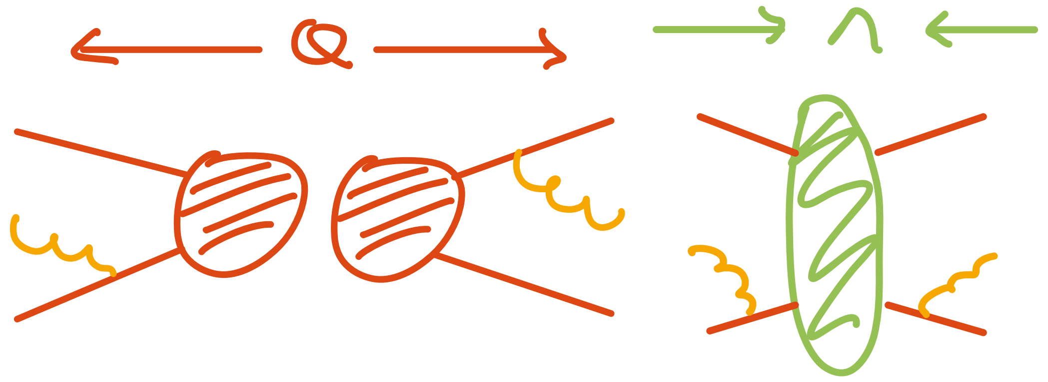
$$\mathbf{X}_n \mathbf{E}_n^{(s)} \circ \mathbf{E}_n^{(s)\dagger} \mathbf{X}_n^{\dagger} - \mathbf{F}_n^{(s)} \mathbf{Z}_{n-s} \circ \mathbf{Z}_{n-s}^{\dagger} \mathbf{F}_n^{(s)\dagger} - \sum_{t=1}^{s-1} \mathbf{F}_n^{(t)} \mathbf{E}_{n-t}^{(s-t)} \circ \mathbf{E}_{n-t}^{(s-t)\dagger} \mathbf{F}_n^{(t)\dagger} = 0$$

dressing of hard process ~ parton shower

soft evolution ~ measurement and hadronization model

$$\sum_n \int \alpha_S^n \, \mathrm{Tr}\left[(\mathbf{A}_n + \mathbf{\Delta}_n) \mathbf{S}_n\right] \mathrm{d}\phi(Q) \prod_{i=1}^{n} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$

$\alpha_s$ corrections to tower of logarithms in A — truncation error of relation of Z factors



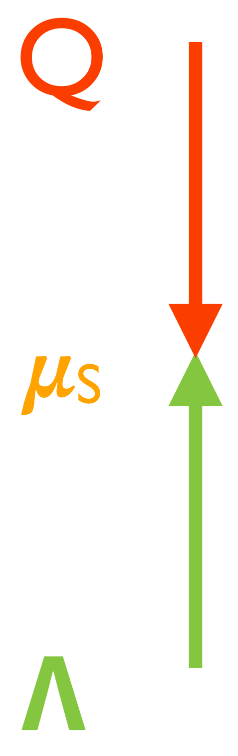
# Infrared subtractions

How do we consistently hadronize in light of (improved) shower algorithms?

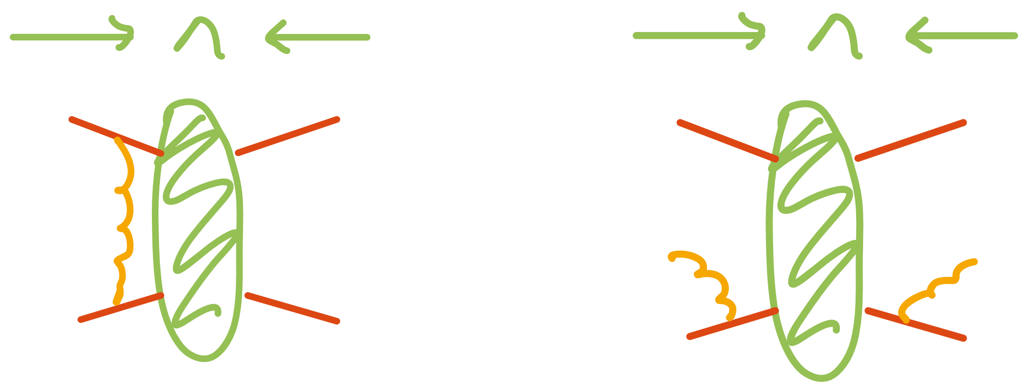
How to do this at subleading N and higher order shower evolution?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Hard factor reproduces CVolver algorithm and predicts key features of second order evolution.




$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^\dagger_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1}\alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s} \prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$



$$\partial_S \mathbf{A}_n = \mathbf{\Gamma}_{n,S}\mathbf{A}_n + \mathbf{A}_n\mathbf{\Gamma}^\dagger_{n,S} - \sum_{s\geq 1}\alpha_S^s \mathbf{R}^{(s)}_{S,n}\mathbf{A}_{n-s}\mathbf{R}^{(s)\dagger}_{S,n}$$

# Infrared subtractions

How do we consistently hadronize in light of (improved) shower algorithms?

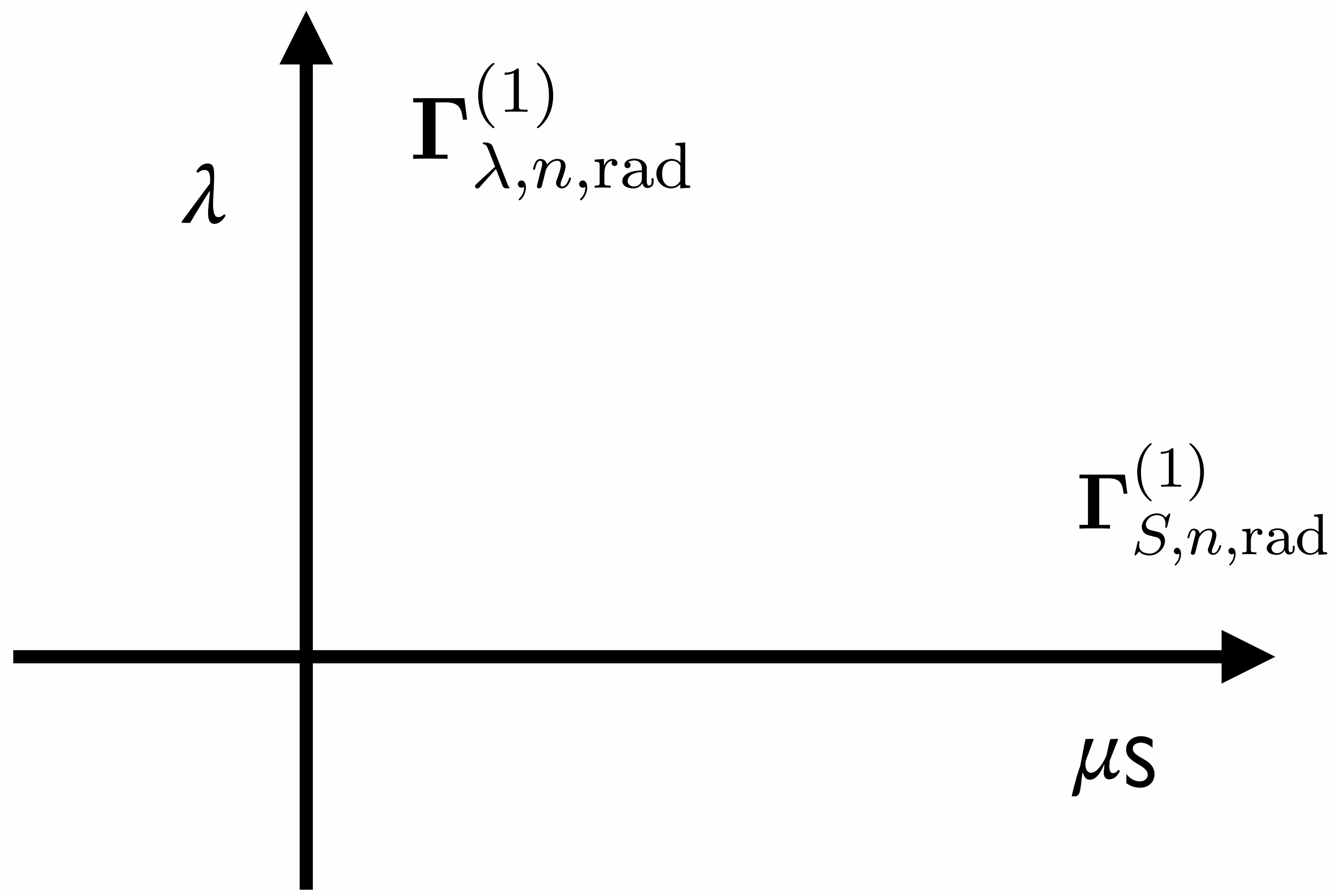
How to do this at subleading N and higher order shower evolution?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

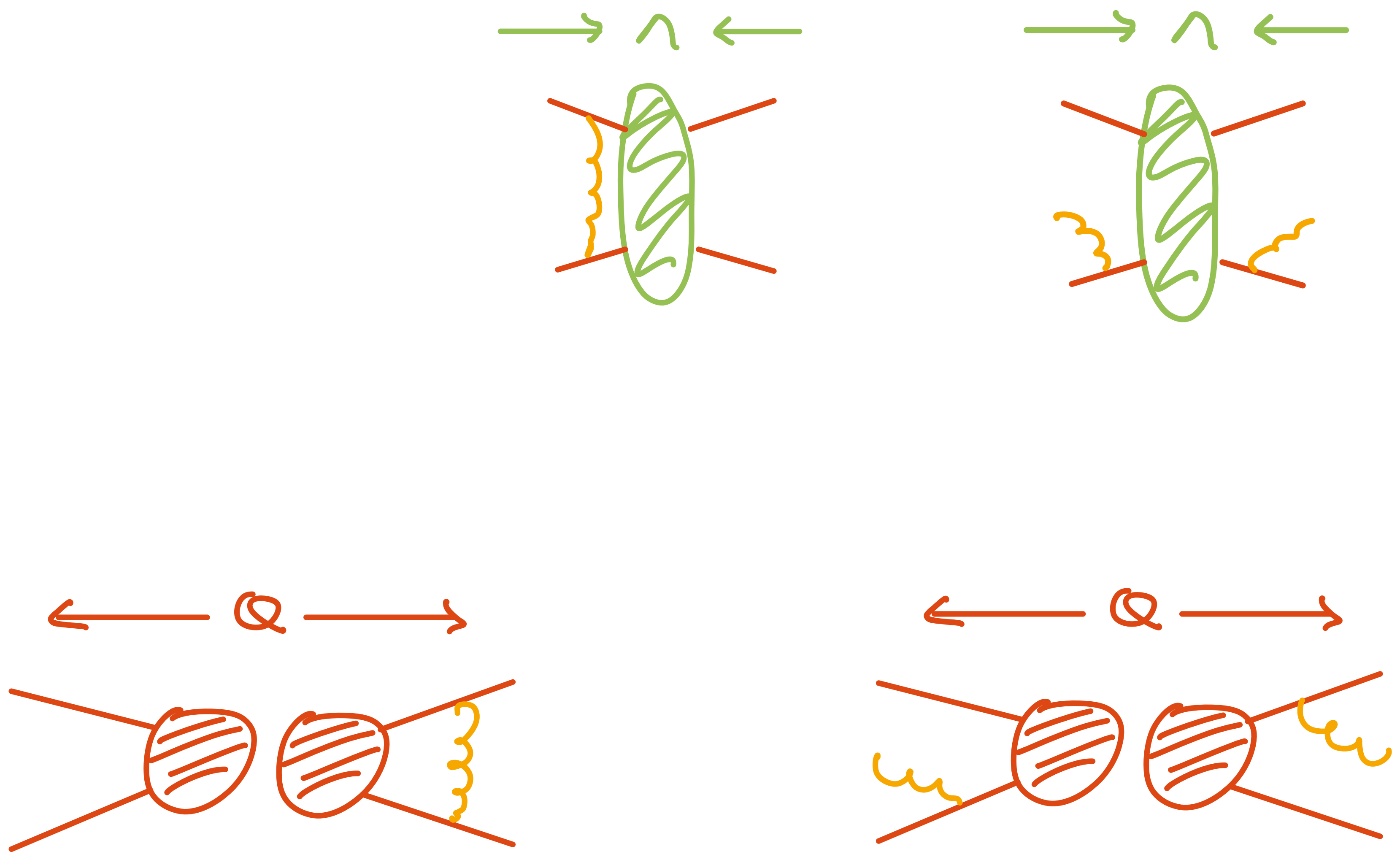
Hard factor reproduces CVolver algorithm and predicts key features of second order evolution.

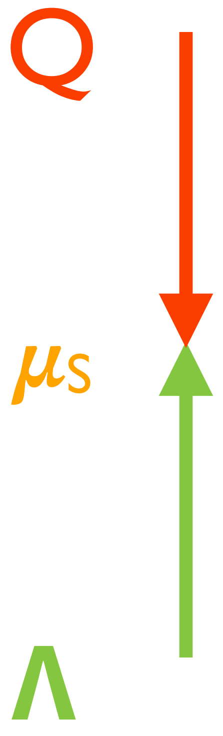
$\partial_S \mathbf{S}_{,}$ $(p_i)$

$\partial_S \mathbf{A}$




$$\Theta_{n,1} = 1 - \hat{\Theta}_{n,1}\theta(E_n - \mu_S)$$





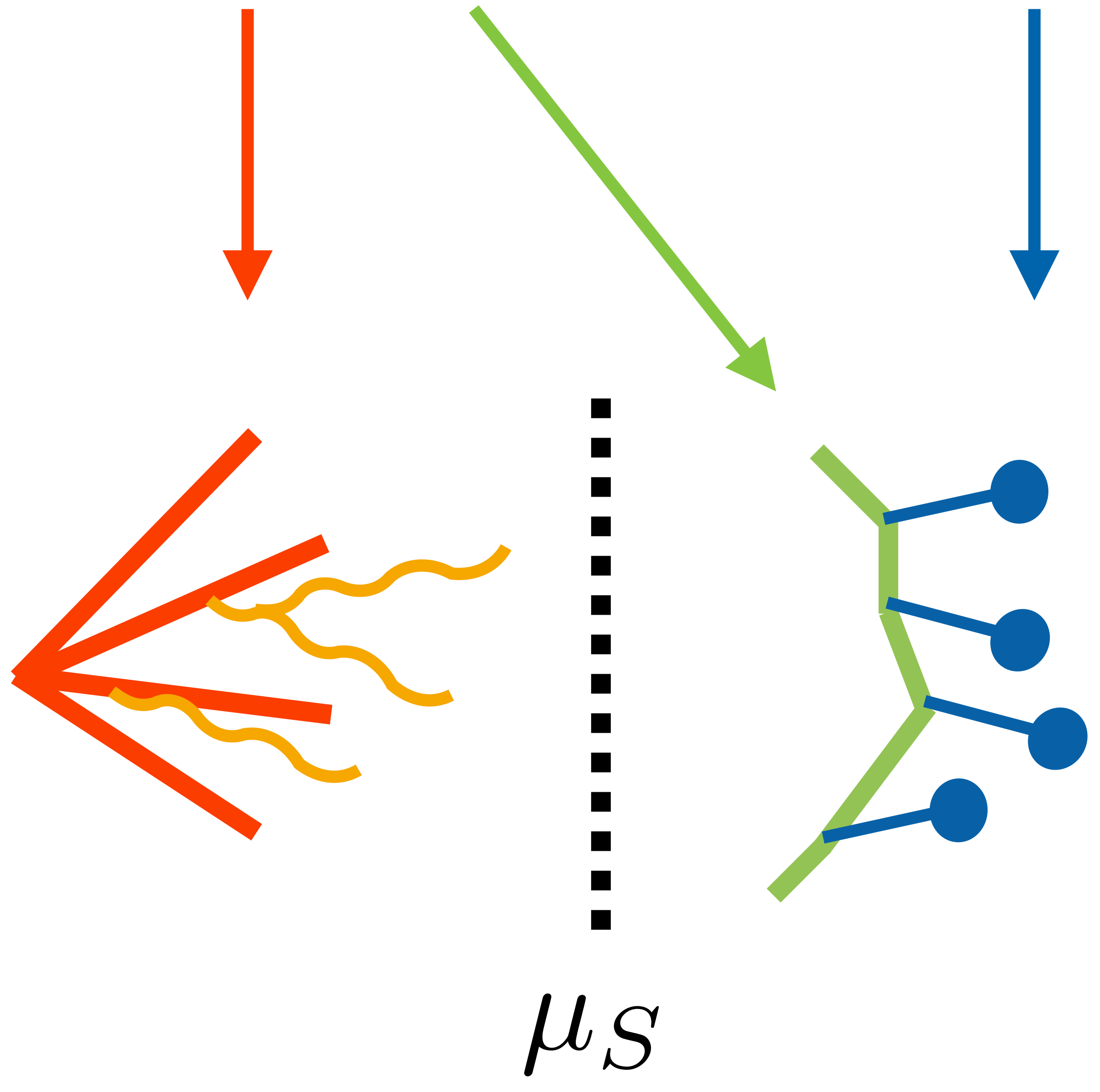
# Constructing evolution algorithms

How do we consistently hadronize in light of (improved) shower algorithms?

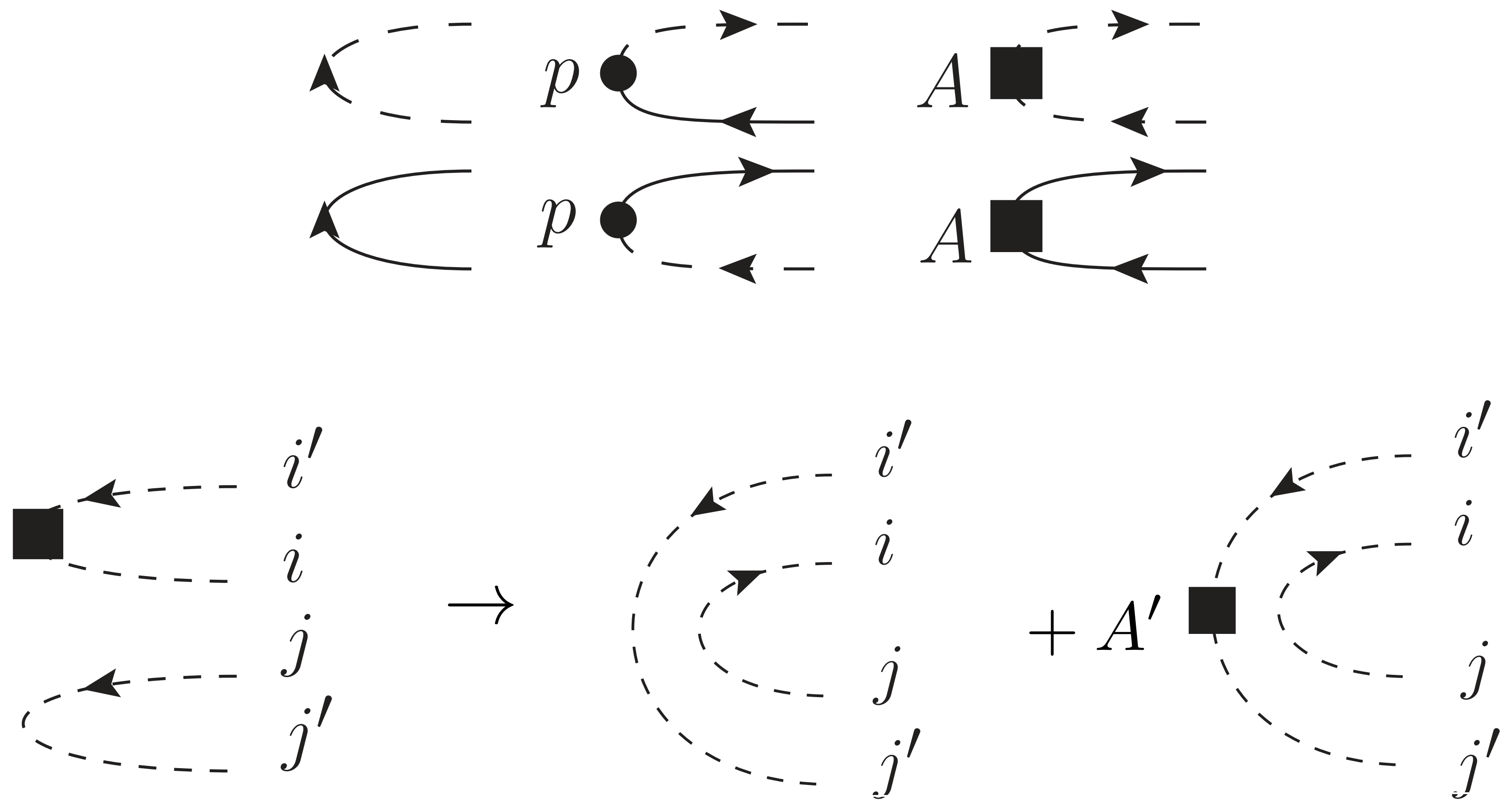
How to do this at subleading N and higher order shower evolution?

$$\sigma = \sum_{n,m} \int\int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$




Construct electroweak evolution.
Measurement projection is ubiquitous.




Basis and mixing of chirality structures.

[Plätzer, Sjödahl — '21]

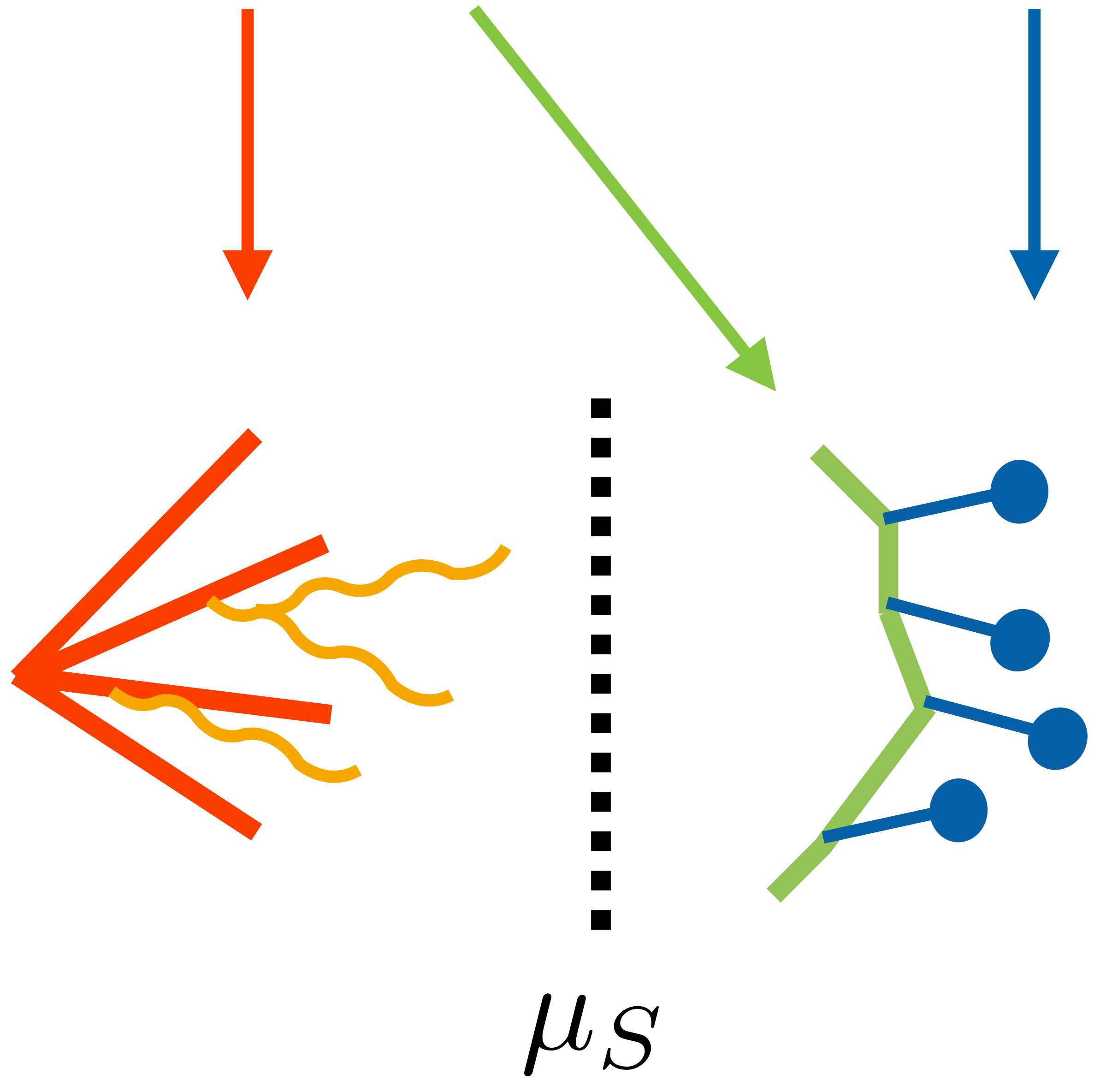
# Constructing evolution algorithms

How do we consistently hadronize in light of (improved) shower algorithms?

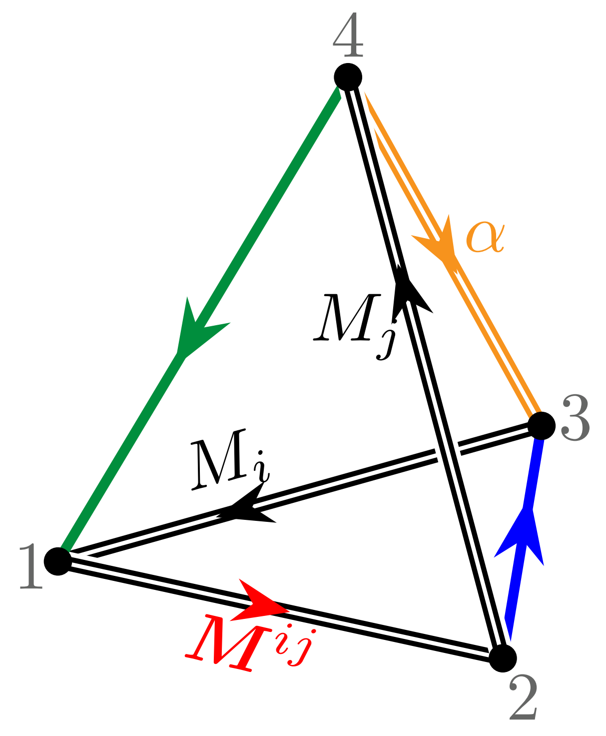
How to do this at subleading N and higher order shower evolution?

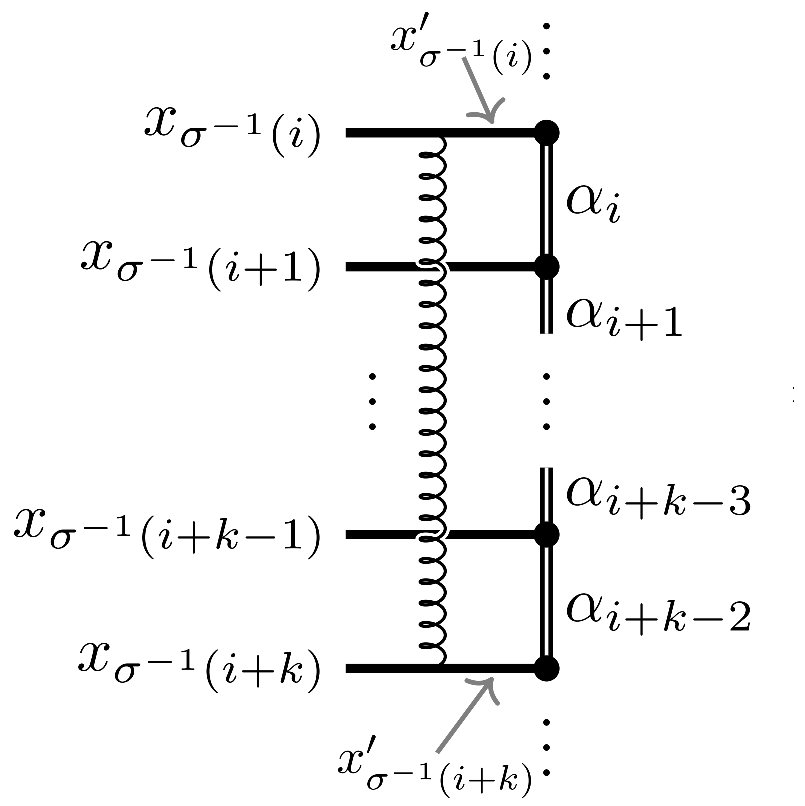
$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$




Understand colour multiplets for many legs.




[Alcock-Zeilinger, Keppeler, Plätzer, Sjödahl – '22 & in progress]

# Constructing evolution algorithms

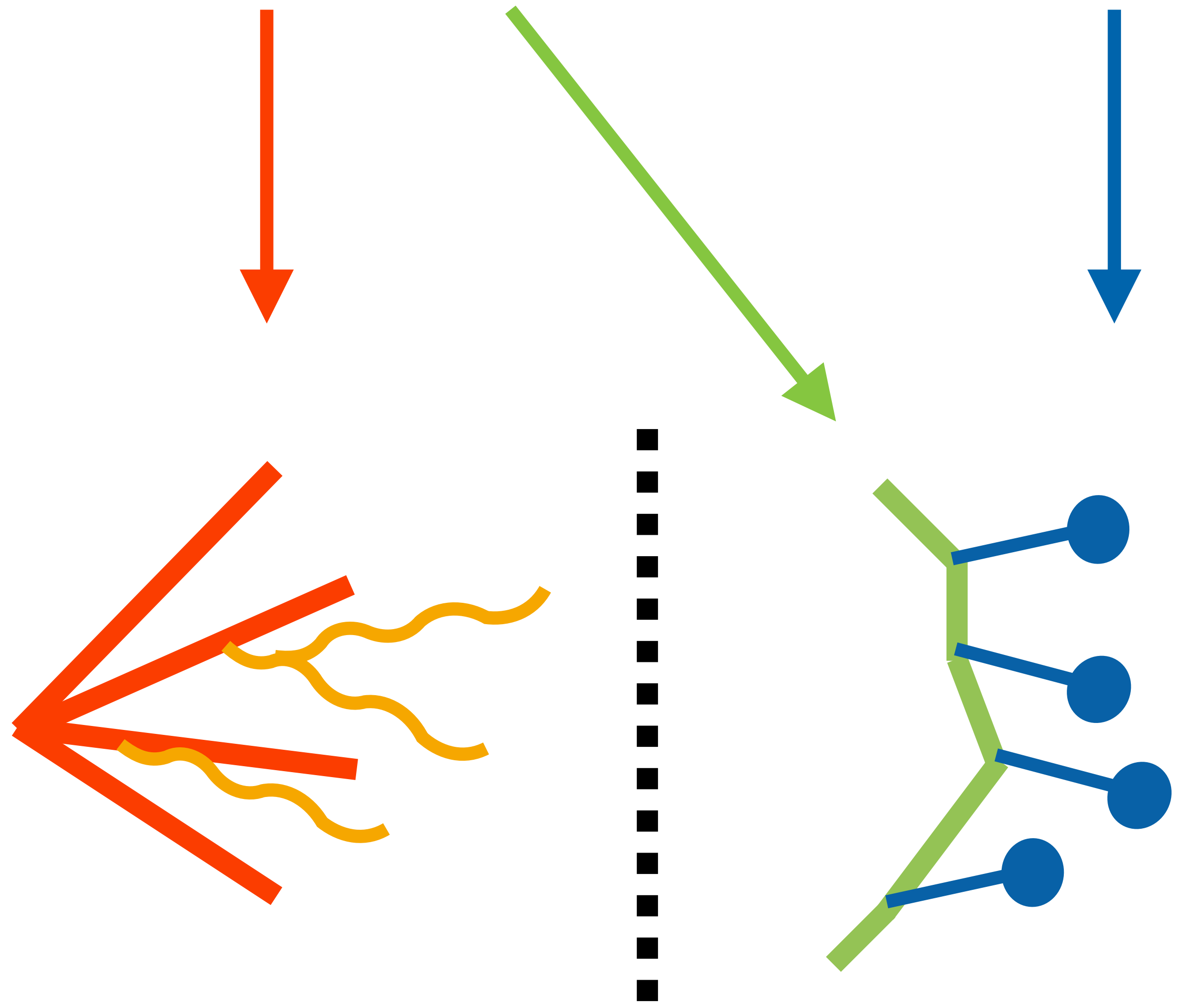
How do we consistently hadronize in light of (improved) shower algorithms?

How to do this at subleading N and higher order shower evolution?

[Plätzer – '22]

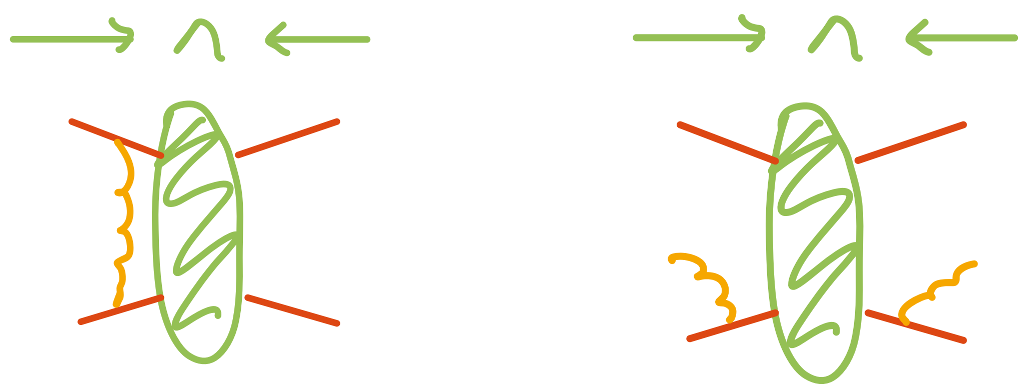
$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

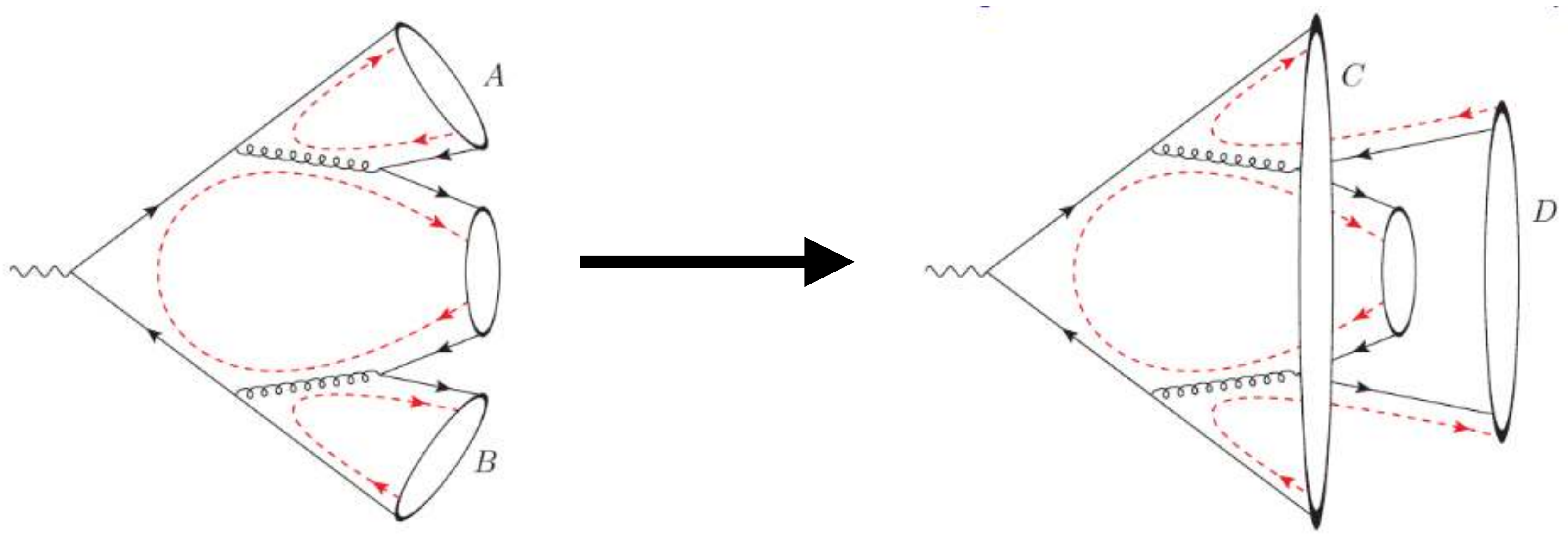
$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$



$\mu_S$

Construct perturbative end of hadronization.

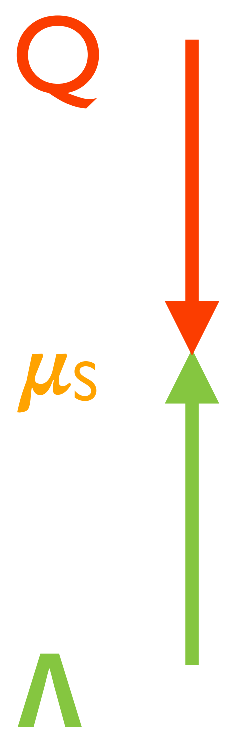




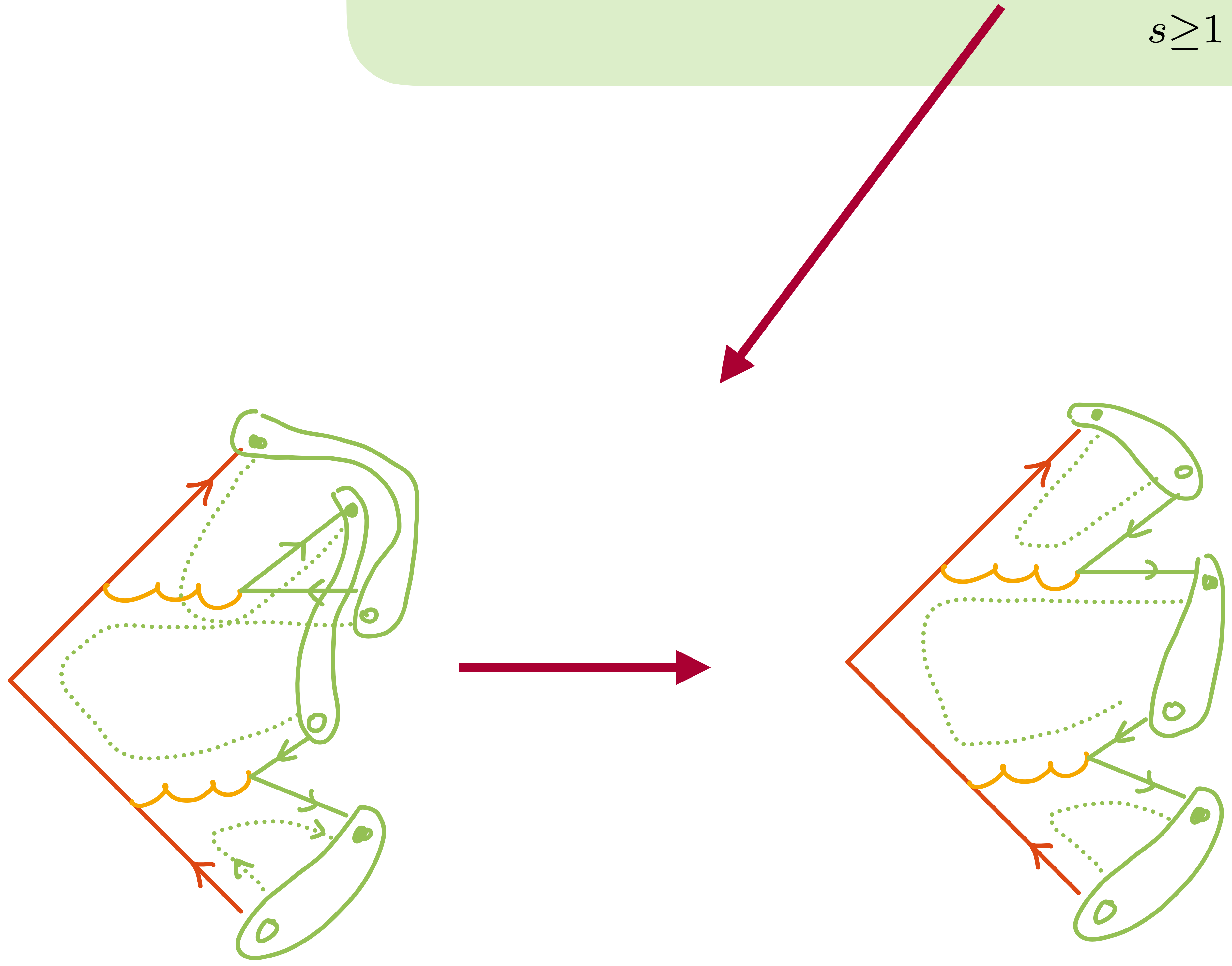

e.g. colour reconnection *implied* just as observed in [Gieseke, Kirchgaesser, Plätzer – '18 …]

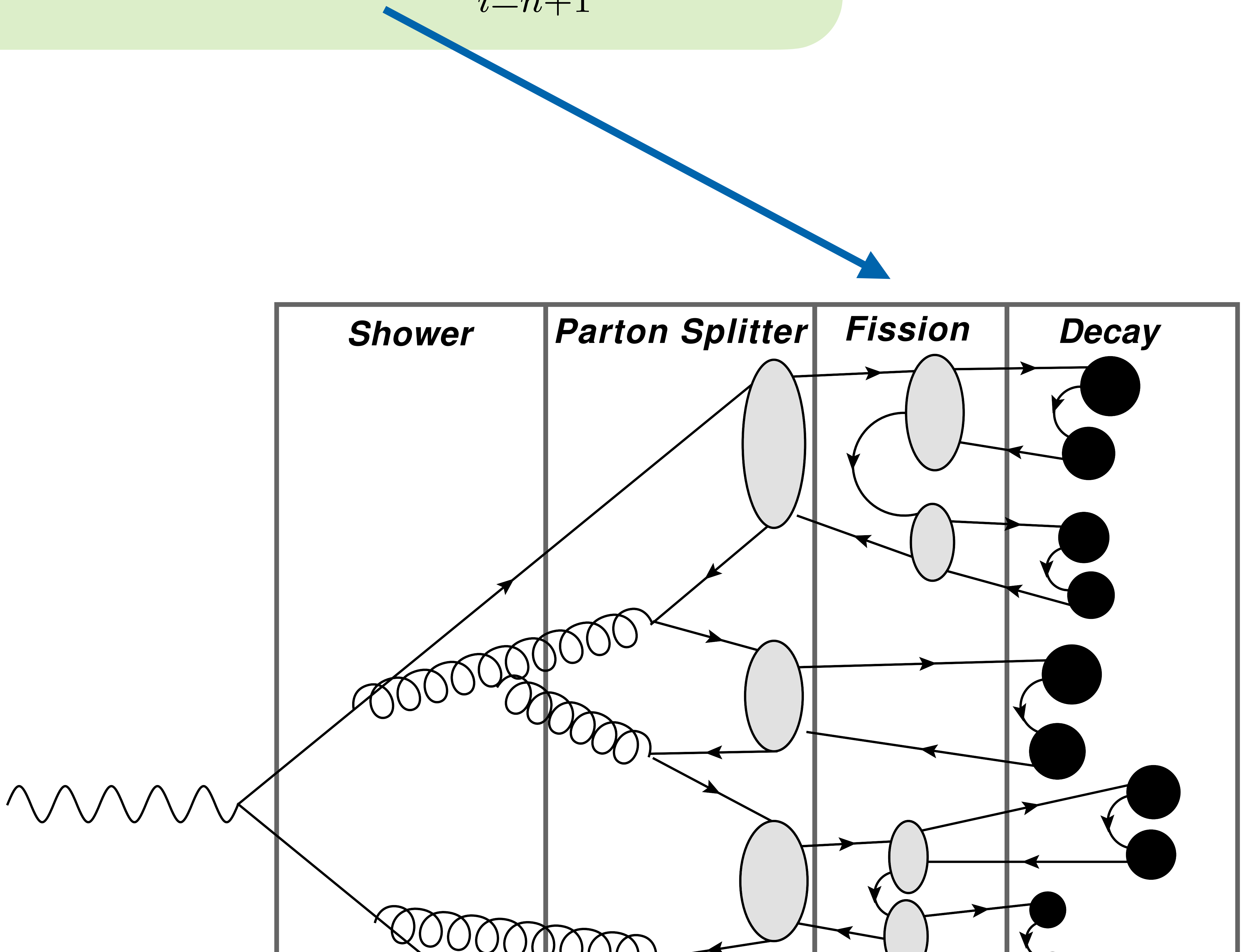
# Hadronization models

$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$




$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^\dagger_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1}\alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s}\prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$



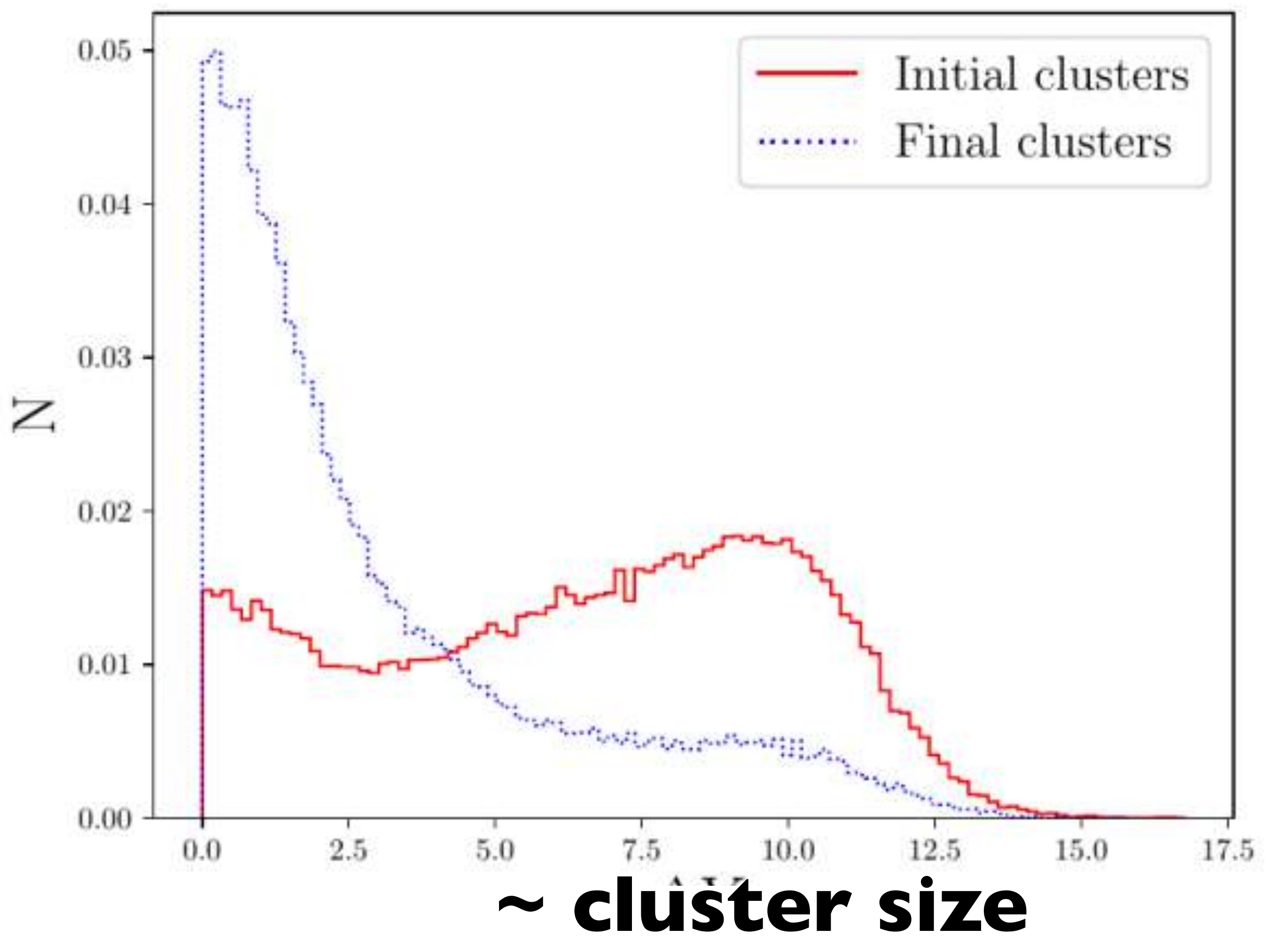


# Hadronization models

$$d\sigma \sim L \times d\sigma_H(Q) \times PS(Q \to \mu) \times MPI \times Had(\mu \to \Lambda) \times \ldots$$

$\partial_S S$

**amplitude evolution**

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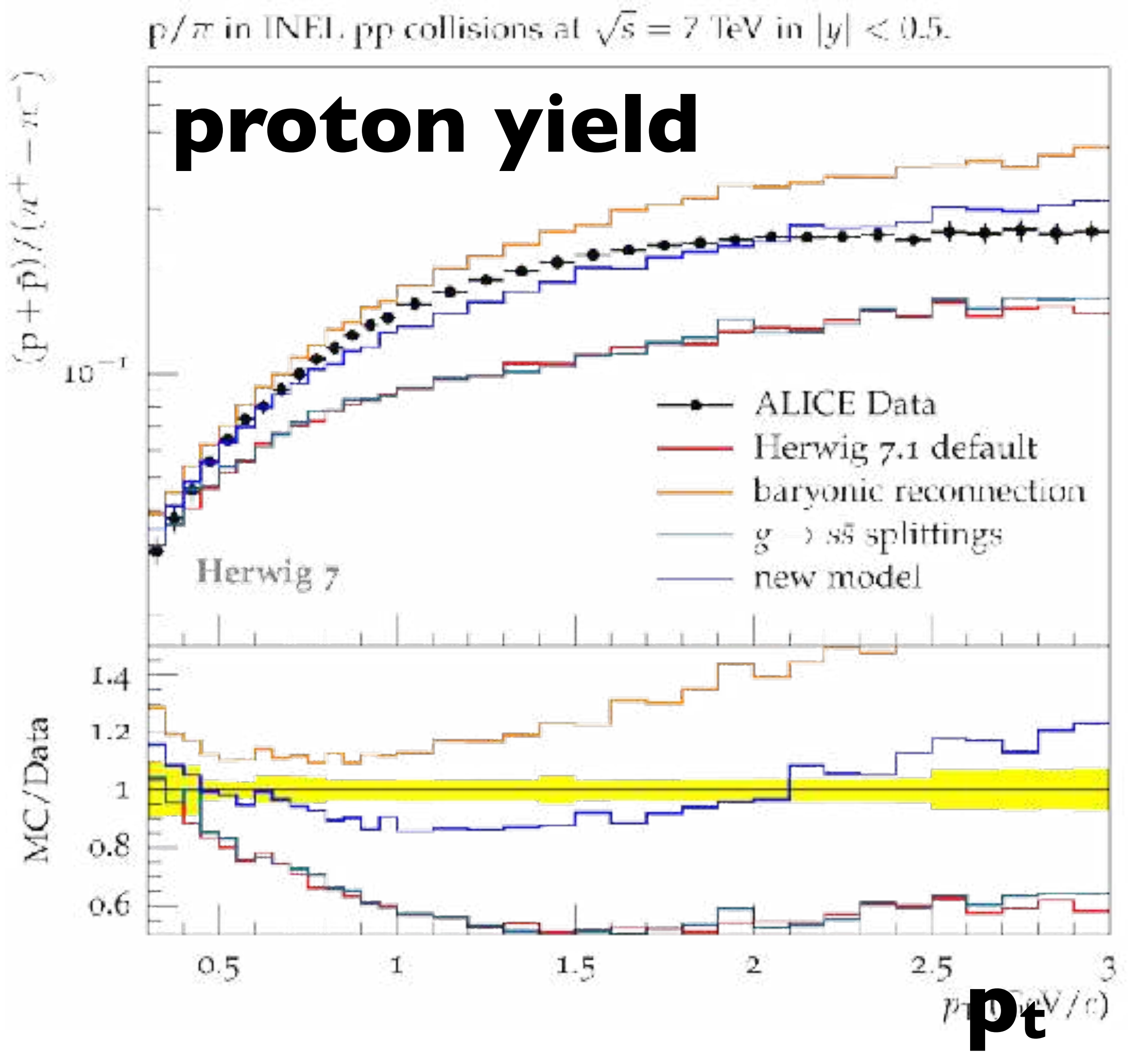
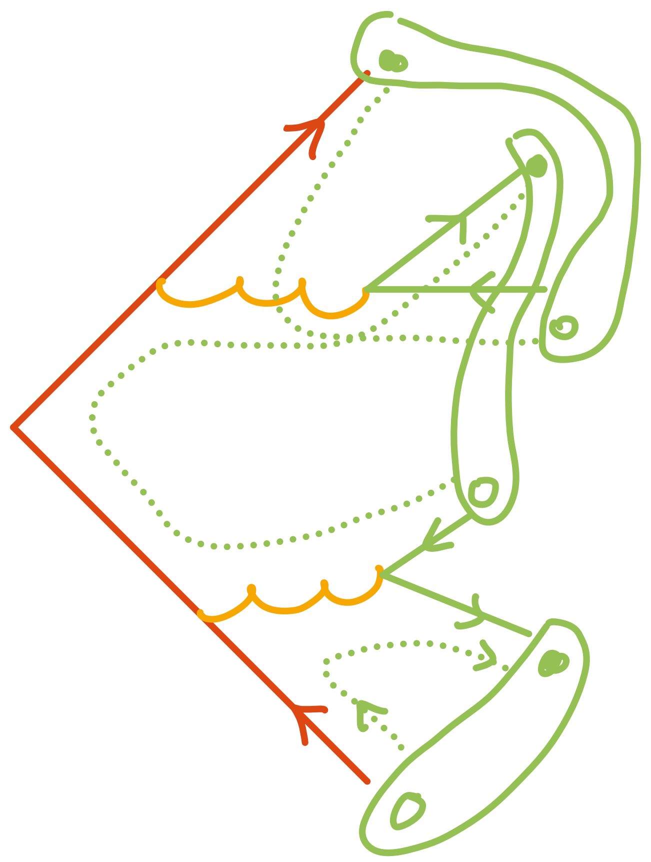
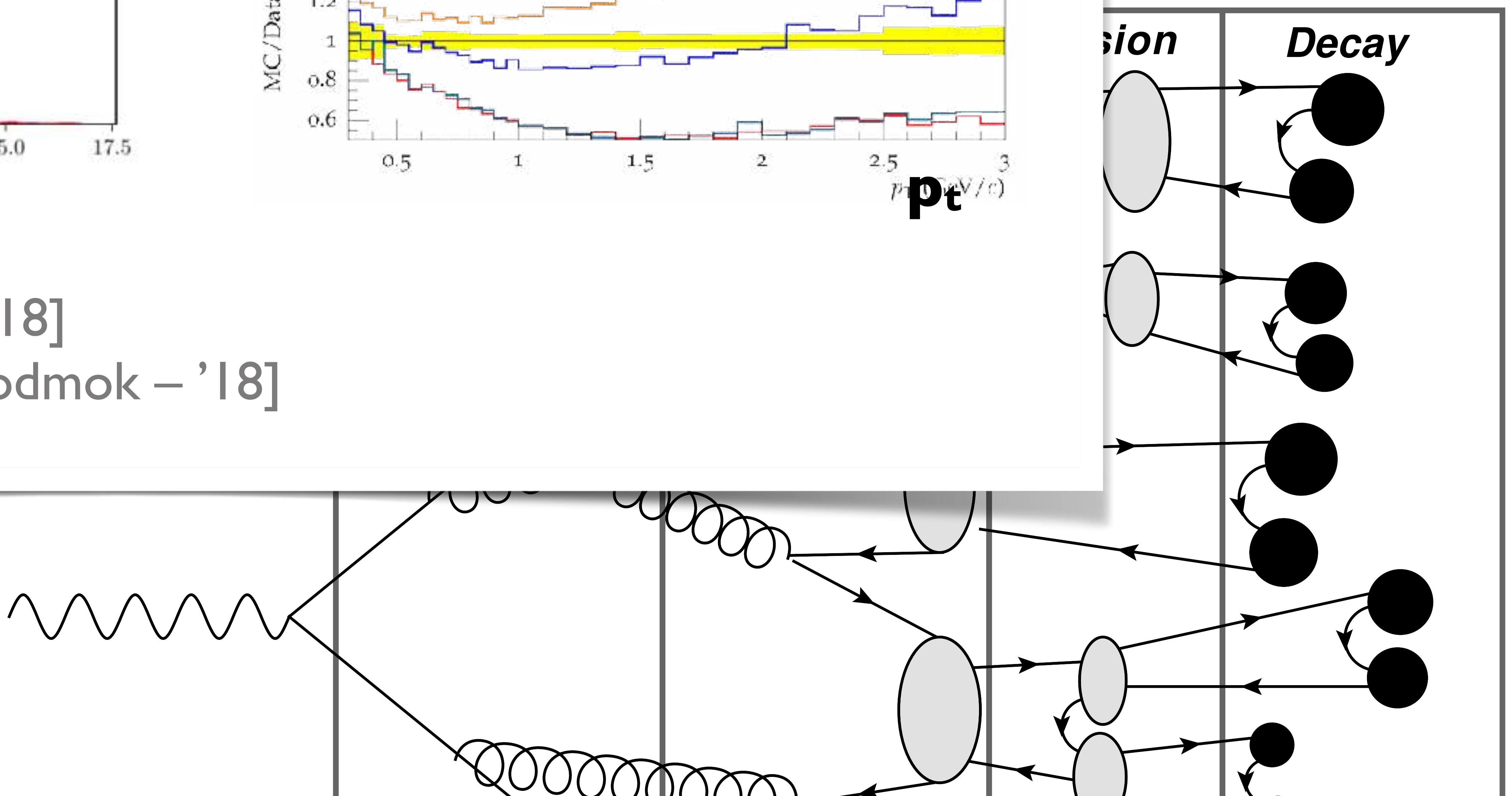



[Gieseke, Kirchgaesser, Plätzer – '18]
[Gieseke, Kirchgaesser, Plätzer, Siodmok – '18]






Q

$\mu_S$

Λ

# Summary

Amplitude evolution is much more than just studying subleading-N effects.
We use to as a theoretical tool and algorithm in its own right.

We can address the structure of evolution algorithms, at leading and higher orders.
Systematic break down in large-N allows us to solidify structure of new algorithms.

[in progress for second order]

Infrared cutoff is the factorisation scale to hadronization models and allows us to construct their high-energy end from perturbative considerations, including colour reconnection.

explored for non-globals and in Herwig, see Stefan's talk
[Hoang, Plätzer, Samitz — in progress]
[Gieseke, Kiebacher, Plätzer, Priedigkeit — in progress]

This framework is set to be extended to the initial state and to JIMWLK.

[Plätzer, Weigert — in progress]

C.Stadler/Bwag

July 1 Room for informal meetings

# 2024 Graz July 2-4

# Parton Showers and Resummation

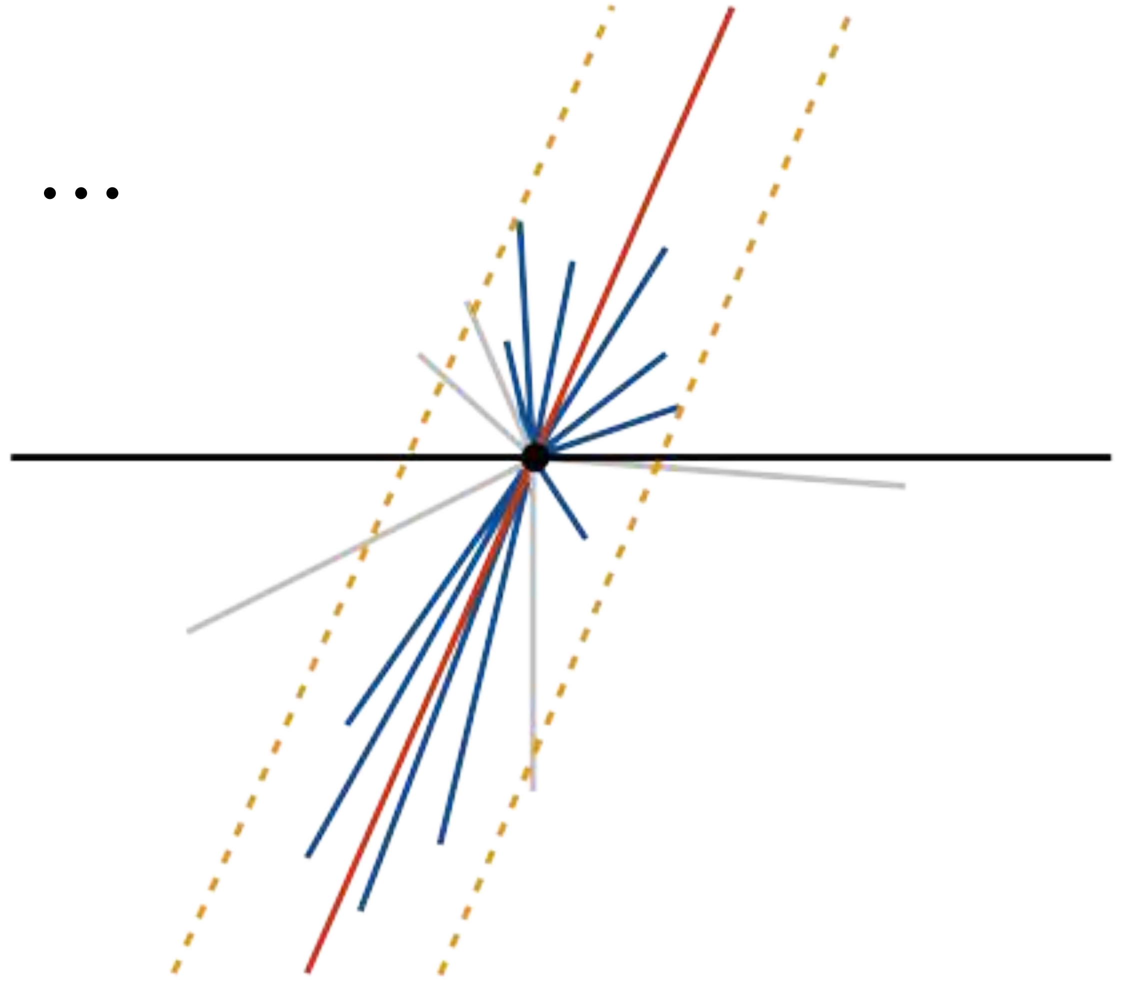
July 5 A fresh look at hadronization

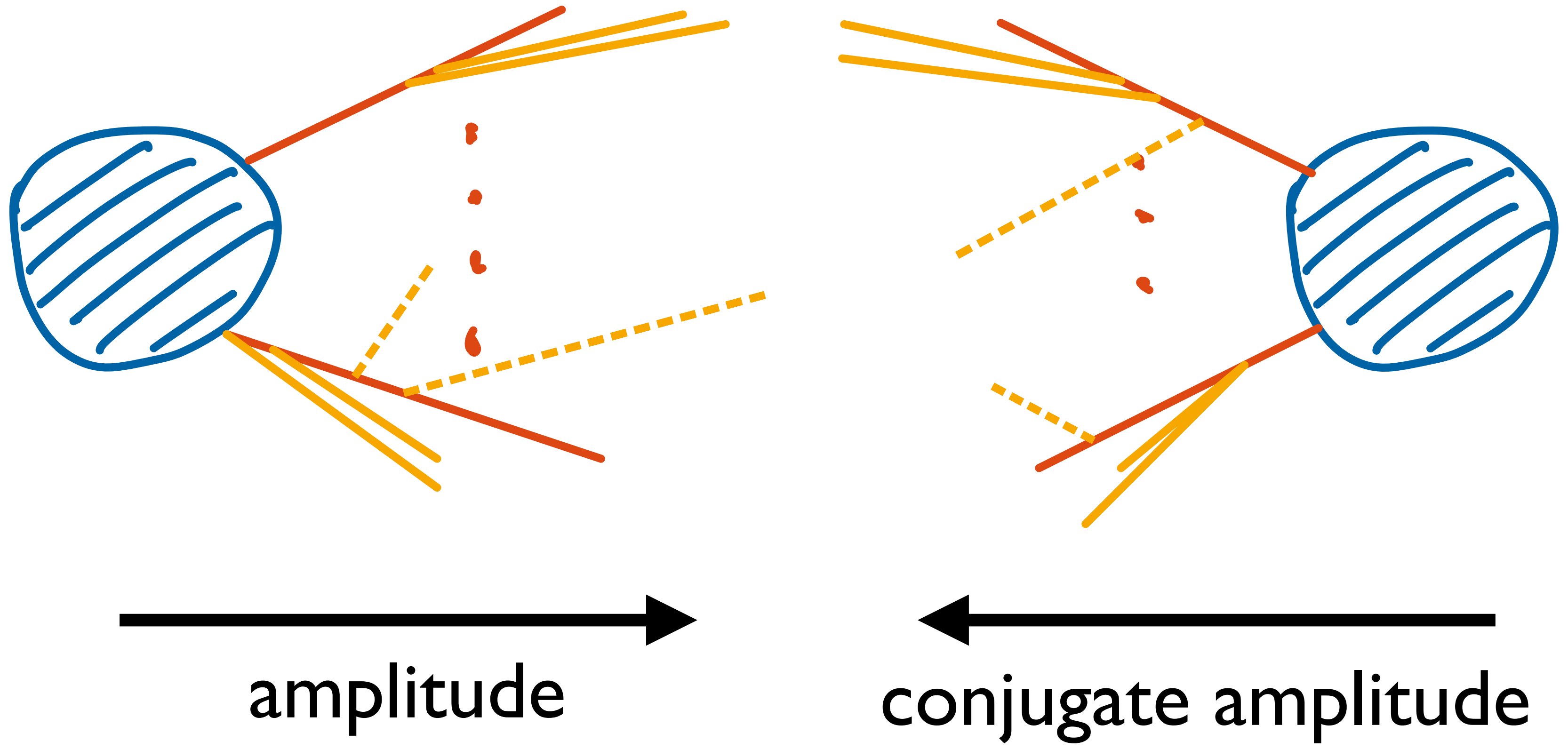
Organised by J. Forshaw, S. Plätzer and M. Sjödahl

# Thank you.

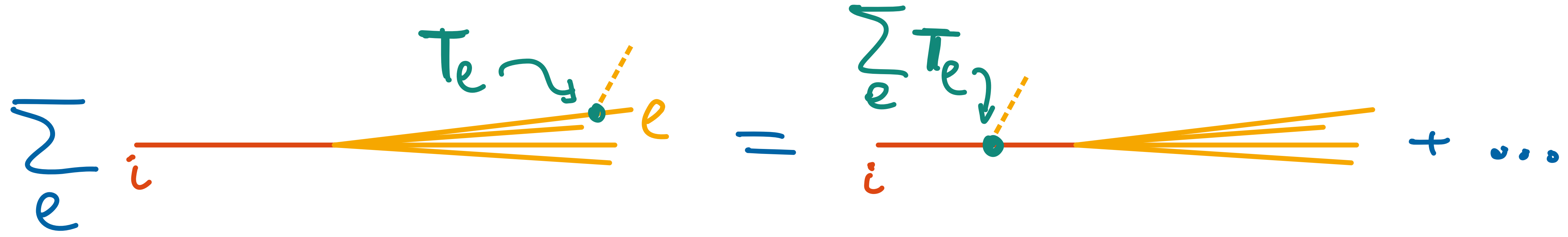
# Coherent branching parton showers

$$d\sigma \sim L \times d\sigma_H(Q) \times PS(Q \to \mu) \times MPI \times Had(\mu \to \Lambda) \times \ldots$$



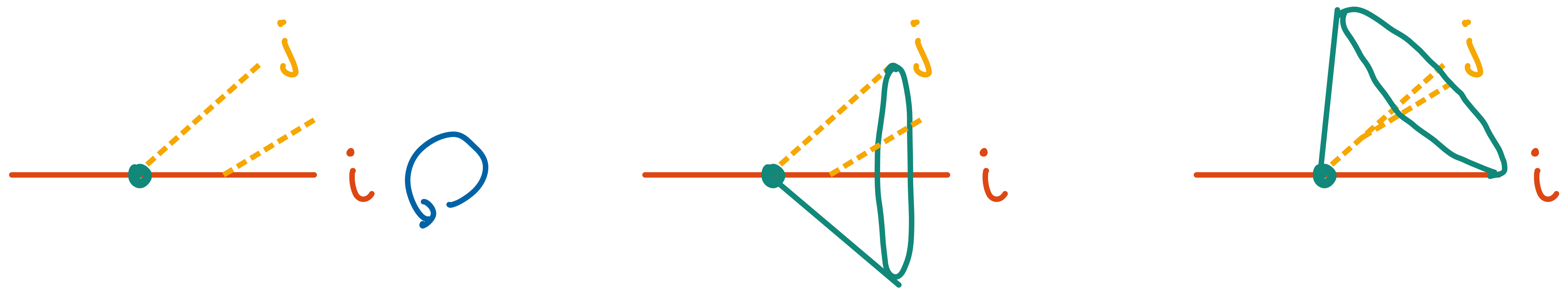






Move soft colour charges towards hard process and use angular ordering for azimuthal average around jet axes:

$$T_j T_l T_i \cdot T_i T_m T_j = C_i T_j T_l \cdot T_m T_j$$

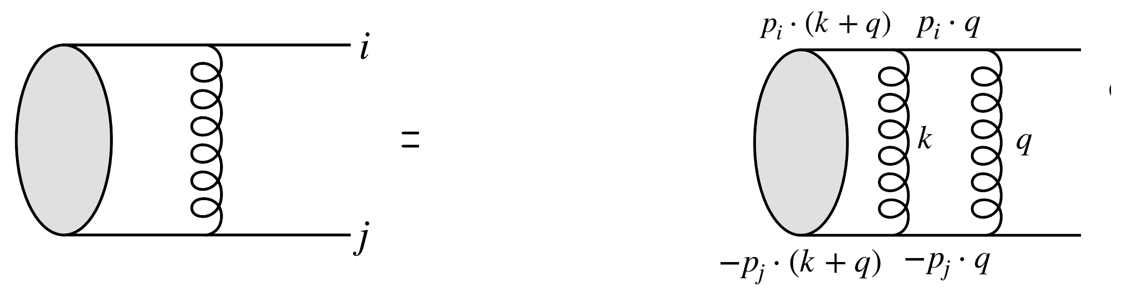


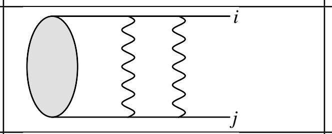
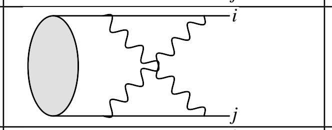
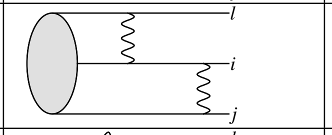
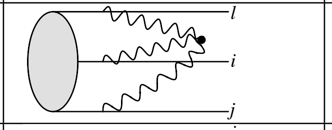
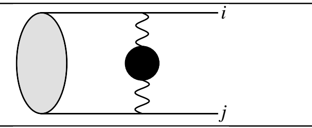
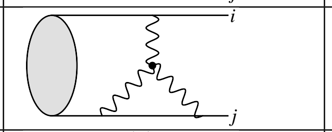
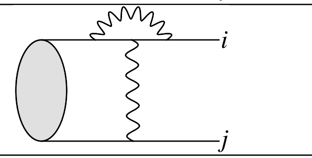
# (Soft) factorisation of amplitudes

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[ \mathbf{M}_n \mathbf{U}_{nm} \right] \mathrm{d}\phi_m u(\phi_m)$$

## Factorisation of virtual contributions

$$\mathbf{M}_n^{(l)} = \mathbf{V}^{(1)}\mathbf{M}_n^{(l-1)} + \mathbf{M}_n^{(l-1)}\mathbf{V}^{(1)\dagger} + \mathbf{V}^{(1)}\mathbf{M}_n^{(l-2)}\mathbf{V}^{(1)\dagger} + \mathbf{V}^{(2)}\mathbf{M}_n^{(l-2)} + \mathbf{M}_n^{(l-2)}\mathbf{V}^{(2)\dagger} + \dots$$

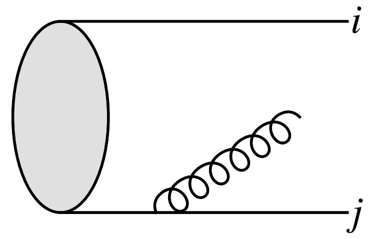


| Coefficient | Diagram | Colour-factor |
|---|---|---|
| $\Omega_{ij}^{(2)}$ |  | $(\mathbf{T}_i \cdot \mathbf{T}_j)(\mathbf{T}_i \cdot \mathbf{T}_j)$ |
| $\tilde{\Omega}_{ij}^{(2)}$ |  | $\mathbf{T}_i^a \mathbf{T}_i^b \mathbf{T}_j^b \mathbf{T}_j^a$ |
| $\Omega_{ijl}^{(2)}$ |  | $(\mathbf{T}_i \cdot \mathbf{T}_l)(\mathbf{T}_i \cdot \mathbf{T}_j)$ |
| $\hat{\Omega}_{ijl}^{(2)}$ |  | $i f^{abc} \mathbf{T}_i^a \mathbf{T}_j^b \mathbf{T}_l^c$ |
| $\Omega_{ij,\mathrm{self-en.}}^{(2)}$ |  | $T_R(\mathbf{T}_i \cdot \mathbf{T}_j)$ |
| $\Omega_{ij,\mathrm{vertex-corr.}}^{(2)}$ |  | $T_R(\mathbf{T}_i \cdot \mathbf{T}_j)$ |
| $\hat{\Omega}_{ij}^{(2)}$ |  | $\mathbf{T}_i^b \mathbf{T}_i^a \mathbf{T}_i^b \mathbf{T}_j^a$ |

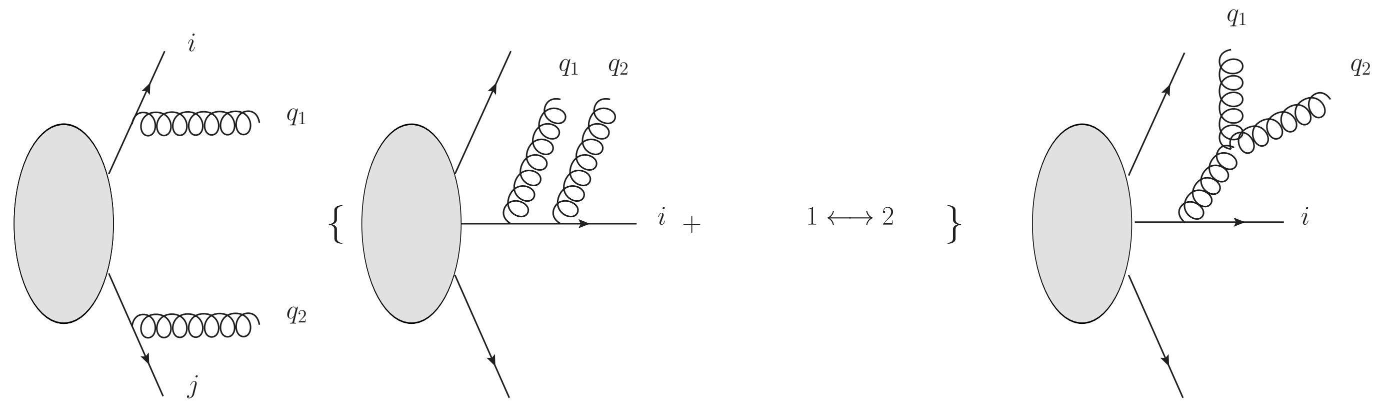
Handle virtual as phase-space type integrals to remove divergencies with subtractions.

[Plätzer, Ruffa — '21]

## Factorisation of real contributions



$$\mathbf{M}_n^{(l)} = \mathbf{D}_n^{(1,0)}\mathbf{M}_{n-1}^{(l)}\mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,1)}\mathbf{M}_{n-1}^{(l-1)}\mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,0)}\mathbf{M}_{n-1}^{(l-1)}\mathbf{D}_n^{(1,1)\dagger} + \mathbf{D}_n^{(2,0)}\mathbf{M}_{n-2}^{(l)}\mathbf{D}_n^{(2,0)\dagger} + \dots$$

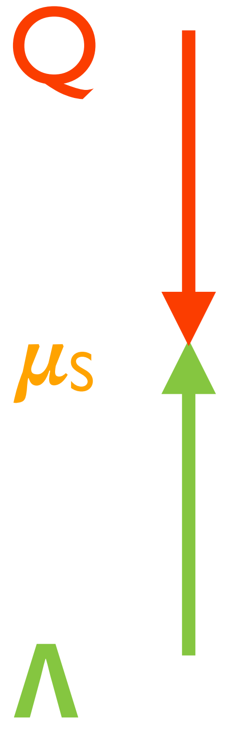


$$\sum_{(a,b),(c,d)} \sum_{i,j,k,l=1}^{n} \omega_{ijkl}^{abcd}\, \mathrm{T}_i^{(a)}\mathrm{T}_j^{(b)} \circ \mathrm{T}_k^{(c),\dagger}\mathrm{T}_l^{(d),\dagger}$$

[Majcen — M.Sc. thesis 2022]
based on Catani & Grazzini

# Infrared subtractions

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[ \mathbf{M}_n \mathbf{U}_{nm} \right] \mathrm{d}\phi_m u(\phi_m)$$




Subtractions necessitate a resolution:
what is it we call 'unresolved'?
Encompass all singular regions!

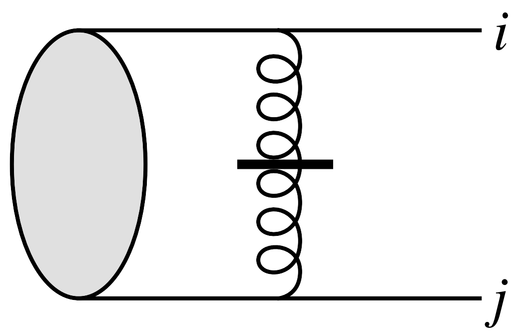
$$\mathbf{U}_n = \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$

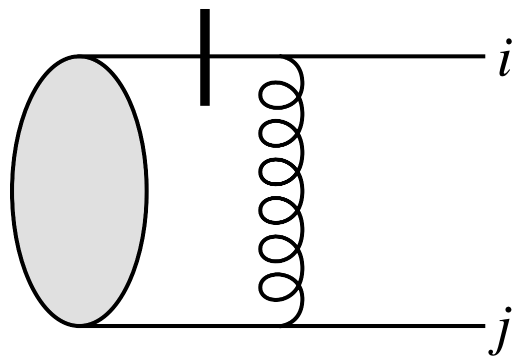
resolution function for (cut) loop momenta

$$\mathbf{X}_n^{(1)} = \hat{\mathbf{V}}_n^{(1)} [\Xi_{n,1}]$$

$$\hat{\mathbf{V}}_n^{(l)} [\Xi_{n,l}] = \sum_\alpha \int \mathcal{I}_{n,\alpha}^{(l)} (p_1, ..., p_n; k_1, ..., k_l) \; \Xi_{n,l}^{(\alpha)} \prod_{i=1}^{l} \mu_R^{2\epsilon} [\mathrm{d}k_i]$$

$$\mathbf{F}_n^{(1,0)} \circ \mathbf{F}_n^{(1,0)\dagger} = \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

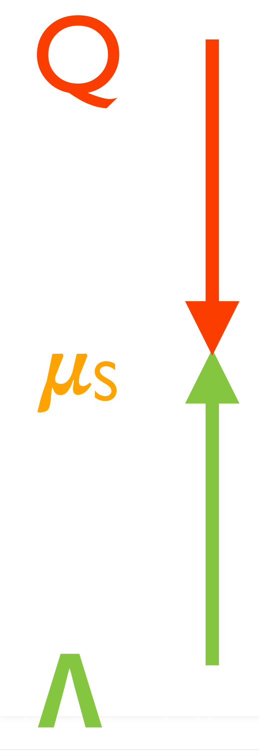



resolution function for real emission

# Infrared subtractions

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$




Subtractions necessitate a resolution:
what is it we call 'unresolved'?
Encompass all singular regions!

$$\mathbf{U}_n = \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$

resolution function for (cut) loop mome

$$\mathbf{X}_n^{(1)} = \hat{\mathbf{V}}_n^{(1)}[\Xi_{n,1}]$$

$$\mathbf{F}_n^{(1,0)} \circ \mathbf{F}_n^{(1,0)\dagger} = \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

resolution function for real emission

## Continues to higher orders …

$$\mathbf{X}_n^{(2)} = \hat{\mathbf{V}}_n^{(2)}[\Xi_{n,2}] - \hat{\mathbf{V}}_n^{(1)}[\Xi_{n,1}]\hat{\mathbf{V}}_n^{(1)}$$

$$\begin{aligned}\mathbf{F}_n^{(1,1)} \circ \mathbf{F}_n^{(1,0)\dagger} =\ & \mathbf{D}_n^{(1,1)}\left[\Xi_{n-1,1}\right] \circ \mathbf{D}_n^{(1,0)\dagger}\Theta_{n,1} \\ & + \mathbf{D}_n^{(1,1)}\left[1 - \Xi_{n-1,1}\right] \circ \mathbf{D}_n^{(1,0)\dagger}\Theta_{n,1} + \mathbf{D}_n^{(1,1)}\left[\Xi_{n-1,1}\right] \circ \mathbf{D}_n^{(1,0)\dagger}(1 - \Theta_{n,1}) \\ & - \hat{\mathbf{V}}_n^{(1)}\left[\Xi_{n-1,1}\right] \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} + \mathbf{D}_n^{(1,0)} \hat{\mathbf{V}}_{n-1}^{(1)} \circ \mathbf{D}_n^{(1,0)\dagger}\Theta_{n,1}\end{aligned}$$

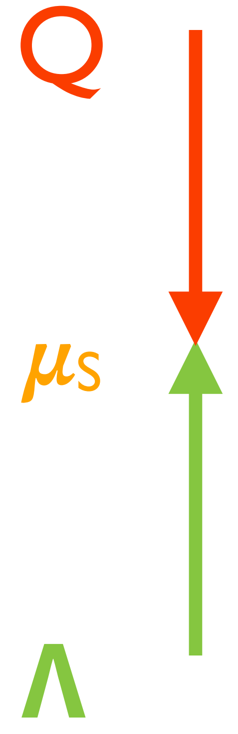
$$\mathbf{F}_n^{(2,0)} \circ \mathbf{F}_n^{(2,0)\dagger} = \mathbf{D}_n^{(2,0)} \circ \mathbf{D}_n^{(2,0)\dagger}\Theta_{n,2} - \mathbf{D}_n^{(1,0)}\mathbf{D}_{n-1}^{(1,0)} \circ \mathbf{D}_{n-1}^{(1,0)\dagger}\mathbf{D}_n^{(1,0)\dagger}\Theta_{n,1}$$

# Infrared subtractions

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Subtractions necessitate a resolution:
what is it we call 'unresolved'?
Encompass all singular regions!

$$\mathbf{U}_n = \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$




resolution function for (cut) loop momenta

$$\mathbf{X}_n^{(1)} = \hat{\mathbf{V}}_n^{(1)} [\Xi_{n,1}]$$

$$\mathbf{F}_n^{(1,0)} \circ \mathbf{F}_n^{(1,0)\dagger} = \mathbf{D}_n^{(1,0)} \circ \mathbf{D}_n^{(1,0)\dagger} \Theta_{n,1}$$

resolution function for real emission

Resolution functions introduce cutoff dependence, e.g. energy ordering:

$$\Theta_{n,1} = 1 - \hat{\Theta}_{n,1} \theta(E_n - \mu_S)$$

"soft or collinear"

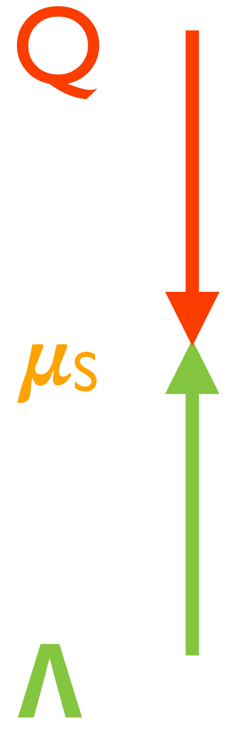
$$\Theta_{n,2} = 1 - \hat{\Theta}_{n,2} \theta(E_{n-1} - \mu_S) \theta(E_n - \mu_S)$$

# Infrared subtractions

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[ \mathbf{M}_n \mathbf{U}_{nm} \right] \mathrm{d}\phi_m u(\phi_m)$$

Subtractions necessitate a resolution:
what is it we call ‘unresolved’?
Encompass all singular regions!

$$\mathbf{U}_n = \mathbf{X}_n^\dagger \mathbf{S}_n \mathbf{X}_n - \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{F}_{n+s}^{(s)\dagger} \mathbf{S}_{n+s} \mathbf{F}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$




Enters the re-definition of observables, e.g. demanding a jet cross section

use unitarity and pick equal resolutions $\int \mathbf{D}_{n+1}^{(1,0)\dagger} \mathbf{D}_{n+1}^{(1,0)} \Theta_{n,1} \mu_R^{2\epsilon} [\mathrm{d}p_{n+1}] \tilde{\delta}(p_{n+1}) = -\frac{1}{2} \hat{\mathbf{V}}_n^{(1)} [\Theta_{n,1}]$

$$\mathbf{U}_n = \mathbf{1}_n u(p_1, \ldots, p_n) - \alpha_s \int \mu_R^{2\epsilon} [\mathrm{d}p_{n+1}] \tilde{\delta}(p_{n+1}) \hat{\mathbf{D}}_{n+1}^{(1,0)\dagger} \hat{\mathbf{D}}_{n+1}^{(1,0)} \Theta_{n,1} \left[ u(p_1, \ldots, p_n, p_{n+1}) - u(p_1, \ldots, p_n) \right] + \mathcal{O}(\alpha_s^2)$$

Proof this to vanish or to generate a power correction.

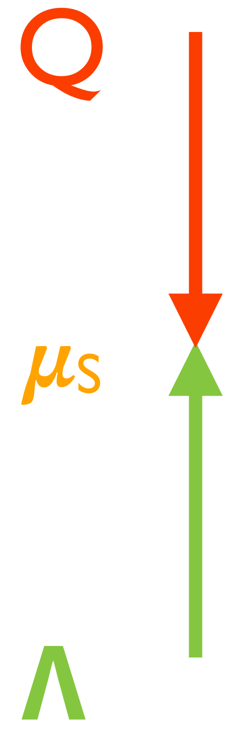
# Constructing evolution algorithms

How do we consistently hadronize in light of (improved) shower algorithms?

How to do this at subleading N and higher order shower evolution?

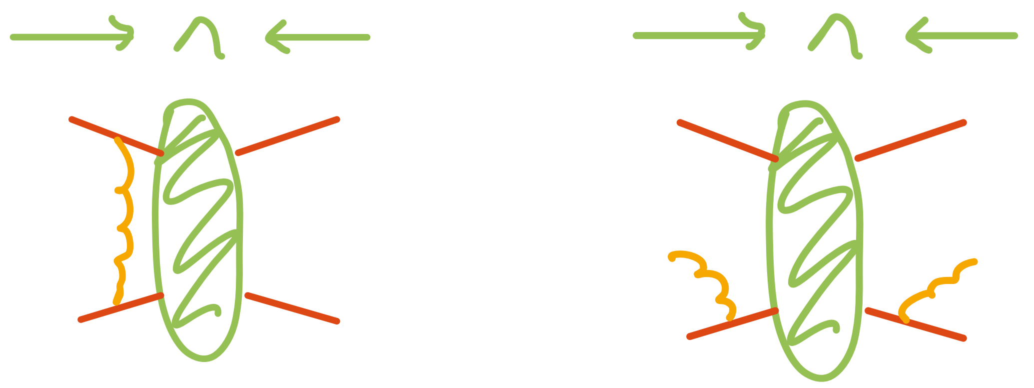
$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Hard factor reproduces CVolver algorithm and predicts key features of second order evolution.




Subtract iterated contribution in ordered phase space.

$$\mathbf{R}_n^{(2,0)} \circ \mathbf{R}_n^{(2,0)\dagger} = \Big(\hat{\mathbf{D}}_n^{(0,2)} \circ \hat{\mathbf{D}}_n^{(0,2)\dagger} \hat{\Theta}_{n,2} - \hat{\mathbf{D}}_n^{(0,1)} \hat{\mathbf{D}}_{n-1}^{(0,1)} \circ \hat{\mathbf{D}}_{n-1}^{(0,1)\dagger} \hat{\mathbf{D}}_n^{(0,1)\dagger} \hat{\Theta}_{n-1,1} \hat{\Theta}_{n,1}\Big) \times \theta(E_{n-1} - \mu_S)\delta(E_n - \mu_s) + \hat{\mathbf{D}}_n^{(0,2)} \circ \hat{\mathbf{D}}_n^{(0,2)\dagger} \hat{\Theta}_{n,2} \theta(E_n - \mu_S)\delta(E_{n-1} - \mu_S)$$



Use full double gluon matrix element outside.

Similar consequences for virtual corrections.

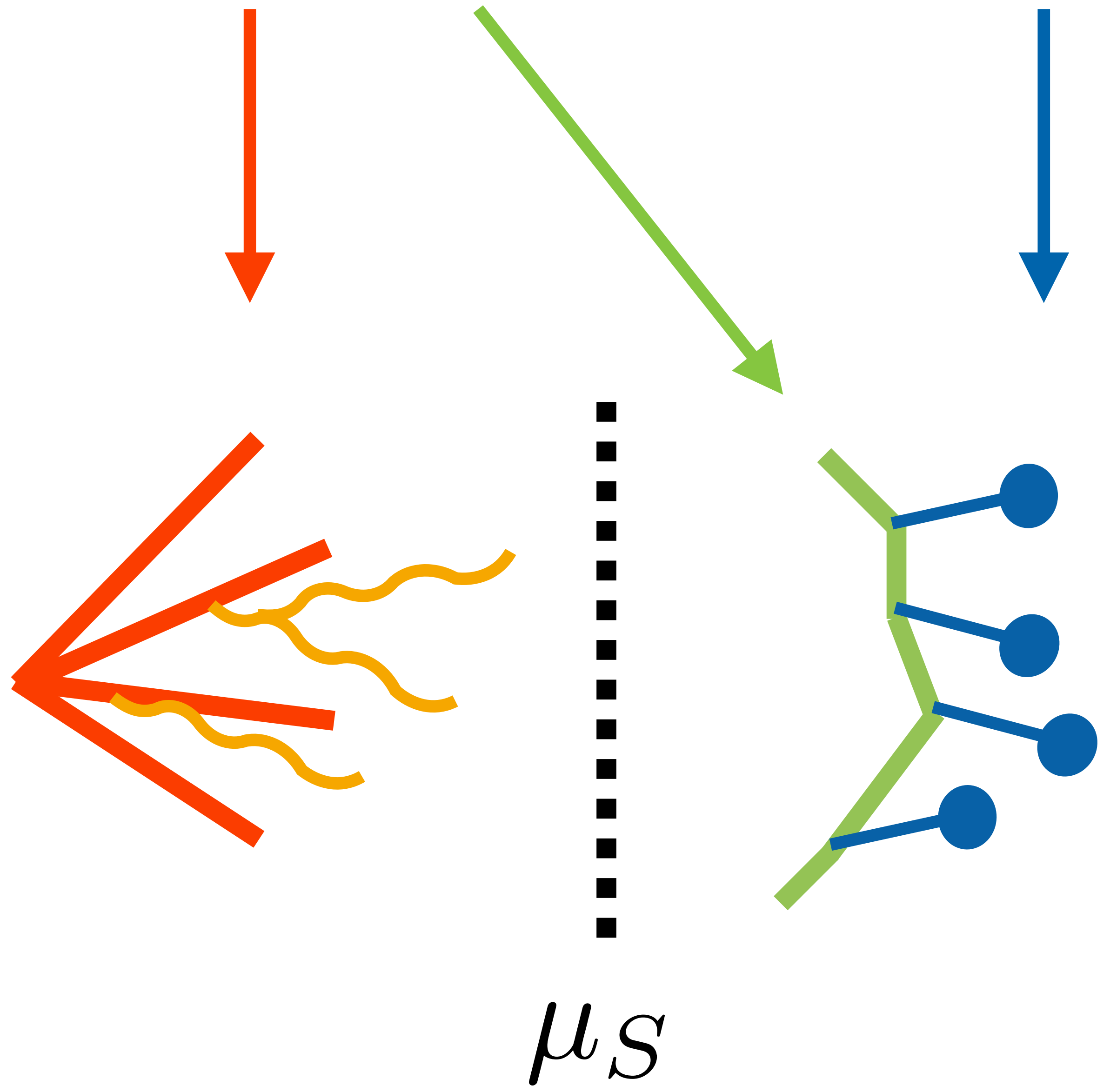
# Constructing evolution algorithms

How do we consistently hadronize in light of (improved) shower algorithms?

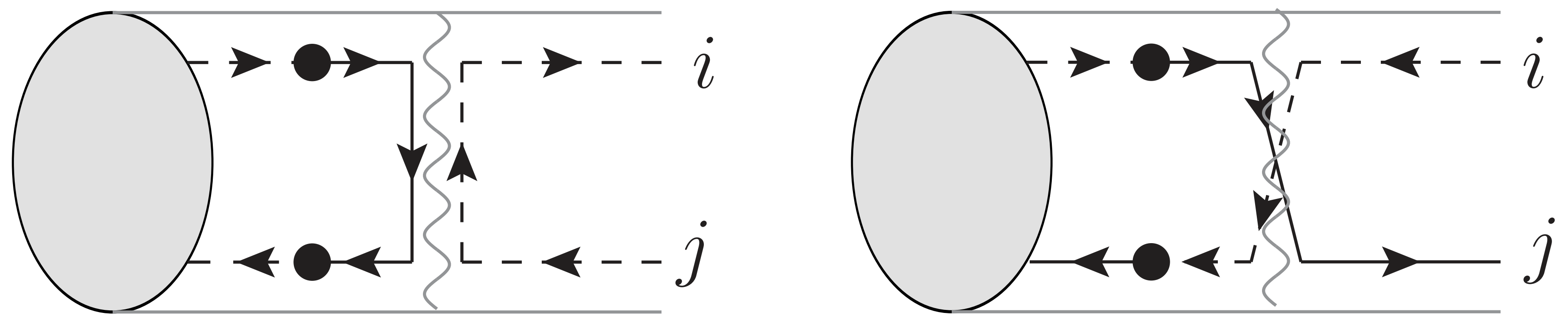
How to do this at subleading N and higher order shower evolution?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$




Construct electroweak evolution.
Measurement projection is ubiquitous.




Factorisation and kinematics.

[Plätzer, Sjödahl — '21]

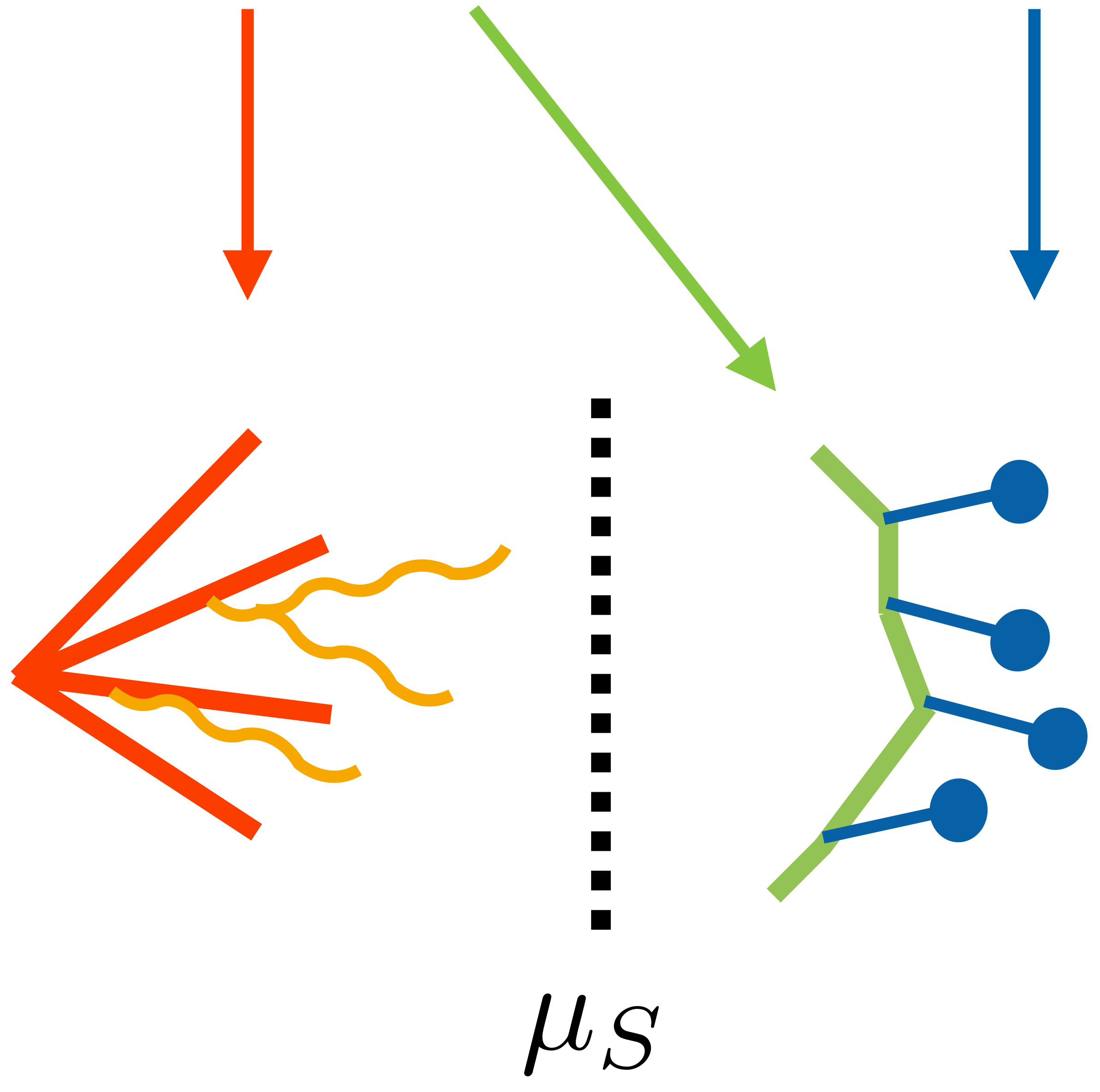
# Constructing evolution algorithms

How do we consistently hadronize in light of (improved) shower algorithms?

How to do this at subleading N and higher order shower evolution?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

$$0 = \frac{\mathrm{d}}{\mathrm{d}\mu_S}$$




Construct electroweak evolution.

Cutting indicates that subtraction terms refer to different final states — unitarity?

$$\frac{1}{k^2 - m^2 - im\Gamma\ \mathrm{sign}(T \cdot k)} =$$

$$\frac{1}{k^2 - m^2 + im\Gamma} + 2i \frac{m\Gamma}{(k^2-m^2)^2 + m^2\Gamma^2}\theta(T \cdot k)$$

Factorisation and kinematics.

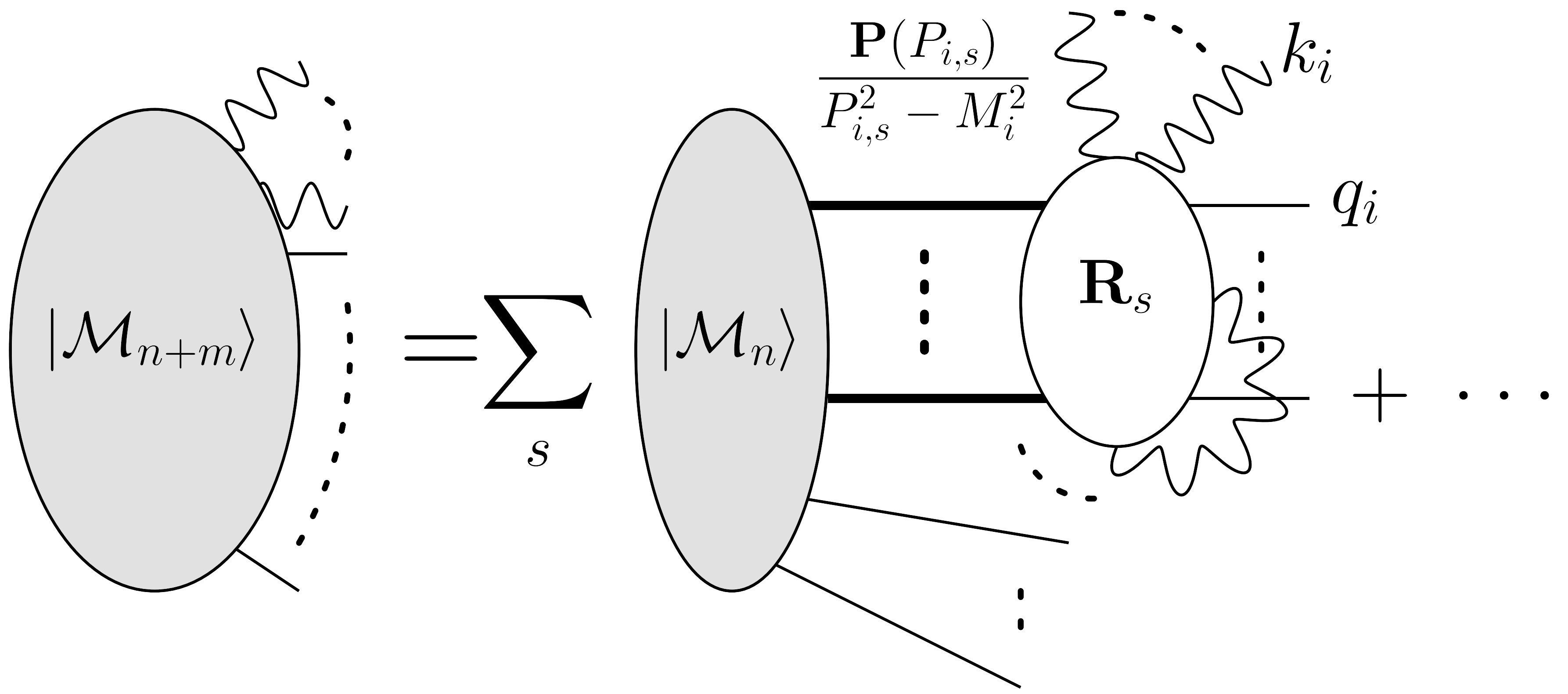
[Plätzer, Sjödahl — '21]

# Electroweak evolution

Momentum remapping closely tie in with how factorisation is performed.
Eikonal approximation needs to separate true soft degrees of freedom.

$$(q_i + K_{i,s})^2 - M_i^2 = 2p_i \cdot Q_{i,s}$$

$$p_i \cdot Q_{i,s} \ll p_i \cdot n_{i,s} \equiv S_{i,s}$$




$$K_{i,s}^\mu = \Lambda^\mu{}_\nu \left(Q_{i,s}^\nu + \delta_{i,s}\, n_{i,s}^\nu\right)$$

$$q_i^\mu = \Lambda^\mu{}_\nu \left(\alpha p_i^\nu + \frac{(1-\alpha^2)M_i^2 + p_i \cdot Q_{i,s}}{2\alpha\, n_{i,s} \cdot p_i} n_{i,s}^\nu\right) - K_{i,s}^\mu$$

$$\sum_{n=0}^{\infty} \left( \frac{\mathbf{P}(q_i + K_{i,s}, M_i)}{(q_i + K_{i,s})^2 - \tilde{M}_{R,i}^2} \mathbf{\Sigma}(q_i + K_{i,s}) \right)^n \frac{\mathbf{P}(q_i + K_{i,s}, M_i)}{(q_i + K_{i,s})^2 - \tilde{M}_{R,i}^2} = \frac{1}{2p_i \cdot Q_{i,s}} \frac{\Psi(\Lambda p_i, M_i)\bar{\Psi}(\Lambda p_i, M_i)}{1 - \Sigma'(M_i^2)} + \mathcal{O}(\lambda) \; ;$$

[Plätzer, Sjödahl — '21]